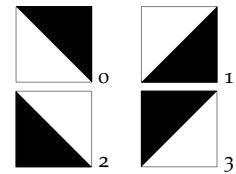


TRUCHET

4×4 patterns with 90° rotational symmetry

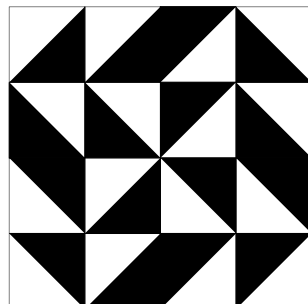
Introduction

Traditionally, Truchet tiles are square tiles that are divided by a diagonal line, and coloured with two colours with a different colour on either side of the diagonal. Each tile can be rotated to one of four positions. Patterns are formed by placing tiles next to each other, rotating individual tiles to create repeated motifs. This booklet presents a complete listing of 4×4 Truchet tile patterns with 90° rotational symmetry (256 patterns). Treating these 4×4 tile patterns as tiles themselves allows for larger decorative patterns to be constructed from them. For example, a uniform frieze made from a single 4×4 tile can actually produce interesting secondary patterns which help illustrate some interesting relationships that exist among the tile patterns.



Each 4×4 Truchet tile pattern with rotational symmetry has a core 2×2 pattern in one of its quadrants that is rotated to produce the overall pattern. In this booklet, the core pattern, or prototile, is assumed to be in the lower left. Each pattern can be identified as a sequence of 4 digits (a, b, c, d) , or more succinctly, $abcd$, that list the rotational positions of each tile in the lower left quadrant. This sequence $abcd$ will be referred to as the *signature* of the tile pattern.

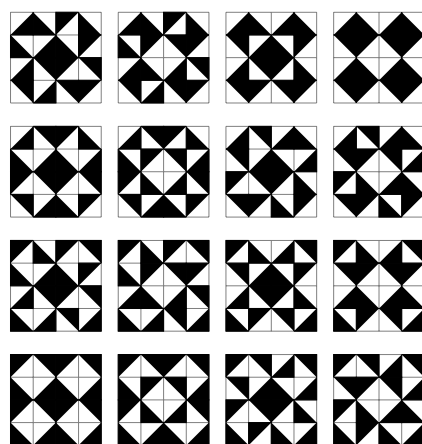
p	q	c	e
c	p	p	q
b	d	d	c
a	c	b	a



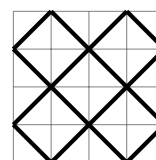
The 0011 pattern

Pattern families

We can group the 4×4 Truchet tile patterns with rotational symmetry into families where tile patterns are considered to be in the same family if they would look the same without colour – if each corresponding tile shares the same diagonal direction. The sequence that represents the family of a tile pattern can be found by taking the sequence of the tile pattern *modulo* 2. So, for example, the 16 tile patterns below are all members of the 0110 family.



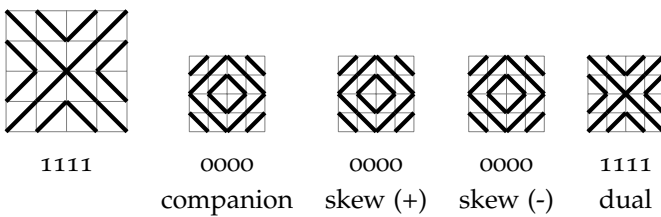
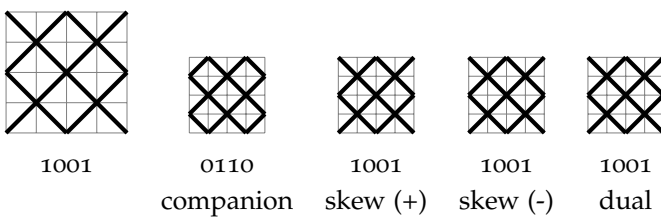
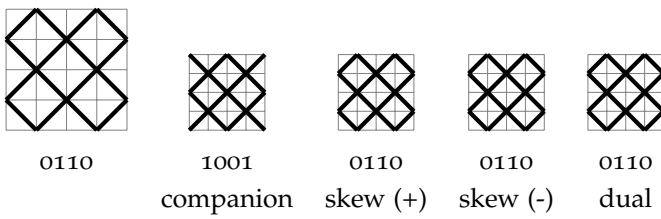
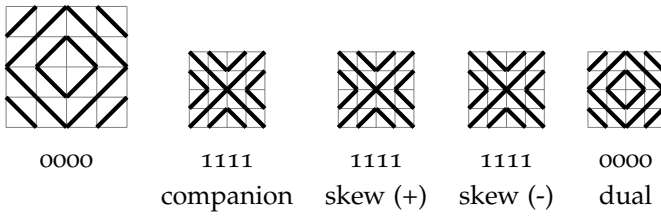
The 0110 pattern family



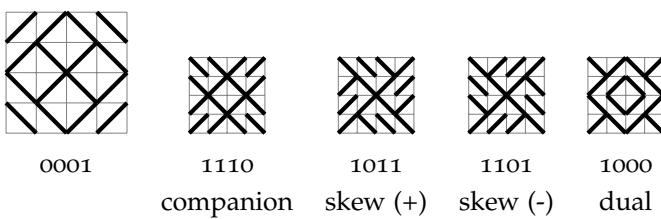
The 0110 family pattern

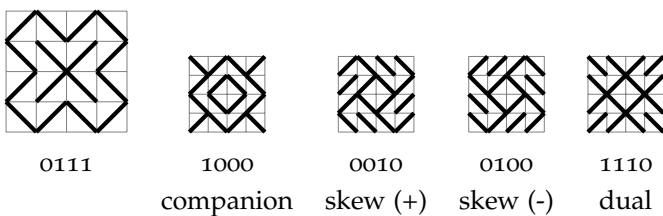
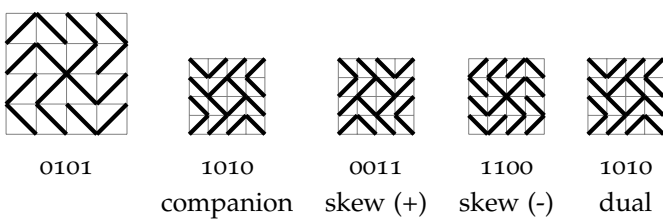
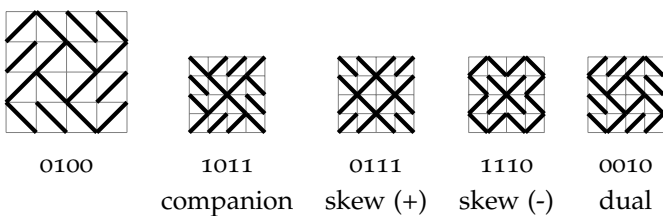
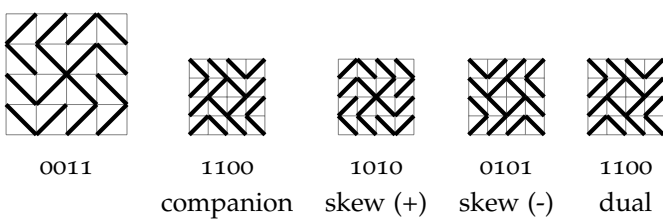
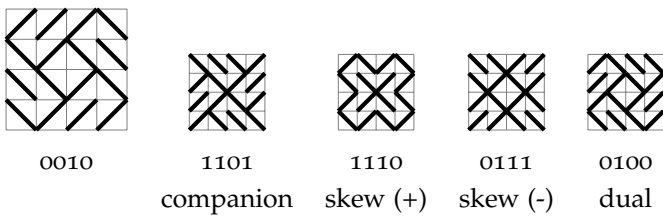
For a given family, there is corresponding *companion* family, the family of patterns formed by rotating each square in a member of the original family by 90° . There are also two *skew* families, formed by taking the upper left and lower right quadrants of an original family tile pattern as a founding pattern and a *dual* family, formed by taking the upper right quadrant as a founding patterns. A family is always different than its companion, and each family has a distinct companion, but it can happen that skew and duals can coincide. Self-dual families, where the dual family is the same as the original are of particular interest in the frieze patterns of the next chapter.

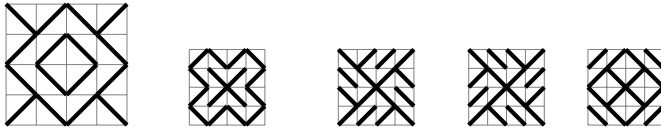
Self-Dual families



Non self-dual families







1000

0111

1101

1011

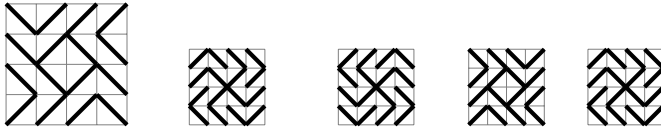
0001

companion

skew (+)

skew (-)

dual



1010

0101

1100

0011

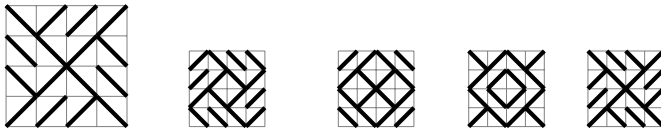
0101

companion

skew (+)

skew (-)

dual



1011

0100

1000

0001

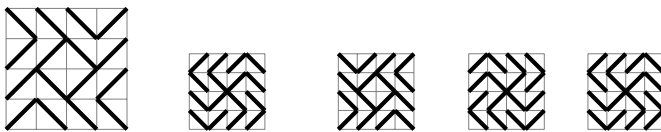
1101

companion

skew (+)

skew (-)

dual



1100

0011

0101

1010

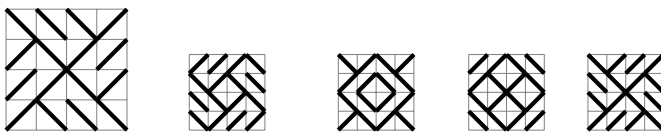
0011

companion

skew (+)

skew (-)

dual



1101

0010

0001

1000

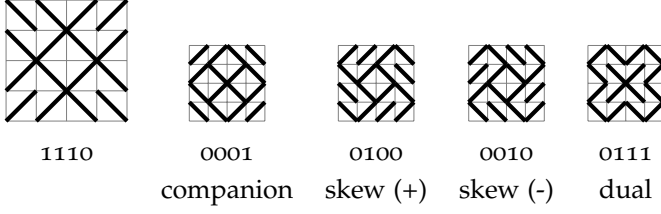
1011

companion

skew (+)

skew (-)

dual

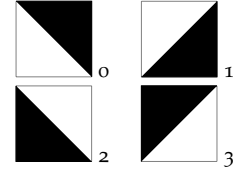


Family and tile pattern mappings

Related families and tiles can be obtained from applying simple mappings on the signature of the tile pattern.

Family mappings

$$\begin{aligned}
 \text{companion} : (a, b, c, d) &\mapsto (a+1, b+1, c+1, d+1) \pmod{2}; \\
 \text{skew}+ : (a, b, c, d) &\mapsto (c+1, a+1, d+1, b+1) \pmod{2}; \\
 \text{reverse} : (a, b, c, d) &\mapsto (d, c, b, a) \pmod{2}; \\
 \text{skew}- : (a, b, c, d) &\mapsto (b+1, d+1, a+1, c+1) \pmod{2};
 \end{aligned}$$

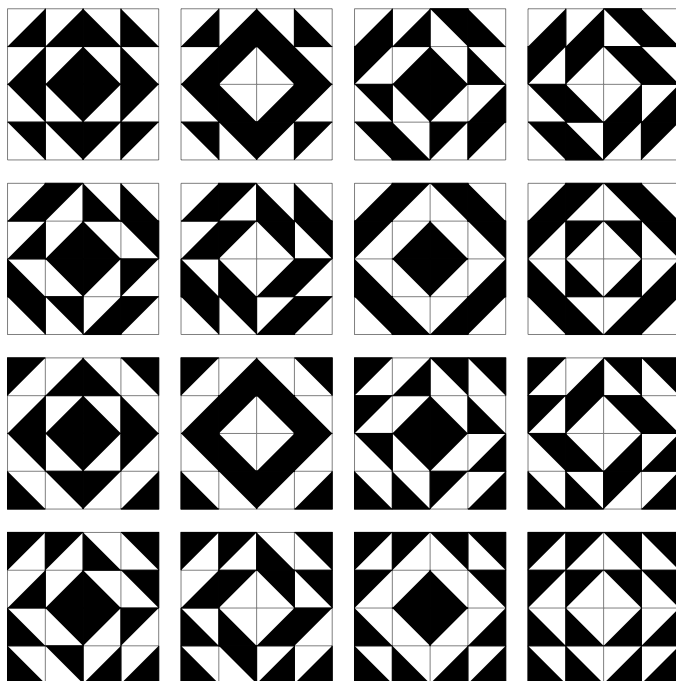


Tile pattern mappings

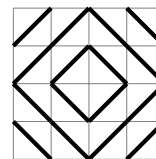
$$\begin{aligned}
 \text{skew}+ : (a, b, c, d) &\mapsto (c+1, a+1, d+1, b+1) \pmod{4}; \\
 \text{dual} : (a, b, c, d) &\mapsto (d+2, c+2, b+2, a+2) \pmod{4}; \\
 \text{skew}- : (a, b, c, d) &\mapsto (b+3, d+3, a+3, c+3) \pmod{4}; \\
 \text{opposite} : (a, b, c, d) &\mapsto (a+2, b+2, c+2, d+2) \pmod{4};
 \end{aligned}$$

a	q	c	e
c	p	p	q
b	d	d	c
a	c	b	a

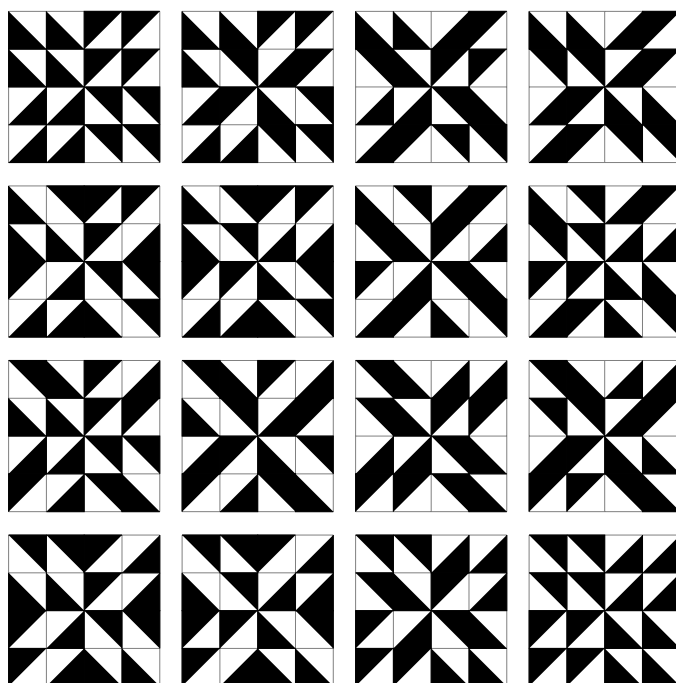
On the following pages each family will be shown along with its corresponding *companion* family, the family of patterns formed by rotating each square in a member of the original family by 90° .



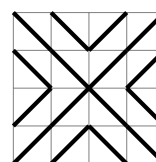
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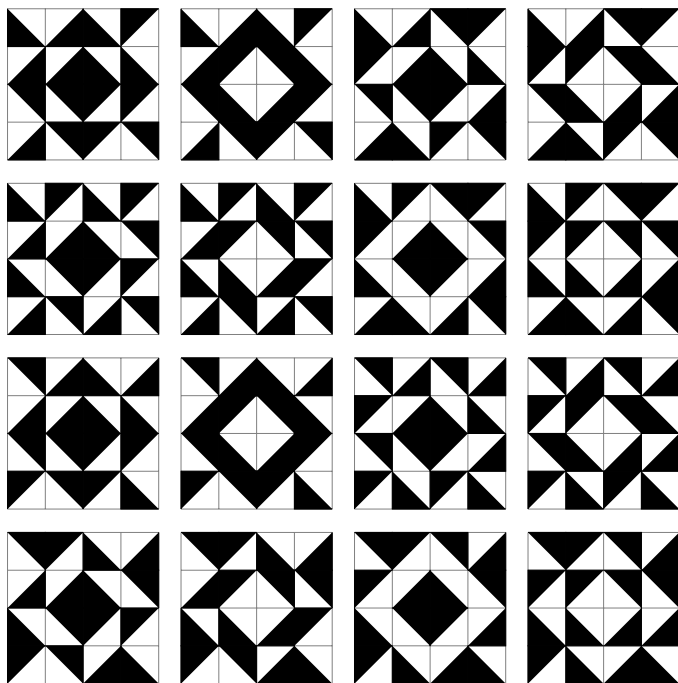
0000	0002	0020	0022
0200	0202	0220	0222
2000	2002	2020	2022
2200	2202	2220	2222



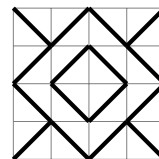
1111



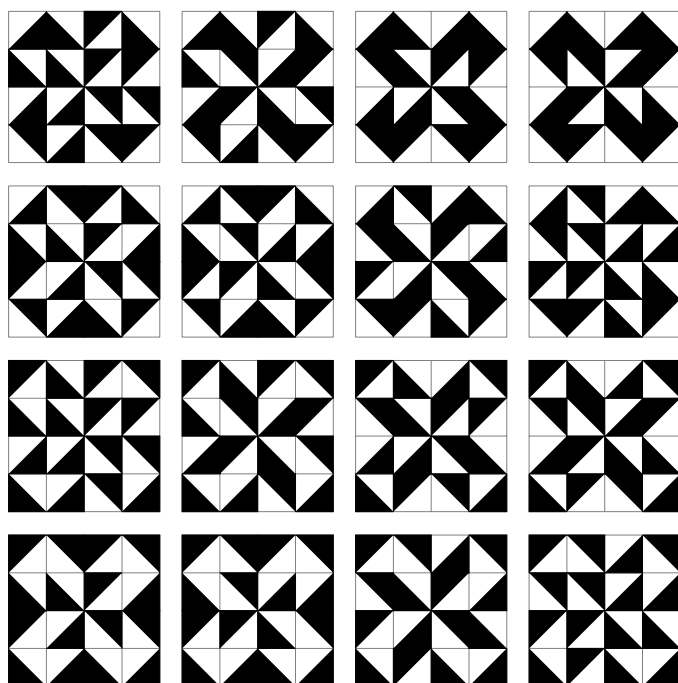
1111	1113	1131	1133
1311	1313	1331	1333
3111	3113	3131	3133
3311	3313	3331	3333



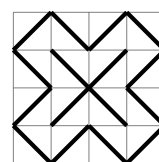
1000



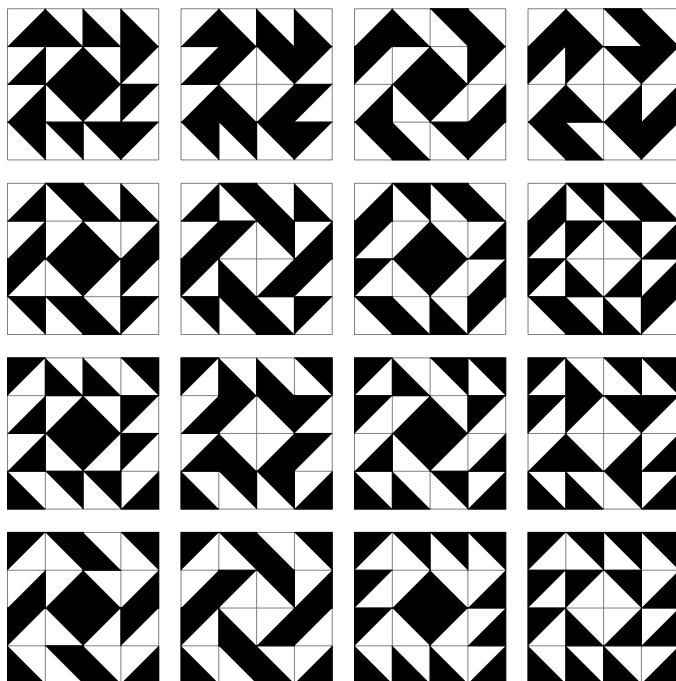
1000	1002	1020	1022
1200	1202	1220	1222
3000	3002	3020	3022
3200	3202	3220	3222



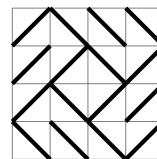
0111



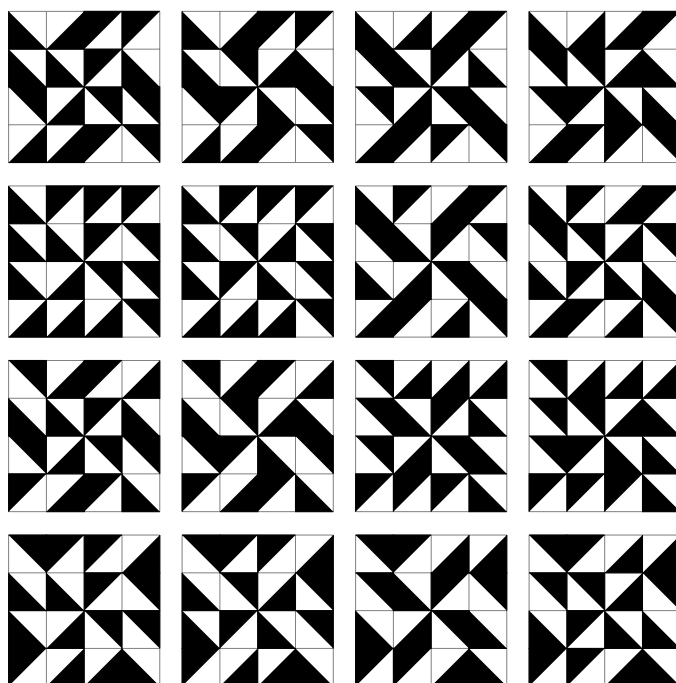
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0311	0313	0331	0333
2111	2113	2131	2133
2311	2313	2331	2333



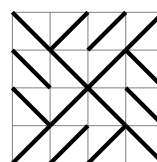
0100



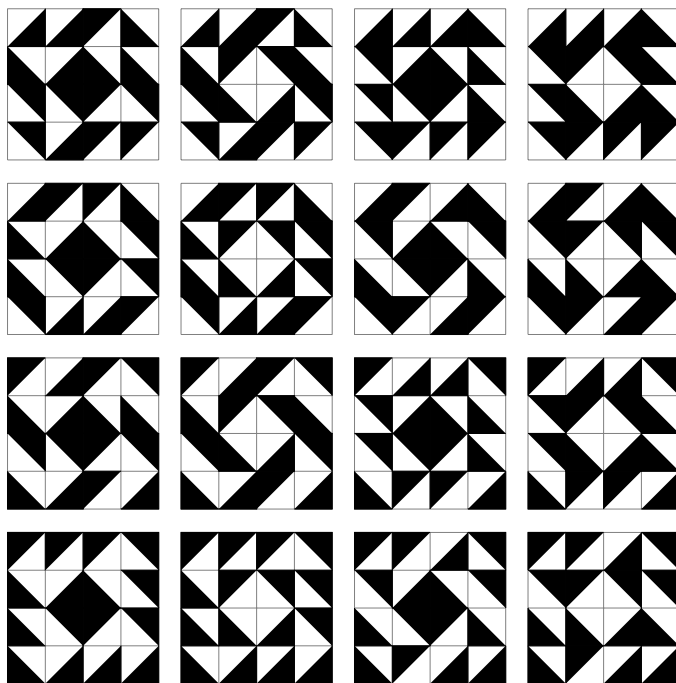
0100	0102	0120	0122
0300	0302	0320	0322
2100	2102	2120	2122
2300	2302	2320	2322



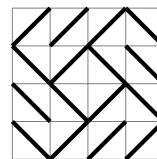
1011



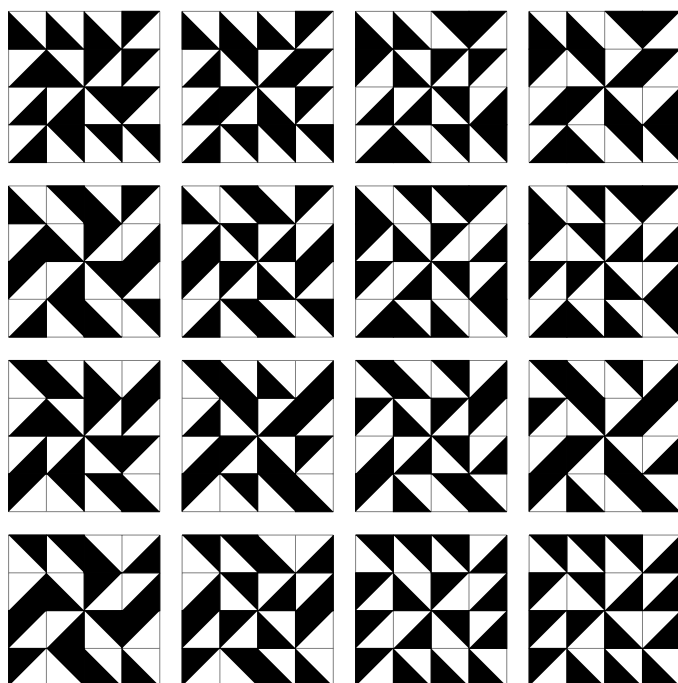
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1211	1213	1231	1233
3011	3013	3031	3033
3211	3213	3231	3233



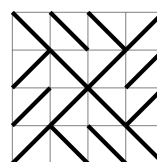
0010



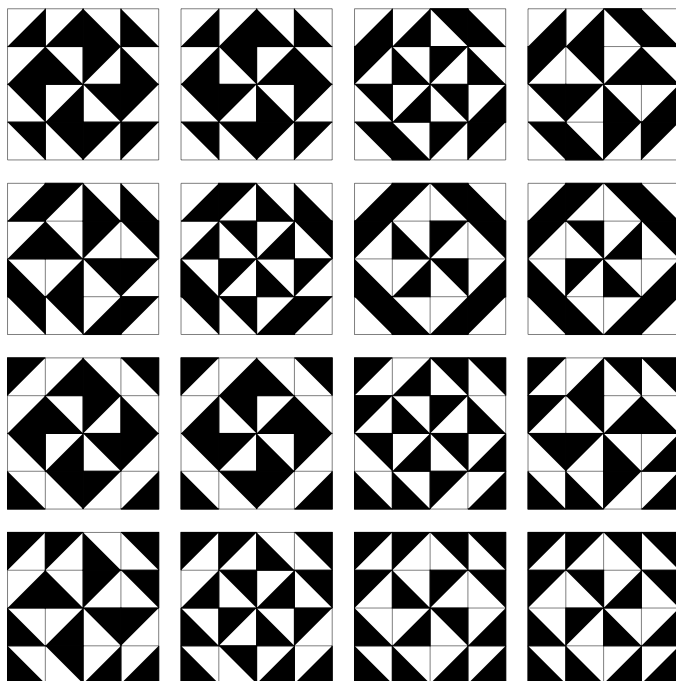
0010	0012	0030	0032
0210	0212	0230	0232
2010	2012	2030	2032
2210	2212	2230	2232



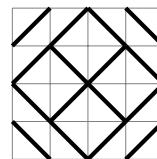
1101



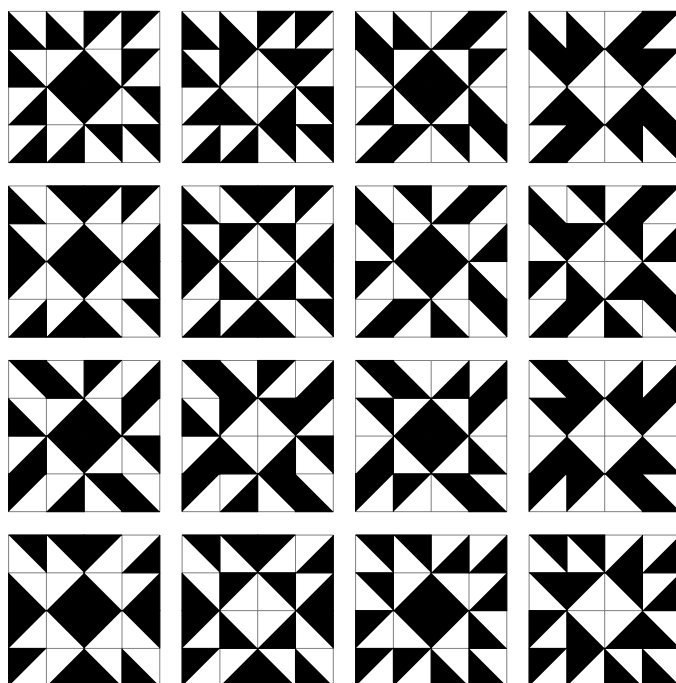
1101	1103	1121	1123
1301	1303	1321	1323
3101	3103	3121	3123
3301	3303	3321	3323



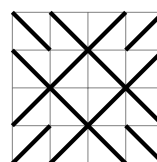
0001



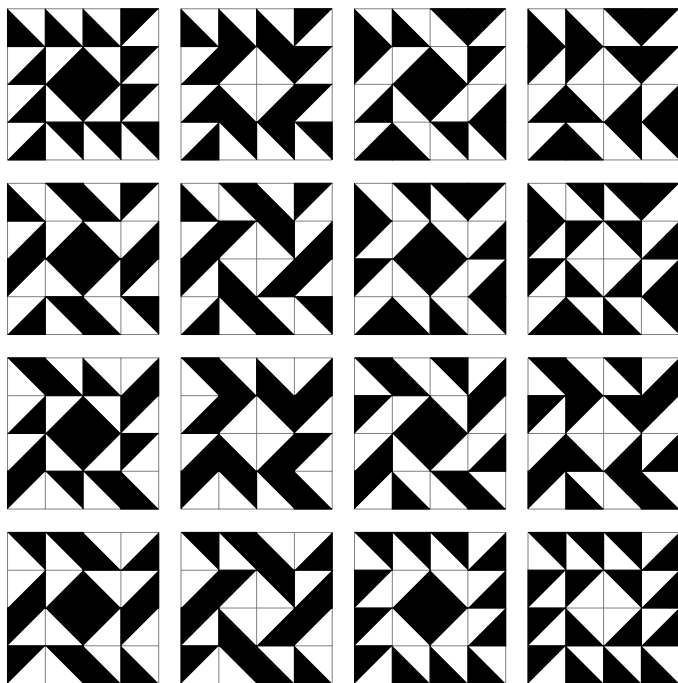
0001	0003	0021	0023
0201	0203	0221	0223
2001	2003	2021	2023
2201	2203	2221	2223



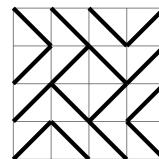
1110



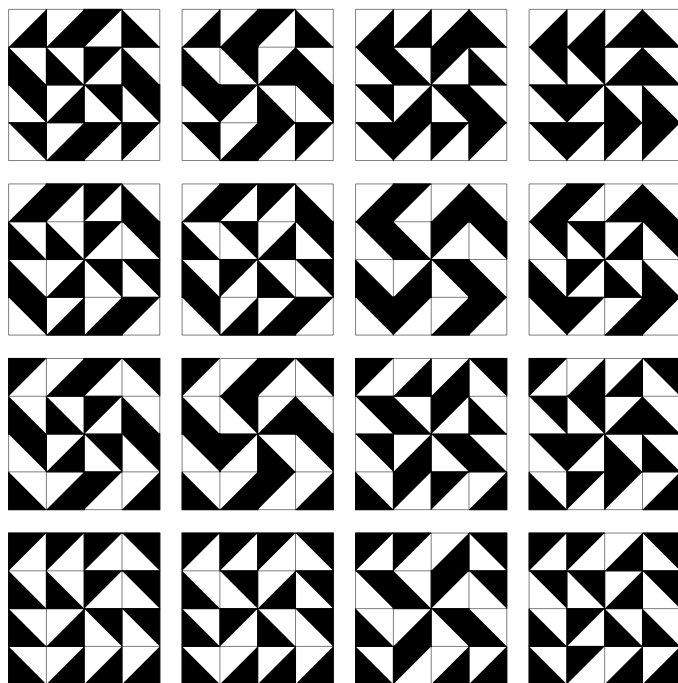
1110	1112	1130	1132
1310	1312	1330	1332
3110	3112	3130	3132
3310	3312	3330	3332



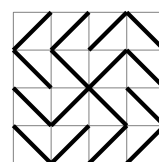
1100



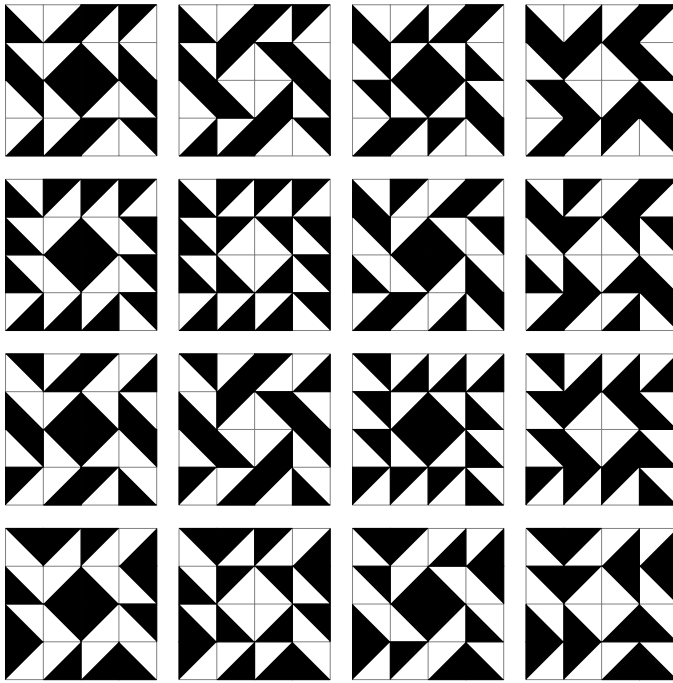
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1300	1302	1320	1322
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3300	3302	3320	3322



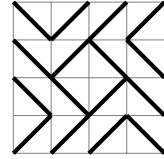
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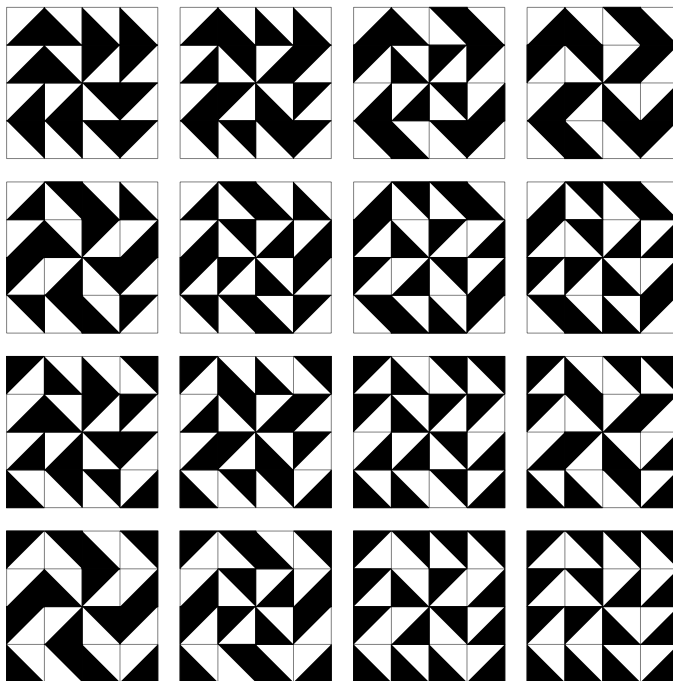
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0211	0213	0231	0233
2011	2013	2031	2033
2211	2213	2231	2233



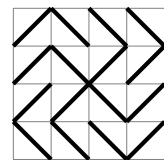
1010



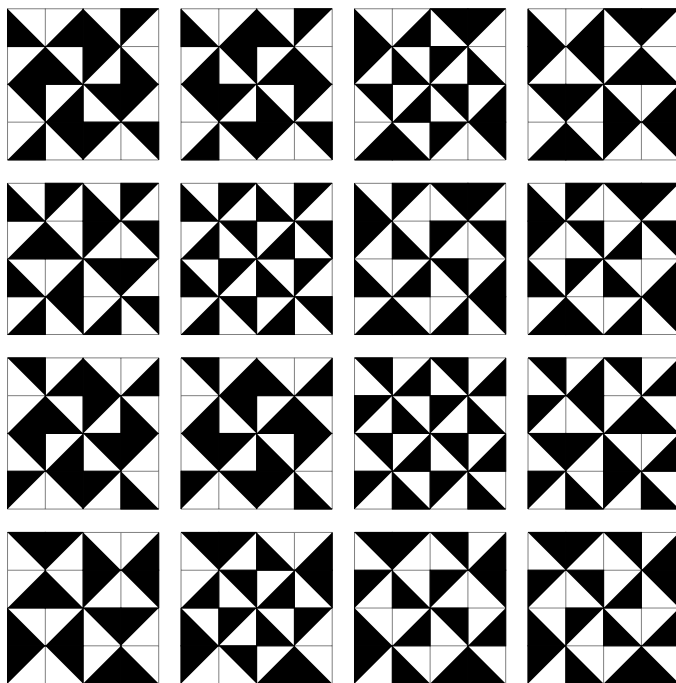
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1210	1212	1230	1232
3010	3012	3030	3032
3210	3212	3230	3232



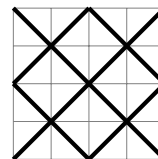
0101



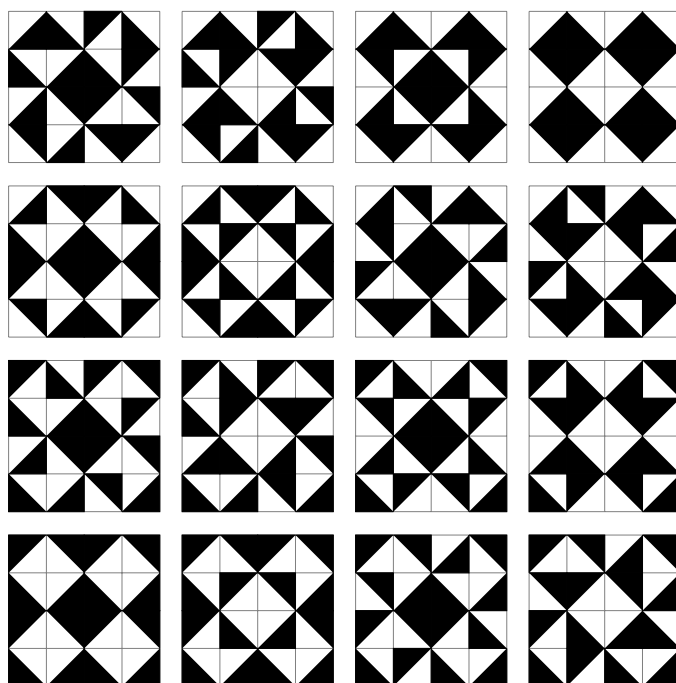
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0301	0303	0321	0323
2101	2103	2121	2123
2301	2303	2321	2323



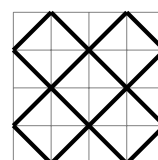
1001



1001	1003	1021	1023
1201	1203	1221	1223
3001	3003	3021	3023
3201	3203	3221	3223



0110

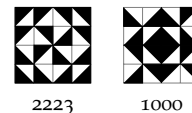
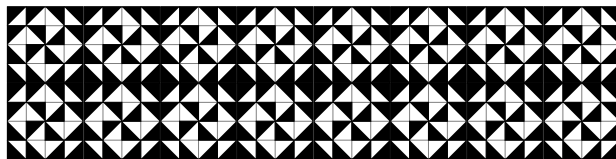


0110	0112	0130	0132
0310	0312	0330	0332
2110	2112	2130	2132
2310	2312	2330	2332

Uniform friezes

Each 4x4 Truchet pattern can be treated like a tile and used in a larger pattern. A uniform *frieze* is a horizontal strip of the same tile pattern repeated. Friezes of 4x4 Truchet pattern tiles with rotational symmetry can be quite striking, and have some interesting characteristics.

A frieze of more than one row of a primary tile reveals a secondary tile pattern that appears as another horizontal strip of 4x4 Truchet tile patterns nestled between the rows of primary tiles. Below, a frieze of 2223 tiles has a secondary pattern of 1000 tiles.



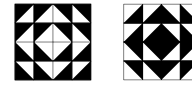
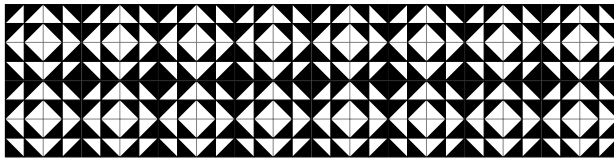
The secondary tile in a frieze pattern is the pattern that has been referred to previously as the *dual* of the original pattern. The dual of a tile pattern is the pattern formed by taking the top right quadrant of the original tile as the prototile of the new tile.

Some tiles are self-dual, and frieze patterns formed by self-dual tiles show a much more uniform pattern, as the extra rows of tiles seemingly nestled between the rows of the original tile are made up of the same original tile. Friezes of self-dual tiles have a third *tertiary* tile pattern with 90° rotational symmetry that appears to overlap between adjacent tiles of the original tile. These tertiary tile patterns are the *skew* of the original tile pattern. Some self-dual friezes are also self-skew, leading to even more uniform patterns.

We can consider the uniform friezes formed by the dual tiles as the same pattern. There are 6 pairs of families where the original and dual are not the same, and these pairs of families yield 16 patterns

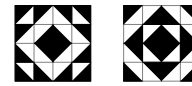
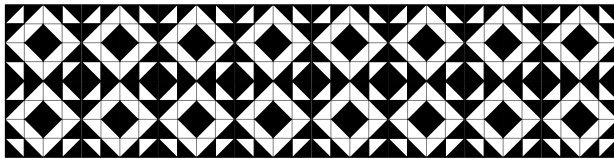
each. The 4 remaining families contain some self-dual patterns, and some patterns that are *opp-dual* (the secondary tile is the opposite tile of the original), also reducing the number of patterns. These 4 remaining families provide 10 distinct frieze patterns each. This means that the 256 tile patterns generate 136 distinct friezes.

Frieze patterns for family 0000 (secondary, 0000)



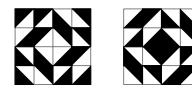
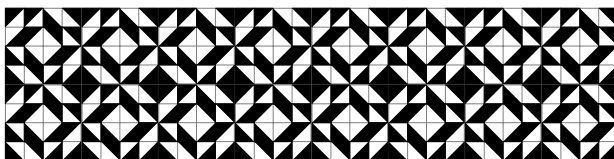
2222

0000



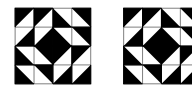
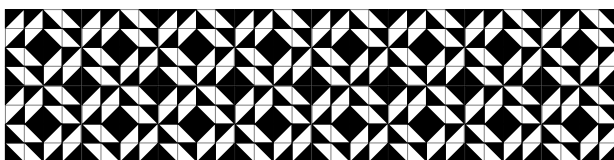
2220

2000



2202

0200

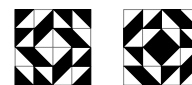
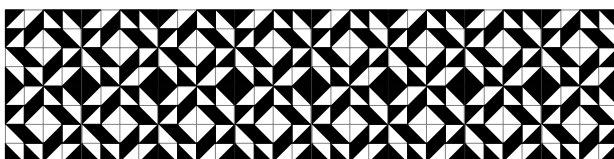


2200

2200

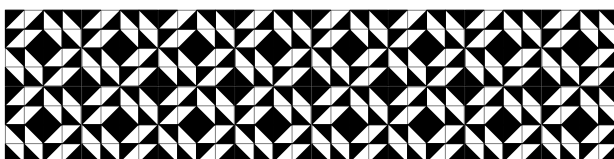


1313



2022

0020

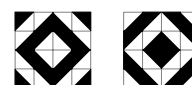
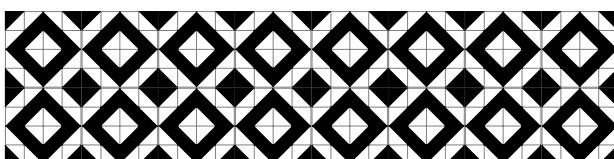


2020

2020

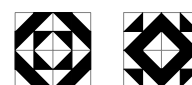
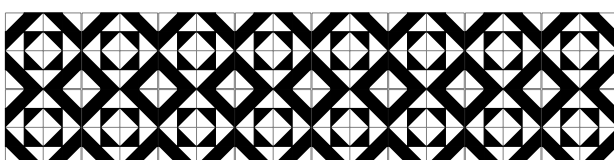


3311



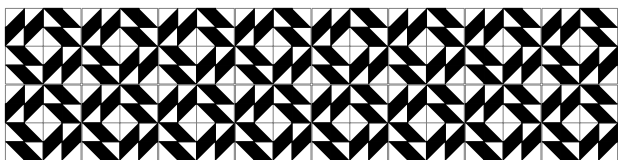
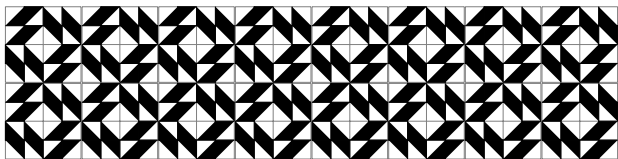
2002

0220



0222

0002



0202



0202



1133



0022

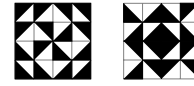
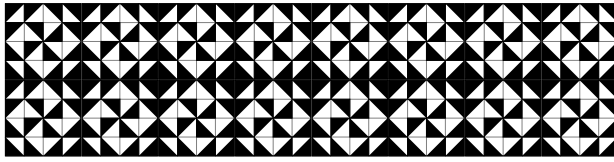


0022



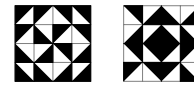
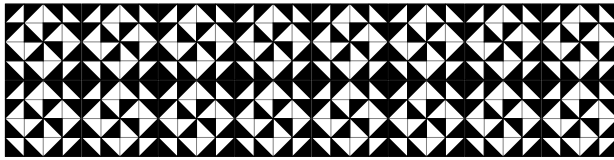
3131

Frieze patterns for family 0001 (secondary, 1000)



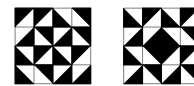
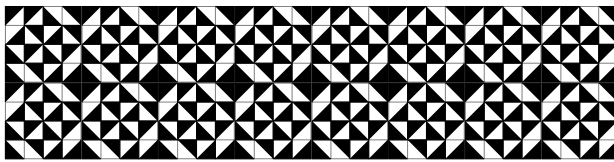
2223

1000



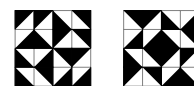
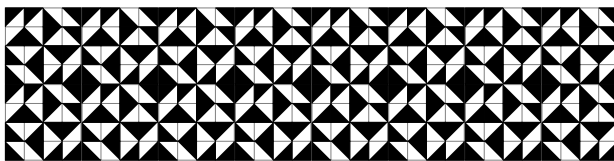
2221

3000



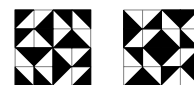
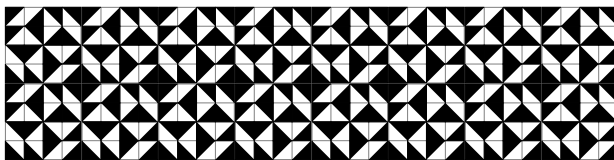
2203

1200



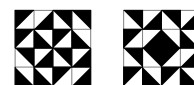
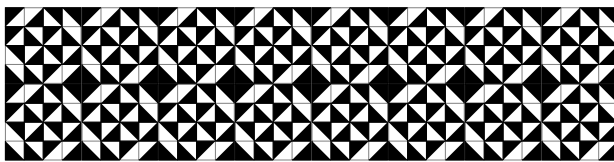
2201

3200



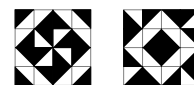
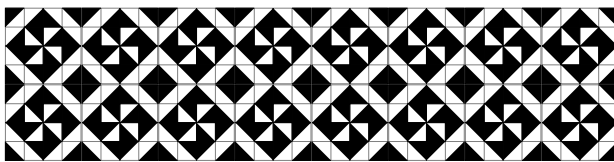
2023

1020



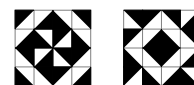
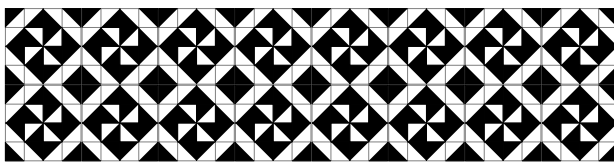
2021

3020



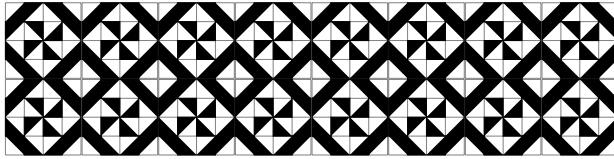
2003

1220



2001

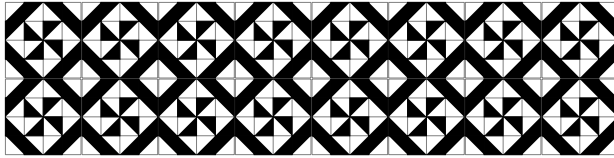
3220



0223



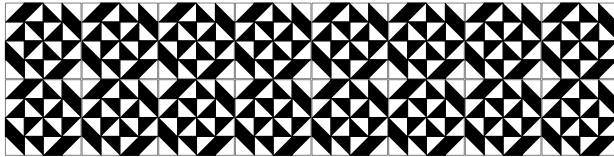
1002



0221



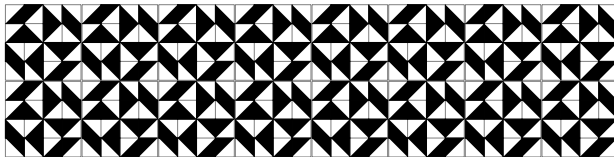
3002



0203



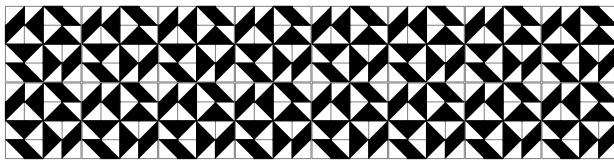
1202



0201



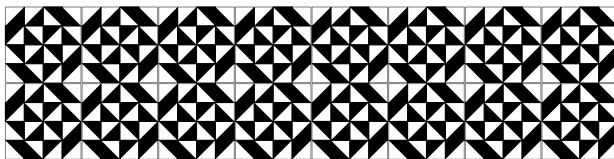
3202



0023



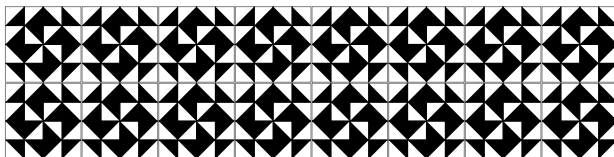
1022



0021



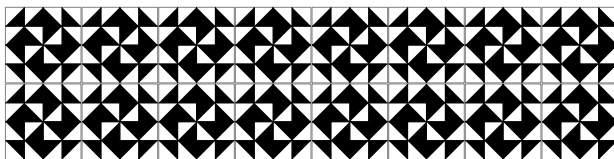
3022



0003



1222

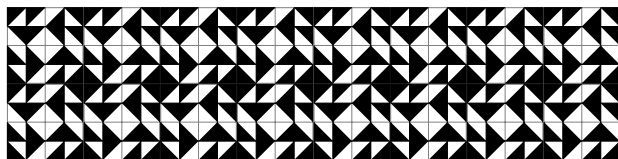


0001



3222

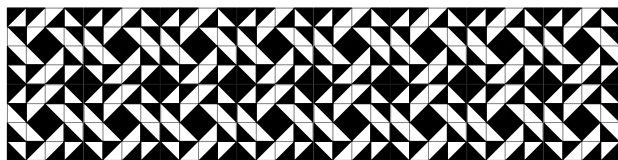
Frieze patterns for family 0010 (secondary, 0100)



2232



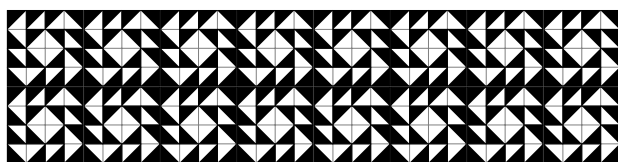
0100



2230



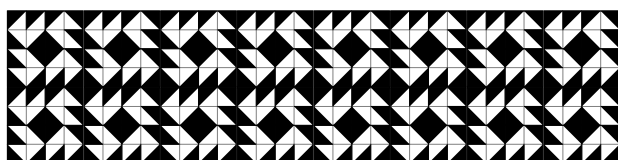
2100



2212



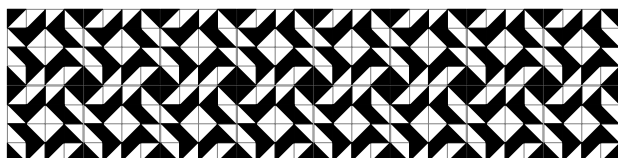
0300



2210



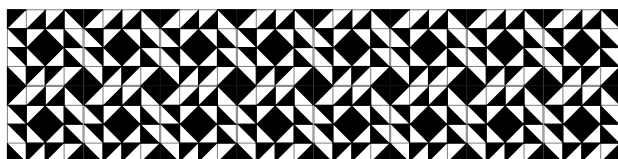
2300



2032



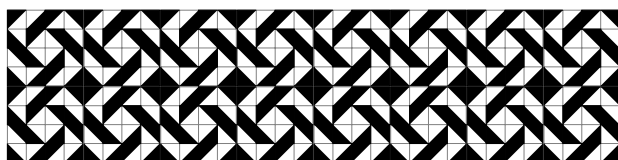
0120



2030



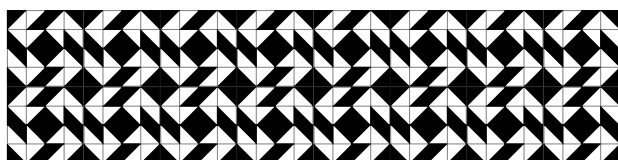
2120



2012



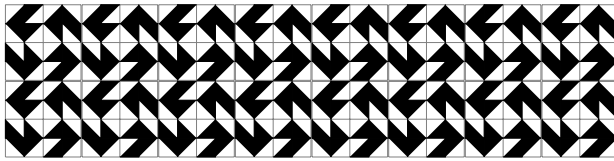
0320



2010



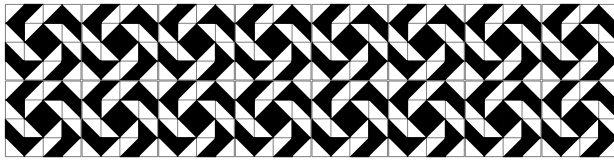
2320



0232



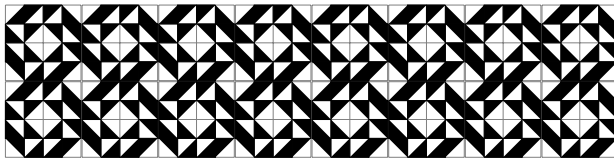
0102



0230



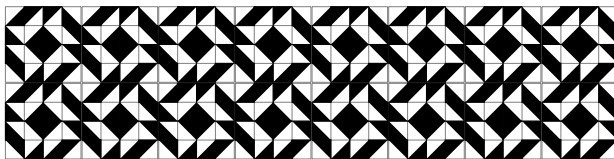
2102



0212



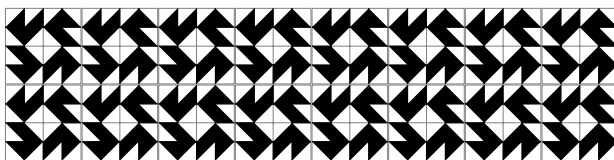
0302



0210



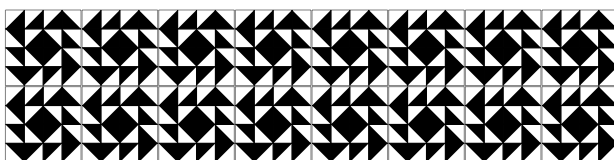
2302



0032



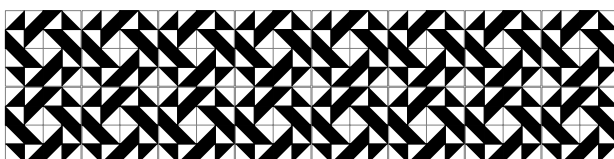
0122



0030



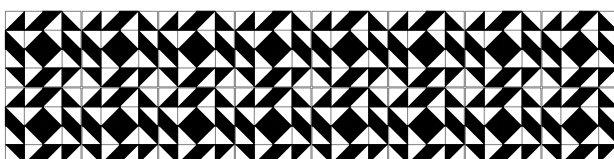
2122



0012



0322

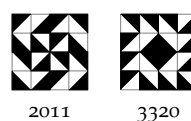
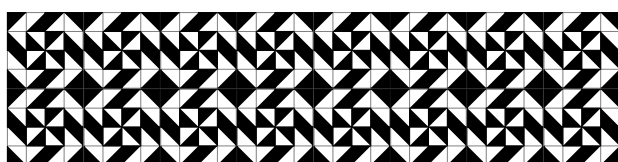
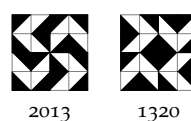
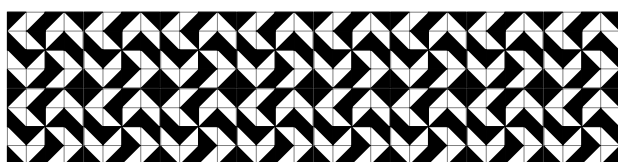
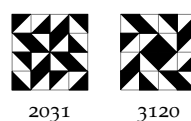
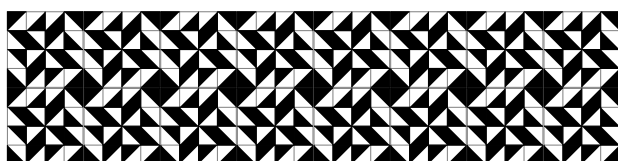
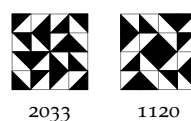
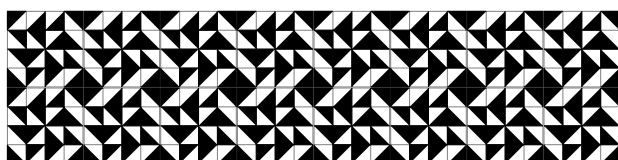
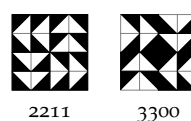
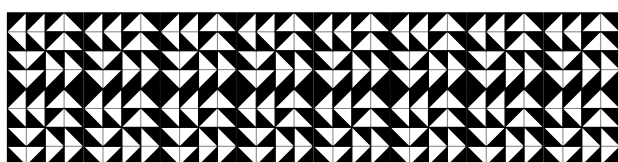
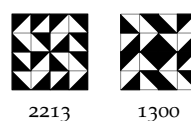
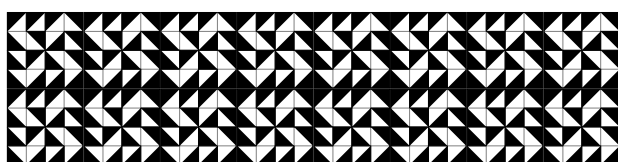
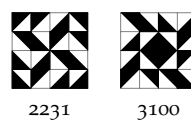
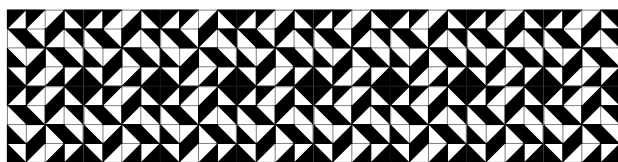
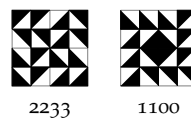
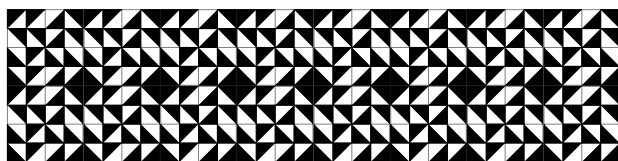


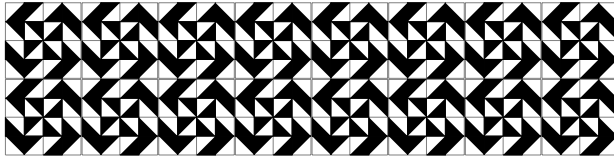
0010



2322

Frieze patterns for family 0011 (secondary, 1100)

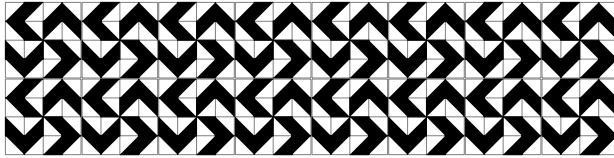




0233



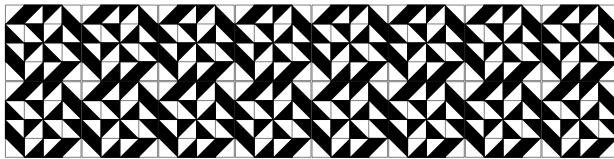
1102



0231



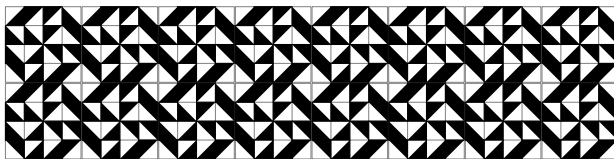
3102



0213



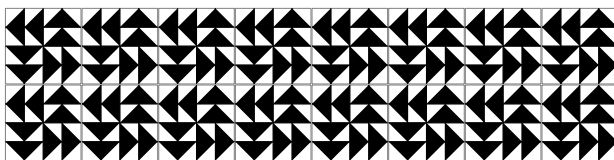
1302



0211



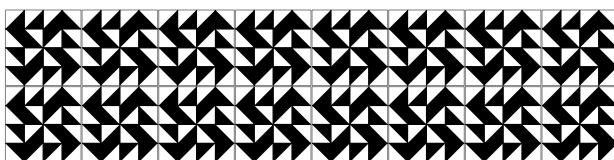
3302



0033



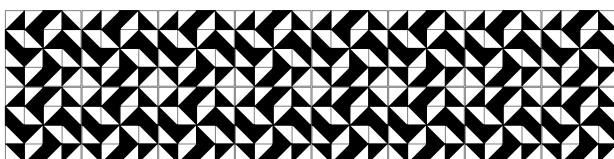
1122



0031



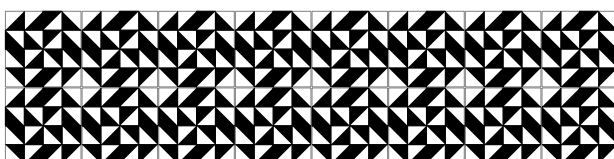
3122



0013



1322

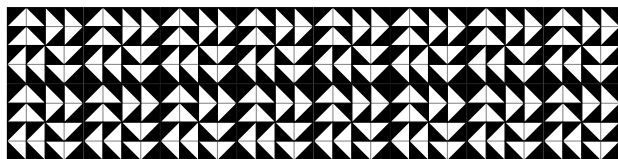


0011



3322

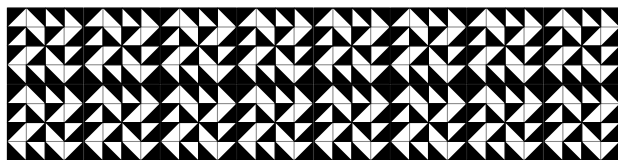
Frieze patterns for family 0101 (secondary, 1010)



2323



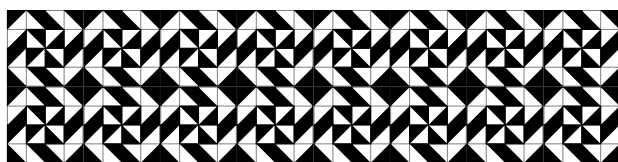
1010



2321



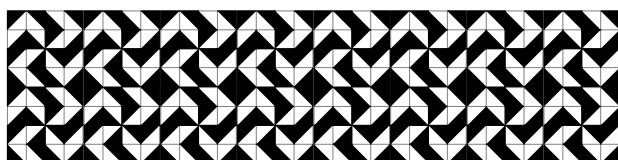
3010



2303



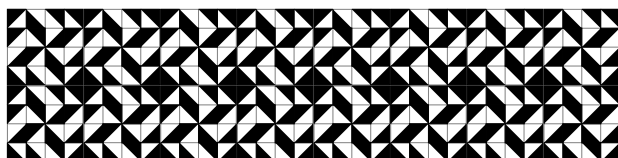
1210



2301



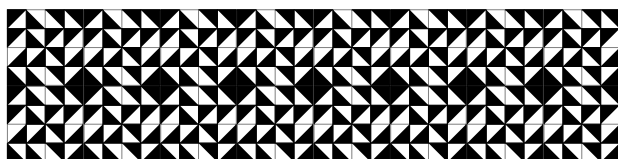
3210



2123



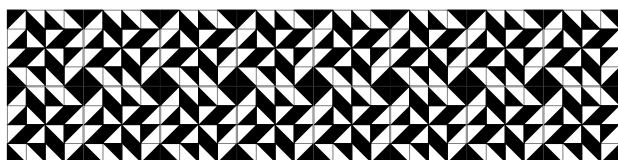
1030



2121



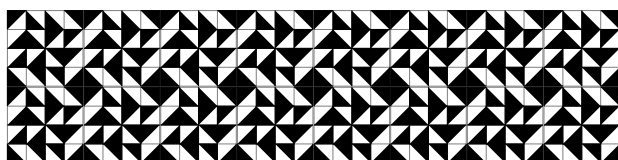
3030



2103



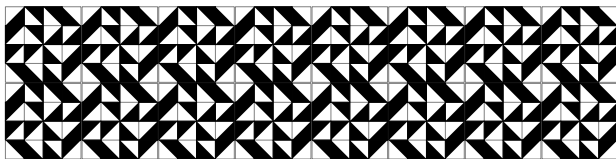
1230



2101



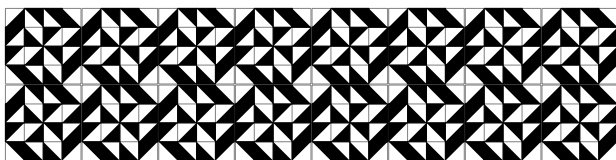
3230



0323



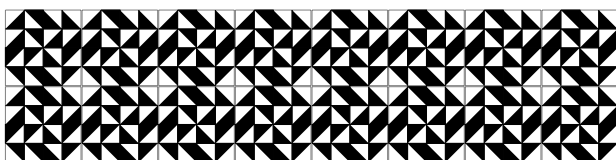
1012



0321



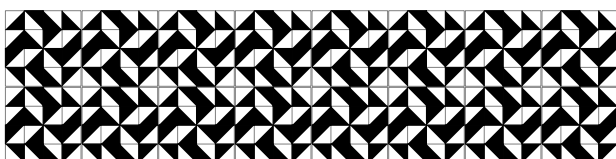
3012



0303



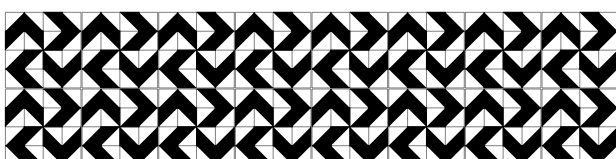
1212



0301



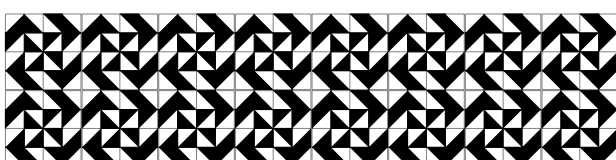
3212



0123



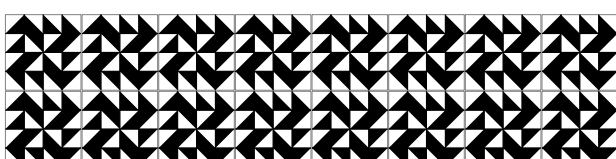
1032



0121



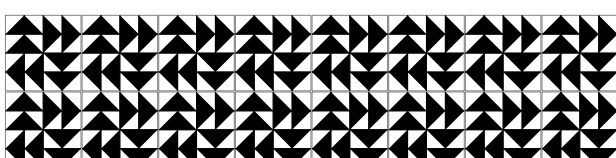
3032



0103



1232

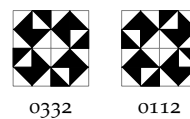
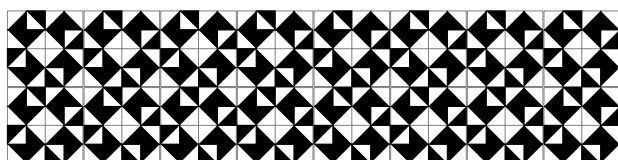
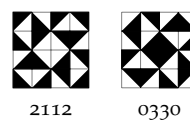
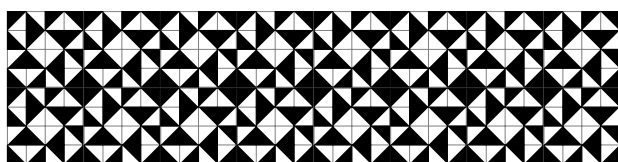
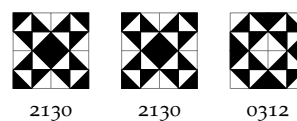
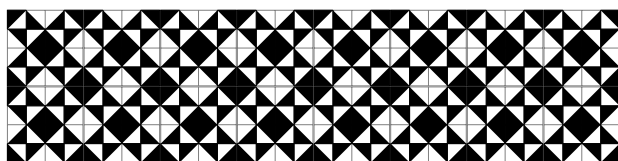
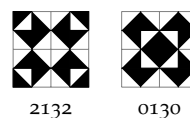
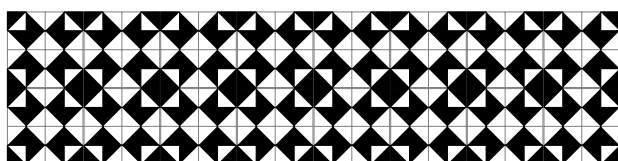
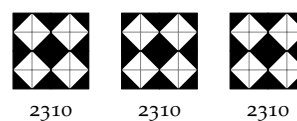
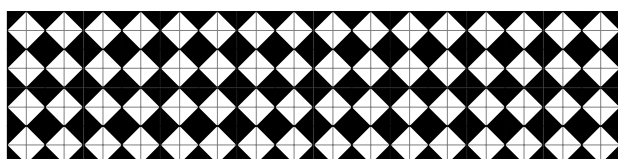
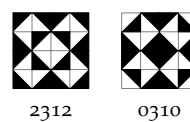
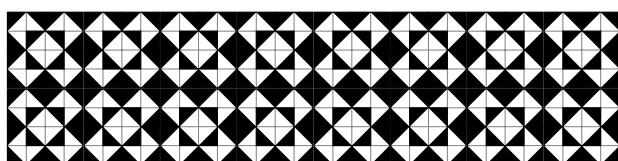
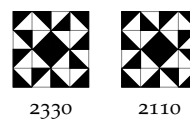
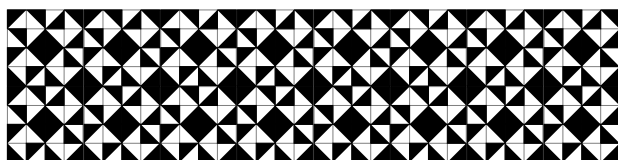
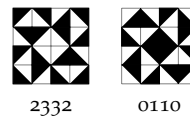
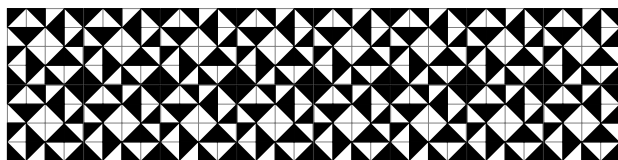


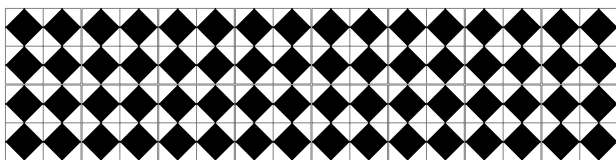
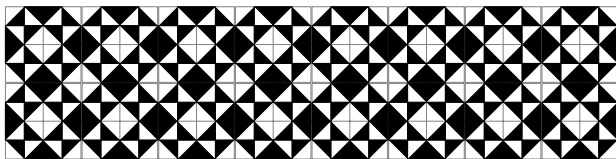
0101



3232

Frieze patterns for family $o110$ (secondary, $o110$)





0312



0312



2130



0132

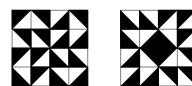
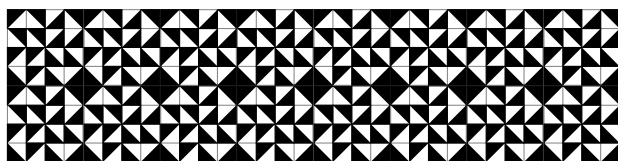


0132



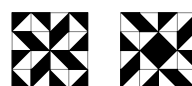
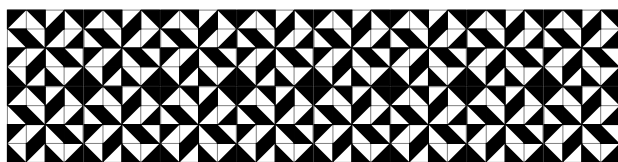
0132

Frieze patterns for family 0111 (secondary, 1110)



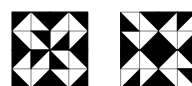
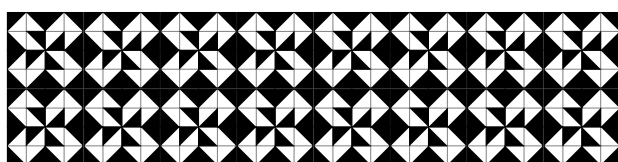
2333

1110



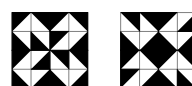
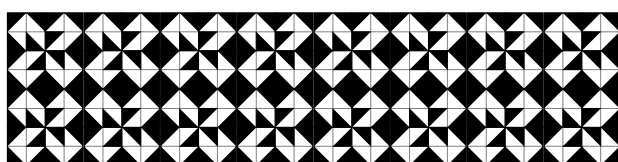
2331

3110



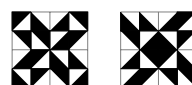
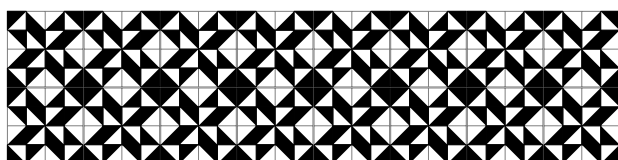
2313

1310



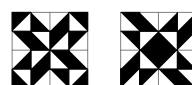
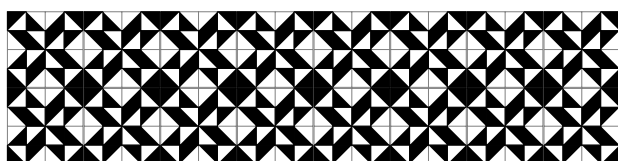
2311

3310



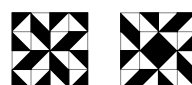
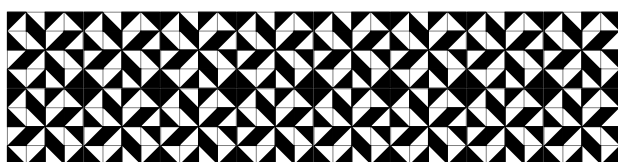
2133

1130



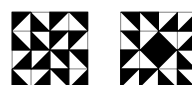
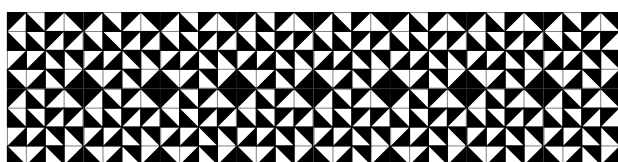
2131

3130



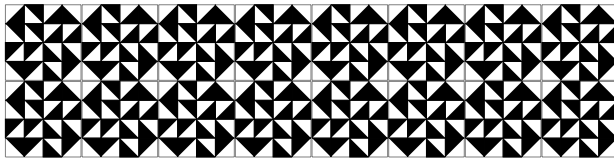
2113

1330



2111

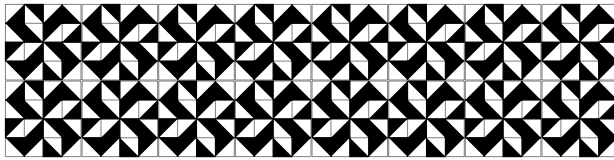
3330



0333



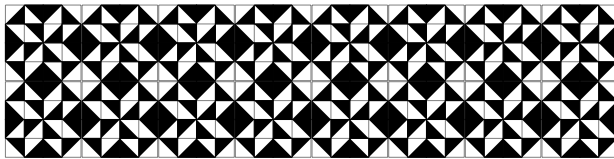
1112



0331



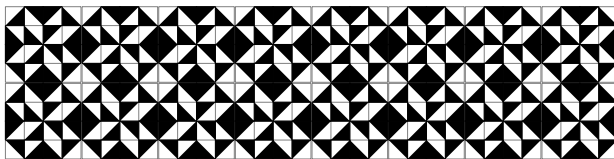
3112



0313



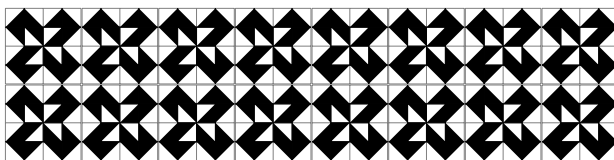
1312



0311



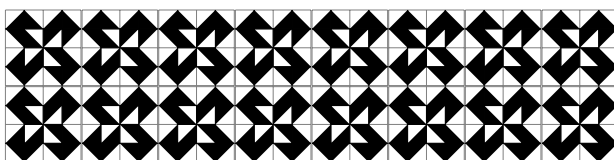
3312



0133



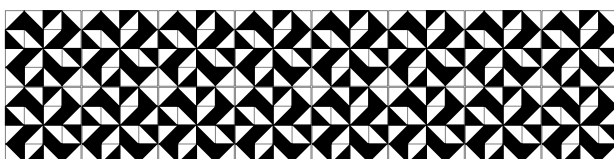
1132



0131



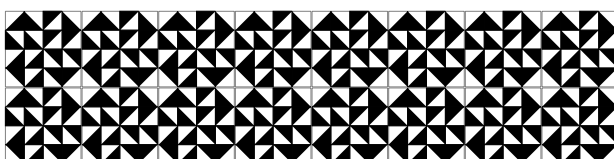
3132



0113



1332

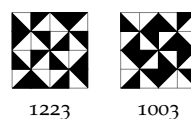
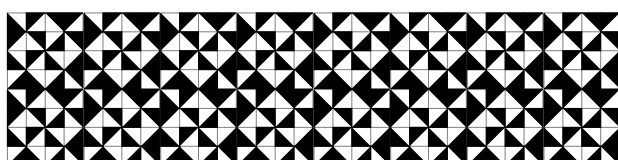
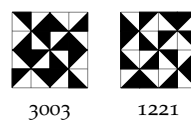
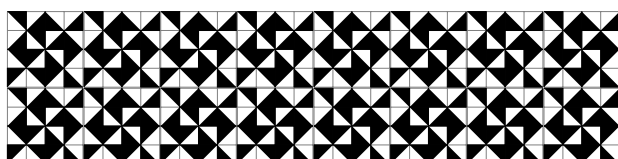
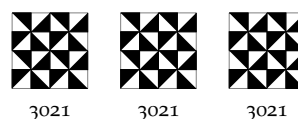
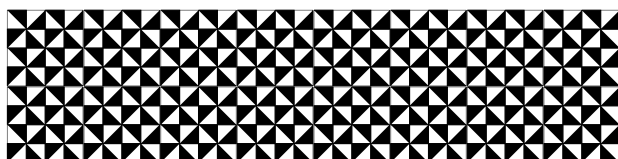
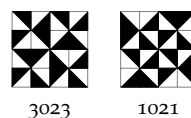
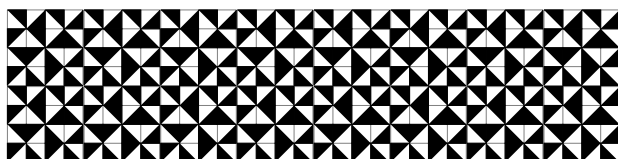
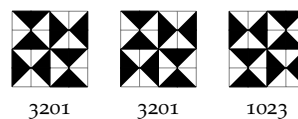
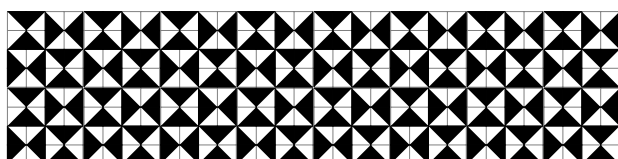
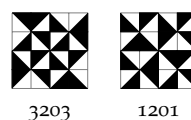
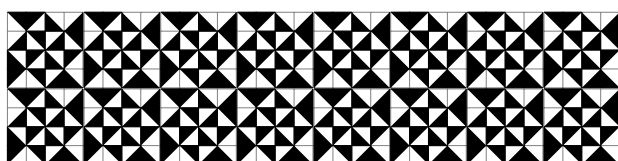
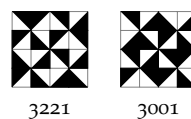
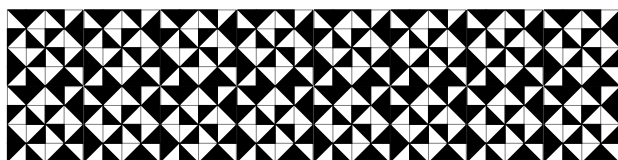
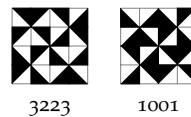
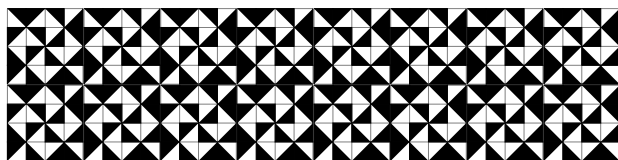


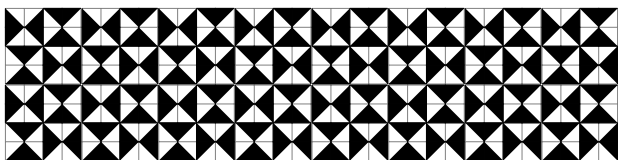
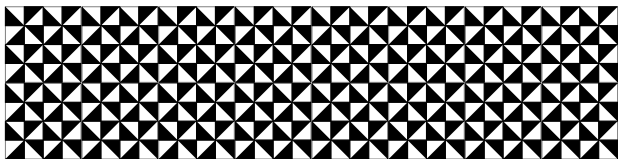
0111



3332

Frieze patterns for family 1001 (secondary, 1001)





1203



1203



1203



1023

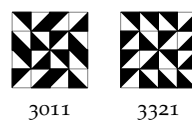
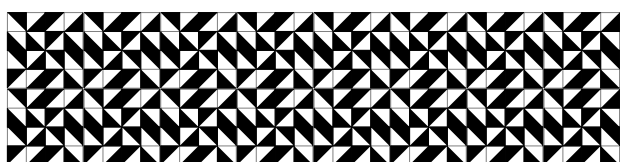
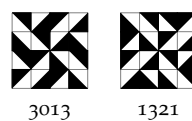
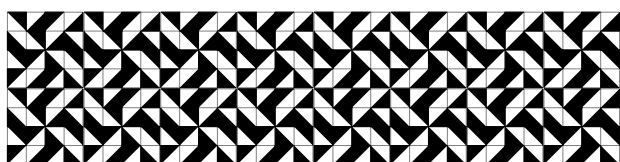
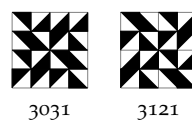
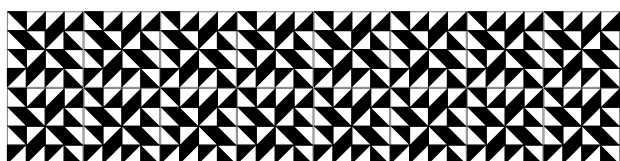
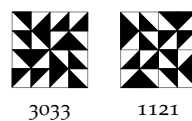
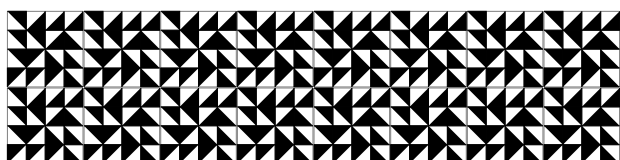
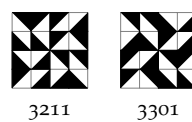
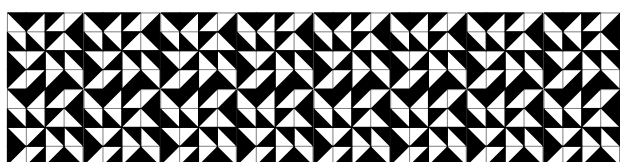
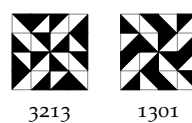
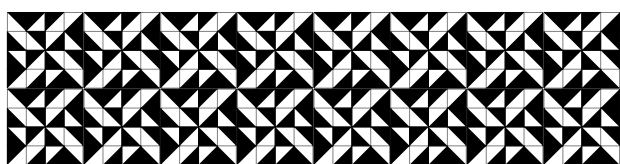
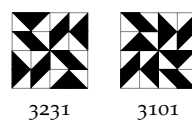
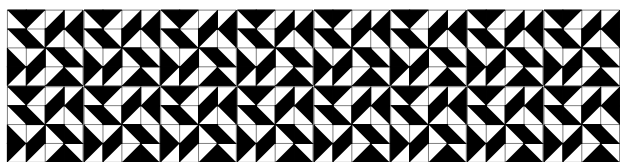
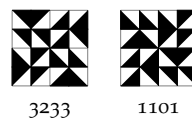
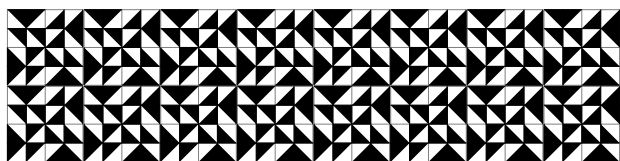


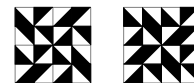
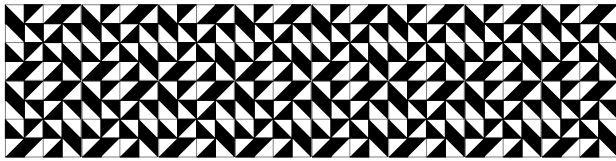
1023



3201

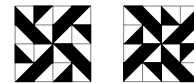
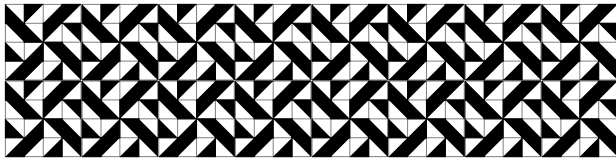
Frieze patterns for family 1011 (secondary, 1101)





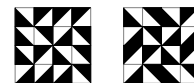
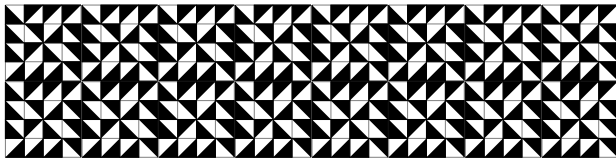
1233

1103



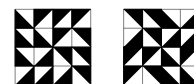
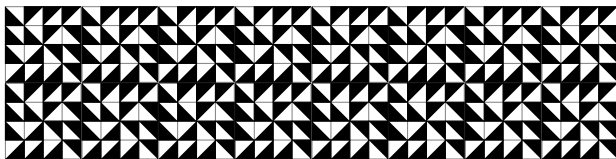
1231

3103



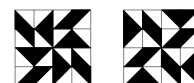
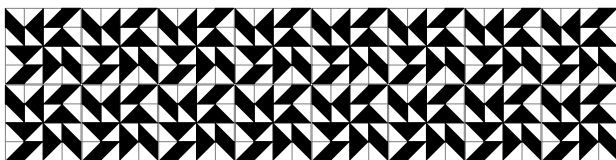
1213

1303



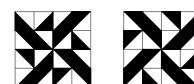
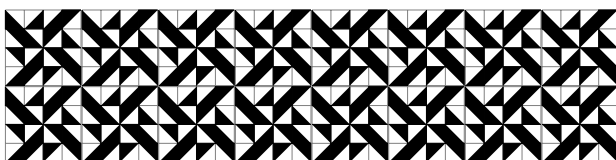
1211

3303



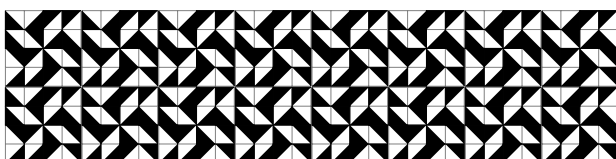
1033

1123



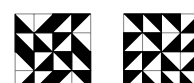
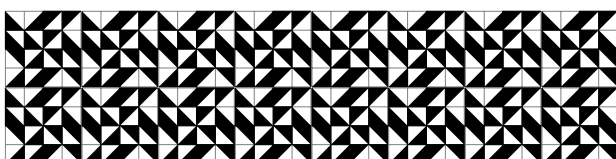
1031

3123



1013

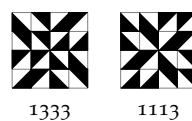
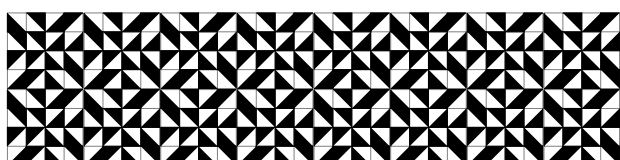
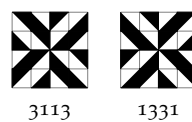
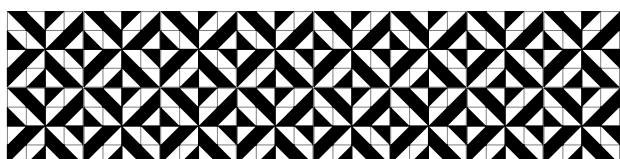
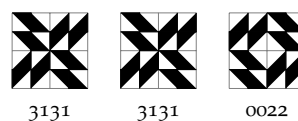
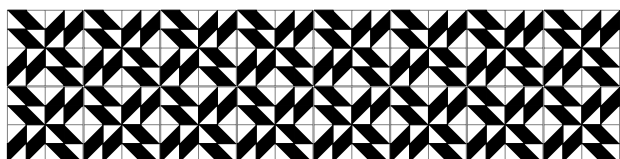
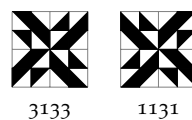
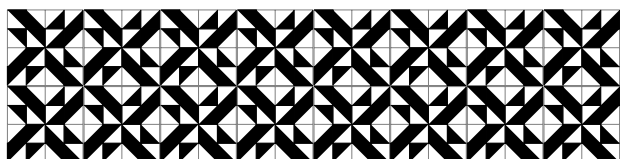
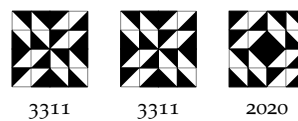
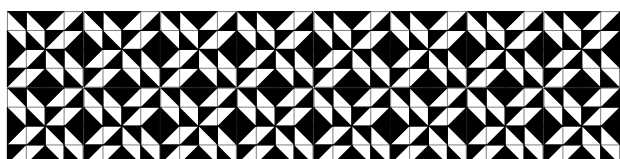
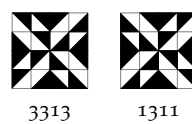
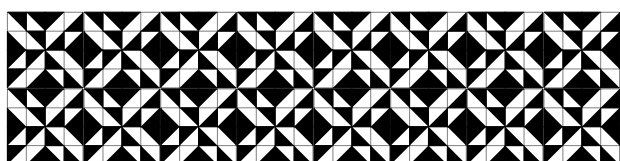
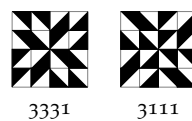
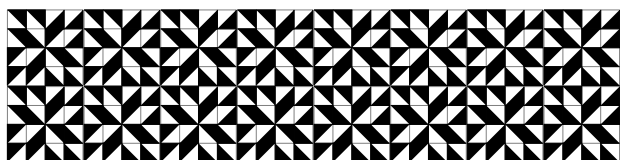
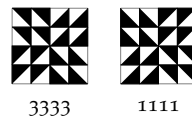
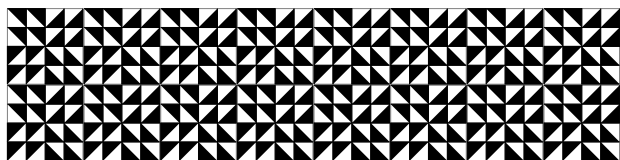
1323

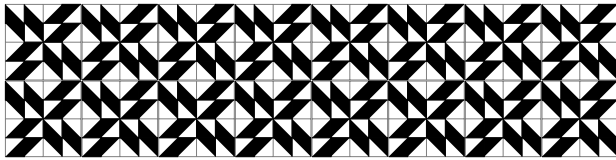
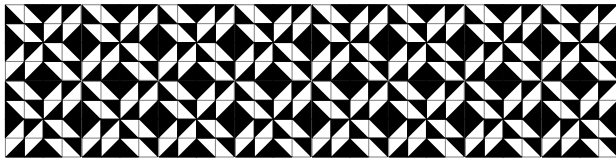


1011

3323

Frieze patterns for family 1111 (secondary, 1111)





1313



1313



2200



1133



1133



0202

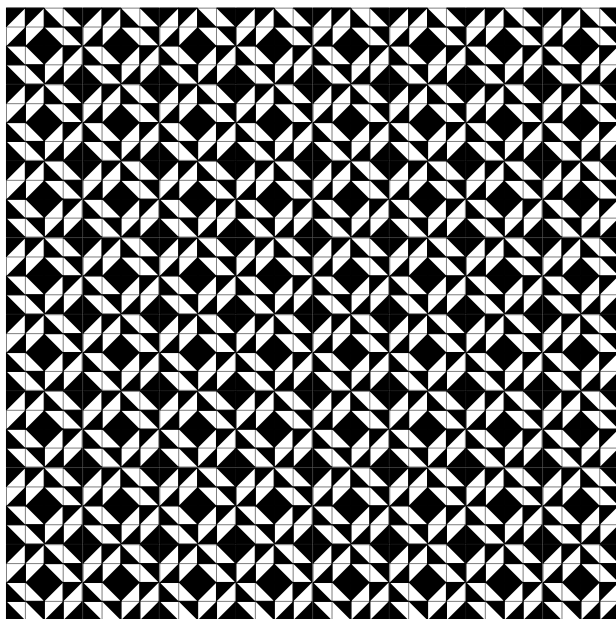
Some designs

This chapter presents some designs made using one or two types of the 4×4 Truchet tile patterns with 90° rotational symmetry.

Self-dual designs

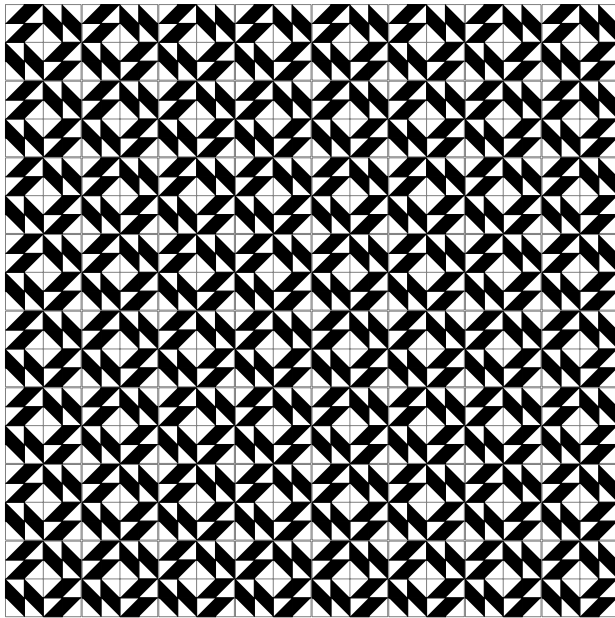
These designs extend the friezes of self-dual patterns to create patterns where copies of the main tile appear as secondary tiles. The other patterns you can see are provided by the tertiary skew tile patterns that seem to overlap between the adjacent pairs of tiles.

Design using 2200



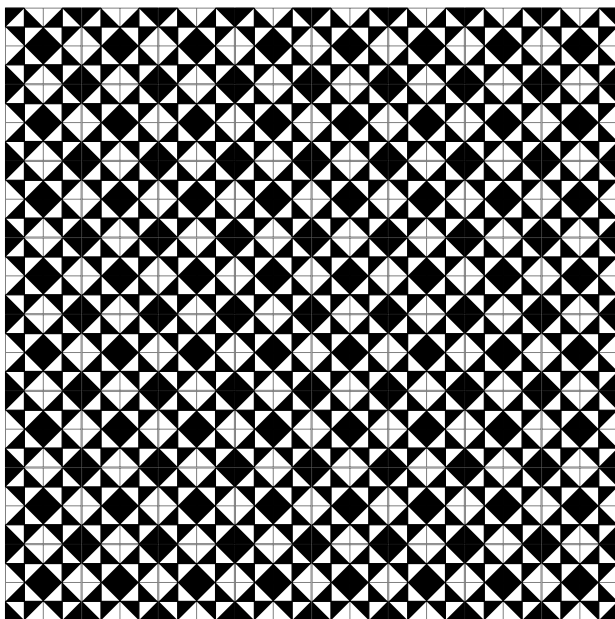
2200

Design using 0202



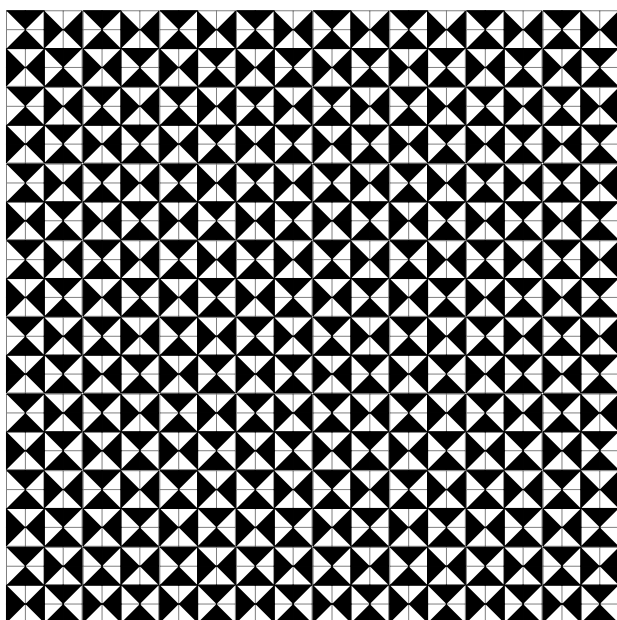
0202

Design using 2130



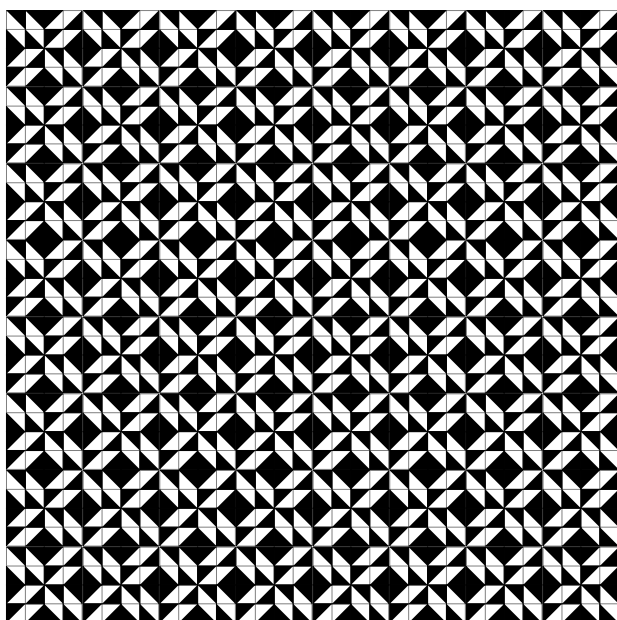
2130

Design using 3201



3201

Design using 3311

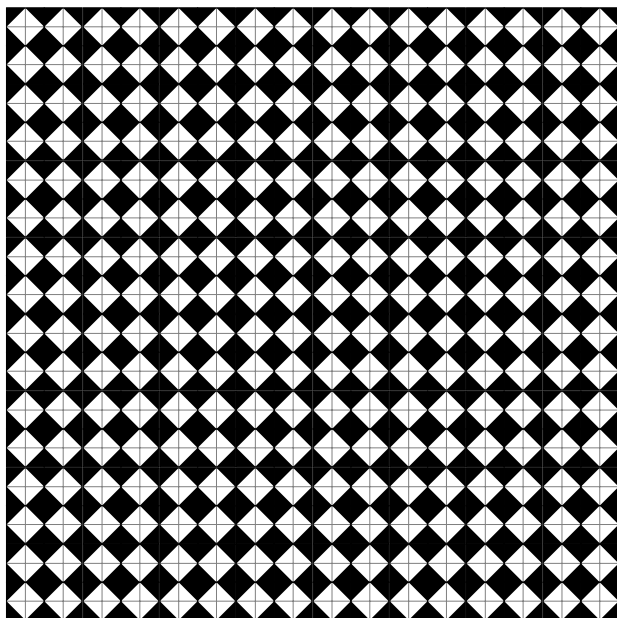


3311

Strongly uniform designs

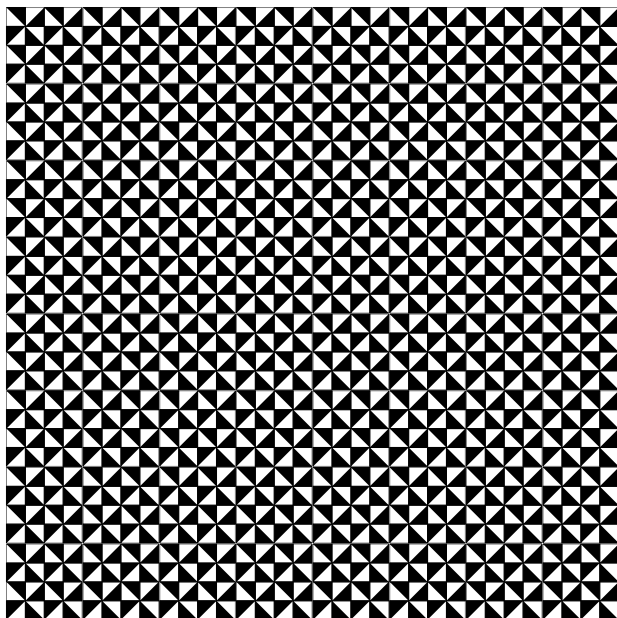
These self-dual tiles are also self-skew, so the pattern that emerges from placing them together is strongly uniform.

Design using 2310



2310

Design using 3021



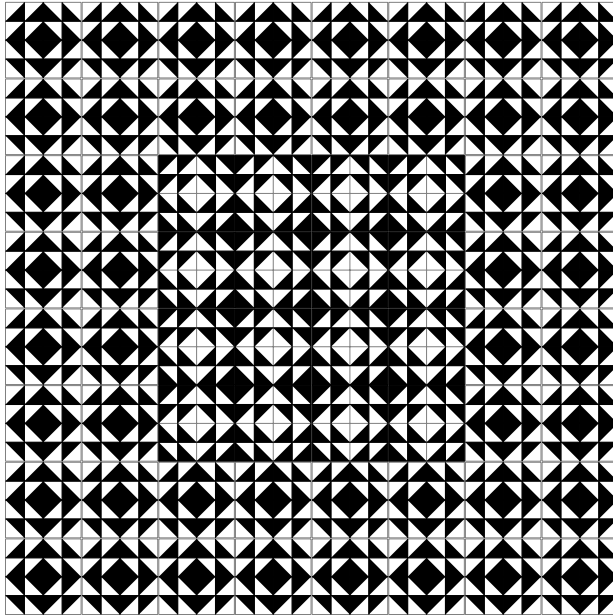
3021

Op-dual designs

These designs include two primary tiles, One tile type is placed in the center of the design in a 4×4 square, and a second tile type is used to create a 2-tile wide border around the central square. The tiles

chosen are opposites of each other, and they are also opp-dual tiles, so we see secondary effects where each tile type appears in between the rows of the other.

Design using 2222 and 0000

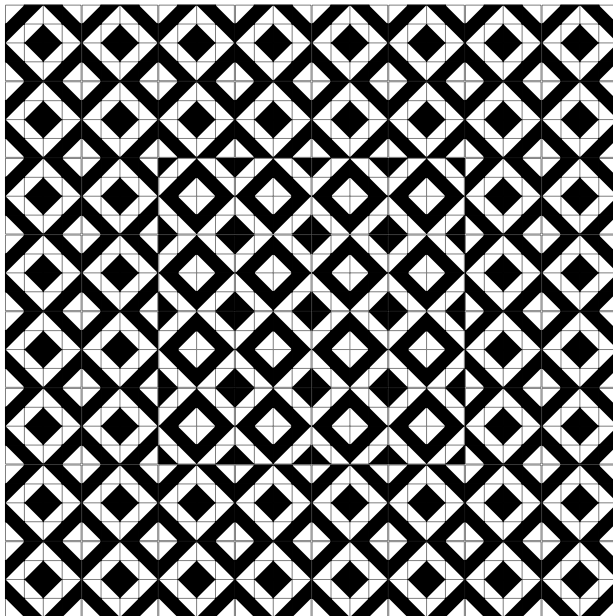


2222



0000

Design using 2002 and 0220

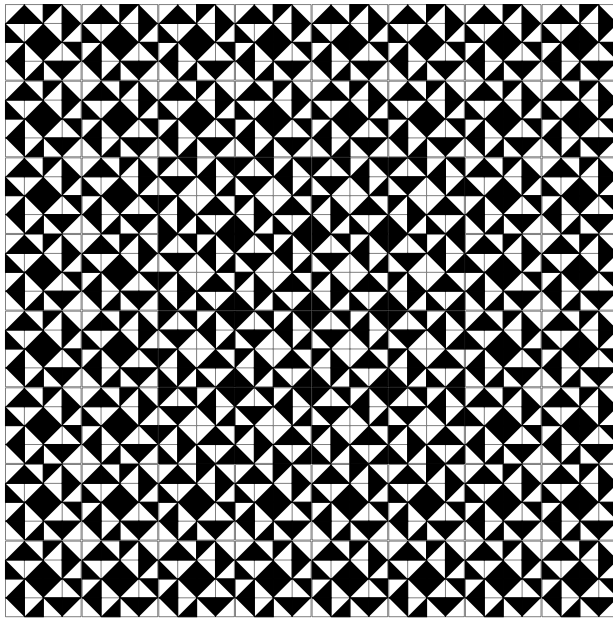


2002



0220

Design using 2332 and 0110

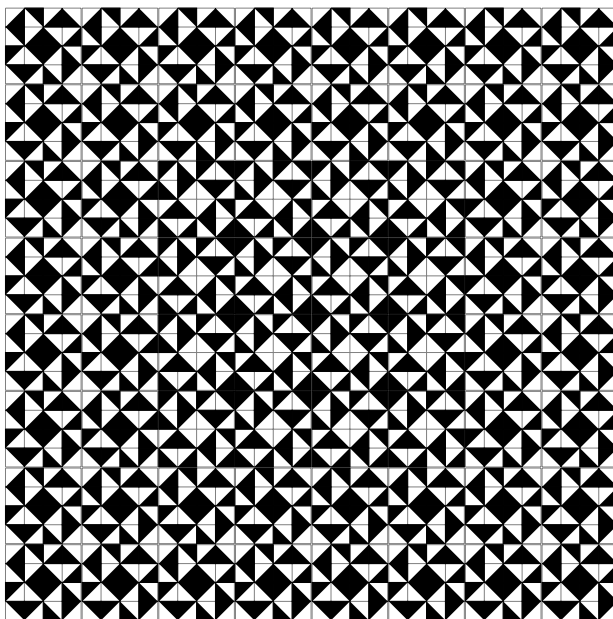


2332



0110

Design using 2112 and 0330

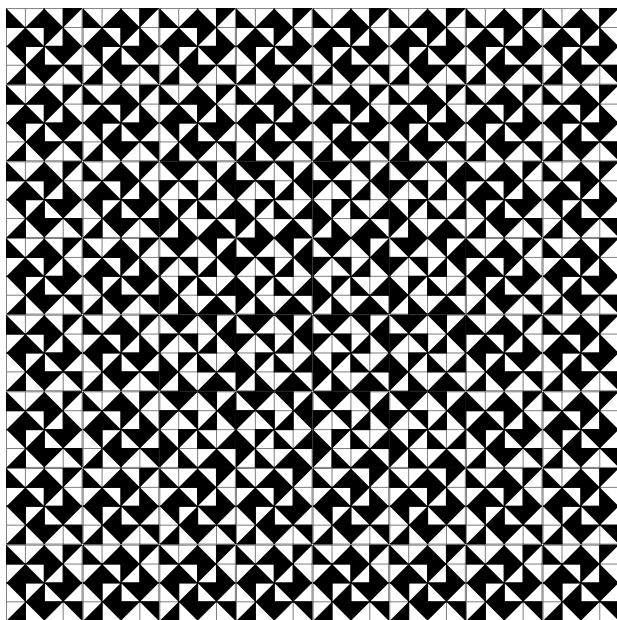


2112



0330

Design using 3223 and 1001

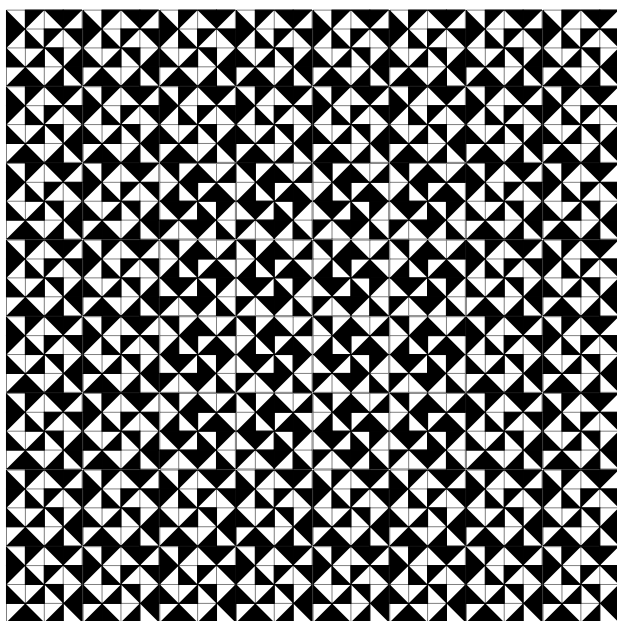


3223



1001

Design using 3003 and 1221

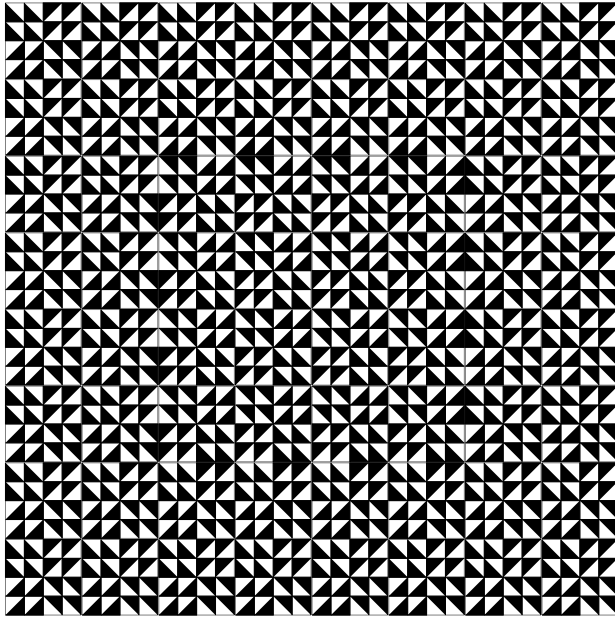


3003



1221

Design using 3333 and 1111

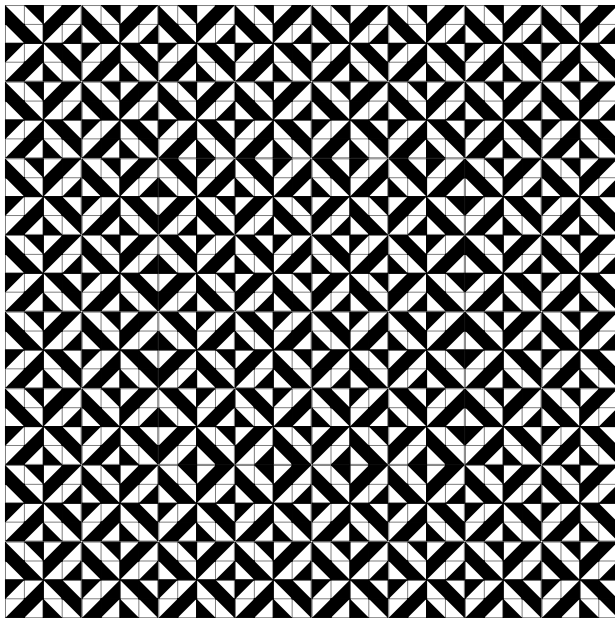


3333



1111

Design using 3113 and 1331



3113

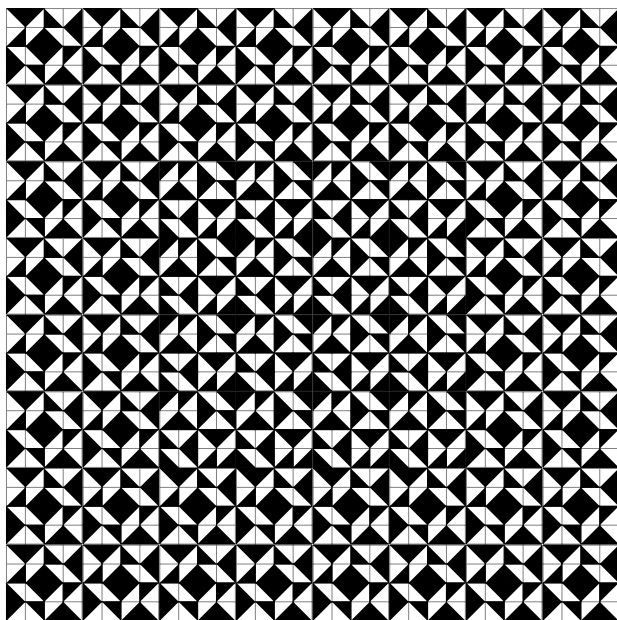


1331

Designs with duals

Like the previous section, these patterns feature a square of one tile type surrounded by another. These tiles are duals of each other, but are not opposite.

Design using 2201 and 3200

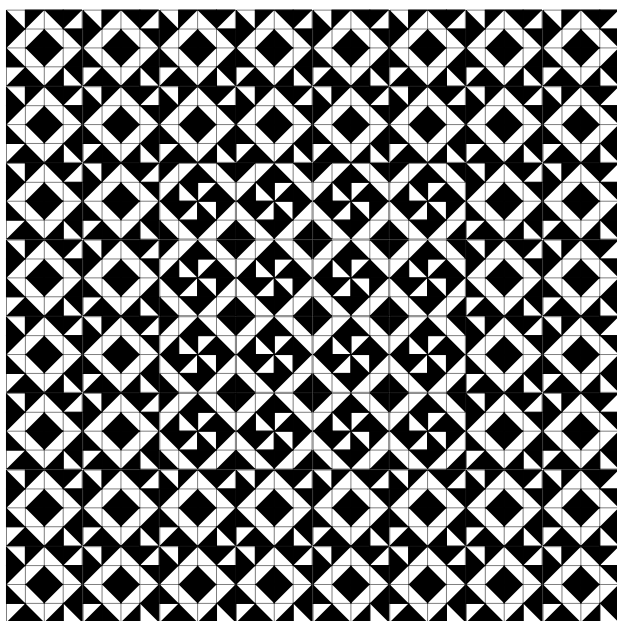


2201



3200

Design using 2003 and 1220

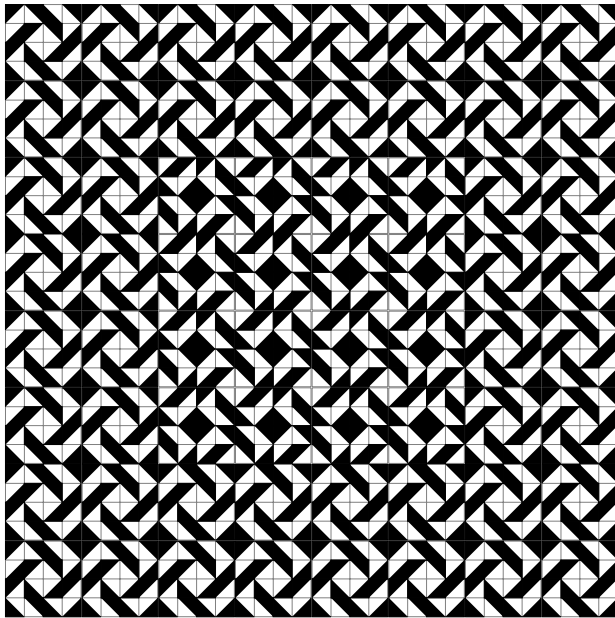


2003



1220

Design using 0210 and 2302

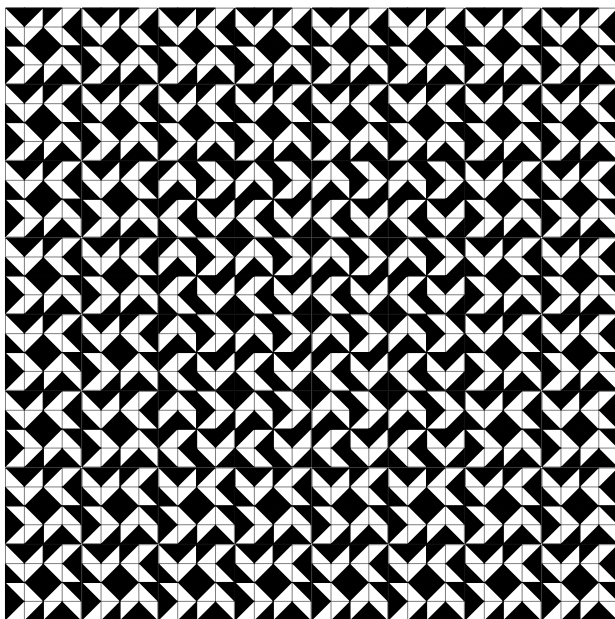


0210



2302

Design using 2301 and 3210

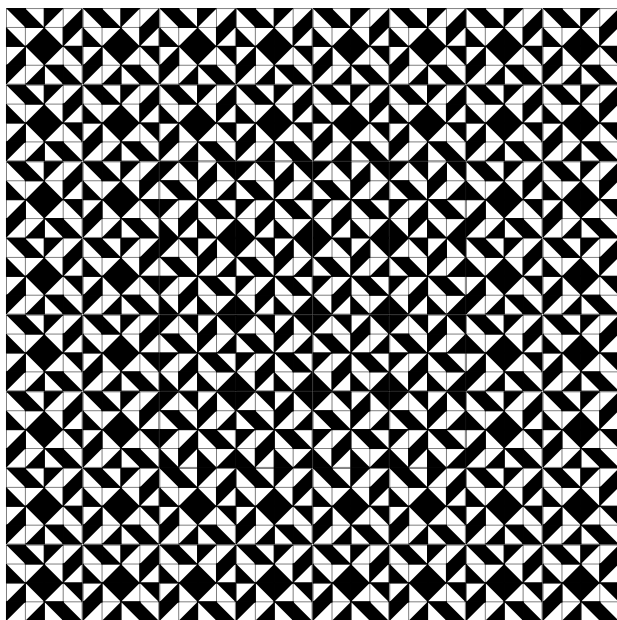


2301



3210

Design using 2331 and 3110

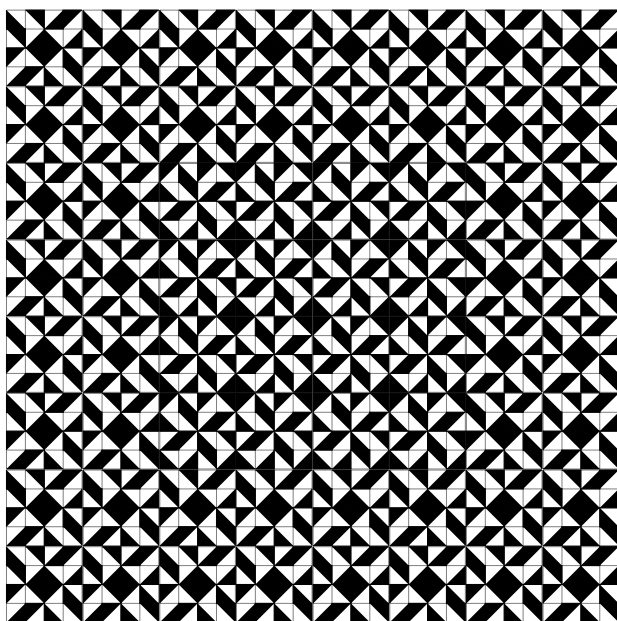


2331



3110

Design using 2113 and 1330

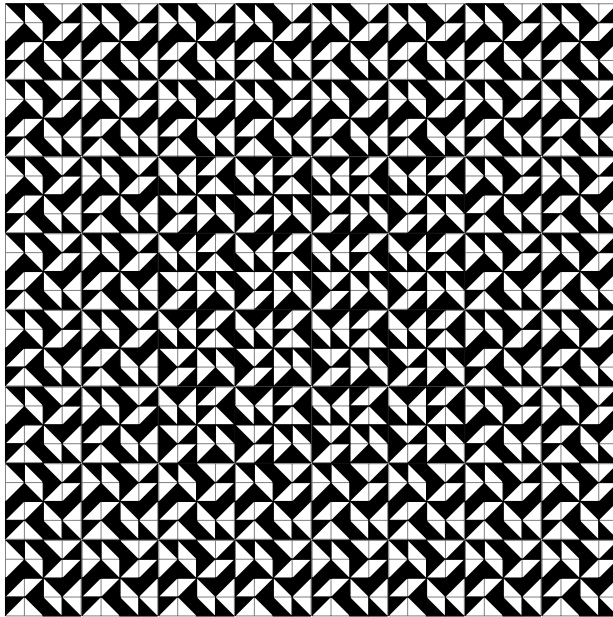


2113



1330

Design using 3211 and 3301

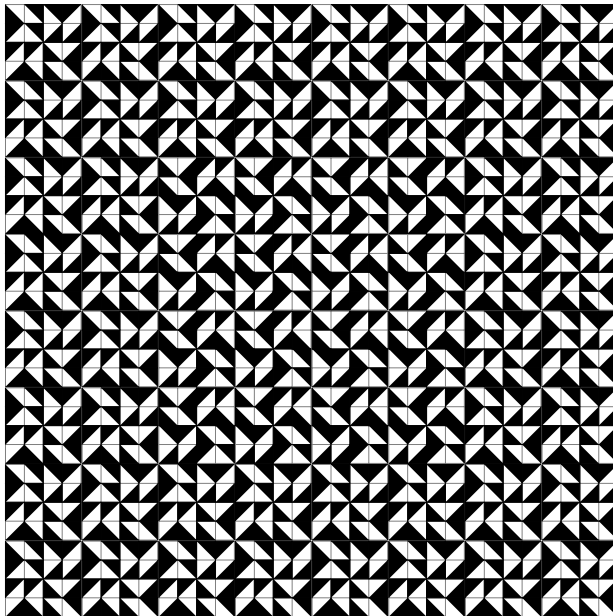


3211



3301

Design using 1013 and 1323



1013

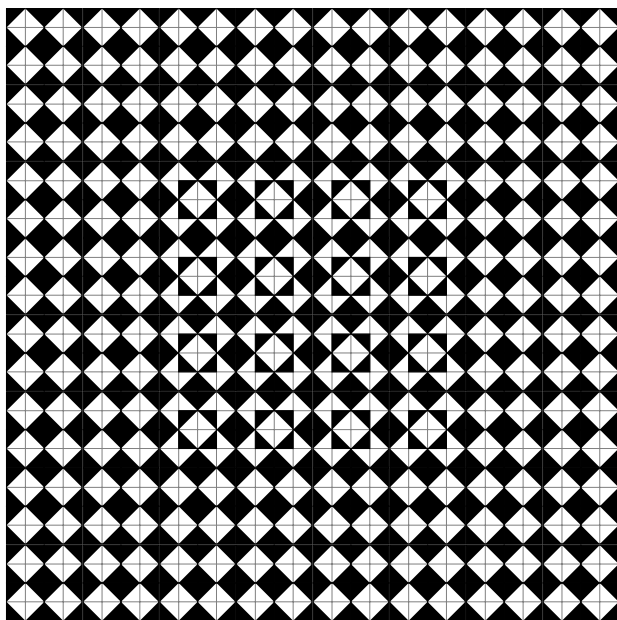


1323

Contrasting designs

The tiles in these square and frame patterns were chosen for their contrast with one another.

Design using 2312 and 2310

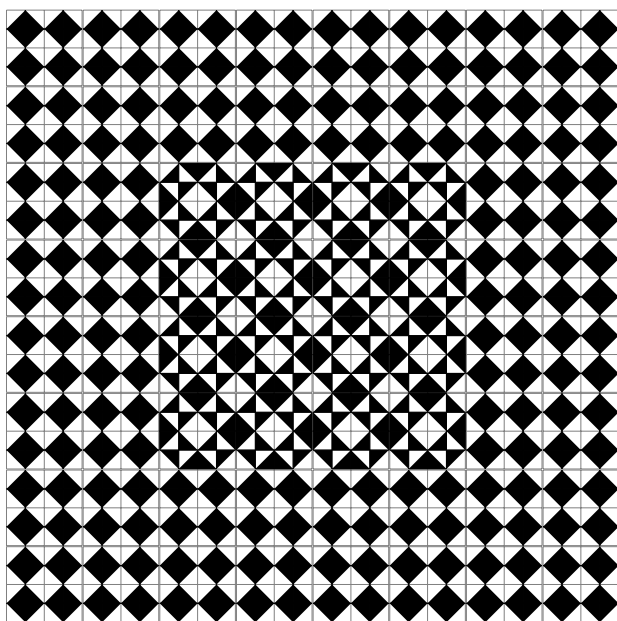


2312



2310

Design using 0312 and 0132

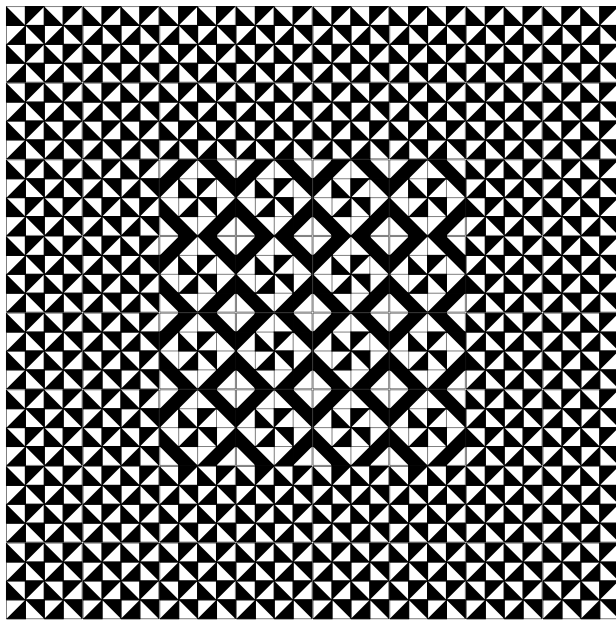


0312



0132

Design using 0221 and 1203

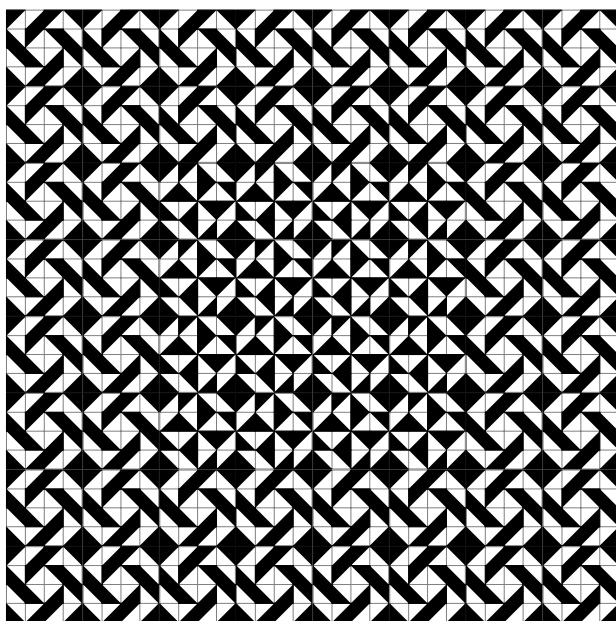


0221



1203

Design using 2201 and 2012

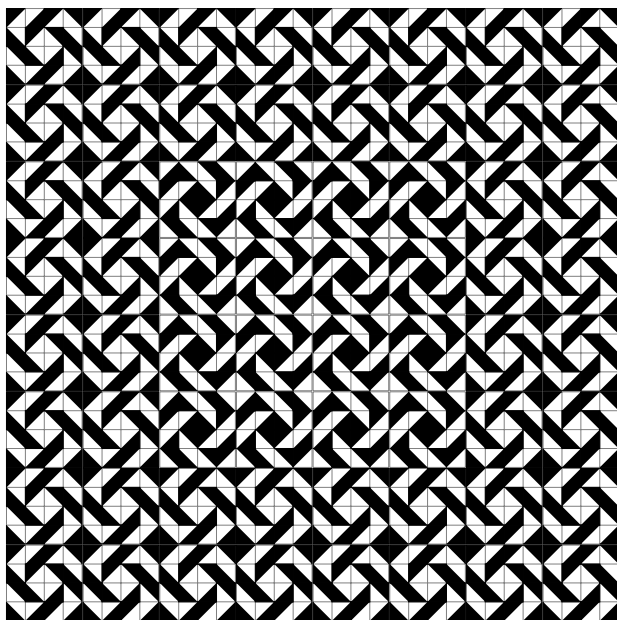


2201



2012

Design using 0120 and 2012

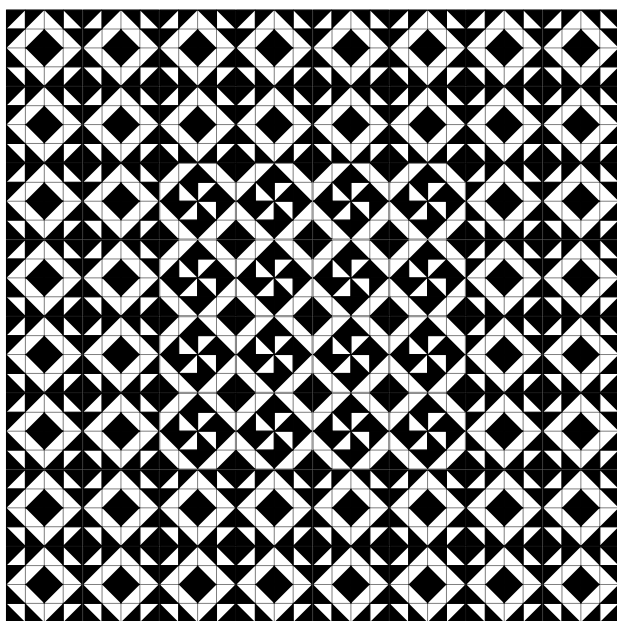


0120



2012

Design using 2003 and 2220

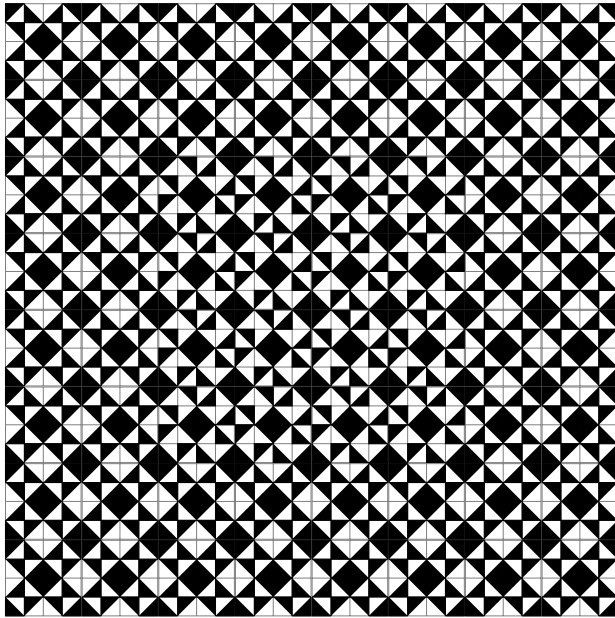


2003



2220

Design using 2330 and 2130

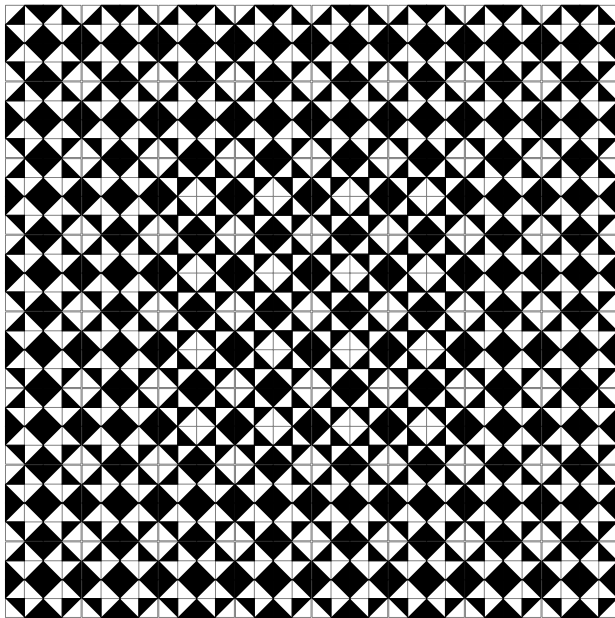


2330



2130

Design using 0312 and 0310



0312



0310

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