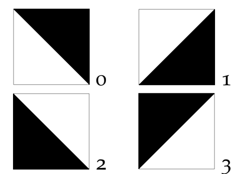
TRUCHET

4×4 patterns with four-fold rotational symmetry

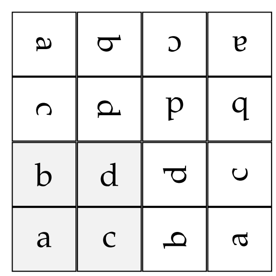
Current printing, February 17, 2025

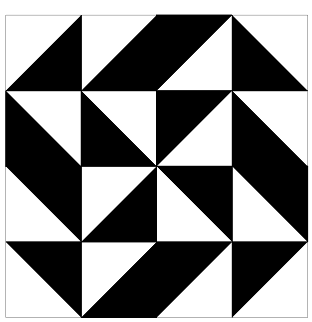
Introduction

Traditionally, Truchet tiles are square tiles that are divided by a diagonal line, and coloured with two colours with a different colour on either side of the diagonal. Each tile can be rotated to one of four positions. Patterns are formed by placing tiles next to each other, rotating individual tiles to create repeated motifs. This booklet presents a complete listing of 4x4 Truchet tile patterns with four-fold (90°) rotational symmetry (256 patterns). Treating these 4x4 tile patterns as tiles themselves allows for larger decorative patterns to be constructed from them. For example, a uniform frieze made from a single 4x4 tile can actually produce interesting secondary patterns which help illustrate some interesting relationships that exist among the tile patterns.



Each 4x4 Truchet tile pattern with rotational symmetry has a core 2x2 pattern in one of its quadrants that is rotated to produce the overall pattern. In this booklet, the core pattern, or prototile, is assumed to be in the lower left. Each pattern can identified as a sequence of 4 digits (a, b, c, d), or more succinctly, *abcd*, that list the rotational positions of each tile in the lower left quadrant. This sequence *abcd* will be referred to as the *signature* of the tile pattern.

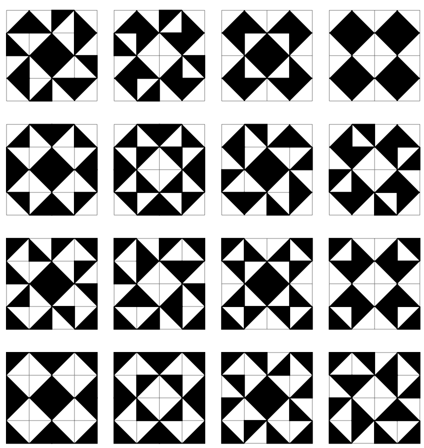




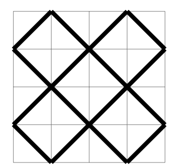
The 0011 pattern

Pattern families

We can group the 4x4 Truchet tile patterns with rotational symmetry into families where tile patterns are considered to be in the same family if they would look the same without colour – if each corresponding tile shares the same diagonal direction. The sequence that represents the family of a tile pattern can be found by taking the sequence of the tile pattern *modulo* 2. So, for example, the 16 tile patterns below are all members of the 0110 family.



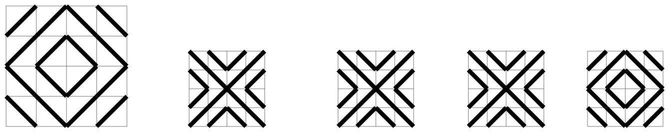
The 0110 pattern family



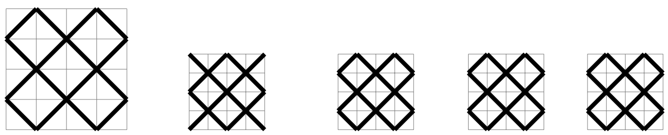
The 0110 family pattern

For a given family, there is corresponding *companion* family, the family of patterns formed by rotating each square in a member of the original family by 90°. There are also two *skew* families, formed by taking the upper left and lower right quadrants of an original family tile pattern as a founding pattern and a *dual* family, formed by taking the upper right quadrant as a founding patterns. A family is always different than its companion, and each family has a distinct companion, but it can happen that skew and duals can coincide. Self-dual families, where the dual family is the same as the original are of particular interest in the frieze patterns of the next chapter.

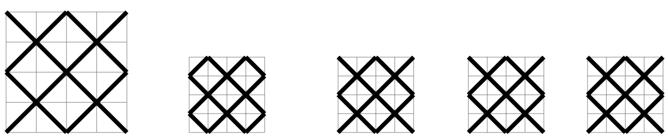
Self-Dual families



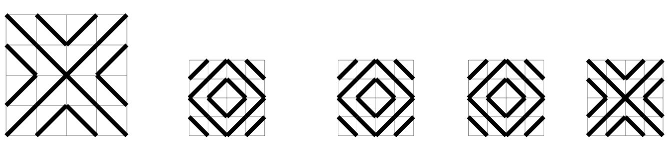
0000	1111	1111	1111	0000
	companion	skew (+)	skew (-)	dual



0110	1001	0110	0110	0110
	companion	skew (+)	skew (-)	dual

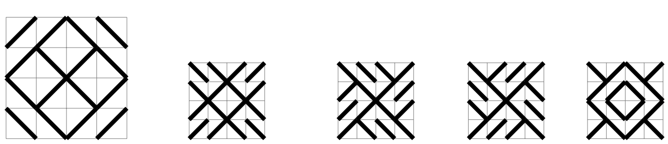


1001	0110	1001	1001	1001
	companion	skew (+)	skew (-)	dual

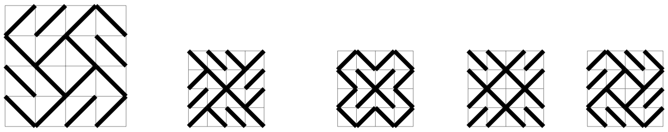


1111	0000	0000	0000	1111
	companion	skew (+)	skew (-)	dual

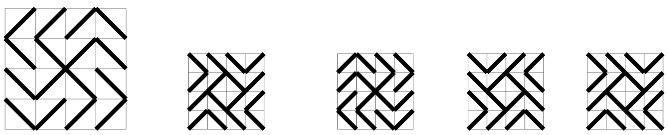
Non self-dual families



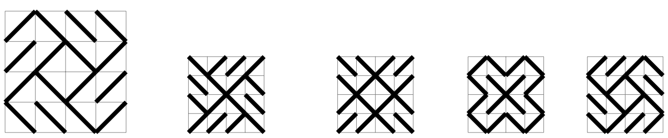
0001	1110	1011	1101	1000
	companion	skew (+)	skew (-)	dual



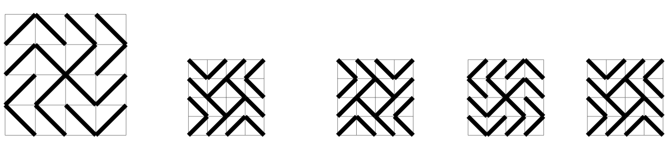
0010	1101	1110	0111	0100
	companion	skew (+)	skew (-)	dual



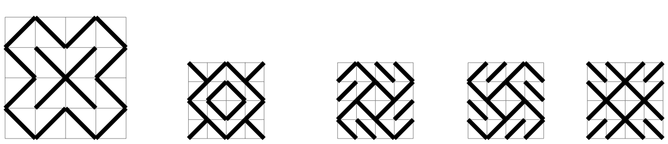
0011	1100	1010	0101	1100
	companion	skew (+)	skew (-)	dual



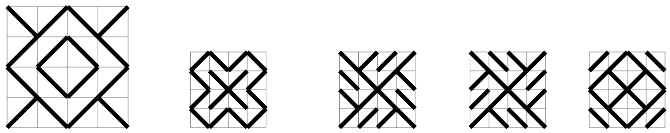
0100	1011	0111	1110	0010
	companion	skew (+)	skew (-)	dual



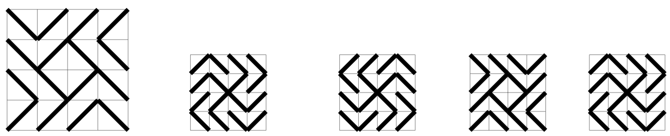
0101	1010	0011	1100	1010
	companion	skew (+)	skew (-)	dual



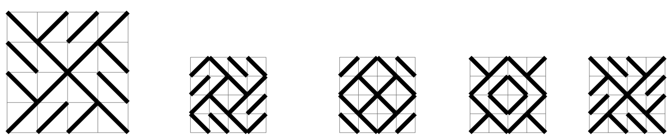
0111	1000	0010	0100	1110
	companion	skew (+)	skew (-)	dual



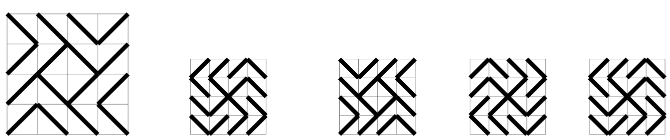
1000	0111	1101	1011	0001
	companion	skew (+)	skew (-)	dual



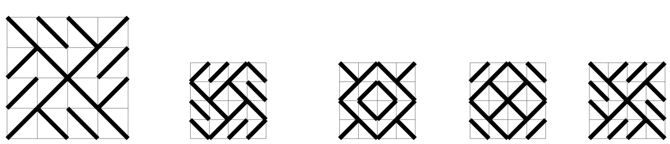
1010	0101	1100	0011	0101
	companion	skew (+)	skew (-)	dual



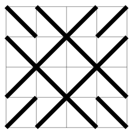
1011	0100	1000	0001	1101
	companion	skew (+)	skew (-)	dual

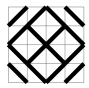


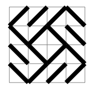
1100	0011	0101	1010	0011
	companion	skew (+)	skew (-)	dual

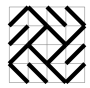


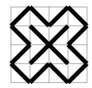
1101	0010	0001	1000	1011
	companion	skew (+)	skew (-)	dual


1110


0001
companion

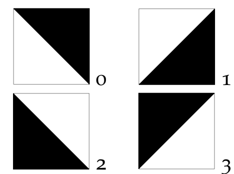

0100
skew (+)


0010
skew (-)


0111
dual

Family and tile pattern mappings

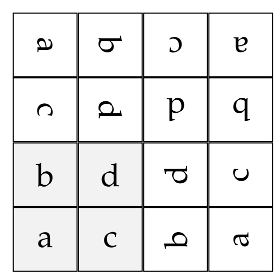
Related families and tiles can be obtained from applying simple mappings on the signature of the tile pattern.



Family mappings

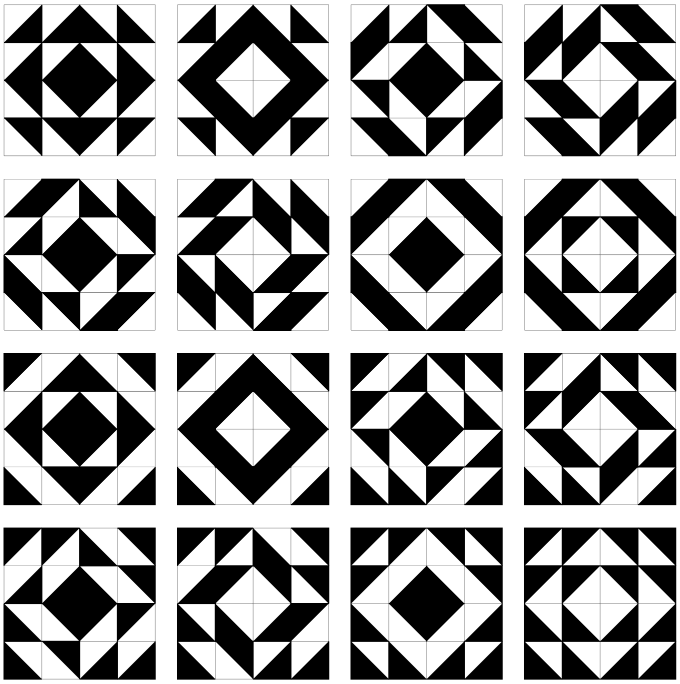
$$\begin{aligned}
\text{companion} &: (a,b,c,d) \mapsto (a+1,b+1,c+1,d+1) \pmod 2;\\
\text{skew+} &: (a,b,c,d) \mapsto (c+1,a+1,d+1,b+1) \pmod 2;\\
\text{reverse} &: (a,b,c,d) \mapsto (d,c,b,a) \pmod 2;\\
\text{skew}- &: (a,b,c,d) \mapsto (b+1,d+1,a+1,c+1) \pmod 2;
\end{aligned}$$

Tile pattern mappings

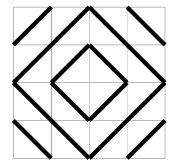


$$\begin{aligned}
\text{skew+} &: (a,b,c,d) \mapsto (c+1,a+1,d+1,b+1) \pmod 4;\\
\text{dual} &: (a,b,c,d) \mapsto (d+2,c+2,b+2,a+2) \pmod 4;\\
\text{skew}- &: (a,b,c,d) \mapsto (b+3,d+3,a+3,c+3) \pmod 4;\\
\text{opposite} &: (a,b,c,d) \mapsto (a+2,b+2,c+2,d+2) \pmod 4;
\end{aligned}$$

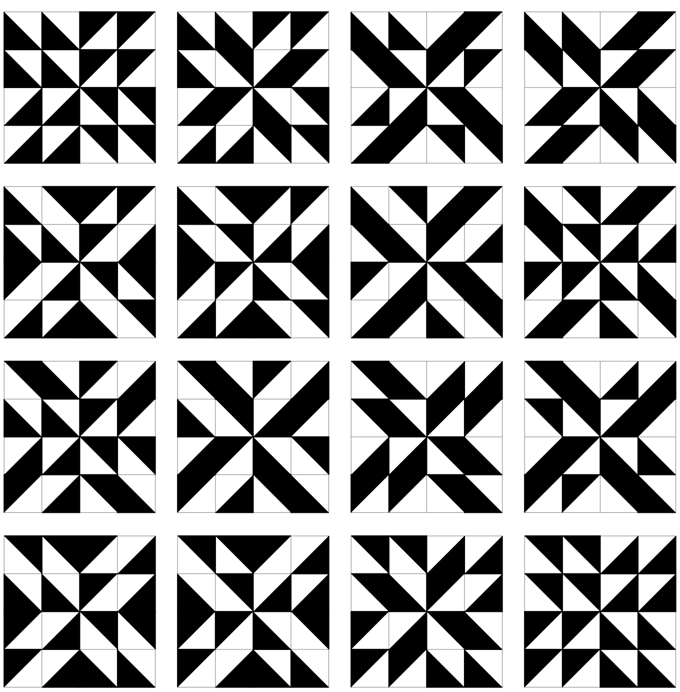
On the following pages each family will be shown along with its corresponding *companion* family, the family of patterns formed by rotating each square in a member of the original family by 90°.



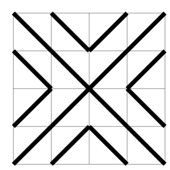
0000



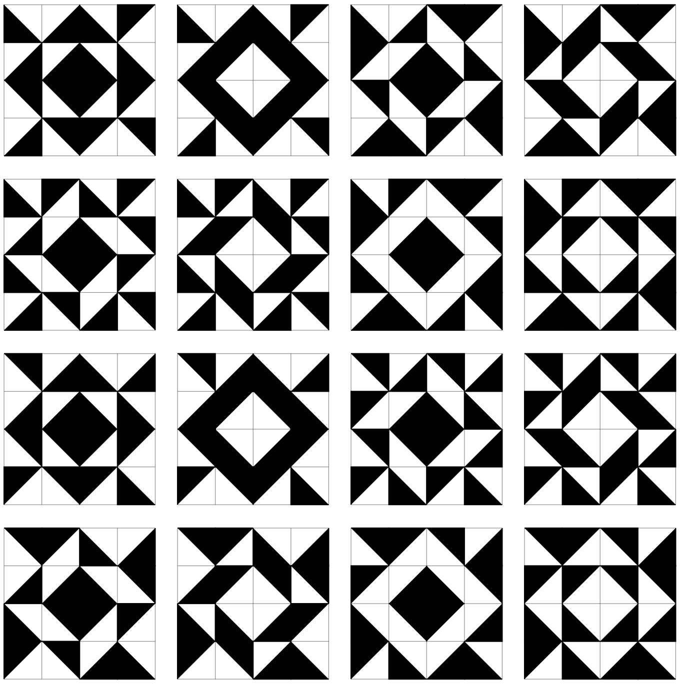
0000	0002	0020	0022
0200	0202	0220	0222
2000	2002	2020	2022
2200	2202	2220	2222



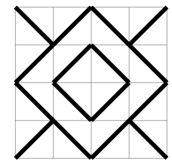
1111



1111	1113	1131	1133
1311	1313	1331	1333
3111	3113	3131	3133
3311	3313	3331	3333



1000



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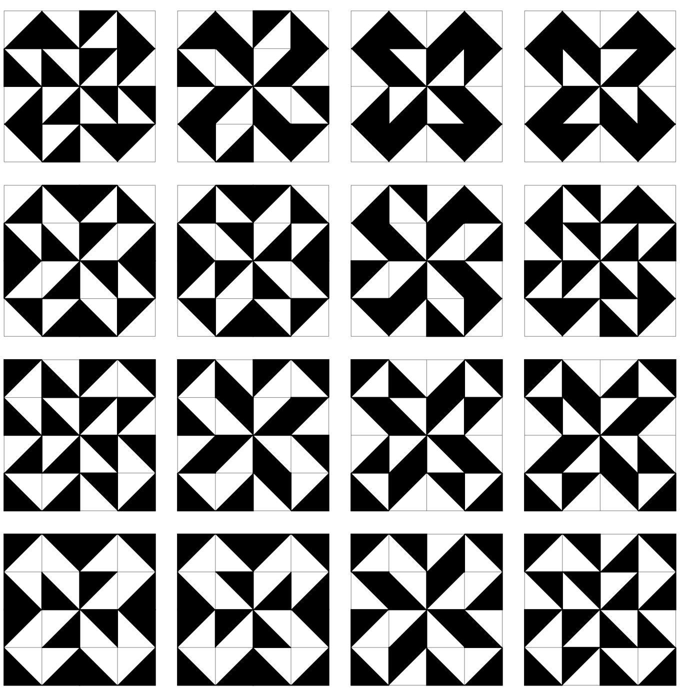
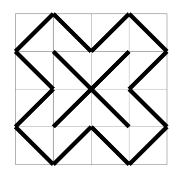
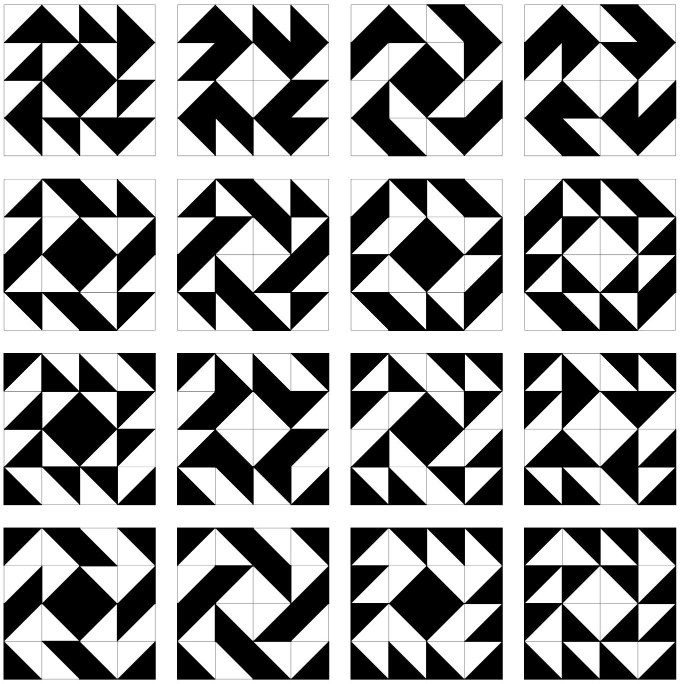
1200	1202	1220	1222
3000	3002	3020	3022
3200	3202	3220	3222



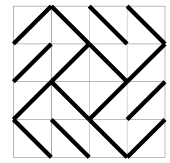
0111



0111	0113	0131	0133
0311	0313	0331	0333
2111	2113	2131	2133
2311	2313	2331	2333



0100



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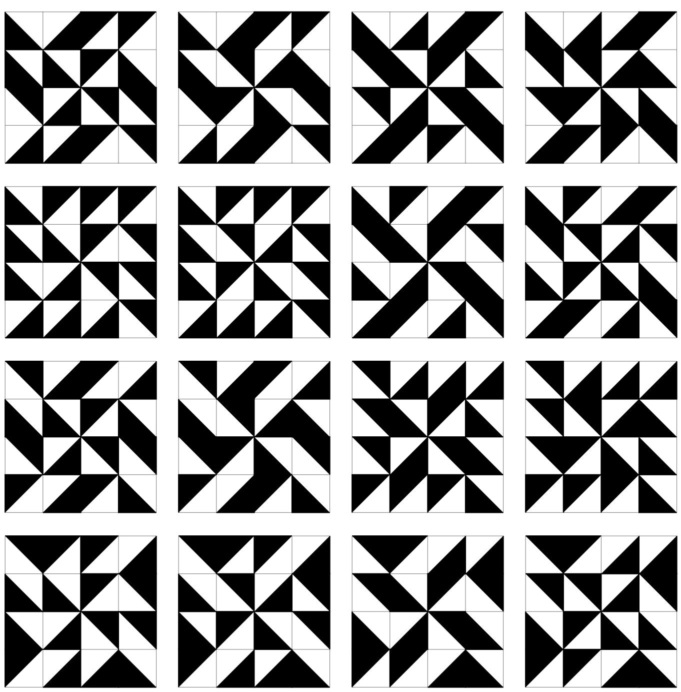
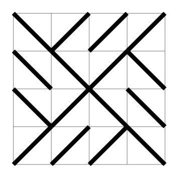
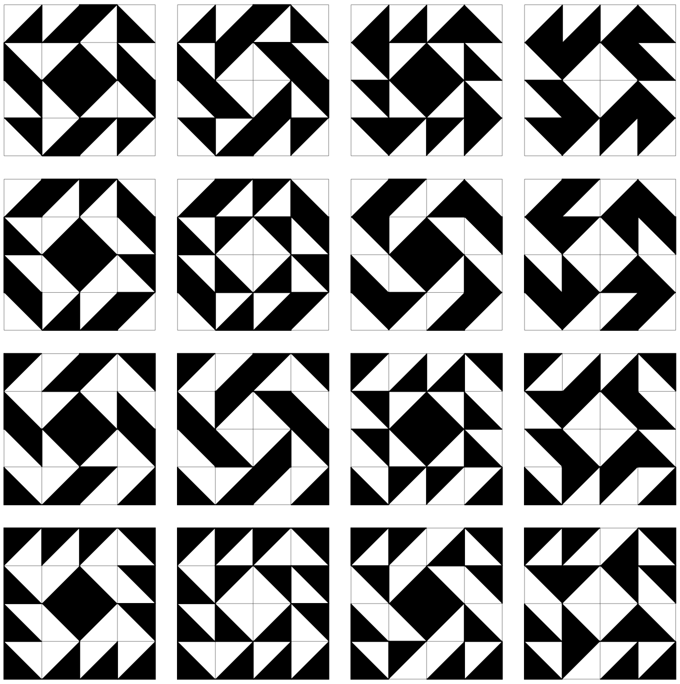
0300	0302	0320	0322
2100	2102	2120	2122
2300	2302	2320	2322



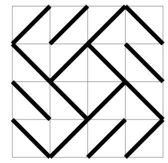
1011



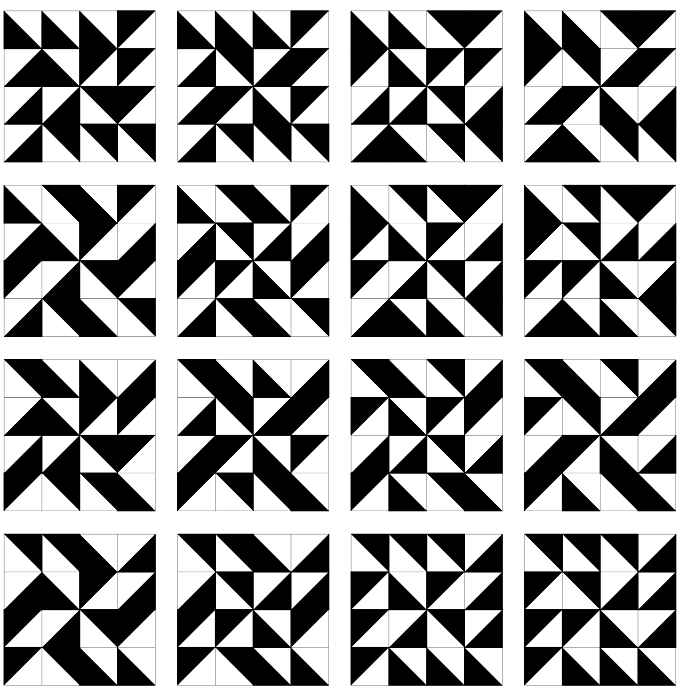
1011	1013	1031	1033
1211	1213	1231	1233
3011	3013	3031	3033
3211	3213	3231	3233



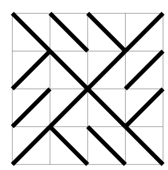
0010



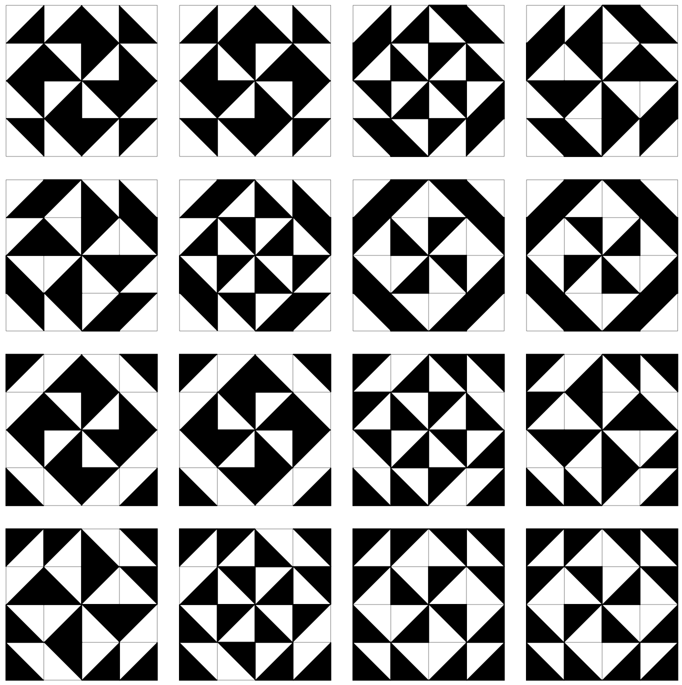
0010	0012	0030	0032
0210	0212	0230	0232
2010	2012	2030	2032
2210	2212	2230	2232



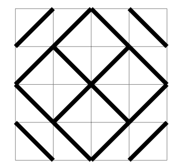
1101



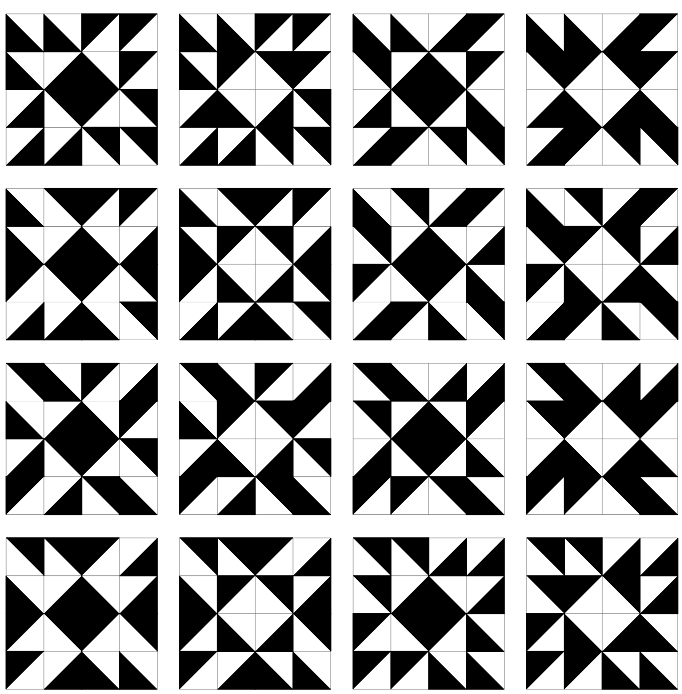
1101	1103	1121	1123
1301	1303	1321	1323
3101	3103	3121	3123
3301	3303	3321	3323



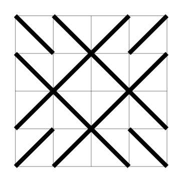
0001



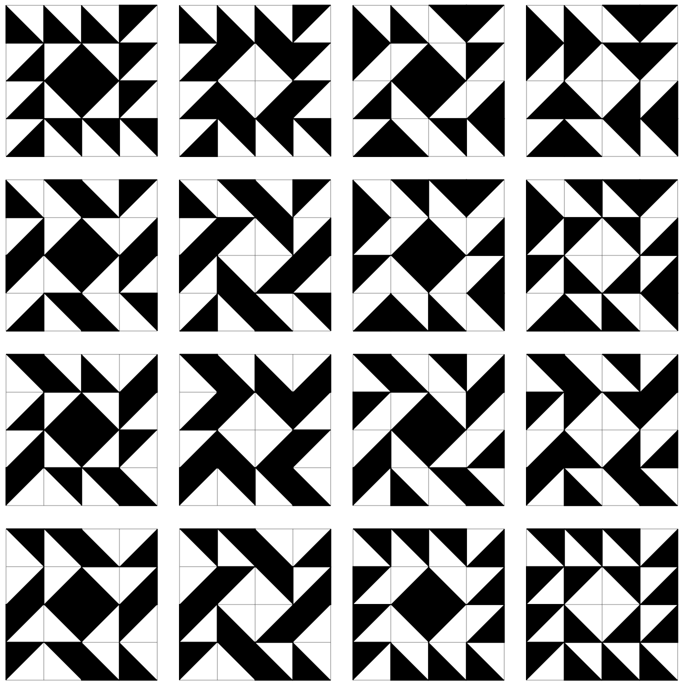
0001	0003	0021	0023
0201	0203	0221	0223
2001	2003	2021	2023
2201	2203	2221	2223



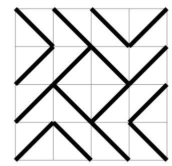
1110



1110	1112	1130	1132
1310	1312	1330	1332
3110	3112	3130	3132
3310	3312	3330	3332



1100



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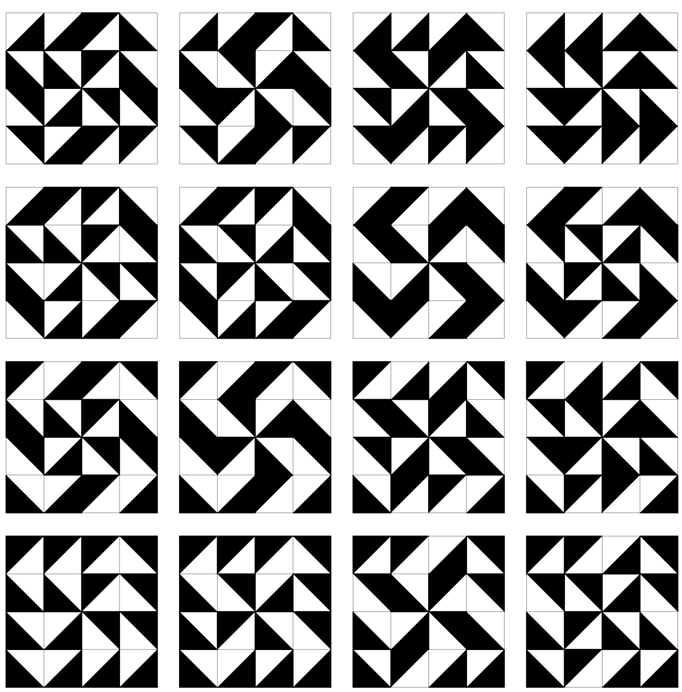
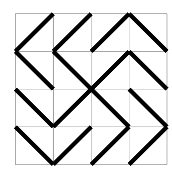
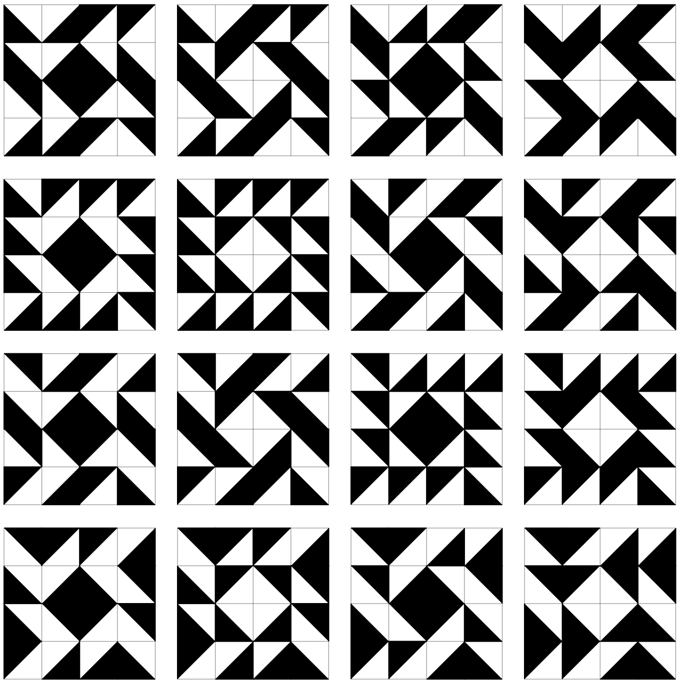
1300	1302	1320	1322
3100	3102	3120	3122
3300	3302	3320	3322



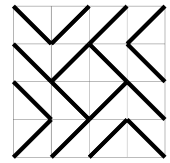
0011



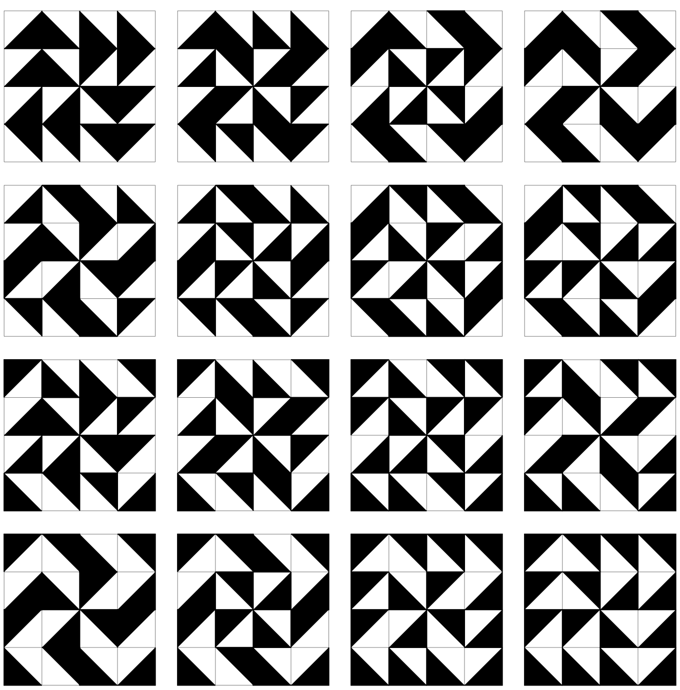
0011	0013	0031	0033
0211	0213	0231	0233
2011	2013	2031	2033
2211	2213	2231	2233



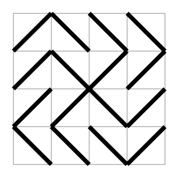
1010



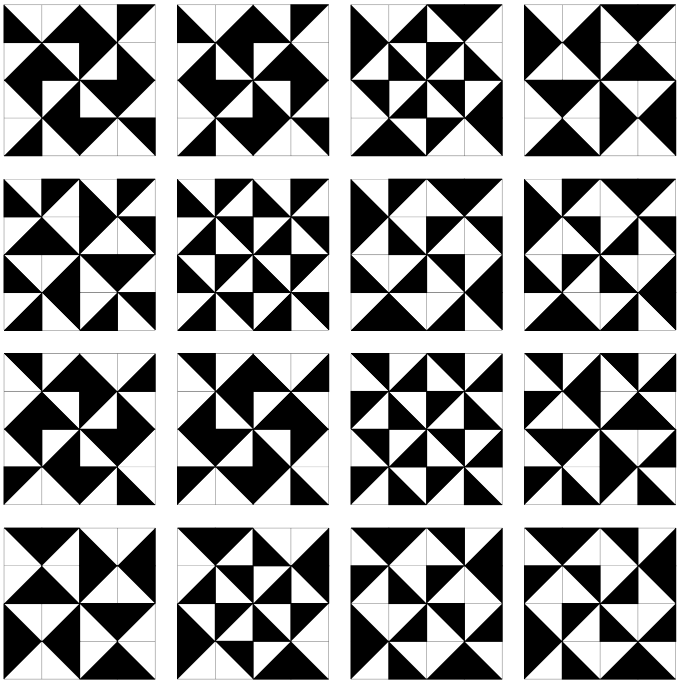
1010	1012	1030	1032
1210	1212	1230	1232
3010	3012	3030	3032
3210	3212	3230	3232



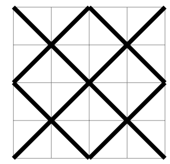
0101



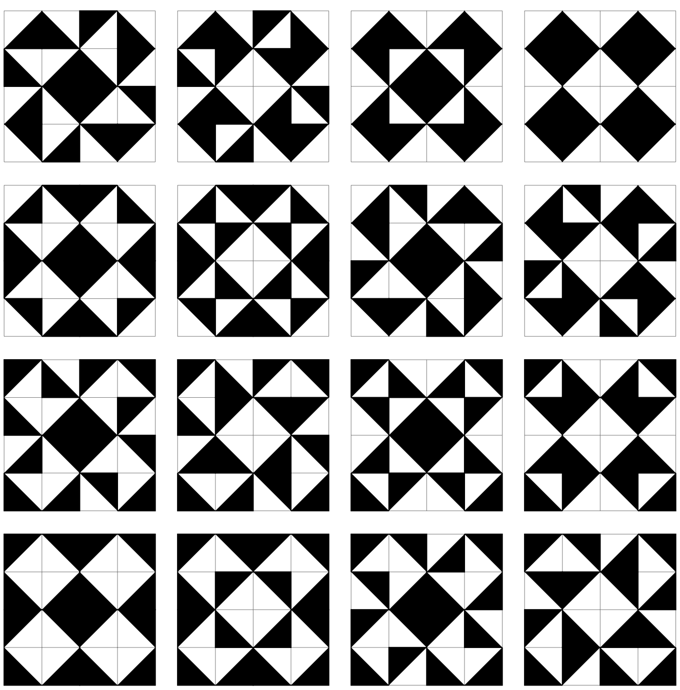
0101	0103	0121	0123
0301	0303	0321	0323
2101	2103	2121	2123
2301	2303	2321	2323



1001



1001	1003	1021	1023
1201	1203	1221	1223
3001	3003	3021	3023
3201	3203	3221	3223



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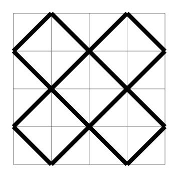
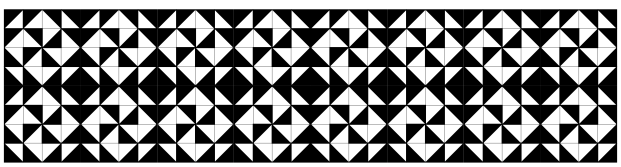


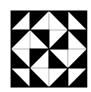
0110	0112	0130	0132
0310	0312	0330	0332
2110	2112	2130	2132
2310	2312	2330	2332

Uniform friezes

Each 4*x*4 Truchet pattern can be treated like a tile and used in a larger pattern. A uniform *frieze* is a horizontal strip of the same tile pattern repeated. Friezes of 4*x*4 Truchet pattern tiles with rotational symmetry can be quite striking, and have some interesting characteristics.

A frieze of more than one row of a primary tile reveals a secondary tile pattern that appears as another horizontal strip of 4*x*4 Truchet tile patterns nestled between the rows of primary tiles. Below, a frieze of 2223 tiles has a secondary pattern of 1000 tiles.





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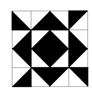


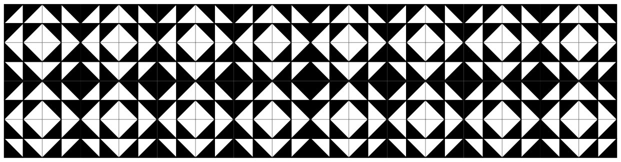
1000

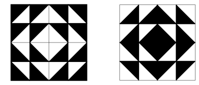
The secondary tile in a frieze pattern is the pattern that has been referred to previously as the *dual* of the original pattern. The dual of a tile pattern is the pattern formed by taking the top right quadrant of the original tile as the prototile of the new tile.

Some tiles are self-dual, and frieze patterns formed by self-dual tiles show a much more uniform pattern, as the extra rows of tiles seemingly nestled between the rows of the original tile are made up of the same original tile. Friezes of self-dual tiles have a third *tertiary* tile pattern with four-fold rotational symmetry that appears to overlap between adjacent tiles of the original tile. These tertiary tile patterns are the *skew* of the original tile pattern. Some self-dual friezes are also self-skew, leading to even more uniform patterns.

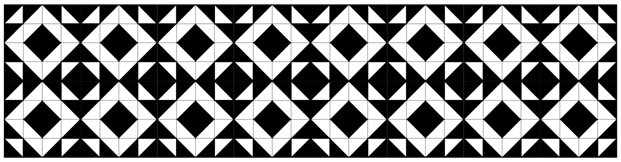
We can consider the uniform friezes formed by the dual tiles as the same pattern. There are 6 pairs of families where the original and dual are not the same, and these pairs of families yield 16 patterns each. The 4 remaining families contain some self-dual patterns, and some patterns that are *opp-dual* (the secondary tile is the opposite tile of the original), also reducing the number of patterns. These 4 remaining families provide 10 distinct frieze patterns each. This means that the 256 tile patterns generate 136 distinct friezes.

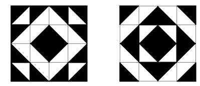
Frieze patterns for family 0000 (secondary, 0000)





2222 0000





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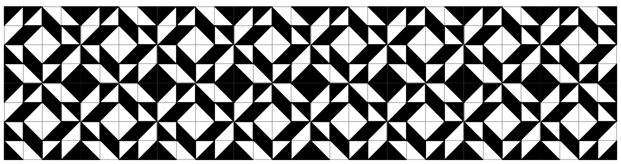
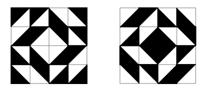
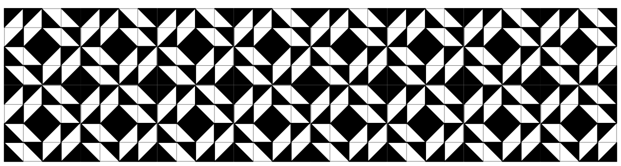
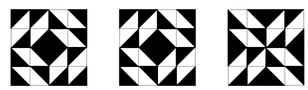




2202 0200





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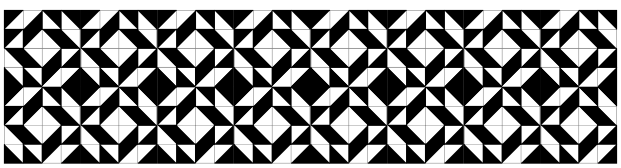

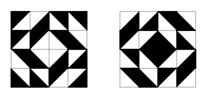

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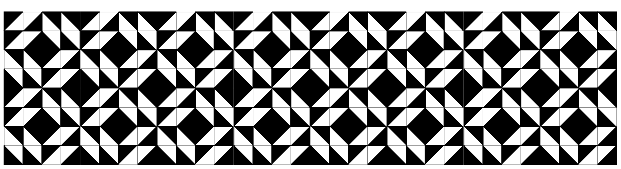

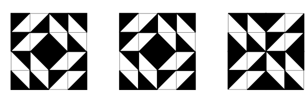

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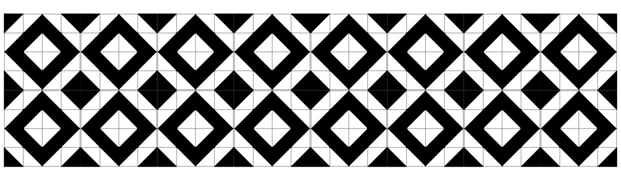

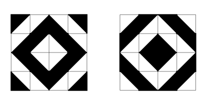

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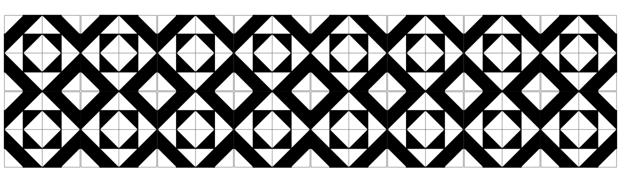

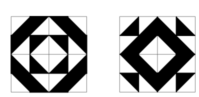

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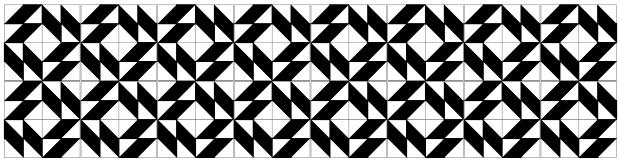

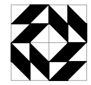
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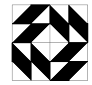
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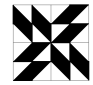
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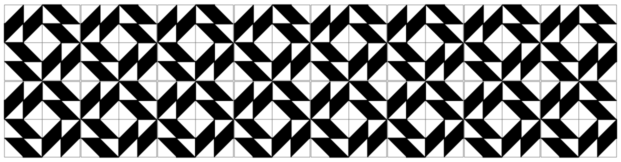

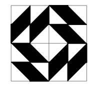
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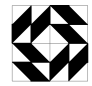
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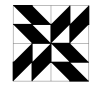
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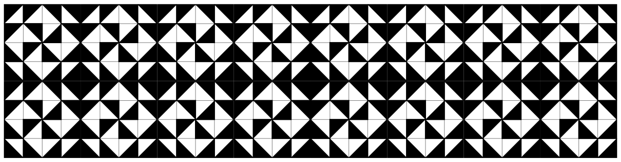
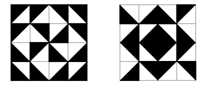
Frieze patterns for family 0001 (secondary, 1000)





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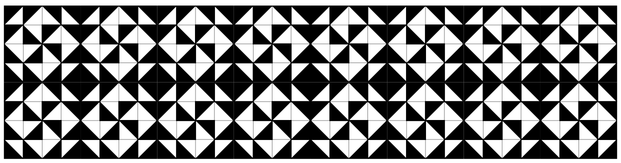

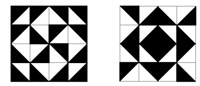

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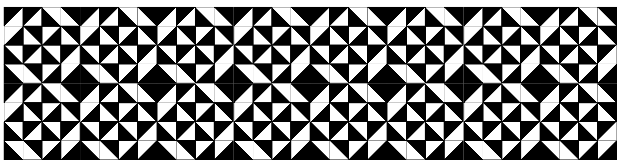

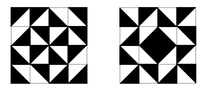

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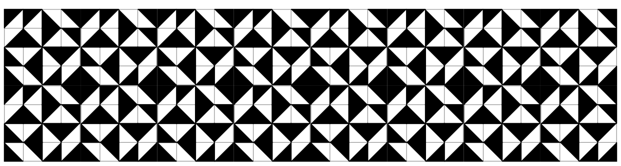

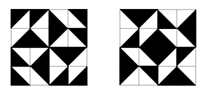

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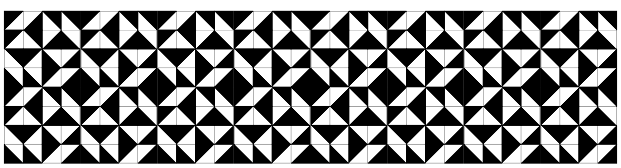

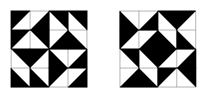

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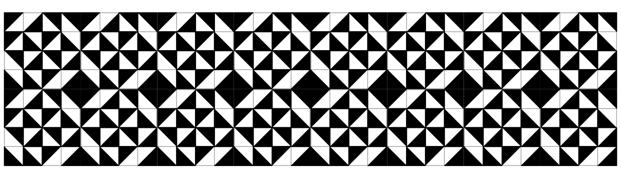

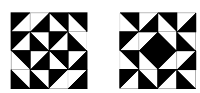

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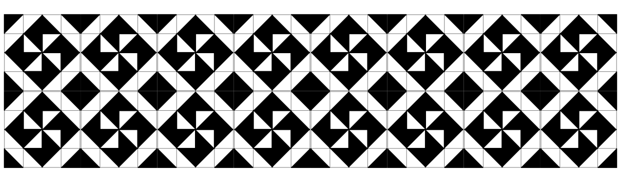

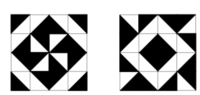

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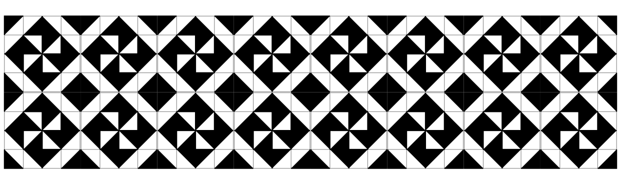

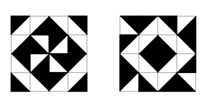

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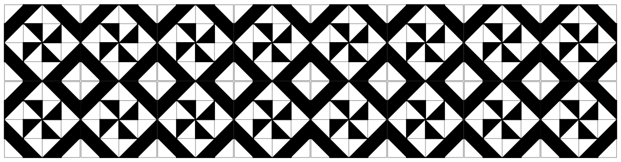

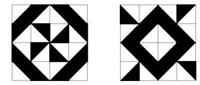

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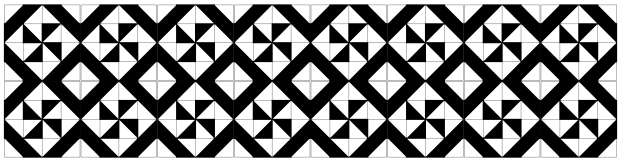

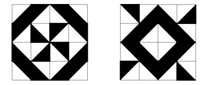

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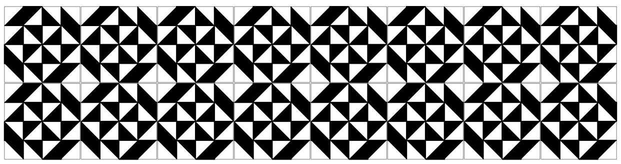

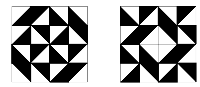

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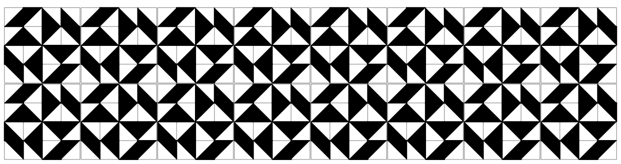

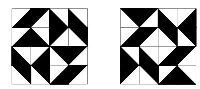

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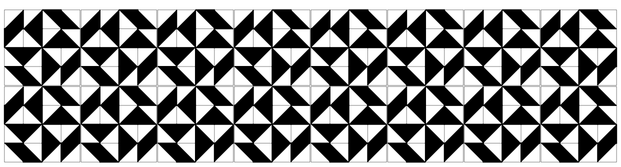
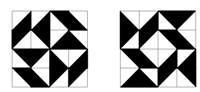
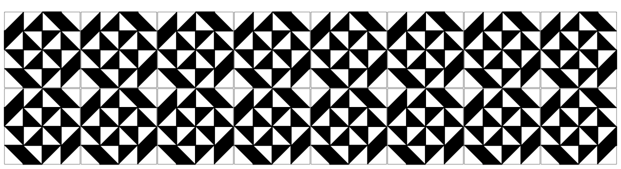
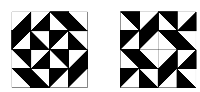




0023 1022





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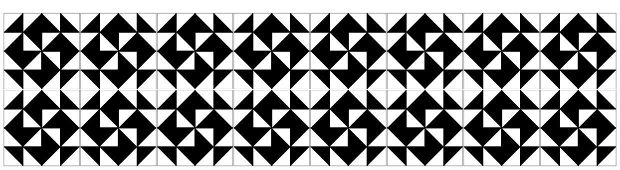

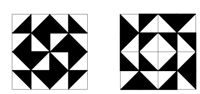

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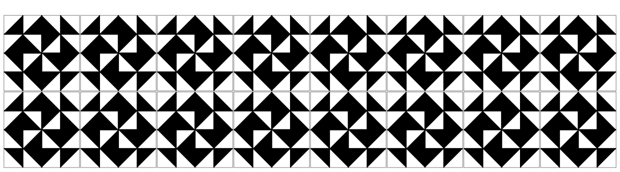
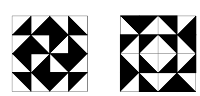
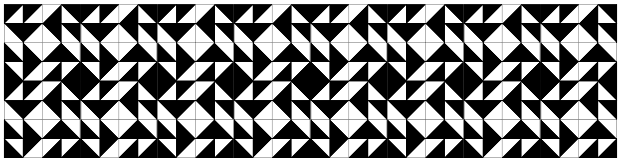
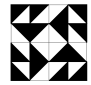




0001 3222

Frieze patterns for family 0010 (secondary, 0100)

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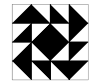
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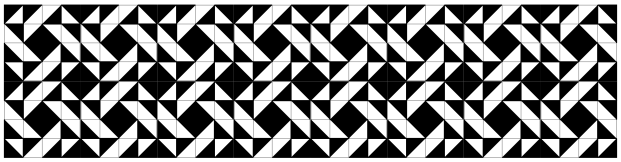

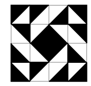
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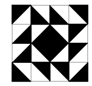
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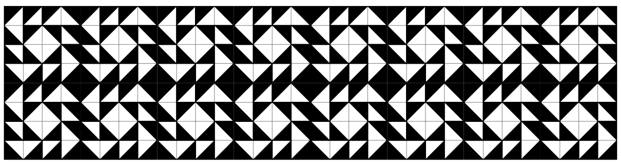

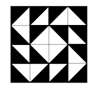
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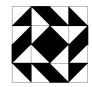
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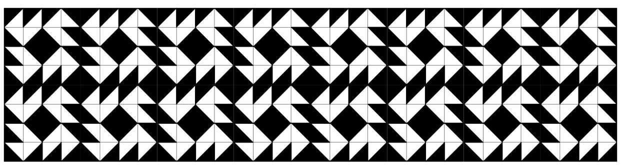

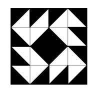
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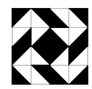
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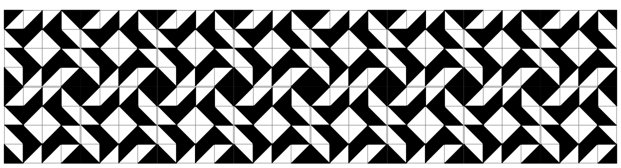

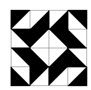
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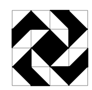
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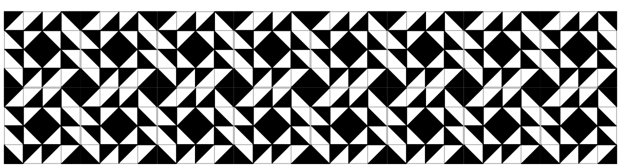

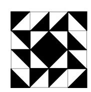
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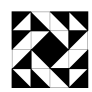
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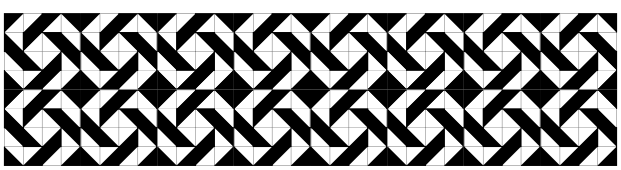

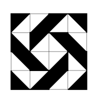
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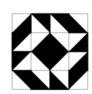
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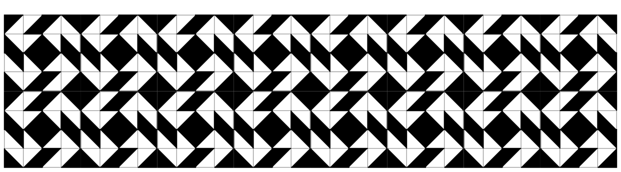

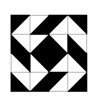
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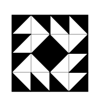
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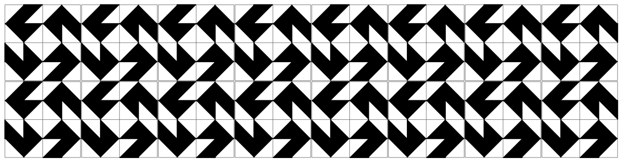

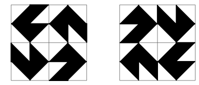

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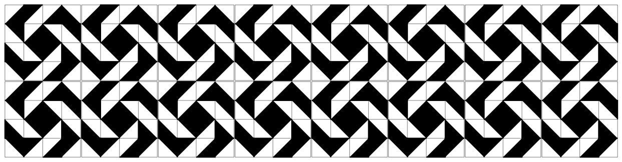

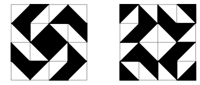

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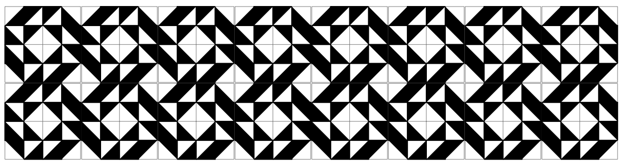

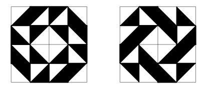

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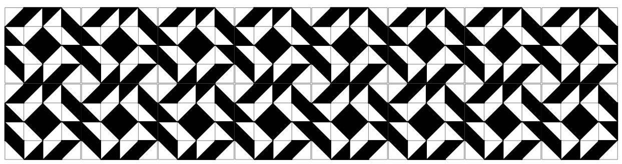

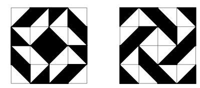

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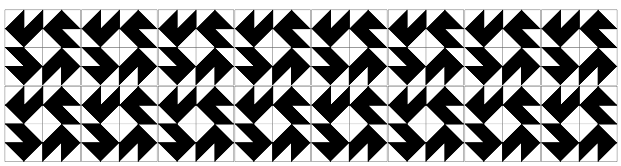

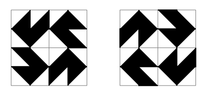

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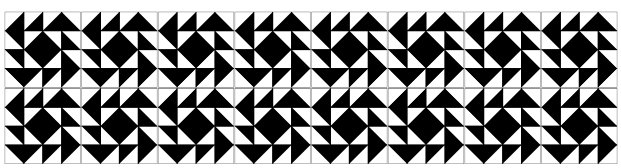

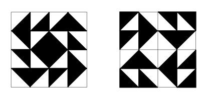

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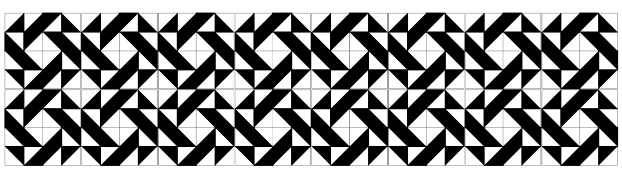

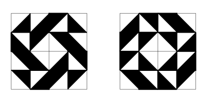

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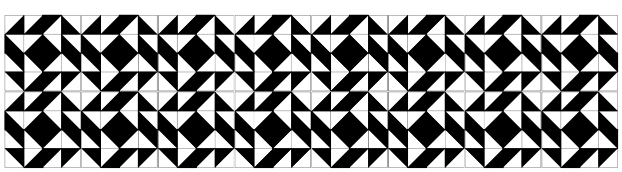
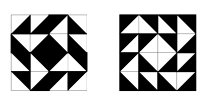
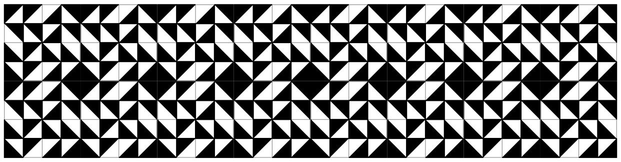
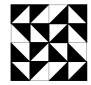




0010 2322

Frieze patterns for family 0011 (secondary, 1100)





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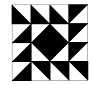

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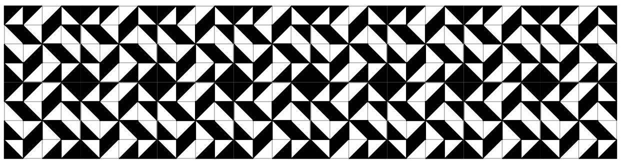

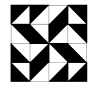

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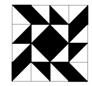

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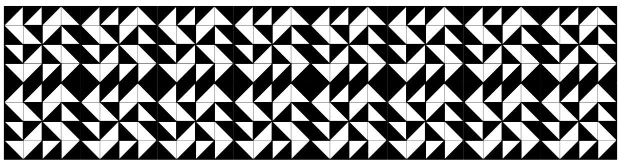

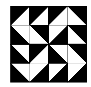

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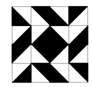

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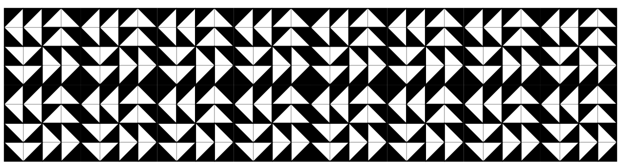

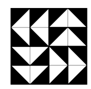

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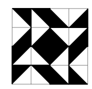

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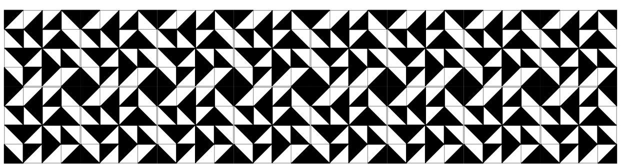

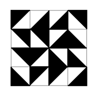

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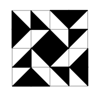

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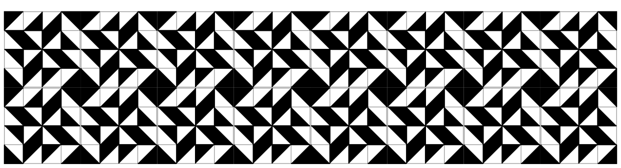

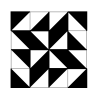

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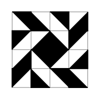

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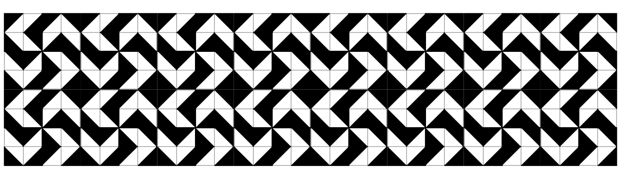

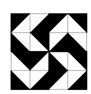

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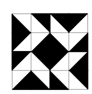

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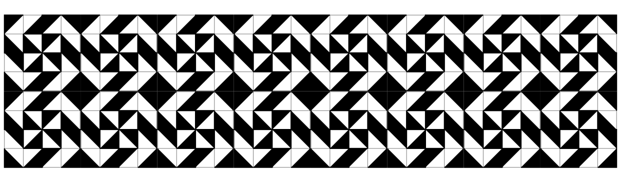

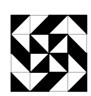

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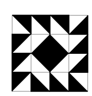

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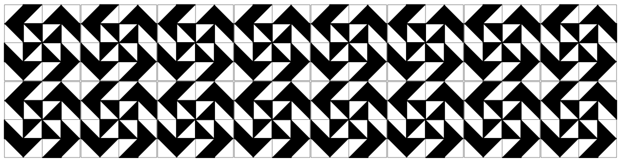

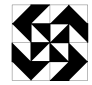
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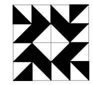
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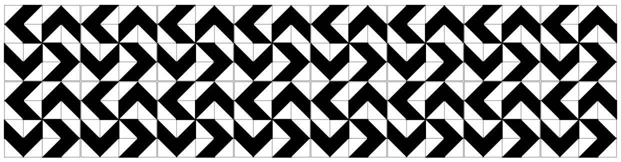

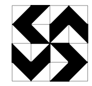
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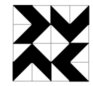
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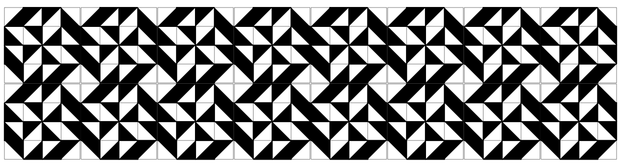

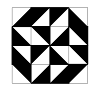
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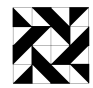
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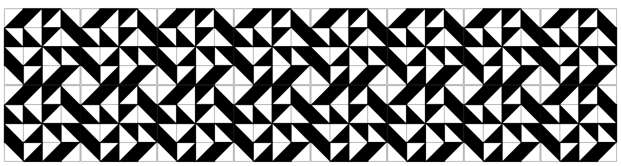

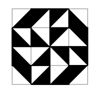
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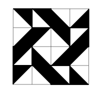
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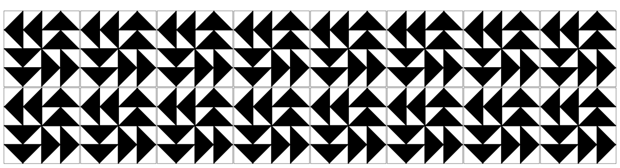

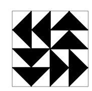
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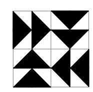
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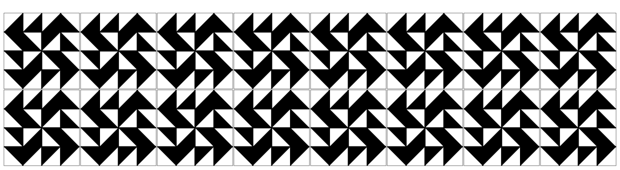

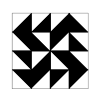
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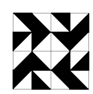
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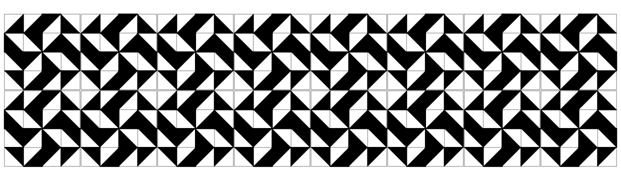

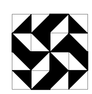
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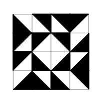
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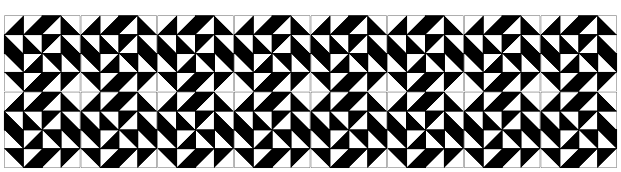

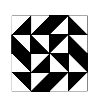
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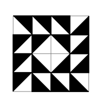
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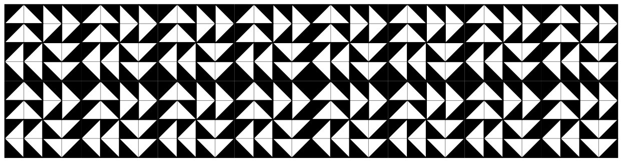
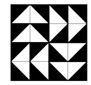
Frieze patterns for family 0101 (secondary, 1010)





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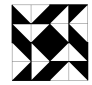

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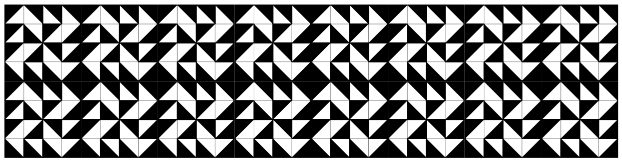

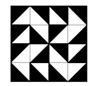

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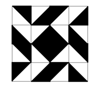

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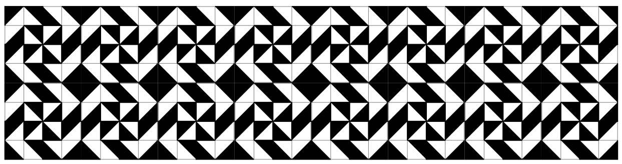

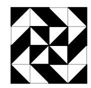

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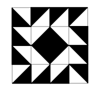

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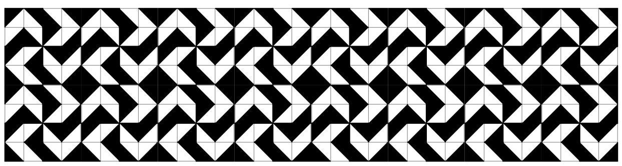

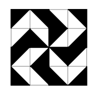

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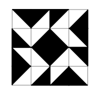

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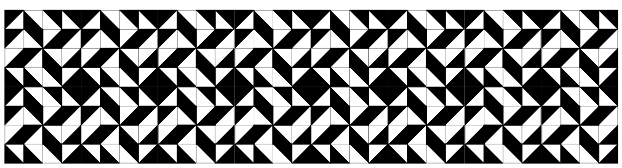

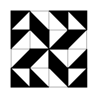

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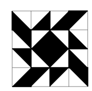

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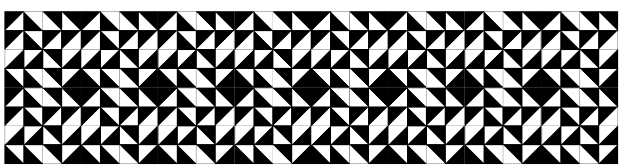

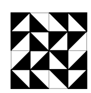

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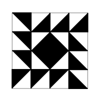

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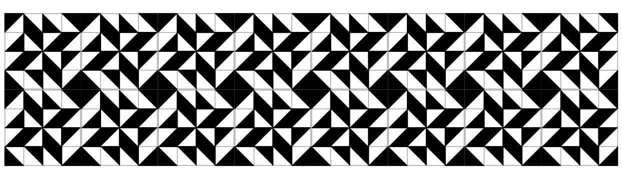

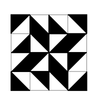

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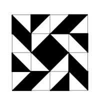

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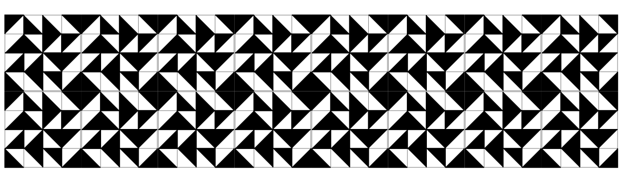

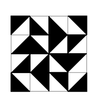

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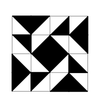

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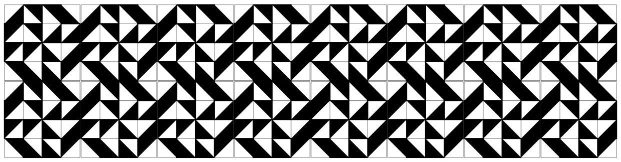
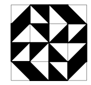
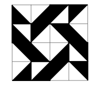

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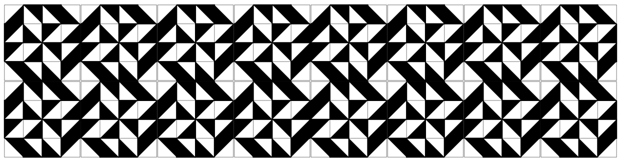
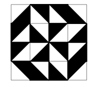
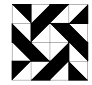

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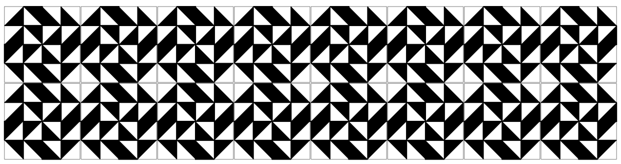
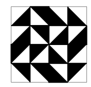
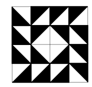

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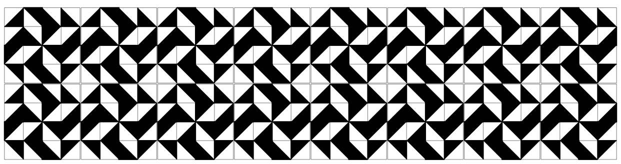
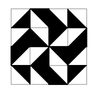
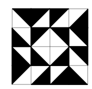

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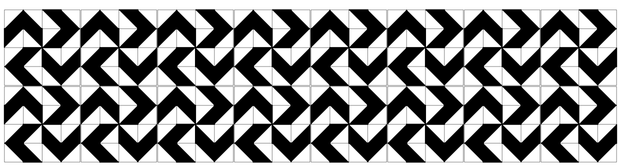
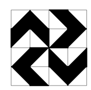
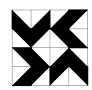

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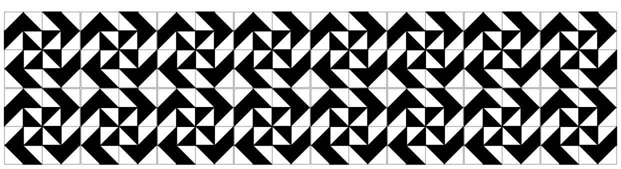
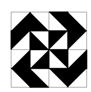
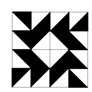

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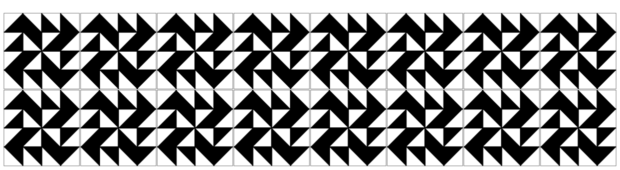
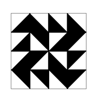
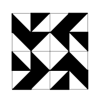

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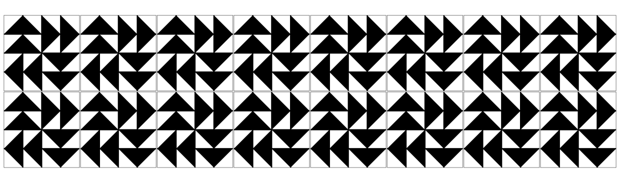
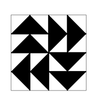
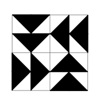
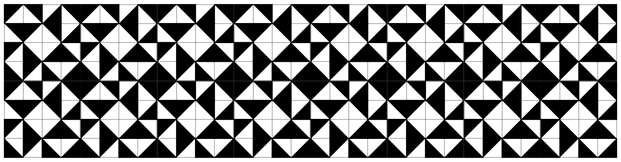
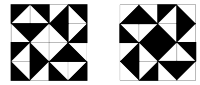




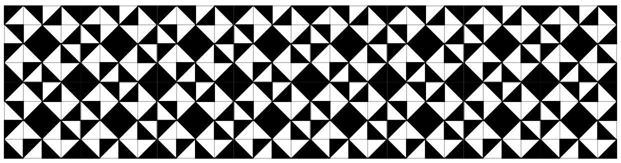
0101 3232

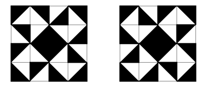
Frieze patterns for family 0110 (secondary, 0110)



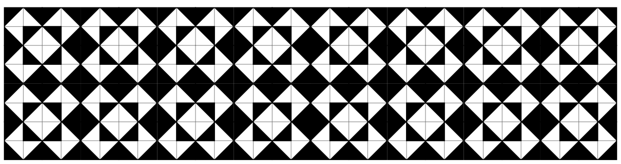


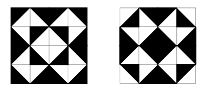
2332 0110



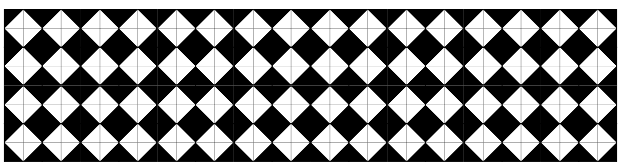


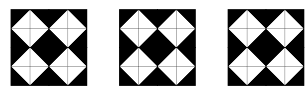
2330 2110



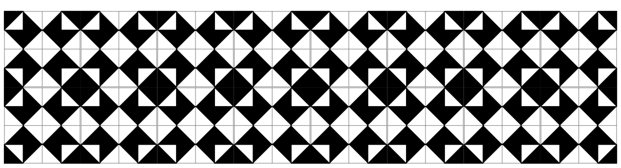


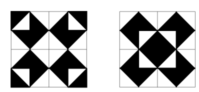
2312 0310



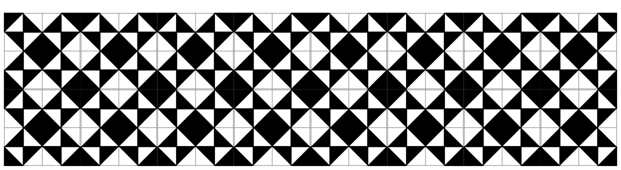


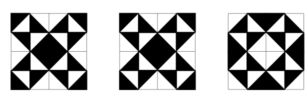
2310 2310 2310



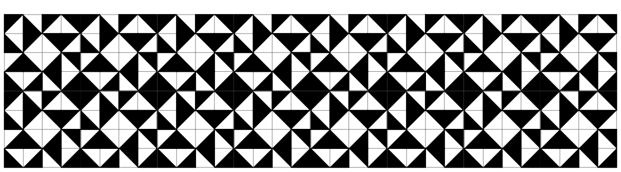


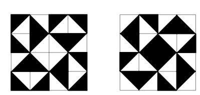
2132 0130



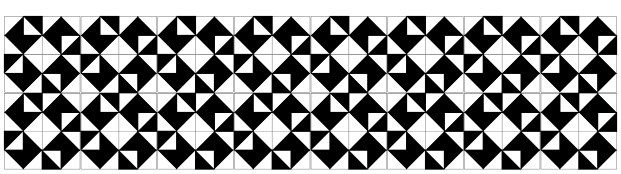


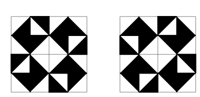
2130 2130 0312



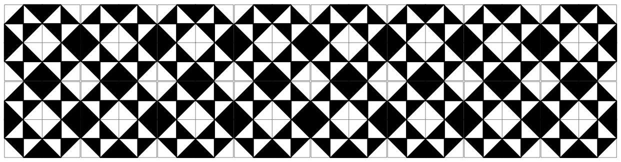


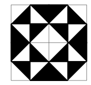
2112 0330





0332 0112





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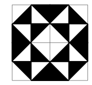

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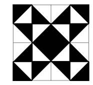

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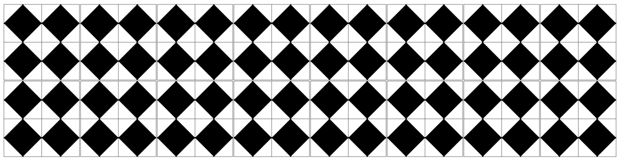
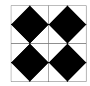
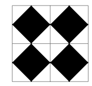




0132



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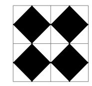
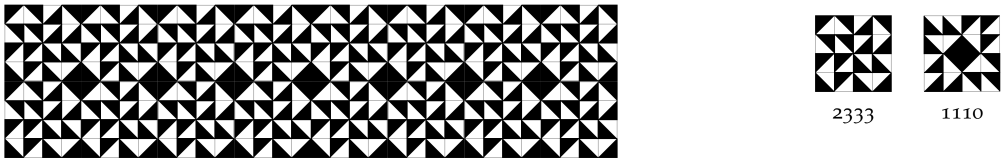
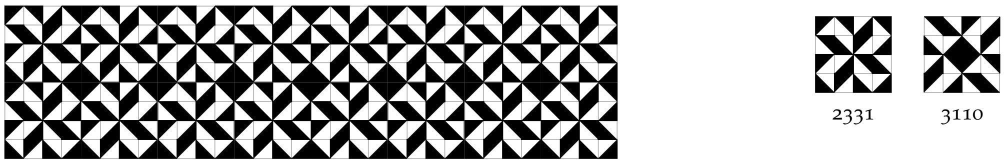


0132

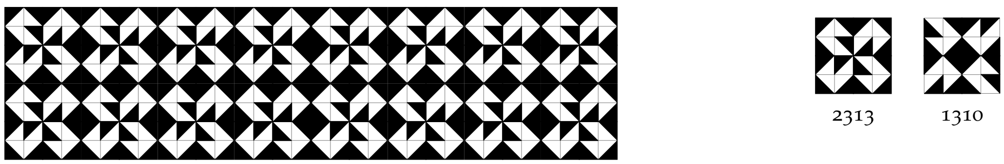
Frieze patterns for family 0111 (secondary, 1110)



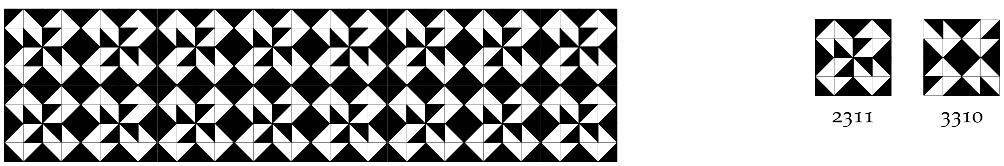
2333 1110



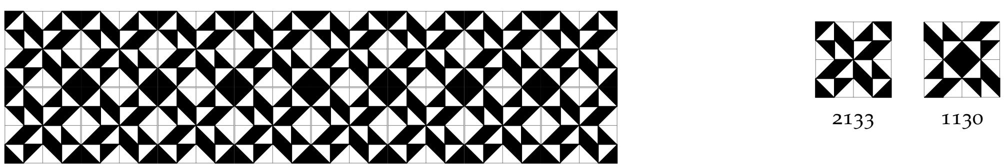
2331 3110



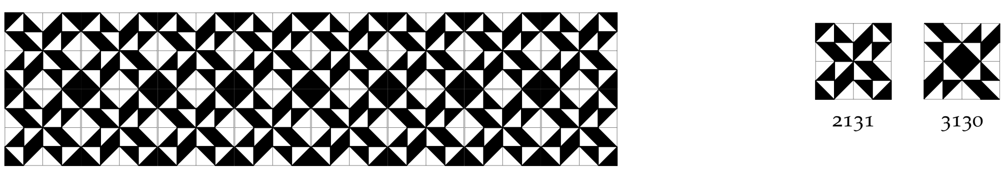
2313 1310



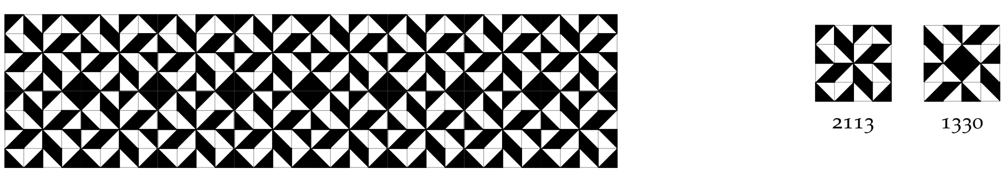
2311 3310



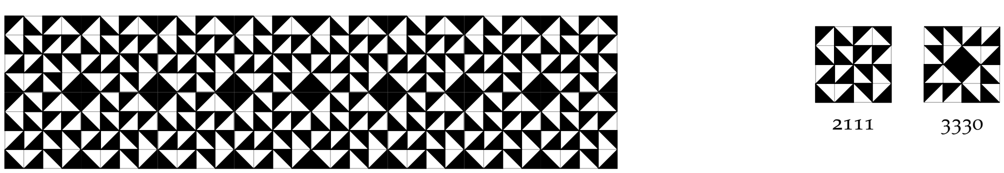
2133 1130



2131 3130



2113 1330



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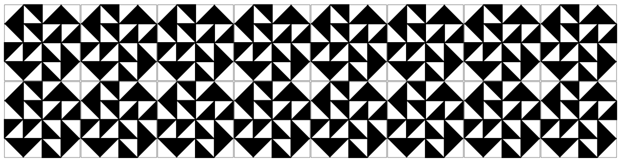

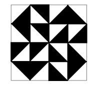

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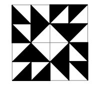

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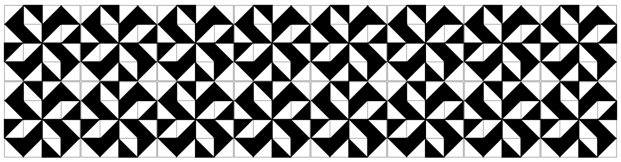

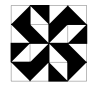

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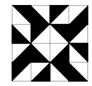

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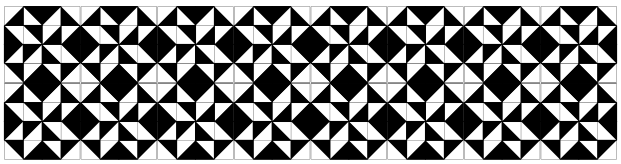

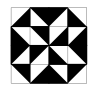

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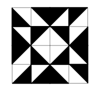

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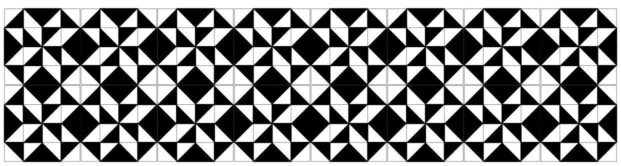

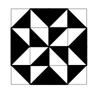

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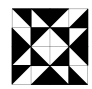

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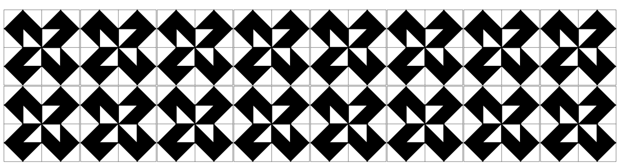

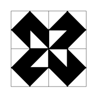

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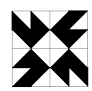

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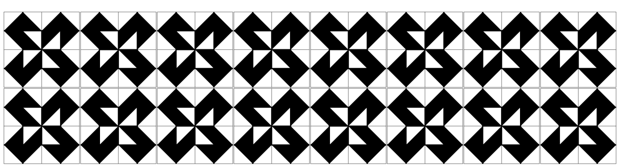

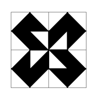

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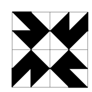

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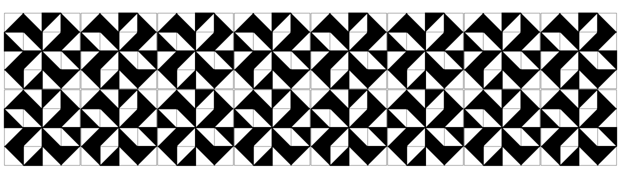

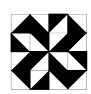

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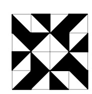

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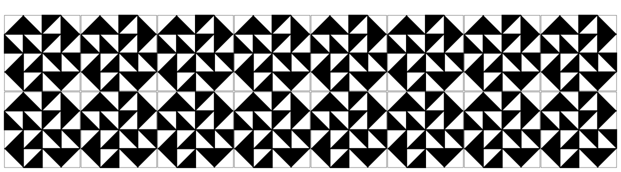

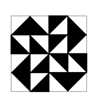

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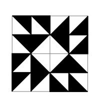
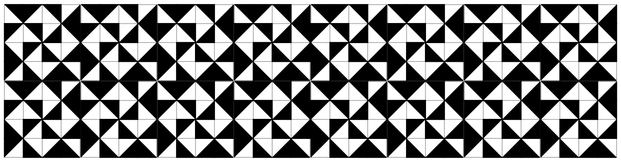
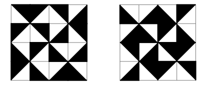


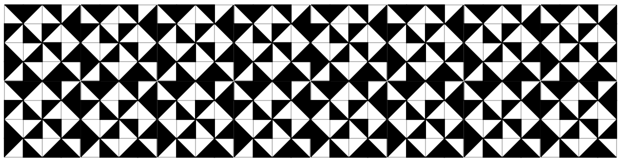
3332

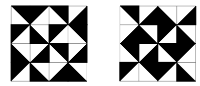
Frieze patterns for family 1001 (secondary, 1001)





3223 1001





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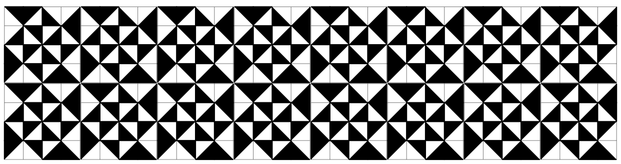

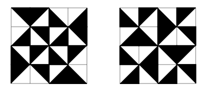

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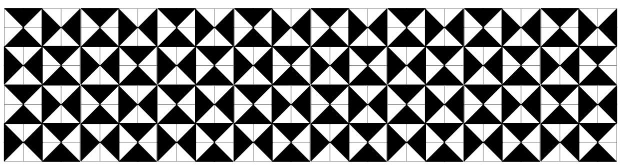
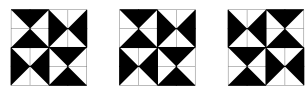
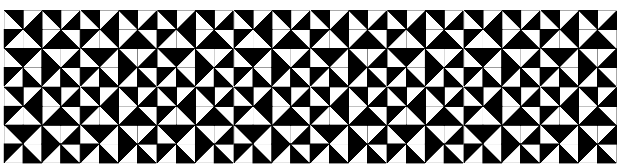
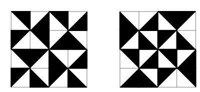




3201 3201 1023





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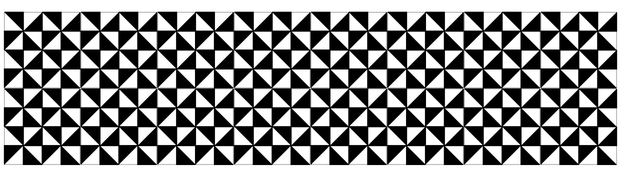
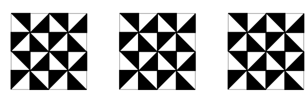
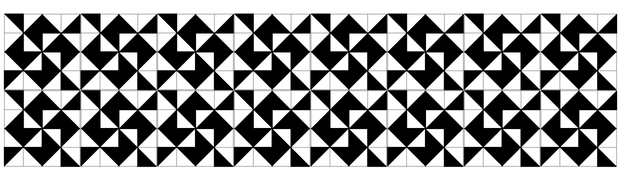
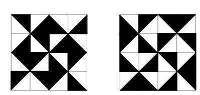




3021 3021 3021





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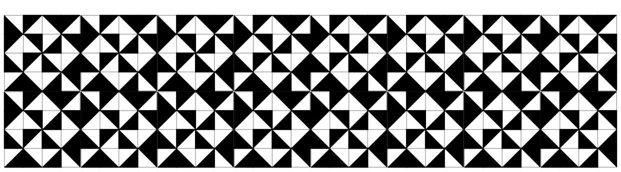
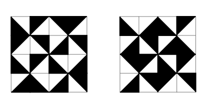
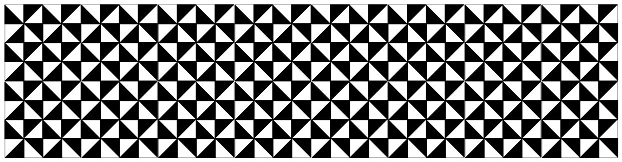
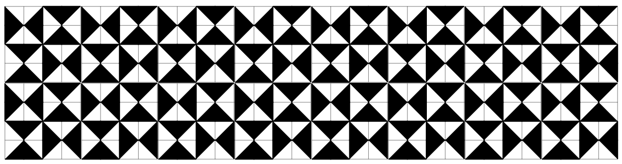
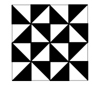


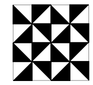


1223 1003

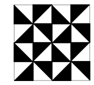


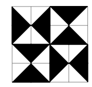


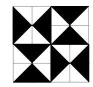


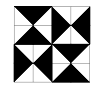


1203 1203 1203



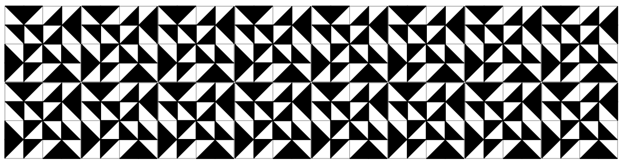


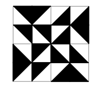




1023 1023 3201

Frieze patterns for family 1011 (secondary, 1101)





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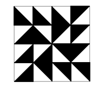
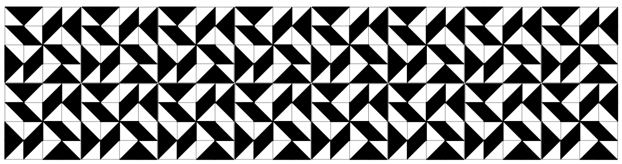
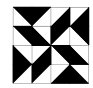


1101





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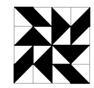

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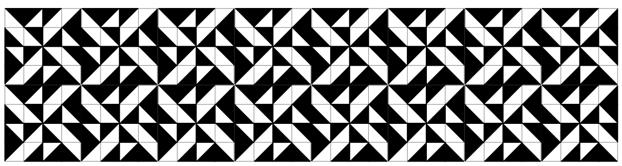

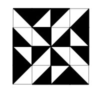

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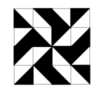
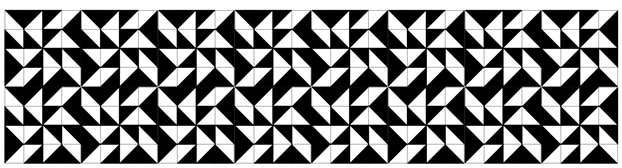
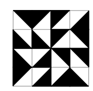


1301





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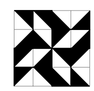

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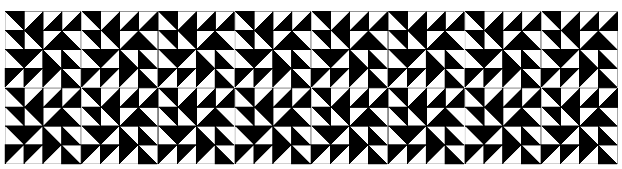

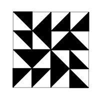

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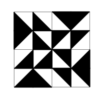
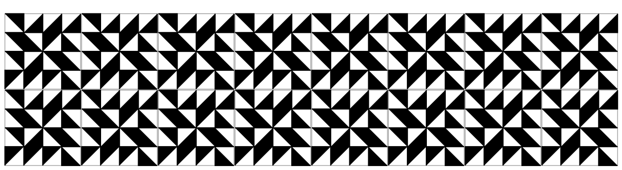
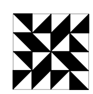


1121





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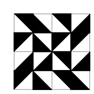

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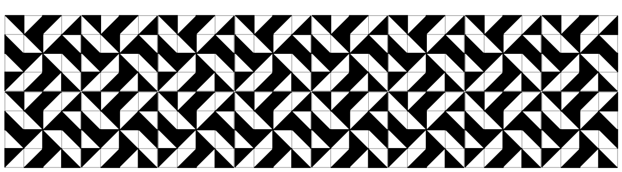

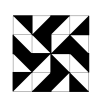

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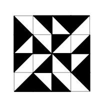

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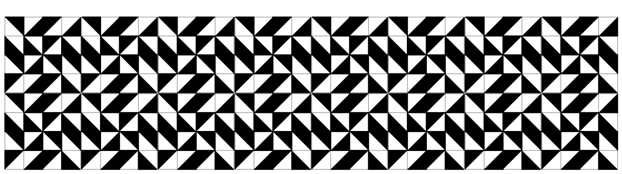

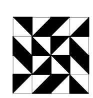

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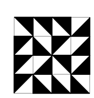

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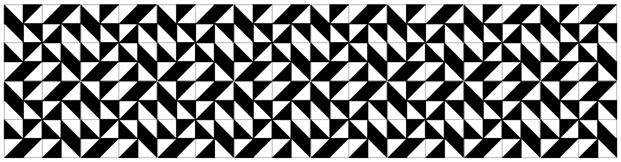

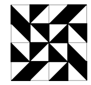
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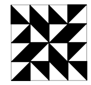
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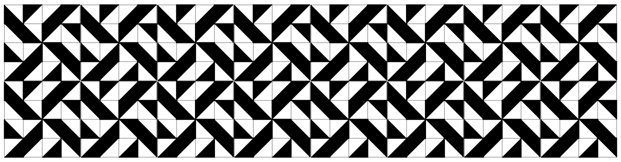

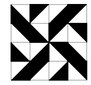
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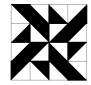
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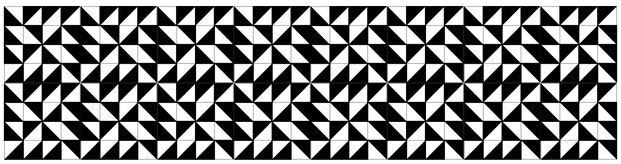

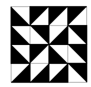
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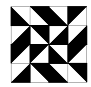
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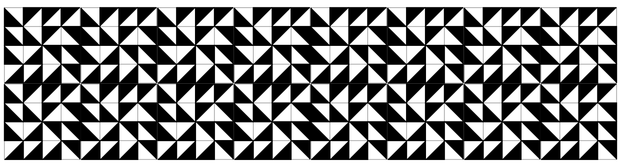

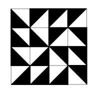
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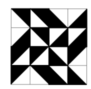
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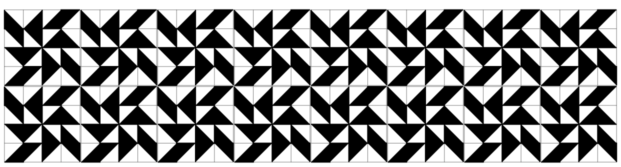

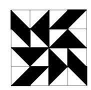
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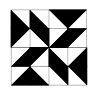
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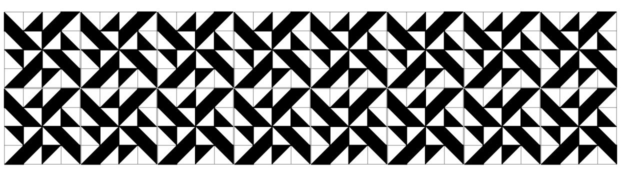

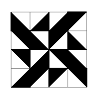
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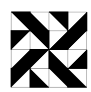
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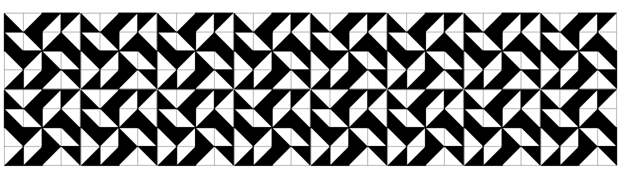

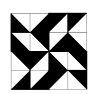
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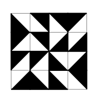
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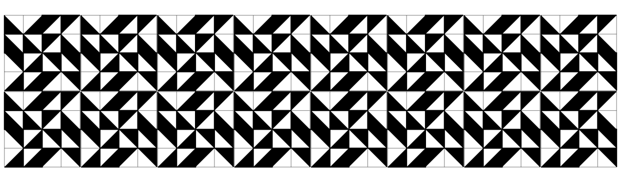

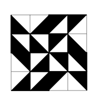
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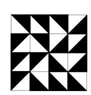
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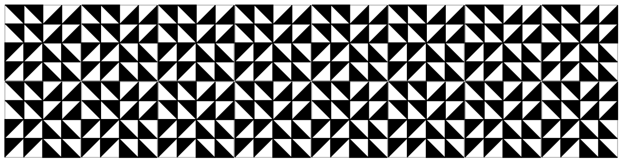
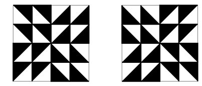
Frieze patterns for family 1111 (secondary, 1111)





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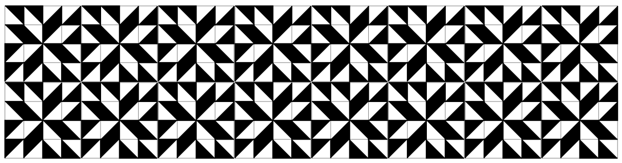

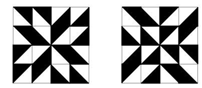

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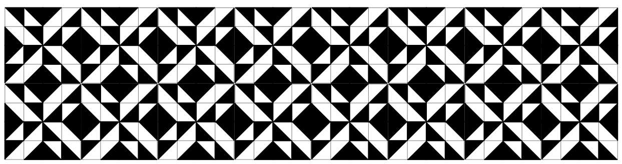

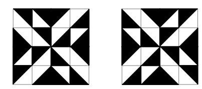

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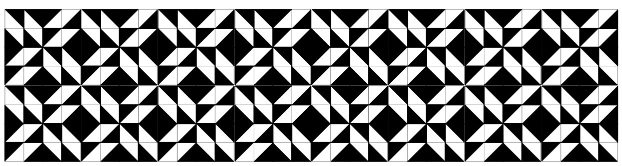

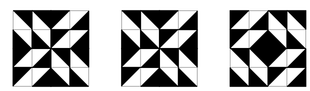

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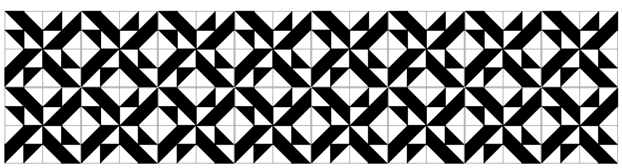

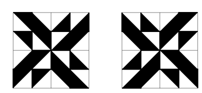

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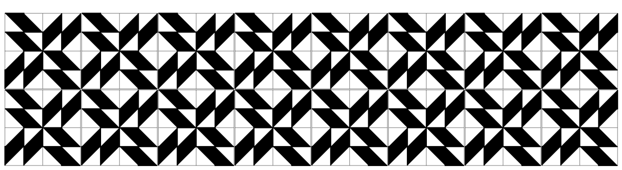

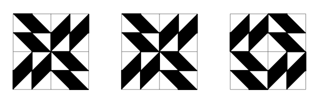

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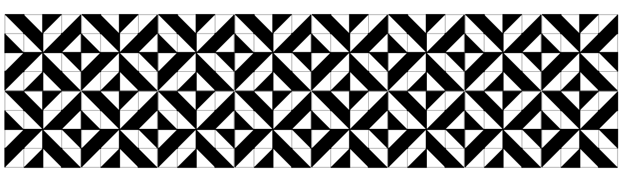

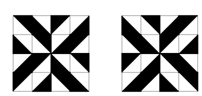

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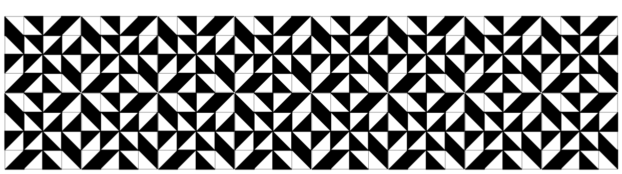

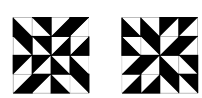

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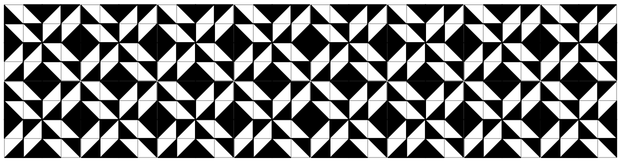

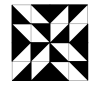

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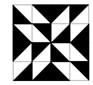

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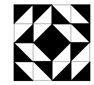

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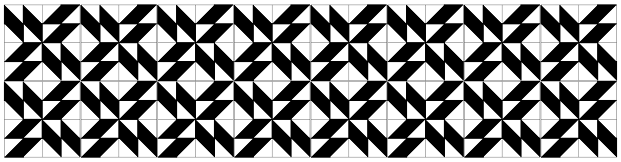

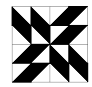

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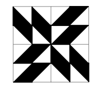

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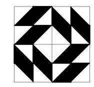


0202

Self-dual tiles

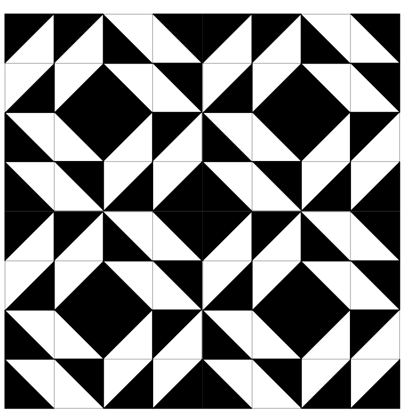
Self-dual tiles are the 4×4 Truchet tile patterns whose 2×2 prototile has two-fold (180°) rotational symmetry. Because of the two-fold rotational symmetry of the prototile, its appearance in the third quadrant of the 4×4 tile is identical to its appearance in the initial quadrant. So the dual tile that emerges when placing four of the tiles together in a larger 2×2 tile array is another copy of the original tile, appearing in the center of the larger 2×2 pattern.

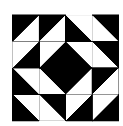
Prototiles with two-fold symmetry

If the original protitle has strict two-fold symmetry, 2×2 patterns made with the four-fold rotationally symmetrical Truchet tile also display another distinct emergent four-fold rotationally symmetrical Truchet tile, which we are calling the *tertiary* tile.

In these patterns, it appears that there are five copies of the primary tile (four placed in a 2×2 array, and another emerging in the center), along with four copies of the tertiary tile pattern.

2200 with 1313





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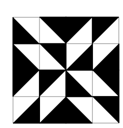
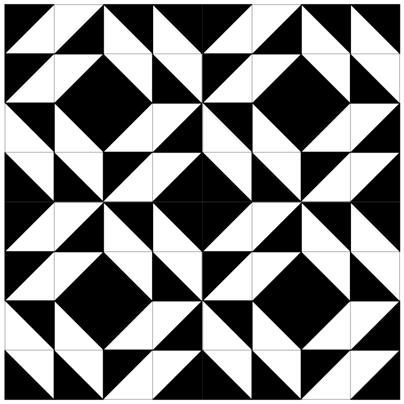
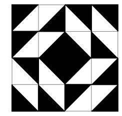


1313

2020 with 3311





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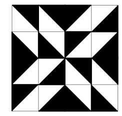
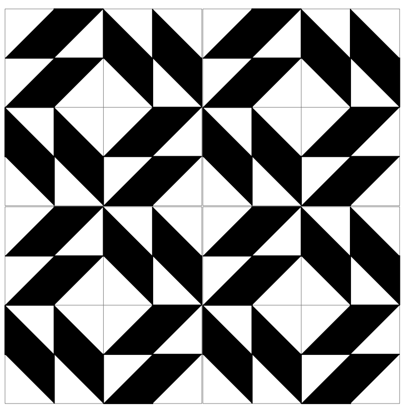
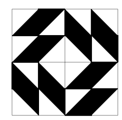


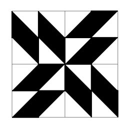
3311

0202 with 1133



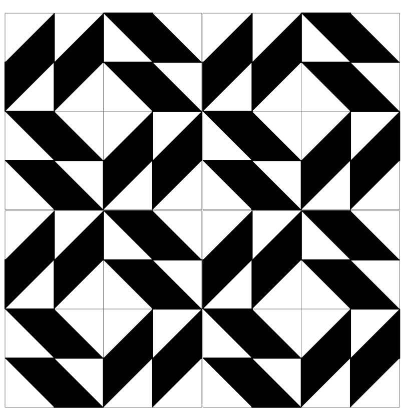


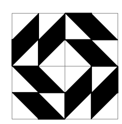
0202



1133

0022 with 3131





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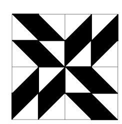
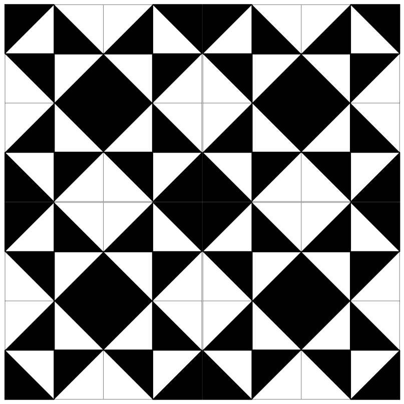
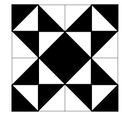


3131

2130 with 0312





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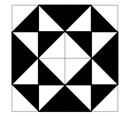
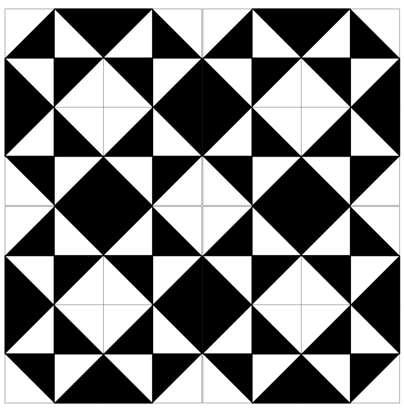
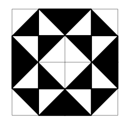


0312

0312 with 2130





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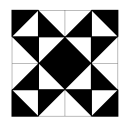
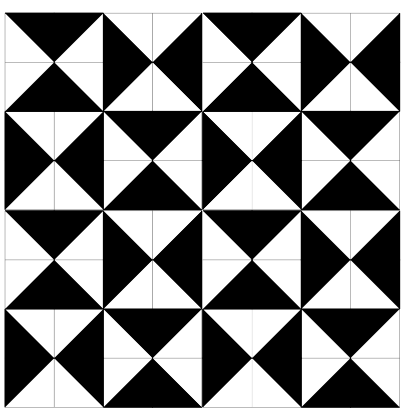
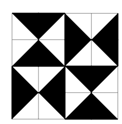


2130

3201 with 1023





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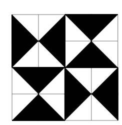
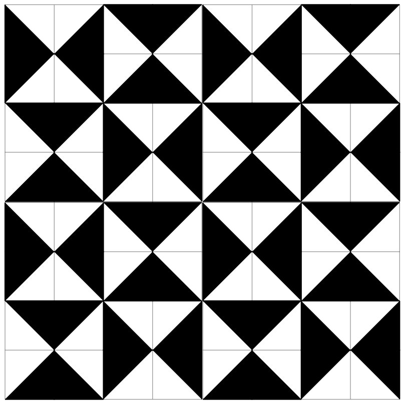
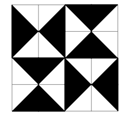


1023

1023 with 3201





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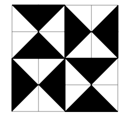
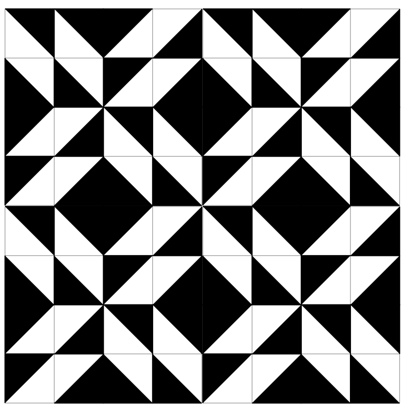
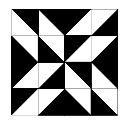


3201

3311 with 2020





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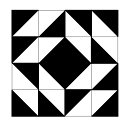
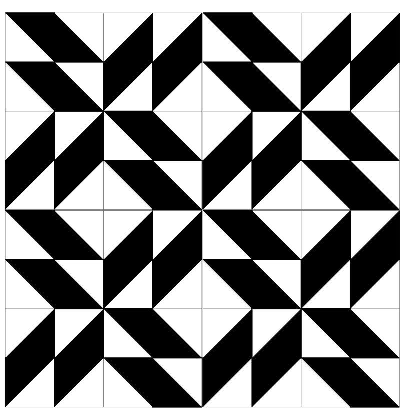
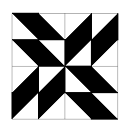


2020

3131 with 0022





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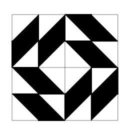
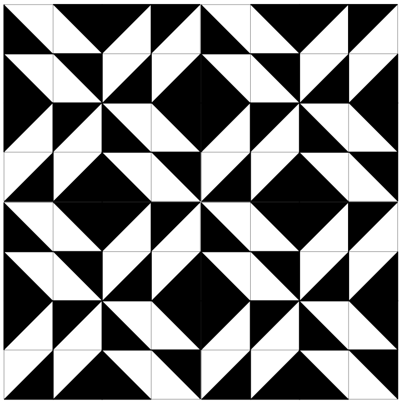
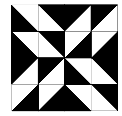


0022

1313 with 2200





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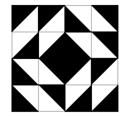
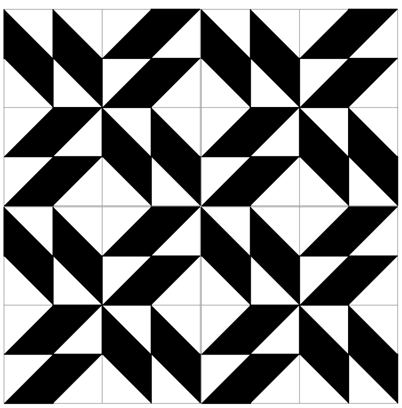
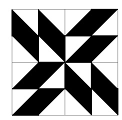


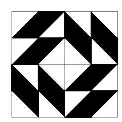
2200

1133 with 0202





1133

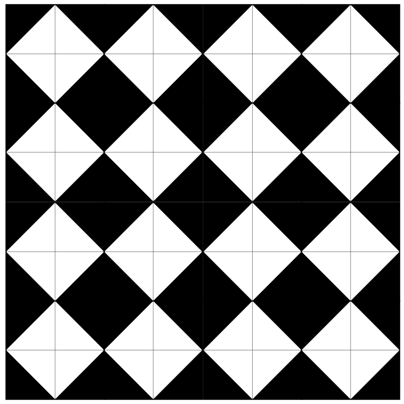


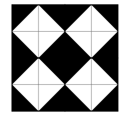
0202

Prototiles with four-fold symmetry

If the original protitle has four-fold symmetry, in 2×2 patterns made with the tertiary tile is another copy of the original four-fold rotationally symmetrical Truchet tile. In this the pattern becomes very uniform, a 4×4 repeating pattern of the underlying prototile.

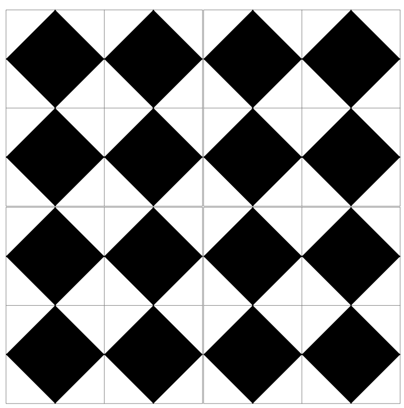
2310

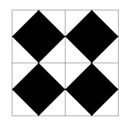




2310

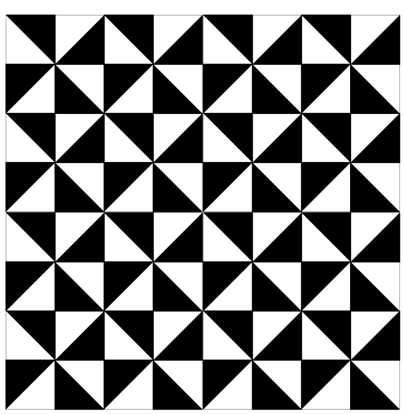
0132

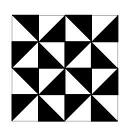




0132

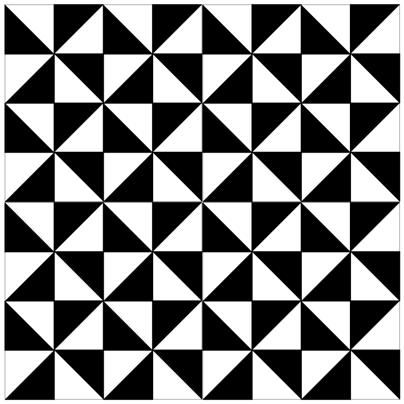
3021

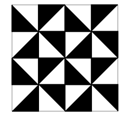




3021

1203

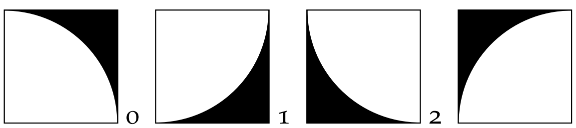




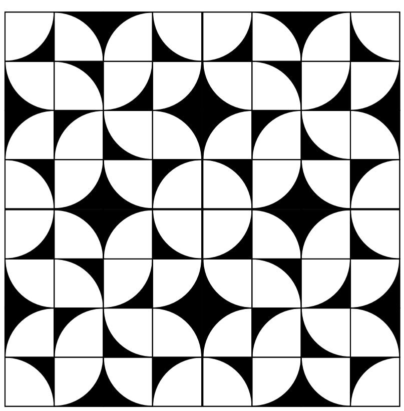
1203

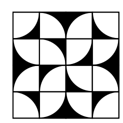
Semicircle Truchet tile patterns

Instead of the traditional Truchet square, any pattern that breaks the rotational symmetry of the square different placement options (2 or 4) can be used. A popular alternative to the traditional Truchet is one where the square is cut by an arc, so that a quarter circle is produced instead of a right triangle.

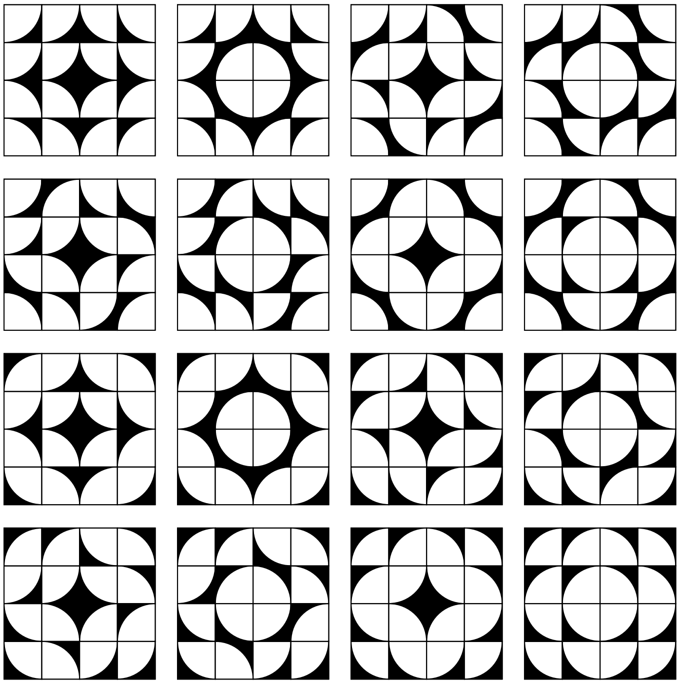

0 1 2 3

In the version used here, the quarter circle is white against a black background, but the opposite colour scheme could also be used.



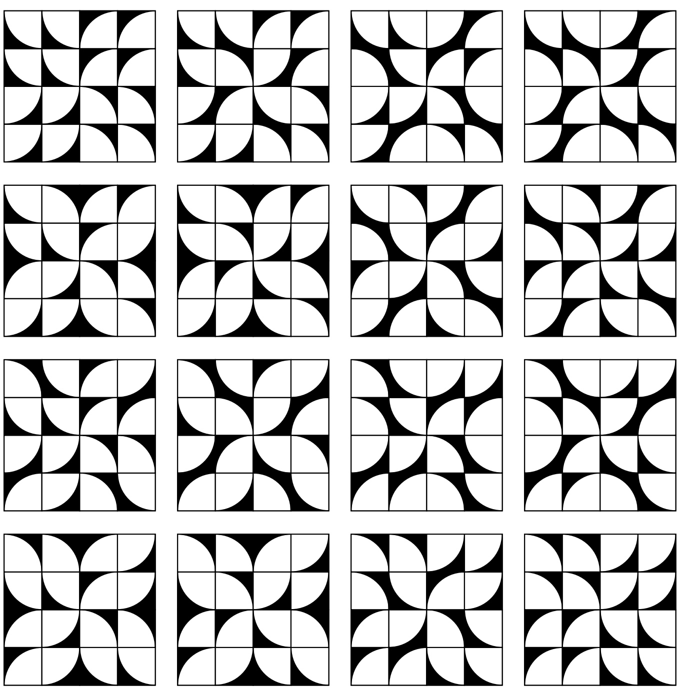

0313

On the following pages, the families of 4*x*4 tiles with rotational symmetry are shown using this semicircle Truchet square variant. With the semicircle version of the Truchet square, there is not an uncoloured family tile that resembles all of the family members, as the colouring is not symmetrical as it is with the traditional Truchet square.



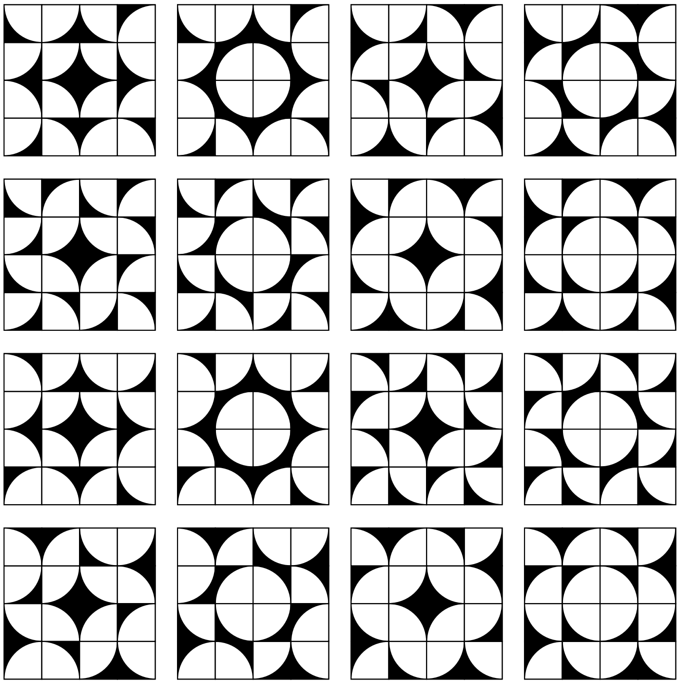
0000

0000	0002	0020	0022
0200	0202	0220	0222
2000	2002	2020	2022
2200	2202	2220	2222



1111

1111	1113	1131	1133
1311	1313	1331	1333
3111	3113	3131	3133
3311	3313	3331	3333



1000

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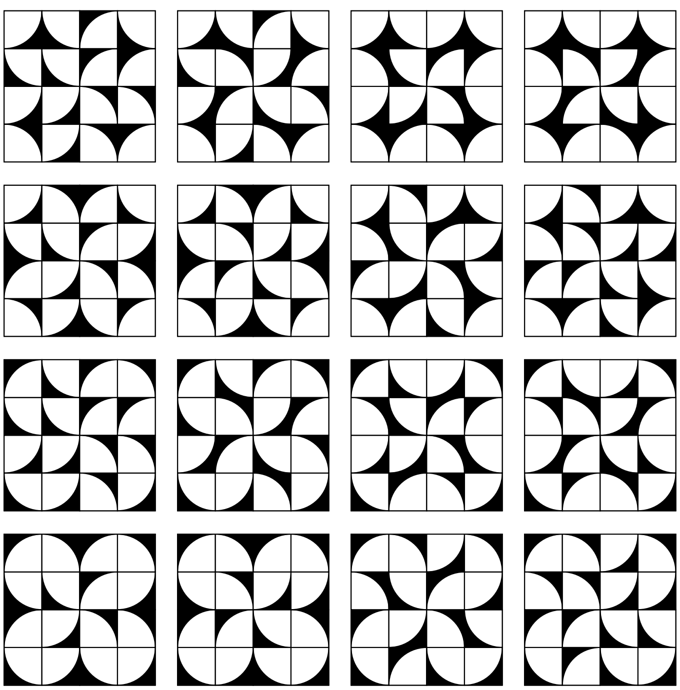
1200	1202	1220	1222
3000	3002	3020	3022
3200	3202	3220	3222



0111

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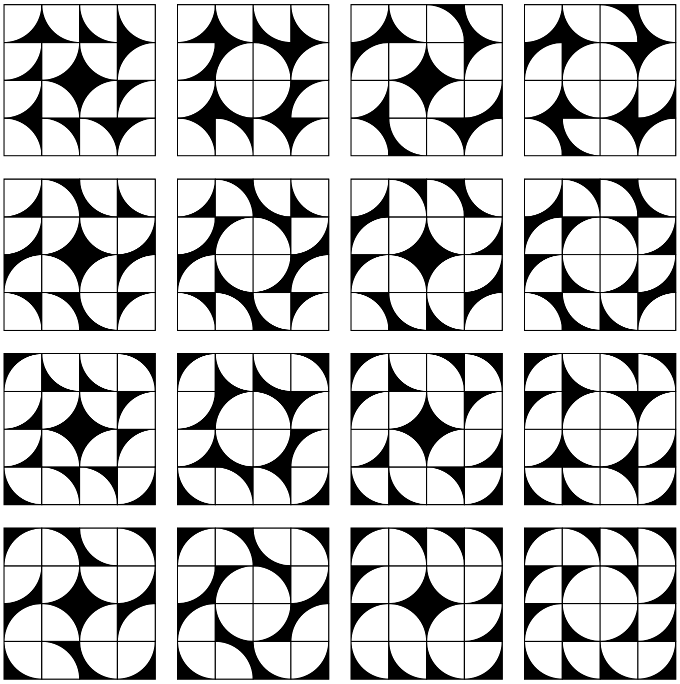
0311	0313	0331	0333
2111	2113	2131	2133
2311	2313	2331	2333



0100

0100	0102	0120	0122
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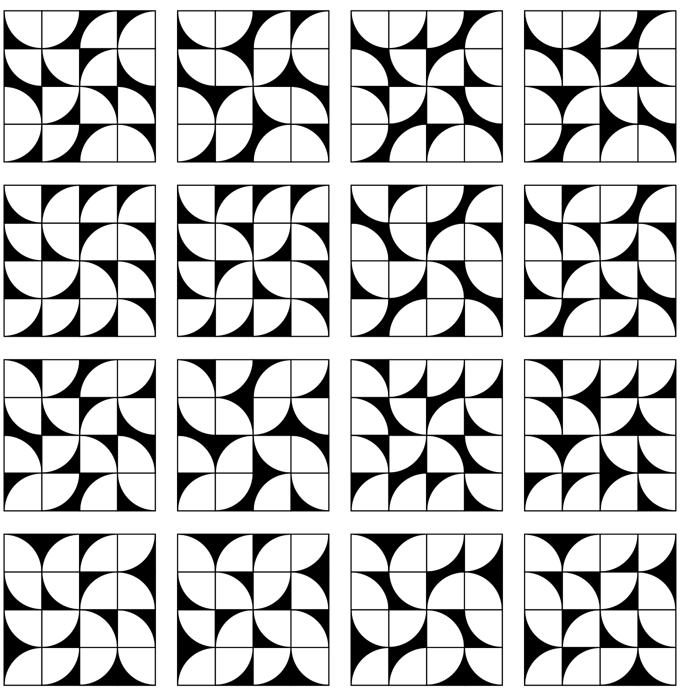
2100	2102	2120	2122
2300	2302	2320	2322



1011

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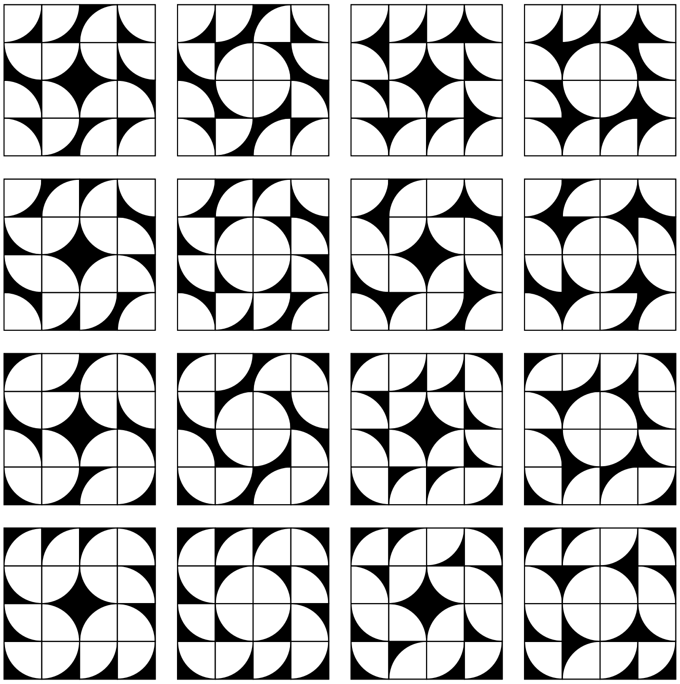
3011	3013	3031	3033
3211	3213	3231	3233



0010

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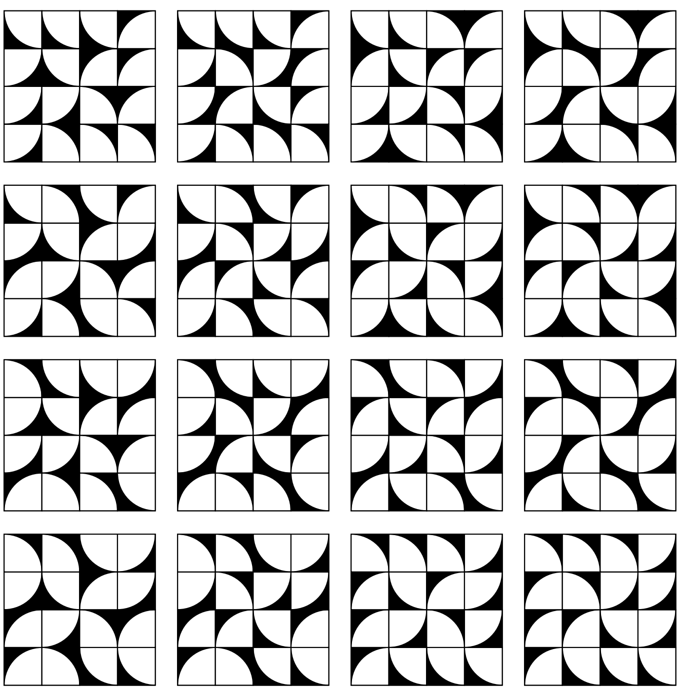
2010	2012	2030	2032
2210	2212	2230	2232



1101

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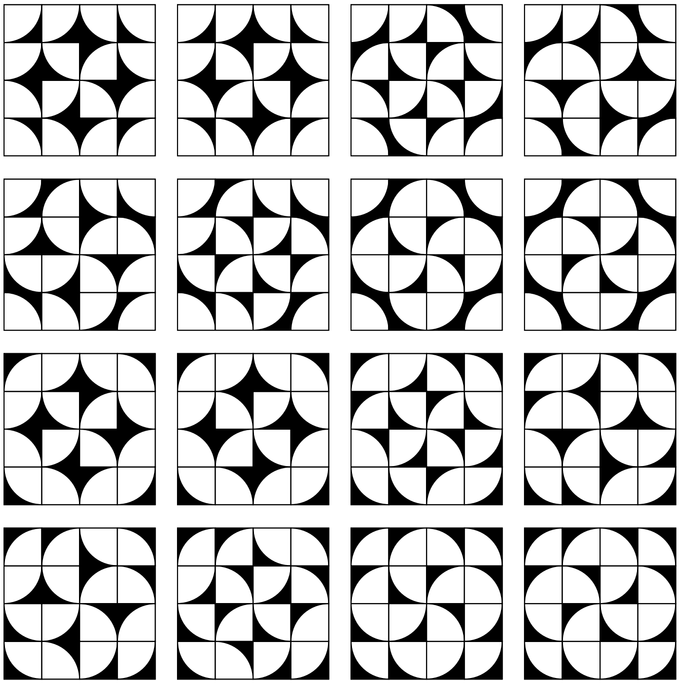
3101	3103	3121	3123
3301	3303	3321	3323



0001

0001	0003	0021	0023
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2001	2003	2021	2023
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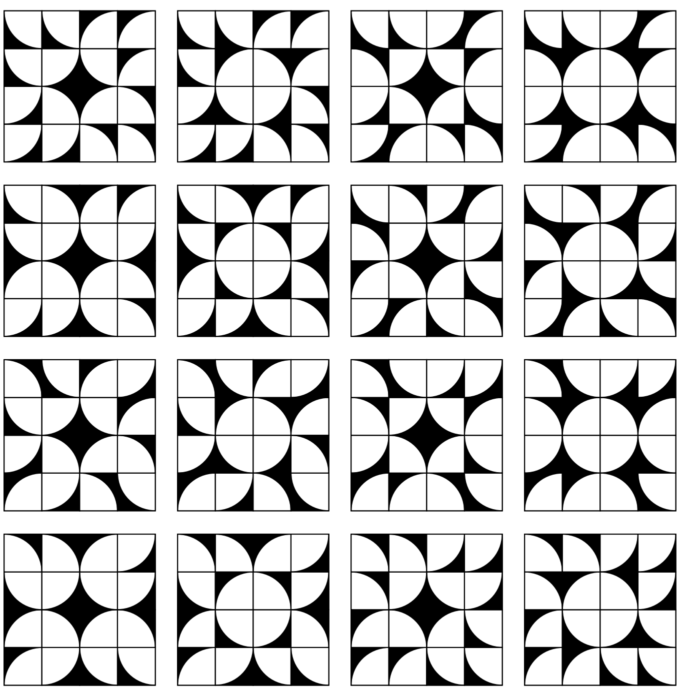

1110

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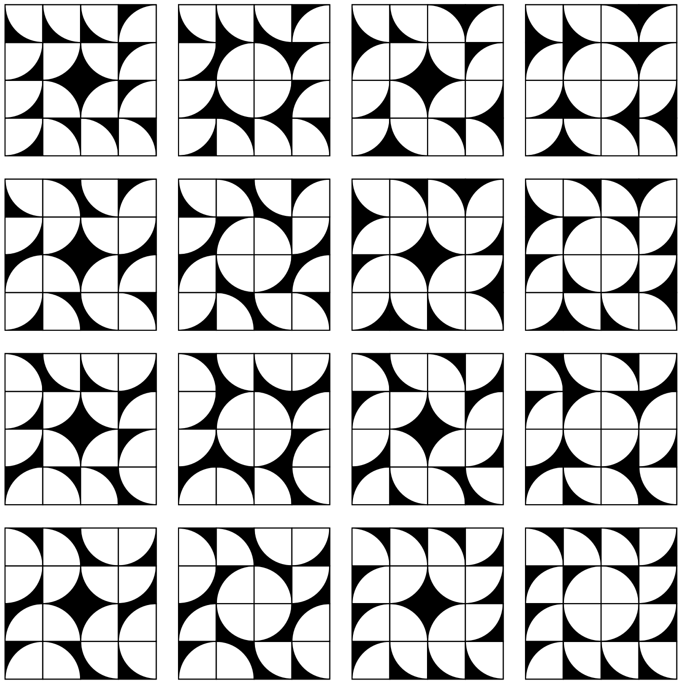

1100

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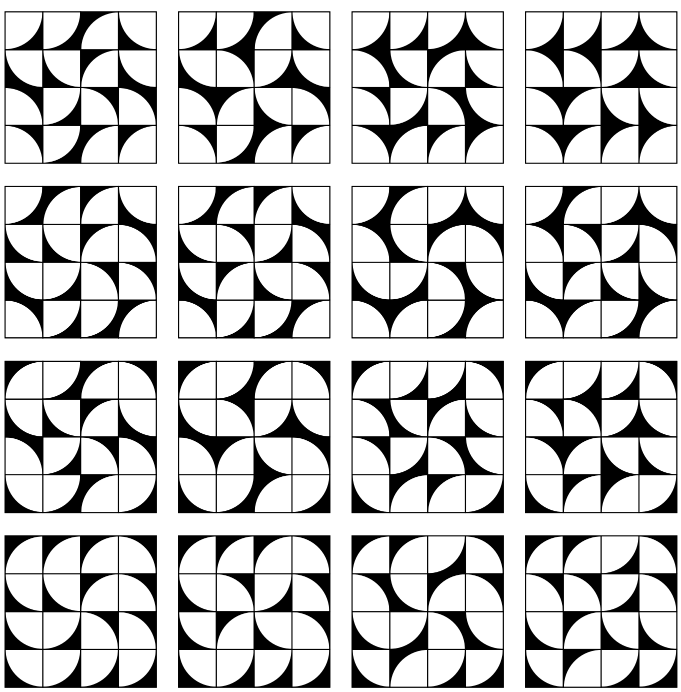
3100	3102	3120	3122
3300	3302	3320	3322



0011

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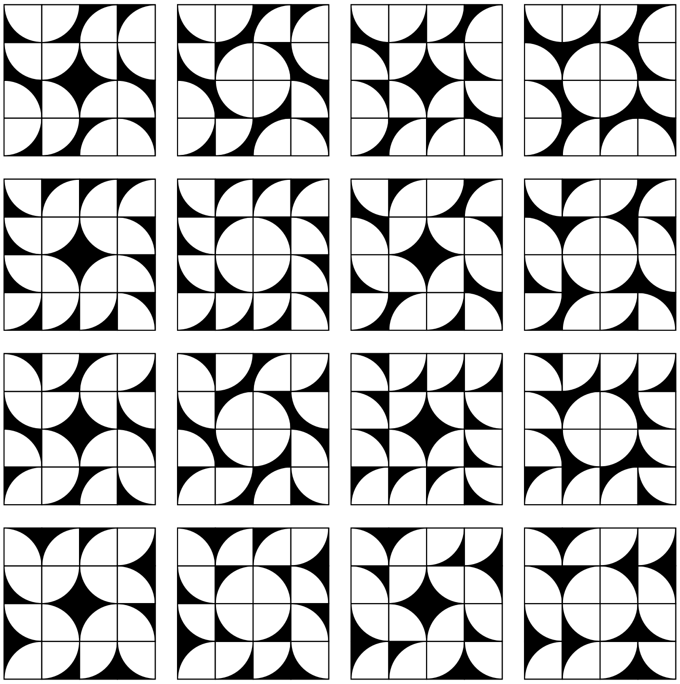
2011	2013	2031	2033
2211	2213	2231	2233



1010

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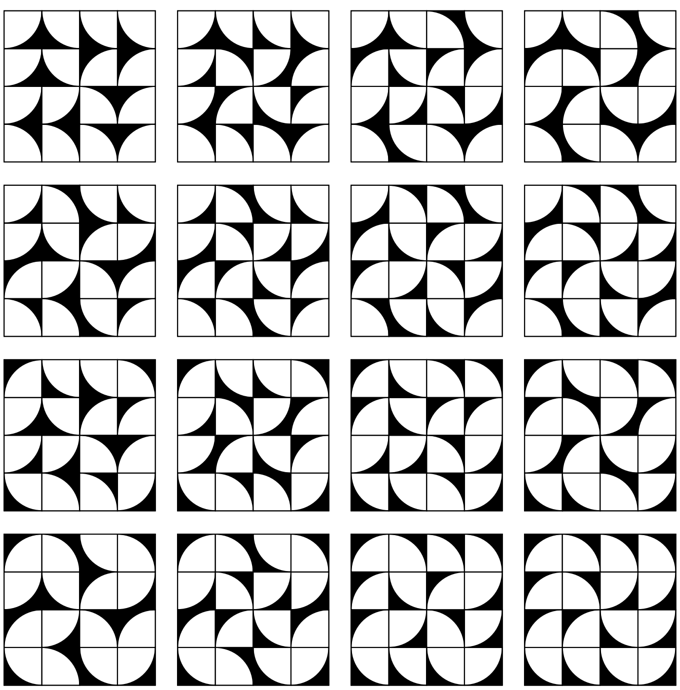

0101

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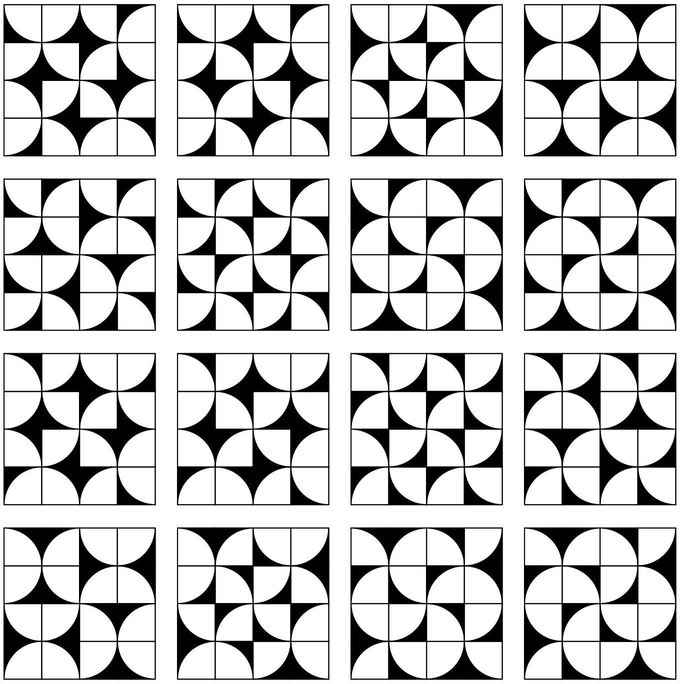

1001

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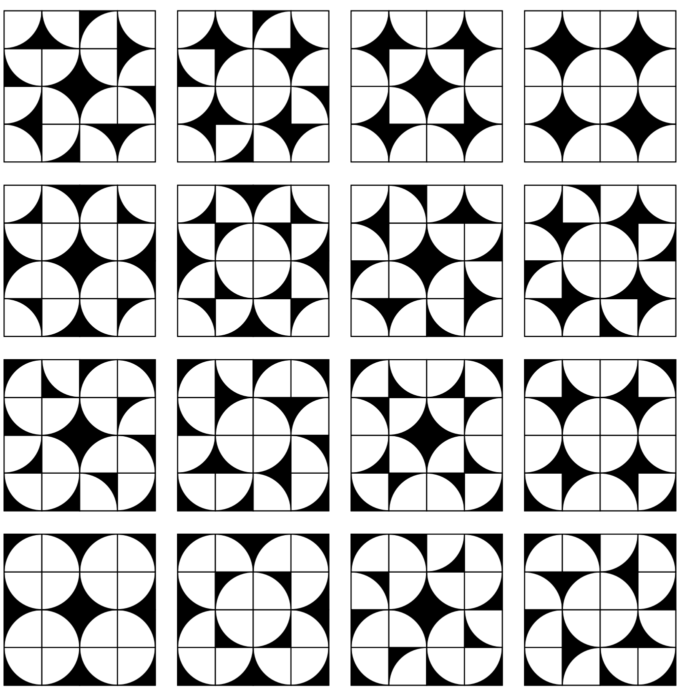

0110

0110	0112	0130	0132
0310	0312	0330	0332
2110	2112	2130	2132
2310	2312	2330	2332

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