SEM Smart cities C.Ghiaus

Assignment 1: Modeling

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Githubclassroom:

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I. Acronyms

Acronym	Meaning
HVAC	Heating, ventilation and air conditioning
DAE	Differential Algebraic Equations
ACH	Air Changes per Hour

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1 Introduction

To study the theory of heat transfer in buildings following a cubic building with 5 identical walls and a transparent wall of glass is considered. Air infiltration and controlling the indoor air temperature with a HVAC system are also taken into account. The description of the building is illustrated in *Figure 1.* To simplify the figure only one of the five concrete walls is illustrated. Objective for the first assignment is to model the heat transfer in the building by a thermal circuit. From this circuit obtains the mathematical model as a system of DAE. Finally the system of DAE is transformed into state-space representation. In the following are described the steps from the toy model to a system of DAE.

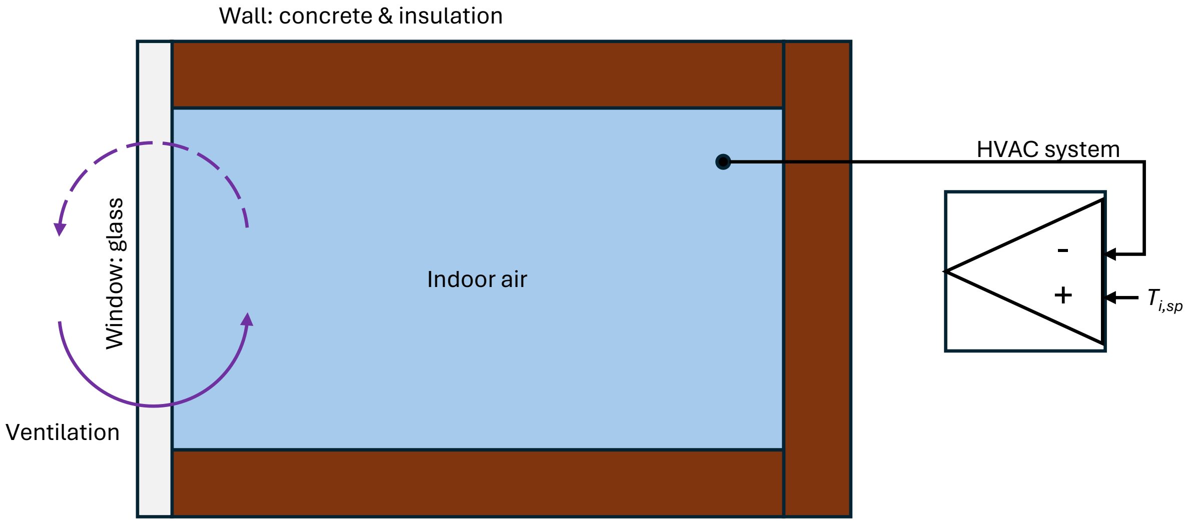


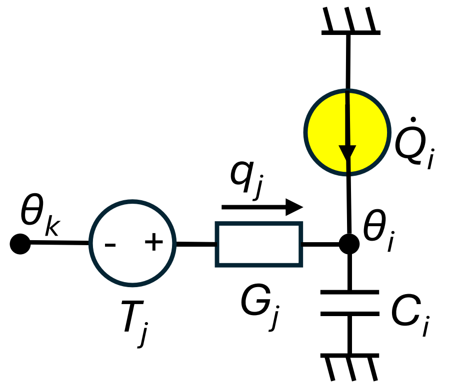
Figure 1: Description of the toy model[1]

[1] Own representation based on C.Ghiaus (Modeling), created on 03.05.2024

2 Modeling

2.1 Thermal circuit

The thermal circuit is a representation of heat transfer, featuring branches, nodes, sources, and thermal capacities. Nodes symbolize the temperatures of various geometries, while oriented branches denote the flow rates of thermal heat between these temperature nodes.



Legend:
θ temperature in node
q flow rate
G thermal conductance
C thermal capacity
T temperature source
$\dot{Q}_i$ heat flow rate source

Figure 2: Basic thermal circuit

2.2 Heat transfer

Heat transfer involves the movement of thermal energy from one body to another due to a temperature difference. This transfer can occur through convection, heat radiation, and/or heat conduction. Following the theory of the thermal conductances [W/K].

2.2.1 Conduction

The thermal conductance for conduction results from:

$$G_{cd} = \frac{\lambda}{w} * S$$

With:

λ - thermal conductivity [W/(m*K)]

w – width of the material [m]

S – surface are of the wall [m^2]

2.2.2 Convection and advection

The conductance for convection results from:

$$G_{cv} = h * S$$

With:

h - convection coefficient [W/m^2*K]

S - surface of the wall [m^2]

The volumetric flow rate of the air results from:

$$\dot{V}_a = \frac{ACH}{3600} * V_a$$

With:

ACH – air infiltration rate [1/h]

3600 – number of seconds in one hour [s/h]

V_a – volume of the air in the thermal zone [m^3]

The net flow rate by advection, which the building receives by ventilation results from:

$$q_v = \dot{m}_a * c_a(T_o - \theta_i) = \rho_a * c_a * \dot{V}_a(T_o - \theta_i)$$

With:

$\dot{m}_a$ – mass flow rate [kg/s]

$\dot{V}_a$ – volumetric flow rate [m^3/s]

c_a – specific heat capacity of the air [J/(kg*K)]

ρ_a – density of air [kg/m^3]

T_o – outdoor air temperature [°C]

θ_i - indoor air temperature [°C]

The conductance of advection, ventilation and/or infiltration results from:

$$G_V = \rho_a * c_a * \dot{V}_a$$

2.2.3 Long wave Radiation

For modeling the radiative heat exchange mostly the method of using view factors between surfaces is used. The view factor is defined as the proportion of radiation that leaves the surface and is intercepted by the surface. The conductances for radiative heat exchange between to surfaces after linearization results from:

$$G_1 = 4 * \sigma * \bar{T}^3 * \frac{\varepsilon_1}{1-\varepsilon_1} * S_1$$

$$G_{1,2} = 4 * \sigma * \bar{T}^3 * F_{1,2} * S_1 = 4 * \sigma * \bar{T}^3 * F_{2,1} * S_2$$

$$G_2 = 4 * \sigma * \bar{T}^3 * \frac{\varepsilon_2}{1-\varepsilon_2} * S_2$$

With:

ε_1 and ε_2 – emmisivities of the surfaces 1 and 2

S_1 and S_2 - areas of the surfaces 1 and 2 [m^2]

$F_{1,2}$ – view factor between surfaces 1 and 2

$\bar{T}$ – mean temperature

σ - Stefan-Bolzmann constant [$W/(m^2 \cdot K^4)$]

2.2.4 Controller

The HVAC system controls the indoor temperature θ_i. It is considered as a proportional controller which adjusts the heat flow rate q_{HVAC} at its setpoint value $T_{i, sp}$. If the gain factor of the proportional controller goes to infinity the controller is perfect and the indoor temperature goes to the setpoint temperature. If the gain factor goes to 0 the controller is not acting, and the building is in free running. This means heat flow rate is 0. The by the HVAC system injected heat flow rate results from:

$$q_{HVAC} = K_p(T_{i,sp} - \theta_i)$$

With:

K_p - proportional gain of the controller [W/K]

$T_{i, sp}$ – setpoint of indoor temperature [°C]

θ_i – indoor temperature [°C]

2.3 Toy building

Examining the toy model results in a thermal circuit. Initially, it's essential to delineate the points of heat transfer. Heat exchange occurs across the two-layered walls (comprising concrete and insulation), through the glass window, via ventilation, from indoor auxiliary sources, and from the HVAC system. There are heat capacities represented in the nodes. The capacity of glass is neglected due to its minimal impact. Each heat transfer mechanism is illustrated in the thermal circuit depicted in Figure 3. Heat transfer from the concrete walls inr red, from the glass in green, from the ventilation in magenta, from the indoor air volume in lightblue and from the HVAC system in black.

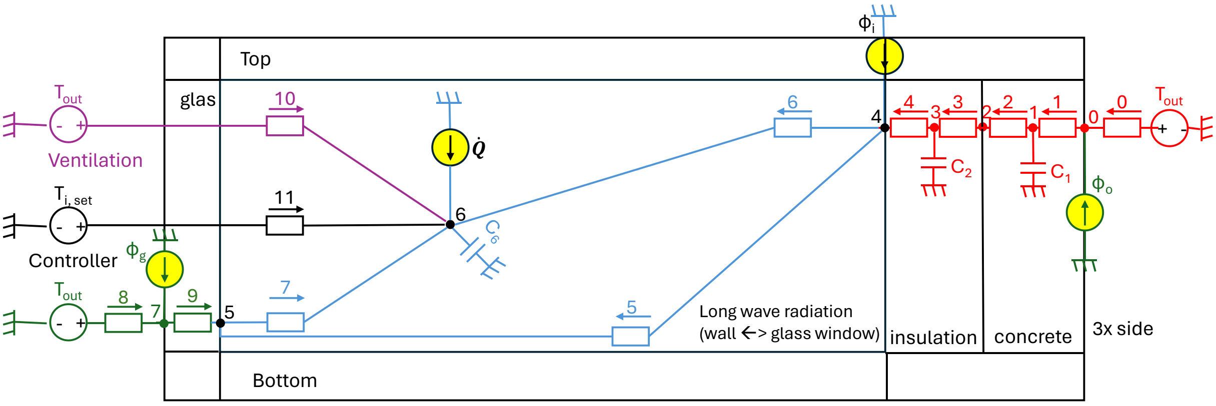


Figure 3: Toy model thermal circuit

The heat transfer occurs in form of conduction, convection and/or radiation. *Table 1* outlines the heat transfer through conductances and specifies whether it pertains to conduction, convection, or radiation.

2.3.1 Overview of kind of heat flow

Table 1: Overview kind of heat flow in toy model

Conductance	Conduction	Convection	Radiation
0		●	
1	●		
2	●		
3	●		
4	●		
5			●
6		●	
7		●	
8		●	
9	●		

10		●	
11		●	

2.3.2 Overview of capacities

- C_1: Capacity of concrete, capability to store heat in outer layer of the wall.
- C_2: Capacity of insulation, capability to store heat in inner layer of the wall.
- C_6: Capacity of the room, capability to store heat in air volume inside.

2.3.3 Overview of temperature sources

- T_{out}: Outside temperature (can vary in course of the year).
- $T_{i,set}$: Aimed room temperature for the regulator.

2.3.4 Overview of heat sources

- $\dot{Q}$: Heat that is emitted by devices and life inside the room.
- Φ_a: Solar radiation that is absorbed by the outer surface of the walls.
- Φ_i: Radiation that is absorbed by the inner surface of the walls.
- Φ_g: Solar radiation that is absorbed by the glass (window).

3 System of algebraic-differential equations

To model a building its necessary to obtain the mathematical model as a system of DAE from the thermal circuit. The analysis of a thermal circuit means to find temperatures in the nodes θ and the heat flows on the branches q. To solve this problem the following formula is used:

$$C\dot{\theta} = -(A^T G A)\theta + A^T G b + f$$

$$q = G(-A\theta + b)$$

With:

θ – temperature vector

q – heat flow vector

$A[n_q, n_\theta]$ – incidence matrix, n_q = number of branches, n_θ = number of nodes

G – conductance diagonal matrix, size n_q x n_q

C – capacity diagonal matrix, size n_θ x n_θ

b – temperature source vector, size n_q (no temperature source on m then $q_m = 0$)

f – heat flow source vector, size n_θ (no heat flow source in then $f_n = 0$)

Following a description how to obtain the mathematical model:

3.1 Temperature vector *θ*

The number of nodes determine the size of the temperature vector. Therefore the size is eight. The vector *θ* is illustrated in *Figure 4.*

3.2 Heat flow vector *q*

The number of branches in the thermal circuit determines the size of the heat flow vector *q.* The vector *q* for the toy building is illustrated in *Figure 4.*

3.3 Incidence matrix *A*[n_q, n_θ]

The A matrix is 2-dimensional and the size is determined from the number of branches n_q and the number of nodes n_θ. It shows how the temperature nodes are connected under considering the orientation of the branches. $A_{m,n}$ is 1 if flow m enters into node n. $A_{m,n}$ is -1 if flow m exits from node n. A is zero if flow m is not connected to branch n. The matrix *A* for the toy building is illustrated in *Figure 4.*

3.4 Conductance diagonal matrix *G*

The size of the conductance diagonal matrix is determined of n_q x n_q, where n_q is the number of flow branches. The matrix *G* for the toy building is illustrated in *Figure 4.*

3.5 Capacity diagonal capacity *C*

The size of the capacity diagonal matrix is determined of n_θ x n_θ, where n_θ is the number of temperature nodes. The matrix *C* for the toy building is illustrated in *Figure 4.*

3.6 Temperature source vector *b*

The size of the temperature source vector is determined of the number of flow branches. If there is no temperature source, then $b_q = 0$. The vector *b* for the toy building is illustrated in *Figure 4.*

3.7 Heat flow source vector *f*

The size of the heat flow source vector is determined of the number of temperature nodes. If there is no heat flow source in the node then $f_\theta = 0$. The vector *f* for the toy building is illustrated in *Figure 4.*

$$
\begin{array}{cc}
 & \begin{array}{cccccccc} \theta_0 & \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 & \theta_7 \end{array} \\
\begin{array}{c} q_0 \\ q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ q_9 \\ q_{10} \\ q_{11} \end{array} &
A = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
\quad
G = diag\begin{bmatrix} G_0 \\ G_1 \\ G_2 \\ G_3 \\ G_4 \\ G_5 \\ G_6 \\ G_7 \\ G_8 \\ G_9 \\ G_{10} \\ G_{11} \end{bmatrix}
\quad
b = \begin{bmatrix} T_{out} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ T_{out} \\ 0 \\ T_{out} \\ T_{i,set} \end{bmatrix}
$$

$$C = diag[0 \quad C_1 \quad 0 \quad C_3 \quad 0 \quad 0 \quad C_6 \quad 0]$$

$$f = [\Phi_o \quad 0 \quad 0 \quad 0 \quad \Phi_i \quad 0 \quad \dot{Q} \quad \Phi_g]^T$$

Figure 4: Matrices and vectors of the system of DAE

4 State space representation

The state-space model is a mathematical representation of a physical system in terms of its inputs, states, and outputs. The DAE is transformed into state-space representation and the following equations result:

$$\dot{\theta} = A_s * \theta_s + B_s * u$$

$$y_{ss} = C_s * \theta_s + D_s * u$$

With:

θ_s – vector of state variables (temperature nodes containing capacities)

u – vector of inputs

y_{ss} – output in steady-state

The matrices in state space (A_s, B_s, C_s, D_s) representation are obtained from the system of DAE with the matrices and vectors of the thermal circuit (*A, G, b, C, f, y*)

5 Conclusion

The Assignment 1 showed how to model the heat transfer in a cubic building. From analyzing the description of the building, the thermal circuit results. The mathematical model from the thermal circuit as a system of DAE is obtained. To solve the heat transfer model and find the temperatures of interest, the state space representation is used. The state variables derive from the DAE.

For the input data we defined the vector *b* and *f*.

$$b = [T_o \quad T_o \quad T_o \quad T_{i,set}]^T$$

$$f = [\Phi_o \quad \Phi_i \quad \dot{Q}_a \quad \Phi_a]$$

We defined the output vector *y* to get the temperature for nodes we are interested in. In case of the cubic building the indoor temperature is interesting and we obtain the following vector *y*:

$$y = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0]^T$$

Which leads to the output of the state-space system:

$$y_{ss} = \theta_6$$

In the assignment 2 we will work out the steady state.

V. References

Literature:

Peter von Böckh Thomas Wetzel (2017)	Wärmeübertragung, Springer Vieweg,Seiten 3-5 Last reference: 16.05.2024

Internet sources:

C.Ghiaus	https://cghiaus.github.io/dm4bem_book/tutorials/02_2_1Toy.html, Last access 21.05.2024