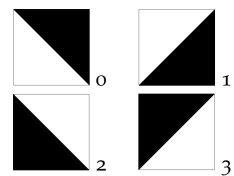
TRUCHET BOOK (I)

4X4 PATTERNS WITH ROTATIONAL SYMMETRY

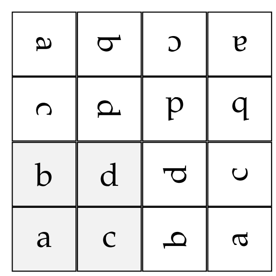
Current printing, January 26, 2025

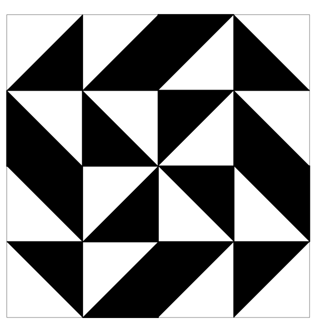
Introduction

Truchet tiles are, traditionally, square tiles that are divided by a diagonal line, and coloured with two colours with a different colour on either side of the diagonal. Each tile can be rotated to one of four positions. Patterns are formed by placing tiles next to each other, often rotating tiles to create repeated motifs. This booklet presents a complete listing of 4*x*4 Truchet tile patterns with rotational symmetry (256 patterns). tiles Treating these 4x4 tile patterns as tiles themselves allows for larger decorative patterns to be built. Interesting relationships among the 4x4 tiles patterns and within the larger patterns created with them can be observed.



Each 4*x*4 Truchet tile pattern with rotational symmetry has a core 2*x*2 pattern in one of its quadrants that is rotated to produce the overall pattern. In this booklet, the core pattern is assumed to be in the lower left. Each pattern can identified as a sequence of 4 digits *abcd* that list the rotational positions of each tile in the lower left quadrant.

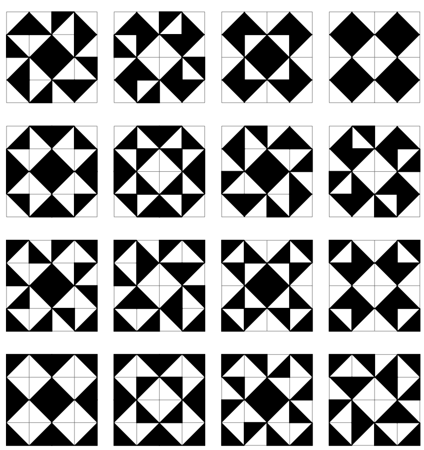




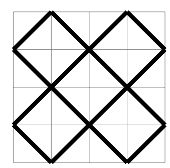
The 0011 pattern

Pattern families

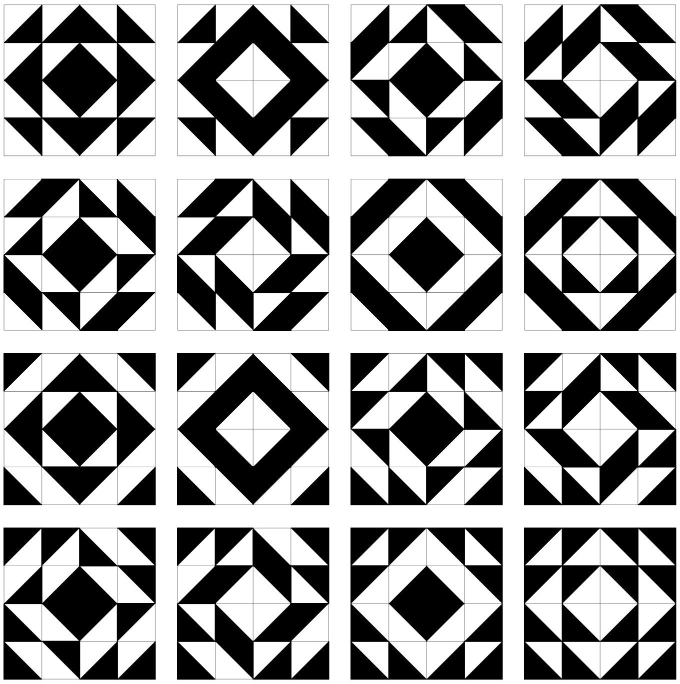
We can group the 4x4 Truchet tile patterns with rotational symmetry into families where tile patterns are considered to be in the same family if they would look the same without colour – if each corresponding tile shares the same diagonal direction. The sequence that represents the family of a tile pattern can be found by taking the sequence of the tile pattern *modulo* 2. So, for example, the 16 tile patterns below are all members of the 0110 family.



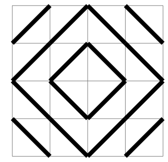
The 0110 pattern family



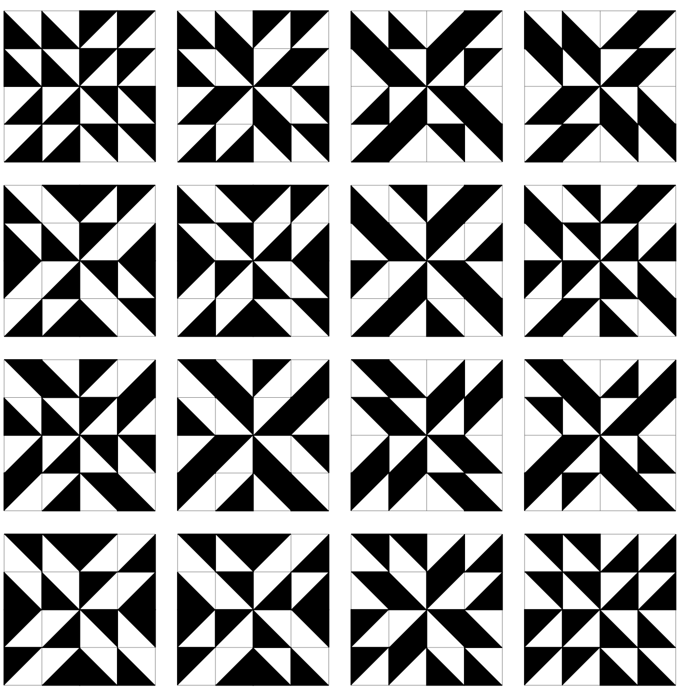
The 0110 family pattern



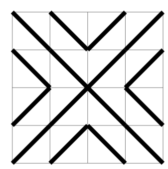
0000



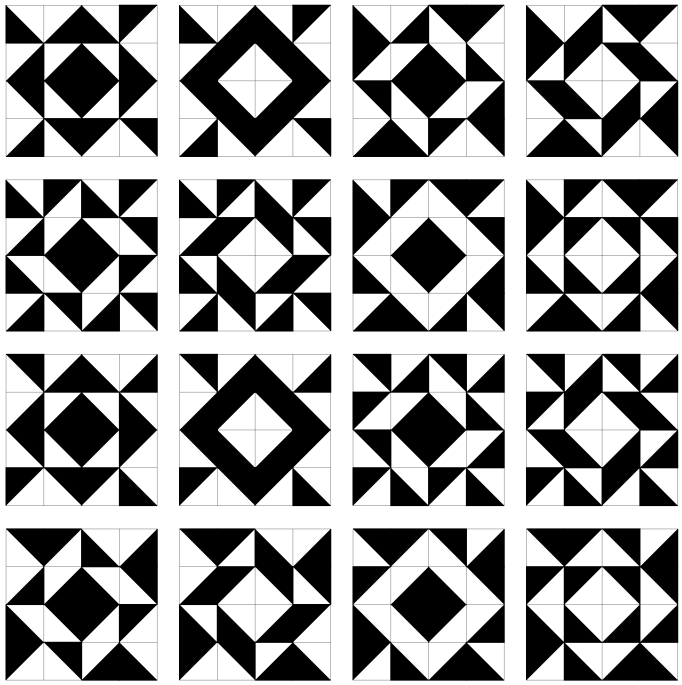
0000	0002	0020	0022
0200	0202	0220	0222
2000	2002	2020	2022
2200	2202	2220	2222



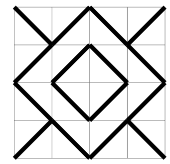
1111



1111	1113	1131	1133
1311	1313	1331	1333
3111	3113	3131	3133
3311	3313	3331	3333



1000



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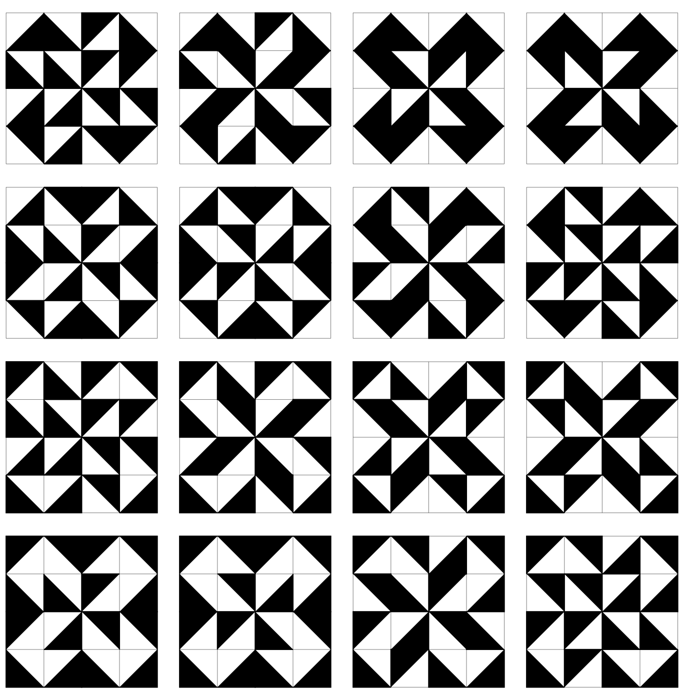
1200	1202	1220	1222
3000	3002	3020	3022
3200	3202	3220	3222



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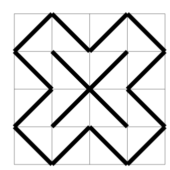

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0311	0313	0331	0333
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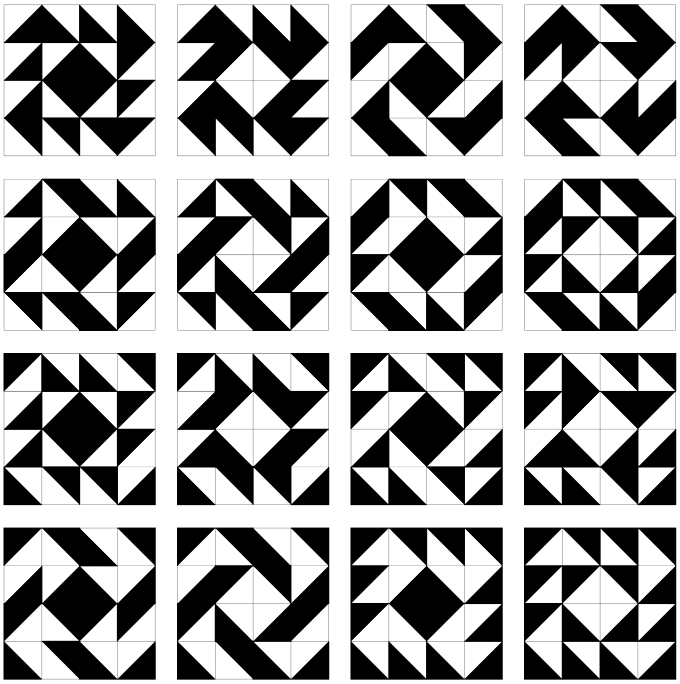
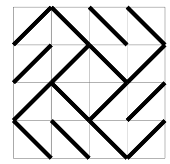
2311	2313	2331	2333



0100



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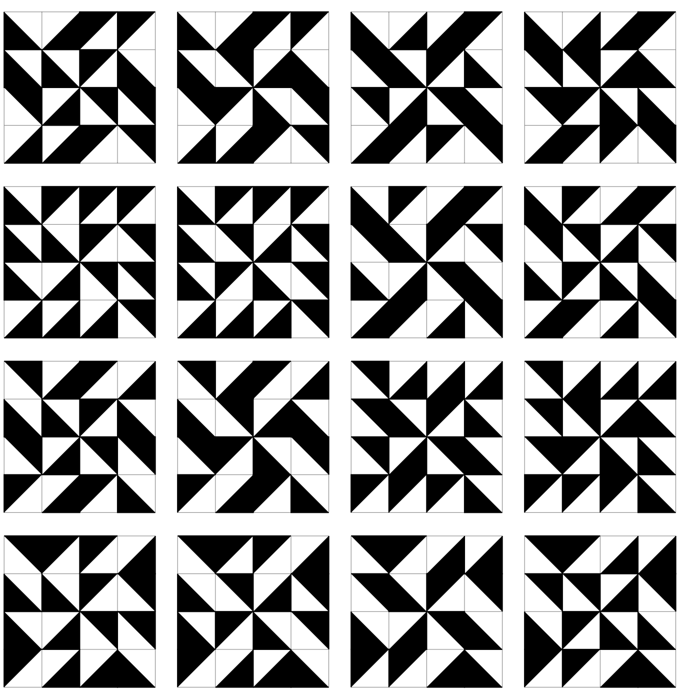
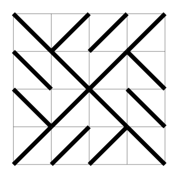
2100	2102	2120	2122
2300	2302	2320	2322



1011



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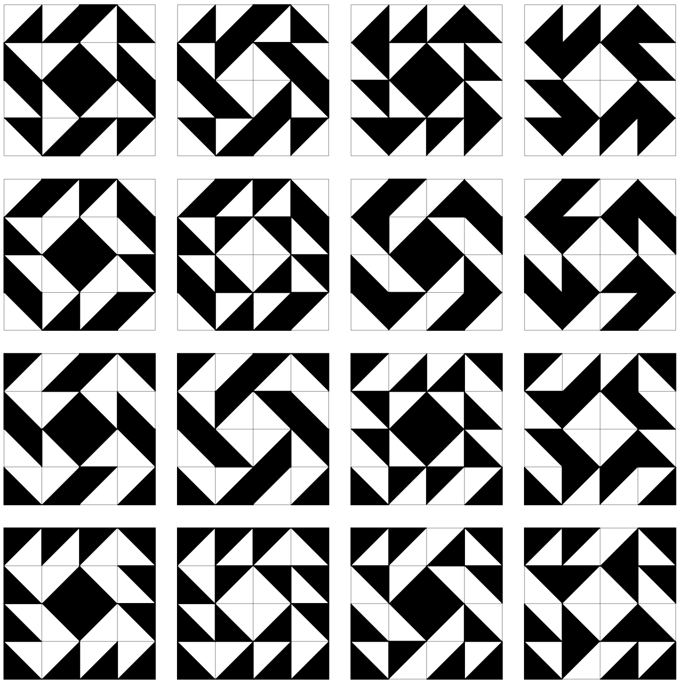
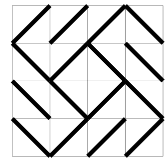
3011	3013	3031	3033
3211	3213	3231	3233



0010



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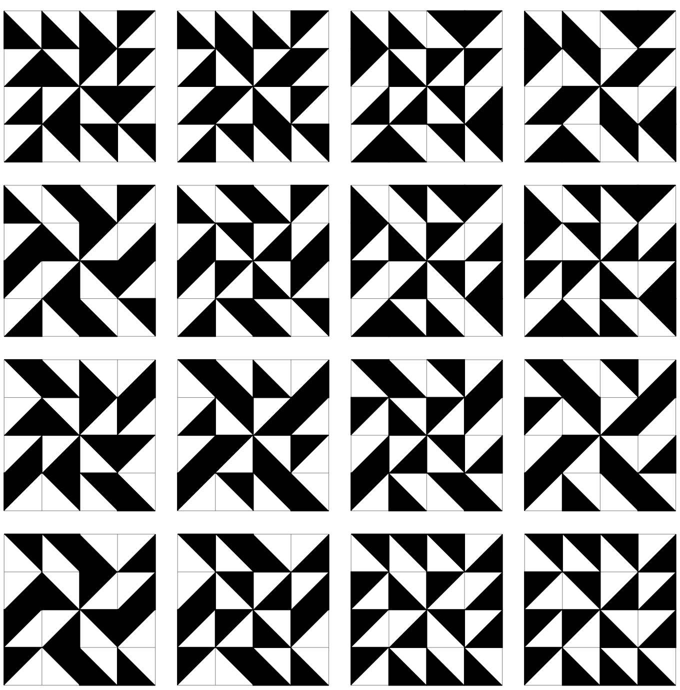
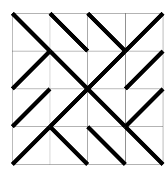
2010	2012	2030	2032
2210	2212	2230	2232



1101



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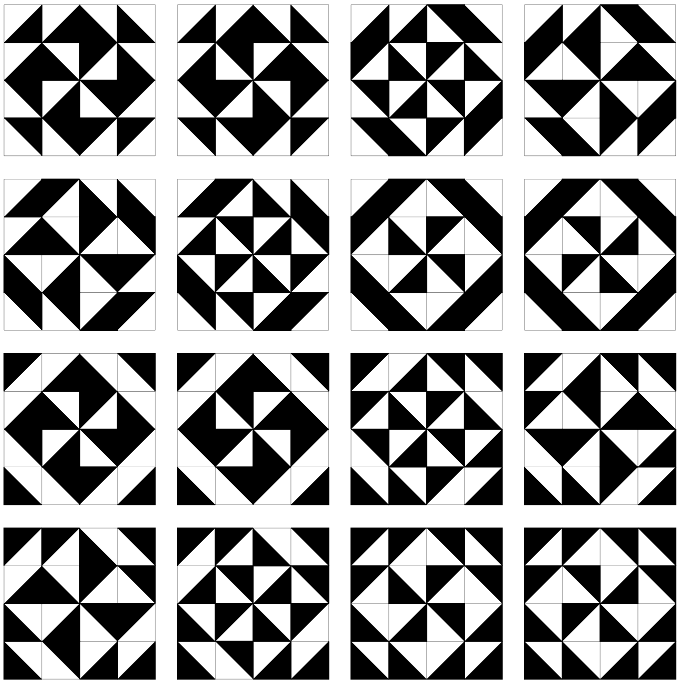
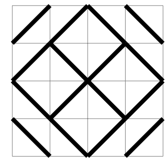
1301	1303	1321	1323
3101	3103	3121	3123
3301	3303	3321	3323



0001



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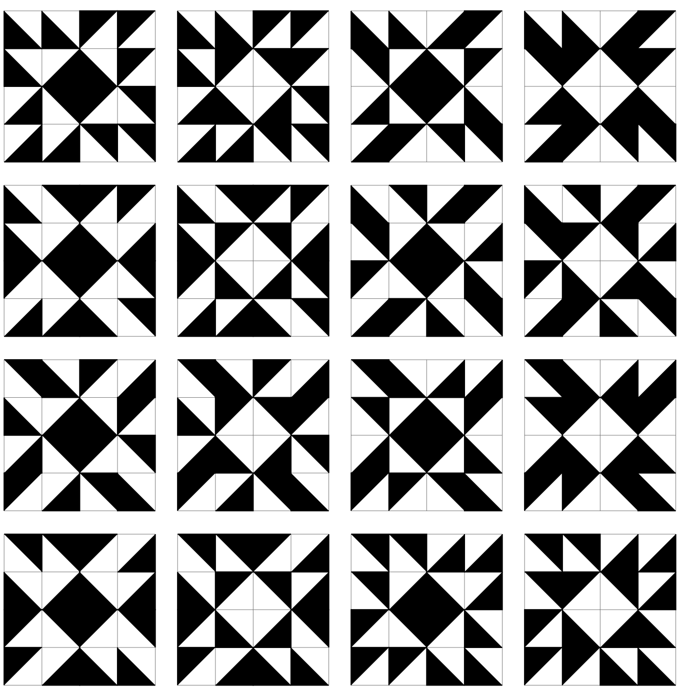

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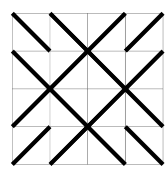

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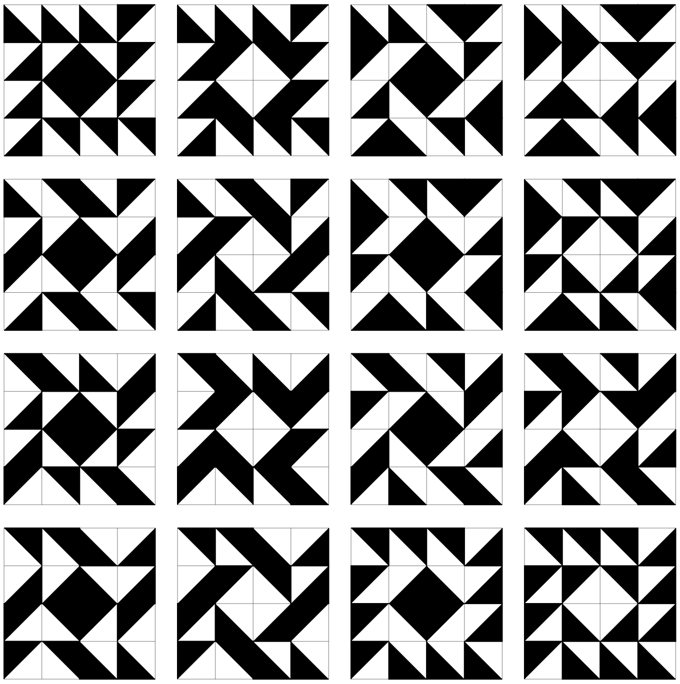

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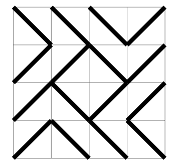

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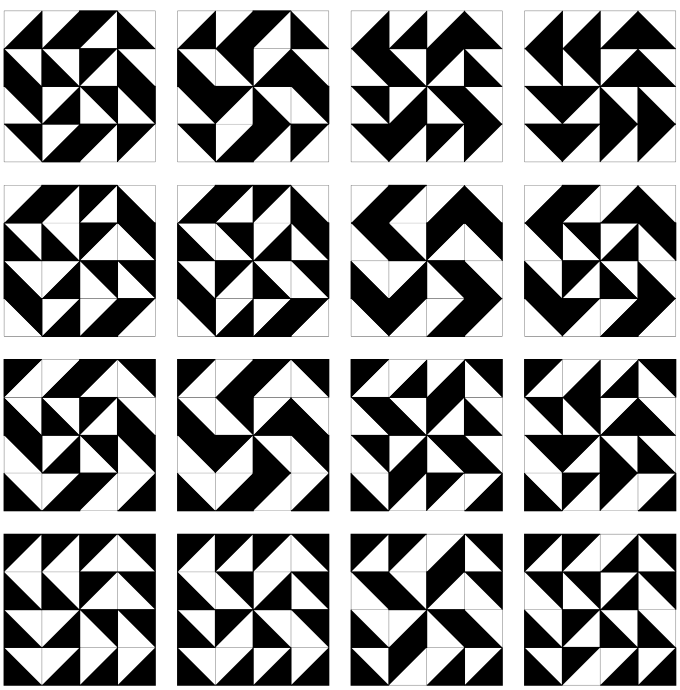
3300	3302	3320	3322



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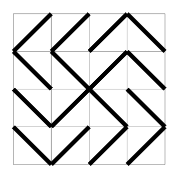

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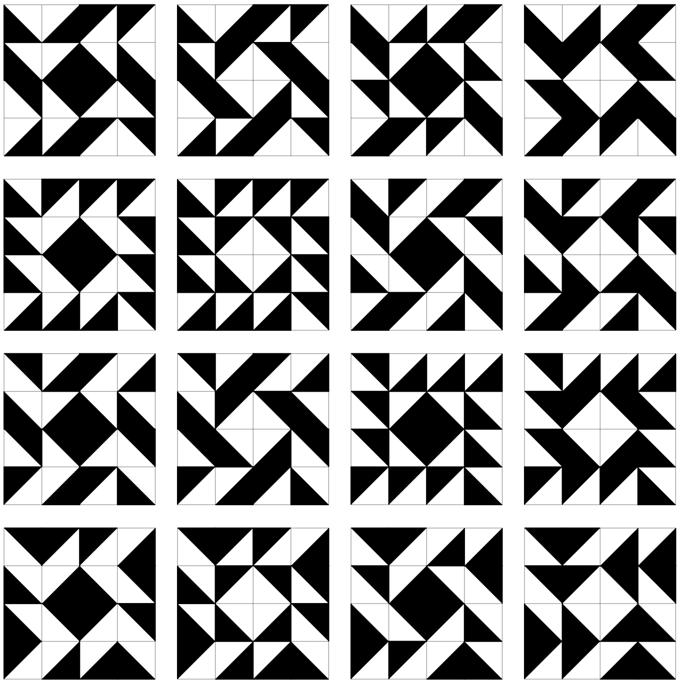
2011	2013	2031	2033
2211	2213	2231	2233



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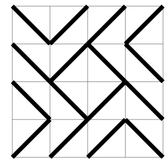

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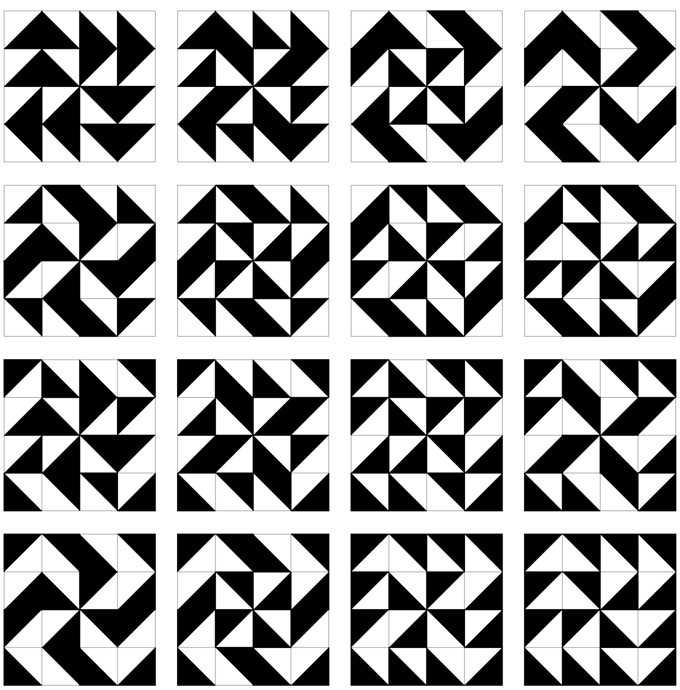
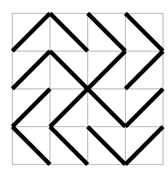


0101



0101	0103	0121	0123
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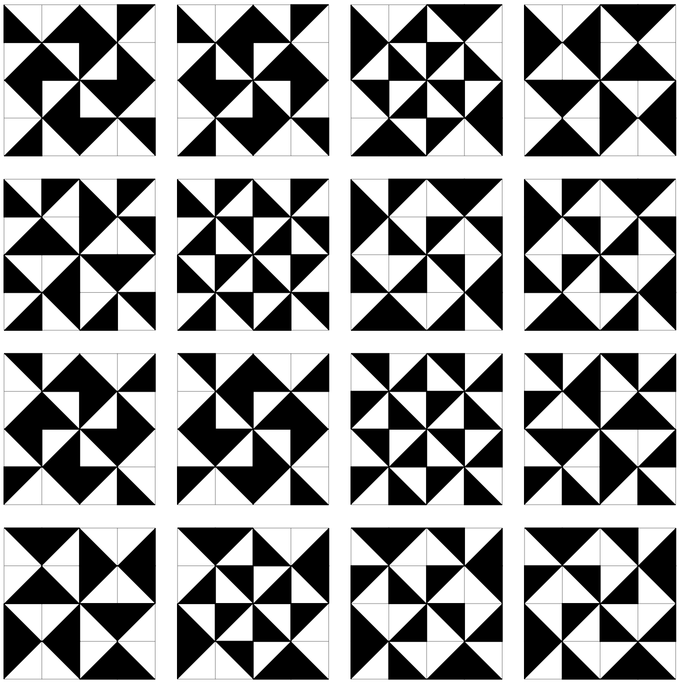
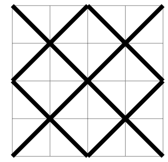
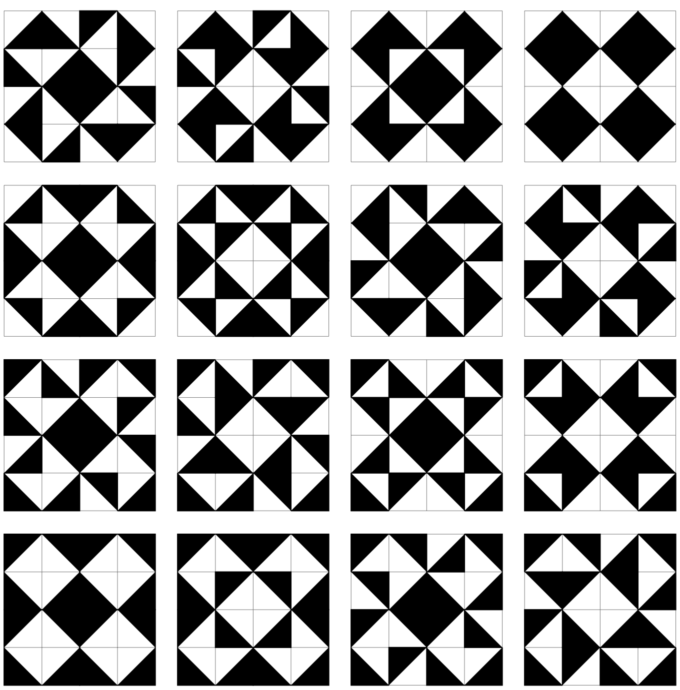
2101	2103	2121	2123
2301	2303	2321	2323



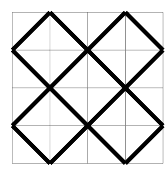
1001



1001	1003	1021	1023
1201	1203	1221	1223
3001	3003	3021	3023
3201	3203	3221	3223



0110

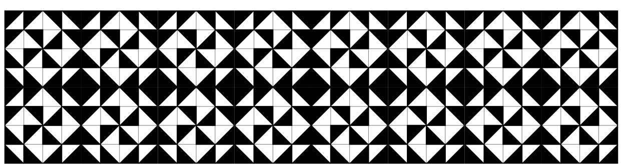


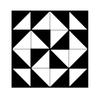
0110	0112	0130	0132
0310	0312	0330	0332
2110	2112	2130	2132
2310	2312	2330	2332

Friezes of Truchet patterns

Each 4x4 Truchet pattern can be treated like a tile and used in a larger pattern. A *frieze* is a horizontal strip of the same tile pattern repeated. Friezes of 4x4 Truchet pattern tiles with rotational symmetry can be quite striking, and have some interesting characteristics.

In frieze of more than one row of a primary tile reveals a secondary tile pattern that appears as another horizontal strip of 4x4 Truchet tile patterns nestled between the rows of primary tiles. Below, a frieze of 2223 tiles has a secondary pattern of 1000 tiles.





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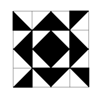
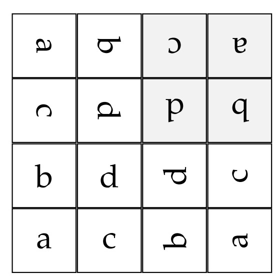


1000

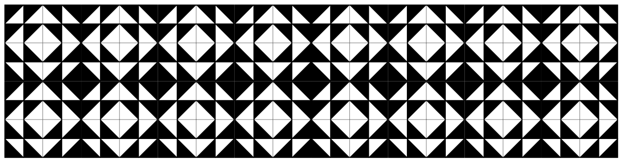
Frieze patterns made from tiles of a particular family will have secondary tiles that also have rotational symmetry and that are from the same family. For example, frieze patters from the family 0001 will have secondary patterns from the family 1000.

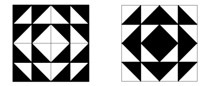


You can compute the secondary frieze tile pattern of a given tile by reversing the digits of the tile label and adding 2 to each digit, modulo 4. This is because the label of the secondary frieze tile pattern comes from the top right corner of the primary tile pattern.

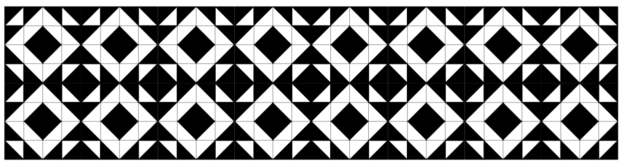
If tile *t* is the primary tile in a frieze pattern and *s* is the secondary, then *t* will be the secondary tile in the frieze pattern formed by *s*. This effectively reduces the total number of unique frieze patterns to 136, these patterns are shown on the pages that follow.

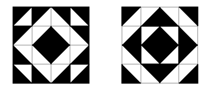
Frieze patterns for family 0000 (secondary, 0000)



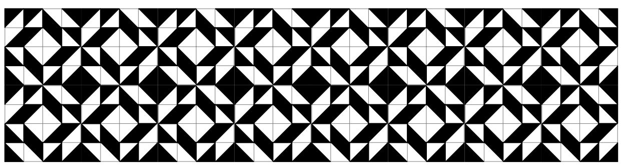


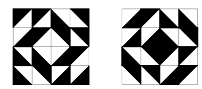
2222 0000



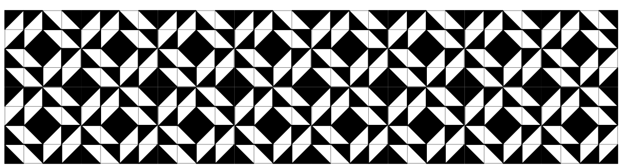


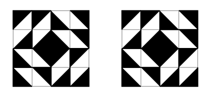
2220 2000



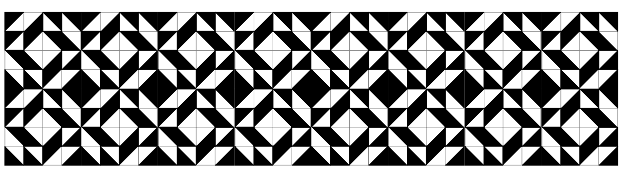


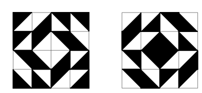
2202 0200



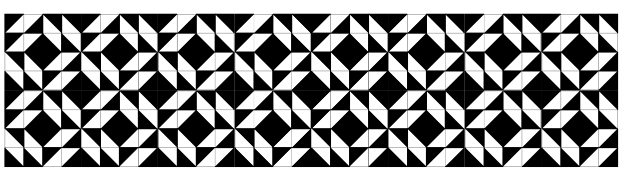


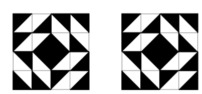
2200 2200



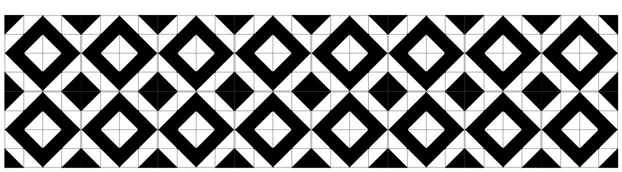


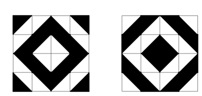
2022 0020



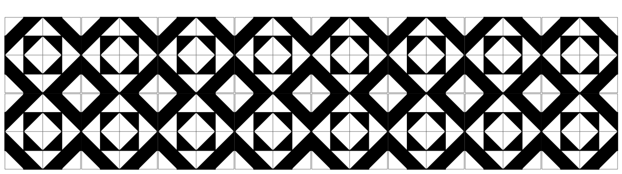


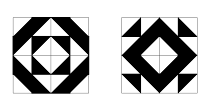
2020 2020



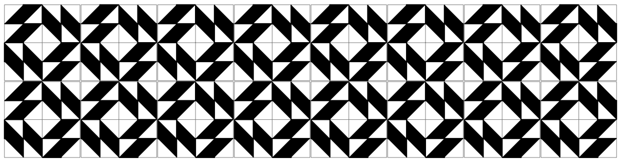


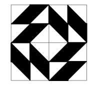
2002 0220



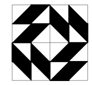


0222 0002

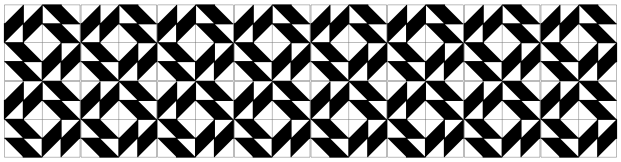


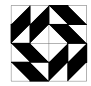


0202

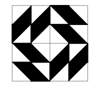


0202



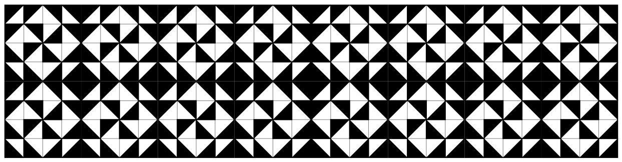


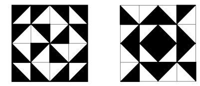
0022



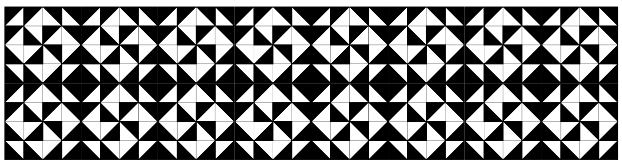
0022

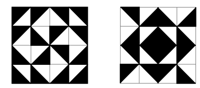
Frieze patterns for family 0001 (secondary, 1000)





2223 1000





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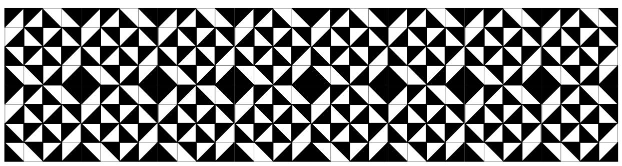
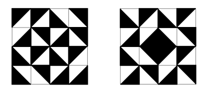
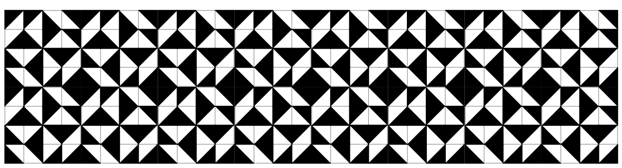
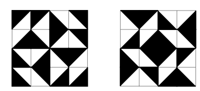


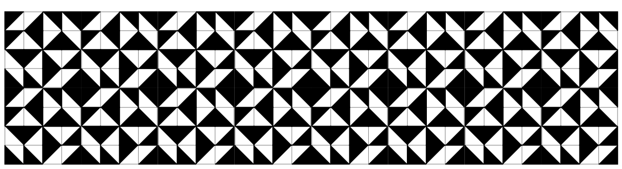


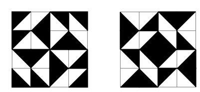
2203 1200



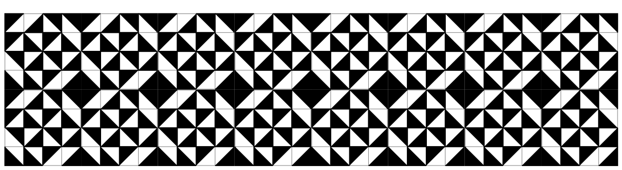


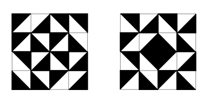
2201 3200





2023 1020





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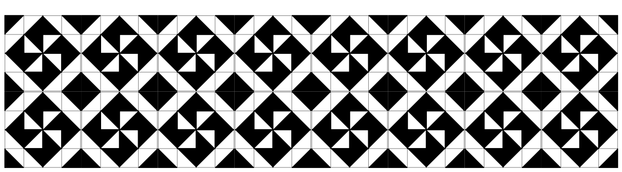
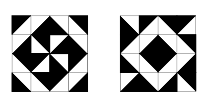
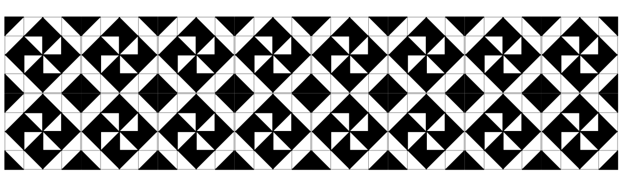
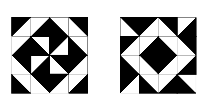


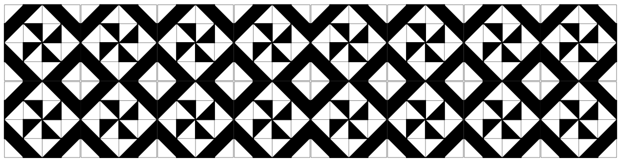


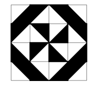
2003 1220





2001 3220

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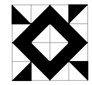
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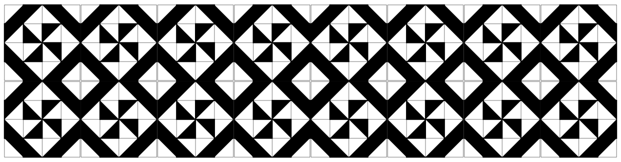

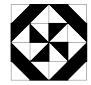
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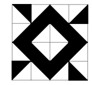
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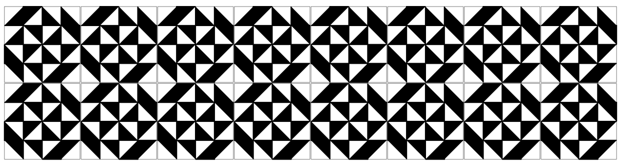

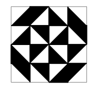
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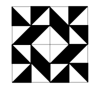
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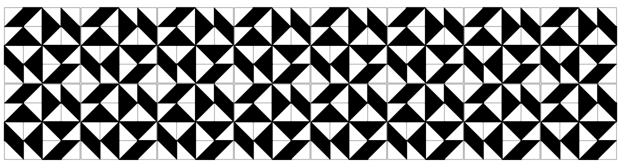

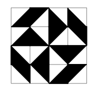
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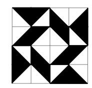
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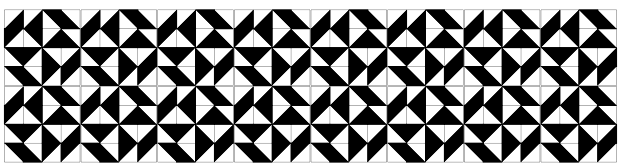

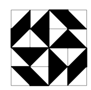
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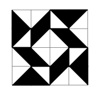
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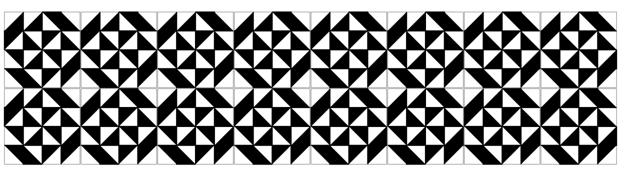

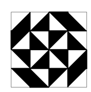
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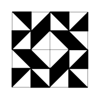
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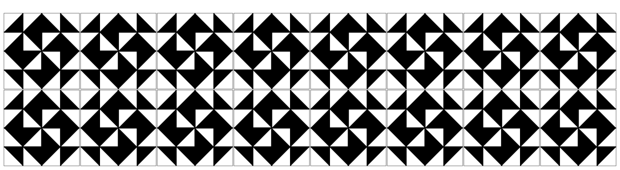

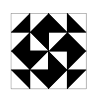
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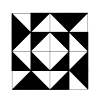
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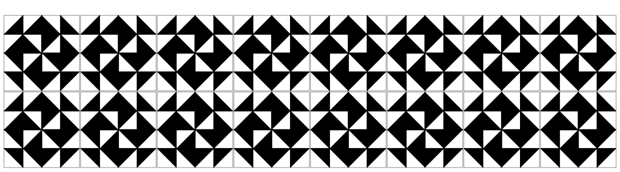

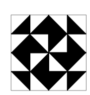
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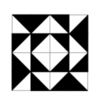
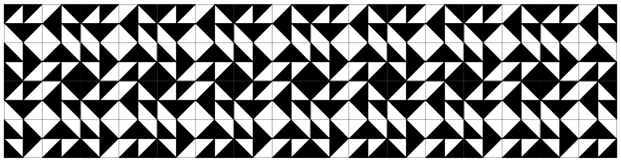
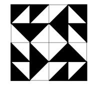

3222

Frieze patterns for family 0010 (secondary, 0100)





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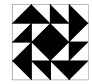
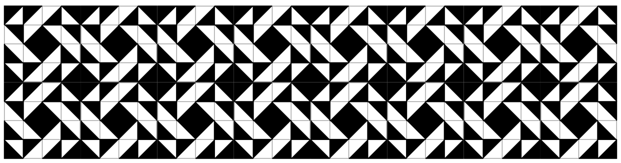
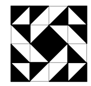


0100





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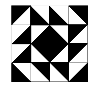

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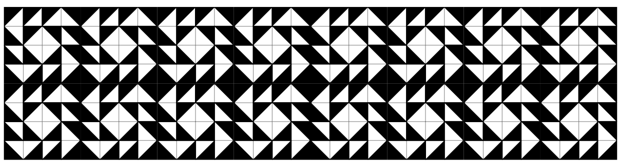

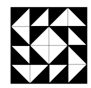

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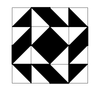

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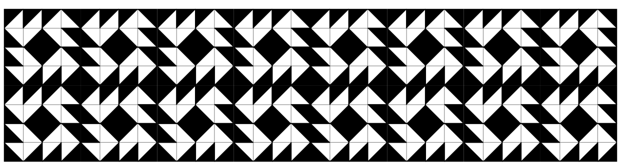

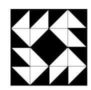

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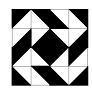

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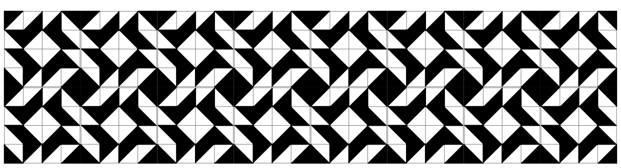

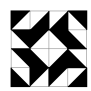

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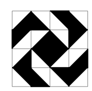
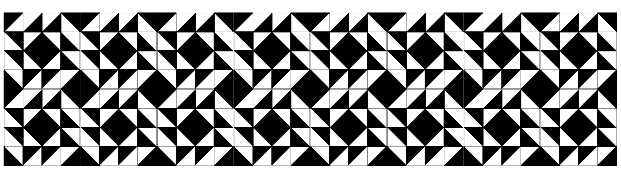
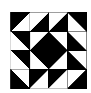


0120





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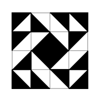

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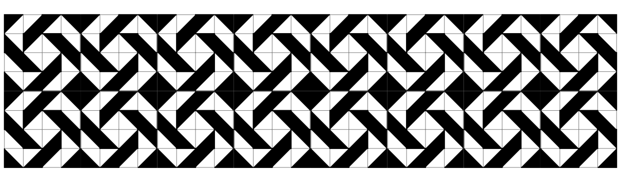
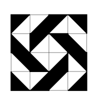
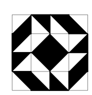




2012



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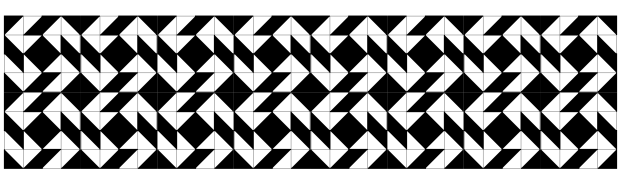
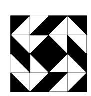
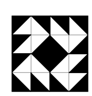




2010



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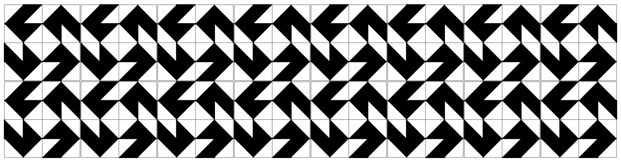

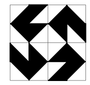
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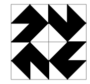
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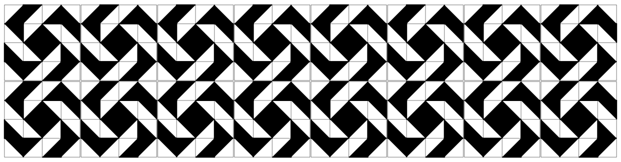

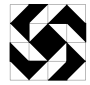
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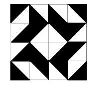
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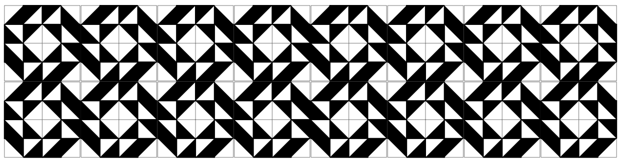

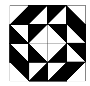
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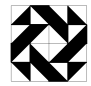
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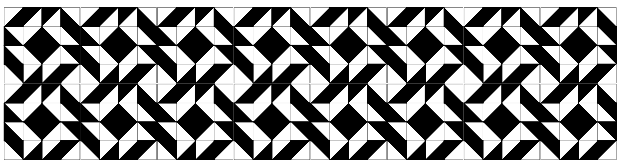

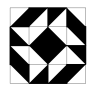
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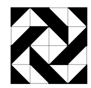
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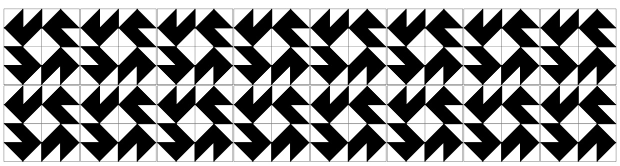

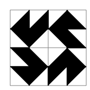
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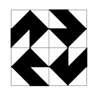
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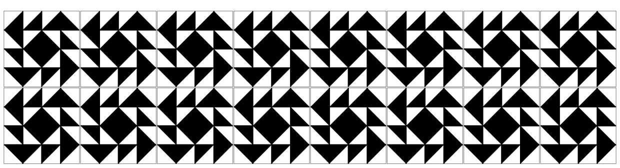

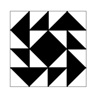
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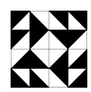
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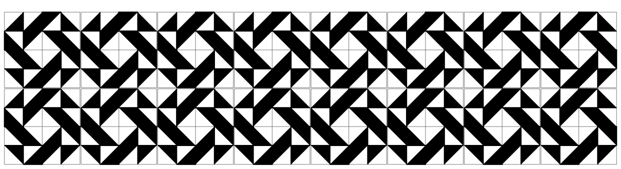

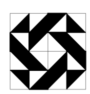
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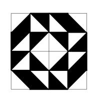
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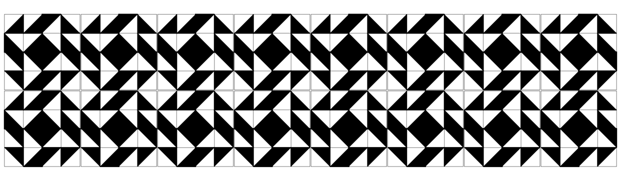

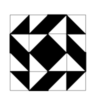
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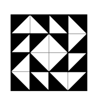
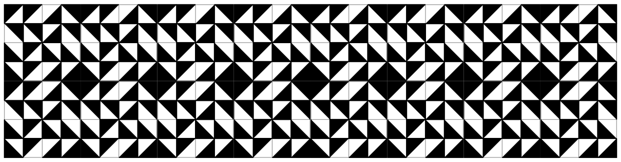
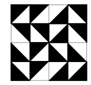

2322

Frieze patterns for family 0011 (secondary, 1100)

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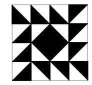
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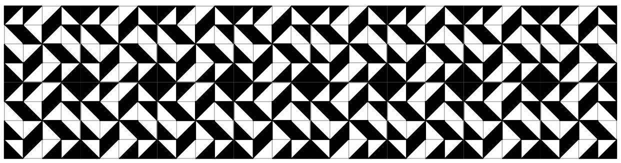

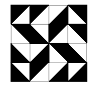
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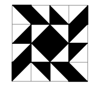
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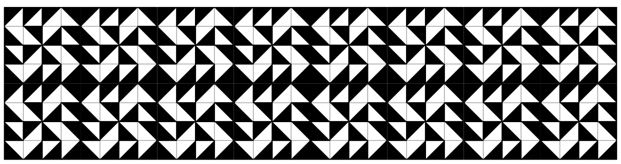

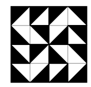
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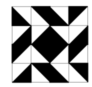
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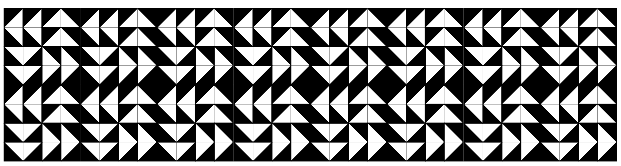

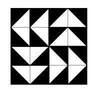
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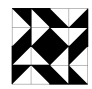
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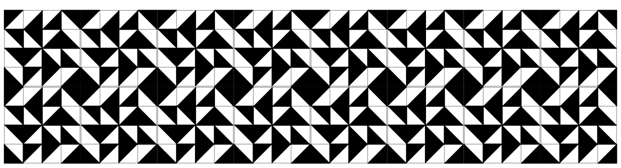

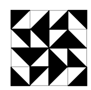
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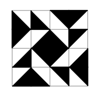
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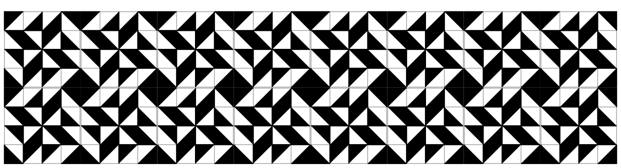

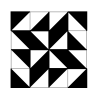
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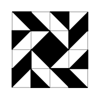
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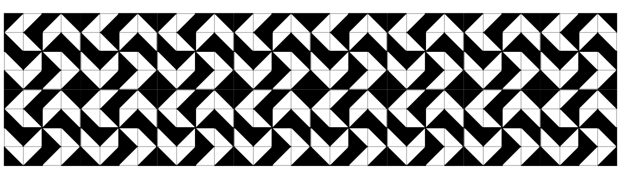
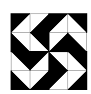
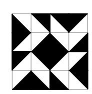



2013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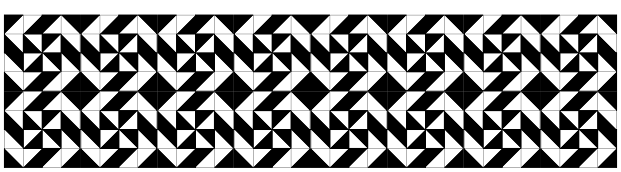

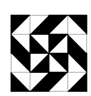
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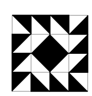
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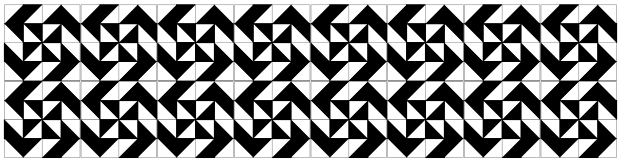

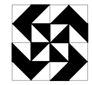
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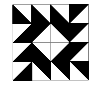
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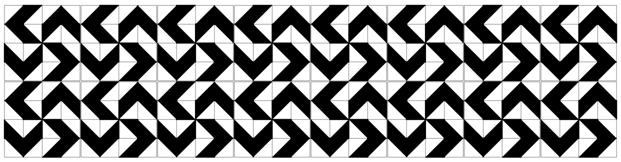

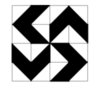
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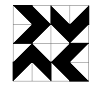
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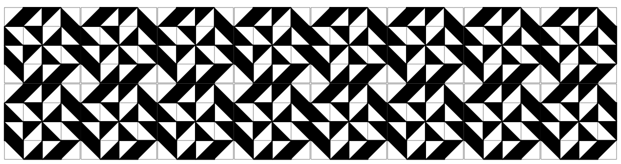

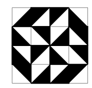
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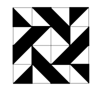
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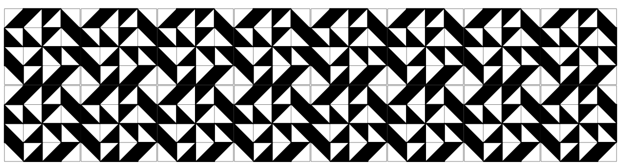

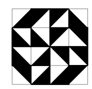
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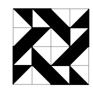
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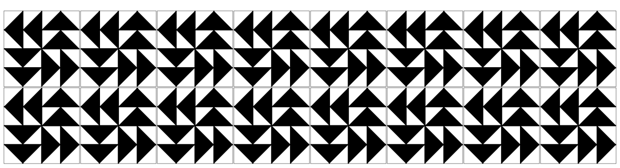

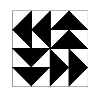
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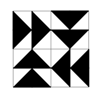
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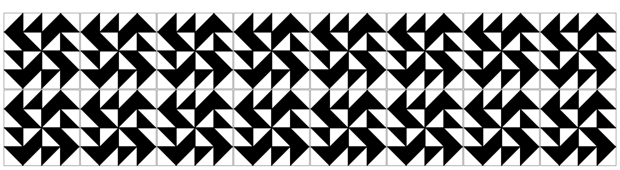

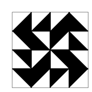
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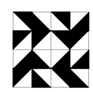
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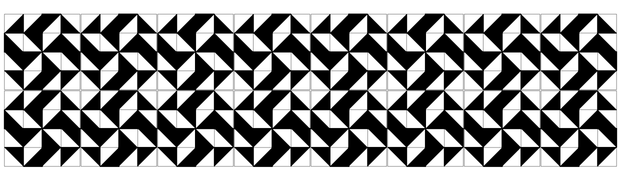

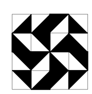
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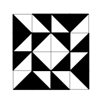
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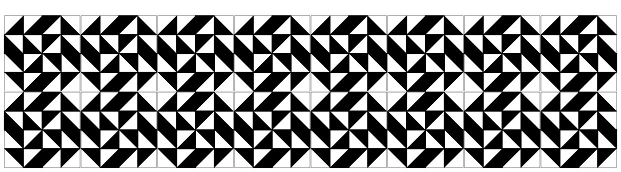

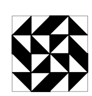
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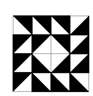
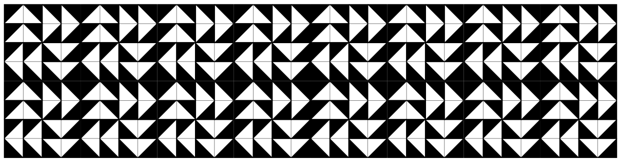
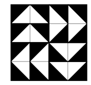
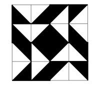

3322

Frieze patterns for family 0101 (secondary, 1010)

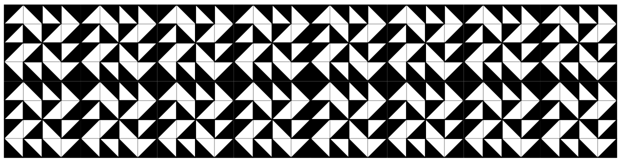


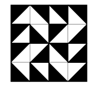


2323



1010





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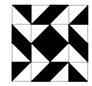

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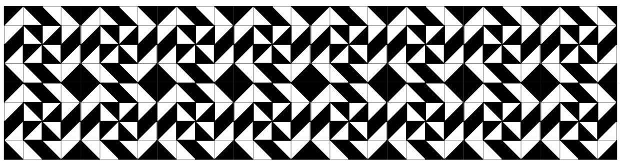
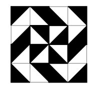
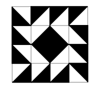
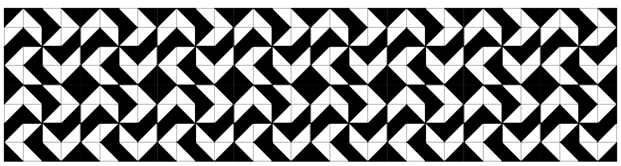


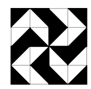


2303

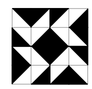


1210

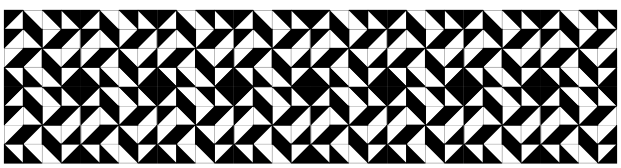


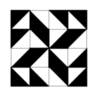


2301



3210





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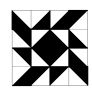

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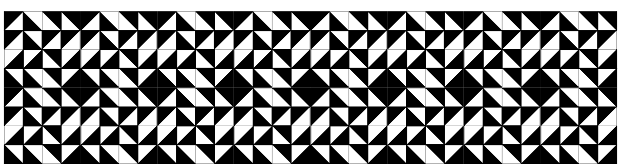

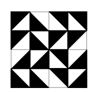

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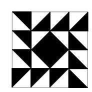

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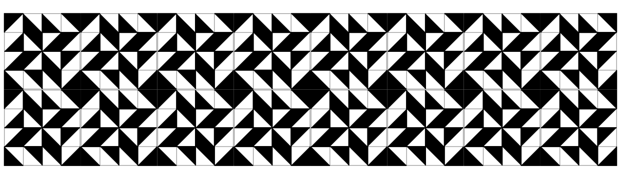

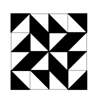

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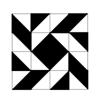

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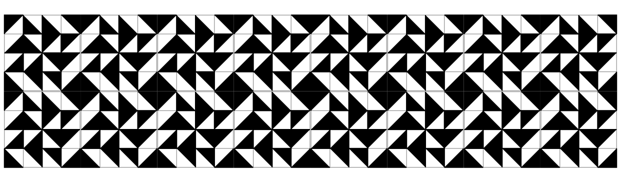

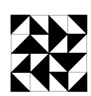

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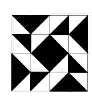

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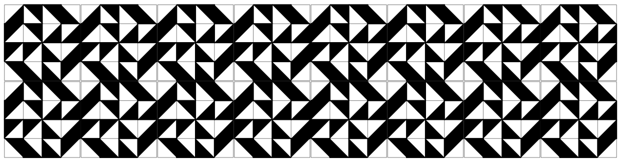

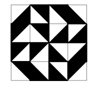

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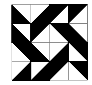

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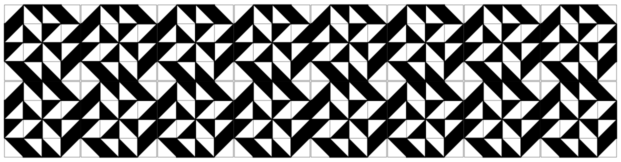

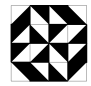

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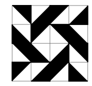

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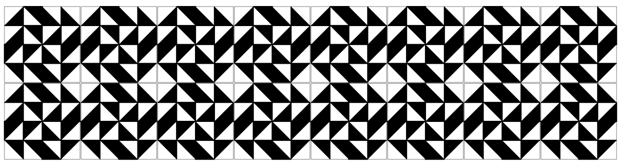

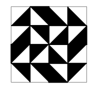

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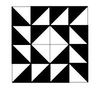

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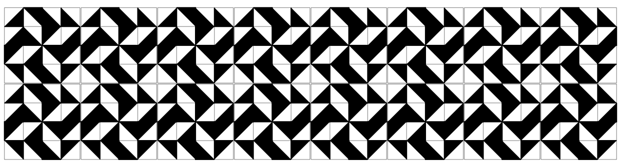

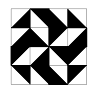

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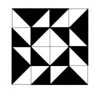

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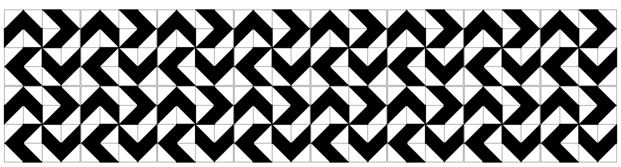

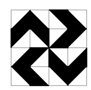

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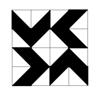

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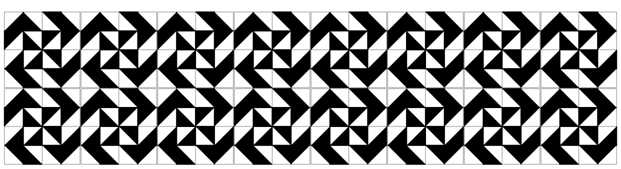

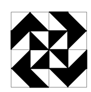

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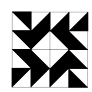

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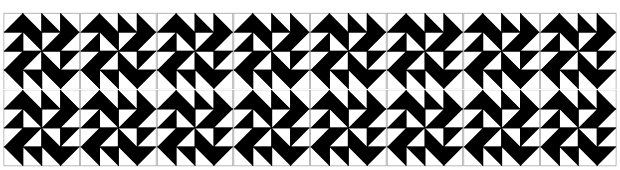

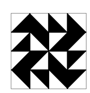

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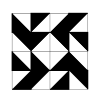

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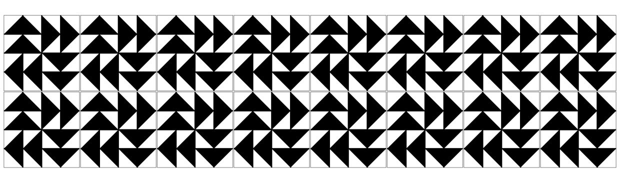

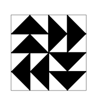

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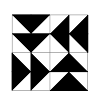
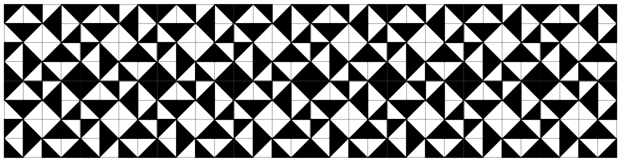
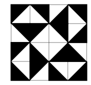


3232

Frieze patterns for family 0110 (secondary, 0110)





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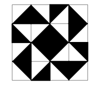

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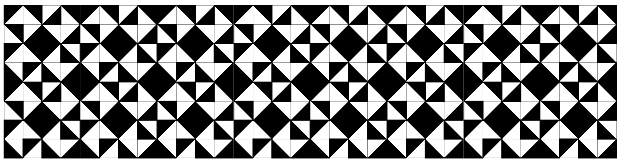

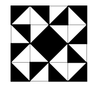

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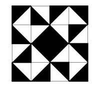

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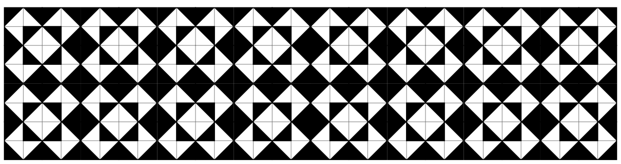

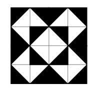

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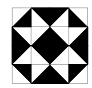

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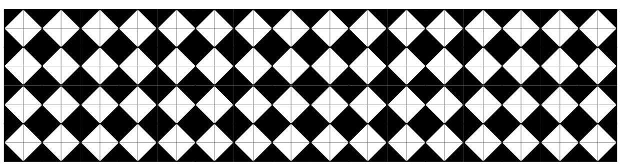

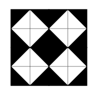

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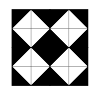

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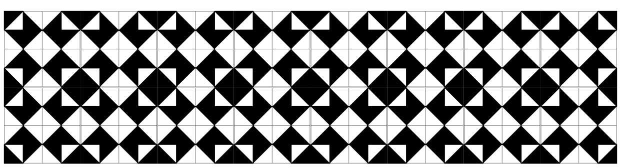

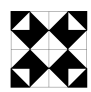

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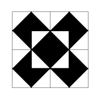

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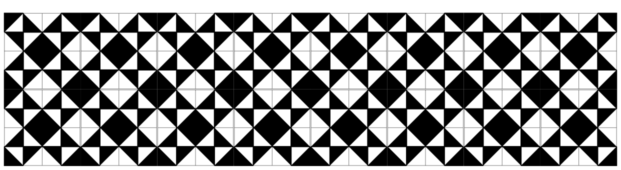

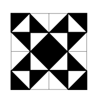

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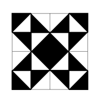

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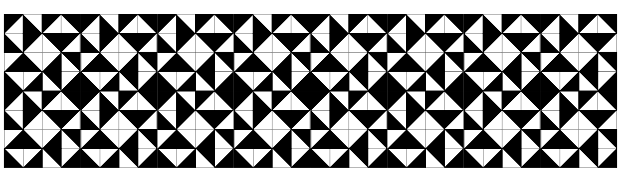

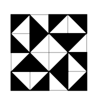

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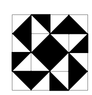

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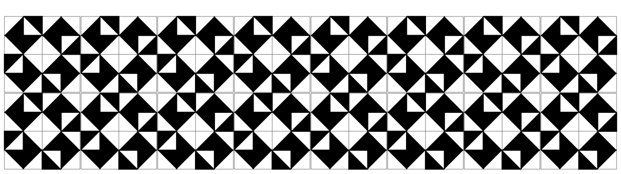

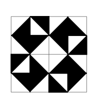

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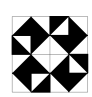

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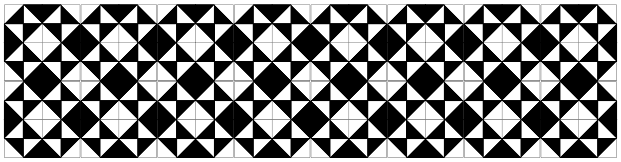

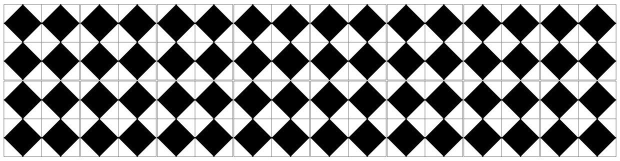

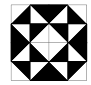

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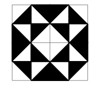

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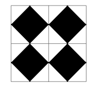

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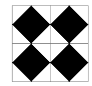
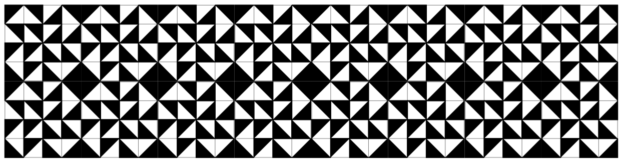
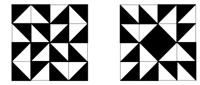


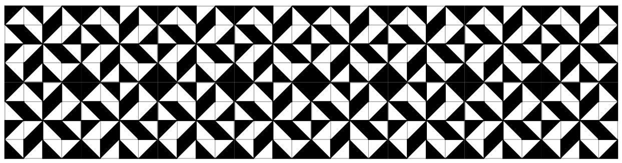
0132

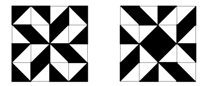
Frieze patterns for family 0111 (secondary, 1110)



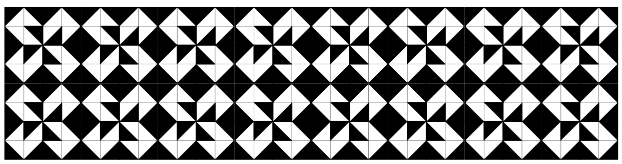


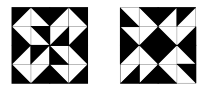
2333 1110





2331 3110





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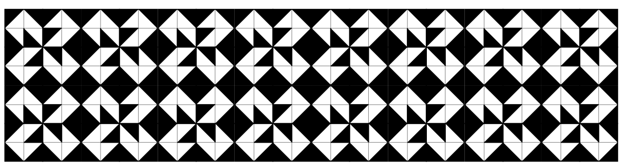
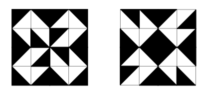
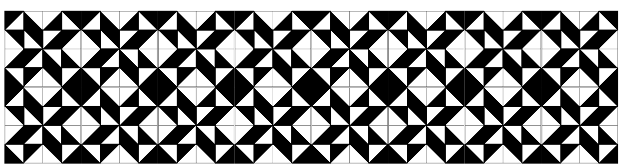
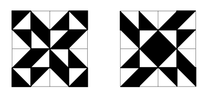




2311 3310





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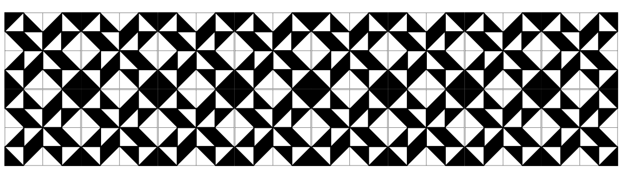

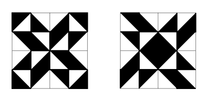

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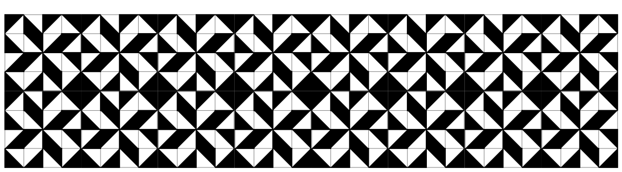

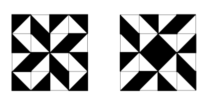

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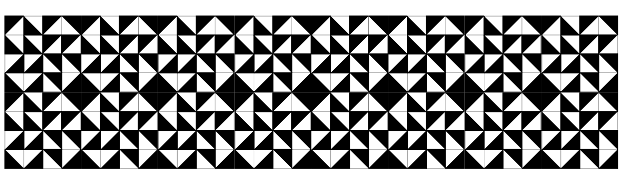

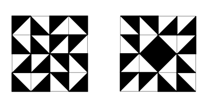

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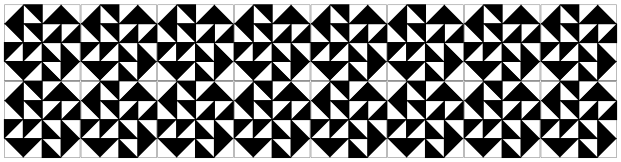

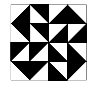
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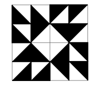
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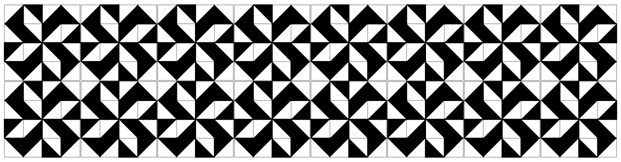

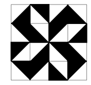
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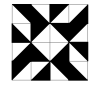
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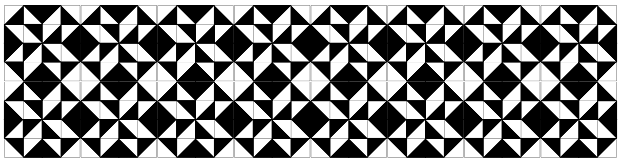

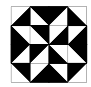
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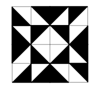
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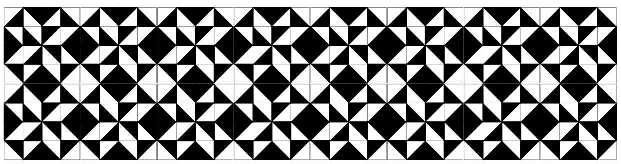

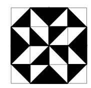
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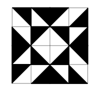
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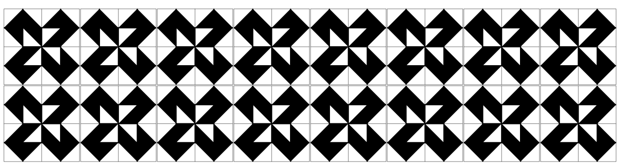

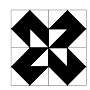
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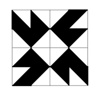
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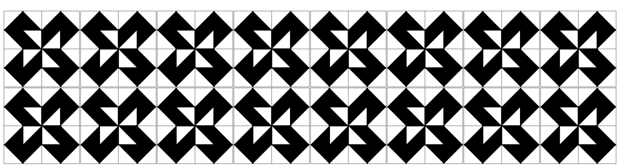

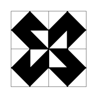
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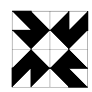
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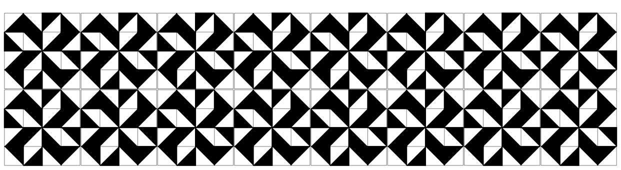

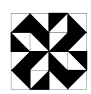
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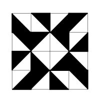
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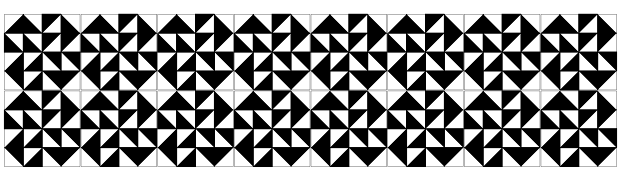

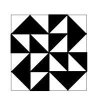
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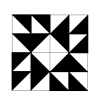
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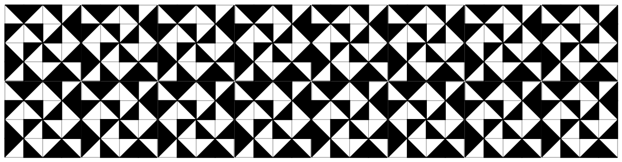
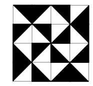
Frieze patterns for family 1001 (secondary, 1001)





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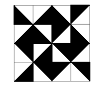

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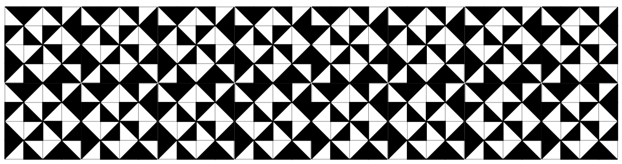

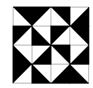

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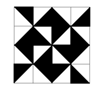

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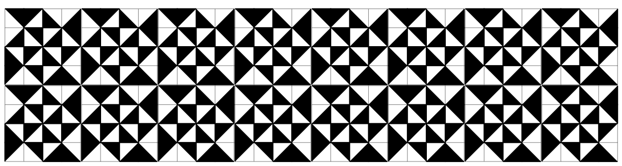

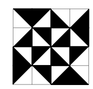

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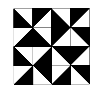

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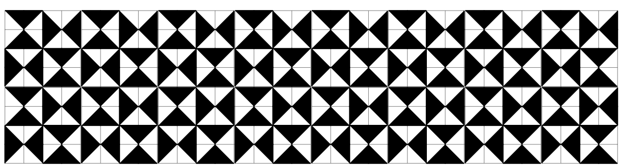

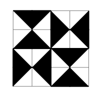

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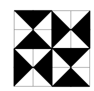

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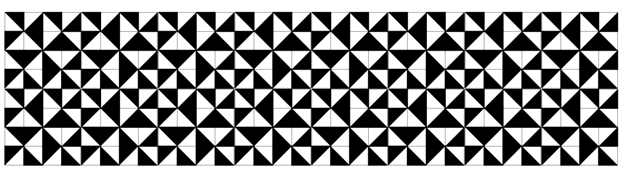

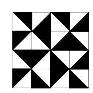

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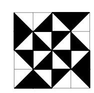

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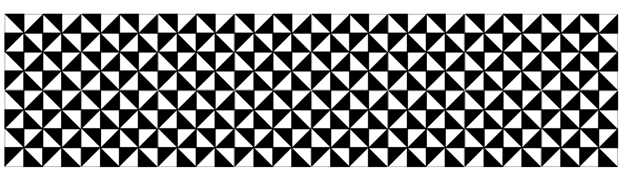

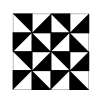

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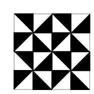

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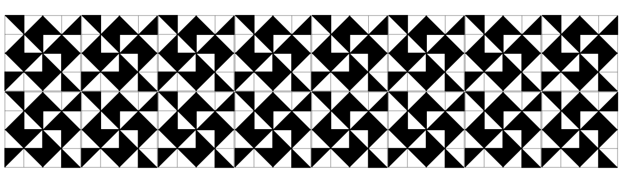

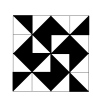

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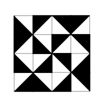

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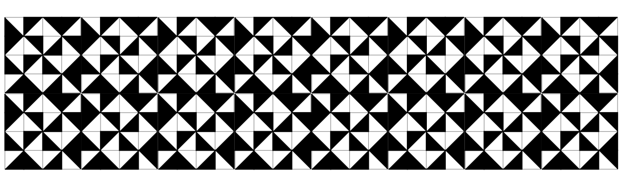

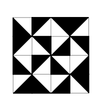

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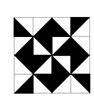

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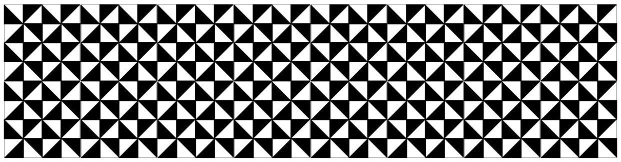

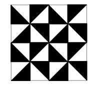

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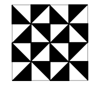

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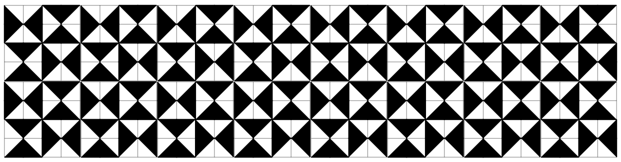

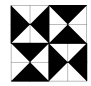

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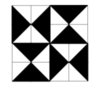
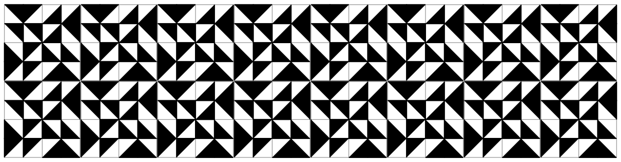
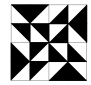


1023

Frieze patterns for family 1011 (secondary, 1101)





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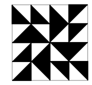

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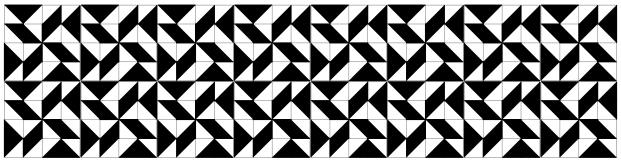

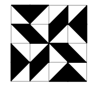

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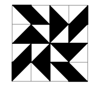

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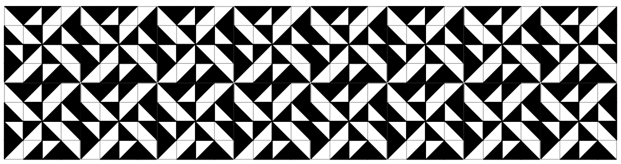

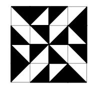

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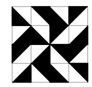

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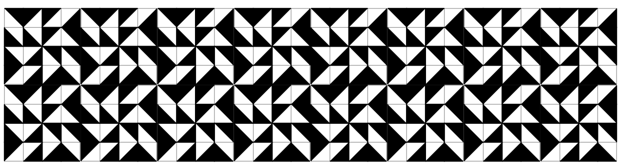

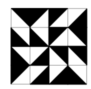

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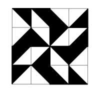

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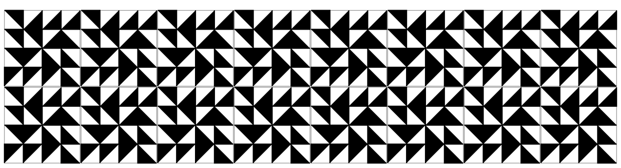

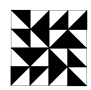

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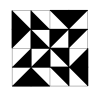

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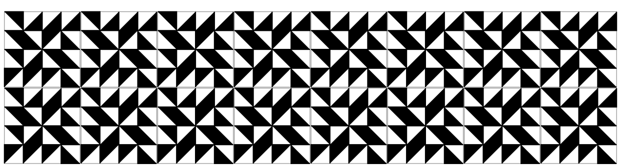

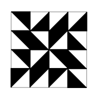

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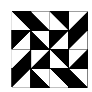

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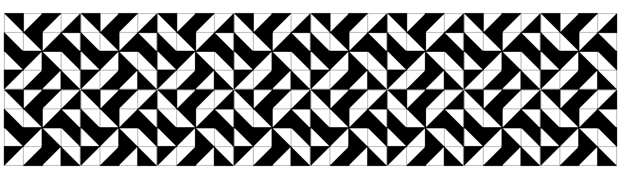

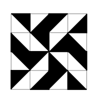

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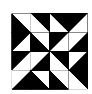

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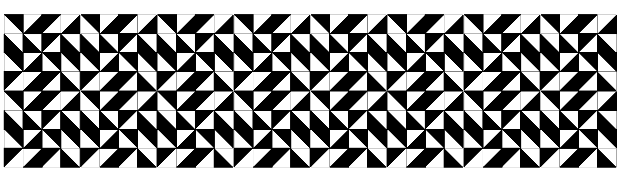

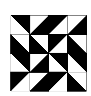

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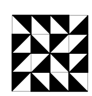

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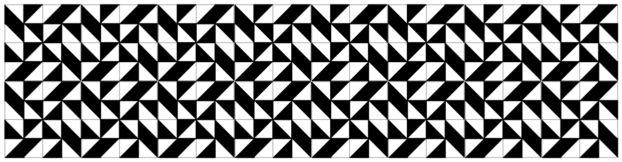

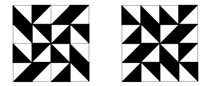

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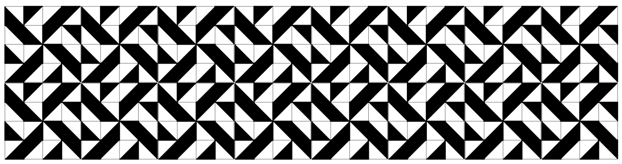

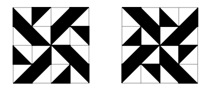

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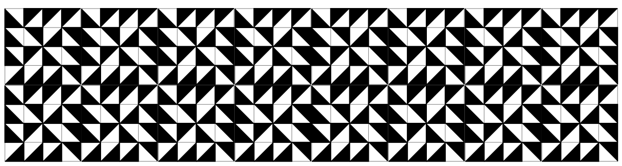

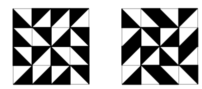

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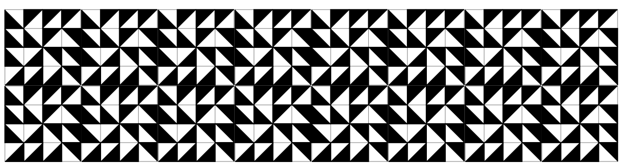

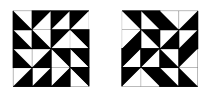

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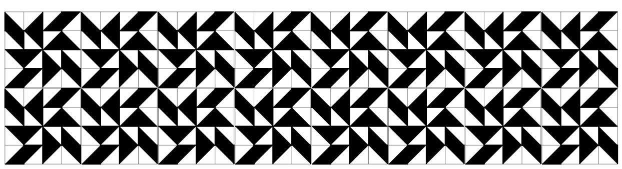

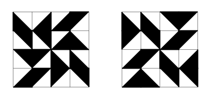

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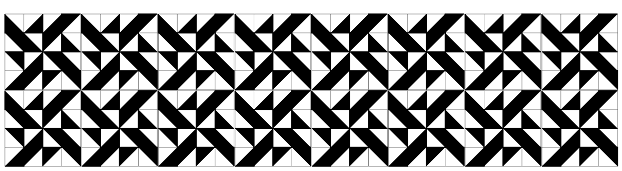

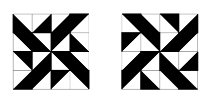

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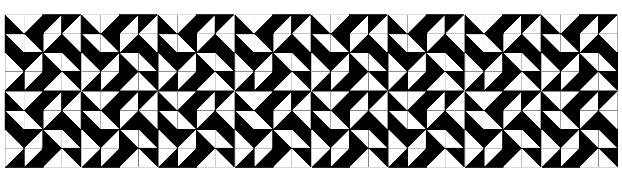

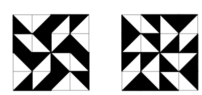

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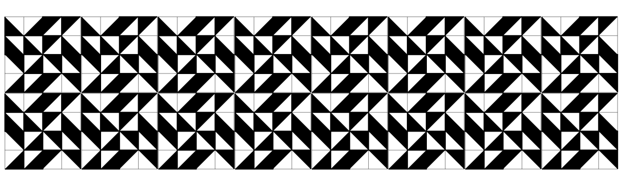

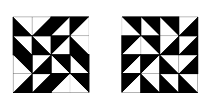

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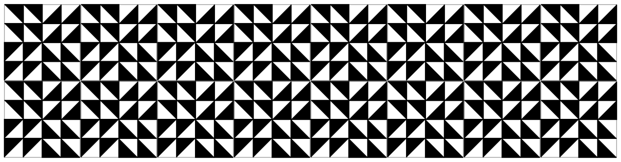
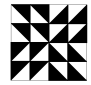
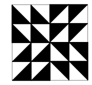
Frieze patterns for family 1111 (secondary, 1111)

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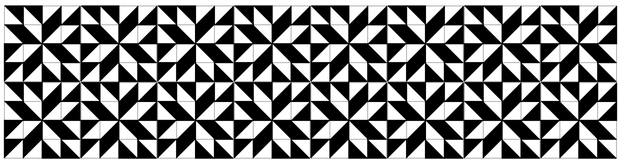
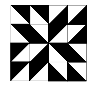
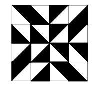

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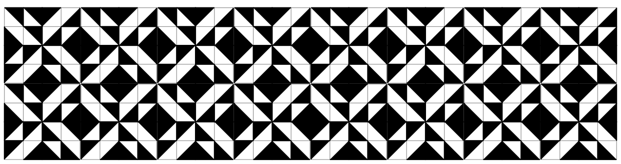
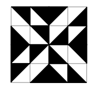
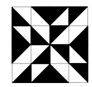

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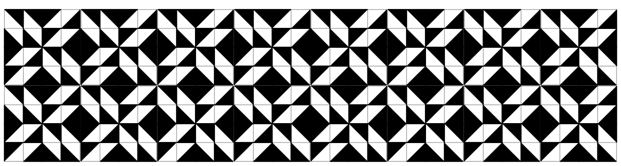
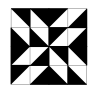
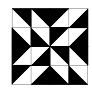

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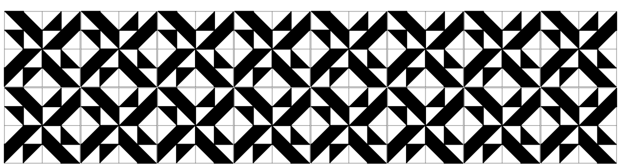
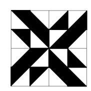
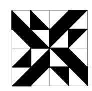

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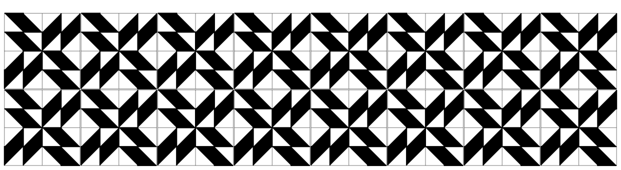
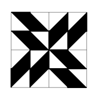
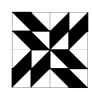

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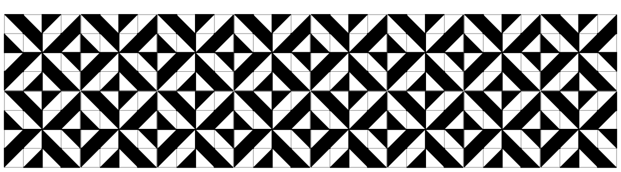
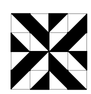
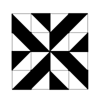

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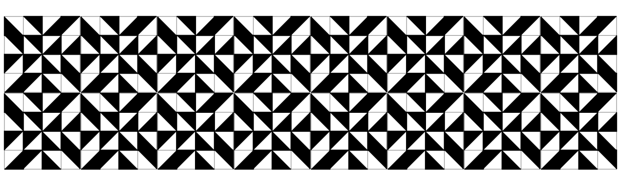
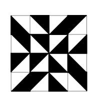
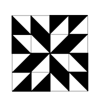

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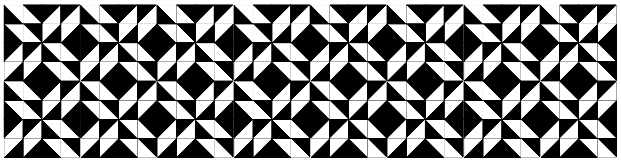

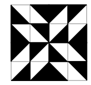

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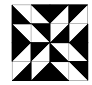

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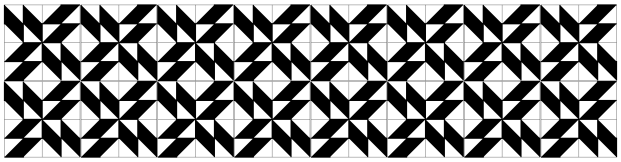

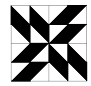

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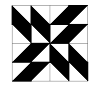

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