

DAN MACKINNON

TRUCHET

4×4 patterns with four-fold rotational symmetry

[HTTPS://GITHUB.COM/DMACKINNON1/TRUCHET-BOOK](https://github.com/dmackinnon1/truchet-book)

Current printing, April 30, 2025

Source, <https://github.com/dmackinnon1/Truchet-Book>

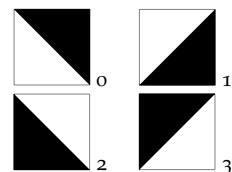
Contact, dmackinnon1@gmail.com

Introduction

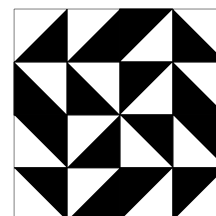
A plain square has four-fold rotational symmetry: it looks the same after being rotated by 90° . Decorating a square can change its appearance and break this symmetry, so that the square looks different when rotated. Traditionally, Truchet squares are rendered asymmetrical by being divided by a diagonal line and coloured differently on either side of the diagonal. With this decoration, a Truchet square loses the plain square's rotational symmetry and can be rotated to four distinguishable positions. Patterns can be formed by tiling a surface with Truchet squares, rotating individual squares into different positions to create repeated motifs. Among the more pleasing patterns that can be made with Truchet squares are ones that express a restored symmetry that the individual squares lack.

This booklet presents a complete listing of 4×4 Truchet tiling patterns with four-fold (90°) rotational symmetry (256 patterns). Treating these 4×4 patterns as tiles themselves allows for larger decorative patterns to be constructed from them. For example, a uniform frieze made from a single 4×4 tile can actually produce interesting secondary patterns which help illustrate some interesting relationships that exist among the tile patterns.

Each 4×4 Truchet tile pattern with rotational symmetry has a core 2×2 pattern in one of its quadrants that is rotated to produce the overall pattern. In this booklet, the core pattern, or prototile, is assumed to be in the lower left. Each pattern can be identified as a sequence of 4 digits (a, b, c, d), or more succinctly, $abcd$, that list the rotational positions of each square in the lower left quadrant. This sequence $abcd$ will be referred to as the *signature* of the tile pattern.



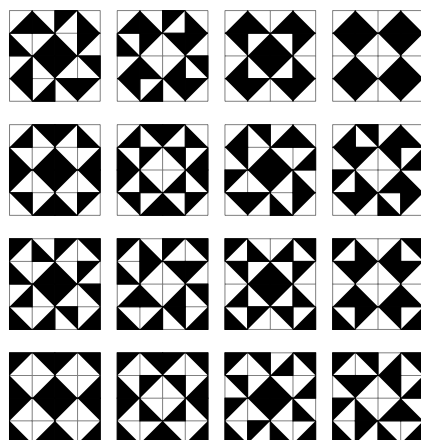
a	d	c	b
c	b	b	d
b	d	d	c
a	c	b	a



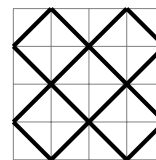
The 0011 pattern

Pattern families

We can group 4×4 Truchet tile patterns with rotational symmetry into families where tile patterns are in the same family if they are identical with the colouring removed, leaving behind just the diagonal line in each square. The squares that make up a colourless family pattern have two-fold rotational symmetry, so the signature that represents the family of a tile pattern can be found by taking the signature of the tile pattern *modulo* 2. For example, the 16 tile patterns below are all members of the 0110 family.

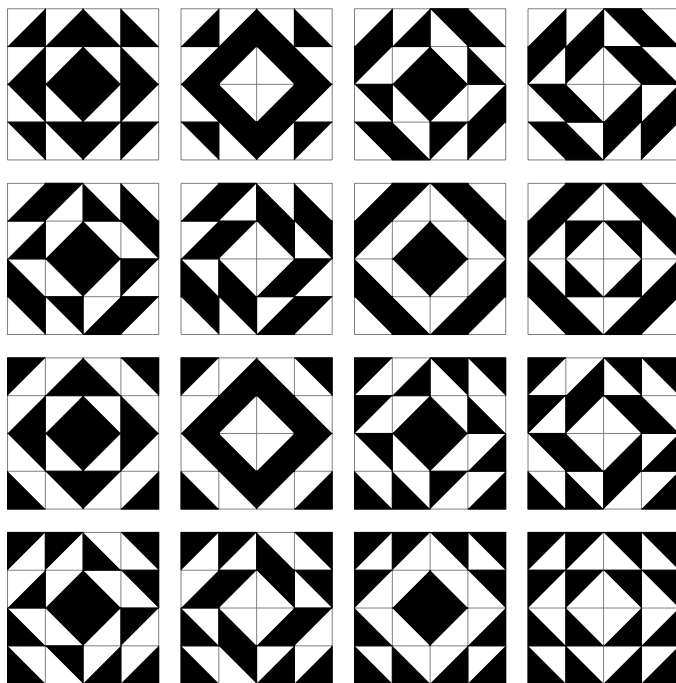


The 0110 pattern family

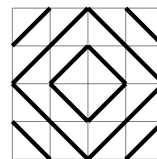


The 0110 family pattern

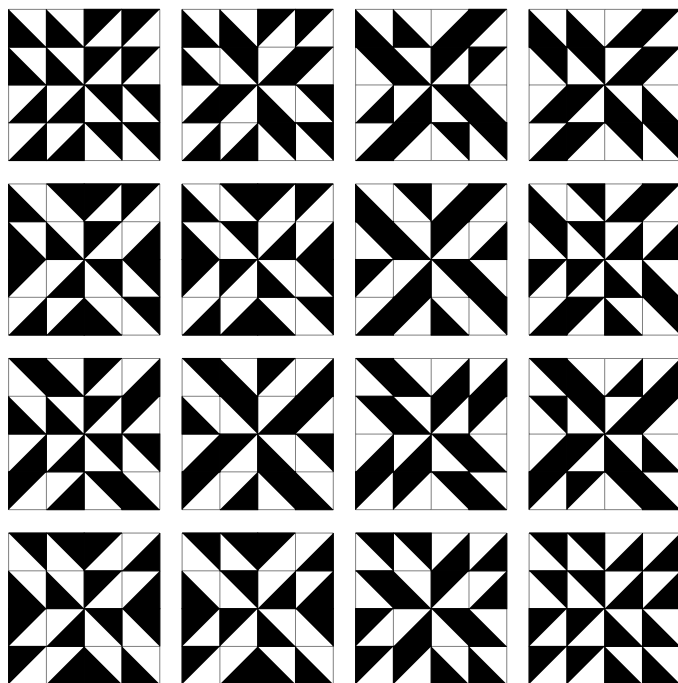
For a given family, there is *companion* family, the family of patterns formed by rotating each square in a member of the original family by 90° . In this way, the companion family represents a set of patterns that are “as different as possible” from the original family. On the following pages each family will be shown along with its *companion* family, providing a complete listing of all 4×4 Truchet tile patterns with rotational symmetry.



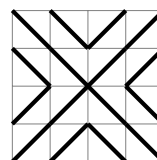
0000



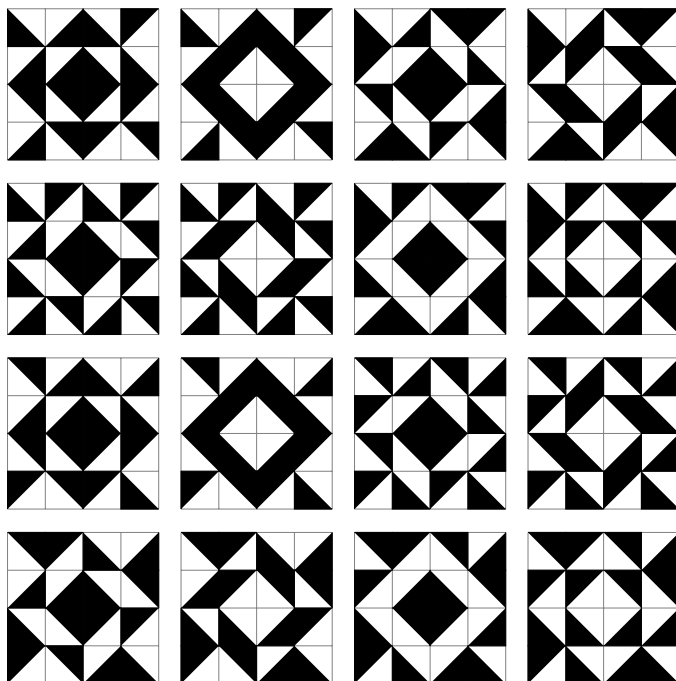
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0200	0202	0220	0222
2000	2002	2020	2022
2200	2202	2220	2222



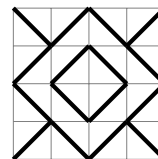
1111



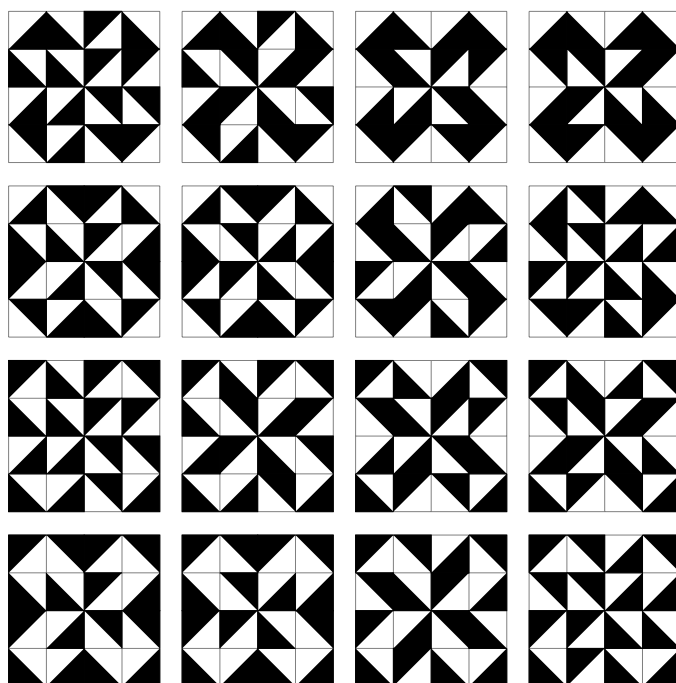
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1311	1313	1331	1333
3111	3113	3131	3133
3311	3313	3331	3333



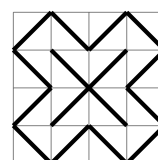
1000



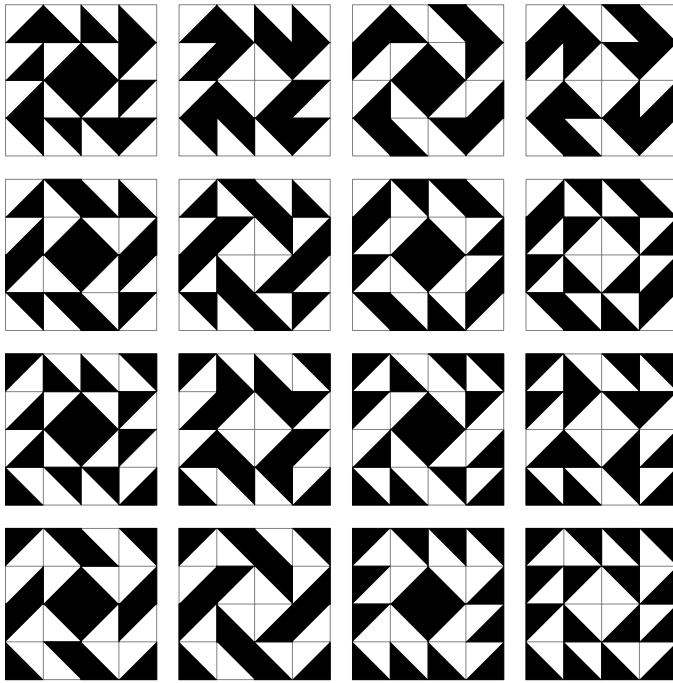
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3000	3002	3020	3022
3200	3202	3220	3222



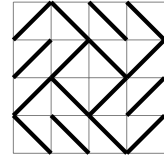
0111



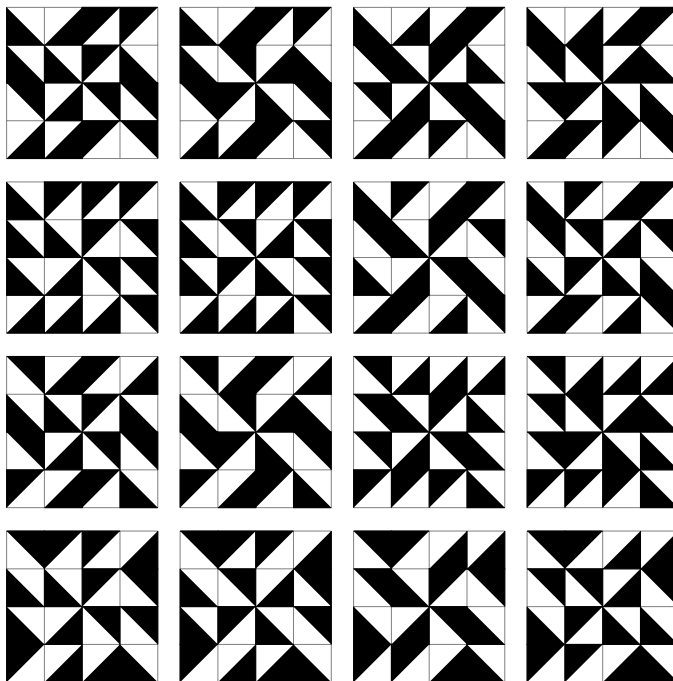
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2311	2313	2331	2333



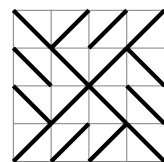
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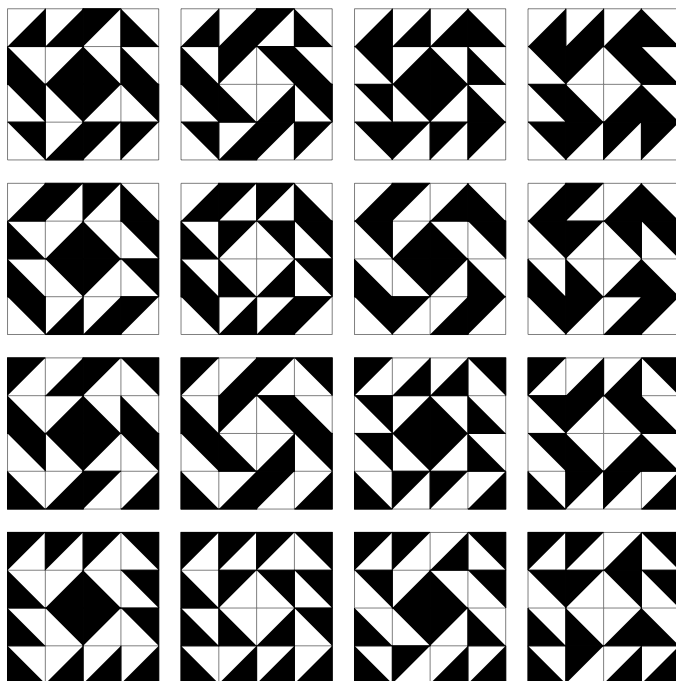
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0300	0302	0320	0322
2100	2102	2120	2122
2300	2302	2320	2322



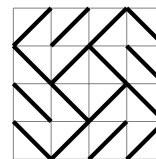
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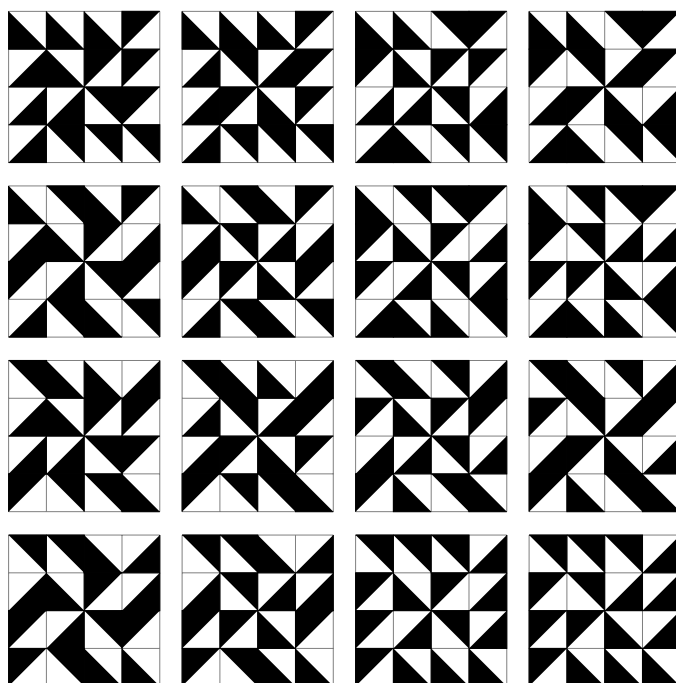
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3211	3213	3231	3233



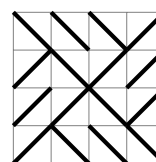
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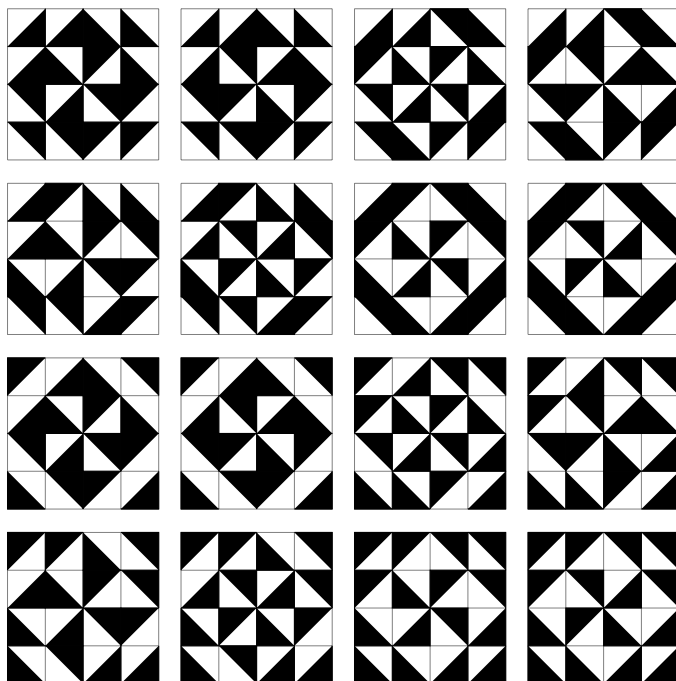
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2210	2212	2230	2232



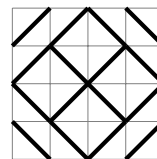
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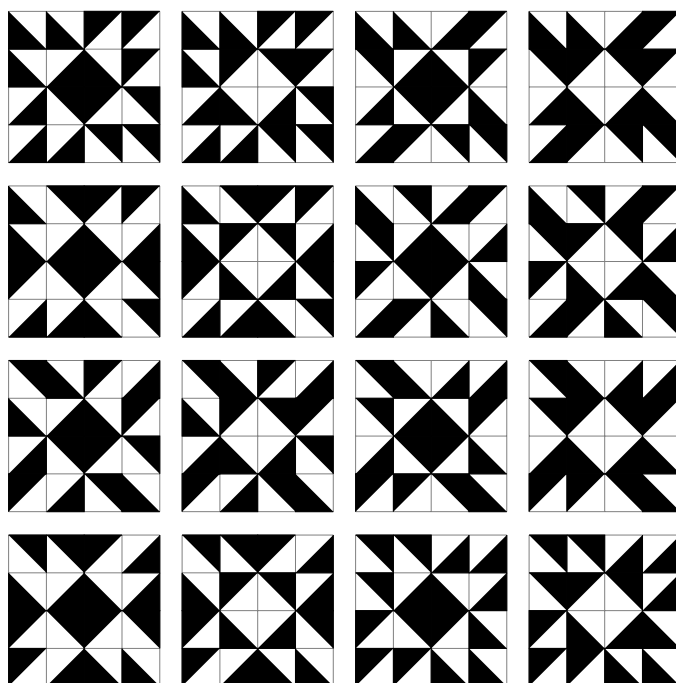
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3301	3303	3321	3323



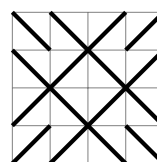
0001



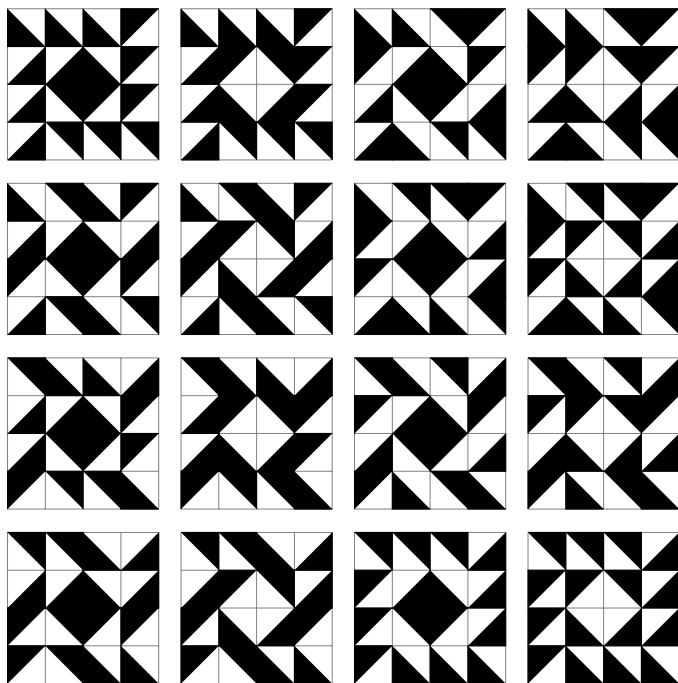
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2001	2003	2021	2023
2201	2203	2221	2223



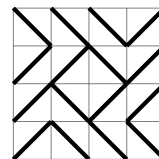
1110



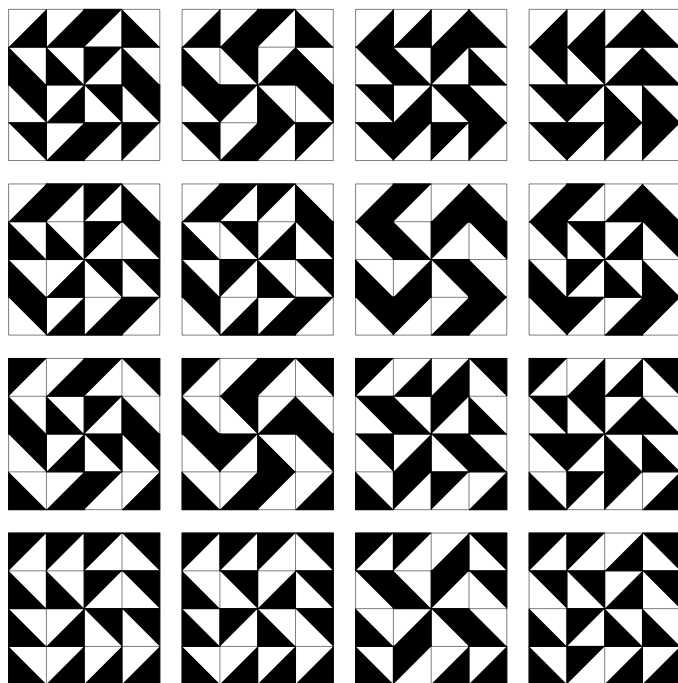
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1310	1312	1330	1332
3110	3112	3130	3132
3310	3312	3330	3332



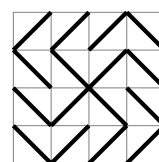
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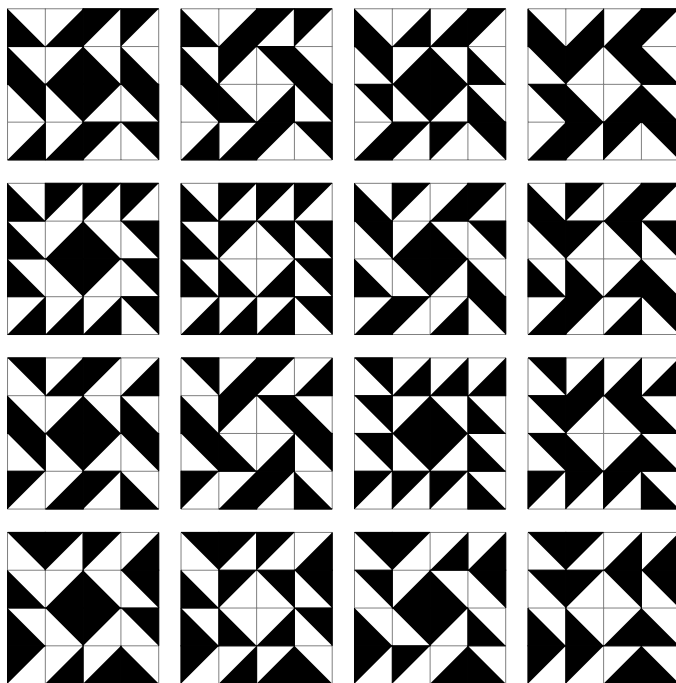
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3100	3102	3120	3122
3300	3302	3320	3322



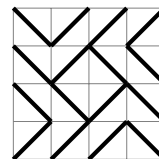
0011



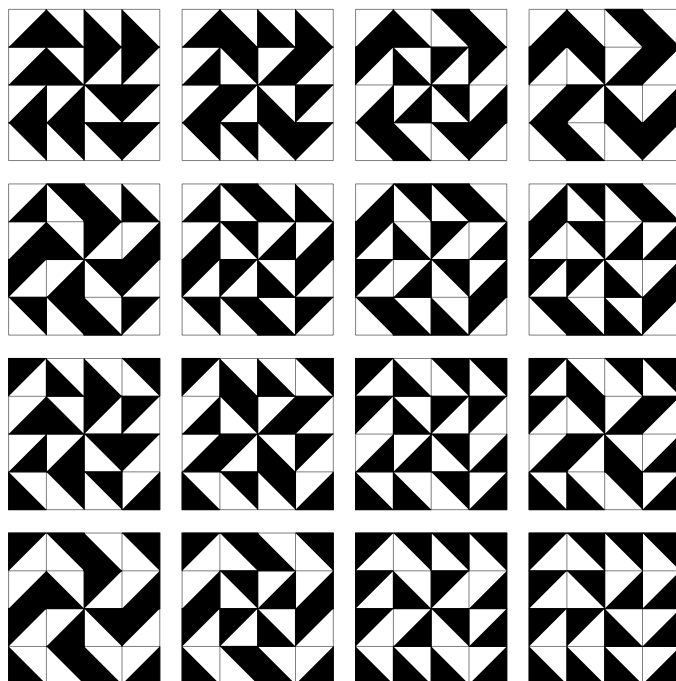
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2011	2013	2031	2033
2211	2213	2231	2233



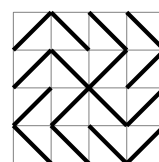
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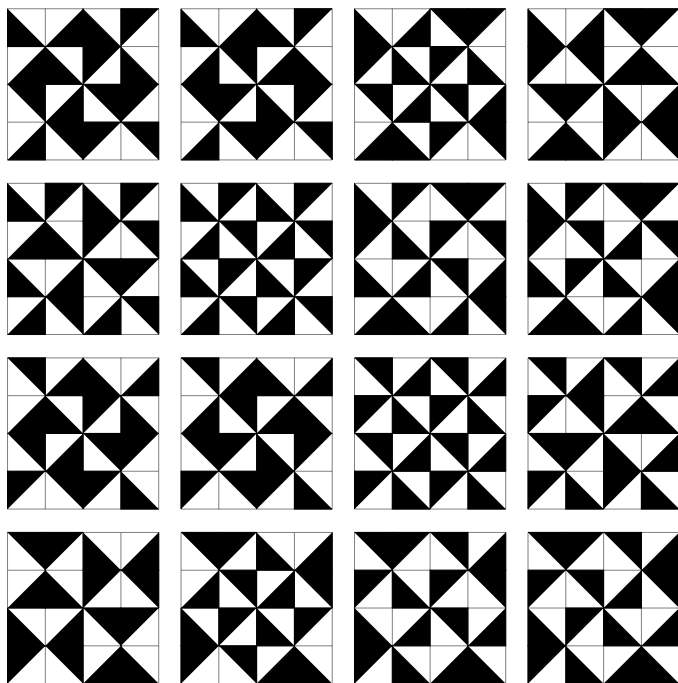
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3210	3212	3230	3232



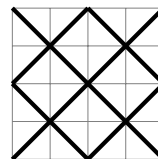
0101



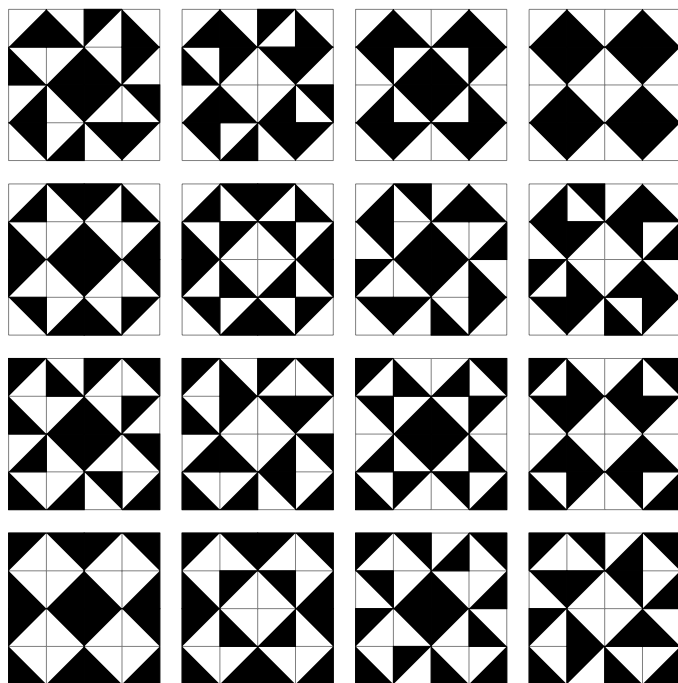
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0301	0303	0321	0323
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2301	2303	2321	2323



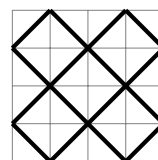
1001



1001	1003	1021	1023
1201	1203	1221	1223
3001	3003	3021	3023
3201	3203	3221	3223



0110

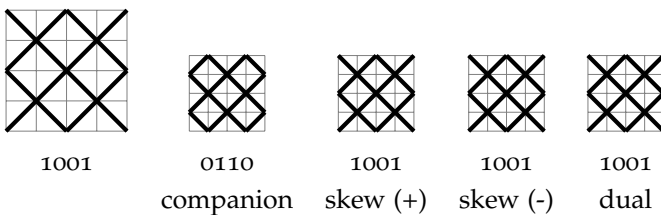
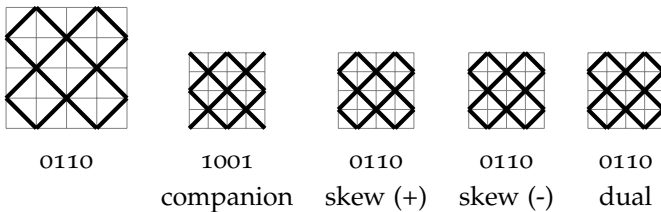
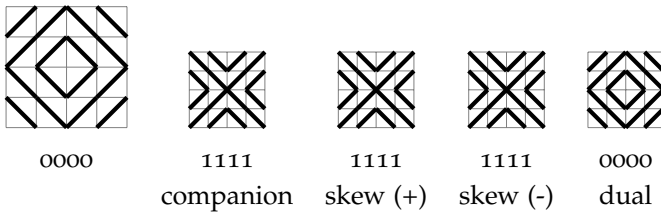


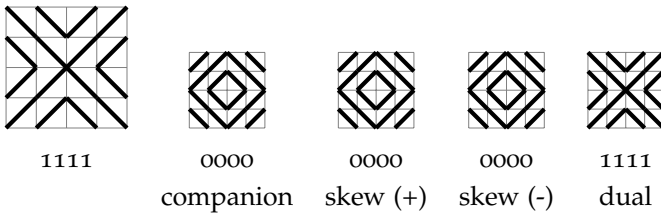
0110	0112	0130	0132
0310	0312	0330	0332
2110	2112	2130	2132
2310	2312	2330	2332

Family and tile pattern mappings

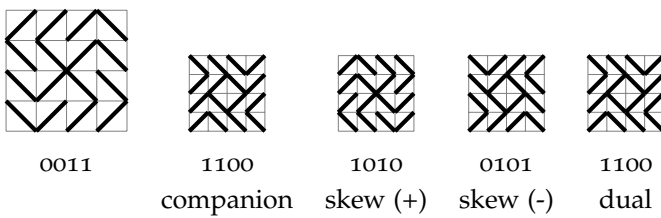
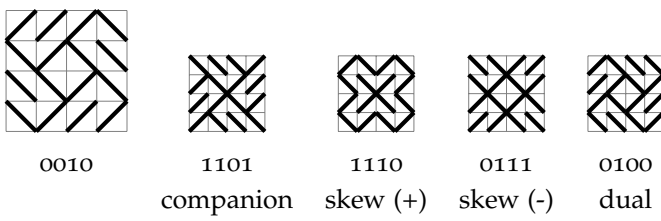
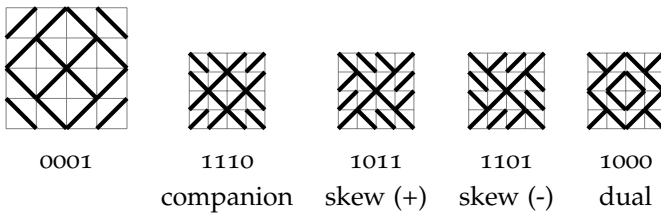
For each family, in addition to the companion family of patterns formed by rotating each square in a member of the original family by 90° , there are also two *skew* families, formed by taking the upper left and lower right quadrants of an original family tile pattern as a founding prototile pattern and a *dual* family, formed by taking the upper right quadrant as a founding prototile pattern. A family is always different than its companion, but it can happen that the family and its skews or duals can coincide. Self-dual families, where the dual family is the same as the original, contain some interesting cases of the frieze patterns explored in the next chapter.

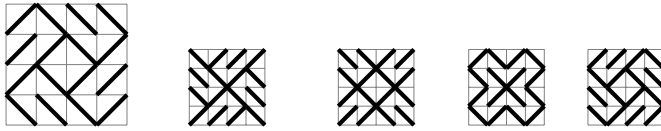
Self-Dual families





Non self-dual families





0100

1011

0111

1110

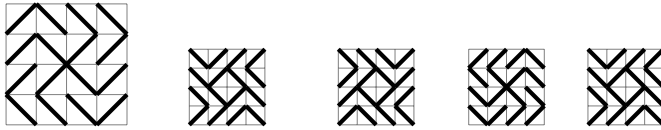
0010

companion

skew (+)

skew (-)

dual



0101

1010

0011

1100

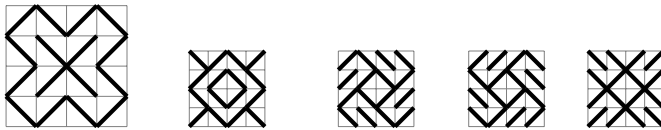
1010

companion

skew (+)

skew (-)

dual



0111

1000

0010

0100

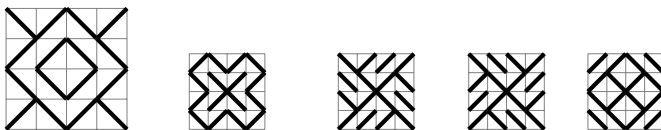
1110

companion

skew (+)

skew (-)

dual



1000

0111

1101

1011

0001

companion

skew (+)

skew (-)

dual



1010

0101

1100

0011

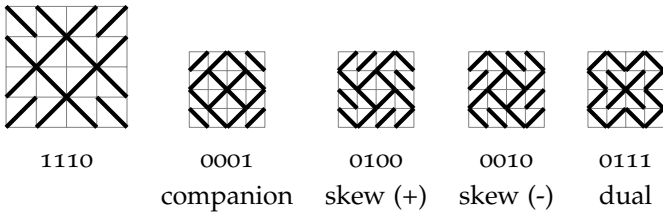
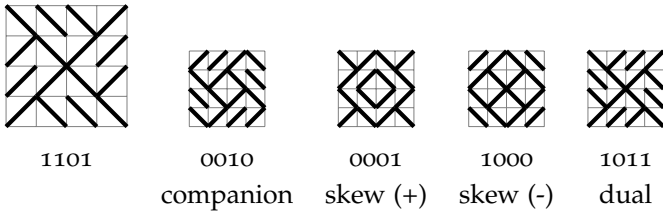
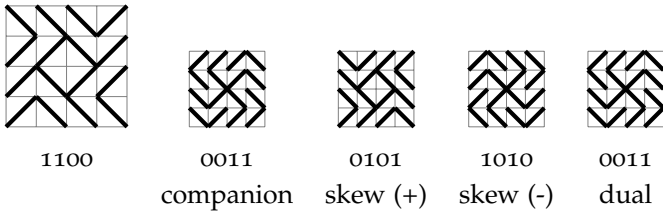
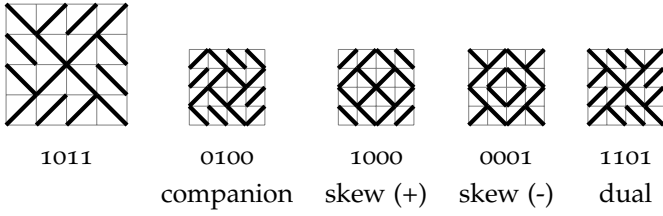
0101

companion

skew (+)

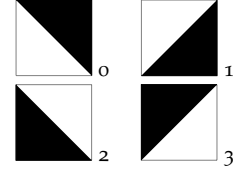
skew (-)

dual



Family and tile mappings

If \mathcal{T} is the set of 4×4 Truchet patters with four-fold rotational symmetry, and \mathcal{F} is the set of 4×4 Truchet tile families, the relationship between tile patterns and their families is a mapping $\mathcal{T} \rightarrow \mathcal{F}$. Both individual 4×4 Truchet tiles patterns and their families can be described by their signatures. As previously mentioned, the signature of the family of a tile can be found by considering the tile's signature (mod 2).



$$\begin{aligned} \text{family} : t &\mapsto F \\ \text{family} : (a, b, c, d) &\mapsto (a, b, c, d) \pmod{2} \end{aligned}$$

Family mappings

Related families ($\mathcal{F} \rightarrow \mathcal{F}$) can be obtained from applying simple operations on the signature a pattern family F .

$$\begin{aligned} \text{companion} : (a, b, c, d) &\mapsto (a + 1, b + 1, c + 1, d + 1) \pmod{2}; \\ \text{skew}+ : (a, b, c, d) &\mapsto (c + 1, a + 1, d + 1, b + 1) \pmod{2}; \\ \text{dual} : (a, b, c, d) &\mapsto (d, c, b, a) \pmod{2}; \\ \text{skew}- : (a, b, c, d) &\mapsto (b + 1, d + 1, a + 1, c + 1) \pmod{2}; \end{aligned}$$

Tile pattern mappings

Mappings between related tile patterns ($\mathcal{T} \rightarrow \mathcal{T}$) can be described in terms of simple operations on the signature of a pattern t .

$$\begin{aligned} \text{skew}+ : (a, b, c, d) &\mapsto (c + 1, a + 1, d + 1, b + 1) \pmod{4}; \\ \text{dual} : (a, b, c, d) &\mapsto (d + 2, c + 2, b + 2, a + 2) \pmod{4}; \\ \text{skew}- : (a, b, c, d) &\mapsto (b + 3, d + 3, a + 3, c + 3) \pmod{4}; \\ \text{opposite} : (a, b, c, d) &\mapsto (a + 2, b + 2, c + 2, d + 2) \pmod{4}; \end{aligned}$$

a	q	c	e
c	p	p	q
b	d	d	c
a	c	b	a

Some theorems

For a tile pattern t in a family F ($t \in F$), we will write t^d , t^+ , t^- , and t^{op} for the dual, positive skew, negative skew, and opposite of t , respectively. Similarly for the family F we will write F^d , F^+ , F^- , and F^c for the dual, positive skew, negative skew, and companion of F . Some simple theorems that describe the relationships among these concepts are fun to state and easy to prove.

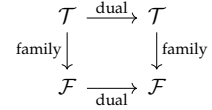
Theorem 1 A tile t is self-dual ($t = t^d$) if and only if its prototile has two-fold rotational symmetry.

Theorem 2 A tile is self-skew ($t = t^+$ or $t = t^-$) if and only if its prototile has four-fold rotational symmetry.

Theorem 3 If a tile t is self-skew ($t = t^+$ or $t = t^-$) then t is also self-dual ($t = t^d$).

Theorem 4 The dual of a tile is a member of the dual of the original tile's family.

$$t \in F \implies t^d \in F^d$$



Theorem 5 A self-dual tile is a member of a self dual family.

$$t \in F, t = t^d \implies F = F^d$$

Theorem 6 A op-dual tile is a member of a self dual family.

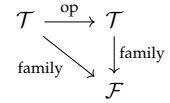
$$t \in F, t^{op} = t^d \implies F = F^d$$

Theorem 7 The companion family of a self-dual family is also self-dual.

$$F = F^d \implies F^c = F^{cd}$$

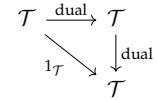
Theorem 8 A tile is self-dual if and only if its positive and negative skew tiles are equal.

$$t = t^d \iff t^+ = t^-$$



Theorem 9 The opposite of a tile t is in the same family as t .

$$t \in F \implies t^{op} \in F$$

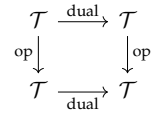


Theorem 10 The dual of a tile's dual is the original tile.

$$(t^d)^d = t$$

Theorem 11 The dual of a tile's opposite is the opposite of the tile's dual.

$$(t^{op})^d = (t^d)^{op}$$

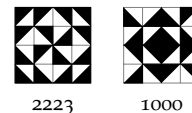
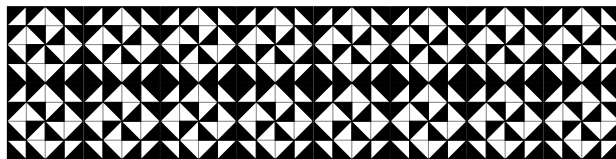


Theorem 12 There are $256 \ 4 \times 4$ Truchet patterns with four-fold rotational symmetry in 16 families of 16 patterns each. There are four self-dual families, each with four self-dual tiles. Two of the self-dual families include two self-skew tiles each.

Uniform friezes

Each 4×4 Truchet pattern can be treated like a tile and used in a larger pattern. A uniform *frieze* is a horizontal strip of the same tile pattern repeated. Friezes of 4×4 Truchet pattern tiles with rotational symmetry can be quite striking, and have some interesting characteristics.

A frieze of more than one row of a primary tile reveals a secondary tile pattern that appears as another horizontal strip of 4×4 Truchet tile patterns nestled between the rows of primary tiles. Below, a frieze of 2223 tiles has a secondary pattern of 1000 tiles.



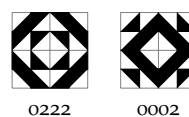
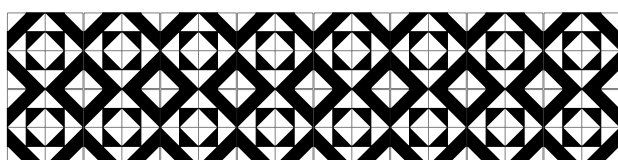
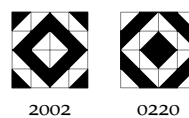
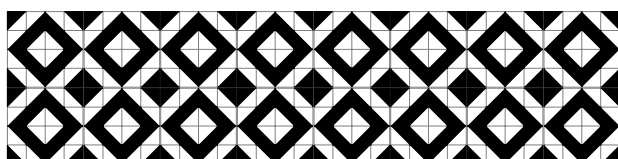
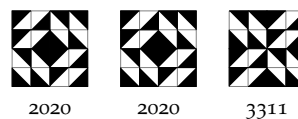
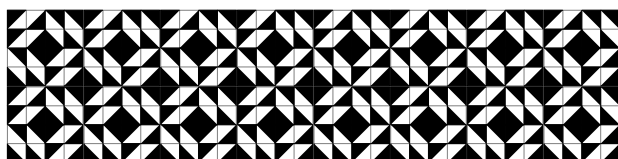
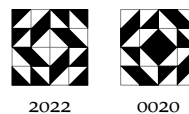
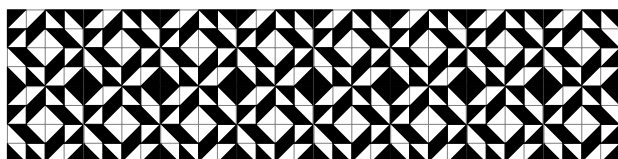
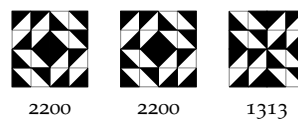
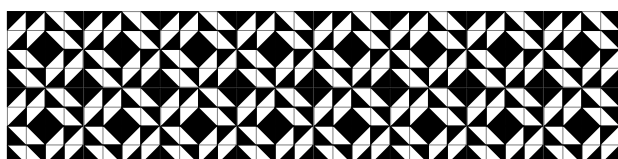
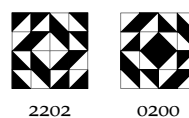
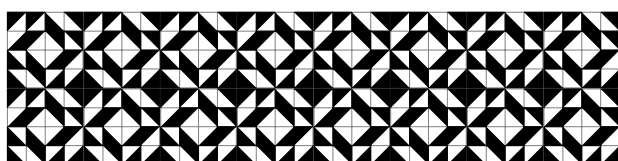
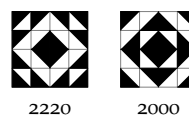
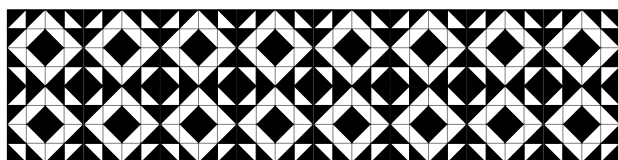
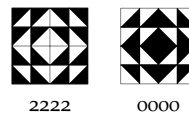
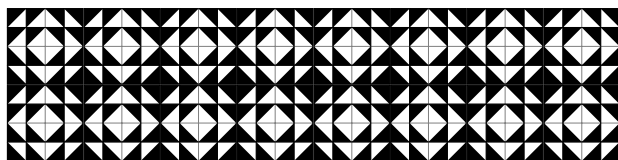
The secondary tile in a frieze pattern is the pattern that has been referred to previously as the *dual* of the original pattern. The dual of a tile pattern is the pattern formed by taking the top right quadrant of the original tile as the prototile of the new tile.

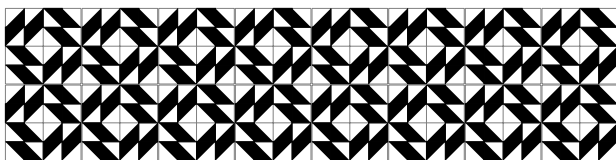
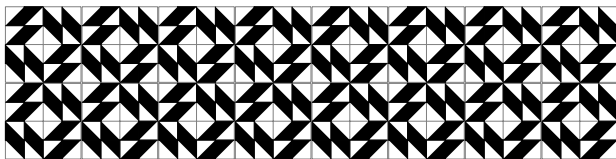
Some tiles are self-dual, and frieze patterns formed by self-dual tiles show a much more uniform pattern, as the extra rows of tiles seemingly nestled between the rows of the original tile are made up of the same original tile. Friezes of self-dual tiles have a third *tertiary* tile pattern with four-fold rotational symmetry that appears to overlap between adjacent tiles of the original tile. These tertiary tile patterns are the *skew* of the original tile pattern. Some self-dual friezes are also self-skew, leading to even more uniform patterns.

We can consider the uniform friezes formed by the dual tiles as the same frieze pattern. There are 6 pairs of families where the original and dual are not the same, and these pairs of families yield 16 pat-

terns each. The 4 remaining families contain some self-dual patterns, and some patterns that are *op-dual* (the secondary tile is the opposite tile of the original), also reducing the number of patterns. These 4 remaining families provide 10 distinct frieze patterns each. This means that the 256 tile patterns generate 136 distinct friezes.

Frieze patterns for family 0000 (secondary, 0000)





0202



0202



1133



0022

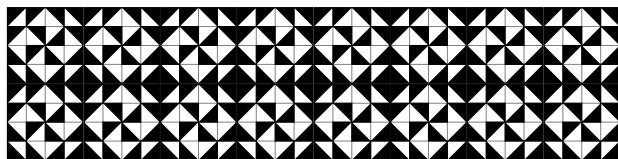


0022



3131

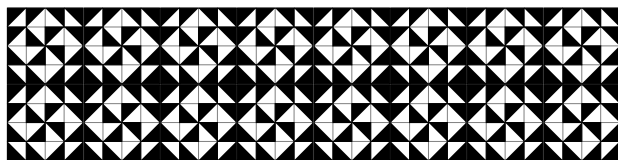
Frieze patterns for family 0001 (secondary, 1000)



2223



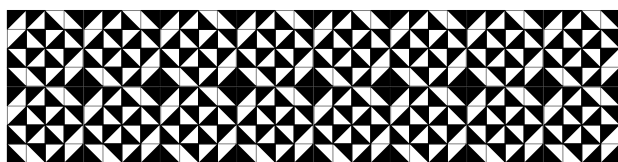
1000



2221



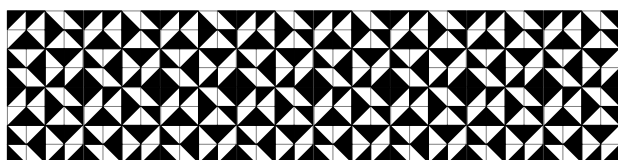
3000



2203



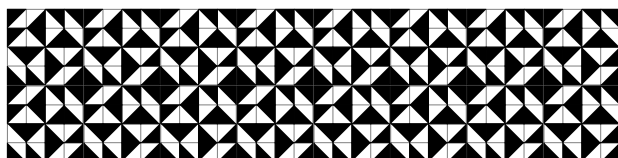
1200



2201



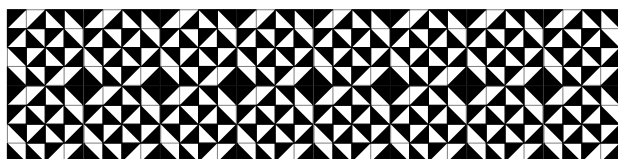
3200



2023



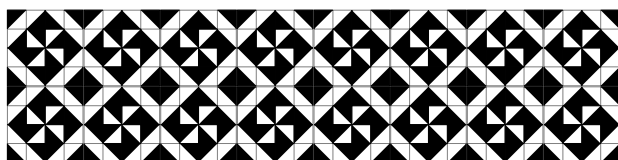
1020



2021



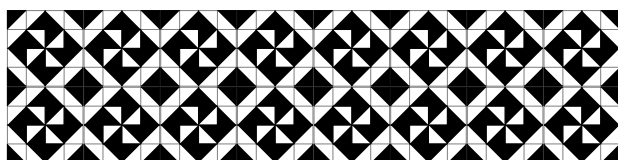
3020



2003



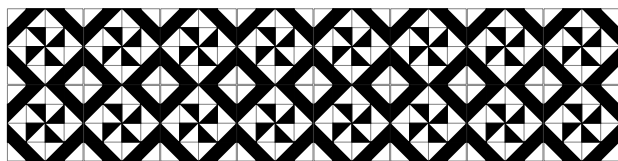
1220



2001



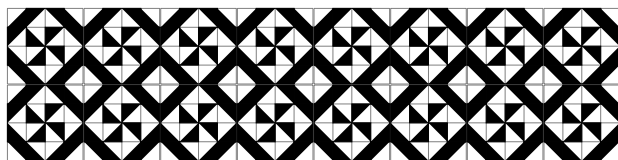
3220



0223



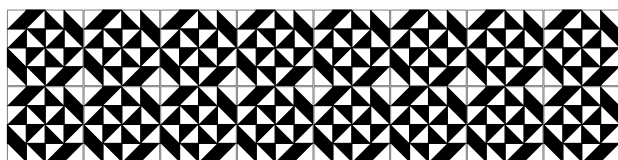
1002



0221



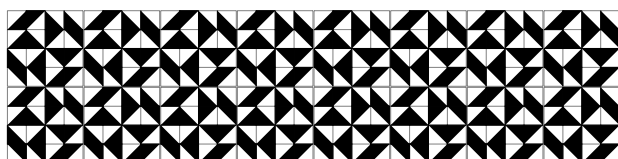
3002



0203



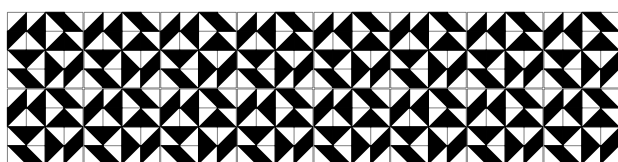
1202



0201



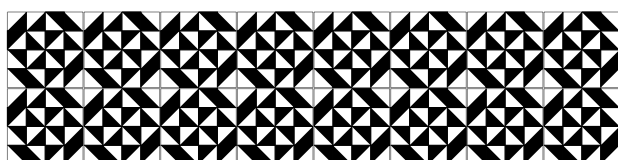
3202



0023



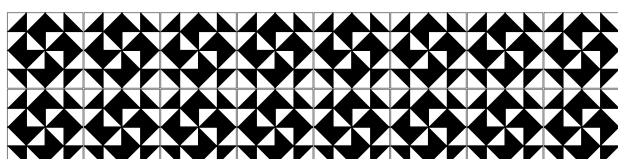
1022



0021



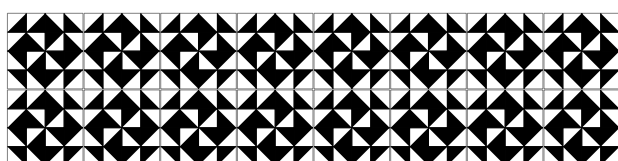
3022



0003



1222

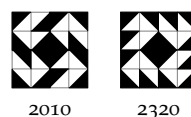
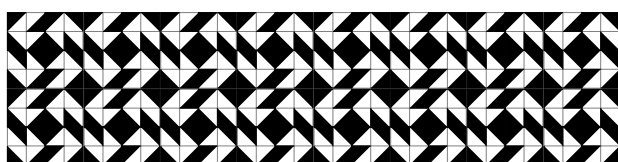
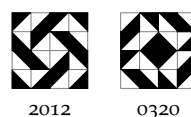
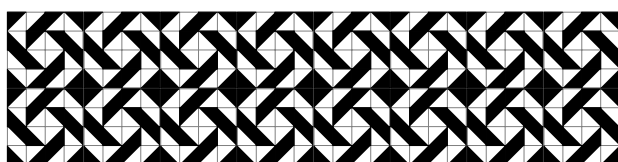
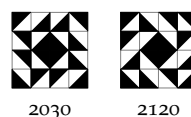
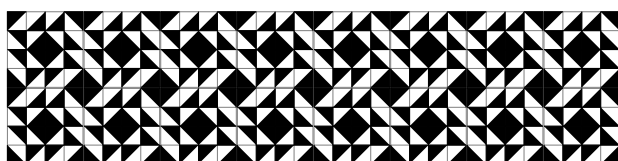
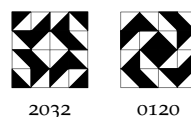
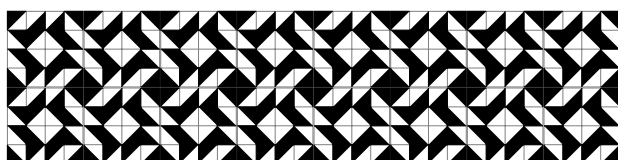
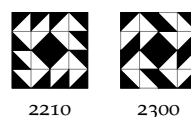
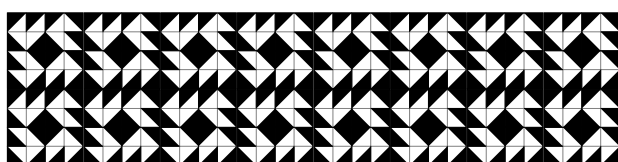
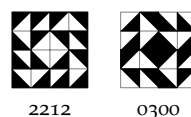
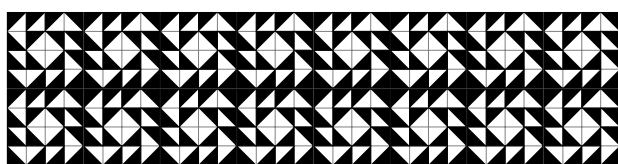
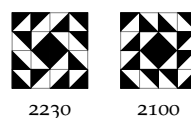
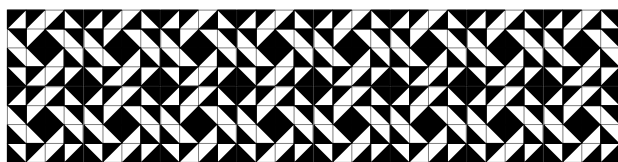
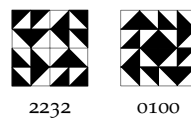
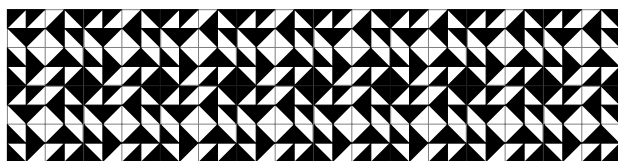


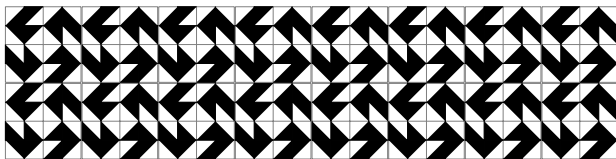
0001



3222

Frieze patterns for family 0010 (secondary, 0100)

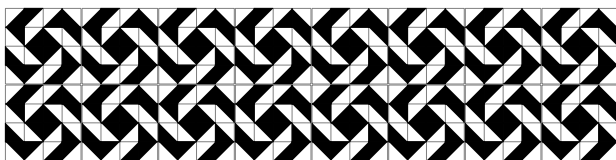




0232



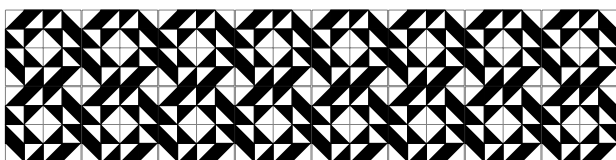
0102



0230



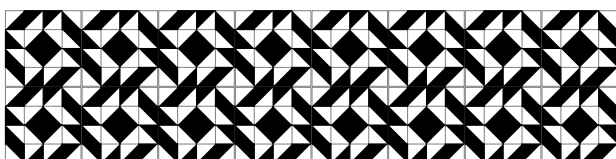
2102



0212



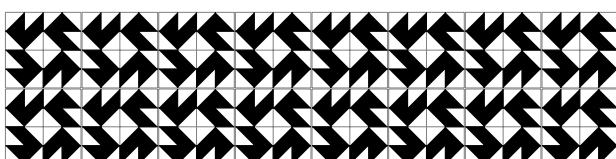
0302



0210



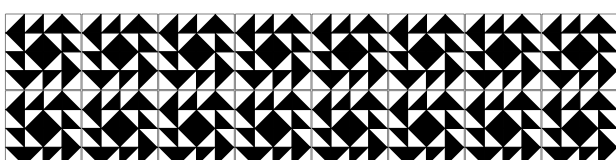
2302



0032



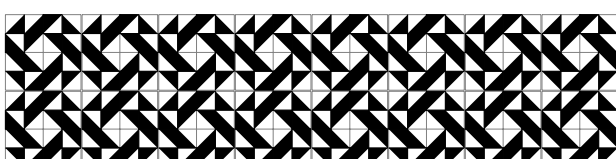
0122



0030



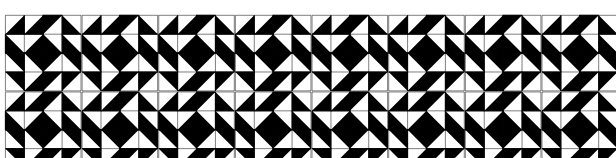
2122



0012



0322

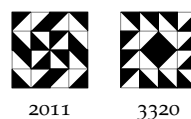
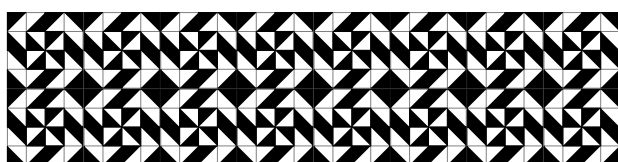
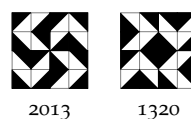
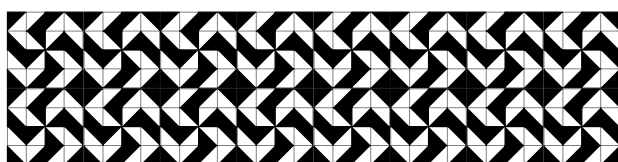
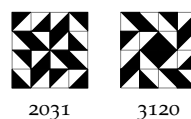
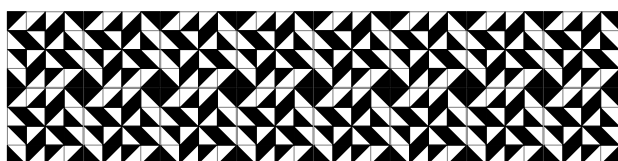
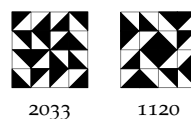
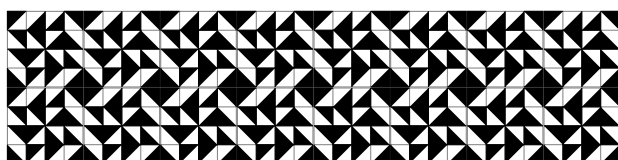
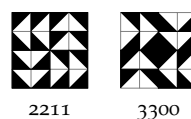
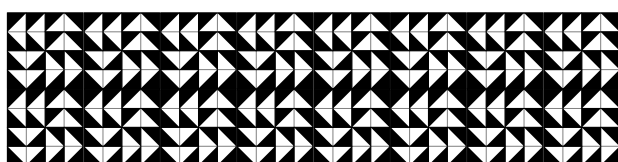
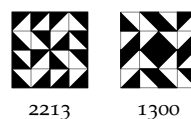
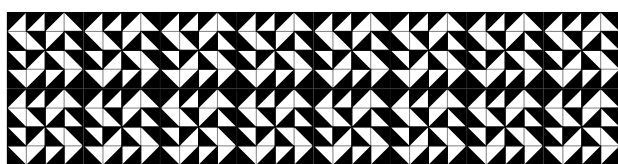
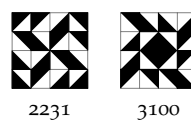
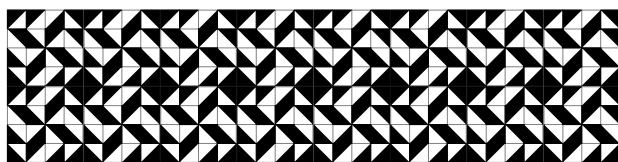
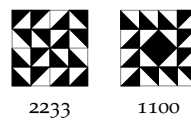
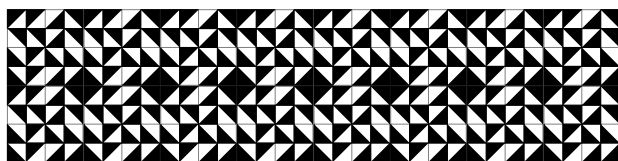


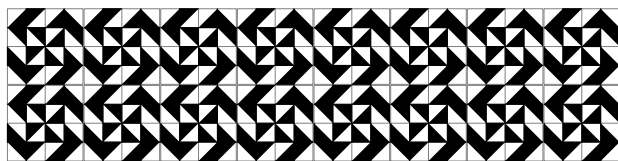
0010



2322

Frieze patterns for family 0011 (secondary, 1100)

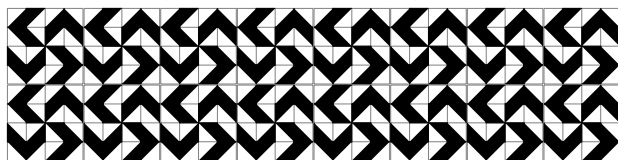




0233



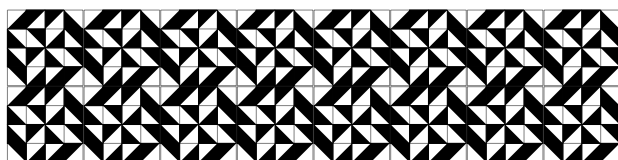
1102



0231



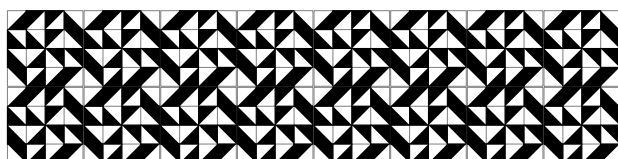
3102



0213



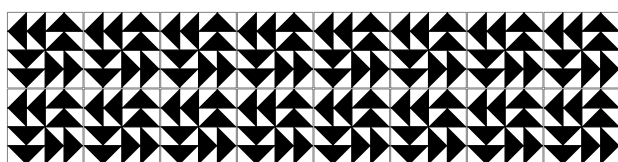
1302



0211



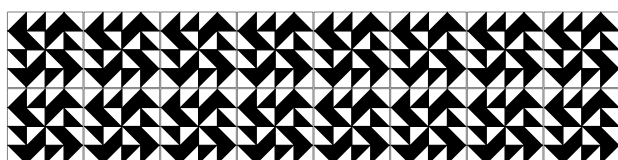
3302



0033



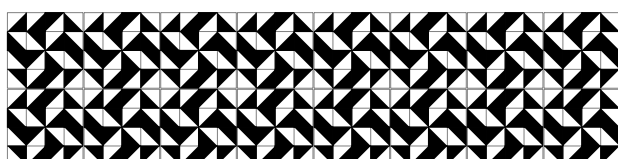
1122



0031



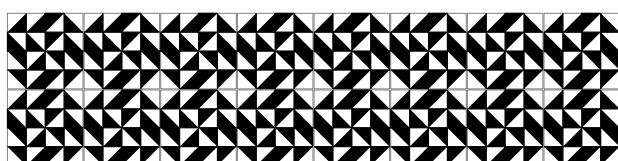
3122



0013



1322

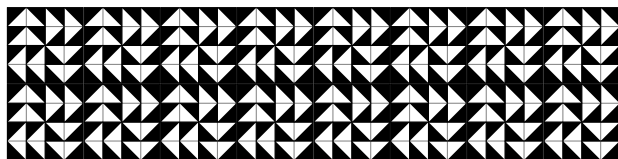


0011



3322

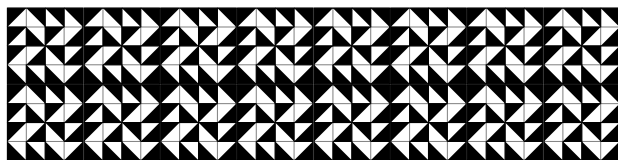
Frieze patterns for family 0101 (secondary, 1010)



2323



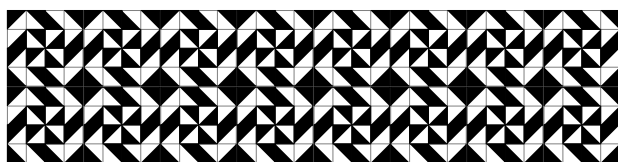
1010



2321



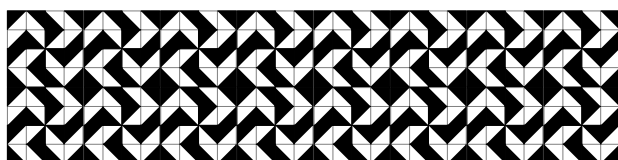
3010



2303



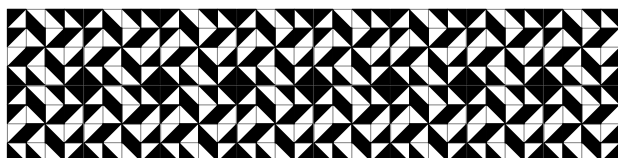
1210



2301



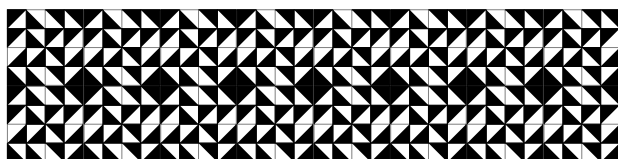
3210



2123



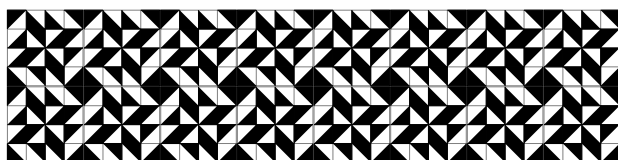
1030



2121



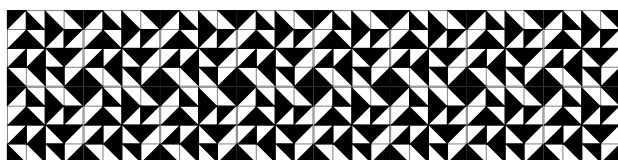
3030



2103



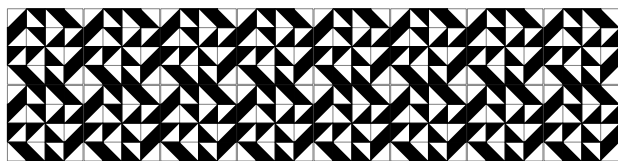
1230



2101



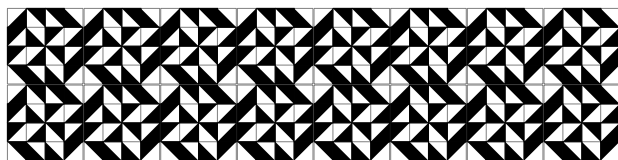
3230



0323



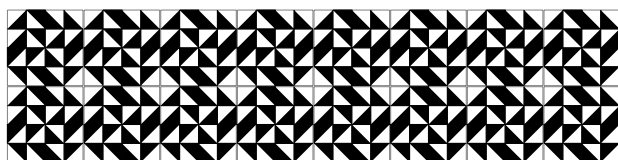
1012



0321



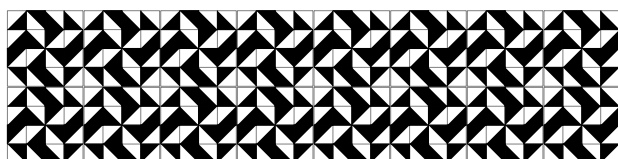
3012



0303



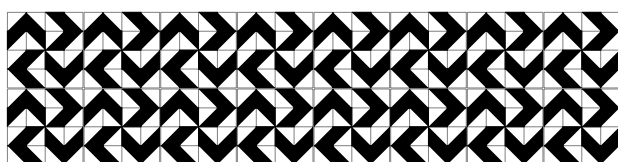
1212



0301



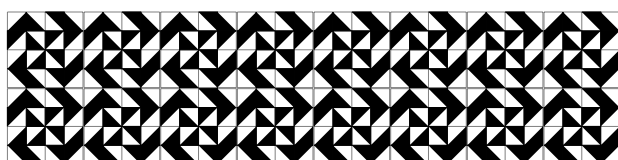
3212



0123



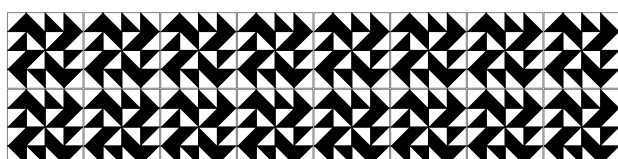
1032



0121



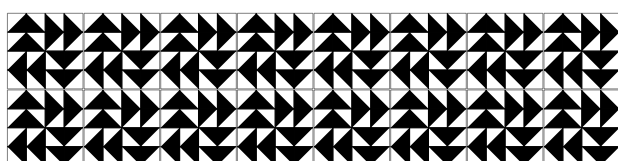
3032



0103



1232

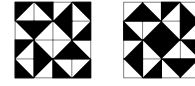
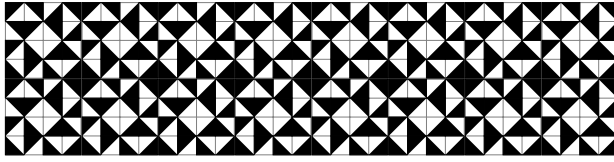


0101



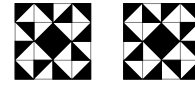
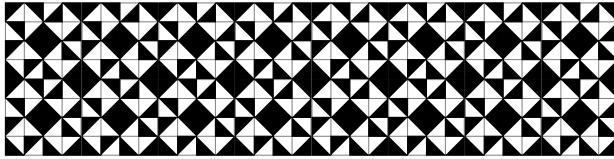
3232

Frieze patterns for family 0110 (secondary, 0110)



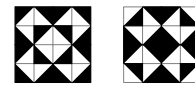
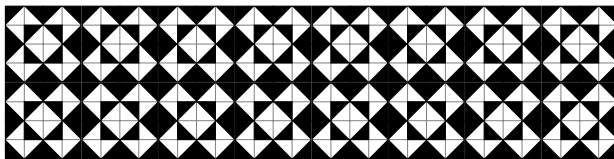
2332

0110



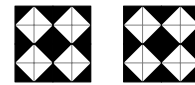
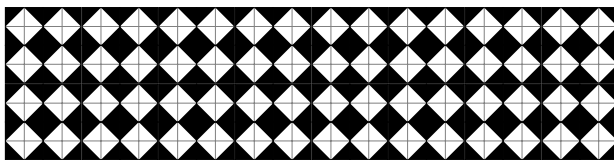
2330

2110



2312

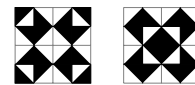
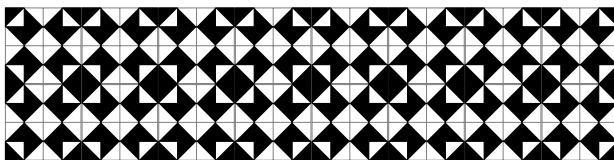
0310



2310

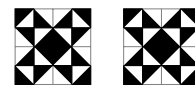
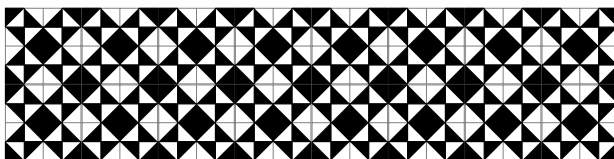
2310

2310



2132

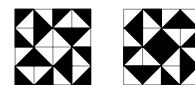
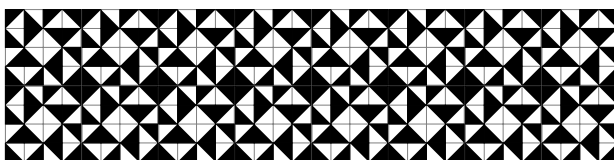
0130



2130

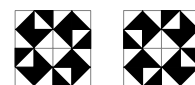
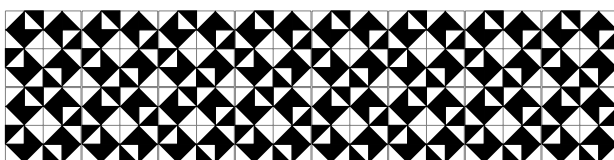
2130

0312



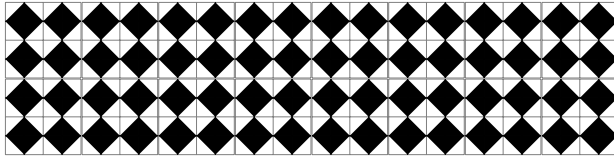
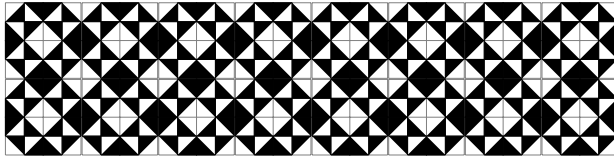
2112

0330



0332

0112



0312



0312



2130



0132

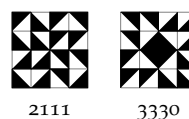
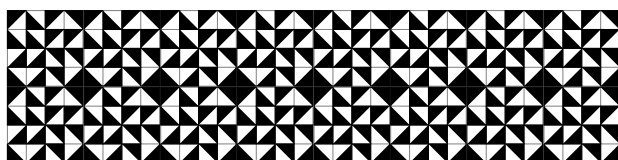
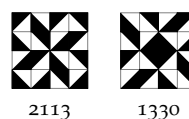
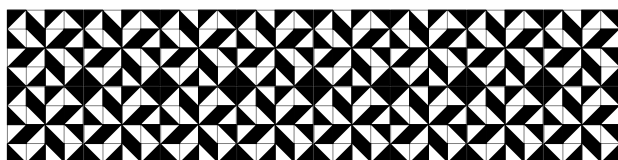
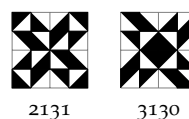
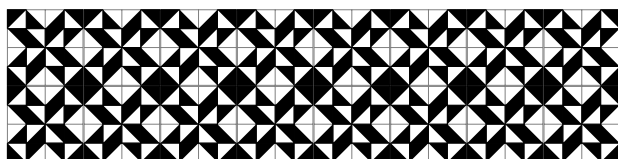
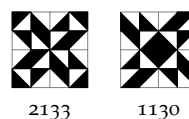
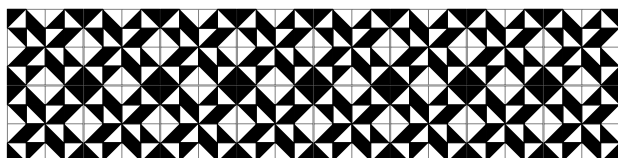
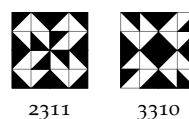
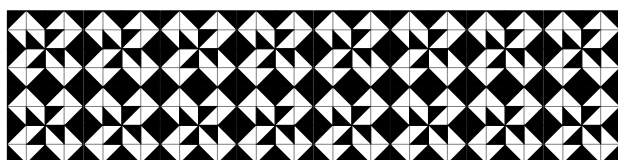
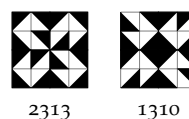
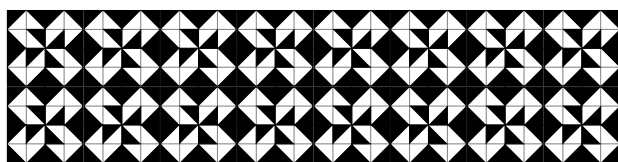
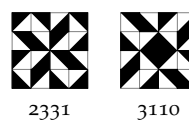
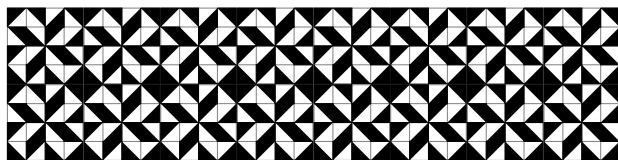
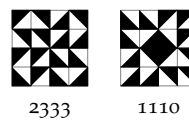
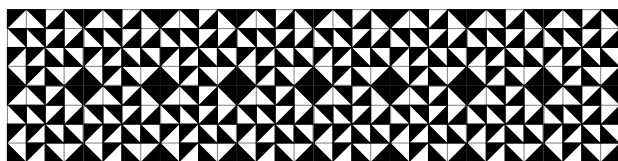


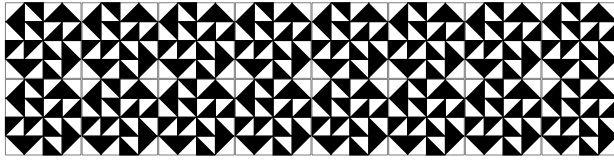
0132



0132

Frieze patterns for family 0111 (secondary, 1110)

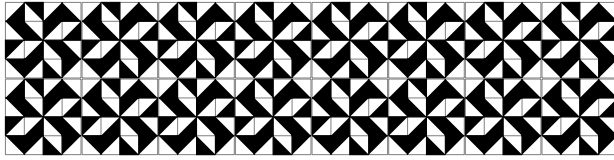




0333



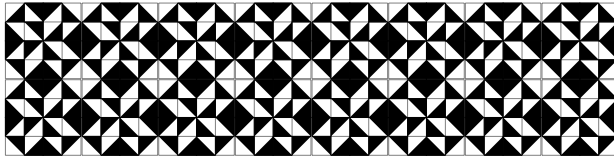
1112



0331



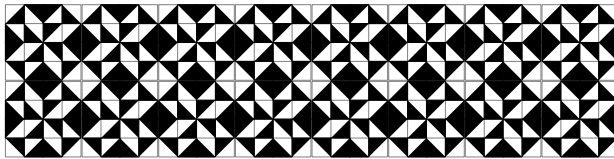
3112



0313



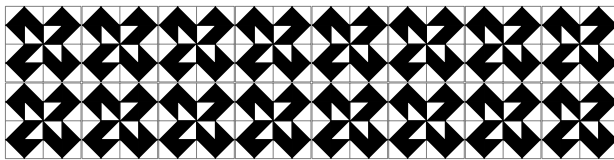
1312



0311



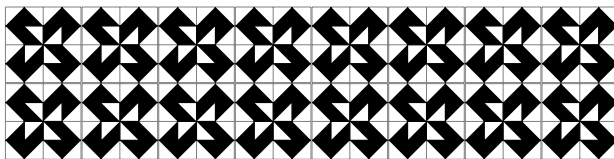
3312



0133



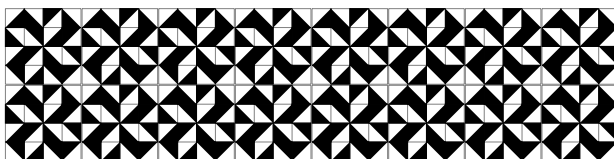
1132



0131



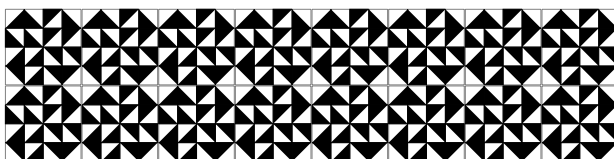
3132



0113



1332

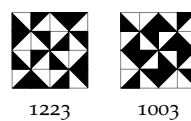
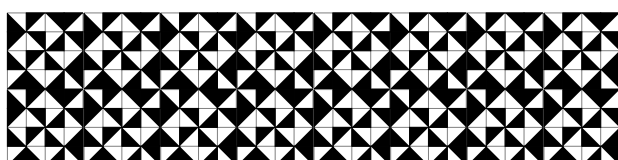
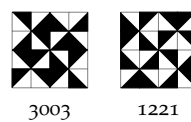
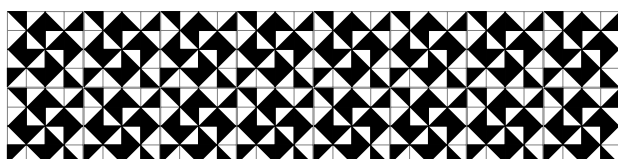
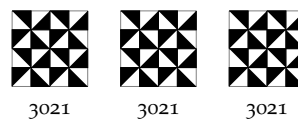
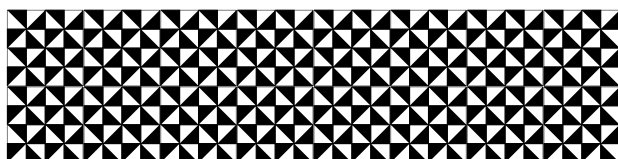
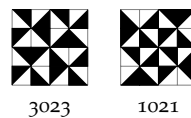
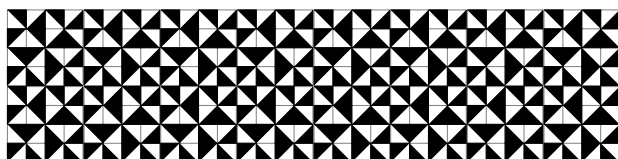
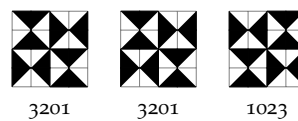
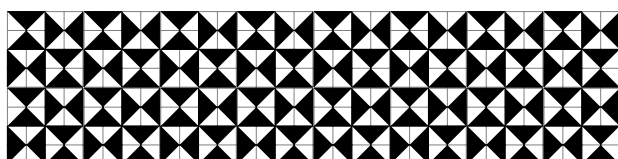
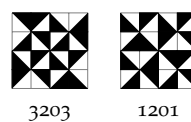
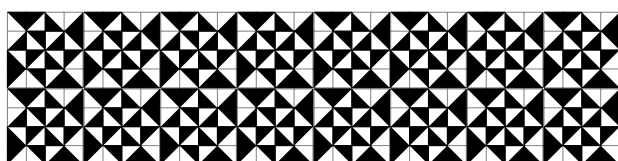
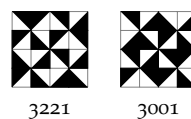
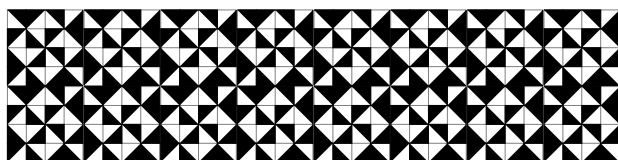
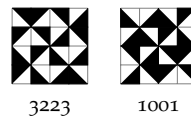
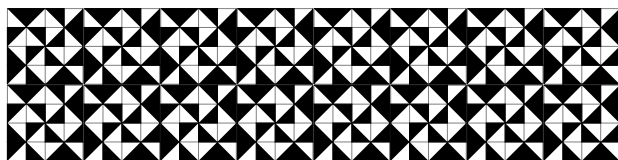


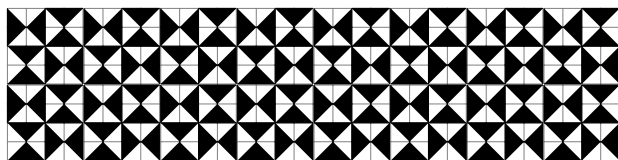
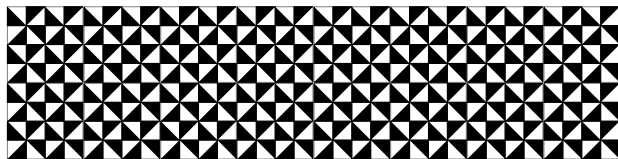
0111



3332

Frieze patterns for family 1001 (secondary, 1001)





1203



1203



1203



1023

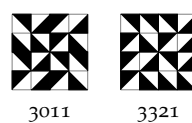
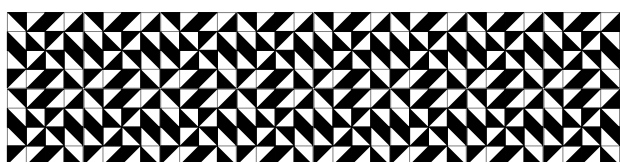
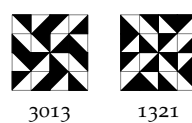
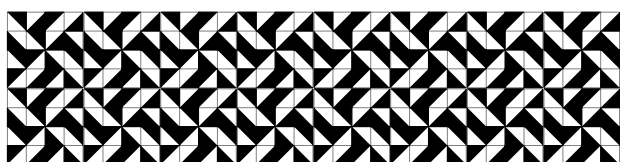
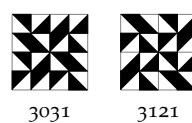
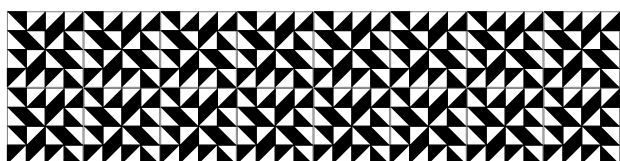
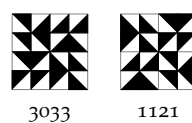
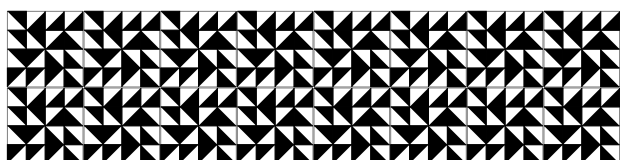
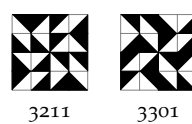
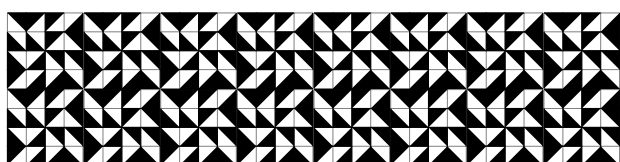
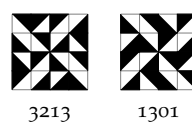
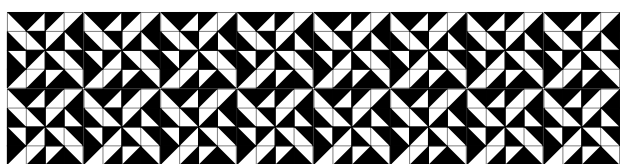
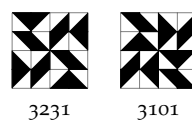
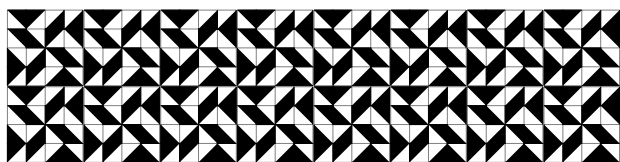
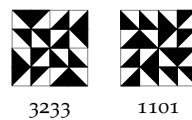
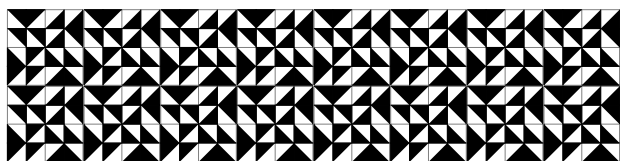


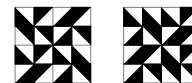
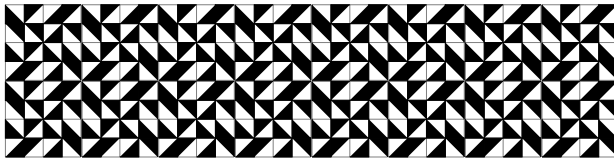
1023



3201

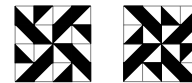
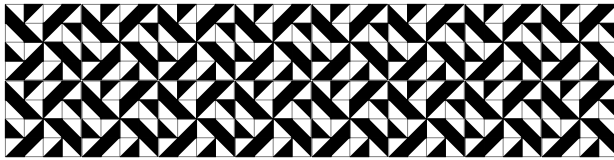
Frieze patterns for family 1011 (secondary, 1101)





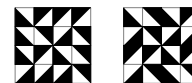
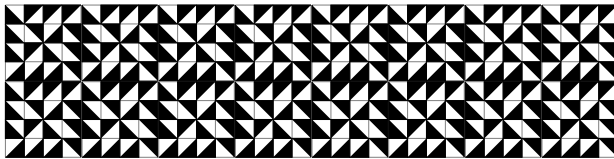
1233

1103



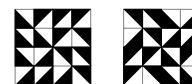
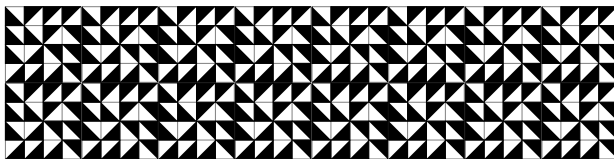
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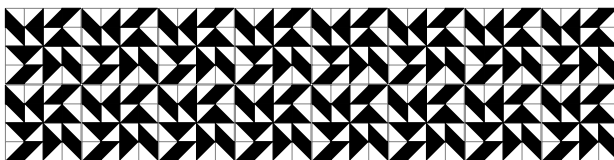
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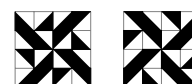
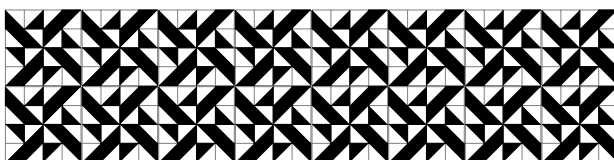
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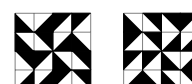
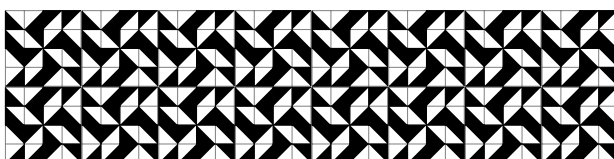
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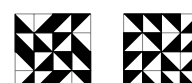
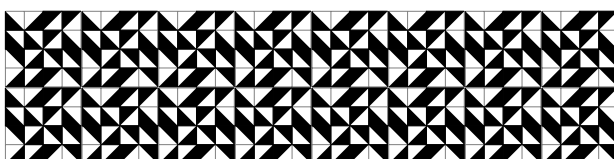
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1013

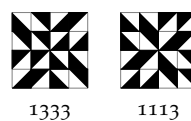
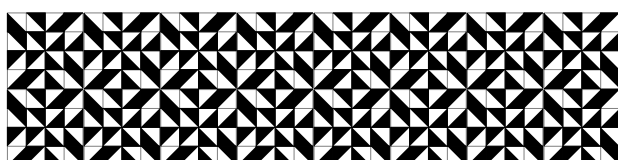
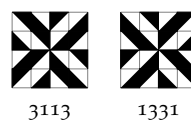
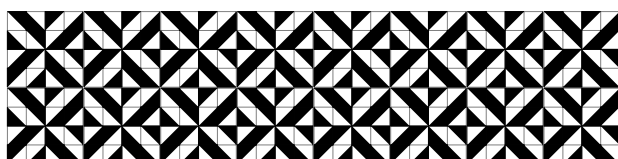
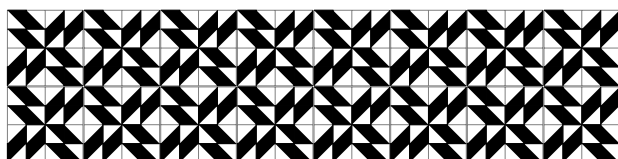
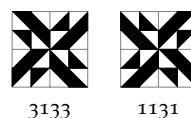
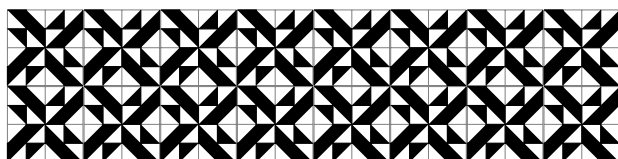
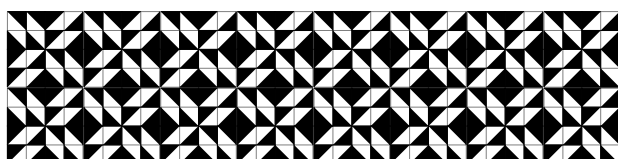
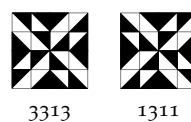
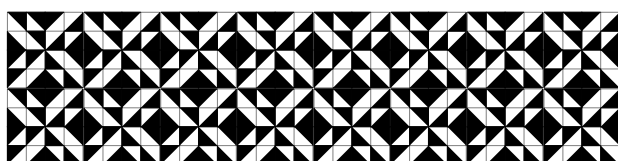
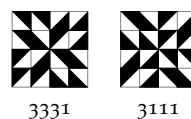
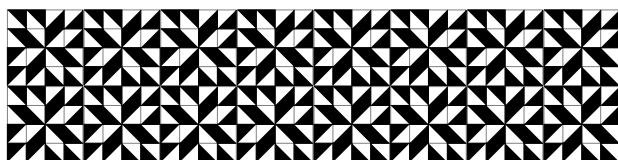
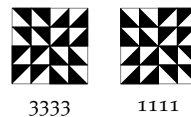
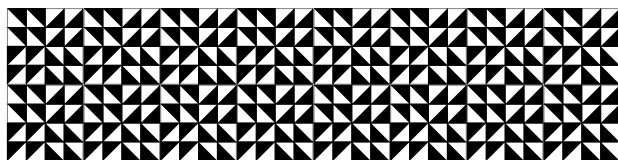
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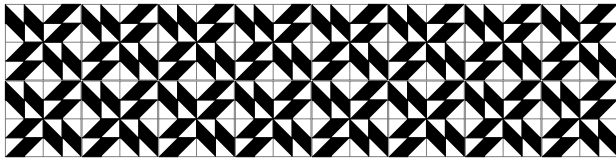
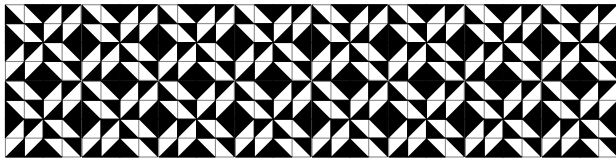


1011

3323

Frieze patterns for family 1111 (secondary, 1111)





1313



1313



2200



1133



1133



0202

Self-dual tiles

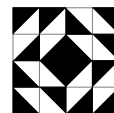
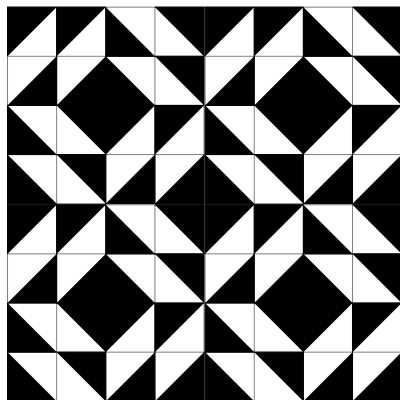
Self-dual tiles are the 4×4 Truchet tile patterns whose 2×2 prototile has two-fold (180°) rotational symmetry. Because of the two-fold rotational symmetry of the prototile, its appearance in the third quadrant of the 4×4 tile is identical to its appearance in the initial quadrant. So the dual tile that emerges when placing four of the tiles together in a larger 2×2 tile array is another copy of the original tile, appearing in the center of the larger 2×2 pattern.

Self-dual tiles that are not self-skew

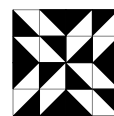
If the original prototile has strict two-fold symmetry, 2×2 patterns made with the four-fold rotationally symmetrical Truchet tile also display another distinct emergent four-fold rotationally symmetrical Truchet tile, which we are calling the *tertiary* tile.

In these patterns, it appears that there are five copies of the primary tile (four placed in a 2×2 array, and another emerging in the center), along with four copies of the tertiary tile pattern.

2200 with 1313

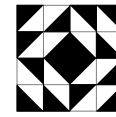
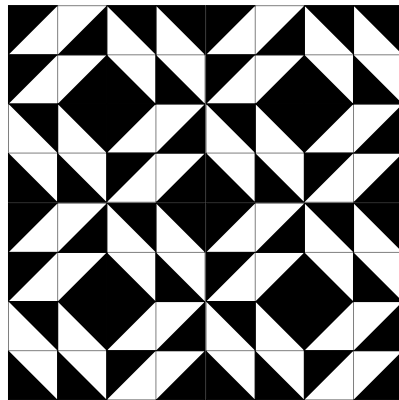


2200

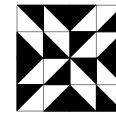


1313

2020 with 3311

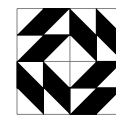
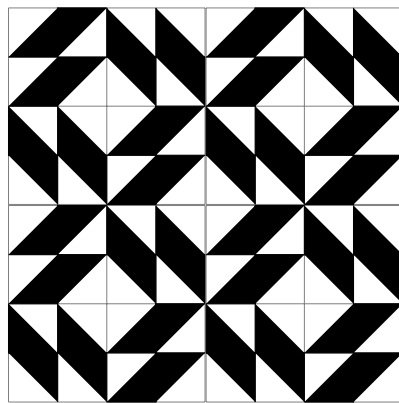


2020

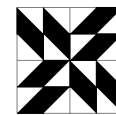


3311

0202 with 1133

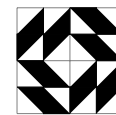
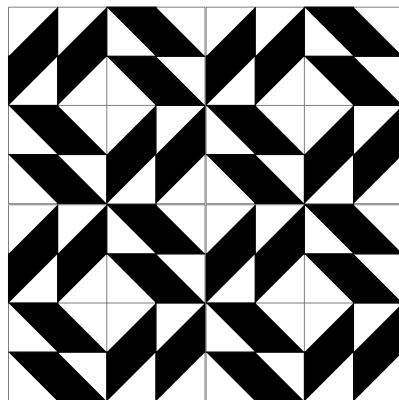


0202

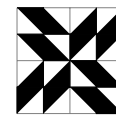


1133

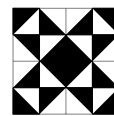
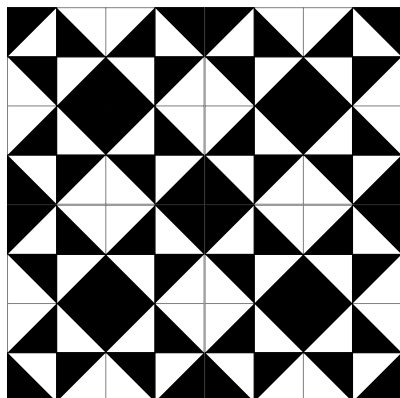
0022 with 3131



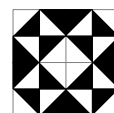
0022



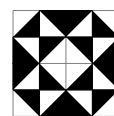
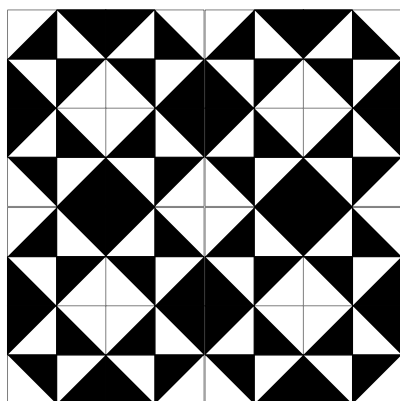
3131

2130 with 0312

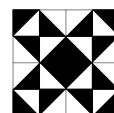
2130



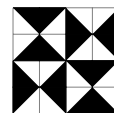
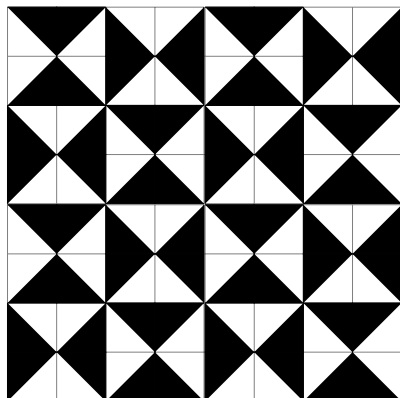
0312

0312 with 2130

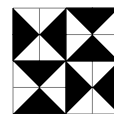
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2130

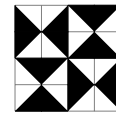
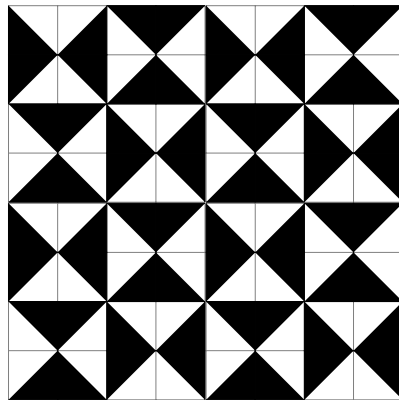
3201 with 1023

3201

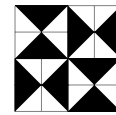


1023

1023 with 3201

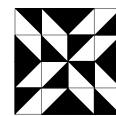
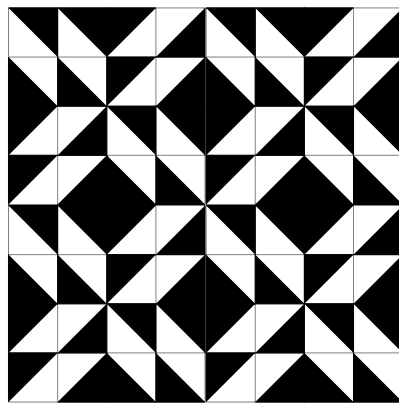


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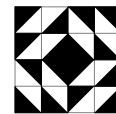


3201

3311 with 2020

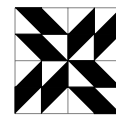
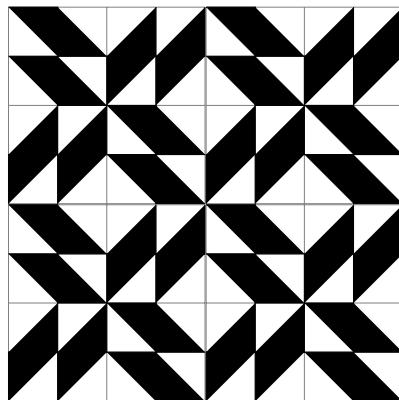


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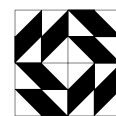


2020

3131 with 0022

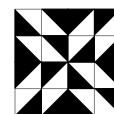
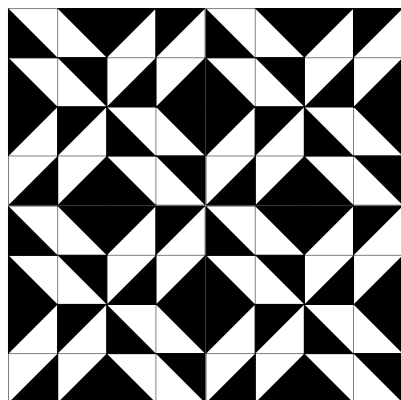


3131

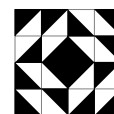


0022

1313 with 2200

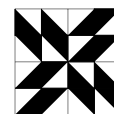
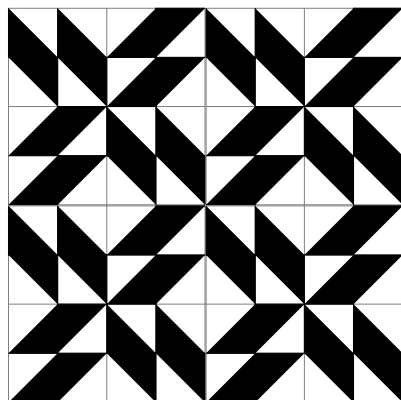


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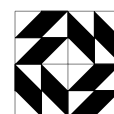


2200

1133 with 0202



1133

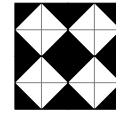
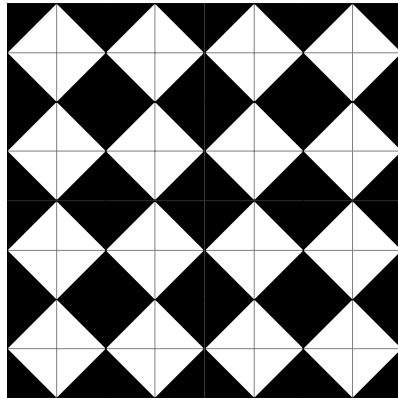


0202

Self-skew tiles

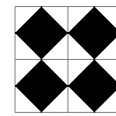
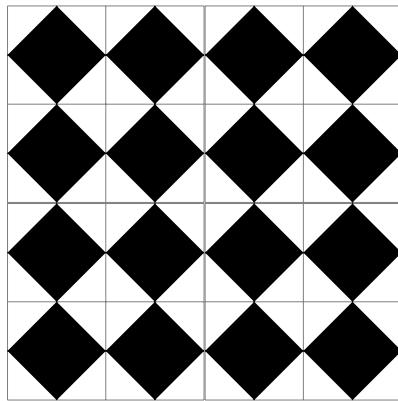
If the original protile has four-fold symmetry, in 2×2 patterns made with the tertiary tile is another copy of the original four-fold rotationally symmetrical Truchet tile. In this the pattern becomes very uniform, a 4×4 repeating pattern of the underlying prototile.

$2\bar{3}10$



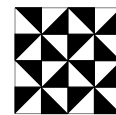
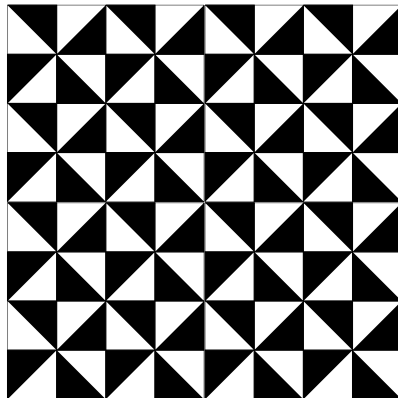
$2\bar{3}10$

$01\bar{3}2$



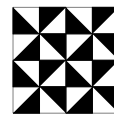
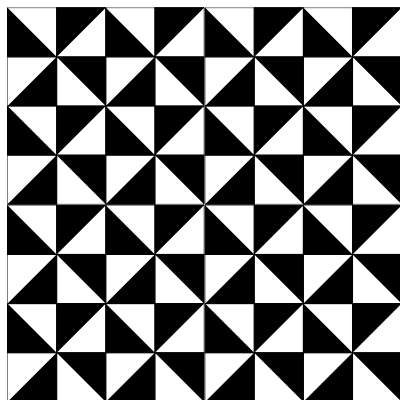
$01\bar{3}2$

$30\bar{2}1$



$30\bar{2}1$

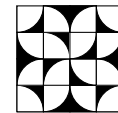
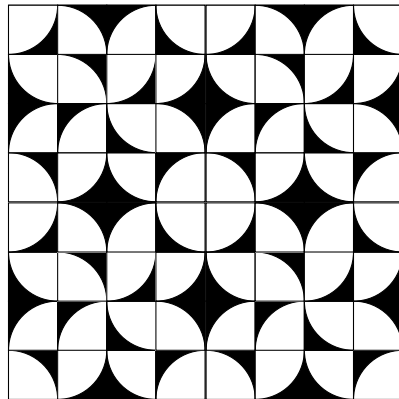
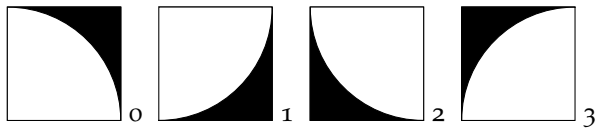
1203



1203

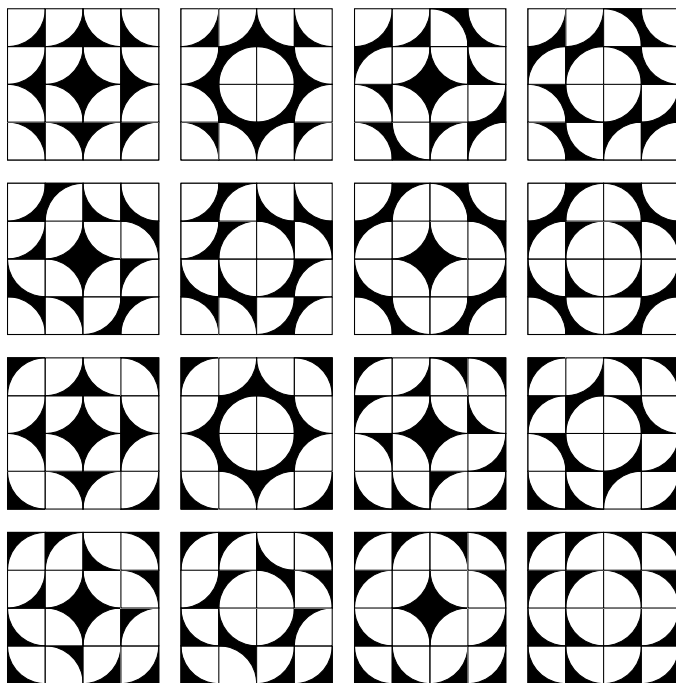
Semicircle Truchet tile patterns

Instead of the traditional Truchet square, any pattern that breaks the rotational symmetry of the square different placement options (2 or 4) can be used. One alternative to the traditional Truchet is to cut the square by an arc, so that a quarter circle is produced instead of a right triangle.



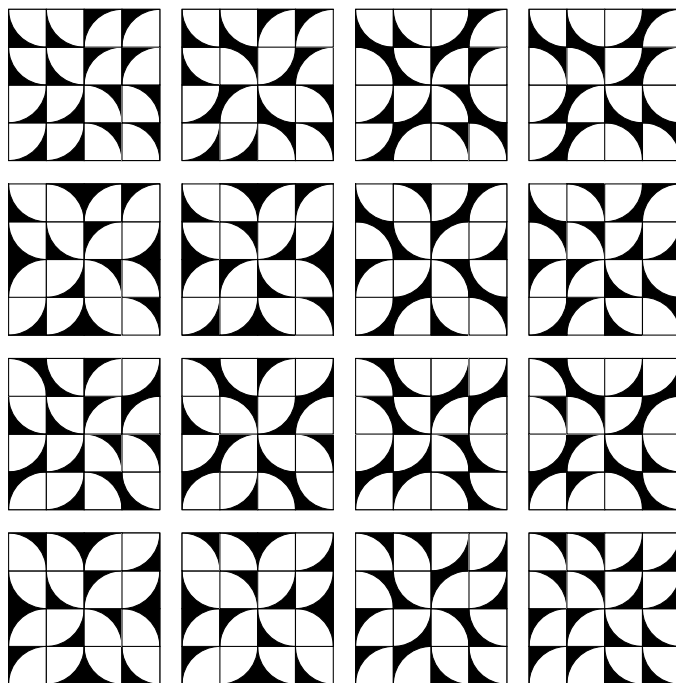
0313

In the version used here, the quarter circle is white against a black background. The unequal colour distribution (favouring either white or black) makes this Truchet variation unbalanced when compared with traditional Truchet tiles. The family groupings no longer bring tile patterns together based on "forgetting" the colour, and the family resemblances among members are somewhat weaker. It is still helpful to group these tiles in the same families as the original tiles, even though the members do not resemble each other as strictly as they do in the traditional case. On the following pages, the families of 4×4 tiles with rotational symmetry are shown using this semicircle Truchet square variant.



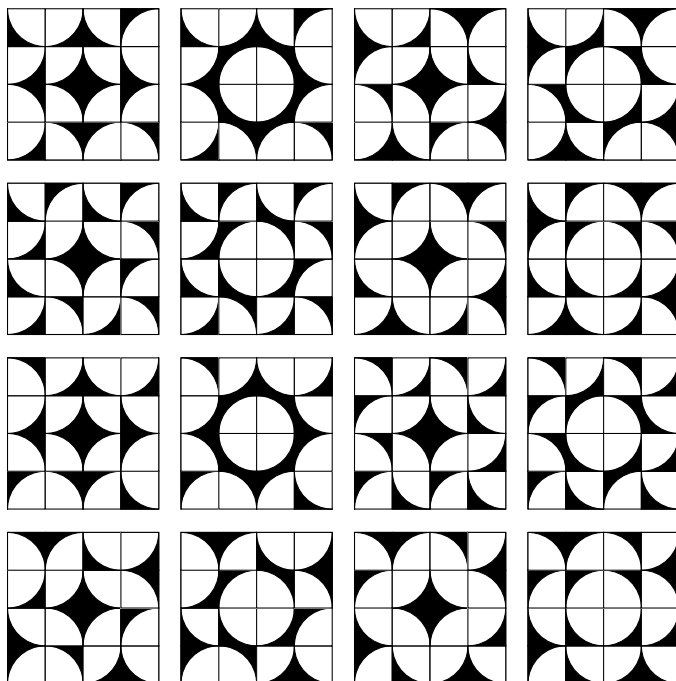
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2000	2002	2020	2022
2200	2202	2220	2222



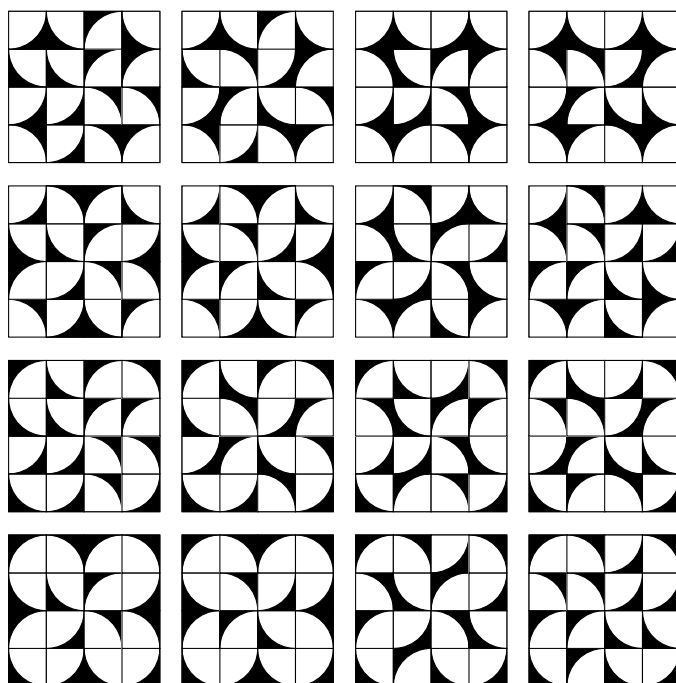
1111

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1311	1313	1331	1333
3111	3113	3131	3133
3311	3313	3331	3333



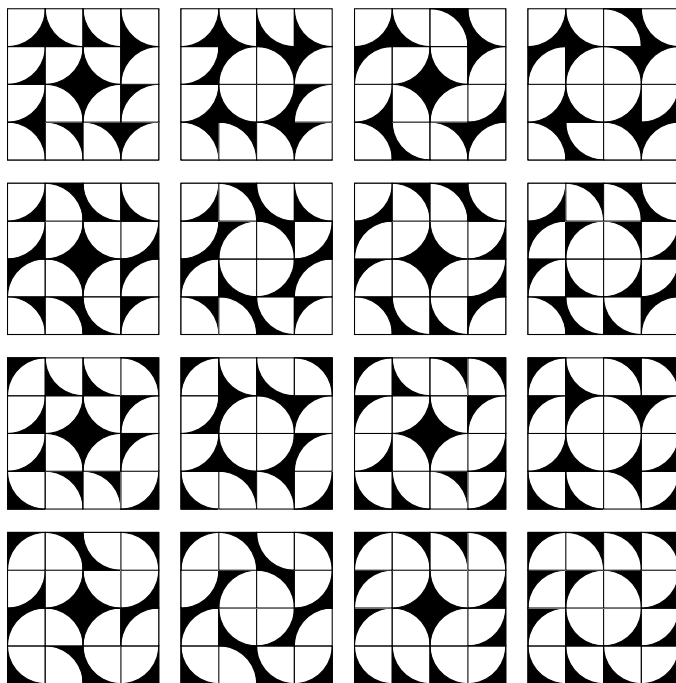
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3200	3202	3220	3222



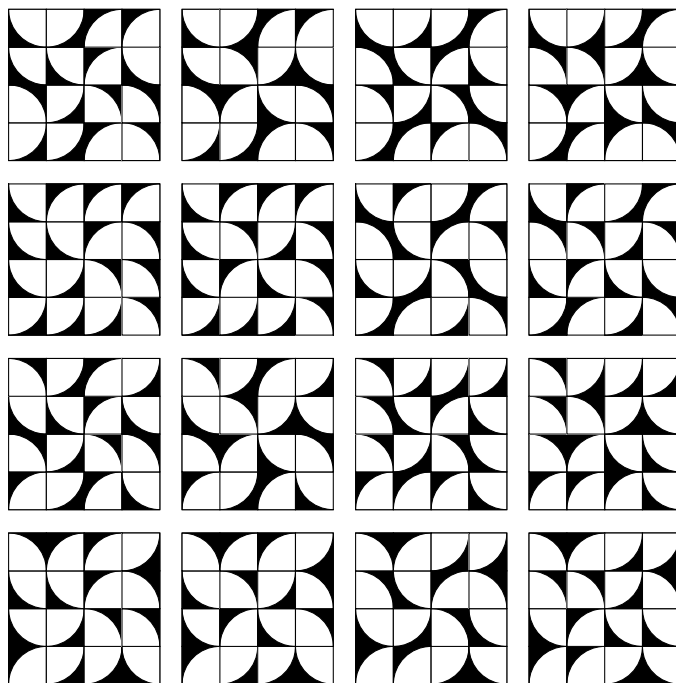
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2111	2113	2131	2133
2311	2313	2331	2333



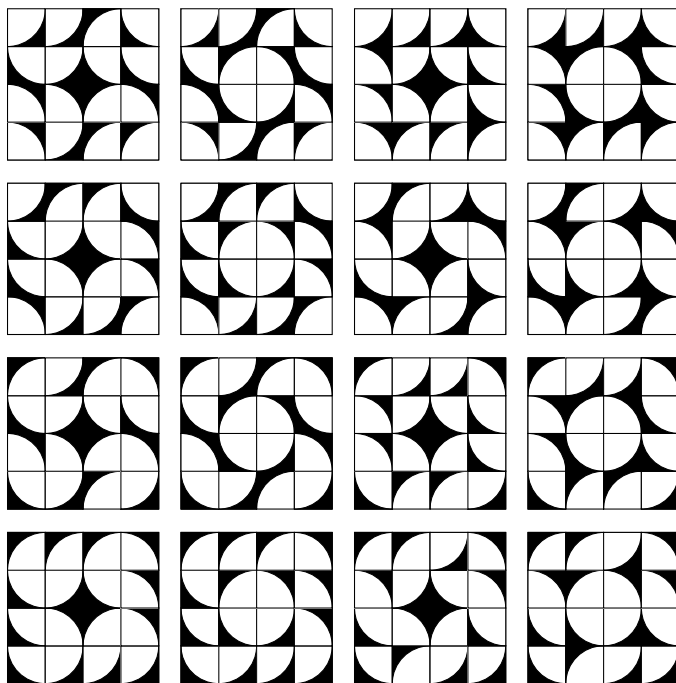
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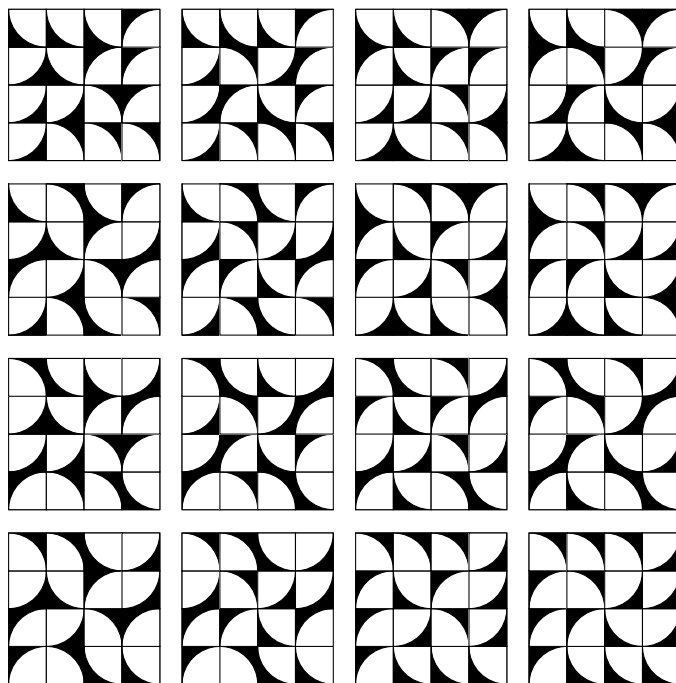
1011

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3011	3013	3031	3033
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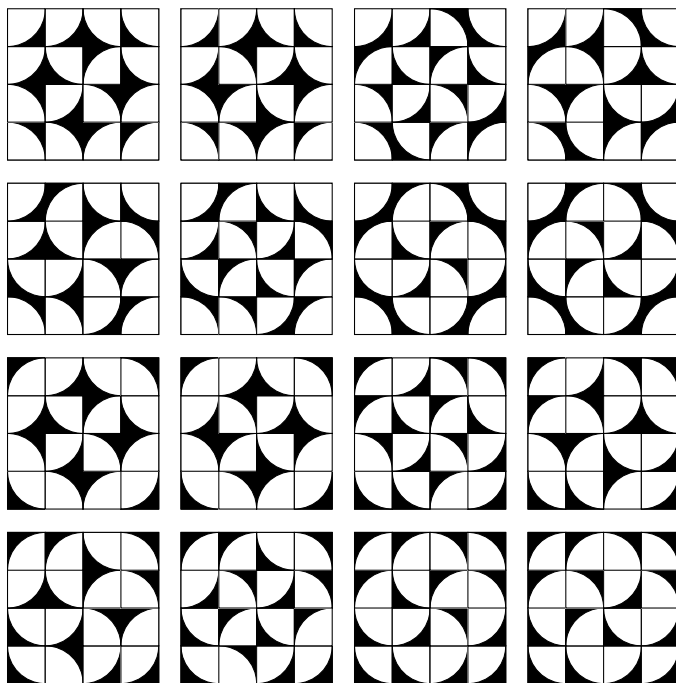
0010

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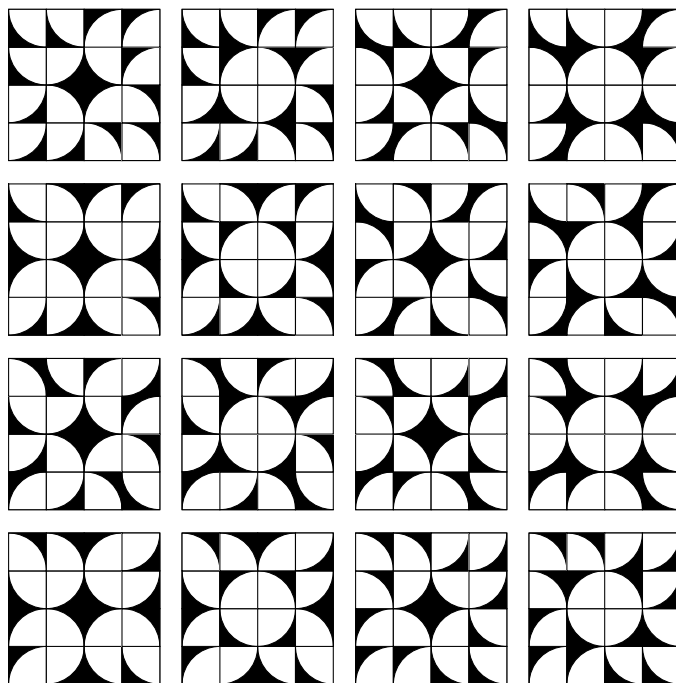
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1301	1303	1321	1323
3101	3103	3121	3123
3301	3303	3321	3323



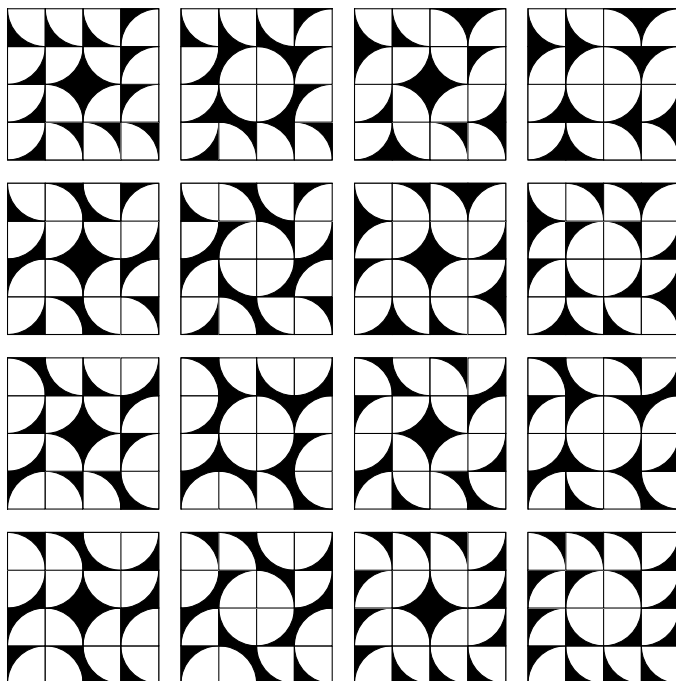
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2001	2003	2021	2023
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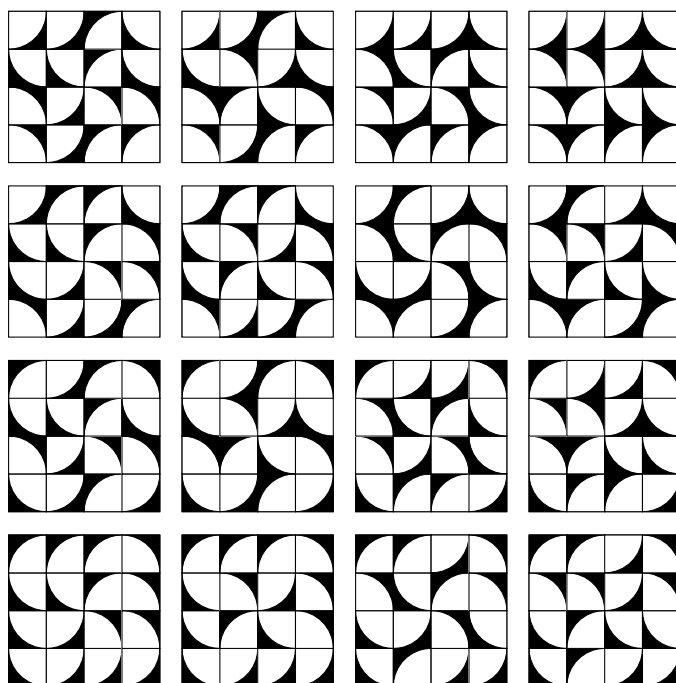
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3110	3112	3130	3132
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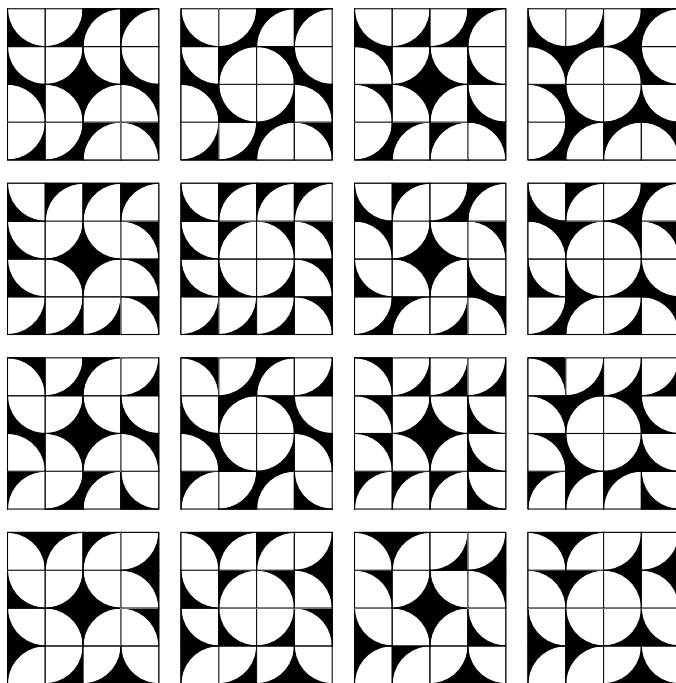
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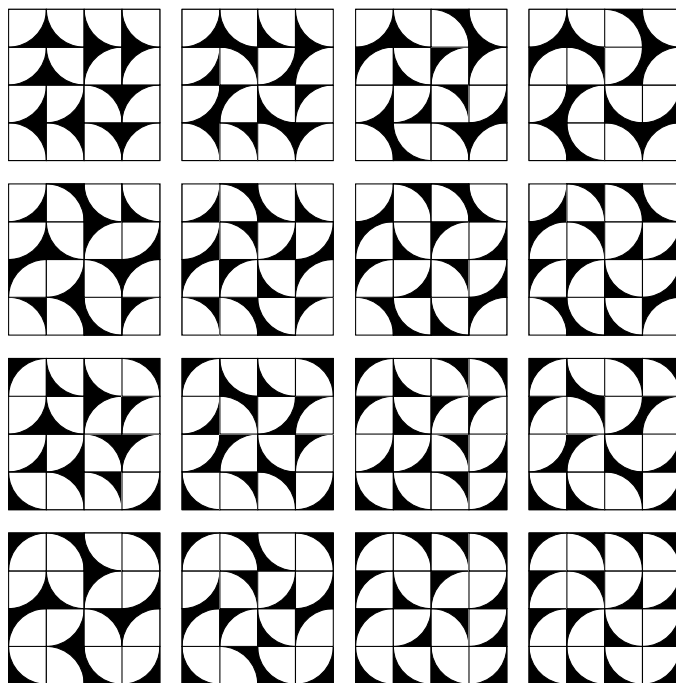
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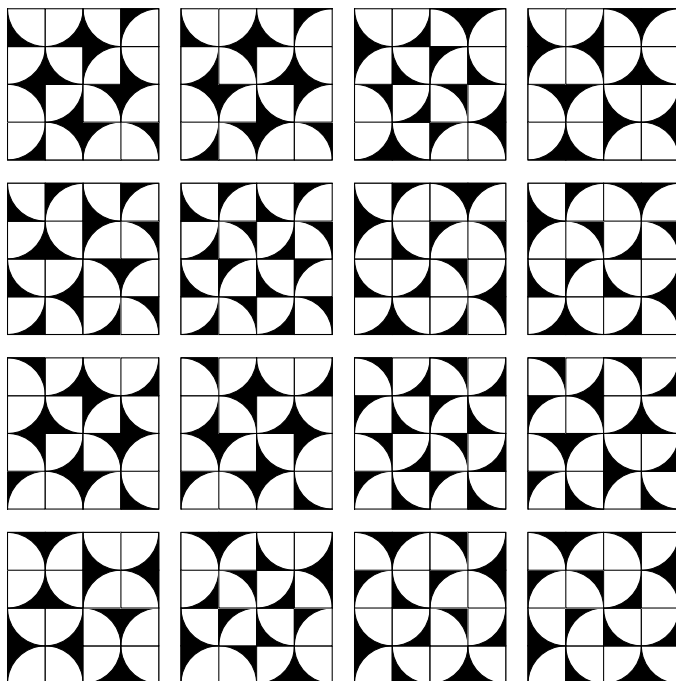
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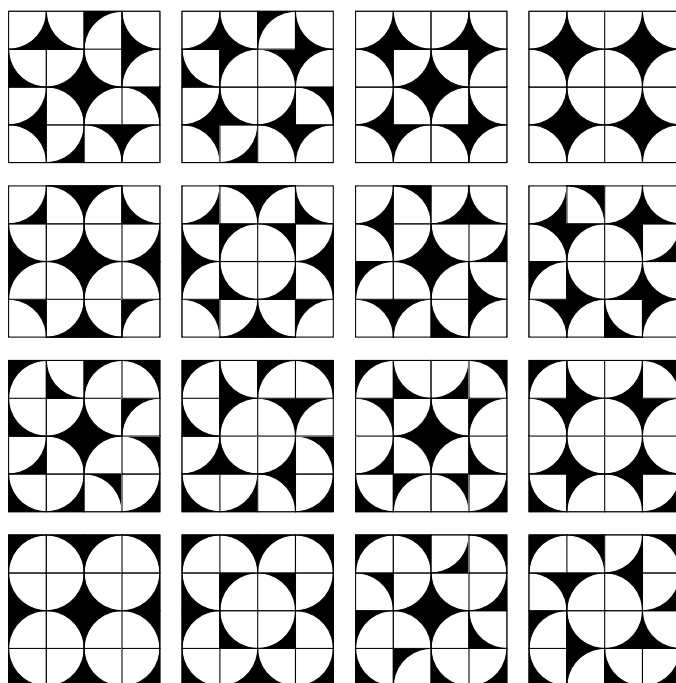
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2101	2103	2121	2123
2301	2303	2321	2323



1001

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1201	1203	1221	1223
3001	3003	3021	3023
3201	3203	3221	3223



0110

0110	0112	0130	0132
0310	0312	0330	0332
2110	2112	2130	2132
2310	2312	2330	2332

Conclusions

The absence of rotational symmetry in the basic Truchet square provides an opportunity express symmetry in larger patterns made from these squares. Truchet tile patterns with 4×4 rotational symmetry are nice to explore - aesthetically appealing, they generate a set that is small enough to completely describe yet large enough to express some interesting variety and relationships.

Looking at Truchet variants suggests that some of the relationships in traditional patterns can be preserved and others altered, opening up new areas to explore.

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