# ME 225 Heat Transfer

Dr. Koylu

# ME 225 HEAT TRANSFER
## Spring 2011

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|---|---|
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| *Office Hours*: | TW: 13:00-14:00; Other times: open door or by appointment |
| *Schedule*: | TTh 9:30-10:45, 254 Toomey |
| *Background*: | Differential Equations, Thermodynamics (Fluid Mechanics +) |
| *Textbook*: | "Introduction to Heat Transfer" by Incropera et al., 5th Edition, Wiley |
| *Suggested Reading*: | Any heat transfer book, e.g., "Heat Transfer" by Cengel, McGraw-Hill |
| *Objective*: | To introduce students to the fundamentals of conduction, convection and radiation heat transfer, and to provide the necessary theoretical/practical tools to tackle real engineering applications involving energy systems |

*Work*:

| | Chapters Covered | Tentative Date† | Weight |
|---|---|---|---|
| Test 1 | 1, 2, 3, 5 | 02/15/11 | 25% |
| Test 2 | 6, 7, 8 | 03/24/11 | 25% |
| Test 3 | All | 05/06/11 | 25% |
| 8 Problem Sets | All | Weekly | 10% |
| 2 Projects | | To be announced | 15% |

*General Outline:*

| Book Chapter | Subject |
|---|---|
| 1 | General Introduction |
| 2 | Introduction to Conduction |
| 3 | One-Dimensional Steady-State Conduction |
| 5 | Transient Conduction |
| 6 | Introduction to Convection |
| 7 | Forced Convection - External Flows |
| 8 | Forced Convection - Internal Flows |
| 11 | Heat Exchangers |
| 12 | Radiation: Processes and Properties |
| 13 | Radiation Exchange between Surfaces |

† It is the student's responsibility to follow the most up-to-date announcements for all tests.

## Course Policies

*Grades*: The grades will be treated on a relative basis, that is, the average score of the entire class, $X$, and the standard deviation, $\sigma$, will determine the final outcome. For a typical class, this approximately corresponds to a "*B*" for $X <$ Grade $\leq X+\sigma$, and a "*C*" for $X-\sigma <$ Grade $\leq X$ at the end of the semester. Any further positive or negative deviations from the mean will result in an "*A*" or a "*D*", respectively. A 60% cut-off may be enforced for a passing grade when applicable. The instructor reserves minor discretion over grades for some critical cases.

*Exams*: There will be three tests, which will be related to the material covered in the lectures/textbook, problems solved in the class, and homework/project assignments. Students are allowed to bring the textbook, their own class notes, handouts, and assignments to each test. Because all exams are required, an unexcused absence will result in a zero for that exam. Therefore, a student, who cannot attend an exam because of illness or emergency, must contact the instructor and receive permission before the exam. After the test, only a last minute real emergency will be considered as a possible excused absence for which the student will be expected to notify the instructor about the situation as soon as possible. In any case, necessary official paperwork must be supplied to prove the emergency. The instructor has the final authority to determine whether a student will be given a make-up exam. Finally, academic dishonesty is a serious offense and will be treated accordingly.

*Assignments*: A total of eight homework assignments will be given on a regular basis. A problem set will be usually due in one week at the end of a class period after it is assigned. Solutions to homework will be e-mailed to students after due dates. It is extremely important that you first attempt to solve the problems on your own, and, if unsuccessful, contact the instructor with your specific questions. Group work is also encouraged. Late assignments are not accepted.

*Projects*: There will be two projects related to the practical and design aspects of heat transfer (details to be given during the semester). For each project, two students will team up to prepare a professional report. Projects are not to be discussed with other students/groups. Similar to assignments, late submissions of projects are strictly unacceptable.

*Attendance*: A regular attendance is expected from students to be successful in this course although it is not directly counted toward students' final grades.

*Expected Performance*: Attend class regularly, participate in discussion and ask questions in the class, solve homework problems on a regular basis, submit assignments and projects on time, and not miss an exam.

*Feedback*: The instructor can provide feedback about your performance in this course and specifically what you need to do to improve it only per your request. He also would appreciate your feedback about how the course material and presentation could be further improved. Please see him confidentially for any suggestions/concerns you may have related to the course.

## — Heat Exchanger —

$$C = \dot{m} C_p \qquad \varepsilon = \frac{q}{q_{max}}$$

$$h_i = \frac{Nu\, k_{fluid}}{D} \qquad q_{max} = C_{min}(T_{hi} - T_{ci})$$

Thin-wall tube: $U = \left[\frac{1}{h_i} + \frac{1}{h_o}\right]^{-1}$

$$Re_D = \frac{V_m \rho D}{\mu}$$

With fouling & wall: $\frac{1}{U_i A_i} = \left[\frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln D_o/D_i}{2\pi k L} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o}\right]$

$$NTU = \frac{UA}{C_{min}}$$

## — Ch. 12 Radiation —

- gray = • $\alpha$ is independent of $\lambda$
  • $\alpha = \varepsilon$
- F factors, Pg 702
- 2-surface enclosures p 793

$$\varepsilon(T_s) = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda b}\, d\lambda}{E_b}$$

$$E_b = \sigma T^4 \; ; \quad E = \varepsilon \sigma T^4$$

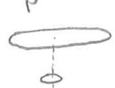
J, radiosity = E + $G_{reflected}$

G, irradiation = all incident rad.

## — Ch. 13 View factors & Rad —

Basic View factors: p775

$A_i << A_j$ p781



$$F_{12} = \frac{A_2}{A_1} F_{21}$$

$F_{11} + F_{12} + F_{13} + F_{14}$ (enclosed) $= 1$

net $q_{12}$, general $= \dfrac{\sigma(T_1^4 - T_2^4)}{\left[\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}\right]}$

Example Problems:

Flat Plates

| | |
|---|---|
| Constant (Tinf-Ts) Re increase, find q ratio | Summer 2010 #3 p3 |
| Boundary layer thickness, local | HW 7.2 |
| Velocity(inf) and T(inf) given | HW 7.11 |
| Convection & solar absorptivity, irradiation | HW 7.20 |
| Cooling time with changed fluid | Spring 2010 #3 p7 |

Tubes

Circular

| | |
|---|---|
| Constant Ts, find Tso | Spring 2010 #4 p8 |
| Annular, find L to cool | HW 8.93 |
| Cross flow, tube conduction | HW 8.50 |
| Pump Power | HW 8.31 |
| Pump power | Fall 2008 #1 p12 |
| Air outside, fluid inside | example p11 |

Noncircular

| | |
|---|---|
| Constant Ts, thin wall, find Tsi, Tso | Summer 2011 #1 p4 |
| Constant q", find Ts, Toutlet | HW 8.74 |

Arbitrary surfaces

| | |
|---|---|
| Velocity profile given, find average Nu | Summer 2010 #1 p1 |
| Temp profile given, find average h | Spring2011 #1 p9 |

Pin fin

| | |
|---|---|
| Diameter doubles, find new q | |
| Diameter doubles, inf long, find new q | HW 7.45 |

Couette flow

| | |
|---|---|
| Surface 1: Constant Ts, Surface 2: no HT | Summer 2010 #2 p6 |
| Both Surfaces constant Ts. Find Tmax | Spring 2010 #2 p10 |

Drag coefficient

| | |
|---|---|
| Cooling spheres | HW 7.73 |

# Exam 3 Help Index

**Chapter 11:**

| | | | |
|---|---|---|---|
| 1 & 2 | Overall U with fouling: | 11.2 | 2008, #1 find fouling factor given q |
| 9 | Shell and Tube HX: (or pg 3) | 11.22 | , also 2008 #3 pg 7 |
| 2, | Concentric Tube: | 11.18 | |
| 4 | Cross-flow HX | 11.34 | |
| 5 | Condensing steam | 11.35 | |

**Chapter 12**

| | | | |
|---|---|---|---|
| 22 | $G_{abs}$ as a function ofF and alpha | 12.46 | 22 step function ε, 12.32 |
| 17 | G with angle of approach, Q received by lens | 12.47 | Redo: |
| 11, | Absorptivity, f(lambda) and Emissive power | 12.51 | |
| 13 | Transmitted through car window | 2008, #2 | |
| 13 | Earth's radiation from sun | Example 12.20 | |
| 15 | Find alpha from $G_{abs}/G$ (step function)<br>find detector incident radiation | 2008 #2 | |

**Chapter 13:**

| | | |
|---|---|---|
| 18, 19 | View factors, all sorts | 13.1 |
| 8, or 19 | Net radiation to A1 in enclosed surface<br>-dot in cylinder  | 13.19 |
| 8, 20 | Net radiation of close parallel plates | 13.41 |
| notes | Radiation shield, flat plates | Ex 13.56 |
| 23 | Radiation shield, q balance w/convection | 13.62 |
| 14, | Long oven view factors | 2005 #3 |
| 21, | Net HT between two enclosed surfaces<br>-side in semicircle | A #3,ii |

## Review for Exam 2

Air properties P. 852

### CH.7 EXTERNAL FLOW

Reminder:

$q$ heat trans. rate, W

$q'$ heat trans rate per length, W/m

$q''$ heat flux, W/m$^2$

$\overline{C_f}$, average friction coefficient

$\delta$, boundary layer thickness (where $\frac{u}{u_\infty} = 0.99$)

$R_{cylinder} = \frac{\ln(r_2/r_1)}{2\pi k L}$

$\alpha$ = absorptivity

$\varepsilon$ = emissivity

Memorize $q'' = A \frac{(T_s - T_\infty)}{R_{tot}}$ $\quad R_{tot} = \left(\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k_a}\right)$ $\quad q = \frac{\Delta T}{R_{tot}}$

$Nu = \frac{h_x X}{k}$

heat transfer correlations Table 7.9 p 431

$T_f$, flow temperature

$C_D$, drag coefficient p 401

$Re_D$ = cross flow = $\frac{V_\infty D}{\nu}$ $\quad Re_{cr} = 5 \times 10^5$

$Re_x = \frac{V_\infty x}{\nu} = \frac{\rho V_\infty x}{\mu}$

### CH 8 INTERNAL FLOW

Energy balance

$T_m(x) = T_{m,i} + \frac{q''_s P}{\dot m c_p} x$

friction factor, MOODY CHART, p 461

Circular pipe, $Re_D = \frac{u_m D}{\nu}$ $\quad Re_{D,c} \approx 2300$

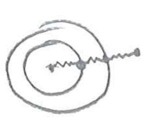
$\dot m$, mass flow $= \rho u_m A_c \Rightarrow u_m = \frac{\dot m}{\rho A_c}$

$\bar h_i = \frac{Nu_D \, k_{fluid}}{D_h}$ (for local & avg Nu) p 489

$D_h \equiv \frac{4A_c}{Perimeter}$

Pumping Pwr $= \Delta P \frac{\dot m}{\rho} = \Delta P \dot{\forall}$ $\quad \Delta P = f\left(\frac{L}{D}\right)\left(\frac{\rho u_m^2}{2}\right)$

$Re_D = \frac{\rho V_\infty D}{\mu} = \frac{V_\infty D}{\nu} = \boxed{\frac{4\dot m}{\pi D_i \mu}}$ circular only



$$\frac{q}{L} = \frac{T_\infty - T_m}{\frac{1}{\bar h_o A_o} + \frac{\ln D_o/D_i}{2\pi k L} + \frac{1}{\bar h_i A_i}}$$

$q'' = A \frac{(T_s - T_\infty)}{R_{tot}}$

$$q = \frac{\Delta T}{R_{tot}}$$

$Re_x$, reynolds by distance p 355 (6.11)
$Re_d$, Reynolds by diameter p457 (8.1)

hydraulic diameter pg. 488. (8.66)

Relate $T_s$ & $T_m$ $\boxed{T_m}$ $T_s$ p.463 (8.27)

Annular tubes
- $T_s$ const p490
- q const p491

h from T(x) function (6.5) p 350 $h = \dfrac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty}$

cooling time t (5.5) (lumped capacitance) $t = \dfrac{\rho V c}{h A_s} \ln\left(\dfrac{T_i - T_\infty}{T - T_\infty}\right)$ p 258

Pipe flow $T_m$ & $T_{\infty, outside}$ NOT q const NOT $T_s$ const $U$ (8.45a) p472 $U = \left(\frac{1}{h_i} + \frac{1}{h_o}\right)^{-1}$

cross flow relationships p431 $\boxed{\text{chapter 7}}$

Notes

| | | hrs |
|---|---|---|
| ME 240 | A | 2 |
| ME 279 | A | 3 |
| EMGT 124 | A | 1 |
| EE 281 | | 3 |
| ME 225 | | 3 |

ME 225

Notes Jan 11, 2011

$$q_x = -kA \frac{dT}{dx}$$ ← Fourier's Law

Temperature gradient: slope of temperature. (Within solid or nonmoving fluid)


Conduction:

- Thermal diffusivity $\alpha = \frac{\text{heat conducted}}{\text{heat stored}} = \frac{k}{\rho c_p}$

$\alpha \uparrow \Rightarrow$ fast response to thermal environment

- In this class: Scott K.
  Zach Jennings
  Stephen Arnaud
  ~~Catzone~~
  ~~Kayla Billadeau~~

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Notes Jan 18,

## Ch 2 Intro to Conduction

- cylindrical coordinates, * radius is changing *

Analysis: determine T field in a solid/stationary fluid resulting from conditions imposed on system boundaries

- define a C.V.
- identify relevant energy transfer processes
- write down governing eqs

} diff eq's & boundary init. conditions ⇓ temp. field

governing eq for pure conduction problems = heat equation

Energy conservation: $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$

$\dot{E}_g = \dot{g}\, d\forall = \dot{g}\, dx\, dy\, dz$

↑ rate of energy gen. per unit volume (W/m³)

$\dot{E}_{st} = mC_p \frac{\partial T}{\partial t} = \rho\, d\forall\, c_p \frac{\partial T}{\partial t} = \rho\, dx\, dy\, dz\, C_p \frac{\partial T}{\partial t}$

$q_x = -k\, dy\, dz \frac{\partial T}{\partial x}$ $\quad q_y = -k\, dz\, dx \frac{\partial T}{\partial y}$ $\quad q_z = -k\, dx\, dy \frac{\partial T}{\partial z}$

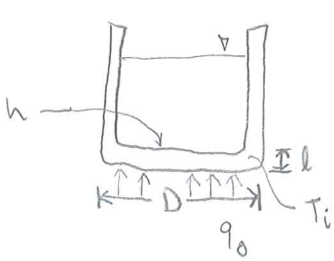
$$(q_x + q_y + q_z) - \left(q_x + \frac{dq_x}{dx}dx + q_y + \frac{dq_y}{\partial y}dy + q_z + \frac{\partial q_z}{\partial z}dz\right) + \dot{g}\, dx\, dy\, dz = \rho C_p \frac{\partial T}{\partial t} dx\, dy\, dz$$

Heat Equation

$$\boxed{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C_p \frac{\partial T}{\partial t}}$$ ← cartesian coords

- also forms for cylindrical & spherical

Example (2.21)




Stage 1: (assuming h.t. is in 1 direction) (no heat gen)

$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$ boundary cond: $q_0 = -k\left(\frac{\pi D^2}{4}\right)\frac{\partial T}{\partial x}\Big|_{x=0}$ ①

b.c ② $-k\frac{\partial T}{\partial x}\Big|_{x=L} = h[T(L,t) - T_\infty(t)]$

initial cond: $T(x,0) = T_i$

Stage 2: $\frac{d^2T}{dx^2} = 0$ $\quad \frac{dT}{dx} = C_1$ $\quad T = C_1 x + C_2$ $\quad$ BC's: $q_0 = -k\left(\frac{\pi D^2}{4}\right)\frac{dT}{dx}\Big|_{x=0}$ $\quad T(L) = T_h$

* get Boundary conditions from Table 2.2.

Jan 20 notes

Ch. 3

- steady state: $\frac{dT}{dt} = 0$
- 1-D $\Rightarrow \frac{\partial T}{\partial y} \approx \frac{\partial T}{\partial z} << \frac{\partial T}{\partial x}$
- no energy generation $\Rightarrow \dot{g} = 0$

H.E.: $\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$

$\frac{dq''_x}{dx} = 0 \Rightarrow q''_x = \text{const}$

$k = \text{const} \Rightarrow \frac{d^2T}{dx^2} = 0 \Rightarrow \frac{dT}{dx} = \text{const.}$

$\Rightarrow \boxed{T = C_1 x + C_2 \text{ (linear)}}$

- Conduction thru wall

$q = \frac{kA}{L}(T_{s_1} - T_{s_2}) = \frac{kA}{L}\Delta T = \frac{\Delta T}{(L/kA)} = \boxed{\frac{\Delta T}{R_{cond}}} = \frac{\text{driving potential for conduction}}{\text{thermal resistance " "}}$

heat transfer

$q = \frac{-kA}{L}(-\Delta T) = kA\frac{\Delta T}{L} = \frac{\Delta T}{L/kA}$

$\boxed{R_{cond} = \frac{L}{kA}}$

- outside the wall

Similarly, $q_{conv} = hA(T_s - T_\infty) = \frac{T_s - T_\infty}{(1/hA)} = \frac{\Delta T}{R_{conv}}$ $\boxed{R_{convection} = \frac{1}{hA}}$

Fig 3.1 $R_{tot} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$ total thermal resistance $\Rightarrow q''_x = \frac{T_{\infty_1} - T_{\infty_2}}{R_{tot}}$

Multilayer Plane walls

i) walls in a series: $R_{tot} = \Sigma R_i$

ii) walls of parallel layers $\frac{1}{R_{tot}} = \Sigma \frac{1}{R_i}$

— Ch 3 slide 6 —

assume $A = 1 m^2$

case a) $q = \frac{\Delta T}{R_{tot}} = \frac{T_{\infty_i} - T_{\infty_o}}{\frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_f}{k_f A} + \frac{L_w}{k_w A} + \frac{1}{h_o A}} = \frac{20 - (-15)}{(0.2 + .059 + 1.92 + .083 + .067)/1}$

$q = \frac{35}{2.33} = \boxed{15.0 \text{ W}}$

b) $q = \frac{T_{\infty_i} - T_{\infty_o}}{\frac{1}{h_i A} + \frac{L_g}{k_g A} + \frac{1}{h_o A}} = \frac{35}{0.269} = \boxed{130 \text{ W}}$

c) $q = \frac{T_{\infty_i} - T_{\infty_o}}{\frac{1}{h_i A} + \frac{L_g}{k_g A} + \frac{L_a}{k_g A} + \frac{L_g}{k_g A} + \frac{1}{h_o A}} = \ldots \boxed{76 \text{ W}}$

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## Notes Jan 25)

- for cylindrical wall, temperature gradient is logarithmic (not Linear)

Overall H.T. coefficient

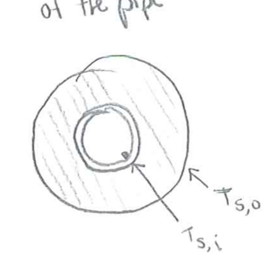
$\varepsilon$ emissivity is $0 <\_< 1$, refers to how much radiation passes thru

Ex.) with insulated tube

Energy balance on outer surface $q'_{cond} = q'_{conv} + q'_{rad}$

* using $q'$ denotes "for 1 m length of the pipe"

$$\frac{T_{s,i} - T_{s,o}}{\frac{1}{h_1 A} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_{st} \cdot 1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_{ins} \cdot 1}} = \frac{T_{s,o} - T_{\infty,o}}{\frac{1}{h_o 2\pi r_3 \cdot 1}} + (2\pi r_3 \cdot 1)(\varepsilon\sigma)(T_{s,o}^4 - T_{surr}^4)$$

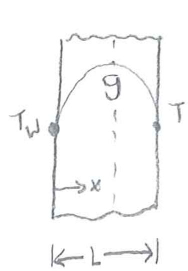



↑ would need this term had they not said "inside wall same temp as fluid"

- will need calculators to solve this

$\Rightarrow r_3 = .394\,m \quad t_{ins} = r_3 - r_2 = 394 - 180 = 214\,mm \qquad q' = 420\ W/m$

ex)




B.C.'s $T(x=0) = T_W \Rightarrow T_W = \cancel{\left(-\frac{\dot{g}}{2k}\right)0^2}^{\,0} + \cancel{C_1 0}^{\,0} + C_2 \Rightarrow C_2 = T_W$

$T(x=L) = T_W \Rightarrow \cancel{T_W} = \left(\frac{-\dot{g}}{2k}\right)L^2 + C_1 L + \cancel{T_W} \Rightarrow C_1 = \frac{\dot{g}L}{2k}$

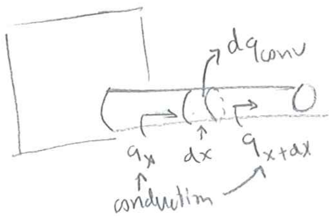
$$T(x) = \frac{-\dot{g}}{2k}x^2 + \frac{\dot{g}}{2k}Lx + T_W$$

Practice $\quad \frac{1}{r}\underbrace{\frac{d}{dr}\left(kr\frac{dT}{dr}\right)} + \dot{g} = 0 \qquad$ Slide #18

## Notes Jan 27)

~~ce~~ cylindrical cooling fin slide 22

(pin fin) • assumed 1 dimensional temperature gradient - only depends on x




Energy balance for dx $\quad q_{cond,x} = q_{cond,x+dx} + dq_{conv}$

$$\cancel{q_x} = -kA_c\frac{dT}{dx} \qquad \cancel{q_x} + \frac{dq}{dx}dx \qquad h(T - T_\infty)\,dA_s$$

$$= -kA_c\frac{dT}{dx} - \cancel{kA_c}\frac{d^2T}{dx^2}dx$$

$$0 = -kA_c\frac{d^2T}{dx^2}\cancel{dx} + h(T - T_\infty)P\,dx \Rightarrow \frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0 \quad \Rightarrow$$

Notes Jan 27

$\theta = T - T_\infty$, $m^2 \frac{hP}{kA_c} \Rightarrow \frac{d^2\theta}{dx^2} - m^2\theta = 0$

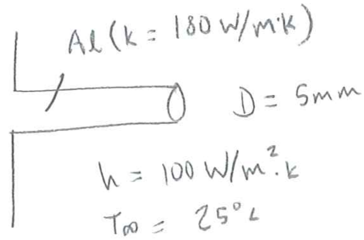
$\theta = C_1 e^{mx} + C_2 e^{-mx}$   1. b.c: $x = 0 \Rightarrow T = T_b$   $T_b - T_\infty = C_1 e^{m.0} + C_2 e^{-m.0}$

$\theta_b = T_b - T_\infty = C_1 + C_2$

2nd b.c: 4 possibilities   slide 24

Note $\cosh x = \frac{e^x + e^{-x}}{2}$   $\sinh x = \frac{e^x - e^{-x}}{2}$

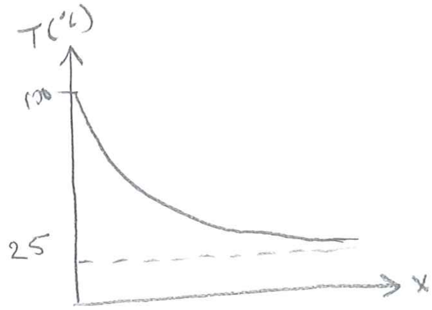
ex) $T_b = 100°C$




Very long fin ($L \to \infty$) $\frac{\theta}{\theta_b} = e^{-mx}$

$\Rightarrow T = T_\infty + (T_b - T_\infty) e^{-mx}$

$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{h}{k} \frac{\pi D}{\frac{\pi D^2}{4}}} = \sqrt{\frac{4h}{kD}} = 21 \Rightarrow T = 25 + 75 e^{-21x}$




$q_f = M = \sqrt{hPkA_c}\,\theta_b$

$q_f = (.075)(75) = 5.6\ W$

* Note; in general, $q_f = q_b$   heat lost by fin = heat entering fin @ base

$= -kA_c \frac{d\theta}{dx}\Big|_{x=0}$   OR   $q_f = \int_{A_f} h(T - T_\infty)\, dA_s$

• what should be the length of a fin with negligible heat loss at the tip that can transfer 80% of $q_{f\,L\to\infty}$ ?

$\frac{q_{f,\text{ adiabatic}}}{q_{f,\,L\to\infty}} = \frac{\cancel{M}\tanh(mL)}{\cancel{M}} = 0.8 \Rightarrow \tanh(mL) = 0.8 \Rightarrow mL = 1.1$

$\Rightarrow L = \frac{1.1}{21} = .05\ m = \boxed{5\,cm}$   $q_f = M\tanh(mL) = 4.5\ W$   plug this into

Jan 27 #3

Engineering Solution:

use corrected fin length $L_c = L + \frac{A_c}{P}$ instead of L in case B (slide 22)

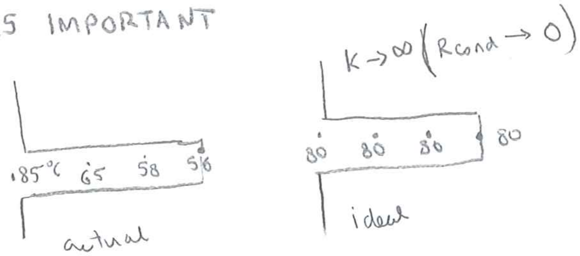

w + $\frac{A_c}{P} \cong \frac{t}{2}$

D $\frac{A_c}{P} = \frac{D}{4}$

— Fin Efficiency —

SLIDE 25 IMPORTANT

ex)




$q_{f,max} = h A_f \theta_b = hPL(T_b - T_\infty)$

$q_f = \eta_f q_{f,max}$ AND $\eta_f = \frac{q_f}{q_{f,max}} = \frac{\sqrt{hPkA_c}\,\theta_b \cdot \tanh(mL_c)}{hPL\,\theta_b}$ $\boxed{\eta_f = \frac{\tan(mL_c)}{mL_c}}$

★ See table 3.5 for $\eta_f$ or figs. 3.18, 3.19

— Fin Effectiveness —

$\varepsilon_f = \frac{q_{f,\,actual}}{q_{no\ fin}}$ $\varepsilon_f \geq 2$ to justify use of fins

max
$\eta_f$
- actual
$\varepsilon_f$
min

$L \to \infty$ (upper limit for $\varepsilon_f$): $\varepsilon_f = \ldots \left(\frac{kP}{hA_c}\right)^{1/2}$

to max. $\varepsilon_f$:
- $k\uparrow$ use metals
- $P/A_c \uparrow$ use thin fins
- $h\downarrow$ use fins on surfaces w/ low h

Feb 3, 2011 | Notes

## — Arrays of fins —

tot. h.t. rate from a surface of N fins

$$q_t = q_{finned} + q_{unfinned} = N q_f + h_{unfinned} A_{unfinned} \theta_b$$

$$= N \eta_f h_{finned} A_f \theta_b + h_{unfinned} A_{unfinned} \theta_b$$

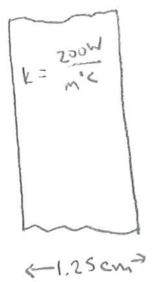
calculate

to study for test 1, consider all configurations of pins/fins on a surface

Ex)

$T_{\infty,1} = 120°C$

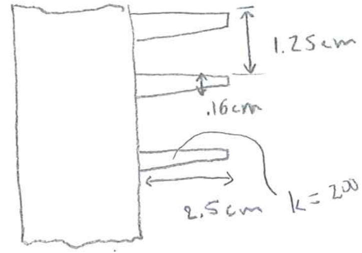
$h_1 = 450 \frac{W}{m^2 °C}$




$T_{\infty 2} = 20°C$

$h_2 = 25 \frac{W}{m^2 °C}$ ⟹

H.T. enhancement due to fin addition?




$$\frac{q_{w/fins}}{q_{w/o\,fins}} = \frac{N \eta_f h A_f \cancel{\theta_b} + h A_{total\,unfinned} \cancel{\theta_b}}{h A_{total\,base} \cdot \cancel{\theta_b}} = \frac{N \eta_f A_f + A_{tot.\,unfinned}}{A_{total\,base}}$$

$$m L_c = \sqrt{\frac{hP}{kA_c}} L_c = \sqrt{\frac{h \cdot 2\cancel{w}}{k \cancel{w} t}} \left(L + \frac{t}{2}\right) \qquad m L_c = \sqrt{\frac{(25)(2)}{(200)(.16 \times 10^{-2})}} \left(2.5 + \frac{.16}{2}\right) \times 10^{-2} = .3225$$

$$\eta_f = \frac{\tanh(mL_c)}{mL_c} = 0.967 \qquad A_f = 2wL_c, \quad A_{total\,unfinned} = w(H - Nt)$$

$$A_{total\,base} = HW$$

$$\frac{q_{w\,fin}}{q_{w/o\,fin}} = \frac{2\eta_f L_c + \frac{H}{N} - t}{\frac{H}{N}} = \boxed{\frac{2\eta_f L_c + S - t}{S}} = \frac{2 \cdot (.967)(2.58 \times 10^{-2}) + 1.25 \times 10^{-2} - .16 \times 10^{-2}}{1.25 \times 10^{-2}}$$

$$= 4.76$$

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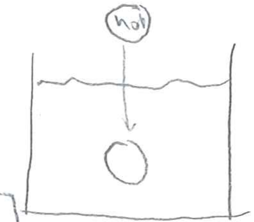



First, neglect spacial variations of temp within the body, $T = T(t)$

$\Rightarrow$ lumped analysis

$\Rightarrow R_{cond} << R_{conv} \Rightarrow \dfrac{q_{conv}}{q_{cond}} = \dfrac{R_{cond}}{R_{conv}} = \dfrac{\frac{L_c}{kA_s}}{\frac{1}{hA_s}} = \boxed{\dfrac{hL_c}{k} = \text{Biot number} = Bi}$

* in this chapter, $L_c$ means "characteristic length", not corrected length

$\Rightarrow Bi << 1 \quad Bi \le 0.1 \Rightarrow$ use the following lump analysis

— Energy balance:


$$\cancel{\dot{E}_{in}}^{\,0} - \dot{E}_{out} + \cancel{\dot{E}_g}^{\,0} = \dot{E}_{st}$$

$$-hA_s(T - T_\infty) = \rho \forall c_p \frac{dT}{dt} \qquad \theta = T - T_\infty \Rightarrow \frac{dT}{dt} = \frac{d\theta}{dt} \qquad -hA_s\theta = \rho \forall c_p \frac{d\theta}{dt}$$

$$\int_0^t -dt = \int_{\theta_i}^{\theta} \frac{\rho \forall c_p}{hA_s}\frac{d\theta}{\theta} \Rightarrow \boxed{t = \frac{\rho \forall c_p}{hA_s} \ln \frac{T_i - T_\infty}{T - T_\infty}} \quad \text{or} \quad \boxed{\frac{T - T_\infty}{T_i - T_\infty} = e^{\left(-\frac{hA_s}{\rho \forall c_p} t\right)}}$$

$$L_c = \frac{\forall}{A_s} = \begin{cases} L & \text{for plane wall of thickness } 2L \\ r_o/2 & \text{for very long cylinder of radius } r_o \\ r_o/3 & \text{for sphere of radius } r_o \end{cases}$$

p 261

$$\text{Now } \frac{hA_s t}{\rho \forall c_p} = \frac{ht}{\rho c_p L_c} = \frac{hL_c}{k}\frac{\alpha t}{L_c^2} = Bi \qquad \text{NONDIMENSIONAL}$$

$$\boxed{\theta^* = \frac{\theta}{\theta_i} = e^{-Bi \cdot F_o}}$$

$F_o$ = Fourier number

$T(t) \Rightarrow q(t) = hA_s[T(t) - T_\infty] = hA_s\theta(t)$

Total heat transfer up to time t $\quad Q = \rho \forall c_p [T_i - T(t)]$

$$Q_{max} = Q_0 = \rho \forall c_p (T_i - T_\infty)$$

Notes

2-8-11

$Bi > .1 \Rightarrow T(x,t)$

Heat equation: $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$

B.C.'s $\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$ (symmetry)

$-k \left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty\right]$

Initial condition: $T(x,0) = T_i$

Feb 10 2011 Notes

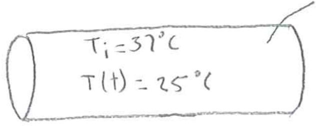
$Bi\# = hl_c/k$

- $Bi \le 0.1$ ⇓ lumped
- $Bi > 0.1$
  - $F_o < 0.2$ ⇓ exact (this case will rarely ever happen)
  - $F_o \ge 0.2$ Heisler charts or approximate solution

Example: CSI Roller

$T_\infty = 20°C \quad h = 8\, W/m^2 °C$

$D = 0.3\, m$




$L = 1.7\, m$

Water, table A-6 ($\bar{T} \approx 305\, K$)

$\rho = 995\, kg/m^3 \quad C_p = 4178\, J/kg \cdot K$

$k = 0.62\, W/m \cdot k \quad \alpha = \frac{k}{\rho C_p} = 1.5 \times 10^{-7} \frac{m^2}{s}$

$$l_c = \frac{\forall}{A_s} = \frac{\frac{\pi D^2}{4} L}{\pi D L + 2\left(\frac{\pi D^2}{4}\right)} = \frac{1}{\frac{4}{D} + \frac{2}{L}}$$

Note: long cyl. $l$ is >> $D \Rightarrow l_c = \frac{D}{4} = \frac{r_o}{2}$

$$Bi = \frac{h l_c}{k} = 0.89 > 0.1 \Rightarrow \text{heisler charts} \qquad \theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{25-20}{37-20} = 0.29$$

$$Bi^{-1} = \frac{k\,1}{h r_o} = 1.04 \Rightarrow F_o = \frac{\alpha t}{r_o^2} \approx 0.6$$

from Fig D.4

$$t = \frac{(0.6)(0.15)^2}{1.5 \times 10^{-7}} = 90{,}000\, s = 25\, hr.$$

? Lumped analysis ?

$$t = \frac{\rho C_p l_c}{h} \ln\left(\frac{T_i - T_\infty}{T - T_\infty}\right)$$

$t = 43815\, s = 12.2\, h$ Error $\cong$ 100% underestimate

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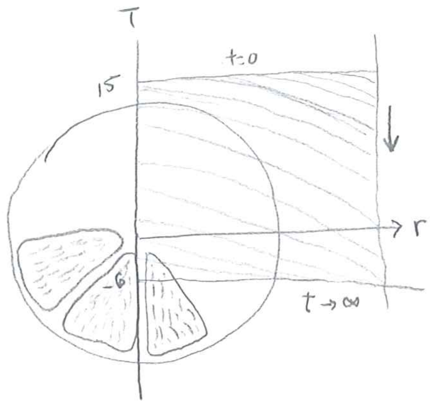
Note: use closest value on tables, no need to interpolate

Ex. oranges in cold weather




$$Bi = \frac{h(r_0/3)}{k} = \frac{30(4\times10^{-3})/3}{613\times10^{-3}}$$

$= 0.65 > 0.1 \Rightarrow$ Heisler

$$Bi^{-1} = \frac{k}{hr_0} = 0.51 \qquad F_0 = \frac{\alpha t}{r_0^2} = \frac{(1.5\times10^{-7})(4\times3600)}{(4\times10^{-2})^2} = 1.35$$

$$\theta_0^* = \frac{T_0 - T_\infty}{T_i - T_\infty} \overset{\text{from Fig. D.7}}{=} 0.007 = \frac{T_0-(-6)}{15-(-6)} \Rightarrow \boxed{T_0 = -5.8\,^\circ C}$$

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$$h = \frac{q_{conv}}{A_s(T_s - T_\infty)} = f(\rho, \nu, k, c_p, V, \text{type of flow, surface geometry})$$

use empirical correlations

solve boundary layer eq's (for 'simple' flows)

fluid viscosity momentum transport $\Rightarrow$ relates gradients with b.l. $\Rightarrow$ drag force

shear stress $\boxed{\tau_{s,x}} = \frac{\text{drag force}}{\text{surface area}} = \frac{F_D}{A_s} = \boxed{\mu \left.\frac{\partial u}{\partial y}\right|_{y=0}}$ $N/m^2$

($\mu$: viscosity)

If you know $u(y)$ w/in b.l. $\Rightarrow \left.\frac{\partial u}{\partial y}\right|_{y=0} \Rightarrow \tau_{s,x} \Rightarrow F_D$

Practical approach: $\tau_{s,x} = C_{f,x} \frac{\rho V_\infty}{2}$ ;

($C_{f,x}$: local friction coef.)

$$F_D = \int_{A_s} \tau_{s,x} \cdot dA_s = \int_{A_s} C_{f,x} \frac{\rho V_\infty^2}{2} dA_s$$

$$= \bar{C}_{f,L} A_s \frac{\rho V_\infty^2}{2} \,(N)$$

$$\bar{C}_{f,L} = \frac{1}{A_s}\int_{A_s} C_{f,x} \cdot dA_s = \frac{1}{L}\int_0^L C_{f,x} \cdot dx$$

· Flow regime depends on Reynolds #

$$Re = \frac{V_\infty L}{\nu} = \frac{\text{inertia forces}}{\text{viscous forces}} \qquad \text{local } Re_x = \frac{V_\infty x}{\nu}$$

nusselt #: $= \frac{q''_{conv}}{q''_{cond}} = \frac{h\,\Delta T}{k_f \frac{\Delta T}{l}} = \frac{hl}{k_f}$

$Nu = 1$, pure conduction

$Nu \uparrow$, more enhanced h.t.

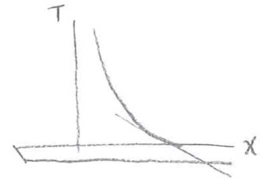
Notes 2-22-11 — Non dimensional #'s —

$Nu = \frac{q_{conv}}{q_{cond}} = \frac{hl}{k_f}$ non-dimensional convection h.t. coeff.

$P_r = \frac{\text{momentum transport}}{\text{heat transport}} = \frac{\nu}{\alpha} = \frac{\frac{\mu}{\rho}}{\frac{k}{\rho c_p}} = \frac{\mu c_p}{k}$

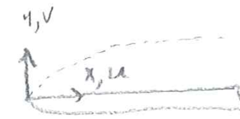
For laminar b.l.'s : $\frac{\delta v}{\delta t} \approx Pr^n$

Convection H.T. coefficient is $\approx$ temperature gradient right at surface




$h \approx \frac{dt}{dx}$

— Boundary layer approximation For incompressible ($\rho$ = const), 2-D, steady flow over surfaces,




- B.L. thickness is typically very small so that

$u >> v, \quad \frac{\partial u}{\partial y} >> \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x} \Rightarrow \tau = \mu \frac{\partial u}{\partial y}$

$\frac{\partial T}{\partial y} >> \frac{\partial T}{\partial x} \Rightarrow q_y'' >> q_x''$

Then, mass: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

x-momentum: $\underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}}_{\text{accelleration}} = \underbrace{-\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}}_{\Sigma F/m}$ ← viscous forces

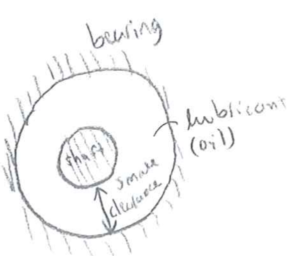
y-momentum: $\frac{\partial P}{\partial y} = 0$

Energy: $\underbrace{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}}_{\text{advection}} = \underbrace{\alpha \frac{\partial^2 T}{\partial y^2}}_{\text{conduction}} + \underbrace{\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2}_{\text{viscous dissipation}} + \dot{g}$

(only important at high speeds)

advection: bulk motion of the fluid

Example of solving bL eqns for simple flows: Couette flow



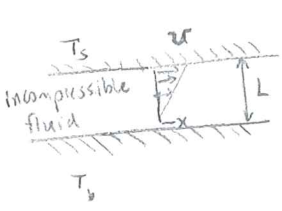

mass: $\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{0} = 0$ so $\frac{\partial u}{\partial x} = 0$ so $u \neq f(x)$

momentum: $\frac{\partial u}{\partial x} = 0, \; v = 0 \; \frac{\partial P}{\partial x} = 0 \Rightarrow \cancel{\nu}\frac{\partial^2 u}{\partial y^2} = 0 \quad \frac{\partial u}{\partial y} = C_1 \quad u(y) = C_1 y + C_2$

B.C.'s $u(y=0) = 0 \Rightarrow C_2 = 0 \quad u(y=L) = U \Rightarrow C_1 = \frac{U}{L}$

$u = U\frac{y}{L}$

≡




Energy $\frac{dT}{dx} = 0 \quad v = 0 \quad \dot{g} = 0 \Rightarrow 0 = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \Rightarrow \frac{d^2T}{dy^2} = -\frac{\mu}{k}\left(\frac{du}{dy}\right)^2$

$= -\frac{\mu}{k}\left(\frac{U}{L}\right)^2 \Rightarrow \frac{dT}{dy} = -\frac{\mu}{k}\left(\frac{U}{L}\right)^2 y + C_3$

$$\boxed{T = -\frac{\mu}{2k}\left(\frac{U}{L}\right)^2 y^2 + C_3 y + C_4}$$

2-22

B.C's $T(y=0) = T_b \qquad T_b = 0+0+C_4 \qquad T_b = C_4$

$T(y=L) = T_s \quad \dots \quad C_3 = \frac{T_s - T_b}{L} + \frac{\mu}{2k}\left(\frac{v}{L}\right)^2$

$\boxed{\text{Thus} \quad T = T_b + \frac{\mu}{2k} v^2 \left(\frac{y}{L} - \frac{y^2}{L^2}\right) + (T_s - T_b)\frac{y}{L}}$

* Prandtl number is a property of the fluid

Local Reynolds number

$\boxed{Re_L = \frac{uL}{\nu}} \quad \boxed{Re_x = \frac{u.x}{\nu}}$

$\boxed{\frac{V_\infty L}{\nu}}$

2-24-11 $\quad \bar{C}_{f_L} = f(Re_L)$

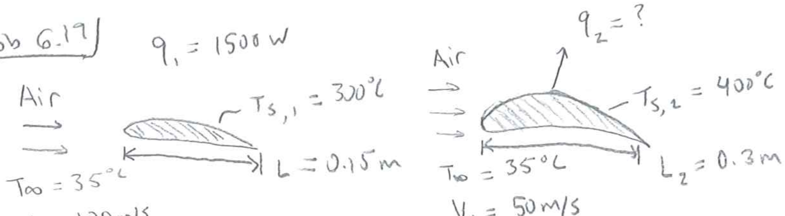
– don't worry about St # in book

$\frac{\bar{C}_f}{2} = \frac{\overline{Nu}_L}{Re_L \cdot Pr^{1/3}} \qquad \overline{Nu}_L = \frac{.0025}{2}(1.23\times10^7)(.72)^{1/3} = 13,780 = \frac{\bar{h}_L L}{k_f}$

Prob. 6.43 $\quad Re_L = \frac{V.L}{\nu} = \frac{100(2)}{16.3\times10^{-6}} = 1.23\times10^7 \quad ; \quad \bar{h}_L = \frac{(13780)(.022)}{2} = 152 \ \frac{W}{m^2 \cdot K}$

$\bar{q}''_{conv} = \bar{h}_L (T_s - T_\infty) = (152)(5-(-123)) = 4260 \ W/m^2$

Prob 6.19




$Re_{L,1} = \frac{V_1 L_1}{\nu_1} = \frac{15}{\nu} \qquad Re_{L,2} = \frac{V_2 L_2}{\nu} = \frac{50(0.3)}{\nu} = \frac{15}{\nu} \Big\} \ \overline{Nu}_{L1} = \overline{Nu}_{L2}$

$\frac{\bar{h}_{L1} L_1}{\cancel{k}} = \frac{\bar{h}_{L2} L_2}{\cancel{k}} \qquad \bar{h}_{L2} = \frac{L_1}{L_2}\bar{h}_{L1} = \frac{L_1}{L_2}\frac{q_1}{A_1(T_s - T_\infty)} \qquad q_2 = \bar{h}_{L_2} A_2 (T_{s,2} - T_\infty)$

$q_1 = \dots = \frac{T_{s,2} - T_\infty}{T_{s,1} - T_\infty} q_1 = 2066 \ W$

March 1 2011

Reynolds analogy connects fluid dynamics to heat transfer

$$\boxed{\frac{C_f}{2} = \frac{Nu}{Re\, Pr^{1/3}}}$$

$C_{f,x} = f(x^*, Re) \Rightarrow \bar{C}_{f,L} = f(Re_L)$

$Nu_x = f(x^*, Re_x, Pr) \Rightarrow \bar{Nu}_L = f(Re_L, Pr$

- be able to solve couette flow problem (using heat flux instead of temp gradient)

## FORED CONVECTION: EXTERNAL FLOWS (Ch.7)

The Empirical Method

measure h for a variety of conditions $\Rightarrow \bar{Nu}_L = C Re_L^m Pr^n$

$m, n, = f($surface geometry, type of flow$)$

assume const. fluid properties at $T_f = \frac{T_s + T_\infty}{2}$

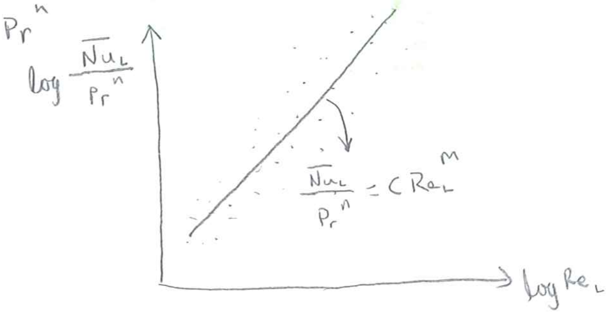
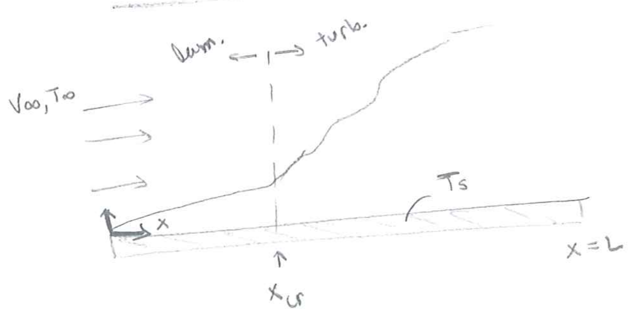



Table 7.9 ← Print this on full page for exam

last 3 under geom WILL NOT USE

Flow over flat plates




$Re_{cr} = \frac{V_\infty x_{cr}}{\nu} = 5 \times 10^5$

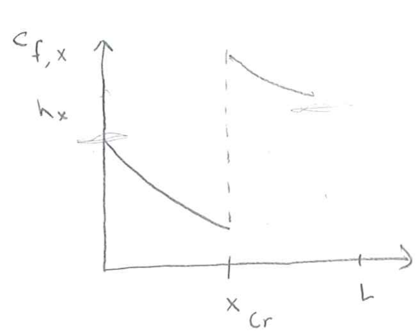
$F_D = \cancel{F_{D\,pressure}} + F_{D\,friction}$

$F_D = \bar{C}_{f,L} \cdot A_s \cdot \frac{\rho V_\infty^2}{2}$

- See table 7.9 for specific correlations p431

For laminar flow, $\delta_{v,t} \sim x^{1/2}$, $C_{f,x} \sim x^{-1/2}$ $Nu_x \sim x^{1/2}$, $h_x \sim x^{-1/2}$

For turbulent flow $\delta_{v,t} \sim x^{4/5}$ $C_{f_x} \sim x^{-1/5}$ $Nu_x \sim x^{4/5}$, $h_x \sim x^{-1/5}$




$\bar{C}_{f,L} = \frac{1}{L}\int_0^L C_{f,x}\, dx$

$\bar{h}_L = \frac{1}{L}\int_0^L h_x dx = \frac{1}{L}\left[\int_0^{x_{cr}} h_{x,lam} \cdot dx + \int_{x_{cr}}^L h_{x,turb} \cdot dx\right]$

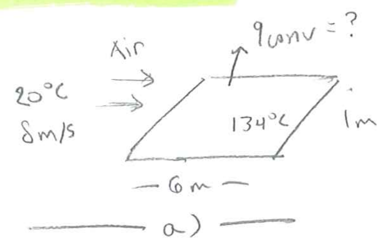
March 1 2011

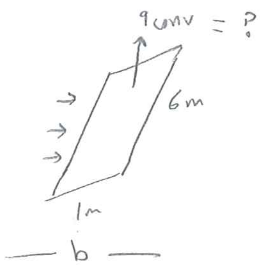
# Flow over flat plates continued

Methodology:

1. calculate $Re_L$ using fluid properties at $T_f = \frac{T_s + T_\infty}{2}$
2. If $Re_L < 5 \times 10^5$ ⇒ entirely laminar flow, use laminar conditions
3. If $Re_L > 5 \times 10^5$ ⇒ mixed laminar & turbulent
   - for local $\delta_v, t$, $C_{f,x}$, $Nu_x$, use either lam or turb local correlations depending on $Re_x$
   - for average $\bar{C}_{f_L}$ & $\bar{Nu}_L$ use mixed avg. correlations
4. $F_D = \bar{C}_{f,L} \cdot A_s \cdot \rho \frac{V_\infty^2}{2}$ $\quad q_{conv} = \bar{h}_L \cdot A_s \cdot (T_s - T_\infty)$ where $\bar{h}_L = \frac{\bar{Nu}_L \cdot k_f}{L}$

Ex)




$T_f = \frac{T_s + T_\infty}{2} = \frac{134 + 20}{2} = 77°C = 350°K$

⇒ $\nu = 20.92 \times 10^{-6}\ m^2/s$ $\quad k = 30 \times 10^{-3}\ W/m \cdot K$

$Pr = 0.7$

a) $L = 6m$ ⇒ $Re_L = \frac{8(6)}{20.92 \times 10^{-6}} = 2.29 \times 10^6$ $\quad Re_{cr} = 5 \times 10^5$ ⇒ mixed flow

P431 find Table 7.9 relationship flat plate ✓, mixed flow ✓ average $\bar{Nu}$ ✓

$\bar{Nu}_L = (0.037 Re_L^{4/5} - 871) Pr^{1/3} = 3249$ ⇒ $\bar{h}_L = \frac{(3249)(30 \times 10^{-3})}{6} = 16.25\ W/m^2 \cdot K$

$q_{conv} = (16.25)(6 \times 1)(134 - 20) = \boxed{11{,}115\ W}$

b) $Re_L = \frac{8(1)}{20.92 \times 10^{-6}} = 3.82 \times 10^5 < Re_{cr}$ ⇒ entirely laminar

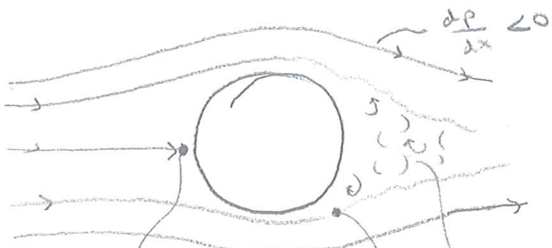
$\bar{Nu}_L = 0.664\ Re_L^{1/2} \cdot Pr^{1/3} = 364$

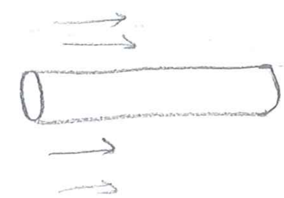
$\bar{h}_L = \frac{(30 \times 10^{-3})(364)}{(1)} = 10.9\ W/m^2 \cdot K$ $\quad \boxed{q_{conv} = 7469\ W}$

Notes 3-3-11

## cross flow

## counter flow



$\boxed{Re = \frac{\rho V D}{\mu} \Rightarrow \frac{\text{inertia}}{\text{viscosity}}}$

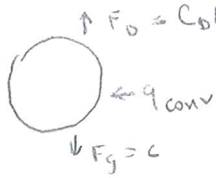
location of Separation


Fig 7.8, important P401

also $\boxed{Re_D = \frac{4\dot{m}}{\pi D_i \mu}} = \boxed{\frac{VD}{\nu}}$

ex) 7.73

$T_\infty = 280°C$
water




Cu sphere, D = 20mm
$T_i(r,0) = 360k$
$T_f(0,t_f) = 320k$

Find: terminal velocity
tank height at point
when $T = T_f$

Balance forces:

$C_D A_f \rho \frac{V^2}{2} = C_D \frac{\pi r^2}{4} \rho \frac{V^2}{2} = g(\rho_{cu} - \rho_{H_2O})$ ....... $\Rightarrow C_D V^2 = \frac{4}{3} D \frac{\rho_{cu} - \rho}{\rho} g$

make a guess and do iterations P401

$C_D V^2 = 2.07\ m^2/s$ $\boxed{\text{Table 7.8} \quad Re_D = \frac{VD}{\nu}}$

$\boxed{V = 2.1\ m/s}$ $Re_D = 29850$

$Nu_D = 2 + (.4 Re_D^{1/2} + .06 Re_D^{2/3})(Pr^{0.4})(\frac{\mu}{\mu_s})^{1/4} = 439$

$\bar{h} = 12775\ W/m^2 \cdot k$

$Bi = h(\frac{r_0}{3})k_{cu} = 12775 \times (\frac{.02}{2\times3}) 398 = 0.11$

$\theta^* = C_1 e^{(-\zeta^2 F_0)} \rightarrow 5.1$

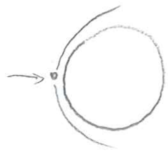
$F_0 = 0.87 = \frac{\alpha t_f}{r_0^2}$

$H = 2.1 \times 0.76 \quad .9890 =$

$\boxed{= 1.6\ m}$

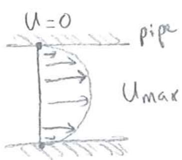
# 3-8-11 Notes

$\bar{C}_{f,L} \rightarrow$  only friction drag

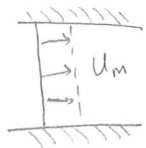
$\bar{C}_D \rightarrow$  friction and pressure drag

} summary for last class

## Momentum Considerations:




actual




- Idealized

$$\dot{m} = \int_{A_c} \rho u(r)\, dA_c = \boxed{\rho u_m A_c}$$

$$Re = \frac{\rho u_m D_h}{\mu} \quad \text{where } D_h = \frac{4A_c}{P} \text{ (hydraulic diameter)}$$

Circular pipe with Diameter $D \Rightarrow D_h = D$

$$\boxed{Re = \frac{4\dot{m}}{P\mu} = \frac{4\dot{m}}{\pi D \mu}}$$

$Re_{cr} \approx 2{,}300$ (onset of turbulence)

Fully turbulent flow: $Re \simeq 10{,}000$

hydrodynamic entry length: $\left(\frac{x_{fd,h}}{D}\right)_{lam} = .05\, Re_D \qquad 10 \lesssim \left(\frac{x_{fd,h}}{D}\right)_{turb} \lesssim 60$

— Fully developed conditions —

pumping power $= \Delta P \cdot \dot{\forall} = \boxed{\Delta P \cdot \frac{\dot{m}}{\rho}}$

pressure drop $= \Delta P = P_1 - P_2 = \boxed{f\left(\frac{L}{D}\right)\left(\frac{\rho u_m^2}{2}\right) \; N/m}$

Friction factor $= f = \dfrac{-(dP/dx)D}{\left(\frac{\rho u_m^2}{2}\right)} = \dfrac{\frac{\Delta P}{L} \cdot D}{\left(\frac{\rho u_m^2}{2}\right)} = f(Re)$ for smooth surfaces

Smooth surfaces {
- laminar flow in circular tube: $f = \frac{64}{Re_D}$
- turbulent flow circular tube: $\longrightarrow f = (0.79 \ln Re_D - 1.64)^{-2}$

- in general; $f = f(Re, e/D) \Leftarrow$ Moody chart (Fig 8.3) p461

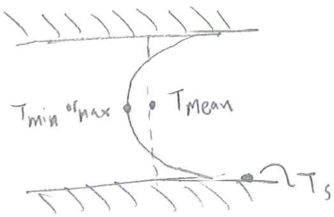
$\uparrow$ relative pipe roughness

$\left.\frac{\partial u}{\partial r}\right|_{r = \pm\frac{D}{2}} \neq f(x) \Rightarrow \tau_w$ constant $\Rightarrow f$ constant for $x > x_{fd,h} \Leftarrow$ fully developed description

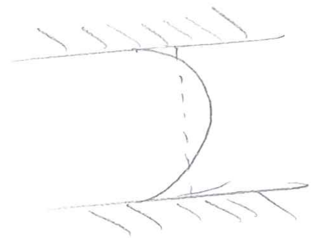
$\frac{\partial u}{\partial x} = 0 \Rightarrow u_m = \text{const.}$

3-8-11 | Notes #2

Thermal considerations:




OR



$\dot{E}_t = \int_{A_c} \rho u c_p T \, dA_c = \dot{m} c_p T_m$ Local heat flux : $q_s'' = h(T_s - T_m)$

Thermal entry length : $\left(\frac{x_{fd,t}}{D}\right) \approx 0.05 Re_D Pr$ $\left(\frac{x_{fd,t}}{D}\right)_{turb} \approx 10$

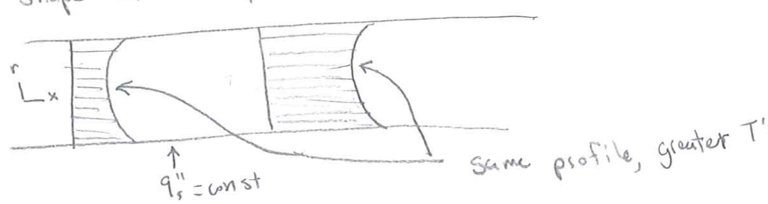
Note: $Pr \approx 1$, then $x_{fd,l} \approx x_{fd,h}$

Fully developed conditions:

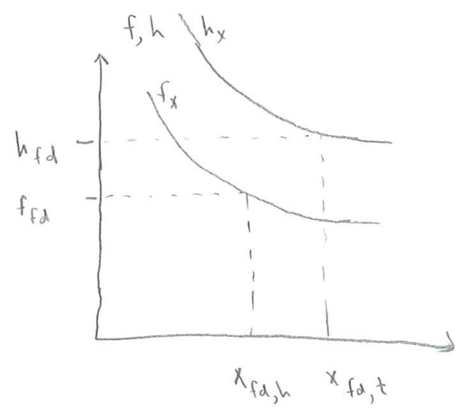
$\frac{\partial u}{\partial x} = 0$ but $\frac{dT_m}{dx} \neq 0$ and $\frac{\partial T}{\partial x} \neq 0$ as $T_m = T_m(x)$

Allthough $T(r) = f(x)$, relative shape of the temperature profile does not change w/x,

ie) $\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right)_{fd,t} = 0$




similar to f, h = const for $x > x_{fd,t}$



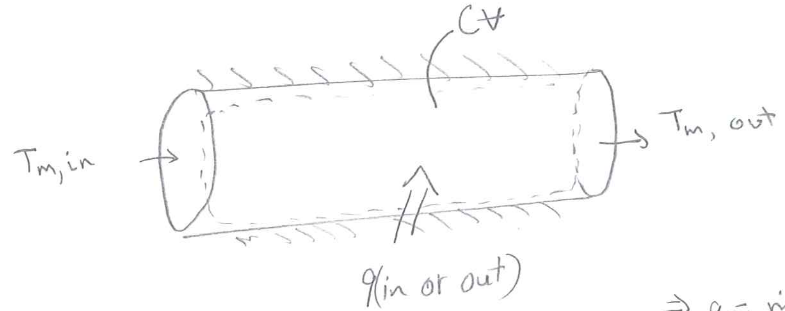
3-15-11 | Notes

— Fully developed internal flows —

effect of friction felt all over x-section

$\frac{\partial u}{\partial x} = 0 \Rightarrow u_M$ const, but; $\frac{\partial T}{\partial x} \neq 0 \Rightarrow T_M = T_m(x) = ?$

$f =$ const (at fully developed) $h =$ const.




$\dot{m} =$ const

$E_{in} = E_{out}$

$q + \underbrace{\dot{m}\, c_p T_{m,i}}_{h} = \dot{m}\, c_p T_{m,o}$

$\Rightarrow q = \dot{m} c_p (T_{m,i} - T_{m,o})$

$dq = \dot{m} c_p \, dT_m$

$q'' p\, dx = \dot{m} c_p \, dT_m \Rightarrow \frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p}$

Solve this. D.E. for 2 surface conditions -

1) $q''_s =$ const

2) $T_s =$ const

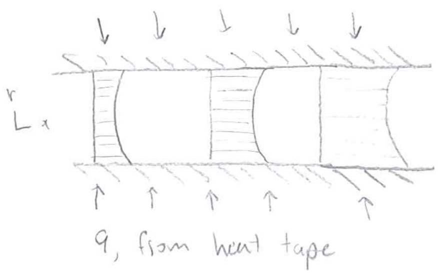
heat tape

$q''_s = h(T_s - T_m) =$ const. $h =$ const, for $x > x_{fd,t} \Rightarrow T_s - T_m = \frac{q''_s}{h} =$ const.

i) $q_{surface} =$ constant

$\frac{dT_m}{dT_x} = \frac{q_s'' \cdot P}{\dot{m} c_p} =$ const $\Rightarrow T_m(x) = T_{m_i} + \frac{q_s'' \cdot P}{\dot{m}, c_p} x$ (linear) $T_{m,o} = T_{m,i} + \frac{q_s'' P \cdot L}{\dot{m} c_p}$

$$\boxed{T_{m,o} = T_{m,i} + \frac{q''_s A_s}{\dot{m} c_p}}$$



3-15-11 | Notes Continued

ii) $T_s = $ const (phase change)

$$\frac{dT_m}{dx} = \frac{q''_s P}{\dot{m} C_p} = \frac{h(T_s - \overbrace{T_m}^{T_m})P}{\dot{m} C_p}$$

$$\Delta T \equiv T_s - T_m \Rightarrow \frac{d(\Delta T)}{dx} = -\frac{dT_m}{dx}$$

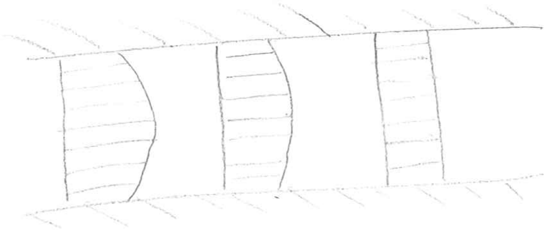
$$-\frac{d\Delta T}{dx} = \frac{h \Delta T P}{\dot{m} C_p} \Rightarrow \frac{d\Delta T}{\Delta T} \; -\frac{P h}{\dot{m} C_p} \cdot dx \qquad \text{integration} \Rightarrow \ln \frac{\Delta T}{\Delta T_i} = \frac{-P\bar{h}}{\dot{m} C_p} x$$

$$\boxed{\frac{\Delta T}{\Delta T_i} = \frac{T_s - T_m(x)}{T_s - T_{m,i}} = e^{\left(\frac{-P\bar{h}}{\dot{m} C_p} x\right)}}$$

or $$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = e^{\left(\frac{-P L \bar{h}}{\dot{m} C_p}\right)}$$

or $$\boxed{\ln \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = -\frac{\bar{h} A_s}{\dot{m} C_p}}$$

or $$T_{m,o} = T_s - (T_s - T_{mi})\, e^{\left(\frac{-\bar{h} A_s}{\dot{m} C_p}\right)}$$



constant surface temp cooling

$\frac{h A_s}{\dot{m} C_p} = NTU$ (number of transfer units)   $L \to \infty$, $NTU \to \infty \Rightarrow T_{m,o} \to T_s$

$$q = \dot{m} C_p (T_{m,o} - T_{m,i}) = \frac{-\bar{h} A_s}{\ln\left(\frac{T_s - T_{m,o}}{T_s - T_{mi}}\right)} (T_{m,o} - T_{mi}) = -\Delta T_{lm}$$ , log mean Temp. difference

- If the external fluid temp $T_\infty$ is constant (instead of pipe surface temp $T_s$ const) then replace $T_s$ w/ $T_\infty$ & $\bar{h}$ with $\bar{U}$

$T_s$   $T_\infty$ const. 25°C

$$\boxed{\frac{T_\infty - T_{mo}}{T_\infty - T_{mi}} = e^{\left(\frac{-A_s \bar{U}}{\dot{m} C_p}\right)}}$$

$$q = \bar{U} A_s \Delta T_{lm}$$

$\uparrow$ overall heat transfer coefficient

$$U A_s = \frac{1}{R_{total}}$$

Review, ex $$R_{tot} = R_{conv} + R_{cond,} + R_{conv,o} = \frac{1}{h_i A} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A}$$

$h = ?$   Table 8.4 summarizes Nu # correlations

For example i) fully developed laminar flow inside circular pipes:

$$Nu_D = \frac{hD}{k} = \begin{cases} 4.36 & \text{for } q''_s = \text{const} \\ 3.66 & \text{for } T_s = \text{const.} \end{cases}$$

for noncircular tubes use hydraulic diameter,

$D_h = \frac{4 A_c}{P}$ w/ Table 8.1

3-15-11

i) fully-developed turbulent flow inside circular pipes

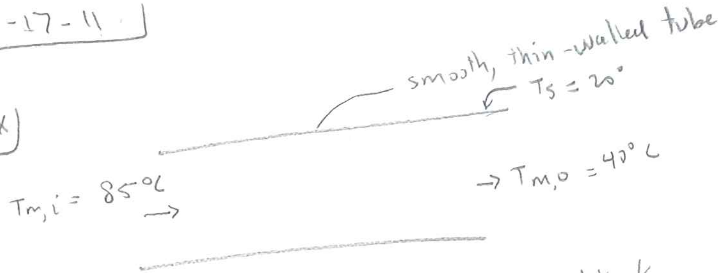
$$Nu_D = 0.023\, Re_D^{0.8}\, Pr^{n} \quad \text{where } n = \begin{cases} 0.4 \text{ for heating } (T_s > T_m) \\ 0.3 \text{ for cooling} \end{cases}$$

Turbulent correlations are valid for both surface conditions of $q''_s = \text{const}$ & $T_s = \text{const}$.

- for noncircular pipes, use the correlations with $\boxed{D_h = \frac{4A_c}{P}}$

3-17-11

ex)




$\rho = 1000\ kg/m^3$
$c_p = 3000\ J/kg \cdot K$
$\mu = 0.04\ Ns/m^2$
$k = 0.26\ W/mK$

$L = ?$ i) $D = 10cm$, $T_s = 20°C$ const $\dot{m} = 1\ kg/s$
$q = ?$
(power) $P = ?$

$$L = \frac{-\dot{m} c_p}{\pi D \bar{h}} \ln\left(\frac{T_s - T_{m,i}}{T_s - T_{m,o}}\right) \qquad q = \dot{m} c_p (T_{m,o} - T_{m,i}) = \bar{h} A_s \Delta T_{lm} \qquad P = \Delta P \dot{\forall} = f \frac{L}{D} \frac{\rho u_m^2}{2} \cdot \frac{\dot{m}}{\rho}$$

$$= 1(3000)(40-85) = -135 kW$$

$$Re = \frac{\rho u_m D}{\mu} ; \quad u_m = \frac{\dot{m}}{\rho \frac{\pi D^2}{4}} = \frac{1}{1000\left(\frac{\pi}{4}\right)(.1)^2} = 0.127\ m/s$$

Table 8.4
p 499

$$Re = \frac{1000(.127)(0.1)}{.04}$$

relationship 8.53 used for Nu

$$Nu = 3.66 = \frac{\bar{h} D}{k_f} \Rightarrow \bar{h} = \frac{(3.66)(.26)}{(0.1)} = 9.5\ W/m^2 k$$

$$L = \frac{1(3000)}{\pi(.1)(9.5)} \ln \qquad \boxed{L = 1182 m}$$

$$P = (0.2) \frac{(1182)}{(.1)} \frac{(1000)(.127)^2}{2} \frac{(1)}{(1000)} \Rightarrow \boxed{P = 19.2\ Nm/s}$$

8.62 $$Nu = \frac{\frac{f}{8}(Re - 1000) Pr}{1 + 12.7 \left(\frac{f}{8}\right)^{1/2} (Pr^{2/3} - 1)} = \bar{h} = 250\ W/m^2 k$$

Calculate this with Re doubled from original - still laminar

$L = 450 m$

$q = -1350$

$P = 1533\ Nm/s$

3-17-2011

ex continued

(ii) a square duct of equal perimeter $P = \pi D = \pi(.1) = .314m = 4a$ $a = .0785$

$D_h = \frac{4A_c}{P} = \frac{\cancel{4}(a^2)}{\cancel{4} \cdot \cancel{a}} = a = .0785$ See table 8.1

$u_m = .162 m/s \Rightarrow Re_{D_h} = \frac{\rho u_m D_h}{\mu} = 318$, laminar

$f = \frac{57}{Re} = .179$ ; $Nu = 2.98 \Rightarrow \bar{h} = 9.9 \ W/m^2K$ ?

$L = 1137m$ $q = -135 kW$ $P = 34 \ Wm/s$

iv) $D = 10cm$ $T_\infty = 20°C$, const $\dot{m} = 1 kg/s$ air, $V_\infty = 10 m/s$

$L = \frac{\dot{m} c_p}{\pi D \bar{U}} \ln\left(\frac{T_\infty - T_{mi}}{T_\infty - T_{mo}}\right)$ $UA = \frac{1}{R_{tot}} \Rightarrow \boxed{U = \left(\frac{1}{h_i} + \frac{1}{h_o}\right)^{-1}}$ $\bar{h}_o = 46.2 \ W/m^2K$

9.5 ch7 46.2

$L = 1425m$

Calculate this on your own, this is most common problem type

Project help - no need to draw air

3-22-11 Notes

Example test prob

$Pr = 4$ $Re_1 = 2000$, circular pipe, laminar, $T_s$ = const $> T_{m,i}$ $\dot{m}$ = const $D_2 = \frac{D_1}{5}$

$\frac{L_2}{L_1} = ?$ $\frac{f_2}{f_1} = ?$ $\frac{P_2}{P_1} = ?$ $Re_2 = 5Re_i = 10{,}000 \rightarrow$ turbulent

$T_{m,i}$ = const $T_{m,o}$ = const

$L = \frac{\dot{m} c_p}{\pi D \bar{h}} \ln\left(\frac{T_s - T_{m,i}}{T_s - T_{m,o}}\right)$ $\frac{L_2}{L_1} = \frac{\frac{1}{D_2 \bar{h}_2}}{\frac{1}{D_1 \bar{h}_1}} = \frac{D_1}{D_2}\frac{\bar{h}_1}{\bar{h}_2} = 5\frac{\bar{h}_1}{\bar{h}_2}$

$\overline{Nu}_2 = .023(10{,}000)^{0.8}(4)^{0.4} = 63.5$

$Nu = \frac{hD}{k} \Rightarrow h = \frac{Nu\, k}{D}$ $\frac{L_2}{L_1} = 5\frac{\frac{Nu\,\cancel{k}}{D_1}}{\frac{Nu\,\cancel{k}}{D_2}} = 5\frac{D_2}{D_1}\frac{\overline{Nu}_1}{\overline{Nu}_2} = \cancel{5} \cdot \frac{1}{\cancel{5}} \frac{\overline{Nu}_1}{\overline{Nu}_2} = \frac{3.66}{63.5} = \boxed{0.06}$

$\frac{f_2}{f_1} = \frac{.316 Re_2^{-1/4}}{64/Re_1} = \frac{0.316(10{,}000)^{-1/4}}{64/2000} = .9875$

$\frac{P_2}{P_1} = \frac{\forall_2 \Delta P_2}{\forall_1 \Delta P_1} = \frac{\frac{\dot{m}_2}{\cancel{\rho}} f_2 \frac{L_2}{D_2} \cancel{\rho} \frac{\bar{u}_{m2}^2}{\cancel{2}}}{\frac{\cancel{\dot{m}_1}}{\cancel{\rho_1}} f_1 \frac{L_1}{D_1} \cancel{\rho} \frac{\bar{u}_{m1}^2}{\cancel{2}}} = \frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\left(\frac{u_{m2}}{u_{m1}}\right)^2 = \frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\frac{D_1^4}{D_2^4}$

$\frac{\frac{\dot{m}_2}{\rho \frac{\pi D_2^2}{4}}}{\frac{\dot{m}_1}{\rho \frac{\pi}{4} D_1^2}} = \frac{D_1^2}{D_2^2}$

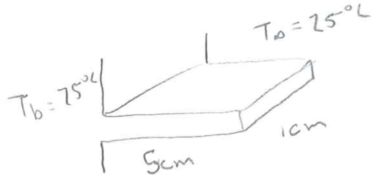
$\frac{P_2}{P_1} = (.9875)(.06)(5)^5$

$\boxed{\frac{P_2}{P_1} = 185}$

Notes $\underline{3\text{-}22\text{-}11}$

We Never use local Nu for cylinder or sphere, sometimes with flat plate

2)




$\eta_f = 0.9$ dissipate $q = 2W = \eta_f \bar{h} A_f (T_b - T_\infty) = (.9)\bar{h}(2 \times 5 \times 10^{-4})(75-25)$

$\bar{Nu} \rightarrow Re \rightarrow V_\infty$

$\Rightarrow \bar{h} = 44.5 \frac{W}{m^2 K}$

what to use for fluid temp

$T_f = \frac{T_s - T_\infty}{2} = 50°C$

$\Rightarrow$ use air @ 300 K

$\nu = 15.89 \times 10^{-6}$

$k = 26.3 \times 10^{-3}$

$Pr = .707$

$\bar{Nu} = \frac{\bar{h}L}{k_{\text{fluid!!!!}}} = \frac{44.5(5 \times 10^{-2})}{26.3 \times 10^{-3}} = .664 Re_L^{1/2} \cdot Pr^{1/3} = .664 Re_L^{1/2} (.707)^{1/3}$

$\Rightarrow Re_L = 20{,}455 = \frac{V_\infty L}{\nu} = \frac{V_\infty \times 5 \times 10^{-2}}{15.89 \times 10^{-6}}$ $\boxed{V_\infty = 6.5 \text{ m/s}}$

ii) water with same velocity above (or use 20 m/s)

$v = \frac{1}{\rho} = 1.011 \times 10^{-3}\ m^3/kg$ $\mu = 577 \times 10^{-6}$ $k = 640 \times 10^{-3}$ W/mK $Pr = 3.77$

$Re_L = \frac{(6.5)(5 \times 10^{-2})}{(577 \times 10^{-6})(1.011 \times 10^{-3})} = 5.5 \times 10^{5} > Re_{cr} \Rightarrow$ mixed flow

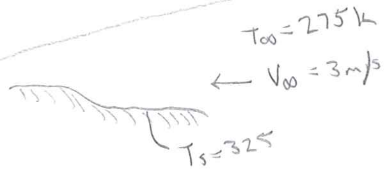
$\rightarrow \nu = \frac{\mu}{\rho} = \mu v$

$\bar{Nu} =$ (expression 7.31) $= 897$ $\bar{h}_L = \frac{640 \times 10^{-3}}{(5 \times 10^{-2})}(897) = 11{,}476\ W/m^2K$

$q_f = (0.9)(11{,}476)(2 \times 5 \times 1 \times 10^{-4})(50) = 516$ W

2008

#3) arbitrary surface, $A_s = 4m^2$ $T_s = 325$ K




$L = 2m$

$Re_{cr} = 10^6$

Temp profile w/in b.l.: $T = 325 + 50e^{-\frac{500y}{x^{1/2}}}$ when flow is locally laminar

$T = 325 - 20{,}000 \frac{y}{x^{1/5}}$ " " " " turbulent

Air at $\frac{325 + 275}{2} = 300K$ $\nu = 15.89 \times 10^{-6}\ m^2/s$ $k = 26.3 \times 10^{-3}$ $Pr = .707$ $\rho = 1.1614\ kg/m^3$

i) $Re_L = \frac{V_\infty L}{\nu} = \frac{(3)(2)}{15.89 \times 10^{-6}} = 3.78 \times 10^5 < Re_{cr} = 10^6$ laminar everywhere

Know definition of Nusselt #

$Nu_x = \frac{h_x x}{k}$

ii) $h(x) = \frac{-k_f \frac{\partial T}{\partial y}\big|_{y=0}}{T_s - T_\infty}$

iii) $\bar{h}_L = \frac{1}{L}\int_0^L h_x\,dx$

iv) $\bar{C}_{f,L} = \frac{2 Nu_L}{Re_L Pr^{1/3}} = \cdots$ (Reynolds analogy)

v) $q = \bar{h}_L A_s (T_s - T_\infty)$

$F_D = \bar{C}_{f,L} \frac{\rho V_\infty^2}{2} A_s$

Notes 4-12-11

## CH 11 Heat Exchangers

Fouling Factor: fluid impurities & rust formation $\Rightarrow$ deposition of a film layer on the surface $\Rightarrow$ decrease heat transfer due to additional conduction resistance

$\downarrow$

$R''_f = f(T, V, t,$ fluid properties

• for an unfinned H.X.: $\frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \boxed{\frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o}}$

a) without fouling $\boxed{\frac{1}{U_o} = \frac{1}{h_i}\frac{D_o}{D_i} + \frac{D_o \ln(D_o/D_i)}{2k} + \frac{1}{h_o}}$ $Re_{D_i} = \frac{4\dot{m}_1}{\pi D_i \mu} = 21{,}220$

☆ See Ch. 11 slide 8 $h_i = \frac{k_f}{D_i}(0.023 Re_{D_i}^{0.8} Pr^{0.4}) = 3400\ W/m^2K$

$U_o = \frac{1}{4.43\times10^{-4}} = 2255\ W/m^2K$

b) with fouling $\left(4.43\times10^{-4} + \frac{D_o}{D_i} R''_f\right)^{-1} = (5.55\times10^{-4})^{-1} = 1800\ W/m^2K$

c) $\dot{m}_h h_{fg} = \dot{m} c_p (T_{mo} - T_{m,i}) \Rightarrow T_{mo} = T_{mi} + \frac{\dot{m}_h h_{fg}}{\dot{m}_c c_p} = 29.4°C$

$\uparrow$ condensation rate

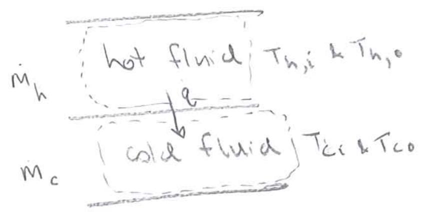
• note that example gives mass flow of water "per tube"

$= 15 + \frac{10(2414\times10^3)}{400(4180)}$

$\boxed{q = \dot{m}_{cond/evap} \cdot h_{fg}}$

### Analysis:

assume
- steady state
- $\dot{m}$ = const.
- negligible $\Delta KE$ & $\Delta PE$
- $C_p$ = const.
- negligible axial heat conduction
- perfectly insulated H.X.




$q = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) = C_h(T_{h,i} - T_{h,o})$

$q = \dot{m}_c \cdot c_{p,c}(T_{c,o} - T_{c,i}) = C_c(T_{c,o} - T_{c,i})$

$C = \dot{m} c_p$ = heat capacity rate

$q = C\Delta T$

$q = UA\Delta T_{lm}$ where $\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$

Parallel flow (PF): $\Delta T_1 = T_{h,i} - T_{c,i}$ $\Delta T_2 = T_{h,o} - T_{c,o}$

Counter flow (CF): $\Delta T_1 = T_{h,i} - T_{c,o}$ $\Delta T_2 = T_{h,o} - T_{c,i}$

- For the same in/outlet temps, $\Delta T_{lm,CF} > \Delta T_{lm,PF}$ Thus
- for the same $U$, $A_{CF} < A_{PF}$ for a prescribed $q$
- note that possibility of $T_{c,o} > T_{h,o}$ is only for a C.F.

Notes 4-12-11) continued

$$q = C_h (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i})$$

Notes 4-14-11)

HX analysis: $\dot{q} = \dot{m}_c c_{p,c} (T_{c,o} - T_{c,i}) = \dot{m} c_{p,h} (T_{h,i} - T_{h,o}) = \underbrace{UA\Delta T_{lm}}_{\text{only for PF \& CF HX's}}$

$\varepsilon$-NTU method:

$\boxed{\varepsilon,\ \text{effectiveness} = \frac{q}{q_{max}}}$ $\quad (0 \le \varepsilon \le 1) \quad q_{max} = C_{min}(T_{h,i} - T_{c,i})$ $\boxed{\text{why?}}$

The fluid with $C_{min}$ experiences the largest temp. difference

$\varepsilon = f(\text{geometry, flow arrangement}) = f(NTU, C_r)$

where $\boxed{NTU = \frac{UA}{C_{min}}}$; $\boxed{C_r = \frac{C_{min}}{C_{max}}}$

Tables 11.3 or Figs 11.10 - 11-15 give $\varepsilon$-NTU relationships

$C \longrightarrow \infty$ for a condensing fluid or phase change

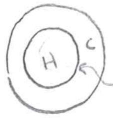
$$\boxed{q = \varepsilon q_{max} = \varepsilon C_{min} (T_{hi} - T_{ci})}$$

During phase change (Boiler or condenser): $C_r \Rightarrow 0 \Rightarrow \varepsilon = 1 - e^{-NTU}$

or $NTU = -\ln(1 - \varepsilon)$

— Discuss prob 11.63 (slides)

$$q = C_c (T_{c,o} - T_{c,i}) = C_h (T_{hi} - T_{ho}) = \varepsilon \underbrace{C_{min} (T_{hi} - T_{ci})}_{q_{max}}$$



surface area $\pi D_i L$

$= UA\Delta T_{lm}$

$\varepsilon = \frac{q}{q_{max}}$ $\qquad U$, overall HT. coefficient

$\varepsilon = 0.65$

$C_r = \frac{C_{min}}{C_{max}} = \frac{0.\cancel{125}(2100)}{\cancel{125}(4200)} = 0.5$ $\qquad NTU = \frac{UA}{C_{min}} \Rightarrow \frac{A_{CF}}{A_{PF}} = \frac{NTU_{CF}}{NTU_{PF}} \cong \frac{1.3}{2.6} \approx 0.5$

fig 11.11 fig 11.10

On slides they solve exactly using $\Delta T_{lm}$, slide 15

Problem 11.38

Notes 4-14-2011

11.38 $A_s = 2\pi D L N$

q for condenser: use Eq. 11.35 instead of figures

Notes 4-19-2011 Ch 12, — RADIATION —

Blackbody:

Stefan Boltzman law: the radiation energy emitted by a blackbody over all λ's per unit time and surface area is

total blackbody emissive power $\rightarrow E_b = \sigma T^4$ (W/m²) (must be in K!)

$\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴

- the radiation emitted by a black body per unit wavelength is given by Planck's Law

$$E_{b\lambda} = \frac{C_1}{\lambda^5 \left(e^{C_2/\lambda T} - 1\right)} \quad (W/m^2 \cdot \mu m)$$

spectral blackbody emissive power

$C_1 = 3.742 \times 10^8$ $W \mu m^4/m^2$

$C_2 = 1.439 \times 10^4$ $\mu m \cdot K$

As T↑, $E_{b\,max}$ shifts towards shorter λ: $\lambda_{max} = f(T)$

$$\frac{\partial E_{b\lambda}}{\partial \lambda} = 0 \Rightarrow \lambda_{max} = T = C_3 = 2897.8\ \mu m \cdot K \quad \text{(Wien's law)}$$

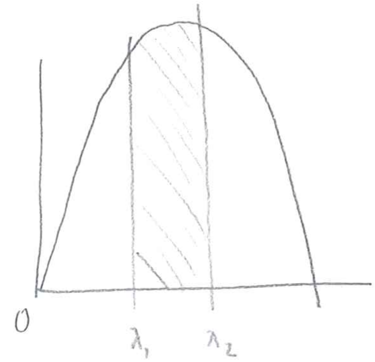
Sun ⇒ T = 5762 K $\lambda_{max} = \frac{2897.8}{5762} \approx 0.50\ \mu m$ (Vis)

T = 298 K ⇒ $\lambda_{max} = 9.72\ \mu m$ (IR)

Flame ⇒ T ≈ 2000 K ⇒ $\lambda_{max} = 1.5\ \mu m$

- fraction of blackbody energy from 0 to λ

$$F_{(0\to\lambda)} = \frac{\int_0^{\lambda} E_{b\lambda}\, d\lambda}{\int_0^{\infty} E_{b\lambda}\, d\lambda} = \frac{\int_0^{\lambda} E_{b\lambda}\, d\lambda}{\sigma T^4} \qquad 0 \le F \le 1$$




- fraction of blackbody energy between $\lambda_1$ & $\lambda_2$

$$F_{(\lambda_1\to\lambda_2)} = \frac{\int_0^{\lambda_2} E_{b\lambda}\, d\lambda - \int_0^{\lambda_1} E_{b\lambda}\, d\lambda}{\sigma T^4} = F_{(0\to\lambda_2)} - F_{(0\to\lambda_1)}$$

Notes 4-19-2011

P. 12.23: fraction of sun's radiation (5800K) is emitted in visible range (VIS 0.4-0.7 μm)

$F_{(0.4-0.7)} = F_{(0\to .7)} - F_{(0\to 4)}$ $\qquad \lambda_1 T = 0.4(5800) = 2320\ \mu m \cdot K$

$\qquad \lambda_2 T = 0.4\ (5800) = 4060\ \mu m \cdot K$

$= 0.48 - 0.14$

$= 0.34 = \boxed{34\%}$

Radiation properties of Real Surfaces:

(i) Emissivity: $\varepsilon = \dfrac{\text{energy emitted by a surface @ T}}{\text{" " " blackbody @ T}}$ $\qquad 0 \le \varepsilon \le 1$

$\varepsilon = 1 \Rightarrow$ blackbody

$\varepsilon_\lambda$ = spectral emissivity

$\varepsilon$ = total emissivity = average $\varepsilon_\lambda$ over all $\lambda$'s

$$\varepsilon_\lambda(T) = \frac{E_\lambda(T)}{E_{b\lambda}(T)} \quad ; \quad \varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{E(T)}{\sigma T^4} = \frac{1}{\sigma T^4}\int_0^\infty \varepsilon_\lambda E_{b\lambda}\, d\lambda$$

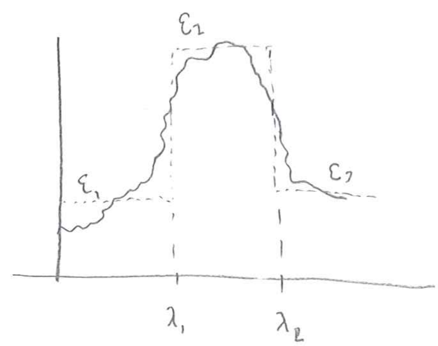
Real surface : $\varepsilon_\lambda \ne$ const $\quad \varepsilon_\theta \ne$ const

Diffuse surface : $\varepsilon_\theta$ = const

Gray surface : $\varepsilon_\lambda$ = const

} diffuse & gray surface : $\varepsilon = \varepsilon_\theta = \varepsilon_\lambda$ = const

Notes 4-21-11


Approximation:

$$\varepsilon_\lambda : \begin{cases} \varepsilon_1 = \text{const. } 1, & 0 \le \lambda \le \lambda_1 \\ \varepsilon_2 \quad " \quad 2 & \lambda_1 \le \lambda \le \lambda_2 \\ \varepsilon_3 \quad\quad 3 & \lambda_2 \le \lambda < \infty \end{cases}$$

$$\varepsilon = \frac{1}{\sigma T^4}\left\{ \varepsilon_{\lambda_1}\int_0^{\lambda_1} E_{b\lambda_1}\, d\lambda + \varepsilon_{\lambda_2}\int_{\lambda_1}^{\lambda_2} \cdots\cdots \right\} \longleftarrow$$

$$\varepsilon = \varepsilon_{\lambda_1} F_{(0\to\lambda_1)} + \varepsilon_{\lambda_2} \underbrace{F_{(\lambda_1\to\lambda_2)}}_{(F_{0\to\lambda_2} - F_{0\to\lambda_2})} + \varepsilon_{\lambda_3} F_{(\lambda_2\to\infty)}$$

Notes 4-21-11 continued

ii) Absorptivity, Reflectivity, and transmissivity:

$\alpha = \frac{\text{absorbed radiation}}{\text{Irradiation}} = \frac{G_{abs}}{G}$ $\quad \rho = \frac{G_{ref}}{G}$ $\quad \tau = \frac{G_{tra}}{G}$

$\sigma = 5.67 \times 10^{-8}$

$$\boxed{G = G_{abs} + G_{ref} + G_{tra}}$$ $$1 = \alpha + \rho + \tau$$

- opaque surface: $\tau = 0 \Rightarrow \alpha + \rho = 1 \quad (G_{tr} = 0)$
- black surface: $\rho = \tau = 0 \Rightarrow \alpha = 1$

| |
|---|
| $\rho$ reflectivity |
| $\varepsilon$ emissivity |
| $\alpha$ absorptivity |
| $\tau$ transmissivity |
| $G$ irradiation |
| $J$ = radiosity |

Spectral properties:

$\alpha_\lambda = \frac{G_{\lambda, abs}}{G_\lambda}$ $\quad \rho_\lambda = \frac{G_{\lambda, ref}}{G_\lambda}$ $\quad \tau = \frac{G_{\lambda, tra}}{G_\lambda}$ $\quad \alpha_\lambda + \rho_\lambda + \tau_\lambda = 1$

$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$ ; $\rho = \frac{\int_0^\infty \rho_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$ $\quad \tau = \frac{\int_0^\infty \tau_\lambda \cdot G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$

Kirchoff's law:
$\alpha_\lambda = \varepsilon_\lambda$

---

ex, 12.69 $\quad$ opaque $\tau_\lambda = 0$ $\quad \alpha_\lambda + \rho_\lambda + \cancel{\tau_\lambda}^{0} = 1$ $\quad \alpha_\lambda = 1 - \rho_\lambda \Rightarrow \rho_\lambda = \begin{cases} 1 - .6 = 0.4 & 0 \le \lambda \le 3 \mu m \\ 1 - .2 = 0.8 & 3 \le \lambda \le \infty \mu m \end{cases}$

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{0.4 \int_1^3 G_\lambda d\lambda + .8 \int_3^6 G_\lambda d\lambda + 0.8 \int_6^8 G_\lambda d\lambda}{\int_1^3 G_\lambda d\lambda + \int_3^6 G_\lambda d\lambda + \int_6^8 G_\lambda d\lambda} = \frac{0.4 \frac{(2)(500)}{2} + .8(3)(500) + .8\frac{(2)(500)}{2}}{2500}$$

$$= \frac{1800}{2500} = 0.72$$ $\quad \varepsilon_\lambda = \alpha_\lambda$

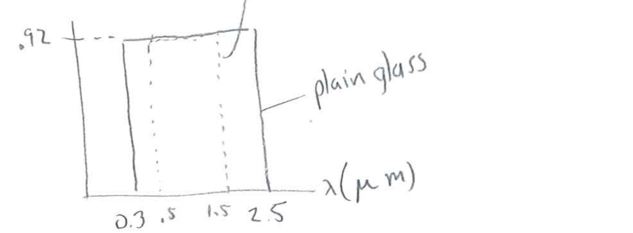
$$\varepsilon = \varepsilon_{\lambda_1} F_{(0\to3)} + \varepsilon_{\lambda_1} F_{(3\to\infty)} = \underbrace{\alpha_{\lambda_1}}_{0.4} \cdot F_{(0\to3)} + \underbrace{\alpha_{\lambda_2}}_{0.8} (1 - F_{(0\to3)})$$

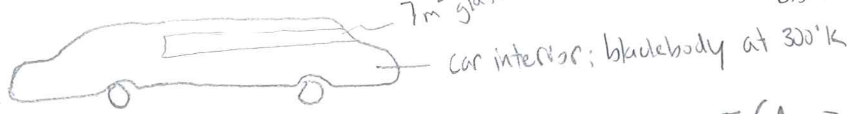
$\lambda T = 3(750) = 2250 \mu m \cdot K \Rightarrow F_{(0\to3)} = 0.1$ $\quad \varepsilon = 0.756$

$$q''_{radiation} = G_{absorbed} - E = \alpha \cdot G - \varepsilon \sigma T_s^4 = (0.72)(2500) - 0.756(5.67 \times 10^{-8})$$
$$= -11{,}763 \text{ W/m}^2$$

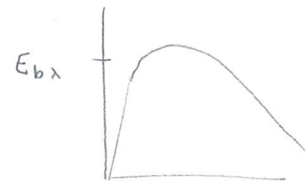
Notes 4-21-11

continued Greenhouse effect:


a) energy of sun transmitted through windows = $\tau G A_s$ = ?




$$\tau = \frac{\int_0^\infty \tau_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{\int_0^\infty \tau_\lambda E_{b\lambda} d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{.92 \int_{.3}^{2.5} E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda}$$

$E_{b\lambda}$ "spectral radiation of the sun"

$$= (0.92)(F_{(0\to 2.5)} - F_{(0\to 0.3)}) = 0.85$$

$\lambda_2 T = 2.5(5800) = 14{,}500\ \mu m \Rightarrow F_{(0\to 2.5)} \cong 0.96$

$\lambda_1 T \quad 0.3(5800) = 1740$ " $\Rightarrow F(0\to .3) \cong 0.04$

b) radiant energy from car's interior transmitted back to the surroundings thru windows $= \tau E_b^{(300K)} A_s = 1.35\times10^{-5}(5.67\times10^{-8})(300^4)(7) = .004\,W$

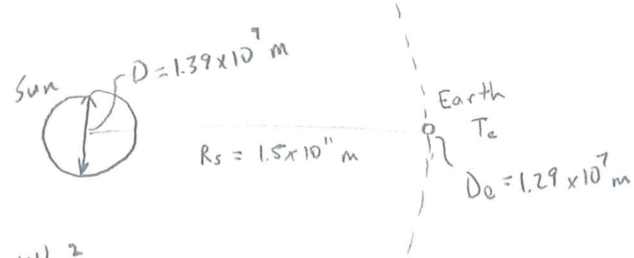
$(2.5)(300) = 750\ \mu m\cdot K \Rightarrow F(0\to 2.5) \cong .000015$

$(.3)(300) = 90\ \mu m\cdot K \Rightarrow F(0\to .3) \cong 0$

$\tau = 1.35\times10^{-5}$

Notes 4-26-2011

Ex) Problem 12.20




Solar flux at outer edge of Earth's atmosphere $q_s'' = 1353$ W/m$^2$

$$E_s \underbrace{(\pi D_s^2)}_{4\pi R_s^2} = q_s'' \, 4\pi (R_{se}^2) \Rightarrow E_s = 6.3\times10^7 \text{ W/m}^2 = \sigma T_s^4 \Rightarrow T_s = 5774 \text{ K}$$

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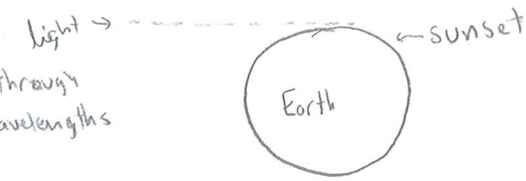
gas molecules & small particles $C_{sca} \sim \frac{1}{\lambda^4}$ ; $C_{abs} \sim \frac{1}{\lambda}$



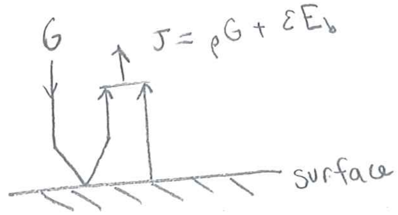

- more absorbtion can occur through a greater length & longer wavelengths will be preserved

— Radiation Exchange Between Surfaces (Ch.13) —

Consider two surfaces separated by a non-participating medium (most gases)

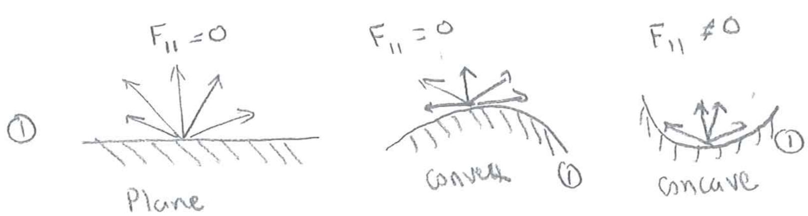
Governing factors
- Temperature
- Radiation Properties
- Geometries & orientations

- Radiosity: (J)




- View factor: fraction of radiation leaving a surface that is intercepted by the other surface
- J: emitted radiation + reflected radiation

$F_{12}$ = how much does surface 1 "see" surface 2.




i) inspection

ii) Reciprocity rule: $A_1 F_{12} = A_2 F_{21}$

iii) Summation rule $\sum_{J=1}^{N} F_{ij} = 1$ for an enclosure w/ N surfaces

iv) Symmetry rule: if surfaces 2 & 3 are symmetric about surface 1 $F_{12} = F_{13}$

N surfaces $\Rightarrow N^2$ view factors but only $\frac{N(N-1)}{2}$ of them are needed directly

## Notes 4-26 continued:

1. Integration:

See tables 13.1-13.2 Figures 13.4-13.6 for selected geometries

2. View factor Algebra

(i)


$F_{11} = 0 \quad F_{22} = 0$

$F_{12} = 1 = F_{21}$

(ii)


$F_{11} = 0 \qquad A_1 F_{12} = A_2 F_{21}$

$F_{12} = 1 \qquad \Rightarrow F_{21} = \dfrac{A_1}{A_2} = \dfrac{D_1}{D_2}$

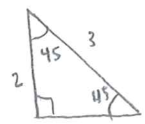
$F_{21} + F_{22} = 1 \Rightarrow F_{22} = 1 - \dfrac{A_1}{A_2}$

iii)


$F_{12} = F_{13} = \frac{1}{2}$

$F_{21} = F_{23} = \frac{1}{2} \ldots$

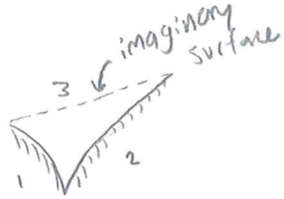
iv)




9 view factors ?

• find these

v)



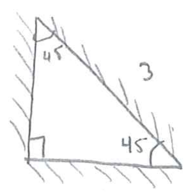

$A_1 F_{12} = A_2 F_{21} = \dfrac{A_1 + A_2 - A_3}{2}$

vi)


$A_1 F_{12} = A_2 F_{21} = \dfrac{\Sigma(\text{crossed strings}) - \Sigma(\text{uncrossed strings})}{2}$

$= \dfrac{(A_5 + A_6) - (A_3 + A_4)}{2}$

## 4-28-2011




$\cancel{F_{11}}^{0} + F_{12} + F_{13} = 1$

$F_{31} + F_{32} + \cancel{F_{33}}^{0} = 1$

$F_{31} = F_{32} = 1/2$

$A_1 F_{13} = A_3 F_{31} = A_3\left(\frac{1}{2}\right)$

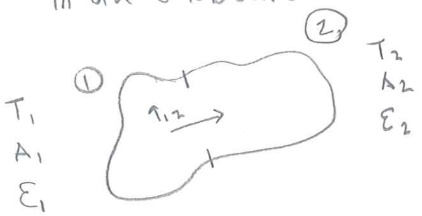
$F_{13} = \dfrac{A_3}{2A_1}$

$A_1 F_{12} = A_2 F_{21} \Rightarrow \ldots$ stopped taking notes

$F_{21} = \dfrac{2A_1 - A_3}{A_2} \quad F$

Notes 4-28-2011

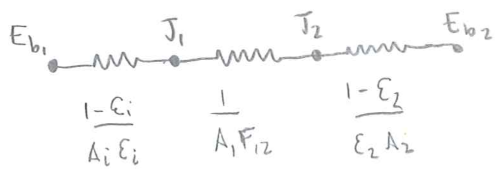
Radiation Exchange Between Two Diffuse, Gray, opaque surfaces in an enclosure




$$q_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$R_i = \frac{1-\varepsilon_i}{A_i\varepsilon_i}$ ; surface resistance to radiation, a measure of departure from blackbody ($\varepsilon = 1 \Rightarrow R = 0$)

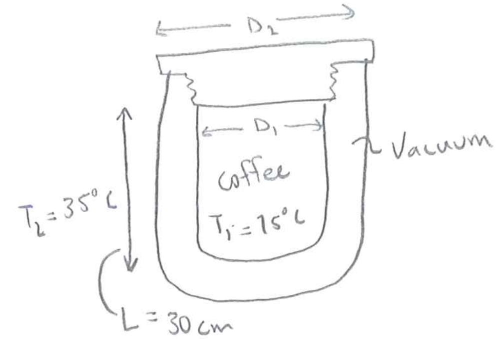
$R_{12} = \frac{1}{A_1F_{12}}$ : a measure of how two surfaces "see" each other




ex) Thermos bottle 13.06, but with vacuum

treat as long concentric cylinder

$$q_{12} = \frac{\sigma \pi D_1 L(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)} = 3.2\text{ W}$$




Thickness of an equivalent conduction insulation (say, $k_{cork} = 0.04 \frac{W}{mK}$)

$$q_{12} = 3.2 = \frac{T_1 - T_2}{\left(\frac{\ln r_2/r_1}{2\pi k L}\right)} \Rightarrow \ln\frac{D_2}{7} = \ldots \quad D_2 = 18\text{ cm}$$

— Radiation Shields —

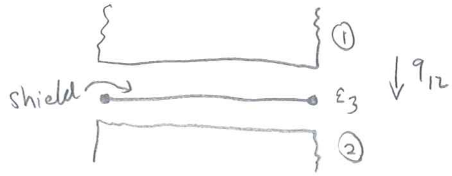
$\varepsilon \downarrow \Rightarrow \frac{1-\varepsilon}{A_1\varepsilon_1} \uparrow \Rightarrow q_{rad} \downarrow$

special case: multiple shields w/ same emissivity : $(q_{12})_{N,\text{ shields}} = \frac{1}{N+1}(q_{12})_{0\text{ shields}}$

Notes 4-28-11 continued

Ex. 13.56

$\varepsilon_1 = \varepsilon_2 = 0.8 \quad F_{13} = F_{12} = 1$

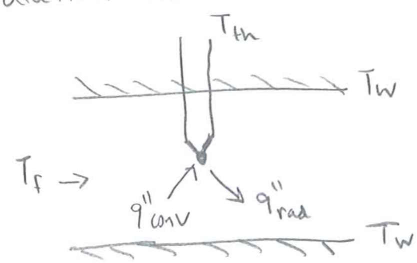



$\varepsilon_3 = ?$

$$(q_{12})_{w/o\ shield} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \qquad (q_{12})_{w/shield} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_3} - 1\right)} \quad (pg\ 793)$$

$$\frac{q_{w\ shield}}{q_{w/out}} = \frac{1}{10} = \frac{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2}{\varepsilon_1} - 2\right)} \Rightarrow \varepsilon_3 = 0.138$$

— Radiation Correction — on temperature measurements




$q_{conv} = q_{rad}$

$h(T_f - T_{th}) = \varepsilon_{th} \sigma (T_{th}^4 - T_w^4)$

$$T_f = T_{th} + \underbrace{\frac{\varepsilon_{th} \sigma (T_{th} - T_w)}{h}}_{\text{radiation correction}}$$

- try to minimize $\varepsilon$

Homework

100

David Malawey

Homework 9

11.2 (a,b), 18, 22, 34(a,b,c), 35(a)

11.2)

$D_i = 22mm$ $R''_{f,i} = .0004 m^2 K/W$

$D_o = 27mm$ $R''_{f,o} = .0002 m^2 K/W$

a) determine overall h.t. coefficient. $U_o$

$$U_o = A_o\left[\frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{\ln D_o/D_i}{2\pi k L} + \frac{R''_{f,o}}{A_o} + \frac{1}{h_o A_o}\right]^{-1}$$

$$\frac{\ln D_o/D_i}{2\pi k L} = .00216$$

$h_o$:

$$Re_{D,o} = \frac{V_\infty D}{\nu} = \frac{20 m/s\,(.027)}{} = 33984$$

$Nu_{D,o} =$ $C = .26$ $m = 0.6$

pg. 431 (7.46) why ?

$$Nu_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr_o^{1/3}}{\left[1 + (0.4/Pr_o)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_{D,o}}{282000}\right)^{5/8}\right]^{4/5}$$

$Nu_{,D,o} = 103$ $\bar{h}_o = \frac{Nu_{D,o} k}{D} = 104\ W/m \cdot k$

$$\frac{1}{U_o} = A_o\left[\underset{R_{cv,i}}{.00429} + \underset{R_{f,i}}{.00579} + \underset{R_w}{.00216} + \underset{R_{f,o}}{.00236} + \underset{R_{cv,o}}{.1134}\right]^{-1}$$

conduction

fouling

convection

Water $T_{m,i} = 75°C \approx 350K$

$k =$

$u_{m,i} = 0.5 m/s$

$$Re = \frac{u_m D}{\nu} = \frac{0.5(.022)}{(1.027\times10^{-7})(365\times10^{-6})} = 29{,}345$$

$Pr = 2.29$

$$Nu_d = .023 Re^{4/5} Pr^{0.3} = 111$$

$$\bar{h}_i = \frac{Nu\,k}{D} = \frac{111(.668)}{.022} = 3370$$

302 stainless steel

$c_p =$

$k = 15.1\ W/m\cdot k$

Air $15°C \sim 300K$

$\nu = 15.89\times10^{-6}$

$R_{tot} = 0.128$

$\boxed{U_o = 92.1}$

57.8

$R_{cv,o,2} = .007689$

$A_o = .0848$

fouling contributes resistance as large as the wall and convection resistances. Convection resistance for air is the greatest.

Part b) on back

David Malawey

11.2 b)

Cross flow water, 15°C, $V_o = 1\,m/s$ $\quad Pr = 7.56 \quad k = .598 \quad D_o = .027m$

$Nu_{D,o}$ : $\quad Re = \dfrac{V_\infty D}{\upsilon \mu} = \dfrac{1(.027)}{(1.001)\times10^{-3}(1080)\times10^{-6}} \Rightarrow Re = 24{,}975 \quad 31{,}xxx$

$Nu_D = C Re_D^m Pr^{1/3} = 198$ $\quad$ Table 7.2

$\bar{h}_o = \dfrac{Nu_{D_o} k}{D_o} = 4385 \Rightarrow R_{cv,o} = \dfrac{1}{h_o A_o} = .002689 \Rightarrow \boxed{U = 681}$

11.18

concentric tube, $A_s = 50m^2$

| | Hot fluid | Cold |
|---|---|---|
| heat capacity rate kW/K | 6 | 3 |
| $T_i$, °C | 60 | 30 |
| $T_o$, °C | $\boxed{48}$ | 54 |

a) find $T_{h,o}$

$q = C_h(T_{h,i} - T_{h,o}) = C_c(T_{co} - T_{ci}) \Rightarrow \boxed{T_{h,o} = 48°}$

b) System is in counterflow because $T_{h,o} < T_{c,o}$

c) find U, $\quad U = \dfrac{q}{A\,\Delta T_{lm}} \quad \Delta T_{lm} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = 19.4 \Rightarrow \boxed{U = .0742}$

check

d) effectiveness, $\varepsilon = \dfrac{q}{q_{max}} = \dfrac{72}{90} \quad C_{min}(T_{hi} - T_{ci}) = 3(60-30) = 90$

$\boxed{\varepsilon = 0.80}$

e) effectiveness if length were very long? $\quad q = C_c(T_{co} - T_{ci})$ & $T_{c,o} \to T_{h,i}$

$\Rightarrow q \to C_c(T_{h,i} - T_{c,i})$

$\Rightarrow \boxed{\varepsilon \to 1}$

David Malawey HW 9

11.22, 34(a,b,c), 35(a)

**11.22** — Shell & Tube H.X.

2.5 kg/s of water

N=10

thin walled

D = 25 mm

tubes make 8 passes ea.

water: $T_{ci} = 15°C$, $T_{c,o} = 85°C$

oil: $T_{hi} = 160°C$, $T_{h,o} = 100°C$

Oil

$h_o = 400\ W/m^2$ outside tubes

$c_p$ @ 130°C (400K) = 2.337 (p 857)

— what is flow rate of oil?

Water

$c_p$ @ 325K = $4.182 \times 10^3$

$\mu = 528 \times 10^{-6}$

$P_r = 3.42$

$k = .645$

$$C_c(T_{co} - T_{ci}) = C_h(T_{ho} - T_{hi})$$

$$2.5(4.182)(85-15) = \dot{m}_{oil}(2.337)(160-100)$$

$$\boxed{\dot{m}_{oil} = 5.22\ kg/sec}$$

— How long must tubes be? $NTU = \frac{UA}{C_{min}}$

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_c(T_{co} - T_{ci})}{C_{min}(T_{hi} - T_{ci})} = \frac{732}{1515}$$

$$\frac{1\ C_{min}}{U} = A$$

$\varepsilon = .483$, $\frac{C_{min}}{C_{max}} = 0.857$ } fig 11.12 NTU=1

$$\frac{1}{UA} = \left[\frac{1}{h_i A_i} + \frac{1}{h_o A_o}\right]$$

$h_i$: $Re = \frac{V_m D}{\nu} = \frac{4\dot{m}}{\pi D_i \mu} = 24114$ $\quad \overline{Nu} = .023\, Re_D^{4/5}\, P_r^{0.4} = 121$

$h_i = \frac{Nu\, k}{D} = 3122$, $h_o = 400$ } $\Rightarrow \frac{1}{U} = [\quad] \Rightarrow U = 355$

$$\frac{NTU}{U}(C_{min}) = 10\pi(.025)L \Rightarrow \frac{1}{355}(2.5)(4.182)\frac{1}{(10\pi(.025))} = \boxed{L = 37.5\ m}$$

11.34 a, b, c  cross-flow hx  5 liter/min

$U = 750\ W/m^2K$

blood

$t_{hi} = 37°C$

$t_{ho} = 25°C$

$\rho = 1050\ kg/m^3$

$c_p = 3740\ J/kg\,K$

ice bath

$t_{ci} = 0°C$

$t_{co} = 15°C$

$c_p = 4.198\ kJ/kg$

a) exhanger heat transfer rate?

$q = \dot{m}_b\, c_{p_b}\,(T_i - T_o)$

$= .005\ m^3/min \left(\frac{1\,min}{60\,sec}\right)(1050\,kg/m^3)(3740)(37-25)$

$q = 3927\ J/sec = \boxed{3.927\ kW}$

b) water flow rate  $\dot{m}_w c_{p_w}(15) = 3.927\ kW$

$\boxed{\dot{m}_w = .0624\ kg/s}$

c) find $A_s$: $q = UA\,\Delta T_{lm}$

$\frac{C_{min}}{C_{max}} = \frac{\dot{m} c_{p_w}}{\dot{m} c_{p_b}} = \frac{262}{327.3} = .800$

$\varepsilon = \frac{q}{q_{max}} = \frac{3.927\ kW}{262(37-0)} = 0.405$

Fig 11.14 NTU = 0.7

$NTU = \frac{UA}{C_{min}} \Rightarrow A_s = \frac{0.7(262)}{750}$

$\boxed{A_s = 0.245}\ m^2$

11.35

Steam (sat) 0.14 bar, condensing $C_s \to \infty$

— Shell & Tube —

- 1 shell pass, 2 tube passes
- 130 brass tubes, 2m per pass

find $U$, $T_{c,o}$, $\dot{m}_{condense}$

$D_i = .0134\,m$

$D_o = .0159\,m$

Steam

$h_{fg} = 2378$ kJ/kg

$T = 52°C$

Cooling $H_2O$

$T_{ci} = 20°c$

$T_{co} =$

$\dot{m} =$

$V = 1.25\,m/s$

$h_o = 13.5\,kW/m^2 K$

$\underline{h_i:}$ $Re = \frac{\rho V_\infty D}{\mu} = \frac{1}{1.005\times10^{-3}} \frac{1.25(.0134)}{769\times10^{6}} = 21{,}673$

water guess ~ 305 K

$\rho = \frac{1}{1.005}$

$\mu$

$Pr = 5.2$

$k = .620$

$C_p = 4178$

$Nu_D = .023\,Re^{0.8}\,Pr^{0.4} = 130.9$

$h_i = \frac{k_f Nu}{D} = 6057$

$$\frac{1}{U A_o} = \left[\frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A_o}\right]$$

$$\frac{1}{U_o} = \left[\frac{1}{h_i}\frac{A_o}{A_i} + \frac{\ln(D_o/D_i) * D_o \cancel{L}}{2\cancel{\pi} k \cancel{L}} + \frac{1}{h_o}\right] = \left[\frac{1}{6057}\left(\frac{159}{134}\right) + \frac{\ln(159/134)\,.0159}{2\,(114)} + \frac{1}{13.5\times10^3}\right]$$

$\boxed{U_o = 3549\ W/m^2 k}$

$\underline{T_{c,o}:}$ $q = \varepsilon q_{max}$ $q_{max} = C_{min}(T_{hi} - T_{ho})$

$\dot{m}_c = 1.25\,m/s\,(\pi\,.0134)(130\text{ tubes})\left(\frac{1}{1.005\times10^{-3}}\,kg/m^3\right) = \boxed{22.8\ kg/sec}$

$\frac{C_{min}}{C_{max}} = 0$

$C_c = 22.8\,kg/sec\,(4178\,J/kg) = 95{,}258 = C_{min}$

$q = C_c(T_{c,o} - T_{c,i}) =$

$NTU = \frac{U_o A_o}{C_{min}}$, $A_o = \pi(.0159)(2m)(130)(2\text{ pass}) = 26.0\,m^2$

$NTU = .969 \Rightarrow$ Fig 11.12 $\Rightarrow \varepsilon = 0.61$

$\varepsilon = 1 - e^{-NTU} \Rightarrow \varepsilon = 0.621$

$T_{c,o} = \frac{q}{C_c} + T_{c,i} =$

$\boxed{T_{c,o} = 39.8°c}$

$q = \varepsilon C_c(T_{hi} - T_{ci}) \Rightarrow q = 1.89\times10^6$

$\dot{m}_{condense} = \frac{\dot{m}_c C_p (T_{co} - T_{ci})}{h_{fg}} = \boxed{0.793\ kg/s}$

EXAM 3 2008

#1) Shell & tube. 2 passes, 1 shell

condensing steam, $C_h = \infty$

$P = 1$ bar

$T = 373$ K

$h = 2000$ W/m$^2$K

tube side:

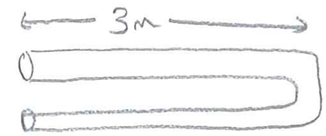
Water

$T_i = 270$ K

$T_o = 320$ K

$\dot{m} = .05$ kg/s

$h = 500$ W/m$^2$K




$D = 2.5$ cm

$\boxed{\text{find: fouling factor}}$

$$\frac{1}{UA} = \left[\frac{1}{h_i A} + \frac{R''_f}{A} + \frac{1}{h_o A}\right] = \frac{1}{U} = \left[\frac{1}{h_i} + R''_f + \frac{1}{h_o}\right]$$

$$q_{per\ tube} = \underline{\dot{m} C_p (T_o - T_i)}$$

$$= .05(4181)(320-270)$$

$$= 5774 \text{ W}$$

$$q_{max} = .05(4181)(373-270) =$$

$$\varepsilon = 1 - e^{(-NTU)}$$

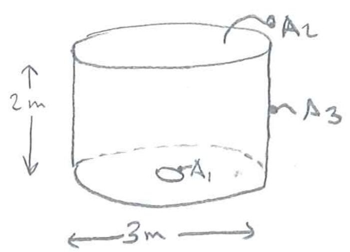
$$\varepsilon = .485 \qquad NTU = .664$$

$$NTU = \frac{UA}{C_{min}} \Rightarrow .664 = \frac{U \; 6 \times \pi(.025)}{.05(4181)} \qquad U = 294.6$$

$$.00339 = \left[\frac{1}{500} + R''_f + \frac{1}{2000}\right] \qquad R = .00089$$

Homework

13.19 — 3 black surfaces




find $F_{13}$, net $q''$ from $A_1 \to A_3$

$A_1 = 0.05\ m^2$
$T_1 = 1000\ k$
$T_3 = 500\ k$

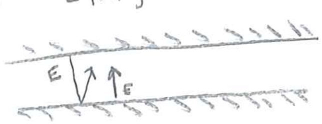
$F_{13} + F_{12} = 1$ $\quad F_{12} = \frac{D^2}{D^2+4L^2}$ (p761) $\quad F_{12} = 0.36$ $\quad \boxed{F_{13} = .64}$

net $q''$: $q_{13} = \frac{\sigma(T_1^4 - T_3^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{13}} + \frac{1-\varepsilon_3}{\varepsilon_3 A_3}} = \frac{\sigma(1000^4 - 500^4)}{\frac{1}{.05(.55)}}$

$\boxed{q_{13} = 1701\ W}$

13.41 2 large diffuse, gray surfaces

Find:
- $G_1$ & $J_1$
- net $q''_{rad}$




$T_1 = 1000$ $\varepsilon = 1$
$T_2 = 500$ $\varepsilon = 0.8$

$G_1 = E_2 + G_1\rho_2 = \varepsilon_2 \sigma T_2^4 + (\varepsilon_1 \sigma T_1^4)\rho_2$
$= \boxed{14175\ W/m^2}$

$J_1 = E_1 + \cancel{E_2\rho_1}^{\,0} = \cancel{\varepsilon_1}^{\,1} \sigma T_1^4 =$
$= \boxed{56{,}700\ W/m^2}$

$q_{1\to2} = G_1 - J_1 = \boxed{42{,}525 \text{ to } A_1}$

HW 9 — Exam Practice —

11.22) Shell-and-tube HX. 10 tubes, shell side


8 passes

Water: $T = 320K$

$\dot{m} = 2.5 kg/s$

$T_i = 15°C$

$T_o = 85°C$

$c_p = 4{,}180$

$v = 1.011 \ m^3/kg \times 10^{-3}$

$\mu = 577 \times 10^{-6}$

$k = .640$

$Pr = 3.77$

engine oil $\dot{m} =$

$T_i = 160°C$

$T_o = 100°C$

$h_o = 400 \ W/m^2k$

$c_p = 2340$

$\dot{m}_{oil}$ ? $\quad 2.5kg/s\,(4180)(85-15) = \dot{m}_{oil}(2340)(160-100) \Rightarrow \boxed{\dot{m}_{oil} = 5.21 \ kg/s}$

$h_i = \frac{\overline{Nu_i}\,k}{}$ $\qquad V = \frac{2.5kg/s\ (.00101\ m^3/kg)}{10 \times \pi \times \frac{(.025)^2}{4}} = 0.514 \ m/sec$

$Re_D = \frac{0.514\,\rho D}{\mu} = 22{,}028$ $\qquad \overline{Nu}_D = .023\,Re_D^{.8}\,Pr^{0.4} = 117$

$h = \frac{Nu\,k}{D} = 3000$

$NTU = \frac{UA}{C_{min}}$ $\qquad \varepsilon = \frac{q}{q_{max}}$ $\qquad q_{max} = C_{min}(T_{hi} - T_{ci}) = 2.5(4{,}180)(160-15)$

$q_{max} = 1.52 \times 10^6 \ W$ $\qquad \frac{C_{min}}{C_{max}} = .857$

$\varepsilon = .481$ $\qquad q = 731{,}500$

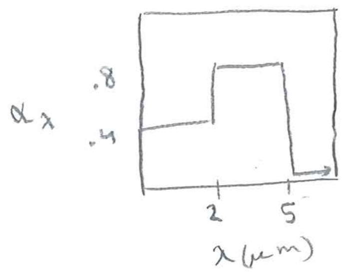
$NTU \cong 1$ (Table 11.12) $\qquad U = \frac{1}{\frac{1}{h_c} + \frac{1}{h_h}} = 353$

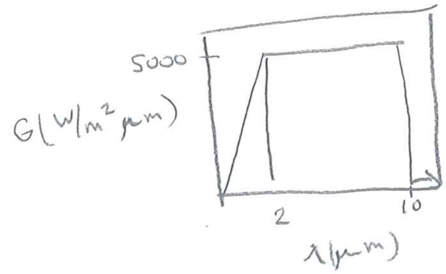
$1 = \frac{353 \times 10 \times \pi \times .025 \times L}{(2.5)(4180)}$ $\qquad \boxed{L = 37.7m}$

HW 10

12.51) Find
- total absorptivity,
- Emissive Power
- Surface temp going ↑ or ↓ ?

$T_{surface} = 1250K$




$$\alpha = \frac{G_{abs}}{G} = \frac{0.4\frac{(2\times5000)}{2} + 0.8((5-2)(5000))}{\frac{2\times5000}{2} + 8\times5000} = \frac{14{,}000}{45{,}000} = \boxed{0.311}$$

$$\varepsilon = 0.4(F_{0\to2}) + 0.8(F_{2\to5}) =$$

$$= 0.4(.161) + 0.8(.754 - .161) = \boxed{0.5388}$$

$$E = \varepsilon E_b = 0.54\,\sigma T^4 = 74{,}750\ W/m^2$$

$$q''_{net} = G_{abs} - E = 14{,}000 - 74{,}750 = -60750$$

$\boxed{\text{temperature is dropping}}$

HW 10

$G_{solar} = 900\,W/m^2$

$\alpha = 0.9$ $\quad h = 20\,W/m^2 \cdot K$

$\varepsilon = 0.1$ $\quad T_{sur} = 290\,K$

Find: $T_{ss}$


q-balance: $G_{abs} - E - q_{conv} = 0$

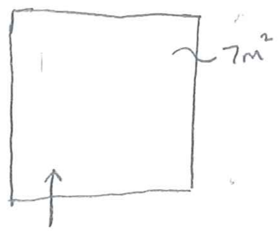
$$\left.\begin{aligned} G_{abs} &= \alpha\,(G_{solar} + G_{surroundings}) \\ E &= 0.1\,\sigma T^4 \\ q_{conv} &= h(T_s - T_{air}) \end{aligned}\right\}$$

$0.9(900\,W/m^2) + (.1)(\sigma)(290)^4 = 0.1\,\sigma T^4 + 20(T - 290K)$

$\Rightarrow T = 329K$

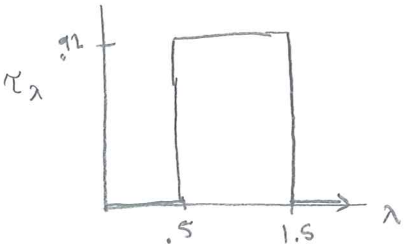
Exam 3 2008

#2, car window




$G_{solar} = 400\ W/m^2$

find • $G_{trans}$
• $E_{interior}$ that is transmitted
• net rad trapped in car in 1 hour




interior, blackbody, 300k

$G_{trans} : \tau = 0.92\ (F_{.5 \to 1.5}) = 0.92(.880 - .250) = 0.580$

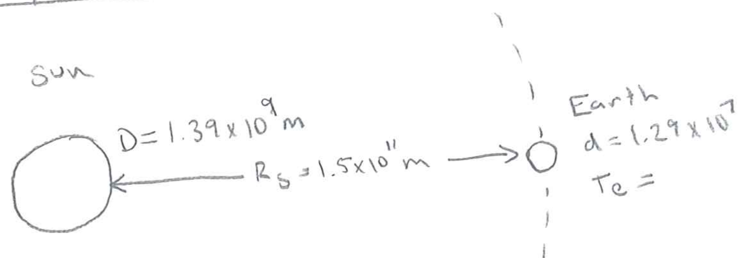
$G_{trans} = \tau(400)(7m^2) = 162\ W$

$E_{trans} = 0.92\ (F_{(1.5 \times 300)} - F_{(0.5 \times 300)}) = (.92) \times 0$

net heat trapped in car $= 162 W = 162\ J/s \times (3600\ s) = 583\ kJ$

Example 12.20

Sphere Surface Area:
$4\pi r^2 = \pi D^2$



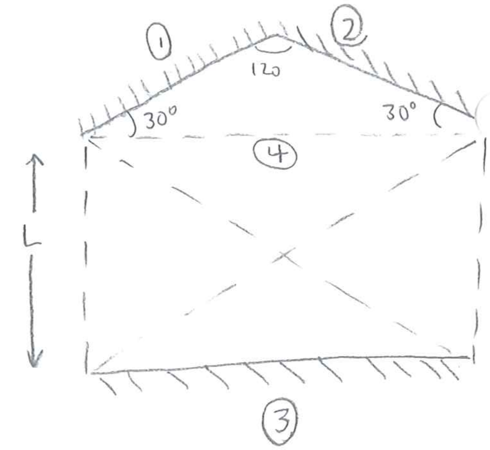

solar flux @ edge of earth atmosphere = $1353\ W/m^2$

$E_s(\pi D_s^2) = q''_{earth} \cdot 4\pi(R_{se})^2$

$T_s = 5774$

Exam 3 2005

#3) long oven $\quad F_{12}, F_{21}, F_{13}, F_{23}, {}_{34}$ ✓ ✓ ✓ ✓




$F_{12}$ : $F_{41} = F_{42} = \frac{1}{2}$ symmetry

$$F_{14} = \frac{A_4}{A_1} F_{41} = 1.73\left(\tfrac{1}{2}\right) = .865$$

$$\cos(30) = \frac{L_4}{2L_1} = 1.73$$

$F_{12} + F_{14} = 1 \Rightarrow$

$\boxed{F_{21} = F_{12} = 0.135}$ ✓

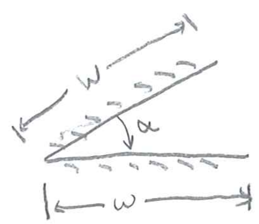
$F_{43} =$ $\quad W_i = 1 \quad W_j = 1 \quad F_{ij} = \dfrac{(2^2+4)^{1/2} - [0^2+4]^{1/2}}{2(1)}$ $\quad F_{43} = 0.414$

$F_{34} = .414 \quad \boxed{F_{31} = F_{32} = 0.207}$

$F_{13} = \dfrac{A_3}{A_1} F_{31} = 1.73(.207) = \boxed{0.358 = F_{13}} \quad \boxed{F_{23} = .358}$

— Finding $F_{14}$ —

Table 13.1 pg 775

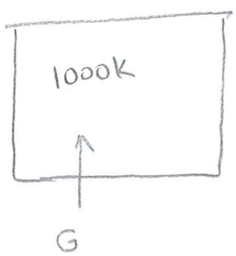


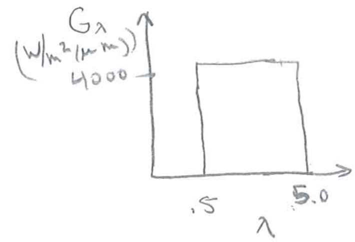

$$F_{ij} = 1 - \sin\left(\frac{\alpha}{2}\right)$$

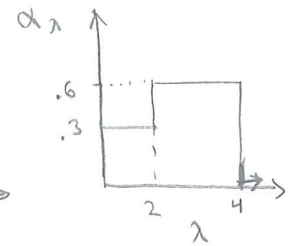
MAY 2008

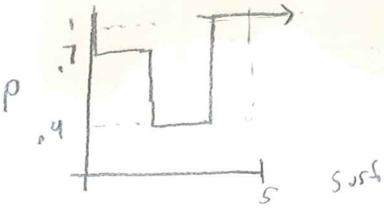
#2

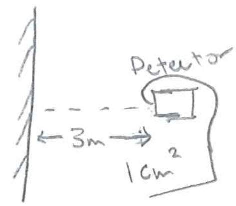
$2\,m^2$ $T = 727°C$




Find: $\alpha$, $\rho$, $\varepsilon$

$$G_{abs} = 0.3(4000)(2-0.5) + 0.6(4000)(4-2) = 6600\ W/m^2$$

$(W/m^2\,\mu m)(\mu m)$

$$\alpha = \frac{G_{abs}}{G} \Rightarrow \alpha = \frac{6600}{18,000} = \boxed{.367}$$

$$\rho_\lambda = 1-\alpha_\lambda \Rightarrow G_{ref.} = 0.7(4000)(2-.5) + 0.4(4000)(4-2) + 1(4000)(5-4)$$

$$= 1140 \Rightarrow \rho = \frac{11400}{18,000} = \boxed{0.633}$$

$$\rho = \frac{G_{ref}}{G} \Rightarrow$$

Find: Temp change over time:

$\varepsilon = \alpha = 0.367$

$$E = \varepsilon\sigma T^4: \quad \varepsilon = 0.3(F_{0\to2}) + 0.6(F_{2\to4})$$

$$= 0.3(.067) + 0.6(.481 - .067) = .269$$

$$E = .269(5.67\times10^{-8})(1000)^4 = 15,252$$

$$q_{\text{entering plate}} = G_{abs} - E = 6,600 - 15,252 = \boxed{-8,650 \text{ plate is cooling}}$$

$J = ?$ $\quad J = G_{ref} + E = 11,400 + 15,252 = 26,650$

Find: radiant power reaching detector

$$q_d = \underbrace{J A_s}\frac{A_d}{\pi L^2} = 26,650\ \frac{2}{\pi}\ \frac{.0001}{9} = \boxed{0.188\ W}$$

heat flux to the detector

$W/m^2 (m^2)\left(\frac{\text{detector area}}{\text{all area}}\right)$

surface of sphere

$= \pi D^2$

225

Suggested problems for last material:

12.47, 13.1, 19, 41, 62

12.47 diffuse, opaque surface, 700 K

radiant flux of 1000 $W/m^2$

uniform $1 < \lambda < 6 \mu m$ is incident

at angle 30° to normal

$\varepsilon_\lambda = 0.5 \quad 3\mu m < \lambda \le 10 \mu m$

$\quad\;\; 0.9 \quad 10 \mu m < \lambda \le \infty$

total radiant Power = ? (received by detector)

for $10^{-4} m^2$ area

arperture = $10^{-5} m^2$, $d = 1m$

$$q_d = I_e + r A_s \cos\theta_d - s \Delta\omega_d - s$$

$$q_d = \frac{\varepsilon \sigma T_s^4 + \rho G}{\pi} A_s \frac{A_d}{L^2}$$

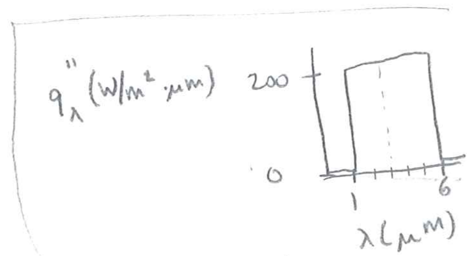
$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda b}(700k)\, d\lambda}{E_b(700k)} = 0.5(0.8081 - .0838) + 0.9(1 - .8081) = 0.535$$

fraction of blackbody radiation in wavelength band

$$\rho = \frac{\int_0^\infty \rho_\lambda G_\lambda d\lambda}{G} = \frac{\int_0^\infty (1-\varepsilon_\lambda) q''_\lambda d\lambda}{q''} = 1 \times F_{(0 \to 3 \mu m)} + 0.5 \times F_{(3\text{-}6 \mu m)}$$

$$= 1 \times (0.4) + 0.5(0.6) = 0.7 = \rho$$

Given:




$$G = q'' \cos\theta_i = 1000 \cos(30°) = 866 \ W/m^2$$

$$q_d = \frac{\varepsilon \sigma T_s^4 + \rho G}{\pi} A_s \frac{A_d}{L^2}$$

arperture $(10^{-5} m^2)$

(1m)

$\sigma = 5.67 \times 10^{-8}$

$$q_d = \frac{7889}{\pi}(1 \times 10^{-9}) = \boxed{2.51 \times 10^{-6} \ W}$$

13.1

a) long duct

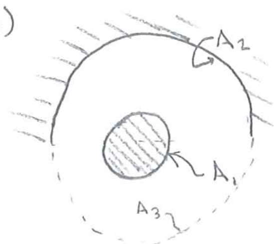

$A_1 F_{12} = A_2 F_{21}$

$F_{12} = \frac{A_2}{A_1} F_{21} \quad \frac{A_2}{A_1} = \frac{\frac{3}{4}\pi \cancel{D}}{\cancel{D}} = \frac{3\pi}{4}$

$\underset{0}{\cancel{F_{11}}} + \boxed{F_{12} = 1}$

$F_{21} = \frac{4}{3\pi}(1) \Rightarrow \boxed{F_{21} = \frac{4}{3\pi} = 0.424}$

b)




$A_2 = 2A_1$

summation: $\underset{0}{\cancel{F_{11}}} + F_{12} + F_{13} = 1$

symmetry $F_{12} = F_{13} = \frac{1}{2}$

$A_1 F_{12} = A_2 F_{21} \quad F_{21} = \frac{A_1}{A_2} F_{12} = \frac{1}{2}\left(\frac{1}{2}\right) = 0.25$

$\boxed{F_{12} = \frac{1}{2} \quad F_{21} = 0.25}$

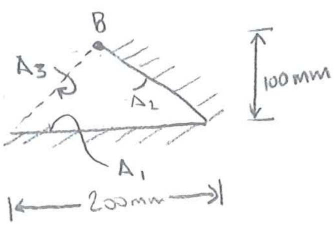
c)


$F_{22} = ?$

$F_{12} = 1 \quad F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\cancel{D}}{\frac{\pi}{2}\cancel{D}}(1) \Rightarrow F_{21} = \frac{2}{\pi}$

$F_{22} = 1 - \frac{2}{\pi} \quad \boxed{F_{22} = 0.363}$

d)




$F_{12} + F_{13} = 1 \Rightarrow F_{12} = F_{13} = \frac{1}{2}$

$A_1 F_{12} = A_2 F_{21} \quad F_{21} = \frac{A_1}{A_2} F_{12} = \frac{200}{\sqrt{100^2 + 100^2}}\left(\frac{1}{2}\right) = 0.707$

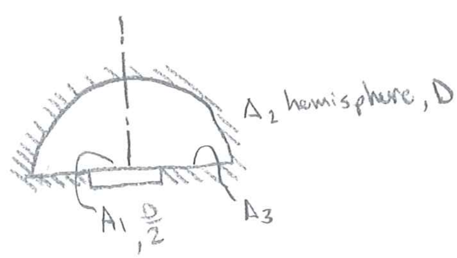
$\boxed{F_{21} = .707} \quad \boxed{F_{12} = \frac{1}{2}}$

e)


$F_{13} + F_{12} = 1 \Rightarrow \boxed{F_{12} = F_{13} = \frac{1}{2}}$

$F_{21} = \frac{A_1}{A_2} F_{12} \rightarrow 0 \quad \boxed{F_{21} = 0}$

f)




$\boxed{F_{12} = 1}$ by obs. $\boxed{F_{32} = 1}$

$F_{21} = \frac{A_1}{A_2} F_{12} \quad \frac{\frac{\pi}{4}\left(\frac{D}{2}\right)^2}{\frac{\pi D^2}{2}} = 0.125$

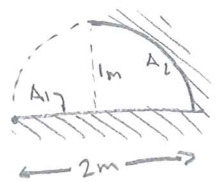
$\boxed{F_{21} = 0.125}$

$F_{23} = \frac{A_3}{A_2} F_{32} = 0.375 \quad \boxed{F_{23} = 0.375}$

13.1

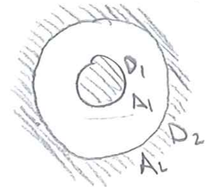
g) long, open channel $\cancel{F_{11}}^{0} + F_{12} + F_{13} = 1$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2 \times L}{\frac{(2\pi 1)}{4} \times L} = \frac{4}{\pi} \times .50 = 0.637$$




$$\boxed{\begin{aligned} F_{12} &= 0.5 \\ F_{21} &= .637 \end{aligned}}$$

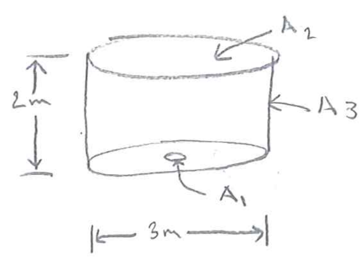
h) concentric cylinders (long)




$F_{12} = 1$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D_1}{\pi D_2}(1) \Rightarrow \boxed{F_{21} = \frac{D_1}{D_2}}$$

13.19




$A_1 << A_2$ & $A_3$ $\qquad A_1 = .05 m^2$

$$r_1 = \left(\frac{A_1}{\pi}\right)^{1/2} = \left(\frac{.05 m^2}{\pi}\right)^{1/2} = .126$$

$F_{12} + F_{13} = 1$

$$F_{12} = \frac{D^2}{D^2 + 4L^2} \text{ (Eq 13.8) pg 781} \qquad F_{12} = 0.36$$

$$F_{13} = 1 - F_{12} = \boxed{0.64}$$

Net radiation H.T. from $A_1$ to $A_3$ $\quad T_1 = 1000K \quad T_3 = 500K \quad$ Black surfaces, $\varepsilon = 1$

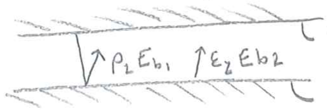
$$q_i = \frac{E_{bi} - \cancel{J_i}^{\,0}}{\cancel{(1-\varepsilon_i)/\varepsilon_i A_i}} \quad \text{Black surfaces}$$

$$q_i = A_1 F_{13} \sigma (T_1^4 - T_3^4)$$

$$= .05 m^2 (0.64)(5.67 \times 10^{-8})(1000^4 - 500^4) = \boxed{1700 \text{ W}}$$

13.41)

diffuse, gray surfaces




$T_1 = 1000$ K $\varepsilon_1 = 1$

$T_2 = 500$ K $\varepsilon_2 = 0.8$

Find:

- irradiation & radiosity for upper plate ?
- radiosity for lower plate ? (J)
- net radiation exchange ?

$$G_1 = \varepsilon_2 E_{b2} + \rho_2 E_{b_1} = \varepsilon_2 \sigma T_2^4 + (1-\varepsilon_2)\sigma T_1^4$$

$$G_1 = .8(5.67\times10^{-8})(500^4) + (0.2)(5.67\times10^{-8})(1000^4) = \boxed{14175 \text{ W/m}^2}$$

$G_1$, irradiation = flux emitted by surface 2 + reflected flux emitted by surface 1

Radiosity = radiant flux leaving the surface by emission & reflection

$$J_1 = E_{b_1} = \sigma T_1^4 = \boxed{56{,}700 \text{ W/m}^2}$$

$$J_2 = \varepsilon_2 E_{b2} + \rho_2 G_2 = 0.8(\sigma T_2^4) + (0.2)(\sigma T_1^4) = \boxed{14{,}175 \text{ W/m}^2}$$

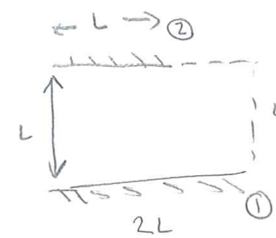
✓ ✓ ✓

$(\rho = 1-\varepsilon)$ $G_2 = E_{b_1}$ (blackbody)

net radiation exchange = $\underset{J_1}{56{,}700} - \underset{J_2}{14{,}175} = \boxed{42{,}525 \text{ towards } A_2}$ $(\text{W/m}^2)$

Exam A

3)

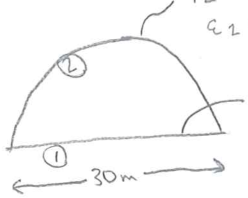



$$F_{ij} = \frac{\left((2+2)^2+4\right)^{1/2} - (4)^{1/2}}{2(2)} = 0.618$$

$$F_{12} = \frac{.618}{2} = .309$$

$$F_{21} = \frac{A_2}{A_1} F_{12} = .618$$

ii)




$T_2 = 20°C = 293$

$\varepsilon_2 = 0.1$

$T_1 = 0°C = 273$

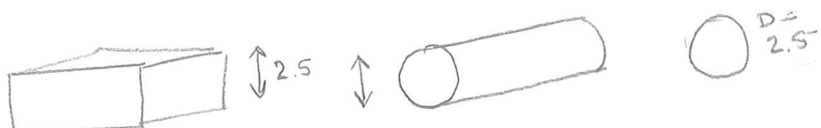
$\varepsilon_1 = 0.8$

$\boxed{F_{12} = 1}$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{1}{\frac{\pi(1)}{2}}(1) = \boxed{.636}$$

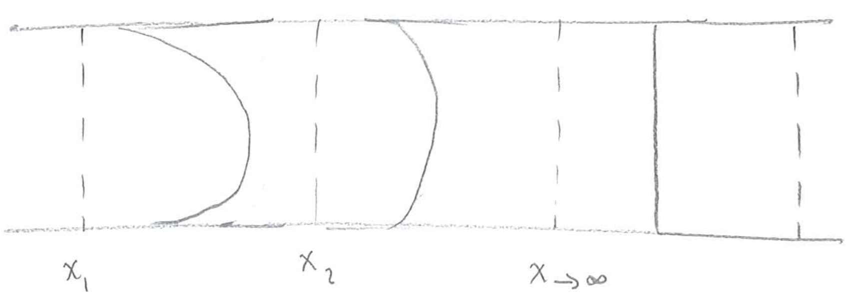
$$q_{12} = \frac{\sigma(273^4 - 293^4)}{\frac{0.2}{30(.8)} + \frac{1}{30} + \frac{0.9}{\pi 15(.1)}} = \boxed{-442 \text{ W}}$$

4) i)




Sphere will cool fastest because it has the shortest distance for heat to escape in all directions. smallest volume too.

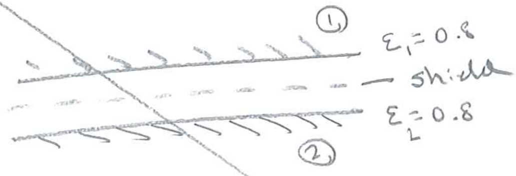
ii



13.56

2 large, diffuse, gray surfaces

Find: $\varepsilon$ shield to reduce $q'' \times 10$




$q$ w/o shield $= 10$ $q$ w/ shield (p792)

$$\frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon}{\varepsilon} + \frac{1-\varepsilon}{\varepsilon}} = 10 \frac{\sigma(T_1^4 - T_2^4)}{\frac{2}{\varepsilon} + \frac{2-2\varepsilon_s}{\varepsilon_s}}$$

$$\frac{1}{\frac{2-1.6}{.8}} = \frac{10}{\frac{2}{.8} + \frac{2-2\varepsilon_s}{\varepsilon_s}}$$

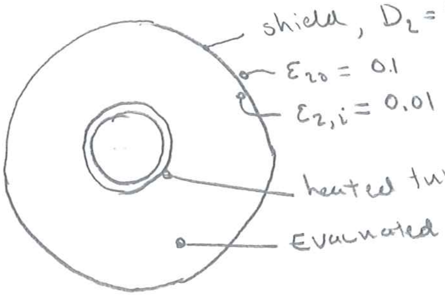
$$2 = \frac{10}{2.5} + \frac{10\varepsilon_s}{2-2\varepsilon_s}$$

$$-2 = \frac{10\varepsilon_s}{2-2\varepsilon_s}$$

$\Rightarrow \varepsilon = 0.444$

13.62

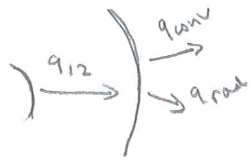
diffuse, gray radiation shield 60mm diam




Walls 17°C

convection, air $h = 10\ W/m^2 K$

shield T = 42° constant




$q_{radiation} + q_{convection\ 2,0} = q_{12}$

$$q_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)^2}$$ Eq (13.21)

$q_{convection} = hA$

$q_{rad} = \varepsilon_{2,0} A_2 \sigma (T_2^4 - T_{wall}^4)$ (13.22), pg 793

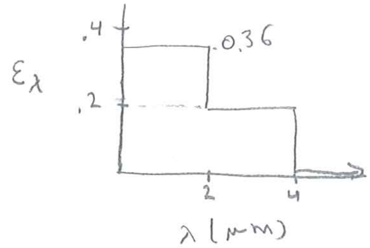
$$\frac{\sigma D_1 (T_1^4 - T_2^4)}{1 + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)^2} = \varepsilon_{2,0} \sigma T_s^4 D_2 + h_o (T_s - T_{amb}) D - \alpha_{2,0} \sigma T_{wall}^4 D$$

$3.34 \times 10^{-11} (T_1^4 - T_2^4) = 3.35 + 9 - 2.41$

$\boxed{T_1 = 745K}$ $\Rightarrow T_1 = 472°C$

HW 10 12.32, 46, 51, 109,

12.32)




a) Find: total hemi emissivity $\varepsilon$ & total emissive power E @ 2000 K

$\varepsilon$ = average $\varepsilon_\lambda$ over all $\lambda$'s

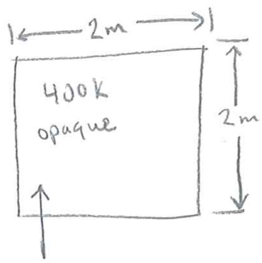
$\varepsilon(T) = F_{(0\to2)} + F_{(2\to4)} = 0.36(.481) + 0.2(.856 - .481)$

$\varepsilon(T) = 0.248$

Total emissive power @ 2000K $= \varepsilon \sigma T^4 = 0.248(5.67\times10^{-8})(2000k)^4$

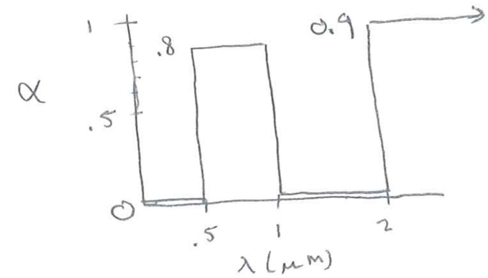
$\boxed{E = 225{,}000 \text{ W}}$

12.46)




diffuse

Find
- absorbed iradiation, $G_{abs}$
- emissive power, E
- radiosity
- net h.t.




$G_{solar} = 1200 \text{ W/m}^2$ $T_{sun} = 5800$k

$G_{abs} = \left[F_{(.5\to1)}\alpha + F_{(2\to\infty)}\alpha\right] G_{solar} \Rightarrow \alpha = 0.43$

$= [0.8(0.720 - .250) + .9(1 - .940)](1200) * \text{Area}$

$= 516 \text{ W/m}^2 * 4\text{m}^2 = \boxed{2064 \text{ W}}$

Emissivity $\varepsilon = \dfrac{\int_0^\infty \varepsilon_\lambda E_{\lambda b}(400)\,d\lambda}{E_b(400K)} = (.8)(F_{0\to.5}) + (.9)(F_{2\to\infty}) = 0.9(1-0) = 0.9$

Emissive Power, E $= \varepsilon \sigma T^4 = 1306 \text{ W/m}^2 \times 4 = \boxed{5224 \text{ W}}$

J, radiosity $= E + \rho G = 5224 + (1-.43)(1200)(4\text{m}^2) = \boxed{7960 \text{ W}}$

$q_{net}$, net radiation heat trans $= 2064 - 5224 = \boxed{3164 \text{ W}}$ (heat leaving surface)

$(G_{abs} - J)$

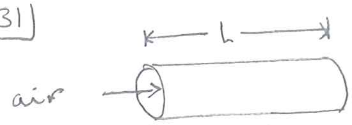
David Malawey HW 8 Due Thurs, March 24

100

8.31, 50, 74, 93, 94

$\nu_{Air,\ 25°C} = 15.89 \times 10^{-6}\ m^2/s$ $\quad P_r = .707$

31)




$k = .15\ W/m \cdot K$

$D_i = 0.15\ m$

$D_o = 0.17\ m$

$T_{\infty, water} = 17°C$

$h_o \approx 15{,}000\ W/m^2 K$ @ outer pipe surface

discharge temp $T_{m,o} = 21°C$

$k_{air} = 26.3 \times 10^{-3}\ W/m \cdot K$

$T_{m,i} = 29°C$ $\quad T_{m,o} = 21°C$

$\dot{V}_i = 0.025\ m^3/s$

smooth surface

find L

$$Re_D = \frac{u_m D}{\nu} = \frac{1.41\,(.15)}{15.89} = 13{,}310 > 2300, \text{ Turbulent}$$

$$\frac{\dot{V}_i}{A} = \frac{.025\ m^3/s}{\pi(.15)^2/4} = u_m = 1.41\ m/s$$

$\cancel{Nu_D = 3.66}$ $\Rightarrow h = \frac{Nu_D k_a}{D_h}$

$$\boxed{R_{tot} = \frac{1}{h_i A} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A}}$$

$A = \pi D_o L$

$$(.267 + .133 + 2.12 \times 10^{-5})\frac{1}{L} = \frac{0.40}{L}$$

$$Nu_D = .023 Re_D^{4/5} Pr^{0.3} = 45.35 \quad \bar{h}_i = 7.95\ W/m^2 K$$

$P\dot{V} = \dot{m} R T$

$\dot{m} = \rho \dot{V} = .0290\ kg/s$

$$(\bar{U} A_s)^{-1} = \frac{1}{L}(R\ total)$$

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = e^{\left(\frac{-A_s \bar{U}}{\dot{m} c_p}\right)} \quad \text{where } \bar{U} A_s = \frac{1}{R\ total}$$

$$\ln\left(\frac{17-21}{17-29}\right) = -\frac{1}{R_{tot}}\left(\frac{1}{\dot{m} c_p}\right) =$$

$$-1.099 = -(2.5\,L)\left(\frac{1}{0.029\,(1007)}\right)$$

$$\boxed{L = 12.84\ m}$$

Pump Power

$$\boxed{\Delta P \cdot \dot{V} = \frac{f \rho_i u_{m,i}^2}{2 D_i} L \cdot \dot{V}_i}$$

$$f = .316(13{,}310)^{-1/4} = .0294$$

$\rho \dot{V} = .0290$

$u_m = 1.41$

$L = 12.84\ m$

$D_i = 0.15\ m$

$$\boxed{\text{Power} = .073\ W}$$

8.50

**134a**

$\dot{m} = .1 \, kg/s$

$T = 240 \, K$

**Teflon tube**

$D_i = .025 \, m$

$D_o = .028 \, m$

**Air**

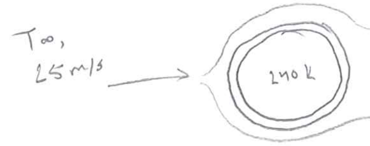
$V = 25 \, m/s$

$T = 300 \, K$

$P_r = .707$

$k = .0263$

find q per unit length




$$q' = \frac{(T_\infty - T_m)}{R_{tot}}$$

$$T_m \quad \frac{1}{h_i A} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi k} + \frac{1}{h_o A} \quad T_\infty$$

$$Re_{D,i} = \frac{\rho V D}{\mu} \; \cancel{(\text{iterations})} = \frac{4\dot{m}}{\pi D_i \mu} = 12126$$

Turbulent

$$Nu = .023 \, Re_D^{4/5} \, P_r^{0.4} = 80.96$$

$$\bar{h}_i = \frac{Nu_D \, k}{D_h} = 347$$

**134a**

$\rho = 1398$

$c_p = 1.267$

$\mu = .042 \times 10^{-2}$

$\nu = .3006 \times 10^{-6}$

$k = .107$

$P_r = 5.0$

**Teflon**

$k = 0.35$

$\bar{h}_o$ ② $Nu_{,D} = C \, Re_{,D}^{m} \, P_r^{1/3}$ Pg 431 lookup

$$= .027 \, (44{,}050)^{.805} (.707)^{1/3} = 131.7$$

① $$Re_D = \frac{VD}{\nu} = \frac{25 \, m/s \, (.028)}{15.89 \times 10^{-6}} = 44{,}050$$

③ $$\bar{h}_o = \frac{k \, Nu_D}{D} = \frac{.0263 \, (132)}{.028} = 124 \, W/m^2 K$$

$$q' = \frac{T_\infty - T_m}{\frac{1}{347 \, \pi \, .025} + \frac{\ln \frac{28}{25}}{2\pi (.35)} + \frac{1}{124 \, \pi \, .028}} = \frac{T_\infty - T_m}{.0367 + .0515 + .0917} = \frac{T_\infty - T_m}{.180} = \frac{300 - 240}{.180}$$

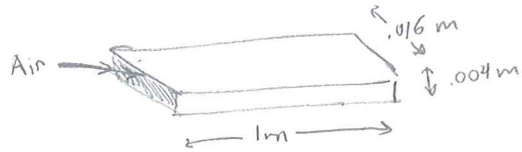
$$\boxed{q' = 333 \, W/m}$$

**8.74**

**Air**

$\dot{m} = 3 \times 10^{-4}$ kg/s

$T = 27°C = 300K$




$q'' = 600\ W/m^2$

Temp of air & surface @ outlet ?

$C_p = 1007$

$\mu = 184.6 \times 10^{-7}$

$k = .0263$ W/mK

$P_r = .707$

$q = q'' \times A_s = 600 \times (.004 \times 1 + .016 \times 1)\,2 = 24W$

$T_{m,o} = T_{m_i} + \dfrac{q}{\dot{m} C_p} = \boxed{379\ K}$

$D_h = \dfrac{4A_c}{P} = \dfrac{4(.004)(.016)}{.04m} = .0064$

$T_{s,o} = T_{m,o} + q''/h$ (8.27) p463

$Re_D = \dfrac{\rho u_m D_h}{\mu} = \dfrac{\dot{m} D_h}{A_c \mu} = 1625$

$h_{local} = \dfrac{k}{D_h} Nu = \dfrac{.0263}{.0064} 5.33 = 21.9$

**Laminar**

$T_{s,o} = 379K + \dfrac{600}{21.9} = \boxed{406\ K}$

**8.93**

Water, $\dot{m} = .02$ kg/s

$T_{m,i} = 20°C$

$T_{m,o} = 75°C$

Tube

$D_i = .025m$

$D_o = .100\ m$

Steam

$T_{s,i} = 100°C$

find length of tube L


Constant $T_{s,i}$

$H_2O$ 320°C

$C_p = 4180$

$\mu = 577 \times 10^{-6}$

$P_r = 3.77$

$k = .640$

$L = -\dfrac{\dot{m} C_p}{P \bar{h}_L} \ln\left(\dfrac{T_s - T_{om}}{T_s - T_{im}}\right)$ (8.41) pg 471

Table 8.2 annular tubes p 490

$\bar{h}_L = \dfrac{\overline{Nu}_i\, k}{D_h}$

$Nu_i = 7.37$

$D_h = D_o - D_i = 0.075m$

$\bar{h}_L = 62.9$

$Re_D = \dfrac{\dot{m} D_h}{A_c \mu} =$

$A_c = \pi \dfrac{D_o^2}{4} - \pi \dfrac{D_i^2}{4} = \dfrac{\pi}{4}(D_o^2 - D_i^2) = .00834 m^2$

$\boxed{Re_D = 353}$

$\boxed{L = 19.7m}$

8.94

how long must annulus be if $\dot{m} = 0.3$ kg/s ?

$Re_D = 5296 > 2300$ turbulent

Water 320°

$Pr = 3.77$

$k = .640$

$Nu_D = .023\, Re_D^{4/5}\, Pr^{0.4} = 33.8$

$\bar{h} = \frac{k}{D_h} Nu = 318$

$L = -\frac{\dot{m} C_p}{P h_L} \ln\left(\frac{T_s - T_{m,o}}{T_s - T_{m,i}}\right)$

$\frac{L_2}{L_1} = \frac{.3}{.02} \frac{62.9}{318} \Rightarrow \boxed{L_2 = 58.5\,m}$

✓

225 HW 7 due Tues March 8 **100**

7.2 a, b, c, 11, 20, 45, 73,

David Malawey

$\nu_{oil, 330k} = 96.6 \times 10^{-6}\ m^2/s$ Table A.5

7.2) oil 100°C

0.1m/s

$Re_L = \frac{uL}{\nu} = \frac{0.1 m/s (1m)}{\cancel{22} \times 10^{-6}\ m^2/s}$ 96.6 $= \cancel{4546}$ 1035 $< 5 \times 10^5$


laminar flow velocity & thermal boundary layer thickness?

a) $\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5(1)}{\sqrt{\cancel{4546}}} = \boxed{0.1554\ m}$  $\delta_t = \delta Pr^{-1/3} = 0.1554 (1205)^{-1/3}$ $\boxed{\delta_t = .0146m}$

b) local heat flux & surface shear stress @ the trailing edge

$C_{f,x} = \frac{\tau_{s,x}}{\rho u_\infty^2/2} = 0.664 Re_x^{-1/2}$  $\tau_{s,x} = 0.664 \frac{1}{\sqrt{1035}} \left(865.8 \frac{kg}{m^3}\right)\left(0.1 \frac{m}{s}\right)^2 \frac{1}{2}$

$\boxed{\tau_s = 0.0894\ N/m^2}$

$P_r = 1205$ (Table A.5)

$k = 141 \times 10^{-3}\ W/m$

heat flux $Nu_x \equiv \frac{h_x x}{k}$  $Nu_x = 0.332 Re_x^{1/2} Pr^{1/3} = 114$

$h_x = \frac{(141 \times 10^{-3} \frac{W}{m})(114)}{(1m)}$ $16.08\ W/m^2 \cdot K$

$q'' = h(T_s - T_\infty) = 16.08 \frac{W}{m^2 k} (20-100) = \boxed{-1.286 \frac{kW}{m^2}}$

c) Total drag & h.t. of plate

$\overline{Nu}_x = .664 Re_x^{1/2} Pr^{1/3} = \frac{\bar{h}_x x}{k} \Rightarrow \bar{h} = 32.05\ W/m^2 \cdot k$

$\bar{C}_f = 1.328 Re_x^{-1/2}$

$q'' = -80(32.05) = 2564 \times 2$ plates sides of

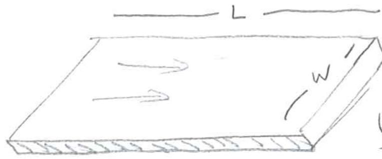
$\boxed{q'' = -5128\ W/m^2}$

$D' = 2L\left(\frac{\rho u_\infty^2}{2}\right) 1.328 Re_L^{-1/2} = \boxed{0.337\ N/m}$ ✓

$D' = 2L \bar{\tau}_{s,L}$

a) find Velocity & BL thickness @ trailing edge

7.111




Given
$u_\infty = 5\,m/s$
$T_\infty = 20°C$

**Air, 300K 1Atm**

$\rho = 1.161\ kg/m^3$

$\nu = 15.89 \times 10^{-6}\ m^2/s$

$k = .0263\ W/m\cdot K$

$P_r = 0.706$

| $H_2O$, liquid 315 K | 310 |
| --- | --- |
| $\rho = 991\ kg/m^3$ | .993 |
| $\nu = 1584\ m^2/s$ | 1439 |
| $k = 0.634$ | |
| $P_r = 5.83$ | |

a) find $\bar{h}$, $q''$, $D'$

$L = 2\,m$ × BOTH SIDES
$W = 2\,m$

Air flow, $T_s = 50°C,\ 80°C$

$$\frac{20+50}{2} = 35° \qquad \frac{20+80}{2} = 40°$$

308 K     313 K

$$Re_L = \frac{u_\infty L}{\nu} = \frac{5\,(2)}{\nu} = 6.293 \times 10^5$$

Turbulent, $> 5 \times 10^5$

$$\bar{C}_f = 0.074\, Re_L^{-1/5} - 1742\, Re_L^{-1}$$
$$= 2.35 \times 10^{-3}$$

$$F_{D,1} = \bar{C}_f \frac{\rho u_\infty^2}{2} A_s \Rightarrow \boxed{F_D = 0.273\,N}$$ For BOTH TEMPERATURES

$$\bar{h}_L = \frac{\overline{Nu}_L \cdot k_f}{L} \qquad \overline{Nu}_{,L} = (0.037\, Re_L^{4/5} - 871)\, P_r^{1/3} = 660.2$$

$$= \frac{660(.0263)}{2} \qquad \boxed{\bar{h}_L = 8.679}$$

$$q_{conv} = \bar{h}_L \cdot A_s (T_s - T_\infty) \qquad \boxed{q_{50°C} = 2083\,W \qquad q_{80°C} = 3471\,W}$$

b) $L = 0.1\,m$ $W = 0.1\,m$ $A_s = .02\,m^2$

$$Re_1 = \frac{u_\infty L}{\nu} = 6.86 \times 10^5 \qquad Re_2 = 9.02 \times 10^5$$

$$\bar{C}_{f,1} = .074\, Re_1^{-1/5} - 1742\, Re_L^{-1} = 2.49 \times 10^{-3} \qquad C_{f_2} = 2.84 \times 10^{-3}$$

$$F_{D_1} = \bar{C}_f \frac{\rho u_\infty}{2} A_s = \boxed{0.620\,N} \qquad \boxed{F_{D_2} = 0.700\,N}$$

$$\overline{Nu}_1 = (.037\, Re_L^{4/5} - 871)\, P_r^{1/3} \qquad Nu_1 = 1450 \qquad Nu_2 = 1952$$

$$\bar{h}_1 = \frac{Nu \cdot k_f}{L} \qquad \boxed{\begin{array}{ll} \bar{h}_1 = 9050\ \frac{W}{m^2 K} & q_1 = 5430\,W \\ \bar{h}_2 = 12{,}600\ \frac{W}{m^2 K} & q_2 = 15100\,W \end{array}}$$

$$q = h_L A_s (T_s - T_\infty)$$

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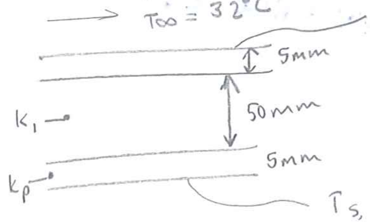
7.20

$u_\infty = 105\ km/h = 29.17\ m/s$

$T_\infty = 32°C$




$T_{s,o}$ $\quad \varepsilon = \alpha_s = 0.5$

$T_{s,i} = -10°C$

$W = 3.5m$

$L = 10m$

$k_p = 180\ w/m\cdot k$

$k_i = 0.026\ w/m\,k$

irradiation $= 750\ w/m^2$

$\nu = 15.89 \times 10^{-6}\ \frac{m^2}{s}$

a) find $T_{s,o}$

$$\alpha_s G_s + q''_{conv} - E = q''_{cond} = \frac{T_{s,o} - T_{si}}{R''_{tot}}$$

$$\alpha_s G_s + \bar{h}(T_\infty - T_{s,o}) - \varepsilon\sigma T_{s,o}^4 = \frac{T_{so} - T_{si}}{2R''_p + R''_i}$$

$R''_p = \frac{t_1}{k_p} = 2.78 \times 10^{-5}$

$R''_i = \frac{t_2}{k_i} = 1.923$

$Re_L = \frac{u_\infty L}{\nu} = 1.84 \times 10^7$

$\bar{h} = \frac{k}{L}\, .037\, Re_L^{4/5}\, P_r^{1/3} = 56.2\ w/m^2\cdot k$

Calculator $\Rightarrow$ $\boxed{T_{s,o} = 307k = 34°C}$

$q = A\,\frac{(T_s - T_\infty)}{R_{tot}} \Rightarrow \boxed{q = 780\ W}$

b) $\alpha = 0.15 \quad \varepsilon = 0.8$

$$\alpha_s G_s + \bar{h}(T_\infty - T_{s,o}) - \varepsilon\sigma T_{s,o}^4 = \frac{T_{so} - T_{si}}{2R''_p + R''_i} \Rightarrow \boxed{T_{so} = 300K = 27°C}$$

$\boxed{q = 675\ W}$

c) no insulation

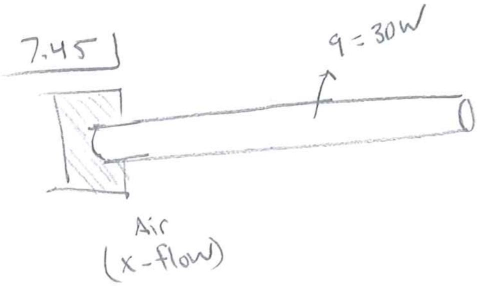
$R''_{tot} = 2 \times (2.78 \times 10^{-5}) = 5.56 \times 10^{-5}$

$\bar{h} = 56.2\ w/m^2 k$

$\alpha_s = \varepsilon = 0.5$

$\boxed{T_{s,o} = 263.1K \quad q = 90.63\,kW}$

7.45




$D = 10mm$
$k = 240\ W/m{\cdot}K$
$\rho = 2700\ kg/m^3$
$c_p = 900\ J/kg{\cdot}K$
$Re = 4000$

infinitely long pin

Ch 3 slide 24

Cylinder, $Re_D$ $.4 < \downarrow < 4\times10^5$ $\overline{Nu}_D = C Re_D^m Pr^{1/3}$ $C = .683$ $m = 0.466$

$\bar{h} \sim D^{-1} D^{.466} = D^{-0.534}$ $q_{conv} \sim (D^{-.534} \cdot D \cdot D^2)^{1/2}$

$q_{conv} = (\bar{h}_L P k A_c)^{1/2} \theta_b \Rightarrow q_{conv} \sim D(^{-.534} \times D \times D^2)^{1/2}$

$q_{conv} \sim D^{1.23}$

$q_2 = q_1 \left(\frac{D_2}{D_1}\right)^{1.23} = 30W(2)^{1.23}$

$\boxed{q_2 = 70.4W}$ ✓

7.73

$\uparrow \boxed{F_D = C_D A_f \frac{\rho V^2}{2}}$ (7.42) p401


$\downarrow \cdot F_b (\rho_{Cu} - \rho_{H_2O}) g \frac{\pi D^3}{6}$

Cu, 350K
$\rho = 8933$
$k = 398$
$C_p = 387$

$H_2O$ 280K
$\rho = 1000$
$\nu = 1/1422\times10^{-6}$
$k = .582$
$Pr = 10.26$

$T_s \approx 340$
$\mu_s = 420\times10^{-6}\ N{\cdot}s/m^2$

$C_D V^2 = \frac{4D}{3} \frac{\rho_{Cu} - \rho_{H_2O}}{\rho} g = 2.07 m^2/s^2$

Fig 7.8 $\rightarrow$ $\boxed{V \approx 2.1 m/s}$

$Re_D = 29580$ $C_D \approx 0.46$ $\overline{Nu}_D = 2 + (.4(29850)^{1/2} + .06(29850)^{2/3})(10.26)^{.4}\left(\frac{1422}{420}\right)^{1/4} = 439$

$\bar{h} = \overline{Nu}_D \frac{k}{D} = 12{,}775\ W/m^2K$

$Bi = \bar{h}\left(\frac{r_o}{3}\right)/k_{Cu} = 0.11$ No lumped method, single term approximation

$\theta_o^* = C_1 e^{(-\zeta_1^2 F_o)}$ $C_1 = 1.0937$ $\zeta_1 = .9472$ $0.5 = 1.094e^{-.9472 F_o} \Rightarrow F_o = 0.87 = \frac{\alpha t_f}{r_o^2}$

$\alpha_{Cu} = \frac{k}{\rho c_p} = 1.15\times10^{-4}\ m^2/s$ $t_f = 0.76s$

$\boxed{H = t_f \cdot V = 1.6m}$ ✓

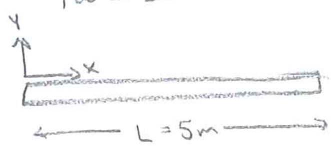
ME225 HW6 David Malawey due 3/1/11

100

6.8, 15, 18, 32, 35

**6.8**

$T_\infty = 20°C$ $\quad u = \rightarrow$




$T(°C)(x) = 20 + 70e^{-600xy}$

find $h(x)$, $\bar{h}$

$\frac{dT}{dy} = -70(600)x e^{-600xy} \Big|_{y=0} \Rightarrow -42000x$

$h = \frac{-k_f \left(\frac{dT}{dy}\right)\big|_{y=0}}{T_s - T_\infty}$

$\bar{T} = \frac{90-20}{2} + 273 = 328K \Rightarrow k_f \approx .03\ W/m \cdot K$ (300K)

$h = \frac{-k_f(-42000x)}{90-20} = \boxed{18x\ W/m^2K = h}$

average coefficient, $\bar{h}$: $= \frac{1}{L}\int_0^L h\,dx = \frac{18}{5}\int_0^5 x\,dx = \frac{18}{5}\left[\frac{x^2}{2}\right]_0^5 = \boxed{45\ W/m^2K = \bar{h}}$

**6.15**

$V_{max} = 50 m/s$ a) find $L_{minimum}$

P = 1 ATM

T = 25°C

$Re_x = 10^8$

$Re_{x,c} = \frac{\rho u_\infty x_c}{\mu} = \frac{u_\infty x_c}{\nu}$ $\quad x_c = \frac{10^8(15.89 \times 10^{-6})}{50 m/s}$

$\boxed{L_{min} = 31.8m}$

$PV = RT$

$\rho = \frac{P}{RT} = \frac{101kPa}{8.314(298)} = .0408\ kg/m^3$

b) $Re_{x,c} = 5\times10^5 \Rightarrow 31.8\left(\frac{5\times10^5}{10^8}\right) = \boxed{0.159m}\ x_c$

**6.18**

L = 1m

$T_s = 400K$

$T_\infty = 300K$

V = 100m/s

$q'' = 20,000\ W/m^2$

$L_2 = 5m$

$T_s = 400K$

$T_\infty = 300$

V = 20m/s

$Re_L = \frac{VL}{\nu}$ $\quad V_1 = V_2 \Rightarrow 5(20) = 1(100) \Rightarrow Re_{L,1} = Re_{L,2}$

$h_1 = \frac{q_{conv}}{A_s(T_s - T_\infty)} = \frac{20,000}{100} = 200\ W/m^2K$

$\bar{h}_2 = \frac{''}{\frac{A_2}{A_1}(")} = (200\ W/m^2K)\frac{q_{conv}}{(T_s-T_\infty)(5A_s)} \Rightarrow \boxed{40\ W/m^2 \cdot K}\ \bar{h}_2$

6.32

$L = 800\ mm$

$t = 6mm$

$V = 70mph = 31.29$

$T_\infty = -15°C$

Table A3: $k_g = 1.4\ W/m \cdot k$

Model: $\overline{Nu}_L = .030\, Re_L^{0.8}\, Pr^{1/3}$

$Re_L = \frac{VL}{\nu}$

$k_{air} = .023\ W/mk$

$\nu = 12.5 \times 10^{-6}\ m^2/s$

$Pr = 0.71$

$T_{DP} = 10°C$

$T_{\infty,i} = 50°C$

$\bar{h}_i$ minimum = ?

$$R_{tot} = \frac{1}{h_i} + \frac{t}{k_g} + \frac{1}{h_o}$$

$$\frac{T_{i\infty} - T_{si}}{1/h_i} = \frac{T_i - T_{\infty,o}}{\frac{t}{kg} + \frac{1}{h_o}} \longrightarrow \frac{50-10}{1/\bar{h}_i} = \frac{10-(-15)}{\frac{.006}{1.4} + \frac{1}{h_o}}$$

$$Re_L = \frac{VL}{\nu} = \frac{31.29\,m/s\,(.8)}{12.5 \times 10^{-6}\,m^2/s} = 2.00 \times 10^6$$

$Pr = 0.71$

$\bar{h}_o = ?$ $\quad Nu = \frac{hl}{kf} = .030(Re_L)^{0.8}\, Pr^{(1/3)}$

$= 2940$

$\Rightarrow \bar{h}_o = 84.5$

$$40(\bar{h}_i) = \frac{25}{.00429 + .0118}$$

$$\boxed{\bar{h}_{i\,min} = 38.8\ W/m^2 \cdot K}$$

6.35

$u_\infty = 15\,m/s$

$T_\infty = 15°C$

$T_s = 140°C$

$A_s = .25\,m^2$

drag force 0.25 N

power needed = ?

$Pr = .707$

$$q_{in} = q_{out} = h A_s (T_s - T_\infty)$$

$$\bar{h}_o = \frac{C_f}{2} Pr^{2/3} \rho u_\infty c_p$$

$$\bar{h}_o = \frac{\tau_s \cancel{(2)}}{\cancel{\rho}(u_\infty^{\cancel{2}})} \frac{\cancel{\rho} \cancel{u_\infty} c_p\, Pr^{-2/3}}{\cancel{2}} = \frac{\tau_s}{u_\infty} c_p\, Pr^{-2/3}$$

$$\bar{h}_o = \frac{F_D\, c_p}{A\, u_\infty Pr^{2/3}} = \frac{0.25}{0.25} \frac{1009}{15} (0.707)^{-2/3}$$

$$\bar{h}_o = 84.8\ \frac{W}{m^2 k}$$

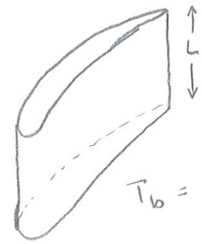
$$q = \frac{T - T_\infty}{\frac{1}{h}}(A) = 85(140-15)(0.25) = \boxed{2.66\,kW}$$

David Malawey 225 HW #4

100

3.116, 129, 134b, 145

adiabatic tip




3.116) $k = 20\ W/m\cdot K$ $T_\infty = 1200°C$

$L = 50\ mm$ $h = 250\ W/m^2\cdot K$

$A_c = 6\times10^{-4}\ m^2$ Perimeter $P = 110\ mm$ $T_{base} = 300°C$ find $T_{tip}$

$$\frac{\theta}{\theta_b} = \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L_c - x)}{\cosh mL_c} = \frac{1}{5.52} = .1811 \Rightarrow T = 1037°C$$

a) $M = \sqrt{hP/kA_c} = 47.87$

$\boxed{1037°C < 1050. \text{ Yes, cooling is satisfactory}}$

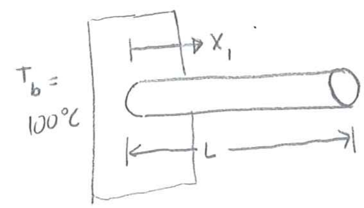
b) $q_f = M \tanh(mL_c) =$

$M = \sqrt{hPkA_c}\,\theta_b = -517.0$

$\boxed{q_f = -508W}$ ✓

3.129) $k_A = 200\ W/m\cdot k$ find $k_b$

$\left.\begin{matrix} T_A = 75°C \\ T_B = 60°C \end{matrix}\right\}$ at $x_1$




Temp. dist. $\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx_{,Al}} = 0.667$ $-mx_{,Al} = -.4055$

$e^{-mx_{,B}} = 0.533$ $-mx_{,B} = -.7621$

$$\frac{-.4055}{-.7621} = \frac{-m_{Al}}{-m_B} = \sqrt{\frac{k_B}{k_A}} \Rightarrow k_B = .2831\, k_A$$

$\boxed{k_B = 56.62\ W/m\cdot k}$ ✓

225 HW 4

134) b)

$T_{max} = 75°C$

4x4 array, Cu pins

$q_c = ?$

$T_c = 75°C$

$D_p = .0015 m$

$L_p = 15 mm$

Conditions, 3.27

$h_o = 1000 W/m^2 \cdot K$

$T_{\infty,o} = 20°C$

$r_{chip/board} = 10^{-4} m^2 \cdot K/W$

$L_{board} = 5 mm$

~~$k_b = 1 W/m \cdot K$~~

air $h_i = 40 W/m^2 \cdot K$ $T_{\infty,i} = 20°C$

$k_{Cu} = 400 W/m \cdot K$

$q = N \eta_f h_{finned} A_f \Theta_b + h_{unfinned} A_{unfinned} \Theta_b$

$(16)(.6768)(1000)(\cancel{1.77 \times 10^{-6}})(75-20)$
$7.25 \times 10^{-5}$

$= 43.2 W$

$+ (1000)(1.33 \times 10^{-4})(75-20) = 7.3 W$

$\boxed{q_{total} = 50.5 W}$

$\eta_f = \frac{\tanh mL_c}{mL_c} = .6768$

$m = \sqrt{hP/kA_c} = 81.65 m^{-1}$

$L_c = L + \frac{D}{4} = .01538$

$A_f = \frac{\pi}{4} D^2 (N)$

$A_{unfinned} = [(12.7 \times 10^{-3})^2 - 16 (\frac{\pi}{4}) .0015^2]$

$= 1.33 \times 10^{-4} m^2$

145) $(L_c)^{3/2} h/kA_p^{1/2} = 0.11$

$q_f = \eta_f h A_f \theta_b =$

$= 2\pi(.97)(40 W/m^2 \cdot K)[.042^2 - .025^2](180)$

a) $\boxed{q_f = 50 W}$

$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \boxed{11.05}$

b) $\bar{q} = N' q_f + h(1 - N't)(2\pi r_1)\theta_b$

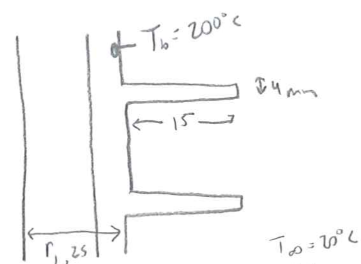
$\boxed{\bar{q} = 6820 W/m}$

$r_{2c} = r_2 + \frac{t}{2} = .042 m$

$r_{2c}/r_1 = 1.68$

$L_c = L + t/2 = .017 m$

$A_p = L_c t = 6.8 \times 10^{-5} m^2$




$K_{Al} = 240 W/m \cdot K$

100

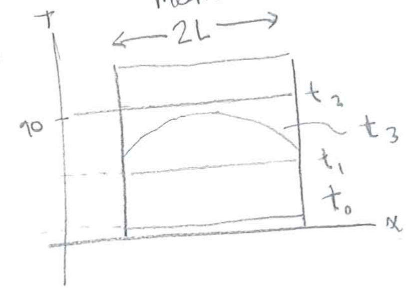
David Malawey 225 HW 5

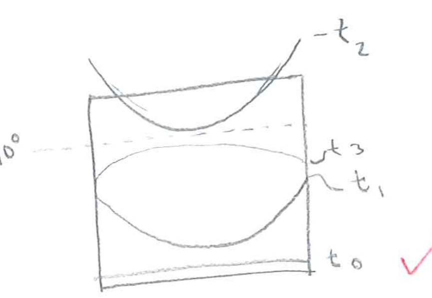
5

5.3, 11, 43(a,b) 57,

5.3)

meat.




— microwave —

— oven —

assuming constant specific heat

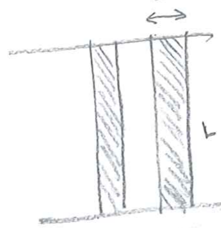
$$\boxed{T_{75\%} = 456°C}$$

5.11) $T_\infty = 600°C$

$h = 100\ W/m^2 \cdot K$

- Time to achieve 75% of capacity
- Temp of aluminum @ 75% ?

$W = .05m$



$L = \frac{W}{2} = .025m$

$k = 237\ W/m \cdot K$

$T_i = 25°C$

$\rho = 2702\ kg/m^3$

$C_p = 1033\ J/kg \cdot K$

$R_{conduction} = \frac{L}{kA} = \frac{.025}{237A}$

$R_{conv} = \frac{1}{hA} = \frac{1}{100A}$

$B_i = \frac{\frac{0.025}{237A}}{\frac{1}{100A}} = .011$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{\left(\frac{-hA_s t}{\rho V c_p}\right)}$$

$$t = \frac{\rho V c_p}{hA_s} \ln(0.75) = \frac{2702(1033)V}{100\ W/m \cdot K\ A} \ln(0.25)$$

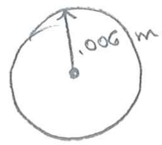
$$\frac{V}{A} = \frac{LkW}{\cancel{kW}} = 0.025m$$

$$\boxed{t = 968s}$$

$$\frac{456 - 600}{25 - 600} = 0.25$$

## 5.5)

sphere




$T_i = 1150$ K

$T_f = 400$ K

$T_\infty = 325$ K

$h = 20$ W/m²K

$\rho = 7800$ kg/m³

$c = 600$ J/kg·K

$k = 40$ W/m·K

find $Bi\# = \frac{hL_c}{k} = \frac{20 \frac{W}{m^2K}\left(\frac{.006\,m}{3}\right)}{40\ W/m\cdot K} = .001$

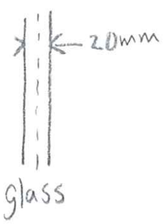
Time required for cooling?

$$t = \frac{\rho \forall c_p}{hA_s} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

$$= \frac{7800\left(\frac{\cancel{\pi}(.012)^{\cancel{3}}}{6}\right)600}{20\ \cancel{\pi}\ \cancel{(.012)^2}} \ln\left(\frac{1150-325}{400-325}\right)$$

$$\boxed{t = 1122 \text{ seconds}}$$

## 5.43)




diffusivity $\alpha = 6\times10^{-7}\ m^2/s$

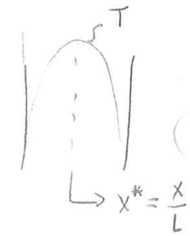
$T_i \to T_s$ suddenly

$L = .01$ m

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty\right]$$

$$Bi = \left.\frac{hL}{k}\right|_{h\to\infty} \to Bi^{-1} = 0$$

heisler charts, $\theta^* = \frac{T_0 - T_\infty}{T_i - T_s} = 0.5$ (to reach 50% of temp. reduction)




$$F_0 = 0.375 = \frac{\alpha t}{L^2} \Rightarrow t = \frac{.375(.01)^2}{6\times10^{-7}} \Rightarrow \boxed{t = 62.5\ s}$$

b) $T_i - T_s = 300°C$, find max temp gradient

$$\theta^* = C_1 e^{(-\zeta_1^2 F_0)} \cos(\zeta_1 x^*) \qquad \frac{\partial\theta^*}{\partial x^*} = C_1 e^{(-\zeta_1^2 F_0)} - \zeta_1 \sin(\zeta x^*)$$

$\zeta_1 =$ (append. B3) 1.5708 $\quad C_1 = 1.2733$ (table 5.1) $\quad x^* = 1 \quad = \left.\frac{\partial\theta}{\partial x^*}\right|_{x^*=1} = \boxed{-0.793}$

$$\frac{\partial\theta^*}{\partial x^*} = \frac{\partial\left(\frac{T-T_\infty}{T_i - T_\infty}\right)}{\partial\left(\frac{x}{L}\right)} = \frac{L}{T_i - T_\infty}\frac{\partial T}{\partial x} \Rightarrow \left.\frac{\partial T}{\partial x}\right|_{x=L} = \frac{T_i - T_\infty}{L}\left.\frac{\partial\theta^*}{\partial x^*}\right|_{x^*=1} = \frac{300}{0.01}(-.0793)$$

$$\frac{\partial T}{\partial x} = \boxed{-23{,}560\ \frac{°C}{m}}$$

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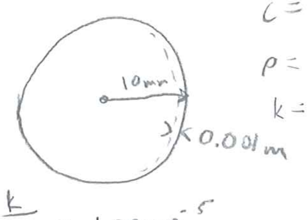
5.57




$c = 500\ J/kg \cdot k$

$\rho = 7800\ kg/m^3$

$k = 50\ W/m \cdot k$

$T_\infty = 1300k$

$h = 5000\ W/m^2$

$T_i = 300K$

$\alpha = \frac{k}{\rho h} = 1.28 \times 10^{-5}$

find t for outer 1 mm to reach 1000 k

$Bi^{-1} = \frac{k}{hr} = \frac{50}{5000(.01)} = 1.0$

$\frac{r}{r_o} = \frac{9}{10} = 0.90 \Rightarrow$ chart D.8 $\quad \frac{\theta}{\theta_o} = 0.475$

surface $\quad \frac{T - T_\infty}{T_o - T_\infty} = 0.6 = \frac{1000 - 1300}{T_o - 1300} \Rightarrow T_o = 800 \quad \theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = \frac{800-1300}{300-1300} = 0.5$

Chart: $Bi^{-1} = 1.0 \quad \theta_o^* = 0.5 \quad F_o = 0.4 \Rightarrow t = \frac{F_o r_o^2}{\alpha} = \frac{.4(.01)^2}{1.28 \times 10^{-5}} = \boxed{3.12\ sec}$

① find Bi, $Bi^{-1}$

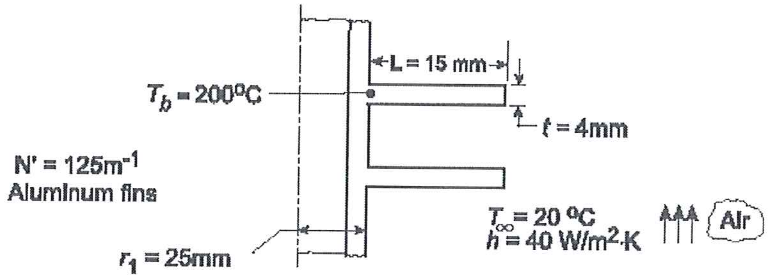
② use chart for surface to find $\frac{\theta}{\theta_o} = 0.6$

## PROBLEM 3.145

**KNOWN:** Dimensions and base temperature of aluminum fins of rectangular profile. Ambient air conditions.

**FIND:** (a) Fin efficiency and effectiveness, (b) Rate of heat transfer per unit length of tube.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional radial conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) Negligible base contact resistance, (6) Uniform convection coefficient.

**PROPERTIES:** *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

**ANALYSIS:** (a) The fin parameters for use with Figure 3.19 are

$$r_{2c} = r_2 + t/2 = 40\text{ mm} + 2\text{ mm} = 0.042\text{ m} \qquad L_c = L + t/2 = 15\text{ mm} + 2\text{ mm} = 0.017\text{ m}$$

$$r_{2c}/r_1 = 0.042\text{ m}/0.025\text{ m} = 1.68 \qquad A_p = L_c t = 0.017\text{ m} \times 0.004\text{ m} = 6.8 \times 10^{-5}\text{ m}^2$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.017\text{ m})^{3/2}\left[40\text{ W/m}^2\cdot\text{K}/240\text{ W/m}\cdot\text{K} \times 6.8 \times 10^{-5}\text{ m}^2\right]^{1/2} = 0.11$$

The fin efficiency is $\eta_f \approx 0.97$. From Eq. 3.86,

$$q_f = \eta_f q_{max} = \eta_f h A_{f(ann)} \theta_b = 2\pi\eta_f h\left(r_{2c}^2 - r_1^2\right)\theta_b$$

$$q_f = 2\pi \times 0.97 \times 40\text{ W/m}^2\cdot\text{K}\left[(0.042)^2 - (0.025)^2\right]\text{m}^2 \times 180^\circ\text{C} = 50\text{ W} \qquad <$$

From Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{50\text{ W}}{40\text{ W/m}^2\cdot\text{K}\; 2\pi(0.025\text{ m})(0.004\text{ m})180^\circ\text{C}} = 11.05 \qquad <$$

(b) The rate of heat transfer per unit length is

$$q' = N'q_f + h(1 - N't)(2\pi r_1)\theta_b$$

$$q' = 125 \times 50\text{ W/m} + 40\text{ W/m}^2\cdot\text{K}(1 - 125 \times 0.004)(2\pi \times 0.025\text{ m}) \times 180^\circ\text{C}$$

$$q' = (6250 + 565)\text{ W/m} = 6.82\text{ kW/m} \qquad <$$

**COMMENTS:** Note the dominant contribution made by the fins to the total heat transfer.

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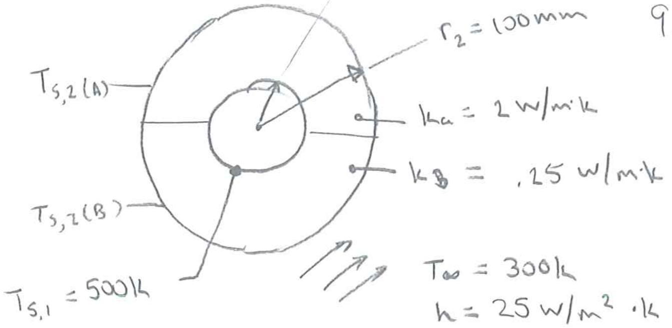
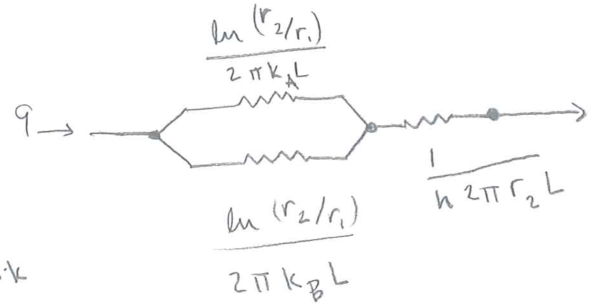
100

3.52)

$r_1 = 50\,mm$




$\ln\frac{100}{50} = .6931$ $\qquad \dot{Q}_r = \frac{\Delta T}{R_{tot}}$

$$\frac{1}{R_{AB}} = \frac{\cancel{2}\pi k_A L}{\ln(r_2/r_1)} + \frac{\cancel{2}\pi k_B L}{\ln(r_2/r_1)}$$

$$\frac{1}{R_{AB}} = \frac{\cancel{2}\pi L(k_A + k_B)}{\ln(r_2/r_1)} \Rightarrow R_{AB} = \frac{\ln(r_2/r_1)}{\cancel{2}\pi L(k_A+k_B)}$$

$$R_{total} = \frac{\ln(r_2/r_1)}{\cancel{2}\pi L(k_A+k_B)} + \frac{1}{h 2\pi r_2 L} = .1617$$

b)

$$\dot{Q} = \frac{500-300}{.1617} = \boxed{1236 \text{ Watts/meter}}$$

find $T_{s_{2,A}}$ $T_{s_{2,B}}$ *

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

above calculations assume uniform temperature outer surface.
- not realistic.

$T(r=0) = T_s$

$T(r=\infty) = 300K$

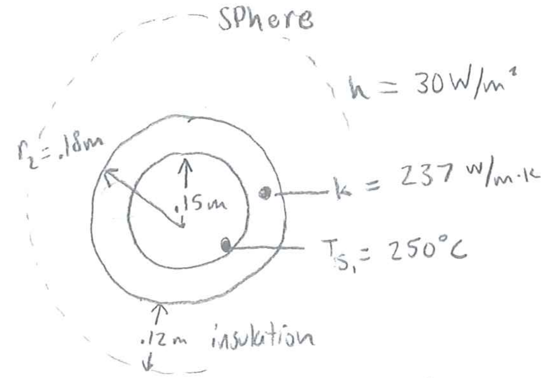
$q_{\text{outside surface}} = \dot{q}_{\text{inside}}$

$$q = \frac{T_s - T_\infty}{R} = h2\pi r L(T_s - T_\infty) = 2\pi(.100)(25\,W/m^2)(T_s - 300) = 1236\,W/m$$

$$\boxed{T_s = 379\text{ K}}$$

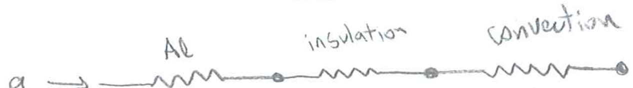
225 HW 3)

3.57)




80 W dissipated
find $k_{insulation}$

$$q = \frac{\Delta T}{R_{eq}}$$




$$\frac{1}{4\pi k}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \qquad \frac{1}{4\pi k}\left(\frac{1}{r_2}-\frac{1}{r_3}\right) \qquad \frac{1}{h\,4\pi r^2}$$

$R_{Al} = 3.731\times10^{-4}$

$R_{ins.} = \frac{1}{k}(.1768)$

$R_{air} = .0295$

$$80\,W = \frac{250-20}{.0295 + 3.731\times10^{-4} + \frac{.1768}{k}}$$

$$\frac{14.14}{k} = 227.61 \Rightarrow \boxed{k = .0621}\ W/mK$$

3.71)

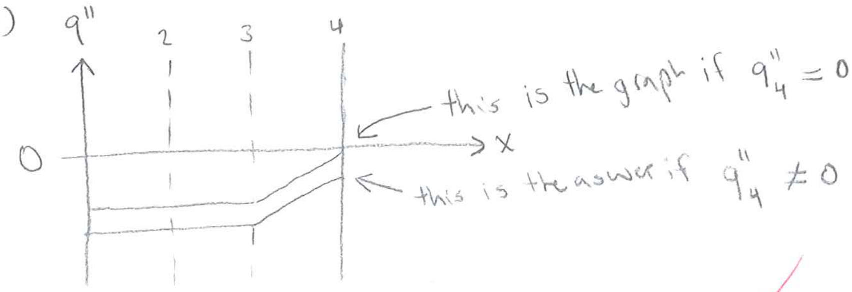
a) $q''_2 = q''_3$ because temp gradients are linear

$q''_3 > q''_4$ because the curved temp indicates heat generation in layer "c"

b) $k_A < k_B$ because slope of temp grad. for A is larger

$k_B > k_C$ because slope of temp gradiant is less but $q''$ is equal

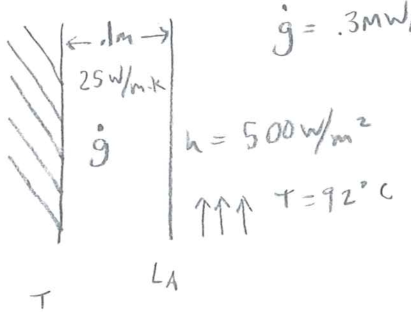
c)



David Malawey 225 HW3

3.72)

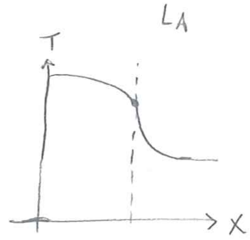
$\dot{g} = .3 MW/m^3$




determine $T_{max}$ in wall

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) = -\dot{g}$$

$$k \frac{d^2T}{dx^2} = -\dot{g} \qquad \frac{d^2T}{dx^2} = -\frac{\dot{g}}{k}$$




$$T = \frac{-\dot{g}}{2k} x^2 + C_1 x + C_2$$

B.C.'s

x(0) $\quad \frac{dt}{dx} = 0 = \frac{-\dot{g}}{k} x + C_1 \Rightarrow C_1 = 0$

at surface $\quad -\dot{q} = \frac{T_\infty - T_L}{R} \qquad \dot{q} R - T_\infty = -T_L \qquad R = \frac{L}{h} \qquad T_L = 92 + \frac{3\times10^6 (0.1)}{500} =$

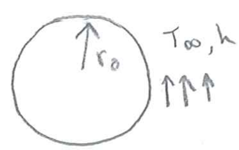
$$T_L = 152°C = \frac{-\dot{g}}{2k} x^2 + C_1 = \frac{-.3\times10^6}{2(25)} (.1)^2 + (C_1)$$

$$152°C + 60 = C_1 = 212 \qquad \boxed{T_0 = 212°C}$$

3.94)

$\dot{g} = \dot{g}_0 [1 - (r/r_0)^2]$

↑ local gen. rate ↑ const ↑ container radius




Find T(r)

use eqn on slide 18 ✗ No.

3.94)

$$\dot g = \dot g_0\left[1-\left(\frac{r}{r_0}\right)^2\right] \quad \text{AND} \quad \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot g}{k}=0$$

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{\dot g_0}{k}\left[1-\left(\frac{r}{r_0}\right)^2\right]r^2$$ INTEGRATE

$$r^2\frac{dT}{dr} = \int\left[-\frac{\dot g_0 r^2}{k}+\frac{\dot g_0 r^4}{k r_0}\right] = -\frac{\dot g_0 r^3}{3k}+\frac{\dot g_0 r^5}{5 k r_0}+C_1$$ DIVIDE BY $r^2$. INTEGRATE

$$T = \int\left[\frac{-\dot g_0 r}{3k}+\frac{\dot g_0 r^3}{5 k r_0}+\frac{C_1}{r^2}\right] = \frac{-\dot g_0 r^2}{6k}+\frac{\dot g_0 r^4}{20 k r_0}-\frac{C_1}{r}+C_2$$

BOUNDARY CONDITIONS

① $$\left.\frac{dT}{dr}\right|_{r=0}=0$$

$$0 = \frac{-\dot g_0 r}{3k}+\frac{\dot g_0 r^3}{5 k r_0}+C_1 \Rightarrow C_1=0$$

② $$\left.-k\frac{dT}{dr}\right|_{r=r_0} = h\left[T(r_0)-T_\infty\right]$$

$$h\left[T(r_0)-T_\infty\right] = \frac{+\cancel{k}\dot g_0 r_0}{3\cancel{k}} - \frac{\dot g_0 r_0^{\cancel{3}\,2}\cancel{k}}{5\cancel{k}\cancel{r_0}}+C_0 = \frac{\dot g_0 r_0}{3}-\frac{\dot g_0 r_0^2}{5}$$

$$\dot g_0\left[\frac{r_0}{3}-\frac{r_0^2}{5}\right] = h\left[-\frac{\dot g}{k}\left(\frac{r_0^2}{6}-\frac{r_0^2}{20}\right)+C_2-T_\infty\right]$$ PLUG IN $T(r)$ & $r_0$ for $r$'s

$$\Rightarrow C_2 = \frac{2 r_0 \dot g}{15h}+\frac{7\dot g_0 r_0^2}{60k}+T_\infty$$

$$\boxed{T(r) = T_\infty + \frac{2 r_0 \dot g_0}{15h} + \frac{\dot g r_0^2}{k}\left[\frac{7}{60}-\frac{1}{6}\left(\frac{r}{r_0}\right)^2+\frac{1}{20}\left(\frac{r}{r_0}\right)^4\right]}$$

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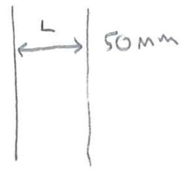
95

225 HW #2

2.23, 25, 46  3.2, 3.15




2.23) therm. conductivity $k = \frac{50W}{m \cdot K}$

$T = a + bx^2$  $a = 200°C$  $b = -2000°C/m^2$

x(meters)

a) $\dot{q} = ?$  heat eq  $\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \cancel{\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)} + \cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)} + \dot{g} = \cancel{\rho C_p \frac{\partial T}{\partial t}}$

$\dot{g} = -\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)$

$\dot{g} = -\frac{\partial}{\partial x}(2bkx) = -2bk$

$\dot{g}(L) - \dot{g}(0) = -2bk(L) + 2bk(0) = -2\left(-2000\frac{°C}{m^2}\right)\left(\frac{50W}{m \cdot K}\right)(.05m)$

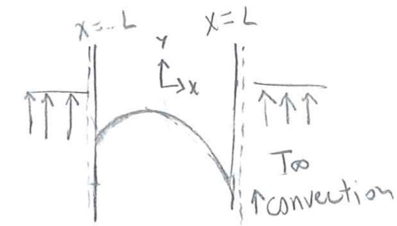
$\boxed{\dot{q} = 10\frac{kW}{m^2}}$

2.25) $2L = 40mm$ (thickness)

$k = 5 W/m \cdot K$

$\dot{q}$ volumetric heat gen rate

a)




$T_x = a + bx + cx^2$  $a = 82.0°C$

x-meters  $b = -210°C/m$

T - °C  $c = -2\times10^4\ °C/m^2$

What is $\dot{q}$ in wall?

$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{g} = 0$

$\frac{\partial}{\partial x}(kb + 2kCx) + \dot{g} = 0$

b) $\dot{g} = -2kc = -2\frac{5W}{m \cdot K}\left(-2\times10^4\frac{°C}{m^2}\right) = \boxed{2\times10^5\frac{W}{m^3}}$

c) $q''_x(x) = -k\left(\frac{dT}{dx}\right) = -kb - 2kCx = -[5(-210) + 2(5)(-2\times10^4)(.02m)]$

$\boxed{q''_x(L) = +5050\ W/m^2 \quad q''_x(-L) = -2950\ W/m^2}$

$T(-L) = 82 - 210(-.02) - 20,000(-.02)^2 = 78.2$

$T(L) = 82 - 210(.02) - 20,000(.02)^2 = 69.8$

d) convection coefficients for surfaces?

$q''_{conv,e} = q''_x(-L) =$

$h_1[T_\infty - T(-L)] = h_1(20 - 78.2)K = -2950\ W/m^2$  $\boxed{h_1 = 51\frac{W}{m^2 \cdot K}}$

$h_1[T(+L) - T_\infty] = h_R(69.8 - 20)K = 5050\frac{W}{m^2}$  $\boxed{h_R = 101\frac{W}{m^2 \cdot K}}$

2.25 continued

e) heat flux distribution $q''_x(x)$

$$q''_x(x) = -k\left(\frac{\partial T}{\partial x}\right) = -k(b+2cx) = -5(-210+2(-2\times10^4)x)$$
$$= 1050 + 2\times10^5 x$$

$$-1050 = 2(10^5)x \qquad x = \frac{-1050}{2(10^5)} \qquad \boxed{\text{flux} = 0 \ @ \ x = -5.25\text{ mm}}$$ $t_{max}$ is here

f) $$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \cancel{\dot{q}}^{\,0} = \rho C_p \frac{\partial T}{\partial t}$$

$$k(2c) = 5\,\frac{W}{m\cdot K}\left(-2(10^4)\,\frac{°C}{m^2}\right) = \boxed{-2\times10^5\,\frac{W}{m^3}}$$

g)

$\rho = 2600\,kg/m^3$ wall will eventually reach $\boxed{20°C}$

$C_p = 800\,\frac{J}{kg\cdot K}$

$$\cancel{E''_{in}}^{\,0} - E''_{out} = \Delta E''_{st} = E''_f - E''_i$$

$$E''_{out} = \rho C_p \int_{-L}^{L} (T_i - T_\infty)\,dx$$
$$= \rho C_p \int_{-L}^{L} [a + bx + cx^2 - T_\infty]\,dx = \rho C_p\left[ax + \frac{bx^2}{2} + \frac{cx^3}{3} - T_\infty x\right]_{-L}^{L}$$
$$= \rho C_p\left[aL(2) + \frac{2cL^3}{3} + -2T_\infty L\right]$$
$$= 2(2600)\,\frac{kg}{m^3}\,800\,\frac{J}{kg\,K}\left[82°C(.02\,m) + \frac{-2\times10^4\,\frac{°C}{m^2}(.02)^3}{3} - 20°C(.02\,m)\right]$$
$$= \boxed{4.94\times10^6\ J/m^2}$$

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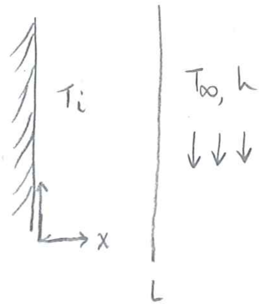
2.46




a) $$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) = \rho c_p \frac{\partial T}{\partial t}$$

$$k \frac{\partial^2 T}{\partial x^2} = \rho c_p \frac{\partial T}{\partial t}$$

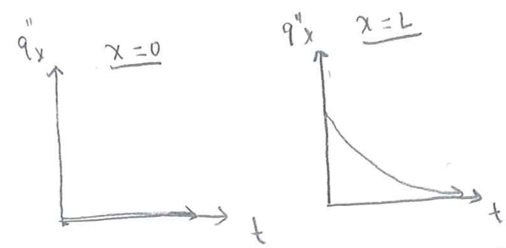
initial conditions $t \le 0, \quad T_x = T_i$

boundary cond. at $x=0, \quad q''_x(0) = 0 \quad \frac{\partial T}{\partial x} = 0$

at $x = L \quad -k\frac{\partial T}{\partial x} = h[T(L,t) - T_\infty]$

b)


c)




d) Total energy transferred to the wall (J/m³)

$$E_{in} = \int_0^\infty q''_{conv} A_s \, dt = h A_s \int_0^\infty (T_\infty - T(L,t))\, dt$$

$$\boxed{\frac{E_{in}}{V} = \frac{h}{L}\int_0^\infty (T_\infty - T(L,t))\, dt}$$

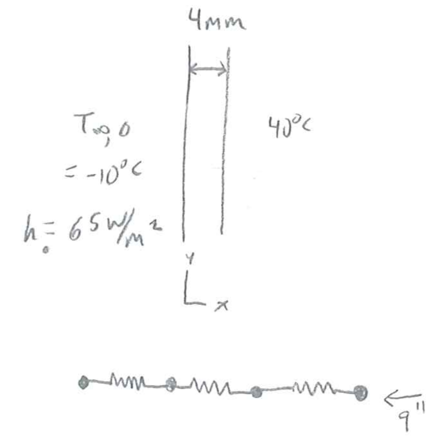
3.2)

$T_{\infty,i} = 40°C$

$h_i = 30\, W/m^2 \cdot K$

find T of glass surfaces

$k_{glass} = 1.4\, W/m \cdot k$

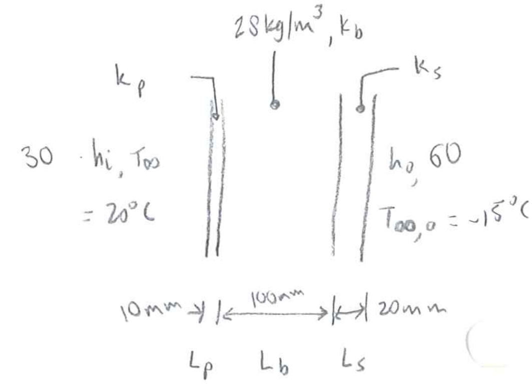



$$q'' = \frac{T_{\infty,i} - T_{\infty,0}}{\frac{1}{h_o} + \frac{L}{h} + \frac{1}{h_i}} = \frac{50°C}{\frac{1}{65} + \frac{.004}{30} + \frac{1}{30}} = 969\, \frac{W}{m^2}$$

$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40 - \frac{969}{30} = \boxed{7.7°C} \text{ inside}$$

$$T_{s,0} = T_{\infty,0} + \frac{q''}{h_o} = -10 + \frac{969}{65} = \boxed{4.9°C} \text{ outside}$$

b)? −5




3.13) a)

$$\boxed{R_{tot} = \Sigma R_i = \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_i A} + \frac{1}{h_o A}}$$

b)

$$q = \frac{20-(-15)}{R_{tot}} = \frac{35}{831 \times 10^{-5}} = \boxed{4.21\, kW}$$

c)

$R_{tot} \Rightarrow 826 \times 10^{-5} \Rightarrow \boxed{q = \uparrow 0.6\%}$ with wind blowing

d) glass fiber blanket is the controlling resistance, $\frac{L_b}{k_b A} = \frac{.1}{.038} = 2.631$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{g}_o\left(1-\left(\frac{r}{r_o}\right)^2\right)}{k}$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=\frac{-rg_o}{k}+\frac{rg_o}{k}\left(\frac{r^2}{r_o^2}\right)$$ integrate

$$r\frac{dT}{dr}=\frac{-r^2}{2}\frac{g_o}{k}+\frac{g_o}{kr_o^2}\left(\frac{r^4}{4}\right)+C_1$$ divide by r & integrate

$$T(r)=\frac{-r^2}{4}\frac{g_o}{k}+\frac{g_o}{kr_o^2}\frac{r^4}{16}+C_1\ln r+C_2$$

B.C.'s: ① $\left.\frac{dT}{dr}\right|_{r=0}=0$

$$\frac{-r}{2}\frac{g_o}{k}+\frac{g_o}{kr_o^2}\left(\frac{r^3}{4}\right)+\frac{C_1}{r}=0 \quad \Rightarrow C_1=0$$

② $T_{(r=r_o)}=T_s$

$$-\frac{r^2}{4}\frac{g_o}{k}+\frac{g_o r^4}{kr_o^2 16}+C_2=T_s$$

$$\frac{-r_o^2 g_o}{4k}+\frac{g_o r_o^2}{k16}+C_2=T_s \Rightarrow C_2=T_s+\left(\frac{3}{16}\right)\frac{g_o r_o^2}{k}$$

$$\boxed{T(r)=\frac{r^4 g_o}{16kr_o^2}-\frac{r^2 g_o}{4k}+\left(\frac{3}{16}\right)\frac{g_o r_o^2}{k}+T_s}$$

mesh 1 $\quad 150 + 20i_1 + 10(i_1 - i_2) = 0$

mesh 2: $\quad 15i_2 - 100 + 10(i_2 - i_1) = 0$

$25i_2 - 10i_1 = 100$

$2.5i_2 = 10 + i_1$

---

heat flux $q''_x$ is the heat transfer rate in x-direction $W/m^2$
per unit area perpendicular to direction of transfer

$q'_x$ = heat transfer rate per unit length $W/m$

$\dot{q}$ = rate of energy generation per unit volume $W/m^3$

---

Pg 133

OK

repeated integration correctly

$$\cancel{\int \left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)\right] dr \rightarrow r\left(\frac{dT}{dr}\right)}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

$$\int \frac{d}{dr}\left(r\frac{dT}{dr}\right) = \int -\frac{\dot{q}r}{k}$$

$$r\frac{dT}{dr} = -\frac{\dot{q}r^2}{2} + C_1$$

$$\int \left(\frac{dT}{dr}\right) = \int\left(-\frac{\dot{q}r}{2} + \frac{C_1}{r}\right.$$

$$T(r) = -\frac{\dot{q}r^2}{4} + C_1 \ln r + C_2$$

$$\iint -\dot{q}r = \int -\frac{\dot{q}r^2}{2} + C = -\frac{\dot{q}r^3}{6} + C_1 r + C_2$$

$$\frac{1}{r}\left(-\frac{\dot{q}r^3}{6} + C_1 r + C_2\right)$$

$$= -\frac{\dot{q}r^2}{6} + C_1 + \frac{C_2}{r}$$

225 math prob.

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)+\dot{g}=0$$

$u=\frac{1}{r}$ $\quad dv=\frac{d}{dr}\left(kr\frac{dT}{dr}\right)$

$du=-r^{-2}$ $\quad v=kr\left(\frac{dT}{dr}\right)$

$$\int = uv-\int v\,du = \frac{1}{r}(kr)\left(\frac{dT}{dr}\right)-\underbrace{\int \frac{kr\left(\frac{dT}{dr}\right)}{r^2}\,dr}_{①}$$

① $\displaystyle\int \frac{kr}{r^2}\left(\frac{dT}{dr}\right)dr = k\int \frac{1}{r}\left(\frac{dT}{dr}\right)dr$

$u=\frac{1}{r}$ $\quad dv=\left(\frac{dT}{dr}\right)dr$

$du=-r^{-2}$ $\quad v=T(r)$ $\quad = \left(\frac{1}{r}\right)T(r)-\underbrace{\int\frac{T(r)}{-r^2}\,dr}_{②}$

② $\displaystyle\int \frac{T(r)}{-r^2}\,dr = \frac{T}{r}+c$

$$\Rightarrow \frac{1}{r}(kr)\left(\frac{dT}{dr}\right)-\left[\left(\frac{1}{r}\right)T-\left(\frac{T}{r}+c\right)\right]$$

$$= k\frac{dT}{dr}+\cancel{\frac{T}{r}}-\cancel{\frac{T}{r}}+c$$

$$= k\frac{dT}{dr}+q'_r+c$$

100

David Malawey

225 HW #1

P. 1.6, 22, 24, 55, 57

1.6

W = 1m $\quad T_{in} = 15°C \quad \dot{Q}_{out} = ?$

H = 2m $\quad T_{out} = -20°C$

5mm thick $\quad k_g = 1.4 W/m \cdot K$

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{(-20-15)}{.005 m} \qquad q''_x = -k\frac{dT}{dx} = -1.4 \frac{W}{m \cdot K}\left(\frac{-35 K}{.005 m}\right)$$

$$q''_x = 9800 \ W/m^2 \qquad q_x = (9800)(2m^2) = 19600 W \quad \boxed{19.6 kW}$$

10 mm air $\quad 10°C, -15°C \quad 1\times2$ m window $\quad k_a = .024 \frac{W}{m \cdot K}$

$$q_x = (-.024)\frac{(-15-10)}{(.01)}(2m^2) = \boxed{120 W}$$

1.22

Plate $T = 225°C \quad \frac{dT}{dt} = -\frac{.022K}{s} \quad T_{air} = 25°C \quad$ plate .3x.3m

W = 3.75 kg

find convection coefficient h ($W/m^2 \cdot K$) $\quad c_p = 2770 \frac{J}{kg \cdot K}$

$$\dot{Q} = c_p \frac{\Delta T}{dt} = \frac{-.022K}{s}(2770) = -60.9 \frac{J}{kg \cdot s}(3.75 kg)$$

$$\dot{Q} = -228.5 \ W$$

$$q'' = h(T_s - T_\infty) \qquad T_s = 225°C \quad T_\infty = 25°C$$

$$h = \frac{-228.5 W}{2\times(.3)^2 m^2}\frac{1}{(200 K)} = \boxed{6.35 \frac{W}{m^2 \cdot K}} = h$$

1.24) $T_{air} = 20°C$ $T_{walls}$ 27°C summer, 14°C winter, $T_{person} = 32°C$

$h = 2 \frac{W}{m^2 \cdot K}$

$q''_{rad} = \varepsilon\sigma(T_s^4 - T_{surr}^4)$ $q''_{summer} = (.9)(5.67\times10^{-8}) \frac{W}{m^2K^4}(305^4 - 300^4) = 28.3\ W/m^2$

$q''_{winter} = (.9)(5.67\times10^{-8})\frac{W}{m^2K^4}(305^4 - 287^4) = 95.4\ W/m^2$

heat is lost from the body at 3.4 times faster in the winter

1.55) absorb from 800 $W/m^2$ $h = 12\ W/m^2 \cdot K$ $\dot{E}_{in} = \dot{E}_{out}$

a) $T_{air} = 20°C$ $T_{roof} = ?$ (steady state) $q''_{rad} - \cancel{q''_{cond}} - q''_{conv} = 0$

$800 \frac{W}{m^2} - h(T_s - 20°C) = 0$

$800 = 12(T_{roof} - 20)$ $\boxed{T_{roof} = 86°C}$

b) surface emissivity $= 0.8$

$q''_{s,abs} A_s - q_{conv} - E \cdot A_s = 0$

$q''_{s,abs} A_s - hA_s(T_s - T_\infty) - \varepsilon A_s \sigma T_s^4 = 0$ $800 \frac{W}{m^2} - 12 \frac{W}{m^2 K}(T_s - 293K) - 0.8(5.67\times10^{-8}) = 0$

$12T_s + 4.536\times10^{-8} T_s^4 = 4316$ $\boxed{T_s = 320K = 47°C}$

c) ?

1.57) $A = 1.8m^2$ 2100kcal 2000k cal thermal energy $T_{air} = 20°C$ $h = 3 \frac{W}{m^2K}$

a) $T_{skin} = ?$ $\varepsilon = 0.95$ $\frac{2000\times10^3\ cal}{0.239\ \frac{cal}{J}(86{,}400\ s/day)} = 96.9\ W$ $\dot{E}_{in} = \dot{E}_{out}$

$96.9W = hA(T_s - T_\infty) + \varepsilon\sigma A(T_s^4 - T_{sur}^4) = 3(1.8)(T_s - 293) + .95(5.67\times10^{-8})(1.8)(T_s^4 - 293^4)$

$\boxed{T_s = 299k = 26°C}$

b) $\dot{E}_{out} = \dot{m} h_{fg}$ $\dot{m} = \frac{\dot{E}_{out}}{h_{fg}} = \frac{96.9W}{2421 kJ/kg} = \boxed{4.0\times10^{-5}\ kg/s}$

$= \boxed{4\times10^{-5}\ l/s}$

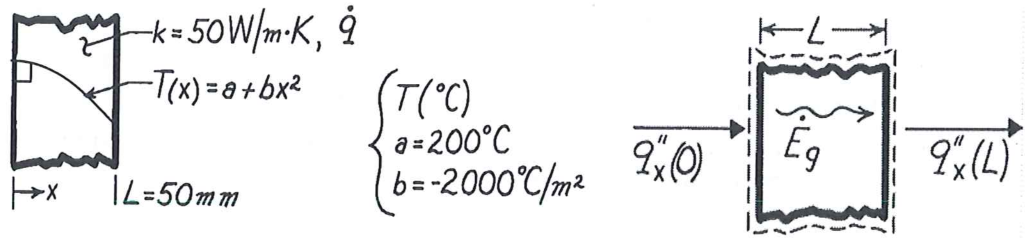
HW 2 Solutions

## PROBLEM 2.23

**KNOWN:** Temperature distribution in a one-dimensional wall with prescribed thickness and thermal conductivity.

**FIND:** (a) The heat generation rate, $\dot{q}$, in the wall, (b) Heat fluxes at the wall faces and relation to $\dot{q}$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

**ANALYSIS:** (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.19 re-written as

$$\dot{q}=-k\frac{d}{dx}\left[\frac{dT}{dx}\right]$$

Substituting the prescribed temperature distribution,

$$\dot{q}=-k\frac{d}{dx}\left[\frac{d}{dx}\left(a+bx^2\right)\right]=-k\frac{d}{dx}\left[2bx\right]=-2bk$$

$$\dot{q}=-2\left(-2000^\circ C/m^2\right)\times 50\ W/m\cdot K=2.0\times 10^5\ W/m^3. \qquad <$$

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x(x)=-k\left.\frac{dT}{dx}\right]_x .$$

Using the temperature distribution T(x) to evaluate the gradient, find

$$q''_x(x)=-k\frac{d}{dx}\left[a+bx^2\right]=-2kbx.$$

The fluxes at x = 0 and x = L are then

$$q''_x(0)=0 \qquad <$$

$$q''_x(L)=-2kbL=-2\times 50 W/m\cdot K\left(-2000^\circ C/m^2\right)\times 0.050m$$

$$q''_x(L)=10,000\ W/m^2. \qquad <$$

**COMMENTS:** From an overall energy balance on the wall, it follows that, for a unit area,

$$\dot{E}_{in}-\dot{E}_{out}+\dot{E}_g=0 \qquad q''_x(0)-q''_x(L)+\dot{q}L=0$$

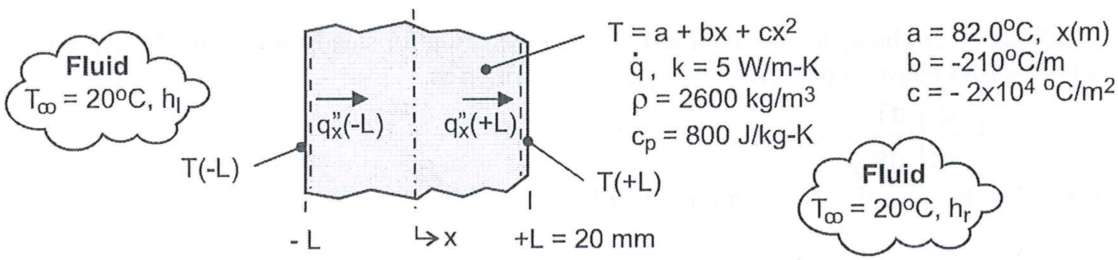
$$\dot{q}=\frac{q''_x(L)-q''_x(0)}{L}=\frac{10,000\ W/m^2-0}{0.050m}=2.0\times 10^5\ W/m^3.$$

## PROBLEM 2.25

**KNOWN:** Analytical expression for the steady-state temperature distribution of a plane wall experiencing uniform volumetric heat generation $\dot{q}$ while convection occurs at both of its surfaces.

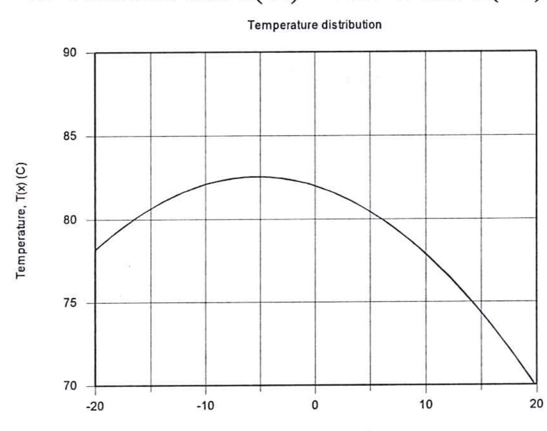
**FIND:** (a) Sketch the temperature distribution, T(x), and identify significant physical features, (b) Determine $\dot{q}$, (c) Determine the surface heat fluxes, $q''_x(-L)$ and $q''_x(+L)$; how are these fluxes related to the generation rate; (d) Calculate the convection coefficients at the surfaces x = L and x = +L, (e) Obtain an expression for the heat flux distribution, $q''_x(x)$; explain significant features of the distribution; (f) If the source of heat generation is suddenly deactivated ($\dot{q}$ = 0), what is the rate of change of energy stored at this instant; (g) Determine the temperature that the wall will reach eventually with $\dot{q}$ = 0; determine the energy that must be removed by the fluid per unit area of the wall to reach this state.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) Uniform volumetric heat generation, (3) Constant properties.

**ANALYSIS:** (a) Using the analytical expression in the Workspace of IHT, the temperature distribution appears as shown below. The significant features include (1) parabolic shape, (2) maximum does not occur at the mid-plane, T(-5.25 mm) = 83.3°C, (3) the gradient at the x = +L surface is greater than at x = -L. Find also that T(-L) = 78.2°C and T(+L) = 69.8°C for use in part (d).




(b) Substituting the temperature distribution expression into the appropriate form of the heat diffusion equation,E q. 2.19, the rate of volumetric heat generation can be determined.

$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad \text{where} \qquad T(x)=a+bx+cx^2$$

$$\frac{d}{dx}(0+b+2cx)+\frac{\dot{q}}{k}=(0+2c)+\frac{\dot{q}}{k}=0$$

Continued .....

$$\dot{q} = -2ck = -2\left(-2\times10^4\,{}^\circ\mathrm{C/m^2}\right)5\ \mathrm{W/m\cdot K} = 2\times10^5\ \mathrm{W/m^3}$$ <

(c) The heat fluxes at the two boundaries can be determined using Fourier's law and the temperature distribution expression.

$$q''_x(x) = -k\frac{dT}{dx} \qquad \text{where} \qquad T(x) = a + bx + cx^2$$

$$q''_x(-L) = -k\left[0 + b + 2cx\right]_{x=-L} = -\left[b - 2cL\right]k$$

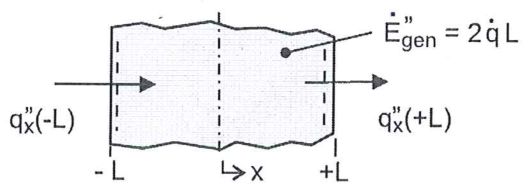
$$q''_x(-L) = -\left[-210^\circ\mathrm{C/m} - 2\left(-2\times10^4\,{}^\circ\mathrm{C/m^2}\right)0.020\mathrm{m}\right]\times 5\ \mathrm{W/m\cdot K} = -2950\ \mathrm{W/m^2}$$ <

$$q''_x(+L) = -(b + 2cL)k = +5050\ \mathrm{W/m^2}$$ <

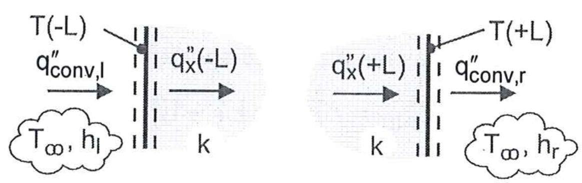
From an overall energy balance on the wall as shown in the sketch below, $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$,

$$+q''_x(-L) - q''_x(+L) + 2\dot{q}L \overset{?}{=} 0 \quad \text{or} \quad -2950\ \mathrm{W/m^2} - 5050\ \mathrm{W/m^2} + 8000\ \mathrm{W/m^2} = 0$$

where $2\dot{q}L = 2\times2\times10^5\ \mathrm{W/m^3}\times0.020\ \mathrm{m} = 8000\ \mathrm{W/m^2}$, so the equality is satisfied




Part (c) Overall energy balance




Part (d) Surface energy balances

(d) The convection coefficients, $h_l$ and $h_r$, for the left- and right-hand boundaries (x = -L and x= +L, respectively), can be determined from the convection heat fluxes that are equal to the conduction fluxes at the boundaries. See the surface energy balances in the sketch above. See also part (a) result for T(-L) and T(+L).

$$q''_{conv,\ell} = q''_x(-L)$$

$$h_l\left[T_\infty - T(-L)\right] = h_l\left[20 - 78.2\right]\mathrm{K} = -2950\ \mathrm{W/m^2} \qquad h_l = 51\ \mathrm{W/m^2\cdot K}$$ <

$$q''_{conv,r} = q''_x(+L)$$

$$h_r\left[T(+L) - T_\infty\right] = h_r\left[69.8 - 20\right]\mathrm{K} = +5050\ \mathrm{W/m^2} \qquad h_r = 101\ \mathrm{W/m^2\cdot K}$$ <

(e) The expression for the heat flux distribution can be obtained from Fourier's law with the temperature distribution

$$q''_x(x) = -k\frac{dT}{dx} = -k\left[0 + b + 2cx\right]$$

$$q''_x(x) = -5\ \mathrm{W/m\cdot K}\left[-210^\circ\mathrm{C/m} + 2\left(-2\times10^4\,{}^\circ\mathrm{C/m^2}\right)\right]x = 1050 + 2\times10^5 x$$ <

Continued .....

The distribution is linear with the x-coordinate. The maximum temperature will occur at the location where $q''_x(x_{max}) = 0$,

$$x_{max} = -\frac{1050\ \mathrm{W/m^2}}{2\times10^5\ \mathrm{W/m^3}} = -5.25\times10^{-3}\,\mathrm{m} = -5.25\,\mathrm{mm} \qquad <$$

(f) If the source of the heat generation is suddenly deactivated so that $\dot{q} = 0$, the appropriate form of the heat diffusion equation for the ensuing transient conduction is

$$k\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \rho c_p \frac{\partial T}{\partial t}$$

At the instant this occurs, the temperature distribution is still $T(x) = a + bx + cx^2$. The right-hand term represents the rate of energy storage per unit volume,

$$\dot{E}''_{st} = k\frac{\partial}{\partial x}\left[0 + b + 2cx\right] = k\left[0 + 2c\right] = 5\ \mathrm{W/m\cdot K}\times 2\left(-2\times10^4\,°\mathrm{C/m^2}\right) = -2\times10^5\ \mathrm{W/m^3} \qquad <$$

(g) With no heat generation, the wall will eventually ($t \to \infty$) come to equilibrium with the fluid, $T(x,\infty) = T_\infty = 20°C$. To determine the energy that must be removed from the wall to reach this state, apply the conservation of energy requirement over an interval basis, Eq. 1.11b. The "initial" state is that corresponding to the steady-state temperature distribution, $T_i$, and the "final" state has $T_f = 20°C$. We've used $T_\infty$ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \qquad E''_{in} = 0.$$

$$E''_{out} = c_p \int_{-L}^{+L} \left(T_i - T_\infty\right) dx$$

$$E''_{out} = \rho c_p \int_{-L}^{+L} \left[a + bx + cx^2 - T_\infty\right] dx = \rho c_p \left[ax + bx^2/2 + cx^3/3 - T_\infty x\right]_{-L}^{+L}$$

$$E''_{out} = \rho c_p \left[2aL + 0 + 2cL^3/3 - 2T_\infty L\right]$$

$$E''_{out} = 2600\,\mathrm{kg/m^3}\times 800\,\mathrm{J/kg\cdot K}\left[2\times82°\mathrm{C}\times0.020\mathrm{m} + 2\left(-2\times10^4\,°\mathrm{C/m^2}\right)\right.$$
$$\left.(0.020\mathrm{m})^3/3 - 2(20°\mathrm{C})0.020\mathrm{m}\right]$$

$$E''_{out} = 4.94\times10^6\ \mathrm{J/m^2} \qquad <$$

**COMMENTS:** (1) In part (a), note that the temperature gradient is larger at x = + L than at x = - L. This is consistent with the results of part (c) in which the conduction heat fluxes are evaluated.

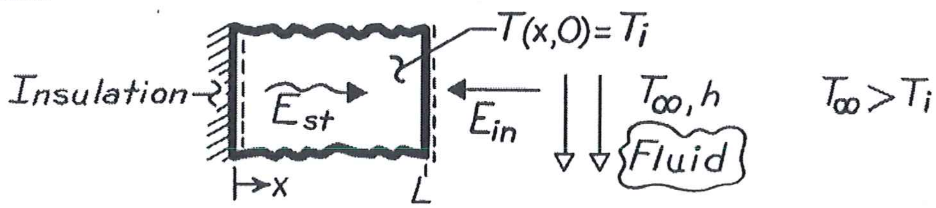
Continued .....

# PROBLEM 2.46

**KNOWN:** Plane wall, initially at a uniform temperature, is suddenly exposed to convective heating.

**FIND:** (a) Differential equation and initial and boundary conditions which may be used to find the temperature distribution, T(x,t); (b) Sketch T(x,t) for these conditions: initial ($t \le 0$), steady-state, $t \to \infty$, and two intermediate times; (c) Sketch heat fluxes as a function of time for surface locations; (d) Expression for total energy transferred to wall per unit volume ($J/m^3$).

**SCHEMATIC:**




**ASSUMPTIONS:** (1) One-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

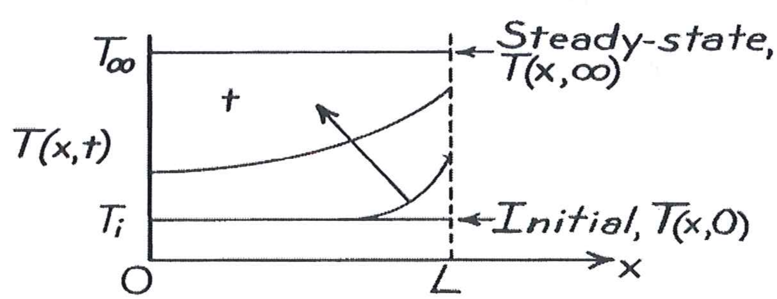
**ANALYSIS:** (a) For one-dimensional conduction with constant properties, the heat equation has the form,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

and the conditions are:

$$\begin{cases} \text{Initial, } t \le 0: & T(x,0) = T_i & \text{uniform} \\ \text{Boundaries:} \quad x=0 & \partial T/\partial x)_0 = 0 & \text{adiabatic} \\ \qquad x=L & -k\partial T/\partial x)_L = h\left[T(L,t) - T_\infty\right] & \text{convection} \end{cases}$$

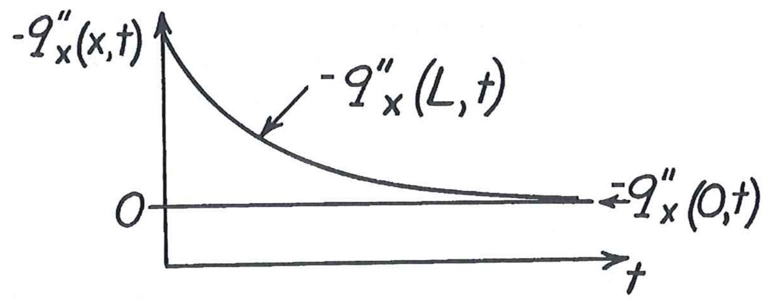
(b) The temperature distributions are shown on the sketch.




Note that the gradient at x = 0 is always zero, since this boundary is adiabatic. Note also that the gradient at x = L decreases with time.

(c) The heat flux, $q''_x(x,t)$, as a function of time, is shown on the sketch for the surfaces x = 0 and x = L.

Continued .....




For the surface at $x = 0$, $q''_x(0,t) = 0$ since it is adiabatic. At $x = L$ and $t = 0$, $q''_x(L,0)$ is a maximum (in magnitude)

$$\left|q''_x(L,0)\right| = h\left|T(L,0) - T_\infty\right|$$

where $T(L,0) = T_i$. The temperature difference, and hence the flux, decreases with time.

(d) The total energy transferred to the wall may be expressed as

$$E_{in} = \int_0^\infty q''_{conv} A_s dt$$
$$E_{in} = hA_s \int_0^\infty \left(T_\infty - T(L,t)\right) dt$$

Dividing both sides by $A_s L$, the energy transferred per unit volume is

$$\frac{E_{in}}{V} = \frac{h}{L} \int_0^\infty \left[T_\infty - T(L,t)\right] dt \qquad \left[J/m^3\right]$$

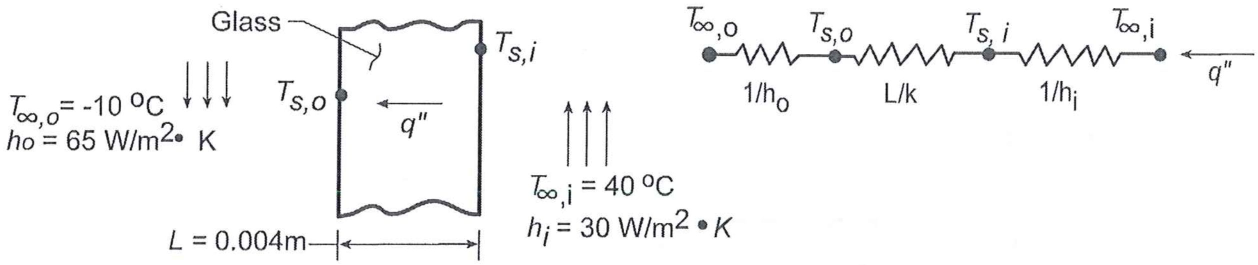
**COMMENTS:** Note that the heat flux at x = L is into the wall and is hence in the negative x direction.

## PROBLEM 3.2

**KNOWN:** Temperatures and convection coefficients associated with air at the inner and outer surfaces of a rear window.

**FIND:** (a) Inner and outer window surface temperatures, $T_{s,i}$ and $T_{s,o}$, and (b) $T_{s,i}$ and $T_{s,o}$ as a function of the outside air temperature $T_{\infty,o}$ and for selected values of outer convection coefficient, $h_o$.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation effects,( 4) Constant properties.

**PROPERTIES:** *Table A-3*, Glass (300 K): k = 1.4 W/m·K.

**ANALYSIS:** (a) The heat flux may be obtained from Eqs. 3.11 and 3.12,

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}} = \frac{40^\circ C - \left(-10^\circ C\right)}{\frac{1}{65\ W/m^2 \cdot K} + \frac{0.004\ m}{1.4\ W/m \cdot K} + \frac{1}{30\ W/m^2 \cdot K}}$$

$$q'' = \frac{50^\circ C}{\left(0.0154 + 0.0029 + 0.0333\right) m^2 \cdot K/W} = 969\ W/m^2.$$

Hence, with $q'' = h_i \left(T_{\infty,i} - T_{\infty,o}\right)$, the inner surface temperature is

$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40^\circ C - \frac{969\ W/m^2}{30\ W/m^2 \cdot K} = 7.7^\circ C \qquad <$$

Similarly for the outer surface temperature with $q'' = h_o \left(T_{s,o} - T_{\infty,o}\right)$ find

$$T_{s,o} = T_{\infty,o} + \frac{q''}{h_o} = -10^\circ C + \frac{969\ W/m^2}{65\ W/m^2 \cdot K} = 4.9^\circ C \qquad <$$

(b) Using the same analysis, $T_{s,i}$ and $T_{s,o}$ have been computed and plotted as a function of the outside air temperature, $T_{\infty,o}$, for outer convection coefficients of $h_o$ = 2, 65, and 100 W/m²·K. As expected, $T_{s,i}$ and $T_{s,o}$ are linear with changes in the outside air temperature. The difference between $T_{s,i}$ and $T_{s,o}$ increases with increasing convection coefficient, since the heat flux through the window likewise increases. This difference is larger at lower outside air temperatures for the same reason. Note that with $h_o$ = 2 W/m²·K, $T_{s,i}$ - $T_{s,o}$, is too small to show on the plot.

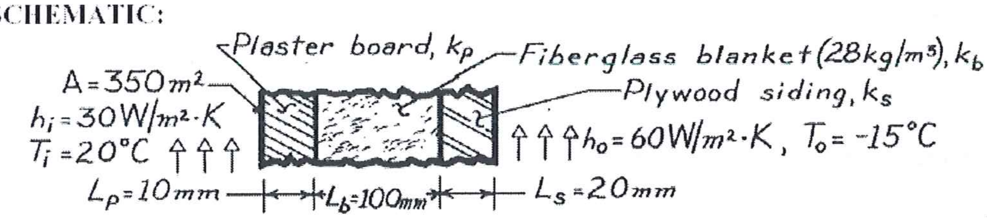
Continued .....

## PROBLEM 3.13

**KNOWN:** Composite wall of a house with prescribed convection processes at inner and outer surfaces.

**FIND:** (a) Expression for thermal resistance of house wall, $R_{tot}$; (b) Total heat loss, q(W); (c) Effect on heat loss due to increase in outside heat transfer convection coefficient, $h_o$; and (d) Controlling resistance for heat loss from house.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) One-dimensional conduction, (2) Steady-state conditions, (3) Negligible contact resistance.

**PROPERTIES:** *Table A-3*, $\left(\bar{T} = (T_i + T_o)/2 = (20-15)^\circ C/2 = 2.5^\circ C \approx 300K\right)$: Fiberglass blanket, 28 kg/m$^3$, $k_b = 0.038$ W/m·K; Plywood siding, $k_s = 0.12$ W/m·K; Plasterboard, $k_p = 0.17$ W/m·K.

**ANALYSIS:** (a) The expression for the total thermal resistance of the house wall follows from Eq. 3.18,

$$R_{tot} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_o A}.$$ <

(b) The total heat loss through the house wall is

$$q = \Delta T / R_{tot} = (T_i - T_o)/R_{tot}.$$

Substituting numerical values, find

$$R_{tot} = \frac{1}{30\text{W/m}^2\cdot\text{K}\times 350\text{m}^2} + \frac{0.01\text{m}}{0.17\text{W/m}\cdot\text{K}\times 350\text{m}^2} + \frac{0.10\text{m}}{0.038\text{W/m}\cdot\text{K}\times 350\text{m}^2}$$
$$+ \frac{0.02\text{m}}{0.12\text{W/m}\cdot\text{K}\times 350\text{m}^2} + \frac{1}{60\text{W/m}^2\cdot\text{K}\times 350\text{m}^2}$$
$$R_{tot} = \left[9.52 + 16.8 + 752 + 47.6 + 4.76\right]\times 10^{-5}\ ^\circ\text{C/W} = 831\times 10^{-5}\ ^\circ\text{C/W}$$

The heat loss is then,

$$q = \left[20-(-15)\right]^\circ\text{C}/831\times 10^{-5}\ ^\circ\text{C/W} = 4.21\text{ kW}.$$ <

(c) If $h_o$ changes from 60 to 300 W/m$^2$·K, $R_o = 1/h_oA$ changes from $4.76 \times 10^{-5}$ °C/W to $0.95 \times 10^{-5}$ °C/W. This reduces $R_{tot}$ to $826 \times 10^{-5}$ °C/W, which is a 0.6% decrease and hence a 0.6% increase in q.

(d) From the expression for $R_{tot}$ in part (b), note that the insulation resistance, $L_b/k_bA$, is 752/830 ≈ 90% of the total resistance. Hence, this material layer controls the resistance of the wall. From part (c) note that a 5-fold decrease in the outer convection resistance due to an increase in the wind velocity has a negligible effect on the heat loss.