

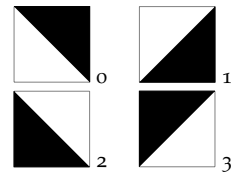
TRUCHET BOOK (I)

4X4 PATTERNS WITH ROTATIONAL
SYMMETRY

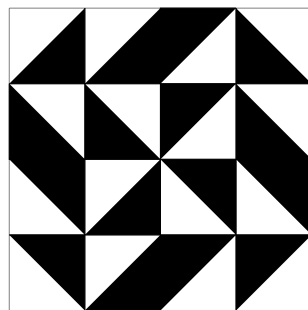
Introduction

Truchet tiles are, traditionally, square tiles that are divided by a diagonal line, and coloured with two colours with a different colour on either side of the diagonal. Each tile can be rotated to one of four positions. Patterns are formed by placing tiles next to each other, often rotating tiles to create repeated motifs. This booklet presents a complete listing of 4x4 Truchet tile patterns with rotational symmetry (256 patterns). Treating these 4x4 tile patterns as tiles themselves allows for larger decorative patterns to be built. Interesting relationships among the 4x4 tiles patterns and within the larger patterns created with them can be observed.

Each 4x4 Truchet tile pattern with rotational symmetry has a core 2x2 pattern in one of its quadrants that is rotated to produce the overall pattern. In this booklet, the core pattern is assumed to be in the lower left. Each pattern can be identified as a sequence of 4 digits *abcd* that list the rotational positions of each tile in the lower left quadrant.



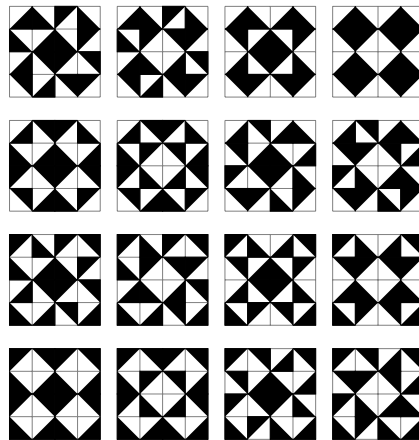
a	q	c	e
c	p	p	q
b	d	d	c
a	c	b	a



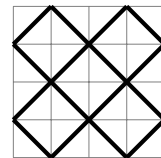
The 0011 pattern

Pattern families

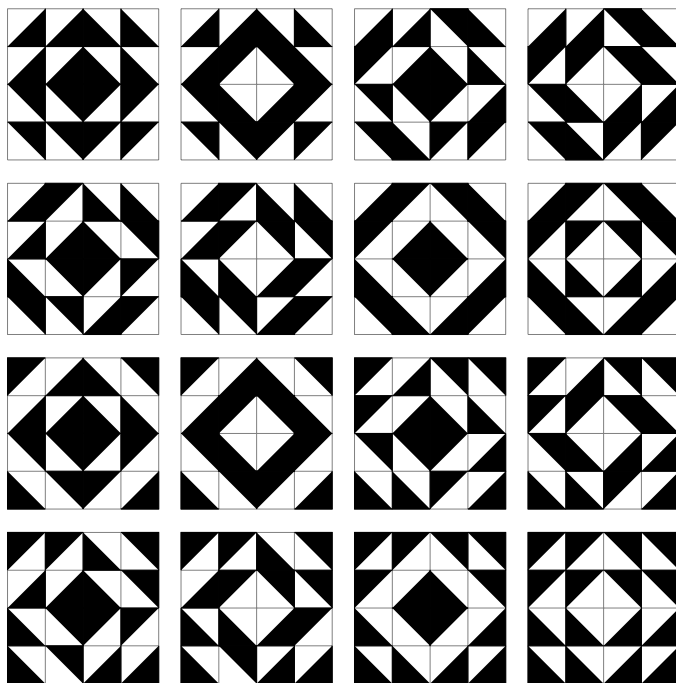
We can group the 4×4 Truchet tile patterns with rotational symmetry into families where tile patterns are considered to be in the same family if they would look the same without colour – if each corresponding tile shares the same diagonal direction. The sequence that represents the family of a tile pattern can be found by taking the sequence of the tile pattern *modulo* 2. So, for example, the 16 tile patterns below are all members of the 0110 family.



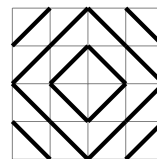
The 0110 pattern family



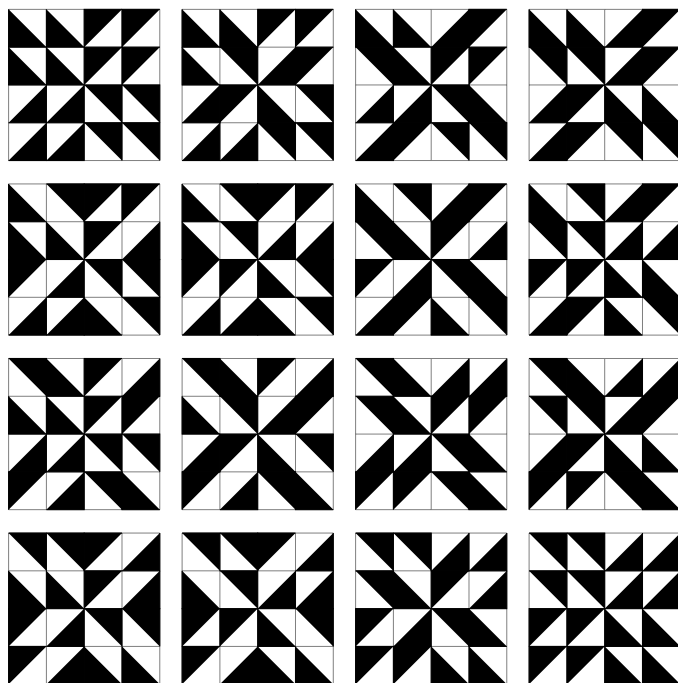
The 0110 family pattern



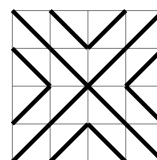
0000



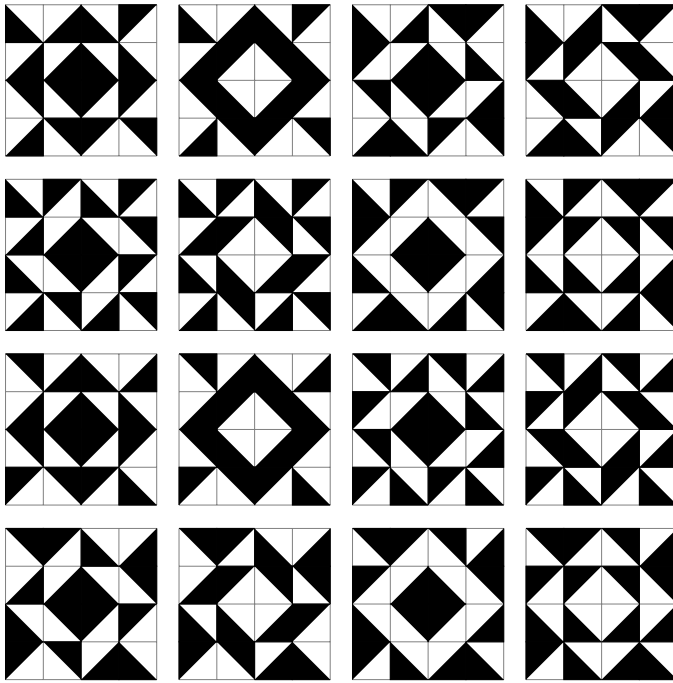
0000	0002	0020	0022
0200	0202	0220	0222
2000	2002	2020	2022
2200	2202	2220	2222



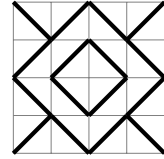
1111



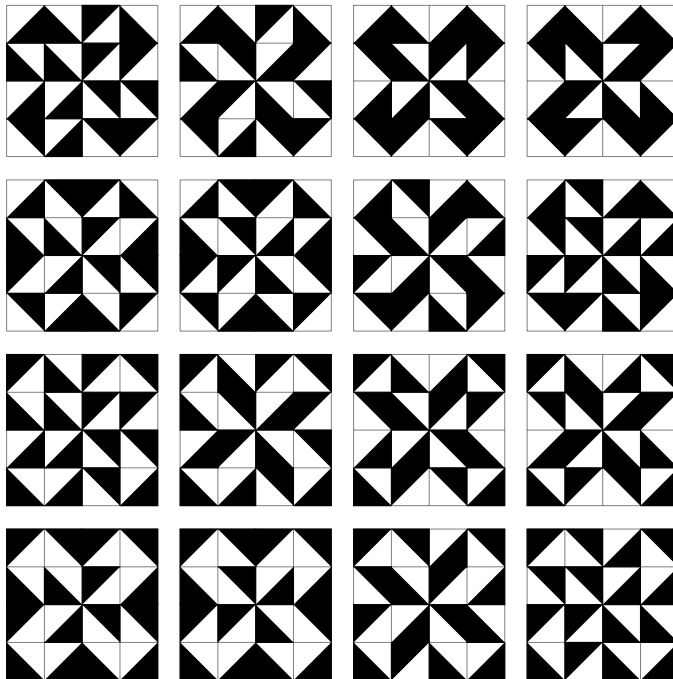
1111	1113	1131	1133
1311	1313	1331	1333
3111	3113	3131	3133
3311	3313	3331	3333



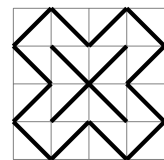
1000



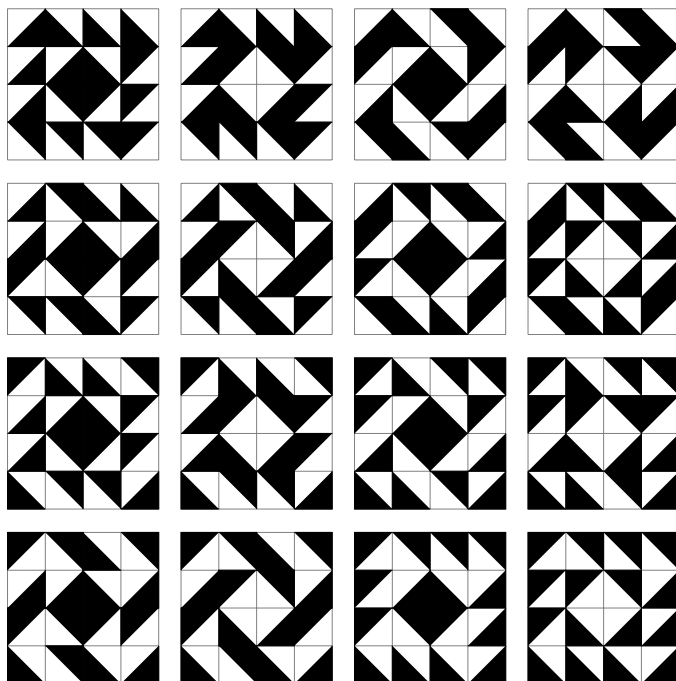
1000	1002	1020	1022
1200	1202	1220	1222
3000	3002	3020	3022
3200	3202	3220	3222



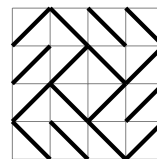
0111



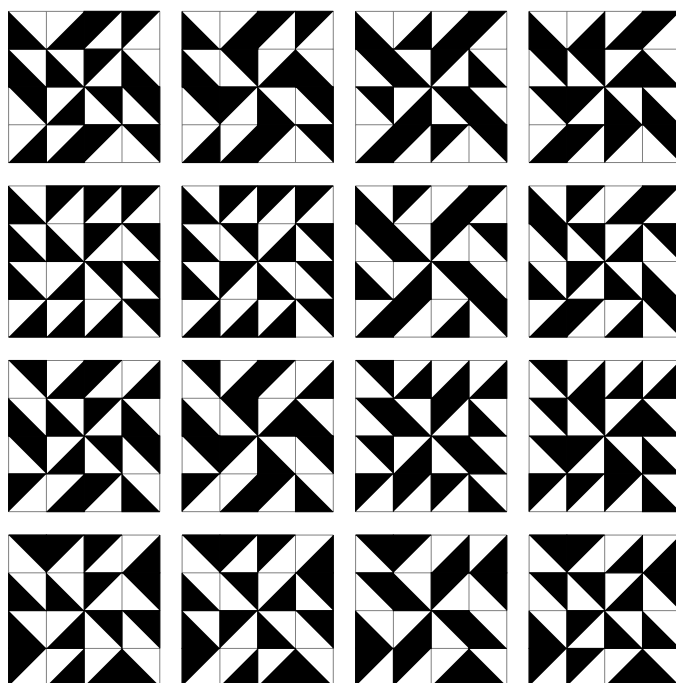
0111	0113	0131	0133
0311	0313	0331	0333
2111	2113	2131	2133
2311	2313	2331	2333



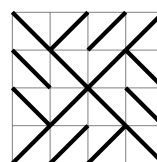
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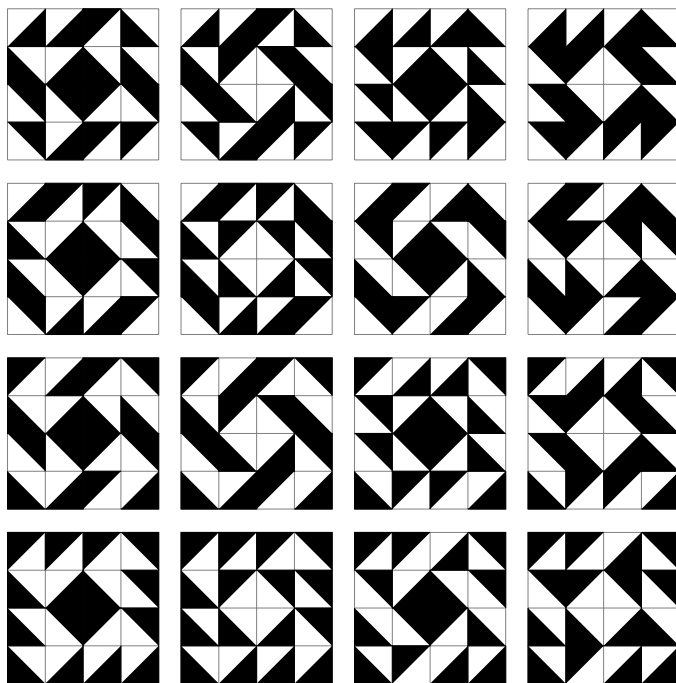
0100	0102	0120	0122
0300	0302	0320	0322
2100	2102	2120	2122
2300	2302	2320	2322



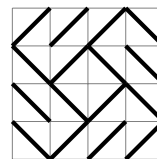
1011



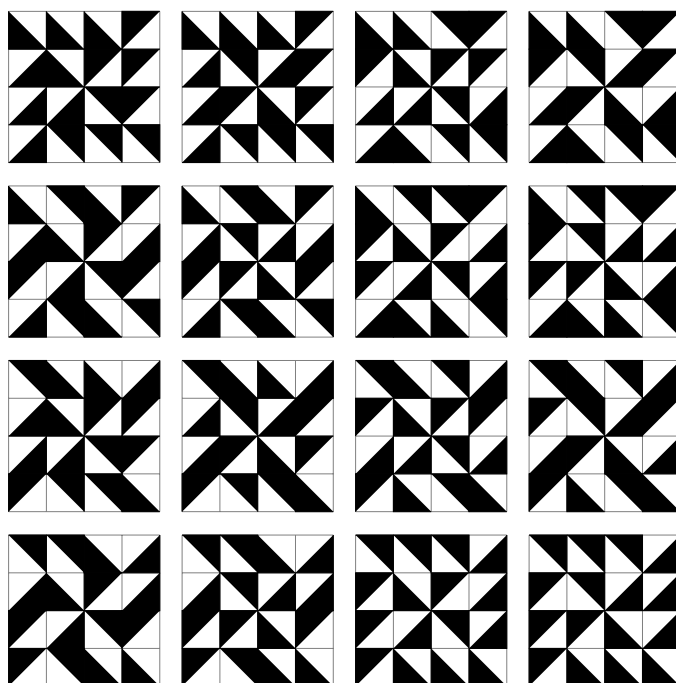
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1211	1213	1231	1233
3011	3013	3031	3033
3211	3213	3231	3233



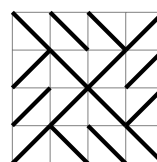
0010



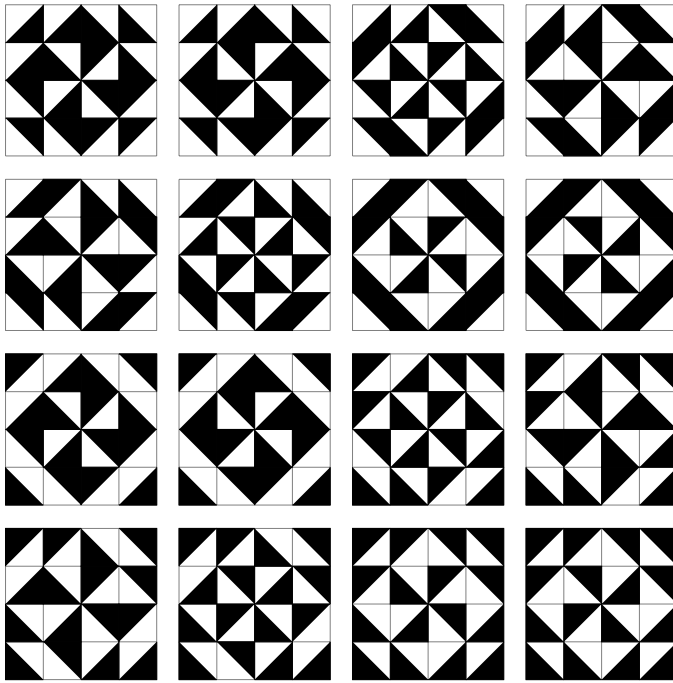
0010	0012	0030	0032
0210	0212	0230	0232
2010	2012	2030	2032
2210	2212	2230	2232



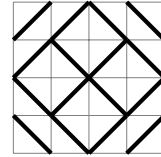
1101



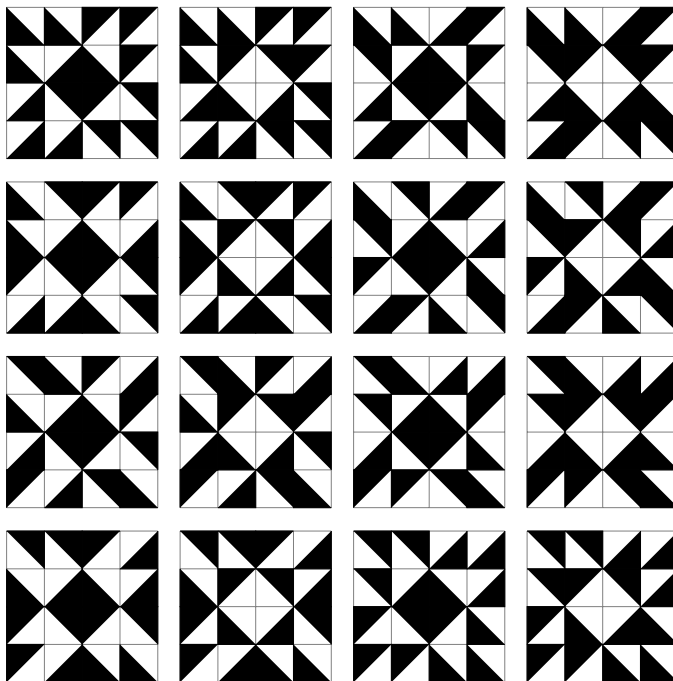
1101	1103	1121	1123
1301	1303	1321	1323
3101	3103	3121	3123
3301	3303	3321	3323



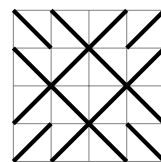
0001



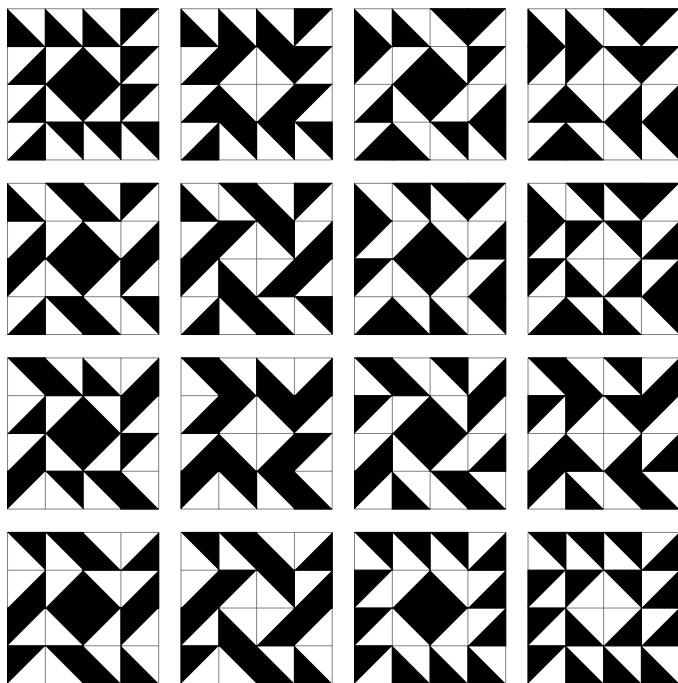
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0201	0203	0221	0223
2001	2003	2021	2023
2201	2203	2221	2223



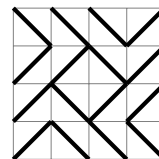
1110



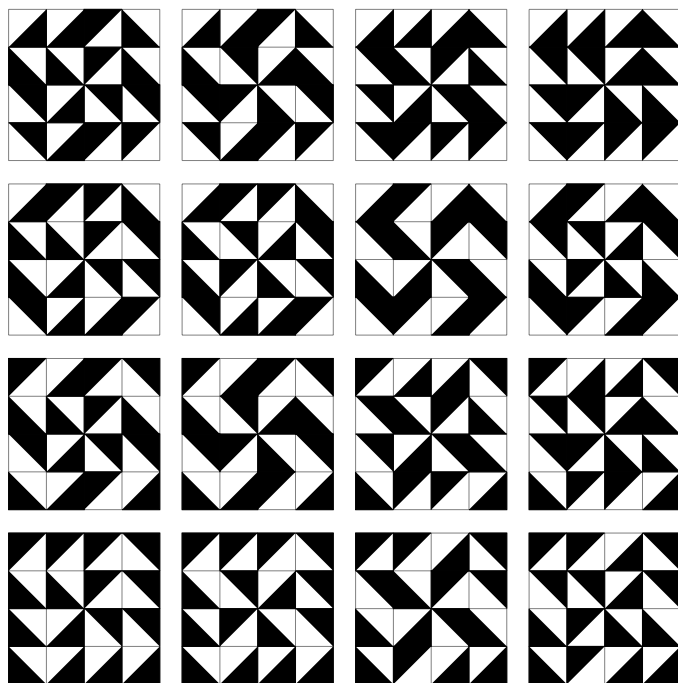
1110	1112	1130	1132
1310	1312	1330	1332
3110	3112	3130	3132
3310	3312	3330	3332



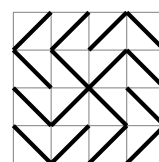
1100



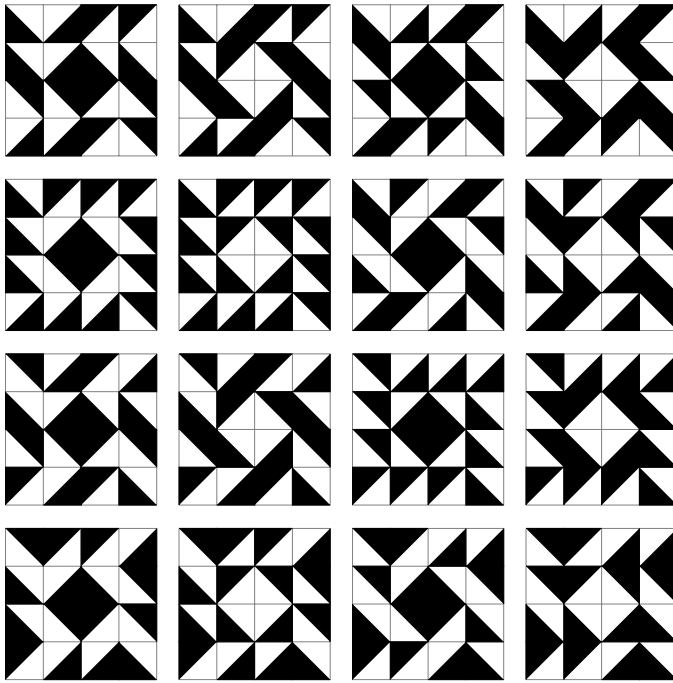
1100	1102	1120	1122
1300	1302	1320	1322
3100	3102	3120	3122
3300	3302	3320	3322



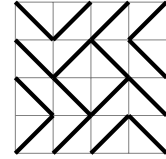
0011



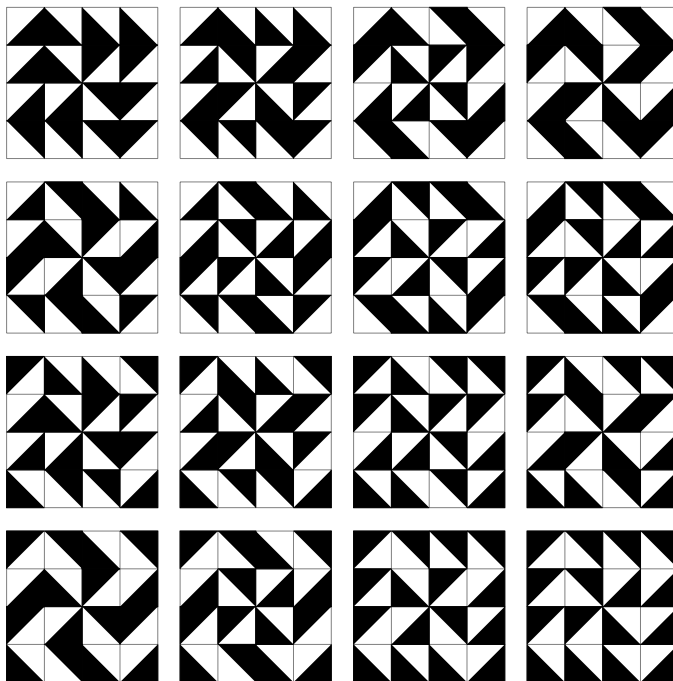
0011	0013	0031	0033
0211	0213	0231	0233
2011	2013	2031	2033
2211	2213	2231	2233



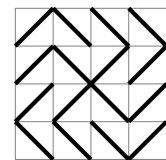
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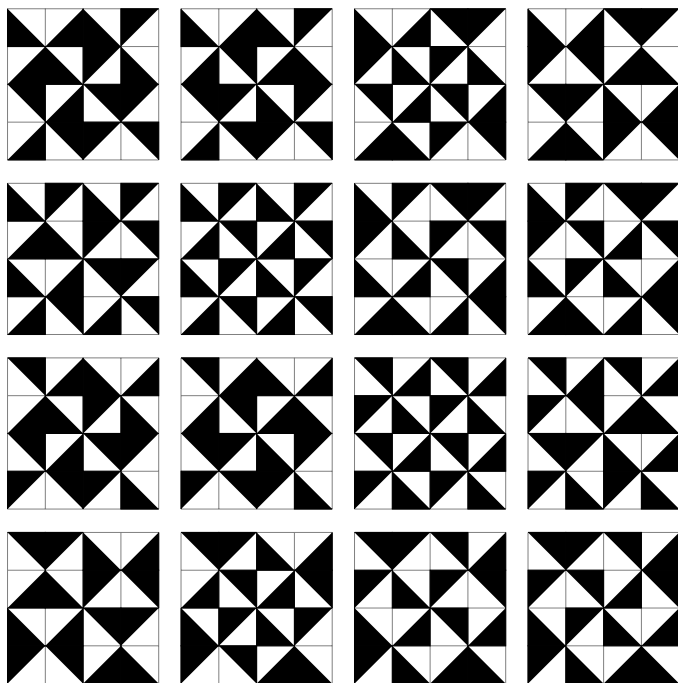
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1210	1212	1230	1232
3010	3012	3030	3032
3210	3212	3230	3232



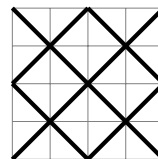
0101



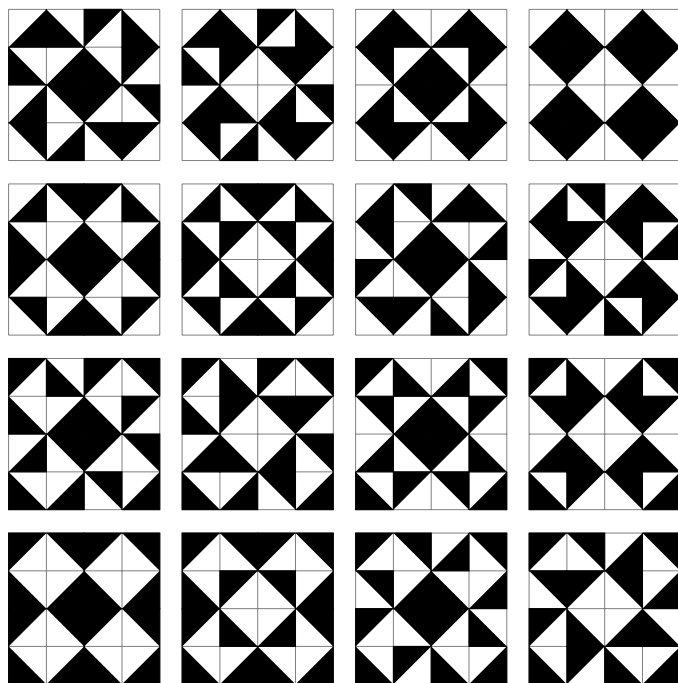
0101	0103	0121	0123
0301	0303	0321	0323
2101	2103	2121	2123
2301	2303	2321	2323



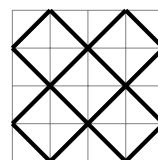
1001



1001	1003	1021	1023
1201	1203	1221	1223
3001	3003	3021	3023
3201	3203	3221	3223



0110

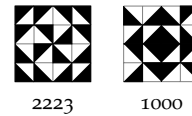
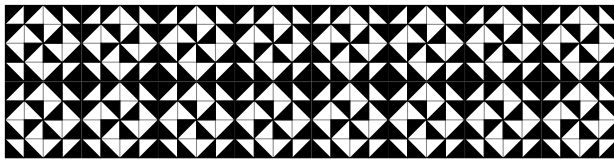


0110	0112	0130	0132
0310	0312	0330	0332
2110	2112	2130	2132
2310	2312	2330	2332

Friezes of Truchet patterns

Each 4x4 Truchet pattern can be treated like a tile and used in a larger pattern. A *frieze* is a horizontal strip of the same tile pattern repeated. Friezes of 4x4 Truchet pattern tiles with rotational symmetry can be quite striking, and have some interesting characteristics.

In frieze of more than one row of a primary tile reveals a secondary tile pattern that appears as another horizontal strip of 4x4 Truchet tile patterns nestled between the rows of primary tiles. Below, a frieze of 2223 tiles has a secondary pattern of 1000 tiles.



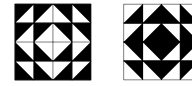
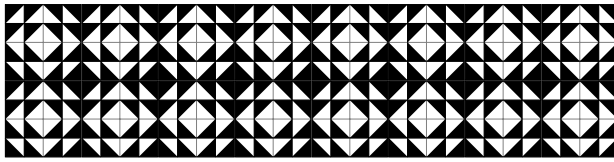
Frieze patterns made from tiles of a particular family will have secondary tiles that also have rotational symmetry and that are from the same family. For example, frieze patterns from the family 0001 will have secondary patterns from the family 1000.

You can compute the secondary frieze tile pattern of a given tile by reversing the digits of the tile label and adding 2 to each digit, modulo 4. This is because the label of the secondary frieze tile pattern comes from the top right corner of the primary tile pattern.

p	q	c	e
c	p	p	q
b	d	d	c
a	c	b	a

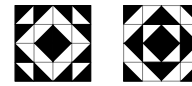
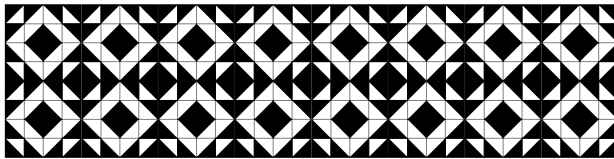
If tile t is the primary tile in a frieze pattern and s is the secondary, then t will be the secondary tile in the frieze pattern formed by s . This effectively reduces the total number of unique frieze patterns to 136, these patterns are shown on the pages that follow.

Frieze patterns for family 0000 (secondary, 0000)



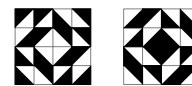
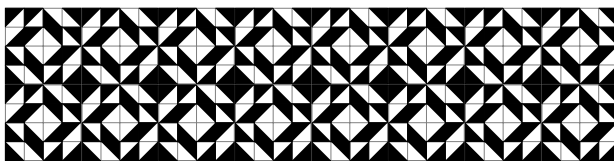
2222

0000



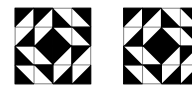
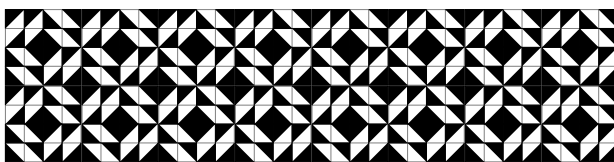
2220

2000



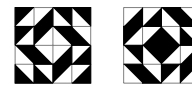
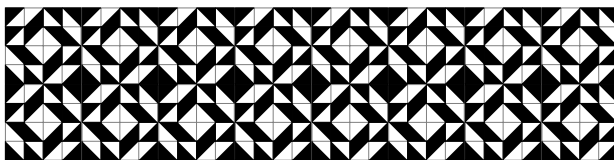
2202

0200



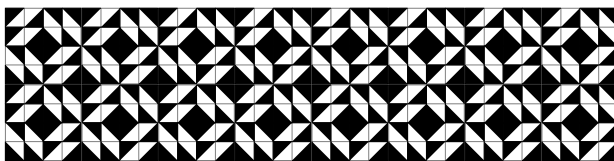
2200

2200



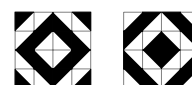
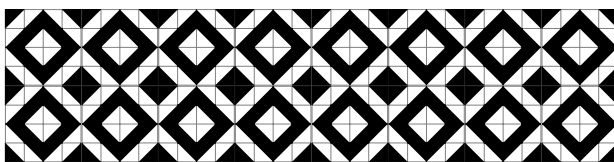
2022

0020



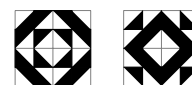
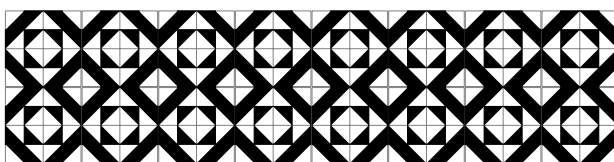
2020

2020



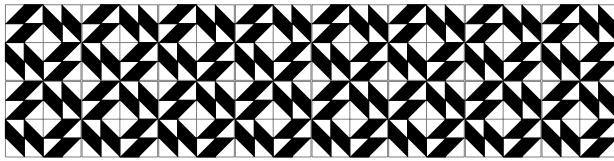
2002

0220



0222

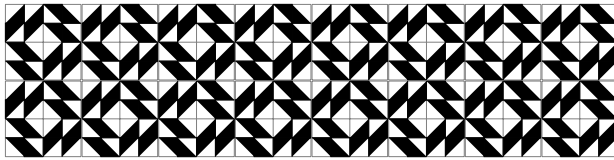
0002



0202



0202

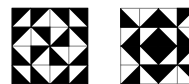
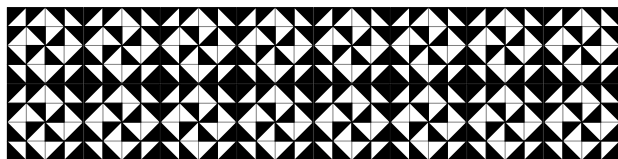


0022



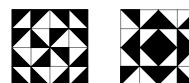
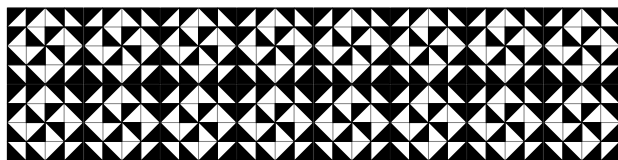
0022

Frieze patterns for family 0001 (secondary, 1000)



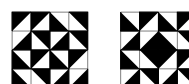
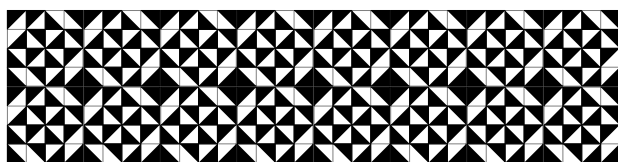
2223

1000



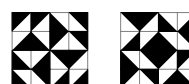
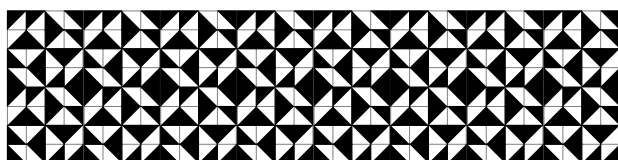
2221

3000



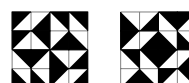
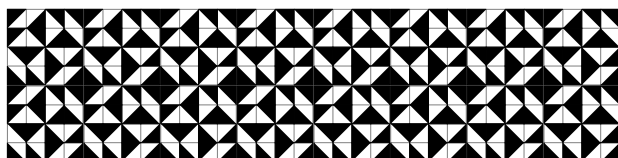
2203

1200



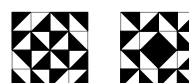
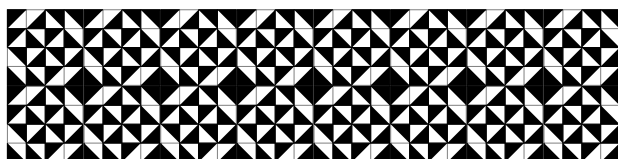
2201

3200



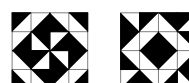
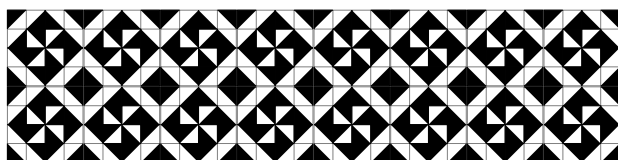
2023

1020



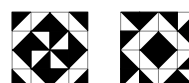
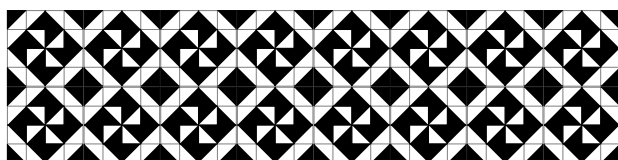
2021

3020



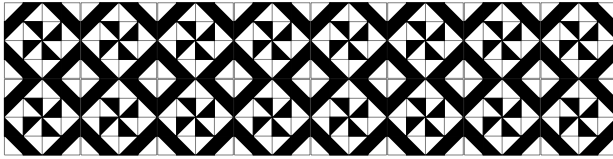
2003

1220



2001

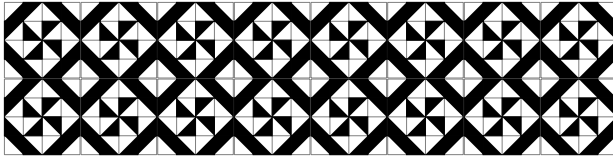
3220



0223



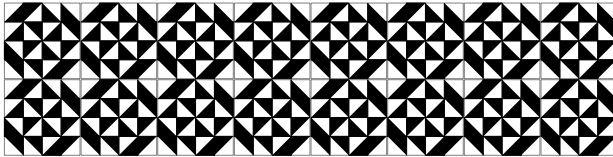
1002



0221



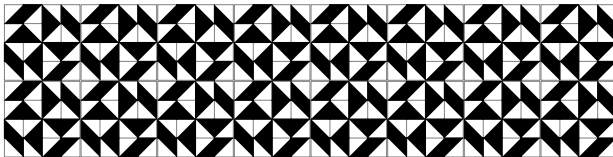
3002



0203



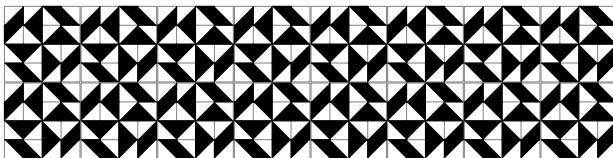
1202



0201



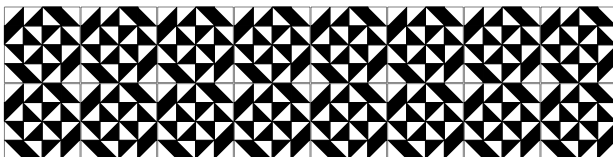
3202



0023



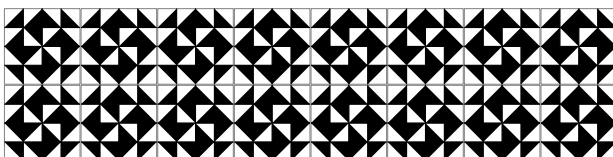
1022



0021



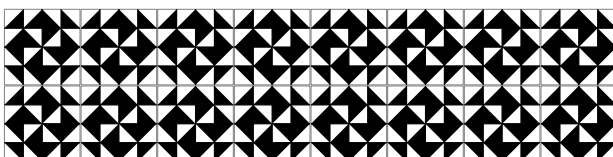
3022



0003



1222

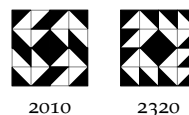
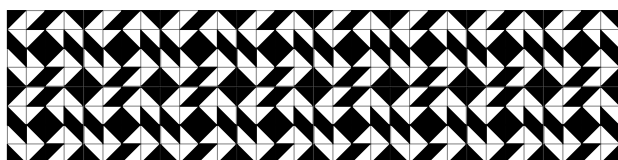
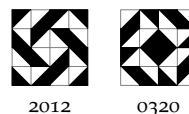
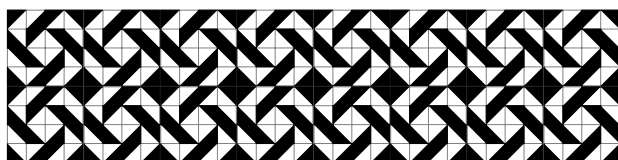
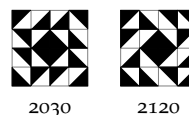
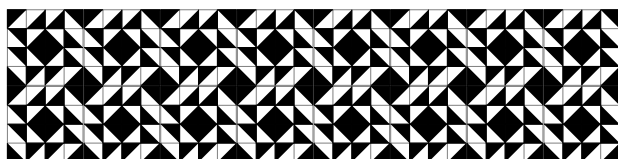
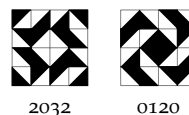
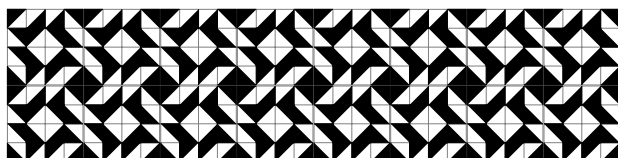
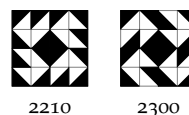
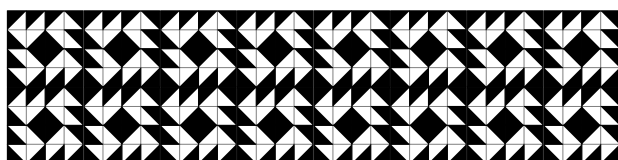
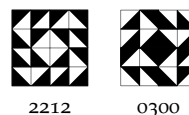
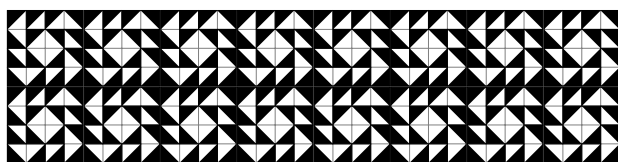
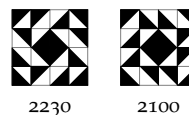
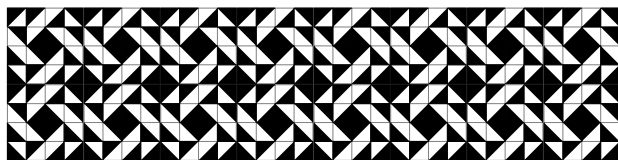
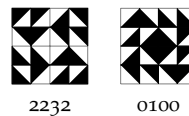
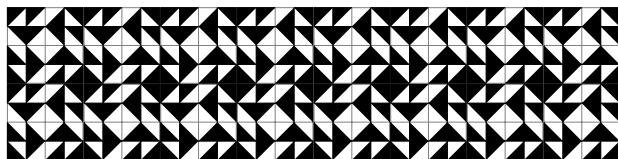


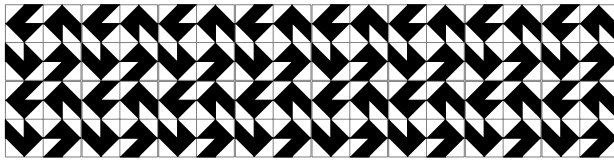
0001



3222

Frieze patterns for family 0010 (secondary, 0100)

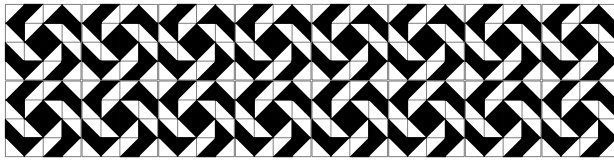




0232



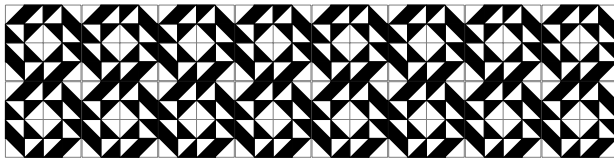
0102



0230



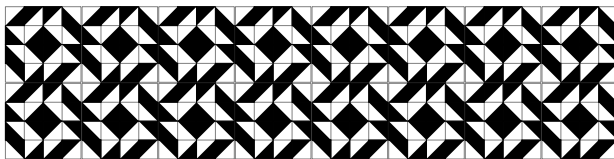
2102



0212



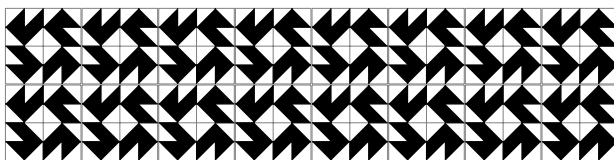
0302



0210



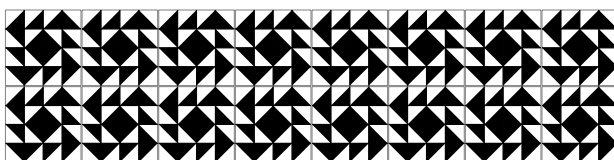
2302



0032



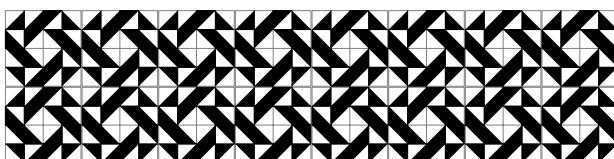
0122



0030



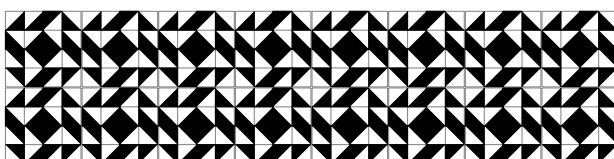
2122



0012



0322

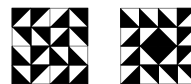
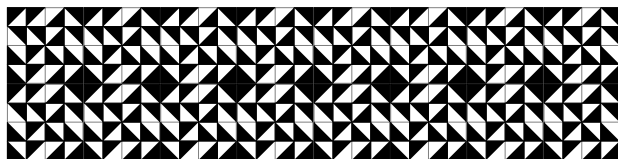


0010



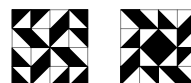
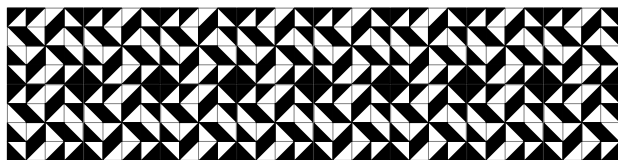
2322

Frieze patterns for family 0011 (secondary, 1100)



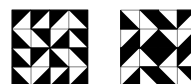
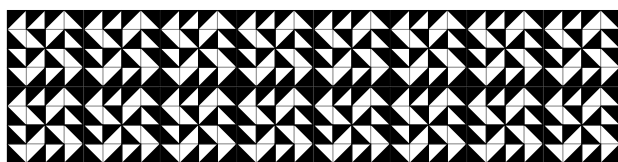
2233

1100



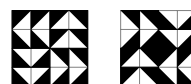
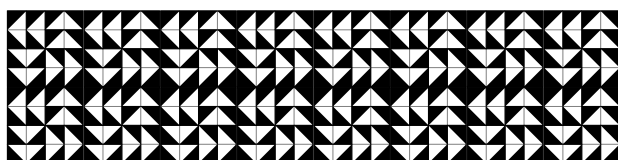
2231

3100



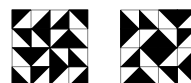
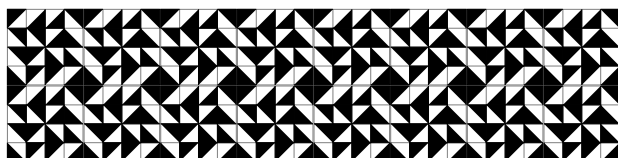
2213

1300



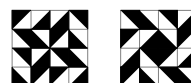
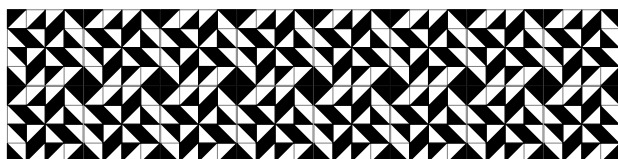
2211

3300



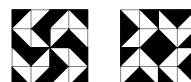
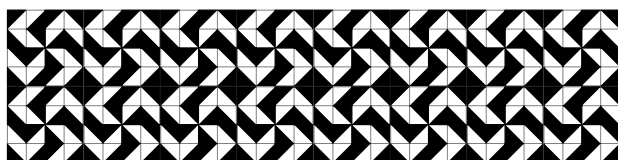
2033

1120



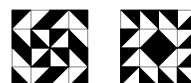
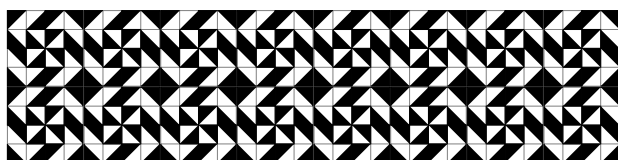
2031

3120



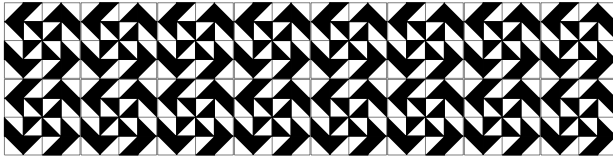
2013

1320



2011

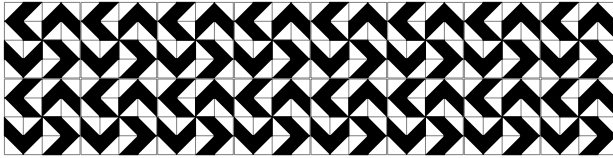
3320



0233



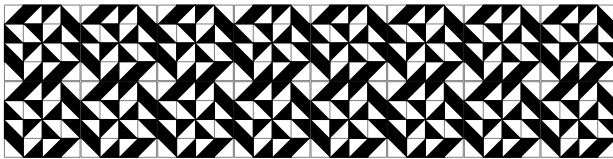
1102



0231



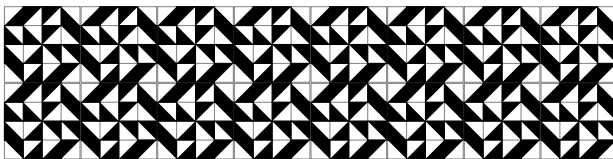
3102



0213



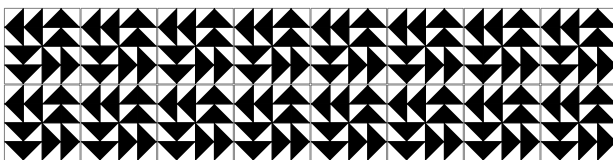
1302



0211



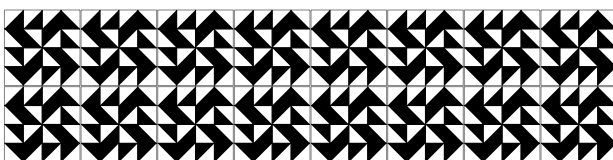
3302



0033



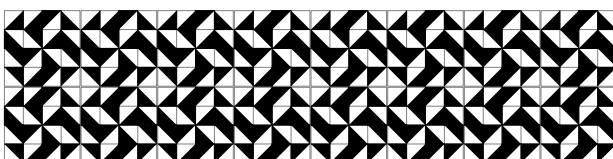
1122



0031



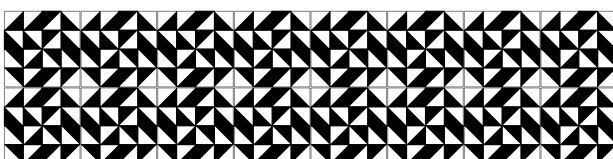
3122



0013



1322

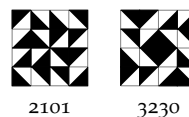
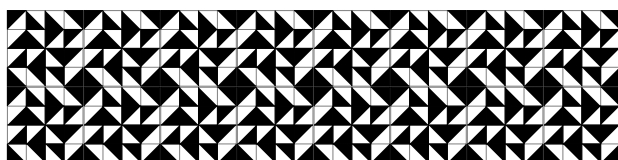
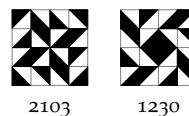
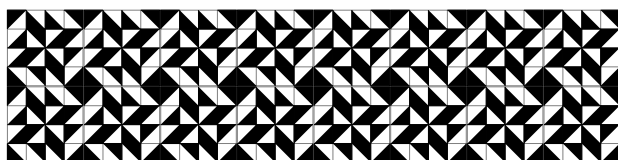
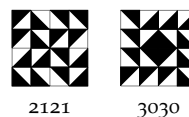
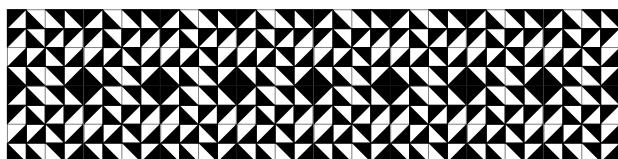
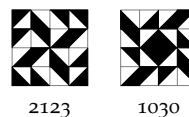
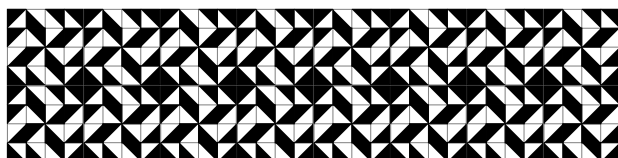
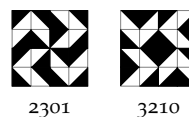
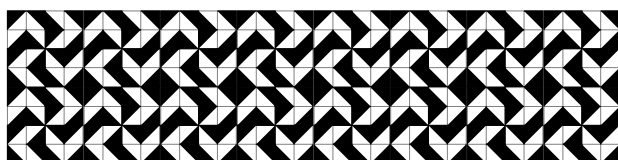
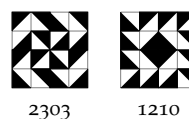
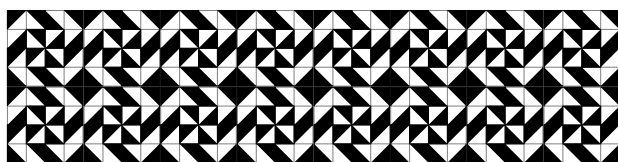
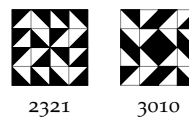
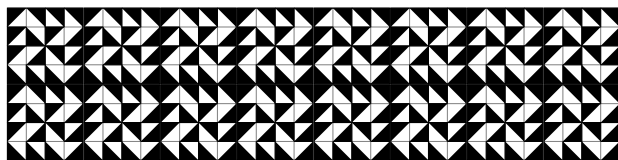
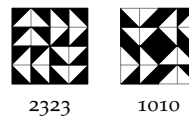
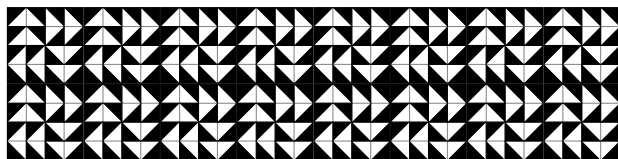


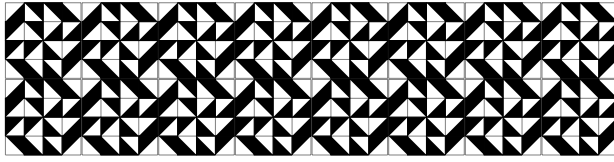
0011



3322

Frieze patterns for family 0101 (secondary, 1010)

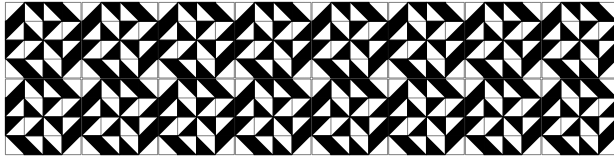




0323



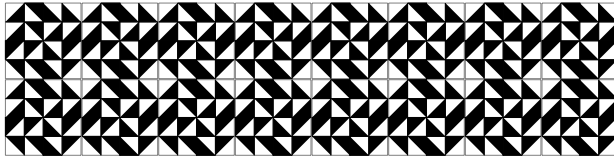
1012



0321



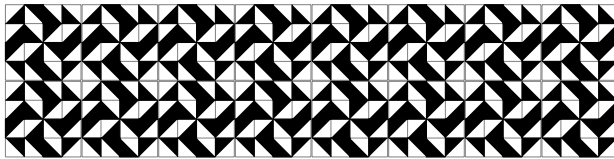
3012



0303



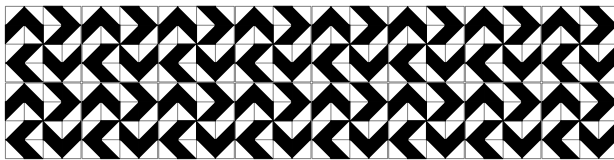
1212



0301



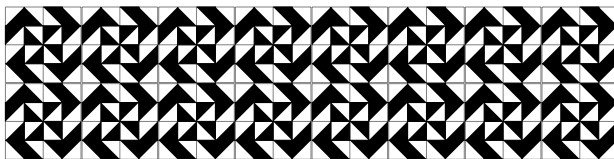
3212



0123



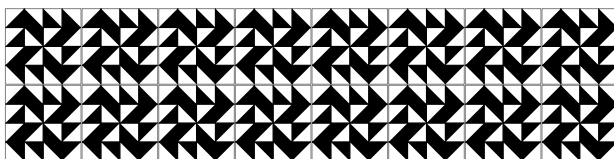
1032



0121



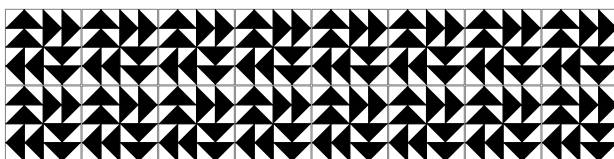
3032



0103



1232

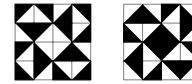
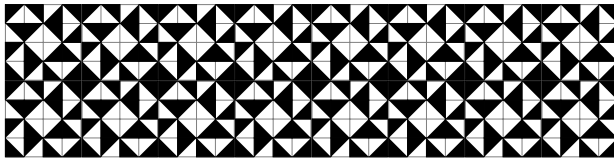


0101



3232

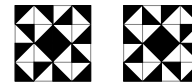
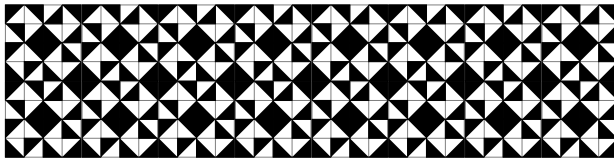
Frieze patterns for family 0110 (secondary, 0110)



2332



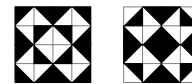
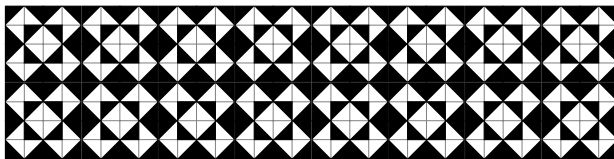
0110



2330



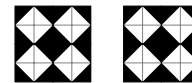
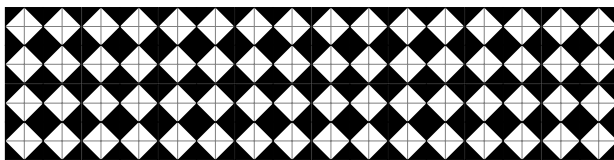
2110



2312



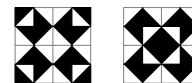
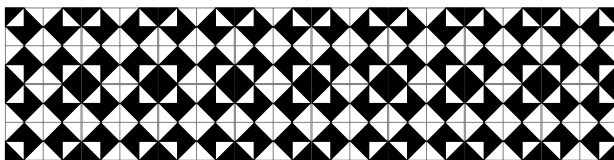
0310



2310



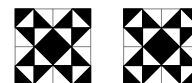
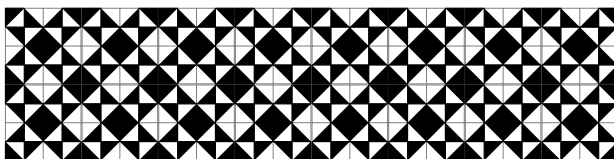
2310



2132



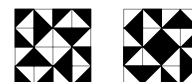
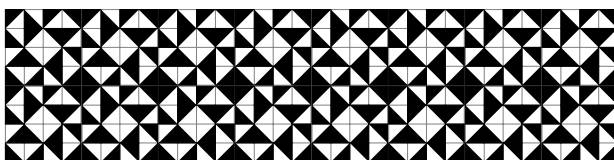
0130



2130



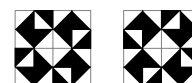
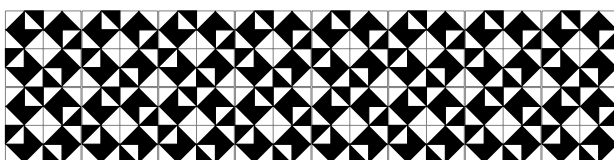
2130



2112



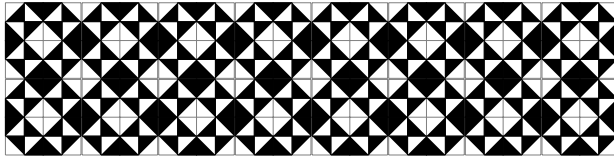
0330



0332



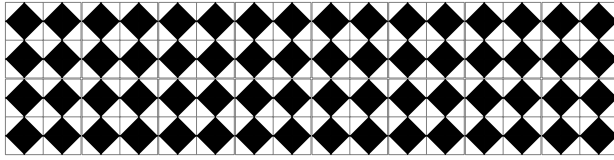
0112



0312



0312

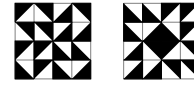
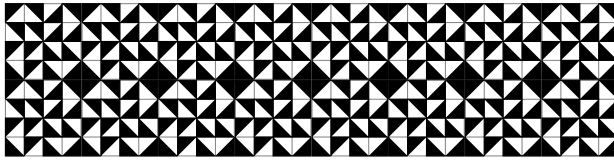


0132



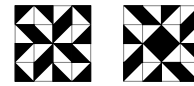
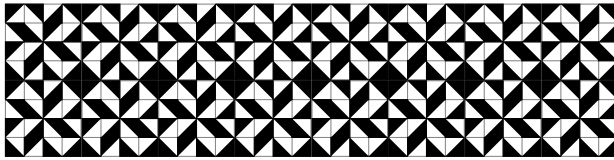
0132

Frieze patterns for family 0111 (secondary, 1110)



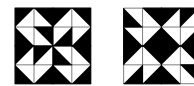
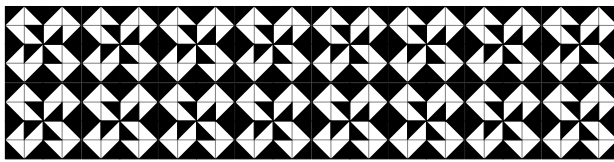
2333

1110



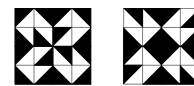
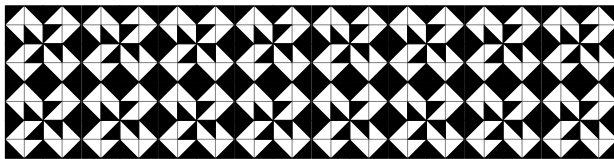
2331

3110



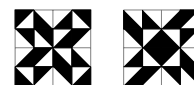
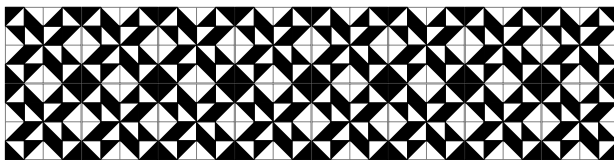
2313

1310



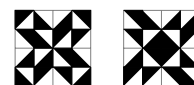
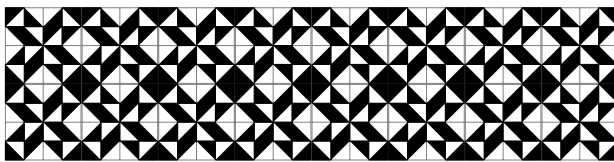
2311

3310



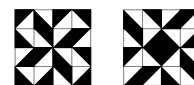
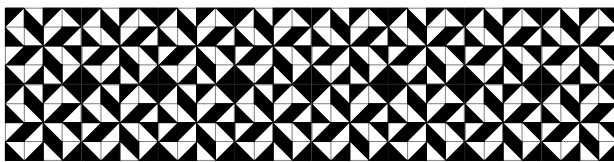
2133

1130



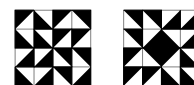
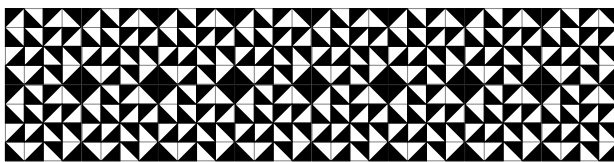
2131

3130



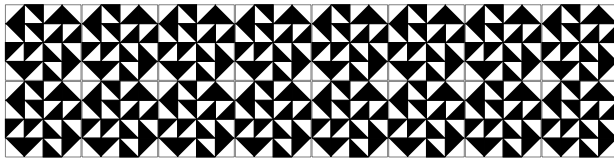
2113

1330



2111

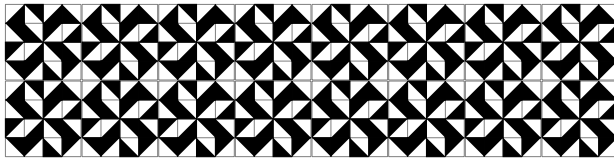
3330



0333



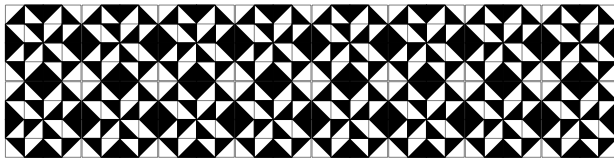
1112



0331



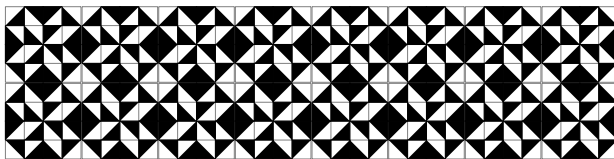
3112



0313



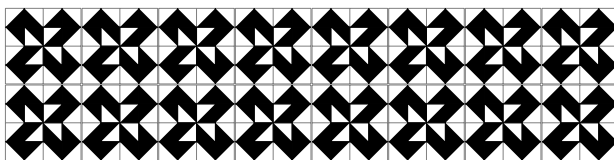
1312



0311



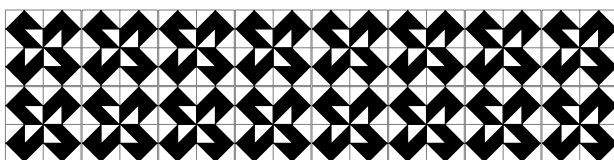
3312



0133



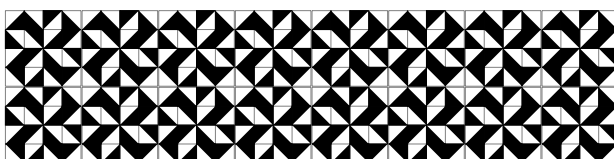
1132



0131



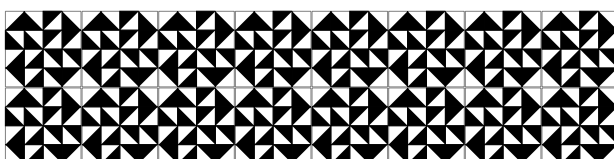
3132



0113



1332

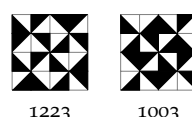
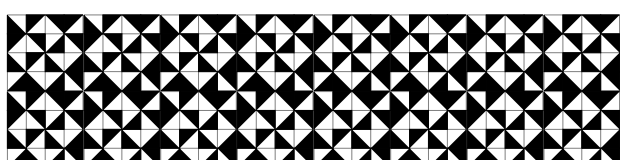
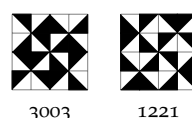
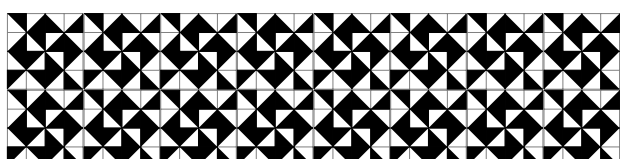
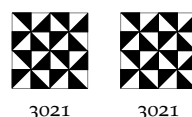
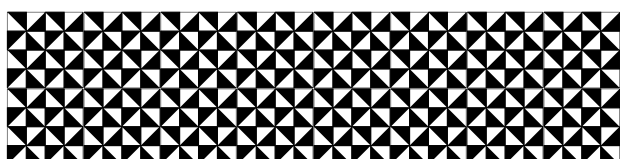
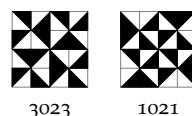
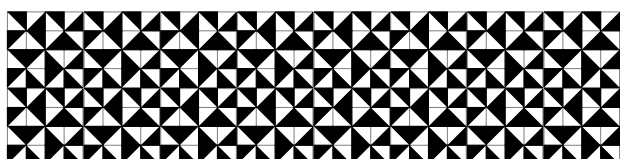
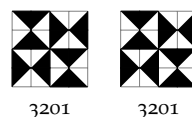
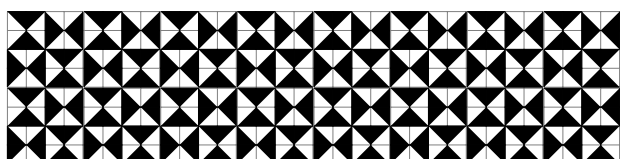
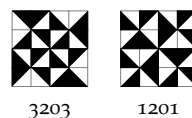
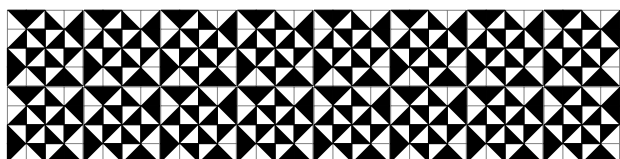
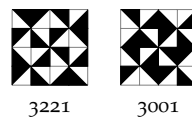
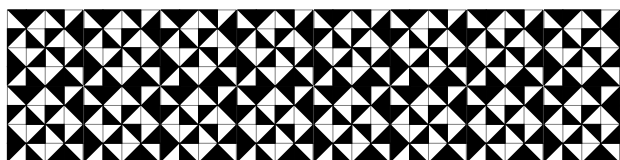
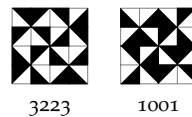
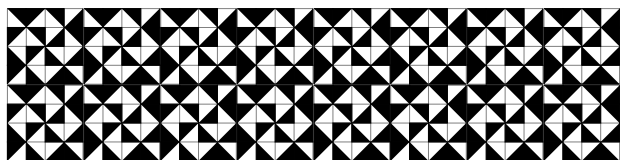


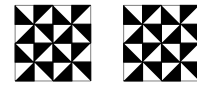
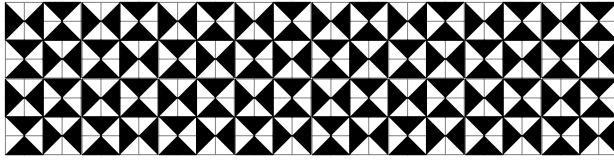
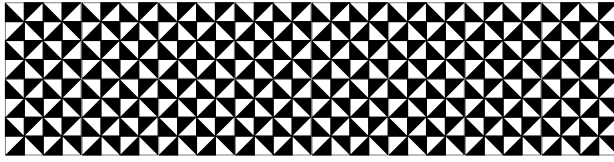
0111



3332

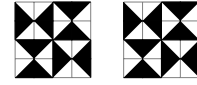
Frieze patterns for family 1001 (secondary, 1001)





1203

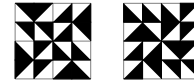
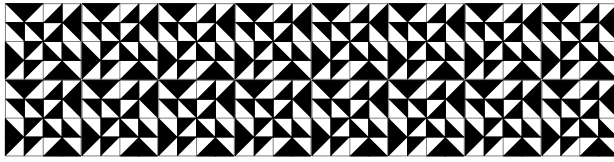
1203



1023

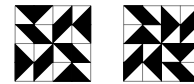
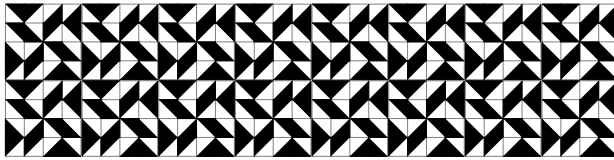
1023

Frieze patterns for family 1011 (secondary, 1101)



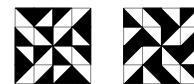
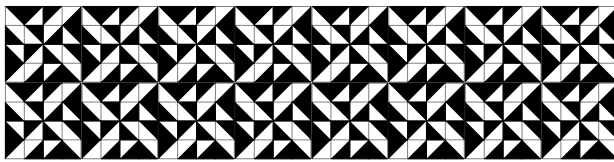
3233

1101



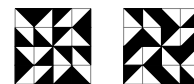
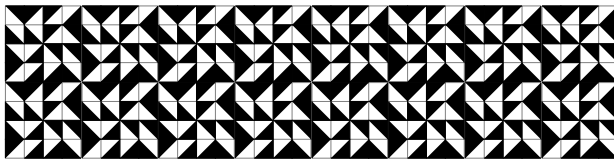
3231

3101



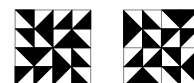
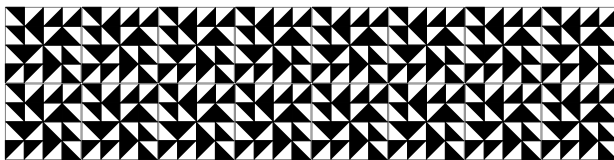
3213

1301



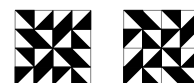
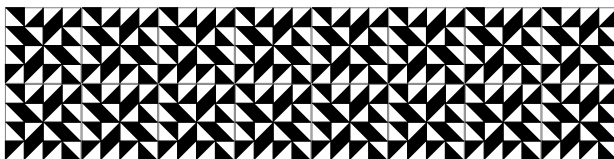
3211

3301



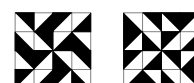
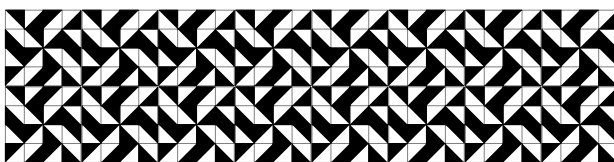
3033

1121



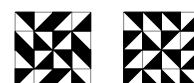
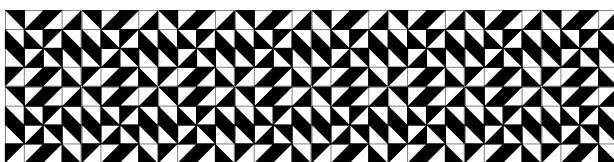
3031

3121



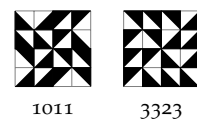
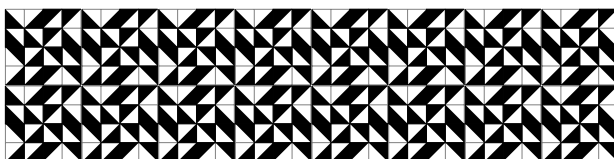
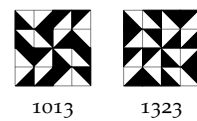
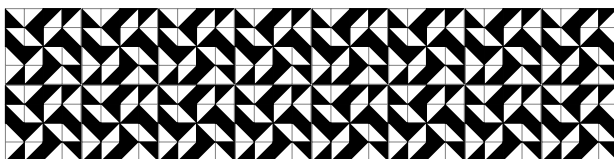
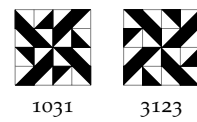
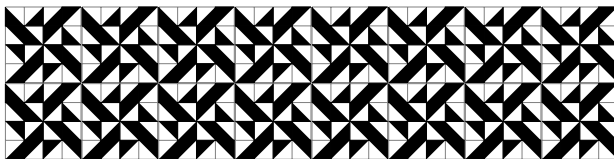
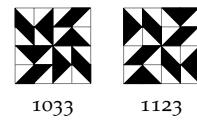
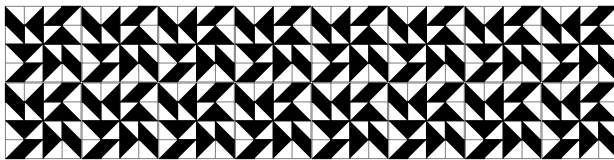
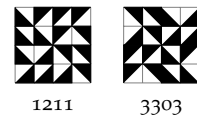
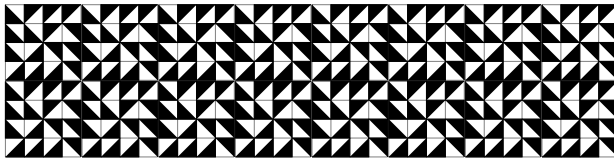
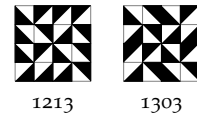
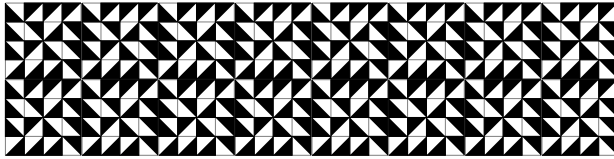
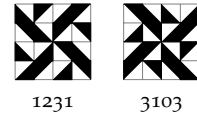
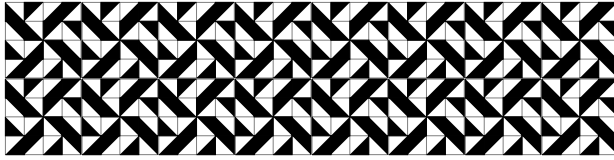
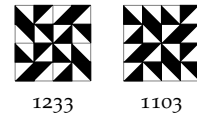
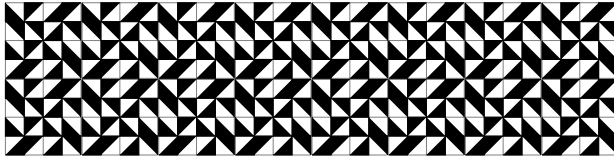
3013

1321

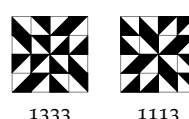
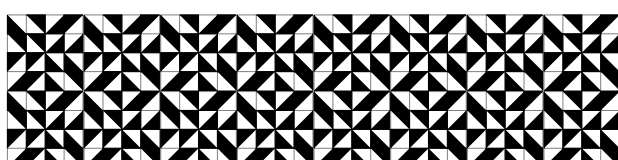
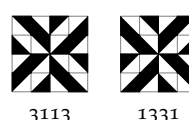
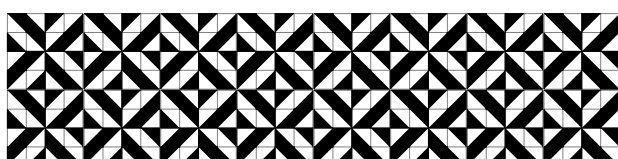
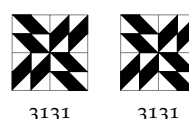
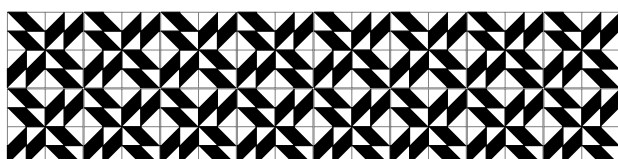
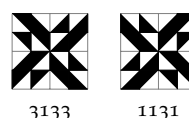
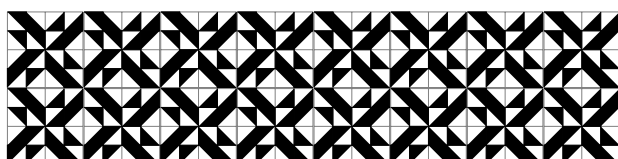
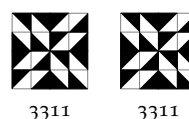
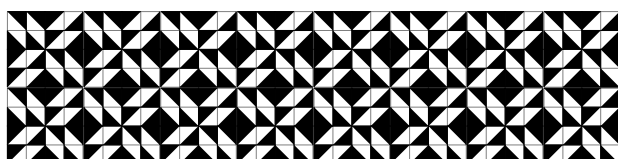
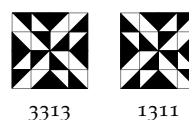
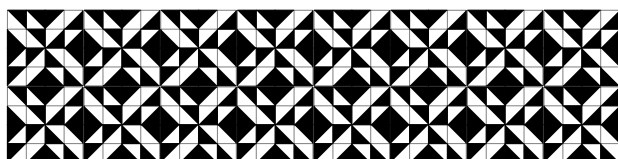
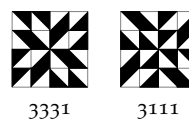
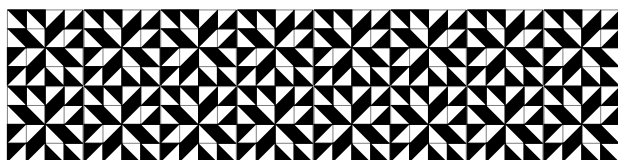
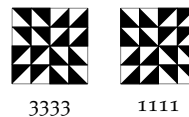
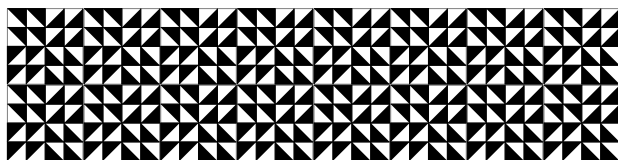


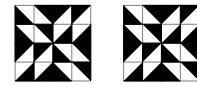
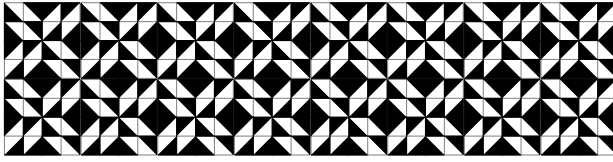
3011

3321



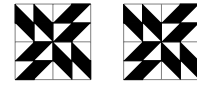
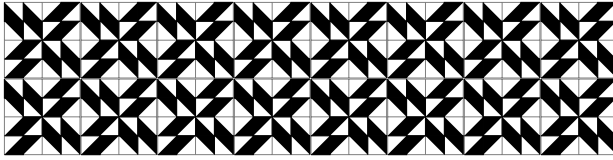
Frieze patterns for family 1111 (secondary, 1111)





1313

1313



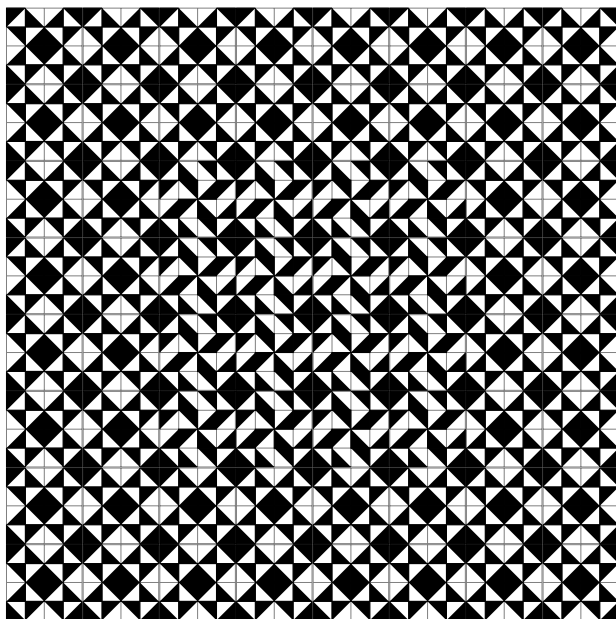
1133

1133

Some designs

This chapter shows some designs made using two types of 4x4 Truchet pattern tiles. One tile type is placed in the center of the design in a 4x4 square (so 16x16 in terms of the original triangle tiles) and a second tile type is used to create a 2-tile wide frame around the central square.

Design using 2123 and 2130

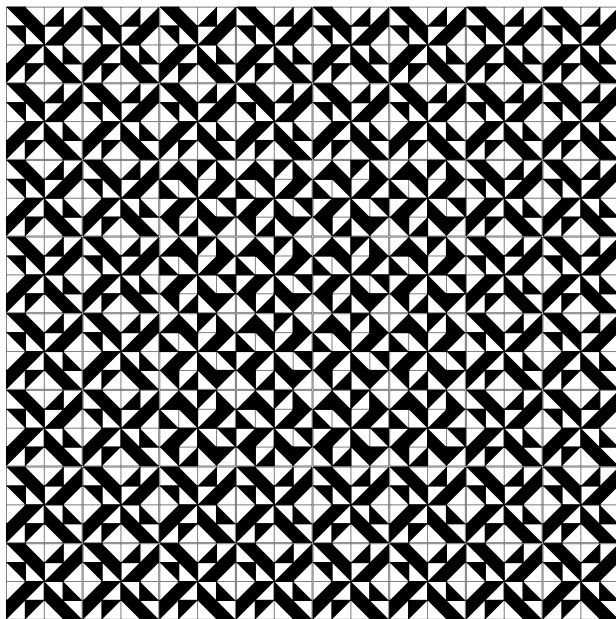


2123



2130

Design using 0113 and 3133

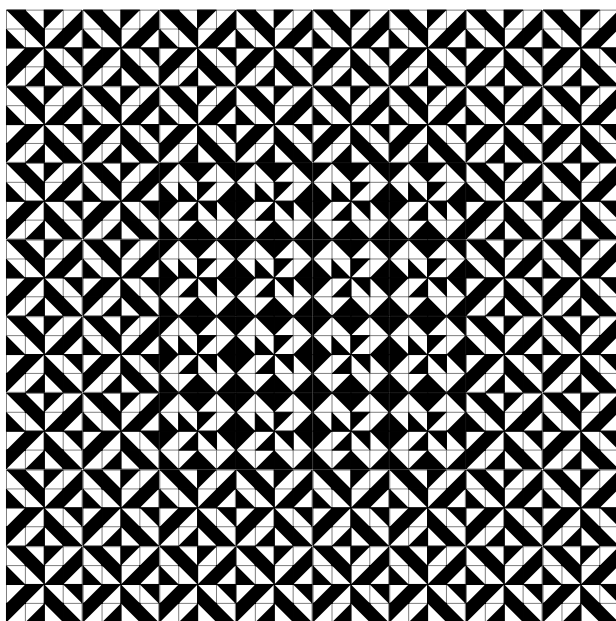


0113



3133

Design using 2311 and 3113

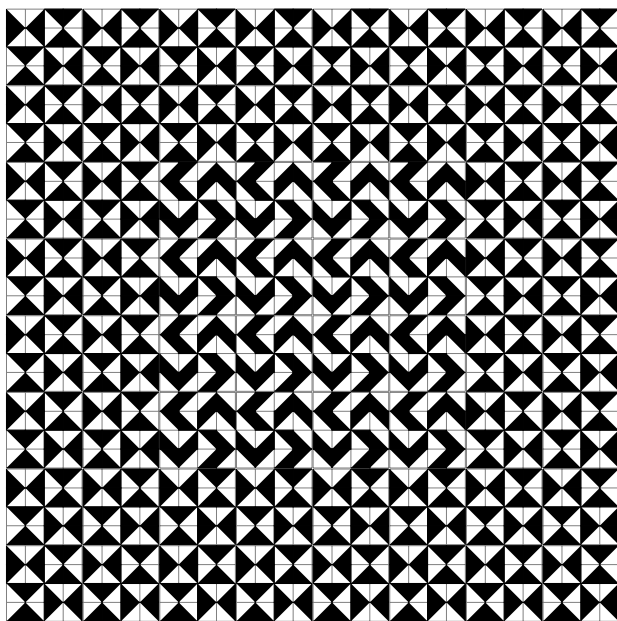


2311



3113

Design using 0231 and 1023

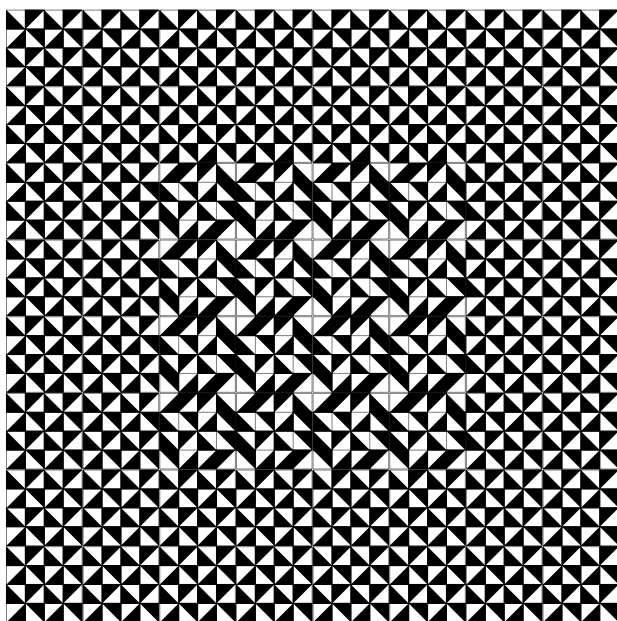


0231



1023

Design using 0213 and 1203

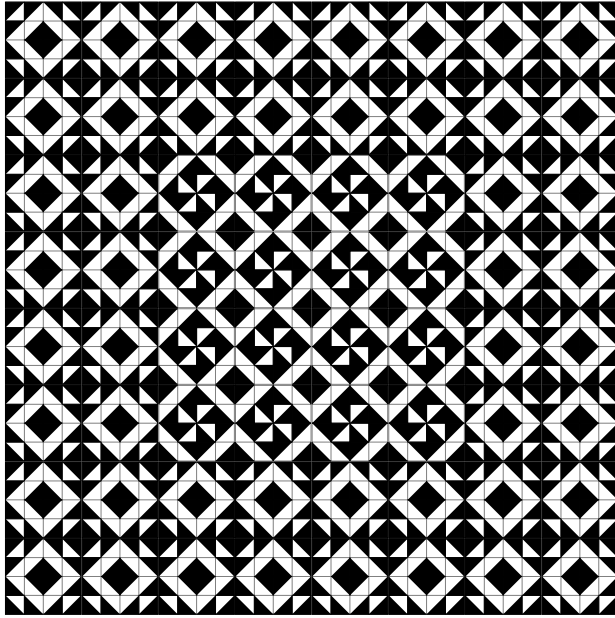


0213



1203

Design using 2003 and 2220

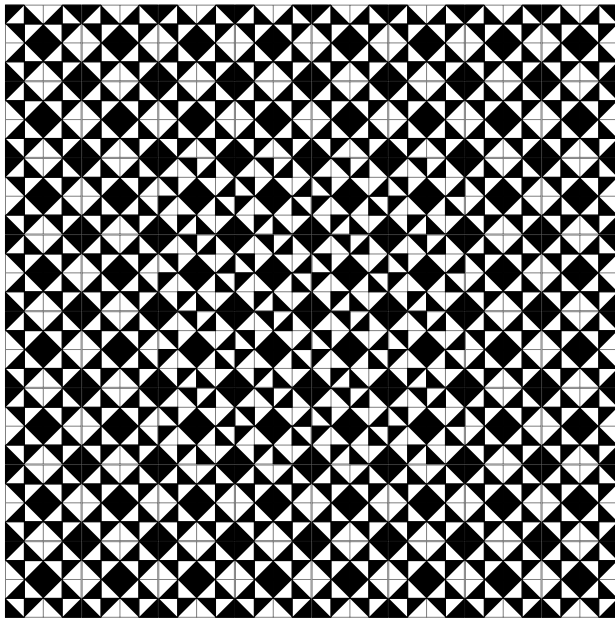


2003



2220

Design using 2330 and 2130

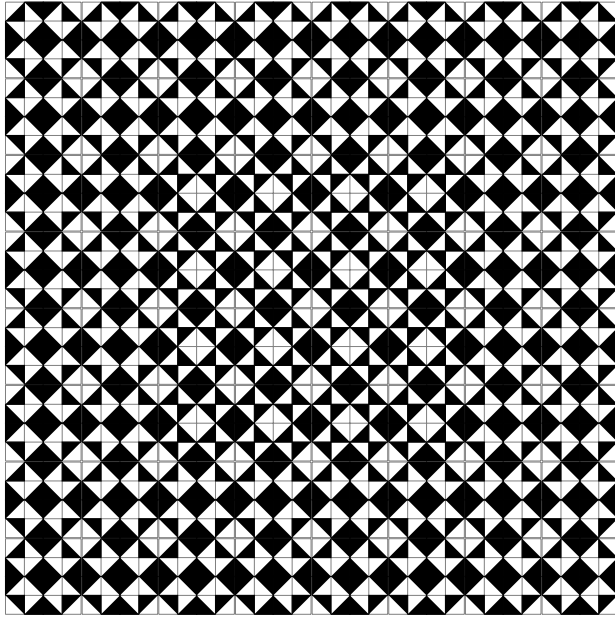


2330



2130

Design using 0312 and 0310

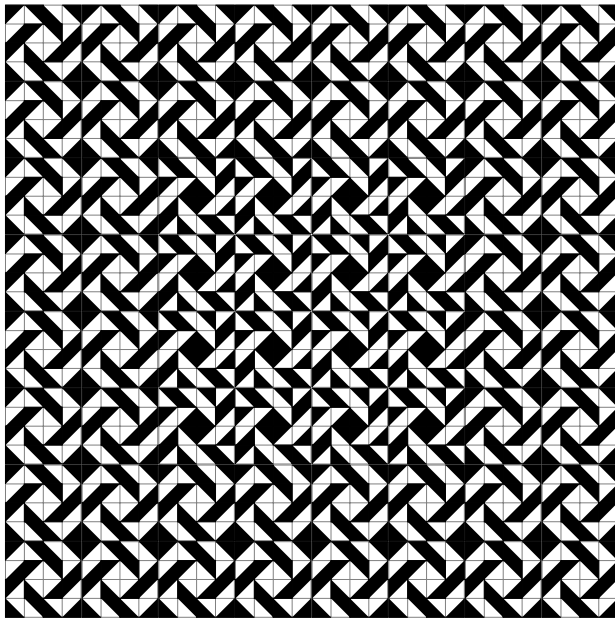


0312



0310

Design using 3120 and 2302

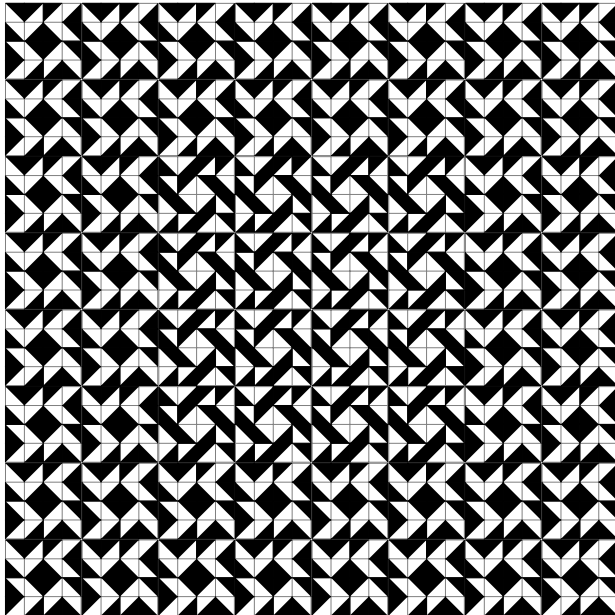


3120



2302

Design using 1012 and 3210

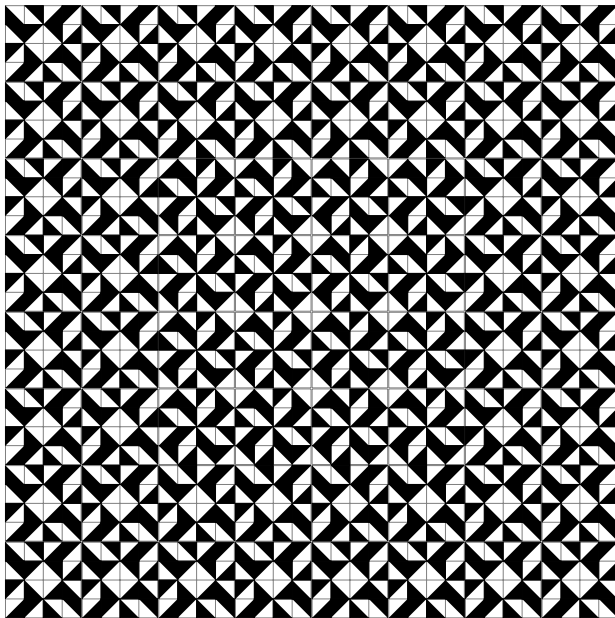


1012



3210

Design using 0113 and 1332



0113



1332

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