SEM Smart cities C.Ghiaus

# Final report cubic buidling

12.06.2024

**Github:** https://github.com/dm4bem/HesTou/tree/main?tab=readme-ov-file

**Binder:** https://mybinder.org/v2/gh/dm4bem/HesTou/HEAD


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# I. Acronyms

| Acronym | Meaning |
|---|---|
| **ACH** | Air Changes per Hour |
| **DAE** | Differential Algebraic Equations |
| **HVAC** | Heating, ventilation and air conditioning |
| **TMY** | Typical meteorological year |

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# 1 Introduction

The theory of heat transfer in buildings is a fundamental aspect of designing energy-efficient and comfortable indoor environments. In this study, we explore this theory by examining a cubic building model, characterized by five identical opaque walls and one transparent glass wall. This model incorporates air infiltration and the control of indoor air temperature using a Heating, Ventilation, and Air Conditioning (HVAC) system. The building's configuration is depicted in *Figure 1*.

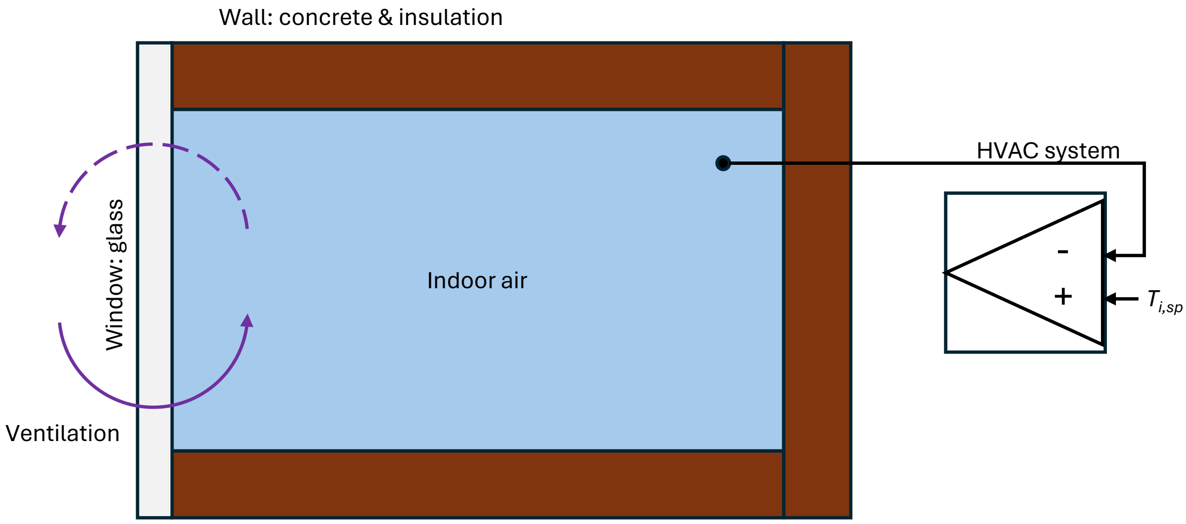


**Figure 1: Description of the toy model**[1]

To simplify the analysis, the walls are discretized in space. The primary objective of this project is to model the heat transfer within the building using a thermal circuit. From this circuit, a mathematical model in the form of a system of Differential-Algebraic Equations (DAE) is derived. This DAE system is subsequently transformed into a state-space representation.

In addition to the basic thermal modelling, the project includes an analysis of the system's step response. This involves examining how the indoor temperature responds to sudden changes in external temperature or internal heat loads. Such

[1] Own representation based on C.Ghiaus (Modeling), created on 03.05.2024

analysis is crucial for understanding the dynamic behaviour of the building's thermal system and for designing effective control strategies.

Furthermore, the model is extended to incorporate real weather data. This allows for the simulation of the building's thermal performance under varying climatic conditions, providing insights into its energy consumption and thermal comfort levels throughout the year.

# 2 Modelling

## 2.1 Thermal circuit

The thermal circuit is a representation of heat transfer, featuring branches, nodes, sources, and thermal capacities. Nodes symbolize the temperatures of various geometries, while oriented branches denote the flow rates of thermal heat between these temperature nodes.

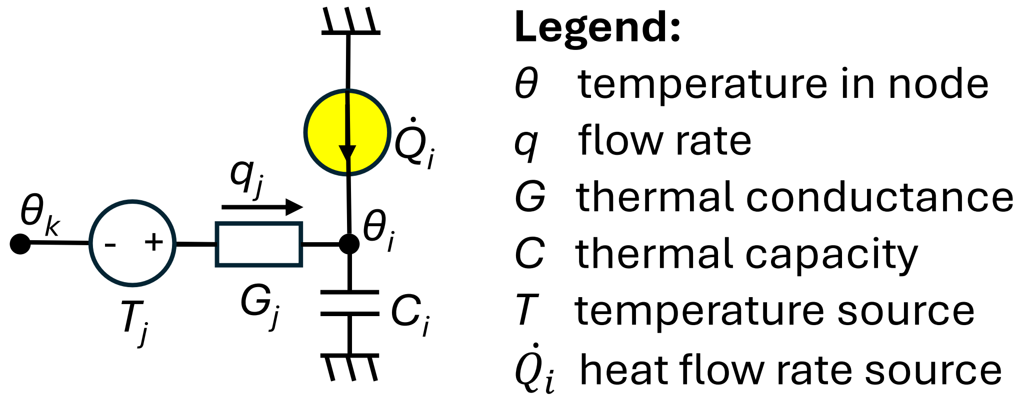


Figure 2: Basic thermal circuit

## 2.2 Heat transfer

Heat transfer involves the movement of thermal energy from one body to another due to a temperature difference. This transfer can occur through convection, heat radiation, and/or heat conduction. Following the theory of the thermal conductances [W/K].

### 2.2.1 Conduction

The thermal conductance for conduction results from:

$$G_{cd} = \frac{\lambda}{w} * S$$

With:

$\lambda$ - thermal conductivity [W/(m*K)]

$w$ – width of the material [m]

$S$ – surface of the wall [m$^2$]

### 2.2.2 Convection and advection

The conductance for convection results from:

$$G_{cv} = h * S$$

With:

*h* - convection coefficient [W/m$^2$*K]

*S* - surface of the wall [m$^2$]

The volumetric flow rate of the air results from:

$$\dot{V}_a = \frac{ACH}{3600} * V_a$$

With:

*ACH* – air infiltration rate [1/h]

3600 – number of seconds in one hour [s/h]

$V_a$ – volume of the air in the thermal zone [m$^3$]

The net flow rate by advection, which the building receives by ventilation results from:

$$q_v = \dot{m}_a * c_a(T_o - \theta_i) = \rho_a * c_a * \dot{V}_a(T_o - \theta_i)$$

With:

$\dot{m}_a$ – mass flow rate [kg/s]

$\dot{V}_a$ – volumetric flow rate [m$^3$/s]

$c_a$ – specific heat capacity of the air [J/(kg*K)]

$\rho_a$ – density of air [kg/m$^3$]

$T_o$ – outdoor air temperature [°C]

$\theta_i$ - indoor air temperature [°C]

The conductance of advection, ventilation and/or infiltration results from:

$$G_V = \rho_a * c_a * \dot{V}_a$$

### 2.2.3 Long wave Radiation

For modelling the radiative heat exchange mostly the method of using view factors between surfaces is used. The view factor is defined as the proportion of radiation that leaves the surface and is intercepted by the surface. The conductances for radiative heat exchange between to surfaces after linearization results from:

$$G_1 = 4 * \sigma * \bar{T}^3 * \frac{\varepsilon_1}{1 - \varepsilon_1} * S_1$$

$$G_{1,2} = 4 * \sigma * \bar{T}^3 * F_{1,2} * S_1 = 4 * \sigma * \bar{T}^3 * F_{2,1} * S_2$$

$$G_2 = 4 * \sigma * \bar{T}^3 * \frac{\varepsilon_2}{1 - \varepsilon_2} * S_2$$

With:

$\varepsilon_1$ and $\varepsilon_2$ – emissivities of the surfaces 1 and 2

$S_1$ and $S_2$ - areas of the surfaces 1 and 2 [$m^2$]

$F_{1,2}$ – view factor between surfaces 1 and 2

$\bar{T}$ – mean temperature

$\sigma$ - Stefan-Boltzmann constant [$W/(m^2 \cdot K^4)$]

### 2.2.4 Controller

The HVAC system controls the indoor temperature $\theta_i$. It is considered as a proportional controller which adjusts the heat flow rate $q_{HVAC}$ at its setpoint value $T_{i,sp}$. If the gain factor of the proportional controller goes to infinity the controller is perfect and the indoor temperature goes to the setpoint temperature. If the gain factor goes to 0 the controller is not acting, and the building is in free running. This means heat flow rate is 0. The by the HVAC system injected heat flow rate results from:

$$q_{HVAC} = K_p\left(T_{i,sp} - \theta_i\right)$$

With:

$K_p$ - proportional gain of the controller [W/K]

$T_{i, sp}$ – setpoint of indoor temperature [°C]

$\theta_i$ – indoor temperature [°C]

## 2.3 Toy building

Examining the toy model results in a thermal circuit. Initially, it's essential to delineate the points of heat transfer. Heat exchange occurs across the two-layered walls (comprising concrete and insulation), through the glass window, via ventilation, from indoor auxiliary sources, and from the HVAC system. There are heat capacities represented in the nodes. The capacity of glass is neglected due to its minimal impact. Each heat transfer mechanism is illustrated in the thermal circuit depicted in *Figure 3*. Heat transfer from the concrete walls in red, from the glass in green, from the ventilation in magenta, from the indoor air volume in light-blue and from the HVAC system in black.

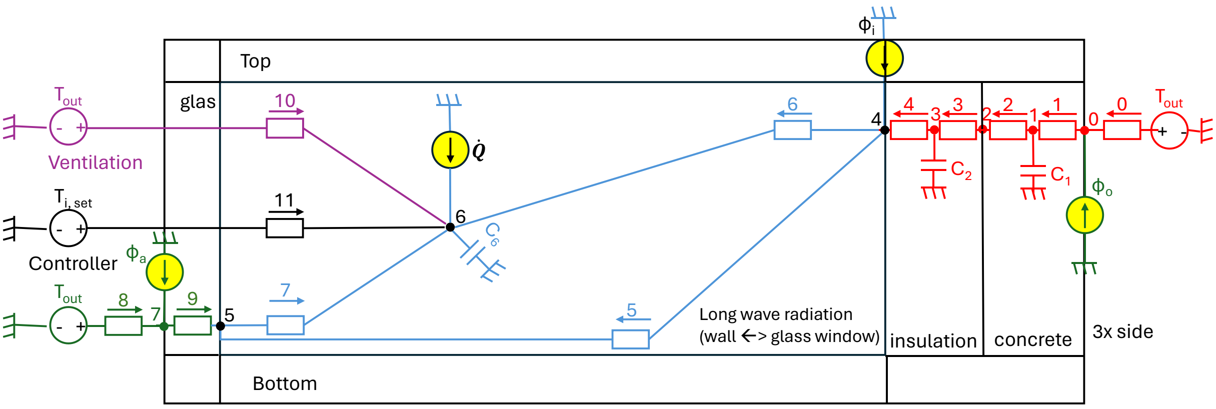


Figure 3: Toy model thermal circuit

The heat transfer occurs in form of conduction, convection and/or radiation. *Table 1* outlines the heat transfer through conductances and specifies whether it pertains to conduction, convection, or radiation.

### 2.3.1 Overview of kind of heat flow

**Table 1: Overview kind of heat flow in toy model**

| Conductance | Conduction | Convection | Radiation |
|---|---|---|---|
| 0 | | ● | |
| 1 | ● | | |
| 2 | ● | | |
| 3 | ● | | |
| 4 | ● | | |
| 5 | | | ● |
| 6 | | ● | |
| 7 | | ● | |
| 8 | | ● | |
| 9 | ● | | |
| 10 | | ● | |
| 11 | | ● | |

### 2.3.2 Overview of capacities

- $C_1$: Capacity of concrete, capability to store heat in outer layer of the wall.
- $C_2$: Capacity of insulation, capability to store heat in inner layer of the wall.
- $C_6$: Capacity of the room, capability to store heat in air volume inside.

### 2.3.3 Overview of temperature sources

- $T_{out}$: Outside temperature (can vary in course of the year).
- $T_{i,set}$: Aimed room temperature for the regulator.

### 2.3.4 Overview of heat sources

- $\dot{Q}$: Heat that is emitted by devices and life inside the room.
- $\Phi_a$: Solar radiation that is absorbed by the outer surface of the walls.
- $\Phi_i$: Radiation that is absorbed by the inner surface of the walls.
- $\Phi_g$: Solar radiation that is absorbed by the glass (window).

# 3 System of algebraic-differential equations

To model a building its necessary to obtain the mathematical model as a system of DAE from the thermal circuit. The analysis of a thermal circuit means to find temperatures in the nodes *θ* and the heat flows on the branches *q*. To solve this problem the following formula is used:

$$C\dot{\theta} = -(A^T G A)\theta + A^T G b + f$$

$$q = G(-A\theta + b)$$

With:

$\theta$ – temperature vector

$q$ – heat flow vector

$A[n_q, n_\theta]$ – incidence matrix, $n_q$ = number of branches, $n_\theta$ = number of nodes

$G$ – conductance diagonal matrix, size $n_q$ x $n_q$

$C$ – capacity diagonal matrix, size $n_\theta$ x $n_\theta$

$b$ – temperature source vector, size $n_q$ (no temperature source on m then $q_m = 0$)

$f$ – heat flow source vector, size $n_\theta$ (no heat flow source in then $f_n = 0$)

Following a description how to obtain the mathematical model:

## *3.1* Temperature vector *θ*

The number of nodes determine the size of the temperature vector. Therefore, the size is eight. The vector *θ* is illustrated in *Figure 4.*

## *3.2* Heat flow vector *q*

The number of branches in the thermal circuit determines the size of the heat flow vector *q.* The vector *q* for the toy building is illustrated in *Figure 4.*

## *3.3* Incidence matrix $A[n_q, n_\theta]$

The A matrix is 2-dimensional and the size is determined from the number of branches $n_q$ and the number of nodes $n_\theta$. It shows how the temperature nodes are connected under considering the orientation of the branches. $A_{m,n}$ is 1 if flow m enters into node n. $A_{m,n}$ is -1 if flow m exits from node n. A is zero if flow m is not connected to branch n. The matrix *A* for the toy building is illustrated in *Figure 4.*

## *3.4* Conductance diagonal matrix *G*

The size of the conductance diagonal matrix is determined of $n_q$ x $n_q$, where $n_q$ is the number of flow branches. The matrix *G* for the toy building is illustrated in *Figure 4.*

## *3.5* Capacity diagonal capacity *C*

The size of the capacity diagonal matrix is determined of $n_\theta$ x $n_\theta$, where $n_\theta$ is the number of temperature nodes. The matrix *C* for the toy building is illustrated in *Figure 4.*

## *3.6* Temperature source vector *b*

The size of the temperature source vector is determined of the number of flow branches. If there is no temperature source, then $b_q = 0$. The vector *b* for the toy building is illustrated in *Figure 4.*

## *3.7* Heat flow source vector *f*

The size of the heat flow source vector is determined of the number of temperature nodes. If there is no heat flow source in the node then $f_\theta = 0$. The vector *f* for the toy building is illustrated in *Figure 4.*

$$
\begin{array}{cc}
 & \begin{matrix} \theta_0 & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 & \theta_7 \end{matrix} \\
\begin{matrix} q_0 \\ q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ q_9 \\ q_{10} \\ q_{11} \end{matrix} &
A = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
\qquad
G = diag\begin{bmatrix} G_0 \\ G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \\ G_6 \\ G_7 \\ G_8 \\ G_9 \\ G_{10} \\ G_{11} \end{bmatrix}
\qquad
\mathrm{b}= \begin{bmatrix} T_{out} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ T_{out} \\ 0 \\ T_{out} \\ T_{i,set} \end{bmatrix}
$$

$$C = diag[0 \quad C_1 \quad 0 \quad C_3 \quad 0 \quad 0 \quad C_6 \quad 0]$$

$$f = [\Phi_o \quad 0 \quad 0 \quad 0 \quad \Phi_i \quad 0 \quad \dot{Q} \quad \Phi_g]^T$$

**Figure 4: Matrices and vectors of the system of DAE**

# 4 State space representation

The state-space model is a mathematical representation of a physical system in terms of its inputs, states, and outputs. The DAE is transformed into state-space representation and the following equations result:

$$\dot{\theta} = A_s * \theta_s + B_s * u$$

$$y_{ss} = C_s * \theta_s + D_s * u$$

With:

$\theta_s$ – vector of state variables (temperature nodes containing capacities)

$u$ – vector of inputs

$y_{ss}$ – output in steady-state

The matrices in state space ($A_s$, $B_s$, $C_s$, $D_s$) representation are obtained from the system of DAE with the matrices and vectors of the thermal circuit ($A$, $G$, $b$, $C$, $f$, $y$)

# 5 Interim conclusion I

The Assignment 1 showed how to model the heat transfer in a cubic building. From analysing the description of the building, the thermal circuit results. The mathematical model from the thermal circuit as a system of DAE is obtained. To solve the heat transfer model and find the temperatures of interest, the state space representation is used. The state variables derive from the DAE.

For the input data we defined the vector *b* and *f*.

$$b = [T_o \quad T_o \quad T_o \quad T_{i,set}]^T$$

$$f = [\Phi_o \quad \Phi_i \quad \dot{Q}_a \quad \Phi_a]$$

We defined the output vector *y* to get the temperature for nodes we are interested in. In case we wanted to know the indoor temperature of the cubic building, we would obtain the following vector *y*:

$$y = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0]^T$$

which would lead to the output of the state-space system:

$$y_{ss} = \theta_6$$

By defining $y = np.ones(8)$ we obtain

$$y = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]^T$$

and therefore, we take into account every node of the system:

$$y_{ss} = \theta$$

In the assignment 2 we will work out the steady state.

# 6 Steady-state and step response

In system analysis we differ between steady-state and step response. The steady state is the behaviour after the system is not changing in time. The step response is the dynamic behaviour of a system after a change of an input from zero to a reference in short time.

## 6.1 Steady state

The steady state is used to test the model. It is important to mention that it is not a verification or validation. In order to test the model, the following assumptions are applied:

- Controller is not active $K_p \rightarrow 0$
- Outdoor temperature set to $T_o = 10°C$
- Indoor temperature setpoint is $T_{i,set} = 20°C$
- All flow rate sources are zero

### 6.1.1 From the DAE

The values for temperature in steady-state are obtained from the system of DAE. From the following equation results with $C\dot{\theta} = 0$ for steady-state:

$$C\dot{\theta} = -(A^T GA)\theta + A^T Gb + f = 0$$

$$\theta_{ss} = (A^T GA)^{-1} * (A^T G_b + f)$$

Considering the indoor temperature it results 10°C on every single node:

$$\theta_{ss} = [10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10 \quad 10]$$

If the sum of auxiliary heat sources in the room is not zero and the outdoor temperature is set to 0, there should be a heat flow in the building. The temperature at $\theta_6$ is the highest, while the surface of the wall at $\theta_0$ is nearly equal to the outdoor temperature, as expected:

$$\theta_{ss} = [0.14 \quad 0.39 \quad 0.65 \quad 5.88 \quad 11.12 \quad 5.57 \quad 12.23 \quad 4.41]$$

A further test of the DAE representation is putting the value of the heat source $\dot{Q}$ inside the room to 1000. The expected steady-state solution would be heat flows in the q vector that distribute the heat provided by $\dot{Q}$ within the building. After setting

$$f = np.zeros(8)$$

$$f[6] = 1000$$

the resulting vector

$$q = [-159.115 \;\; -159.115 \;\; -159.115 \;\; -159.115 - 159.115 \;\;\; 248.878 \;\; -407.992 \\ -481.699 \;\; -730.576 \;\; -730.576 \;\; -110.309 \;\;\; 0]^T$$

shows that the branches 6, 7, 10, 11 sum up to -1000; hence all the heat is distributed. The heat flow in branch 11 is zero, this is logic since the control factor $k_p = 0$. This suggests no incorrectness of the model.

### 6.1.2 From the state space representation

The input vectors *u* in state-space representation results from the temperature sources $b_T$ and flow-rate sources $f_Q$.

$$u = \begin{bmatrix} b_T \\ f_Q \end{bmatrix}$$

With:

$b_T$ – nonzero vector with elements from vector *b*

$f_Q$ – nonzero vector with elements from vector *f*

In steady state the equation changes because of the definition of $C\dot{\theta} = 0$

$$\dot{\theta} = A_s * \theta_s + B_s * u = 0$$

$\Theta_S$ results in:

$$\theta_s = -A_s^{-1} B_s * u$$

With:

$$y_{ss} = C_s * \theta_s + D_s * u$$

The equation for the steady-state output results:

$$y_{ss} = (-C_s * A_s^{-1} B_s + D_s) * u$$

Equivalent to the DAE method, the output is the vector of temperature nodes. Both methods, DAE and state space representation, produce almost identical results.

## 6.2 Step response

Within the step response, one major difference appears compared to the steady state. The temperature and the general state of the model changes. Considering the respective equations (see 6.1.1), $C\dot{\theta}$ is not anymore equal to zero.

$$C\dot{\theta} = -(A^T G A)\theta + A^T G b + f \neq 0$$

Hence, there are different ways of considering the capacities of the room. Due to the comparably small capacity of the glass, it can be neglected. For simulations focusing the wall system, the capacity of the inside air can be neglected as well.
The setting can be adjusted by the Boolean variables $neglect\_glass$ and $neglect\_air$ in the code.

### 6.2.1 Eigenvalues and timesteps

The eigenvalues of the A matrix are used to select a suitable and stable time step for the numerical simulation of the dynamics of the thermal system. This ensures that the simulation is both stable and as efficient as possible. Related to the thermal network the Euler explicit stability criterion is defined as:

$$\Delta t < 2 \min\left(-\frac{1}{\lambda_i}\right) = 2minT_i$$

Settling time is the time required for the system’s response to reach and remain within a certain value of its final value after being subjected to a step input. It obtains from the largest time constant from the eigenvalues.

### 6.2.2 Step response to outdoor temperature

The number of time steps for the step response is calculated from the eigenvalues. Vector $u$ gives the same input data which are used for the steady-state model.

$$u = \begin{matrix} b_T \\ f_Q \end{matrix}$$

With:

$B_T$ - nonzero elements of vector b

$f_Q$ – nonzero elements of vector f

$$b_T = T_0 \quad T_0 \quad T_0 \quad T_{set} = [10 \quad 10 \quad 10 \quad 20]$$

$$f_Q = [\phi_o \quad \phi_i \quad \dot{Q}_a \quad \phi_a] = [0 \quad 0 \quad 0 \quad 0]$$

The step response should eventually approach the steady state values for the same inputs. The temperature curve in function of the time for $\theta_6$, the indoor temperature, is shown in *Figure 5*.

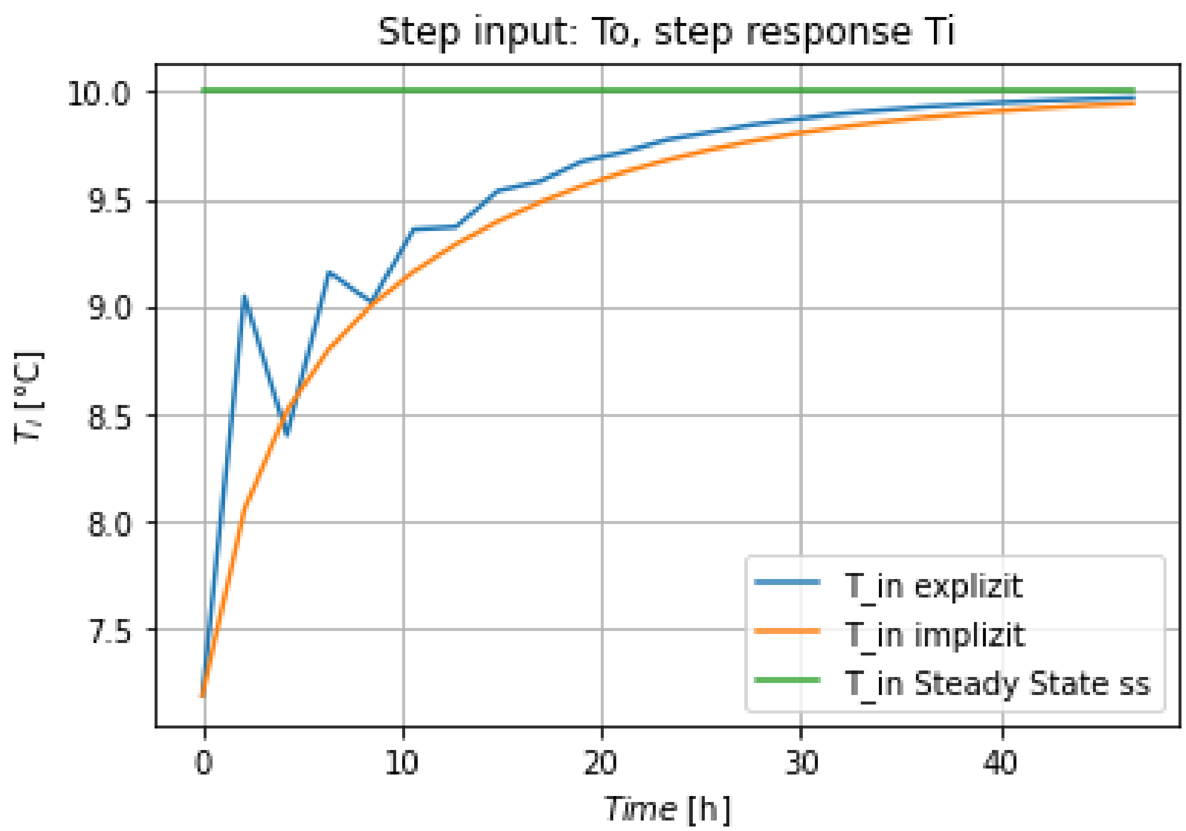


**Figure 5: Step response to indoor temperature (volume air inside neglected)**

*Figure 6* illustrates the step response without neglecting the volume of the indoor air. The settling time is nearly the same as for the neglection of the air volume inside. The time step decreases from 159.8min to 11.3. The steady state temperature is the same, like expected.

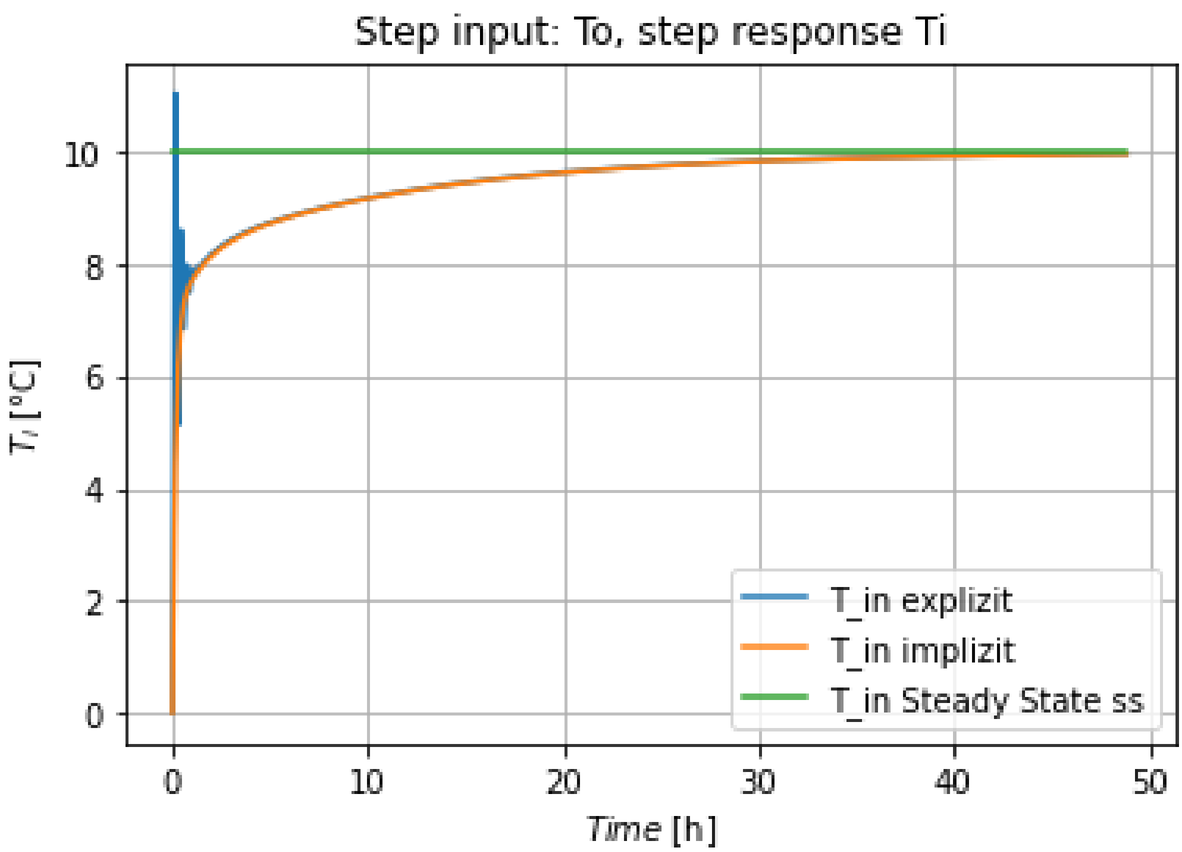


**Figure 6: Step response to indoor temperature (volume air inside NOT neglected)**

### 6.2.3 Step response to internal load

For the step response to internal load, the following conditions are determined:

- Temperature sources = 0
- Flow-rate sources = 0
- Controller is not active $K_p \to 0$
- Auxiliary heat gains $Q_a = \dot{}1000$

The step response to internal load without temperature sources and no flow rates excepted the auxiliary heat gain, follows the steady-state. It obtains a temperature of 12.26°C in the building ($\theta_6$).

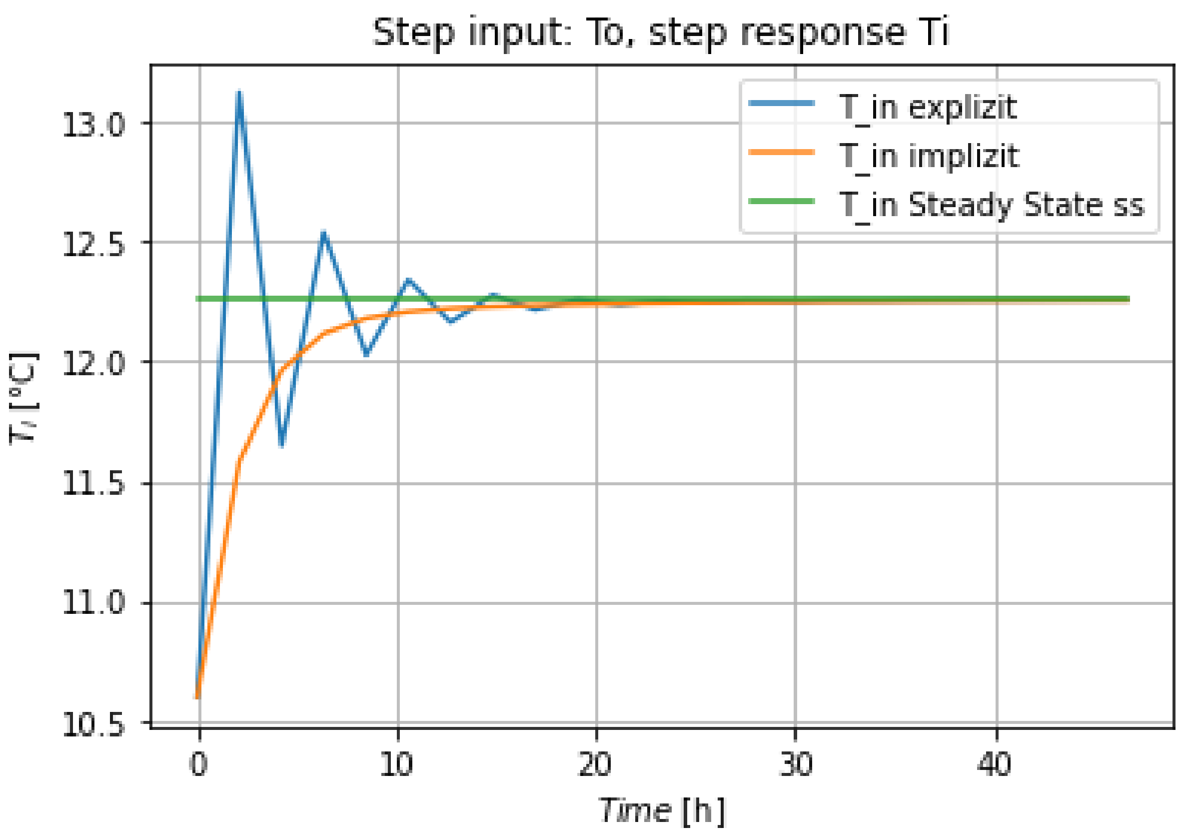


Figure 7: Step response to internal load (volume air inside neglected)

*Figure 8* illustrates the step response without neglecting the volume of the inside air. The settling time is nearly the same as for the neglection of the air volume inside. The time step decreases from 159 min to 11 min. The steady state temperature is the same, like expected.

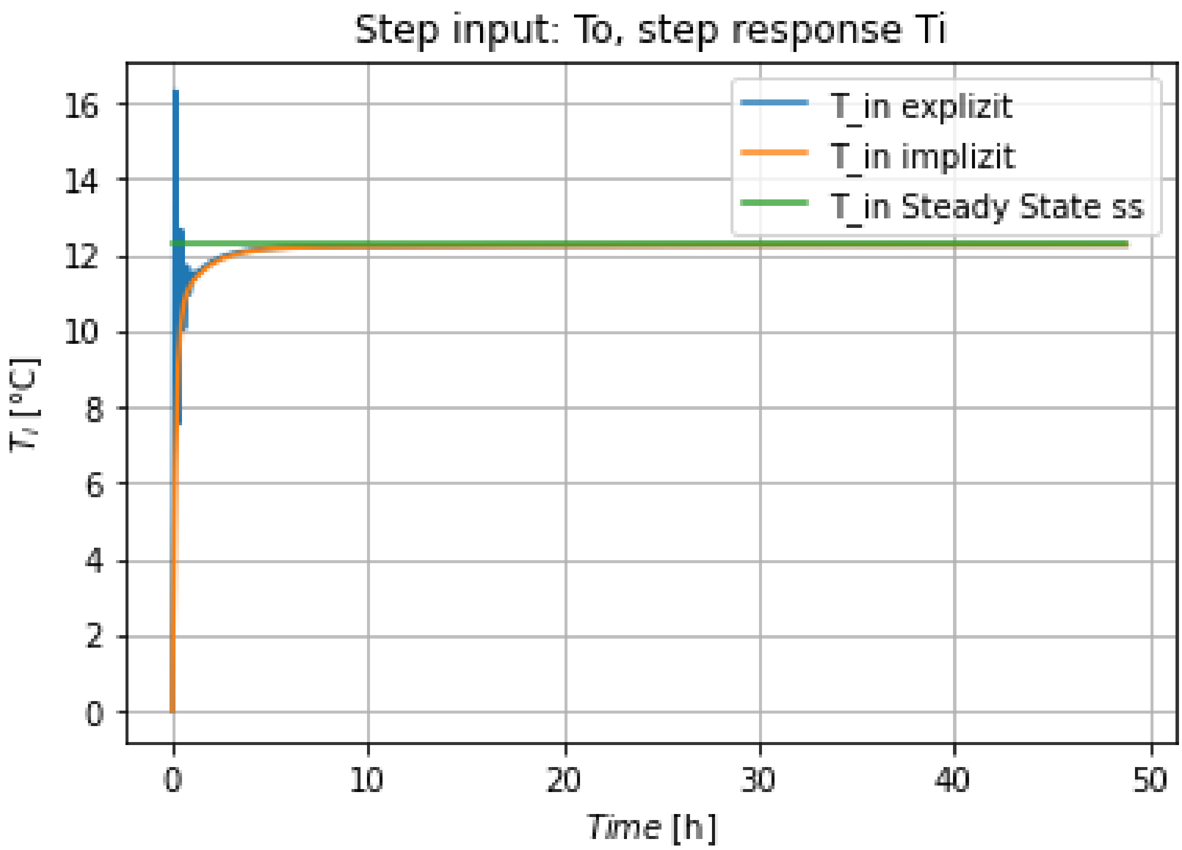


**Figure 8: Step response to internal load (volume air inside NOT neglected)**

# 7 Modelling with weather data

For the steady state and step response modelling the temperature was defined constant. To test the model on real conditions the solar radiation and outdoor temperature is considered in the following. The weather data comes from a Typical Metrological Year file. TMY datasets are created by selecting the most representative weather data for each month from a span of several years. It helps to reduce the impact of cold winters or hot summers. The weather data which is used for the following simulation uses weather data for Grenoble in the span of 2004 to 2018[2]. The considered period is from 03. January 10:00 until the 09. February 18:00

## 7.1 Estimation of degree hours

As a first approximation of the energy used to heat the building, we can model an enveloped volume (by the surface $S\ [m^2]$ ) inside the building that only needs to be heated to a referred temperature $T_{ref}$ when the outside temperature is lower than $T_{ref}$. The needed heat is proportional to degree-hours by the overall heat loss coefficient of the building $U_{bldg}\ [\frac{W}{m^2K}]$ and the surface $S$ :

$$Q_h \sim DHH = \sum_{k=0}^{n} (\theta_{in,k} - [\theta_{out,k}] * \Delta t)$$

Where:

- $Q_h$- energy needed for heating, J;
- $DHH$ – the Degree Hours for Heating
- $\sum_{k=0}^{n}(\theta_{in,k} - [\theta_{out,k}] * \Delta t)$- degree-hours for heating;
- $\theta_{in,k}$- indoor temperature over the time $k$, °C;
- $[\theta_{out,k}]$- outdoor temperature upper bounded by the indoor temperature (i.e., not larger than the indoor temperature), °C,

This implies that the heat balance in steady state can be imaged by:

$$\dot{Q_{HVAC}} = U_{bldg} * S * (\theta_{in} - \theta_{out})$$

Where:

[2] https://climate.onebuilding.org/

- $\dot{Q}_{HVAC}$- power delivered by the Heating, Ventilation and Air Conditioning (HVAC) system to the building to compensate the heat losses in order to maintain the indoor temperature at value $\theta in$, W;
- $U_{bldg} * S * (\theta_{in} - \theta_{out})$- heat losses of the building, W,

Where:

  - $\theta_{out}$- outdoor temperature, °C;
  - $\theta_{in}$ - base (indoor) temperature, °C.

The integral of the time leads to the energy consumed to heat the volume to the referred temperature.

Certainly, the amount of energy resulting from the heat hours needs to be corrected since the inertia of the building is neglected and radiation/ventilation is not taken into account. However, it is a good first indication. In addition, it is possible to create hypothesises of the saving of energy by using varying indoor temperatures.
For example, we can consider different indoor temperatures during day and night.
Then, the degree hours for heating DHH are the sum of the partial DHH during the day

$$DHH_{day} = \sum_k \Delta\theta_{day,k} = \sum_k (\theta_{in,day} - [\theta_{out,k}]_{day})$$

and the night

$$DHH_{night} = \sum_k \Delta\theta_{night,k} = \sum_k (\theta_{in,night} - [\theta_{out,k}]_{night})$$

Hence, we obtain an intermittence value by summing up these two DHH. We can calculate a percentage of saving thanks to the intermittent concept compared to the constant indoor temperature with

$$s = \frac{DHH_{fix} - DHH_{interm}}{DHH\_fix} * 100\%$$

This value represents an upper bound, since the dynamics of the building are neglected, practically they will damp the change of temperature and therefore lower the amount of heat that must be put into the volume to maintain a referred temperature.

As an example for the estimation of heat degree hours and the possible savings with intermittent heating, we consider one average year from January to December:

- Constant set temperature = 20°C
- Intermittent set temperature night = 16°C, from 22h to 06h
- Intermittent set temperature day = 22°C, from 06h to 22h

As a result, we obtain 98415.6 hK for a fixed set-point, and 83716.9 hK for a variable set-point. Regarding the proportionality of degree-hours and heating, that leads to estimated savings of 15 %.

## 7.2 Outdoor temperature and solar radiation

The behaviour of the cubic building, including inside temperature, heat flows and power of the heating source can be modelled based on the weather conditions. It is suggested that after the settling time (see step response), the trace of the model is reliable in the sense that the initial conditions are not important anymore. Thus, the inside temperature and the temperature control is only depending on the given data.

For the first modelling depending on outdoor temperature and radiation data, the following conditions are determined:

- Time period: 03 January, 10h till 09. February, 18h
- Indoor temperature setpoint $T_{i,set} = 20°C$
- Controller is not active: $K_p \to 0$
- Auxiliary heat gains $\dot{Q}_a = 1000$
- Only neglection of glass capacity

After the calculated settling time of 2.04 days, one perceives a seemingly reliable relation between the radiation $\Phi_{total}$ in combination with the outside temperature $T_{outdoor}$ and the inside temperature $T_{inside}$ (*Figure 9*):
$T_{inside}$ mostly follows $T_{outdoor}$, with some exceptions, i.e. days of high radiation, like the days 11 and 15. During these days, the walls and the room additionally heat up

by the power of the sun. The heating source is reasonably zero, since Kp = 0. $T_{inside}$ is always a little higher than $T_{outdoor}$ because of the auxiliary heat gain inside.

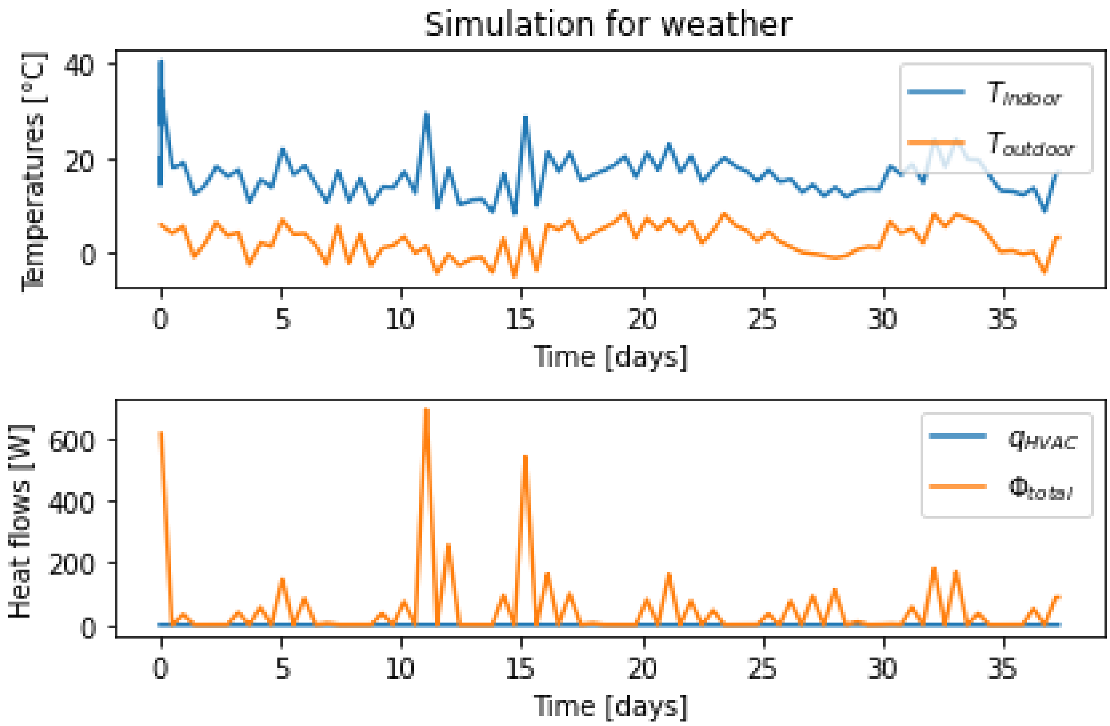


**Figure 9: Simulation of temperature and heating source flow based on weather data, Kp = 0**

As a comparison, we can take the same conditions, and change the controller gain Kp to 500 (*Figure 10*):

The temperature is more constant around the value of $T_{i,set} = 20°C$, and in general less perturbated by outside temperature variations. In addition, the heating source $q_{HVAC}$ is working depending on the gap between outside conditions and inside temperature.

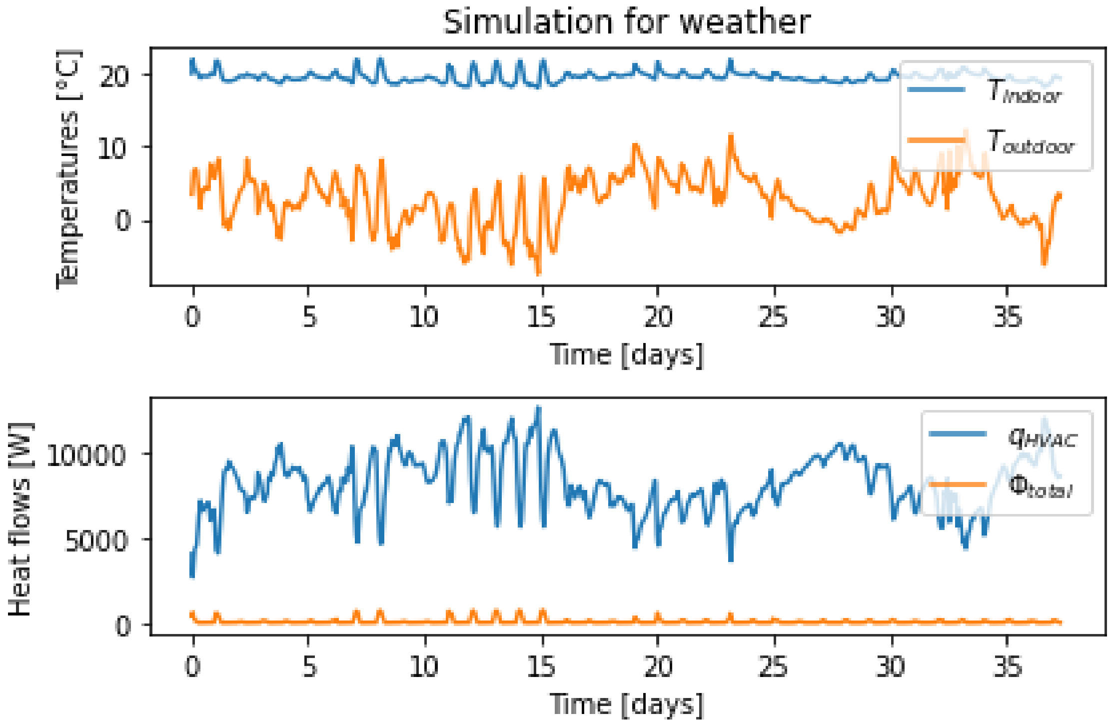


**Figure 10: Simulation of temperature and heating source flow based on weather data, Kp = 500**

# 8 Discussion

Objective of this report was modelling the complex thermal dynamics in a cubic building, emphasizing the effects of external weather conditions on indoor temperatures. The simulations showed that changes in outdoor temperature and solar radiation significantly impact indoor climate, with and without additional heat sources inside.

Using the state-space representation of the building's thermal system allowed us to predict how the building responds to various thermal inputs over time. This method helped us understand the transient behaviour of indoor temperatures and heat flows.

Key results include the performance of the building under different weather conditions and the effectiveness of the HVAC system. The simulations demonstrated that without HVAC control, indoor temperatures could fluctuate widely with external temperature changes, respectively result in steady-state particularly different to comfortable inside temperatures. However, with a proportional controller, indoor temperatures remained significantly more stable. Higher controller gains (Kp) led to better temperature regulation, indicating that fine-tuning the controller and changing the behaviour of heating by intermittent strategy can optimize energy use and maintain comfort.

The study also provided insight into the step responses of the building to sudden changes in outdoor temperature and internal heat loads. These responses showed how fast the indoor environment could stabilize after such changes, which is crucial for designing responsive and efficient HVAC systems.

Despite the useful insights, the study has limitations, such as the assumption of perfect conditions that may not reflect real-world scenarios. Future work should include more detailed modelling to address these limitations and improve accuracy.

# 9 Conclusion

This report comprehensively explores the thermal dynamics of a cubic building model, illustrating the theoretical and practical applications of heat transfer mechanisms. The investigation began with the establishment of a thermal circuit model to represent the building, leading to the formulation of a system of differential-algebraic equations (DAE). The transformation of this system into a state-space representation allowed for the detailed analysis of temperature variations and heat flow within the building.

Through the state-space model, we examined both steady-state and dynamic step responses to various internal and external thermal inputs. Simulations incorporating weather data further highlighted the impact of external conditions on the indoor climate, considering of auxiliary heat gains inside buildings and the role of a proportional controller for maintaining desired indoor temperatures.

The findings underscore the complexity of thermal behaviour in buildings and the necessity of precise modelling for effective climate control.

# V. References


**Literature:**

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