Smart Cities

Dynamic Models for Building Energy Management

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$https://github.com/dm4bem/model-and-steady-state-cristian\_santosh\_elias\_benjamin$

Contents

1 ABSTRACT

This work aims to analyze the thermal behavior of an enclosed room modeled in three dimensions, with the goal of determining indoor temperatures under steady-state conditions. The geometry of the room, appropriately simplified for computational purposes, serves as the basis for discretizing the thermal domain into a network of interconnected thermal nodes, each representing a portion of the building envelope (walls, floor, ceiling, and openings). The study is based on the application of the thermal conduction matrix method to solve the energy balance under steady-state conditions. Specifically, a Python code has been developed to implement a steady-state analysis via differential algebraic equations and via a state-space representation. This approach combines CAD modeling, physical simplification, and numerical resolution to provide an initial evaluation of the thermal parameters of the analyzed building system. The results form a foundation for potential energy optimization and for extending the model to dynamic conditions or more complex building structures.

2 INTRODUCTION

2.1 Description of the Building

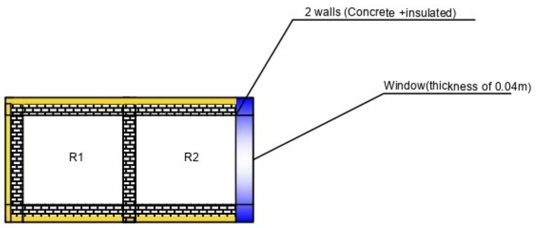

Figure 1: detailed drawing

The selected architectural model (Figure 1 and Figure 2) is composed of two distinct yet interconnected rooms, joined by an internal door. The first room, which we shall refer to as R1, is enclosed predominantly by walls constructed using concrete with integrated insulation. Specifically, four of its enclosing walls are of the "concrete + insulated" type, while a fifth wall features only concrete, lacking the thermal insulation component.

The second room, designated as R2, is similarly enclosed by a heterogeneous combination of wall types. In this case, three of the walls are built with the same "concrete + insulated" construction method employed in R1. One wall, however, is composed solely of concrete material, without any insulation structure. Finally, one side of the room consists of a fully glazed facade—presumably a large window—which contributes to the character of the space by allowing natural light to enter, while also introducing a significant variable in terms of thermal exchange and overall energy performance.

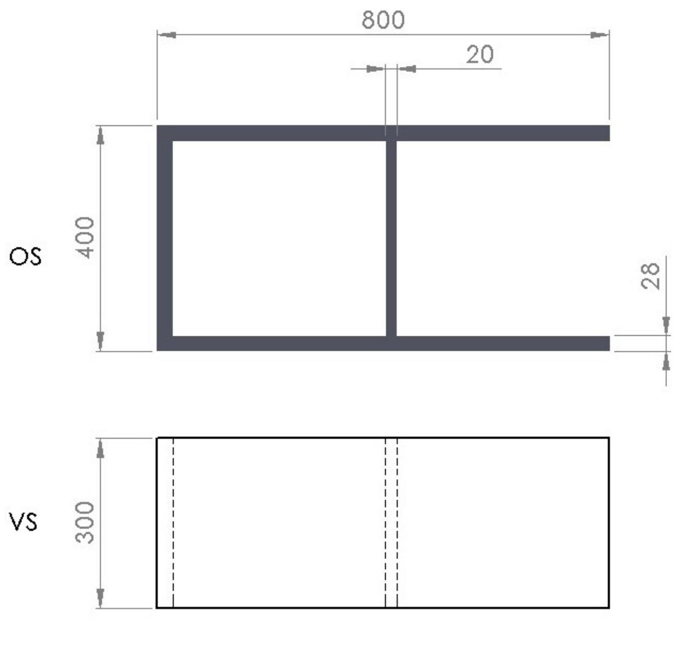

Figure 2: Quotated piece

2.2 Hypothesis

We chose insulation material, characterized by very low thermal conductivity. This will reduce heat transfer through the walls. As a result, rooms with such composite walls are expected to experience reduced heat flux and smaller indoor temperature variations compared to those with bare concrete walls or exposed glass.

Each wall material is defined by its thermal properties. Concrete is modeled with a conductivity of 1.4 W/m·K, a density of 2300 kg/m^3, and a specific heat of 880 J/kg·K. Insulation has a much lower conductivity of 0.027 W/m·K, a density of 55 kg/m^3, and a specific heat of 1210 J/kg·K, making it highly effective in minimizing conductive heat transfer. Glass, used on the façade, has a conductivity of 1.4 W/m·K, density of 2500 kg/m^3, and specific heat of 1210 J/kg·K, and plays a role in radiative and solar heat exchanges due to its partial transparency and surface properties.

Radiative behavior is modeled using the following parameters: long-wave emissivity is set to $\epsilon = 0.85$ for wall surfaces and 0.90 for glass. Short-wave absorptivity is 0.25 for white surfaces and 0.38 for glass, while short-wave transmittance for glass is 0.30, allowing a portion of solar radiation to pass through. These values are critical in estimating the balance between absorbed, emitted, and transmitted solar energy. The Stefan-Boltzmann constant used is $\sigma = 5.67 \times 10^{-8}$ W/m$^2 \cdot K^4$.

The air inside the room is defined with a density of 1.2 kg/m^3 and specific heat of 1000 J/kg·K, contributing to the indoor thermal inertia. The thermal interaction between room air and wall surfaces is influenced by this capacity to store and exchange heat.

In terms of geometry, each wall's surface area is calculated using Sg = L × height, where L and height are the length and height of the glass surface. Room 1 has five opaque walls (Sc1 = 5 × Sg) and Room 2 has four (Sc2 = 4 × Sg). All simulations are performed using a uniform mesh value of 1 for consistency.

Type	Material	Conductivity($\frac{W}{m \cdot K}$)	Specific Heat($\frac{J}{kg \cdot K}$)	Density($\frac{kg}{m^3}$)	Width(m)	Mesh
0	Concrete	1.4	880.8	2300	0.2	1
0	Insulation	0.027	1210	55	0.08	1
1	Glass	1.4	750	2500	0.004	1

Table 1: Wall Material Properties

3 MODELLING

3.1 Thermal Circuit

A thermal model in Figure 3 has been developed to represent the dynamic behavior of heat exchange within a confined space, which is divided into two adjacent volumes, referred to as R1 and R2. The environment is enclosed by walls composed of a composite structure made of insulated concrete, complemented by a glazed surface with a thickness of 0.04 meters. The geometric configuration of the enclosure has been derived from the attached technical drawings, from which it is inferred that the system has overall dimensions of 8.00 meters in length, 4.00 meters in width, and 3.00 meters in height. The wall stratigraphy consists of an internal layer of concrete coupled with an insulating coating, yielding a total thickness of 0.28 meters.

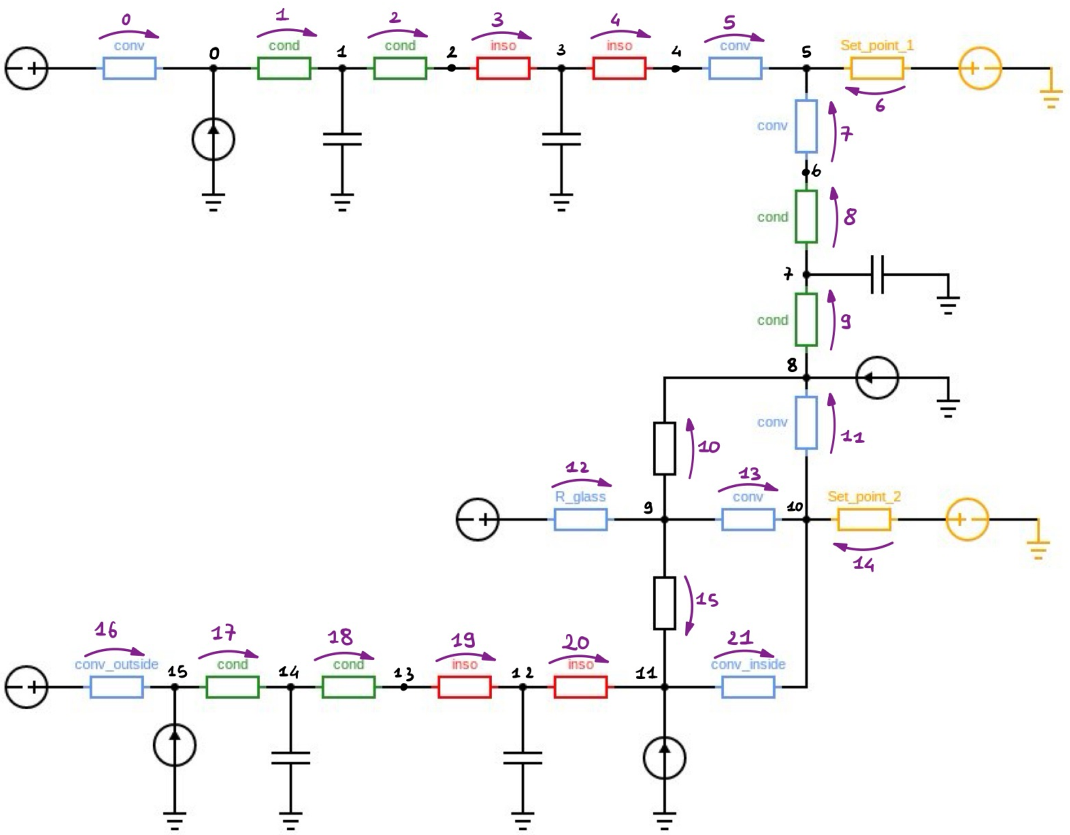

Figure 3: Thermal circuit

To carry out the thermal analysis of the system, a lumped-parameter thermal model has been implemented, represented through an equivalent resistive-capacitive (RC) network, which simulates the heat flow and thermal response of the system. This thermal network consists of a series of thermal resistances modeling the mechanisms of conduction (in the opaque walls and the glass), convection (on both interior and exterior surfaces), and thermal capacitances (to account for energy storage within the materials).

The circuit includes thermal exchange nodes associated with the interfaces between the internal environment and the building envelope, as well as between the two internal rooms, R1 and R2. In particular, two set-point nodes have been defined, namely *Set_point_1* and *Set_point_2*, which represent the temperature control points for each respective room. These allow for monitoring the thermal evolution of the system in response to external stimuli or assigned initial conditions.

Note: the resistor *R_glass* combines both outdoor convection, conduction and capacitance in the window.

The developed model enables an accurate energy analysis of the system, making it possible to evaluate overall thermal transmittance, instantaneous heat fluxes, and the optimization of climate control and insulation strategies. Therefore, it constitutes an effective tool for the study of thermo-physical phenomena in buildings, with a view to energy efficiency and indoor thermal comfort.

3.2 Mathematical Calculations

To implement the thermal model in python, we compute the conductances and capacities of the thermal circuit.

3.2.1 Thermal Conductances

The conductances for conduction are computed as follows:

$$G_{cond_i} = \frac{\lambda_i * S_i}{w_i} \tag{1}$$

The conductances for convection are computed as follows:

$$G_{conv_i} = h_i * S_i \tag{2}$$

The equivalent conductances for radiative long-wave heat exchange between the wall and the glass window are computed as follows:

$$G = \frac{1}{\frac{1}{G_i} + \frac{1}{G_{i,j}} + \frac{1}{G_j}} \tag{3}$$

using a mean temperature Tm of 20°C and the following linearized formulas:

$$
\begin{aligned}
G_i &= 4\sigma T_m^3 \cdot \frac{\epsilon_i}{1-\epsilon_i} \cdot S_i \\
G_{i,j} &= 4\sigma T_m^3 \cdot F_{i,j} \cdot S_i \\
G_j &= 4\sigma T_m^3 \cdot \frac{\epsilon_j}{1-\epsilon_j} \cdot S_j
\end{aligned}
\tag{4}
$$

Due to the 40 mm thickness limit, we opted for double glazing instead of triple glazing. While it includes only one air gap between two glass panes, it still ensures good thermal insulation and meets the design constraints.

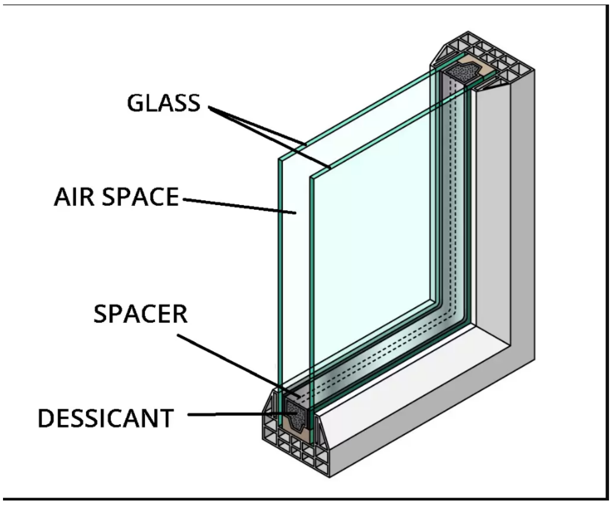

Figure 4: Double Glass Wall

Since the overall thermal resistance R $[\Omega \cdot m^2]$ of the window is known (see Table 2), we can directly calculate the thermal conductance G_{glass}, using the relation $G = \frac{S_{glass}}{R}$.

wall type:external triple glaze				
Layer	Types	d(m)	$\lambda(\frac{w}{mc})$	$R = \frac{d}{\lambda}$
Internal Surface R_i	R_s			0.130
Glass(8mm)	R_h	0.006	1.15	0.005
Air layer(25mm)	R_g			0.18
Glass(8mm)	R_h	0.006	1.15	0.005
External Surface R_e	R_e			0.04
	$R_t(\frac{m^2c}{w})$			0.3
	$G_t(\frac{w}{m^2c})$			2.7

Table 2: double Triple Glass Window

In this first thermal circuit there is no advection taken into accounts.

3.2.2 Thermal Capacities

Since the thermal circuit contains only insulation and concrete walls capacities. We used the following formula to calculate the corresponding capacities:

$$C_i = \rho_i \cdot c_i \cdot S_i \cdot w_i \tag{5}$$

3.3 The Matrices

The matrix A is called the incidence matrix of the thermal network and links the thermal flows q_i with the thermal potentials θ_j (where θ represents the nodal temperatures and q the heat flows through branches). The matrix is constructed based on the principles of *network topology* and the *theory of oriented graphs.*

The matrix G represents the diagonal matrix of *thermal conductances* of the branches in the system. Each diagonal element G_{ii} corresponds to the conductance of a specific thermal element, which can be defined as the inverse of its thermal resistance.

The matrix C is the diagonal matrix of *thermal capacitances*, which associates a thermal inertia to each node in the system. Each entry C_{ii} represents the heat capacity (in J/K) of the corresponding thermal node. This matrix plays a fundamental role in modeling the dynamic response of the system to external excitations.

The vectors b and f are introduced to define the inputs. The vector $\boldsymbol{b}$ characterizes the temperature sources (outdoor temperatures T_o and desired indoor temperatures T_{i_i}, and $\boldsymbol{f}$ represents the flow rate sources.

3.3.1 Incidence matrix A

	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}	θ_{12}	θ_{13}	θ_{14}	θ_{15}
q_0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
q_1	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
q_2	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
q_3	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0	0
q_4	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0	0
q_5	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	0
q_6	0	0	0	0	0	1	0	-1	0	0	0	0	0	0	0	0
q_7	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0
q_8	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0
q_9	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0
] q_{10}	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0
q_{11}	0	0	0	0	0	0	0	0	0	1	0	-1	0	1	0	0
q_{12}	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
q_{13}	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0
q_{14}	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
q_{15}	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0
q_{16}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
q_{17}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1
q_{18}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1
q_{19}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0
q_{20}	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0
q_{21}	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0

3.3.2 G, C, b, f Parameters Table

Index	G (W/K)	C (J/K)	b	f
q_0 / θ_0	1500.000	0.0	T_o	Φ_{o1}
q_1 / θ_1	840.000	24288000.0	0	0
q_2 / θ_2	840.000	0.0	0	0
q_3 / θ_3	40.500	319440.0	0	0
q_4 / θ_4	40.500	0.0	0	0
q_5 / θ_5	600.000	0.0	0	0
q_6 / θ_6	0.000	0.0	T_{i1}	0
q_7 / θ_7	120.000	4857600.0	0	0
q_8 / θ_8	168.000	0.0	0	Φ_{i1}
q_9 / θ_9	168.000	0.0	0	0
q_{10} / θ_{10}	59.260	0.0	0	0
q_{11} / θ_{11}	120.000	0.0	0	Φ_{i2}
q_{12} / θ_{12}	23.715	255552.0	T_o	0
q_{13} / θ_{13}	300.000	0.0	0	0
q_{14} / θ_{14}	0.000	19430400.0	T_{i2}	0
q_{15} / θ_{15}	15.967	0.0	0	Φ_{o2}
q_{16}	1200.000	–	T_o	–
q_{17}	672.000	–	0	–
q_{18}	672.000	–	0	–
q_{19}	32.400	–	0	–
q_{20}	32.400	–	0	–
q_{21}	480.000	–	0	–

Table 3: Unified table of thermal conductances G, capacitances C, source vector b and heat inputs f.

4 STEADY-STATE IMPLEMENTATION

Right now we described the whole model we will use in the following assignments. In order to check the refutability of our model we can implement all the matrices in python. The thermal circuit (TC) is a dictionary containing pandas dataframes for the different matrices.

After defining the matrices, we can build a system of Differential Algebraic Equations (DAE) or transform this system of DAE into a state-space representation. This is easily done with the python code:

$$[As, Bs, Cs, Ds, us] = dm4bem.tc2ss(TC) \tag{6}$$

In steady state we consider that $C\dot{\theta} = 0$ and so:

$$\theta_{steadystate} = (A^T G A)^{-1}(A^T G b + f) \tag{7}$$

4.1 Analysis

The following assumptions are made: The indoor air temperature is not controlled ($K_p = 0$). The outdoor temperature is 10°C and the indoor temperature setpoint is 20°C. In addition, all flow rate sources are zero. The following analysis is a test of falsification of the model. The values of the matrix C or of the conductances in the matrix G can still be wrong.

In steady state the temperature values are obtained from the system of DAE by using equation 7. We find that $\theta_5 = \theta_{10} = 10.0C$. This are the values we expected with the assumptions we made.

The input vector u of the state-space representation is obtained by stacking the vectors b_T (vector of the nonzero elements of vector b $[T_o, T_i, T_o, T_i, T_o]$) and f_Q (vector of the nonzero elements of vector f $[\phi_{o1}, \phi_{i1}, \phi_{i2}, \phi_{o2}]$. This gives $u = [10, 20, 10, 20, 10, 0, 0, 0, 0]$). The steady-state value of the output of the state space representation is obtained when $\dot{\theta} = 0$:

$$y_{ss} = (-C_s A_s^{-1} B_s + D_s)u = 10.0 \tag{8}$$

The error between the steady-state values obtained from the system of DAE and the output of the state-space representation yss is $2.84 * 10^{-14}$°C and $4.97 * 10^{-14}$°C for respectively θ_5 and θ_{10}, the two indoor room temperatures. The errors are practically zero; the slight difference is due to numerical errors.

5 Appendix

5.1 References

The datasheet for the thermal resistance of the window was provided by the company Zmerly Academy. Zmerly Academy provided standard reference values for thermal resistance in building components, ***http://www.academy.zmerly.com/***