ME 225 Heat Transfer

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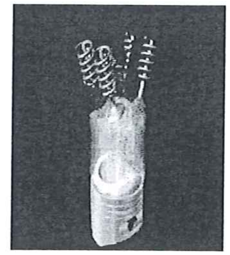
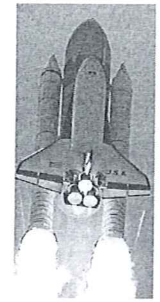
Slides

Chapter 12
Radiation: Processes and Properties

Radiation heat transfer mode is important in applications involving high temperatures.

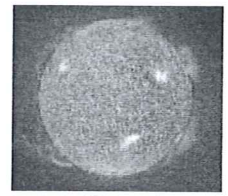


Engines



Rocket nozzles

Fires

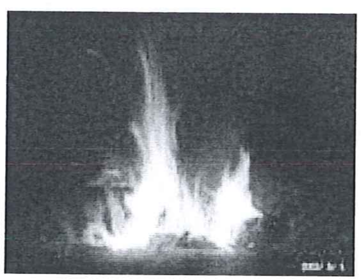


Sun

Atmospheric radiation

Space vehicles

- Unlike conduction and convection, radiation requires no medium.
- Attention is focused on thermal radiation, whose origins are associated with emission from matter at an absolute temperature.
- Emission is due to oscillations and transitions of the many electrons that comprise matter, which are, in turn, sustained by the thermal energy of the matter.
- Emission corresponds to heat transfer from the matter and hence to a reduction in thermal energy stored by the matter.



Radiation heat transfer can take place between two bodies separated by a medium colder than both bodies.

- Radiation may also be intercepted and absorbed by matter.
- Absorption results in heat transfer to the matter and hence an increase in thermal energy stored by the matter.

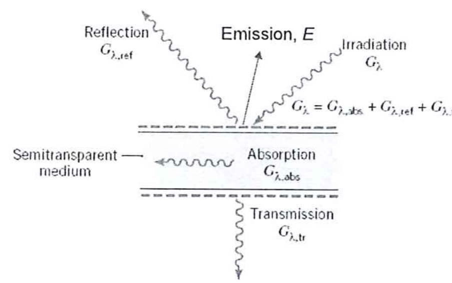


FIGURE 12.20 Absorption, reflection, and transmission processes associated with a semitransparent medium.

- The dual nature of radiation:
  - In some cases, the physical manifestations of radiation may be explained by viewing it as particles (aka photons or quanta).
  - In other cases, radiation behaves as an electromagnetic wave.

- In all cases, radiation is characterized by a wavelength λ and frequency ν, which are related through the speed at which radiation propagates in the medium of interest:

$$\lambda = \frac{c}{\nu}$$

For propagation in a vacuum, $c_0 = 2.998 \times 10^8$ m/s

$c = c_0/n$, where n is index of refraction of the medium

$n \approx 1$ for air and most gases; $n \approx 1.5$ for water and glass

Quantum theory views electromagnetic radiation as the propagation of discrete packets of energy called photons. Each photon of frequency ν has the energy of:

E (eV) = h ν = hc/λ = 1.24/λ (μm)

$h = 6.625 \times 10^{-34}$ J.s ; eV = 1.6022×10^{-19} J

↑
Not Conv. coefficient

Analysis of thermal radiation is much more complex compared to conduction and convection.

Conduction and convection	Radiation
- Short-range phenomena (~ mean free path, ca. 10^{-10} m)	- Long-range phenomena ($10^{-10} - 10^{+10}$ m)
- Properties (k, μ, ρ, etc.) are easily measured.	- Properties are difficult to measure (wavelength dependence).
- Energy balance on an infinitesimal volume, leading to partial differential equation with 4 variables (x, y, z, t)	- Energy balance over the entire volume, leading to integral equation with 7 variables (x, y, z, t, θ, ϕ, λ)

Electromagnetic Spectrum

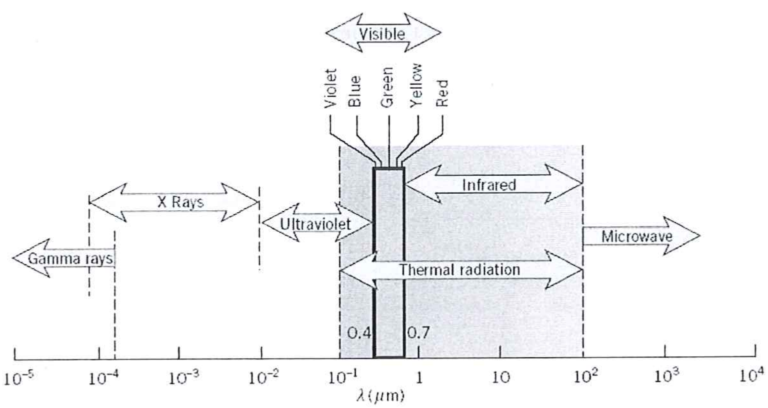


FIGURE 12.3 Spectrum of electromagnetic radiation.

- Thermal radiation is confined to the infrared, visible and ultraviolet regions of the spectrum $(0.1 < \lambda < 100\mu m)$.

Blackbody Radiation

- Fact: Every object emits radiation in all directions over all wavelengths depending on the material, surface properties, and temperature.
- Question: What is the maximum amount of radiation that can be emitted by a surface at a given temperature?
- Answer: Define an idealized body, called "blackbody" to serve as a standard.

Blackbody = perfect emitter and absorber of radiation

➢At a given T and λ, no surface can emit more radiation than a blackbody.

➢Regardless of λ and direction, a blackbody absorbs all incident radiation.

➢A blackbody is a diffuse emitter, i.e., emits radiation uniformly in all directions.

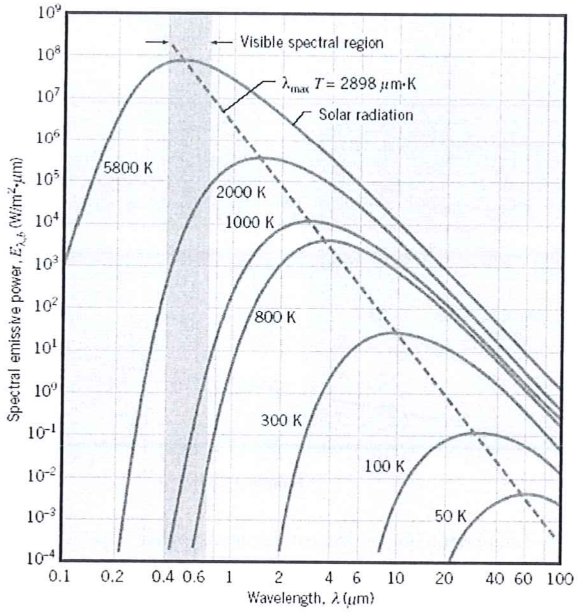


FIGURE 12.12 Spectral blackbody emissive power.

TABLE 12.1 Blackbody Radiation Functions

λT ($\mu m \cdot K$)	$F_{(0\rightarrow\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ ($\mu m \cdot K \cdot sr)^{-1}$	$\dfrac{I_{\lambda,b}(\lambda,T)}{I_{\lambda,b}(\lambda_{max},T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
4,400	0.548796	0.503253	0.696720
4,600	0.579280	0.467343	0.647004
4,800	0.607559	0.433109	0.599610
5,000	0.633747	0.400813	0.554898
5,200	0.658970	0.370580×10^{-4}	0.513043
5,400	0.680360	0.342445	0.474092
5,600	0.701046	0.316376	0.438002
5,800	0.720158	0.292301	0.404671
6,000	0.737818	0.270121	0.373965
6,200	0.754140	0.249723×10^{-4}	0.345724
6,400	0.769234	0.230985	0.319783
6,600	0.783199	0.213786	0.295973
6,800	0.796129	0.198008	0.274128
7,000	0.808109	0.183534	0.254090
7,200	0.819217	0.170256×10^{-4}	0.235708
7,400	0.829527	0.158073	0.218842
7,600	0.839102	0.146891	0.203360
7,800	0.848005	0.136621	0.189143
8,000	0.856288	0.127185	0.176079
8,500	0.874608	0.106772×10^{-4}	0.147819
9,000	0.890029	0.901463×10^{-5}	0.124801

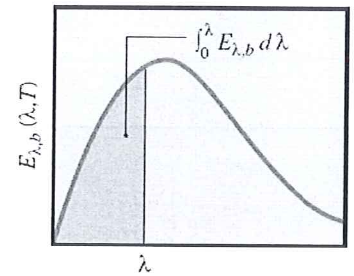


FIGURE 12.13 Radiation emission from a blackbody in the spectral band 0 to λ.

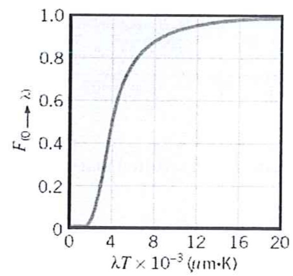


FIGURE 12.14 Fraction of the total blackbody emission in the spectral band from 0 to λ as a function of λT.

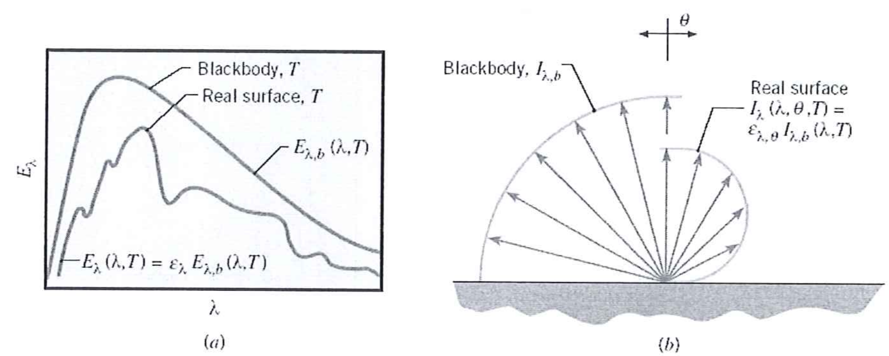


FIGURE 12.15 Comparison of blackbody and real surface emission. (a) Spectral distribution. (b) Directional distribution.

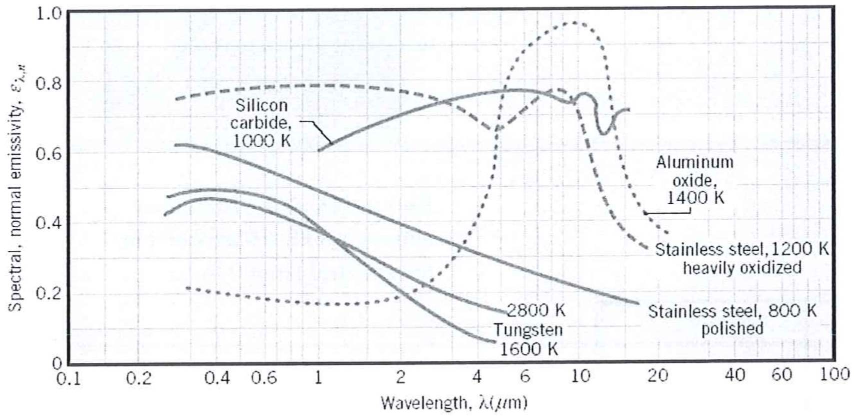


FIGURE 12.17 Spectral dependence of the spectral, normal emissivity $\varepsilon_{\lambda,n}$ of selected materials.

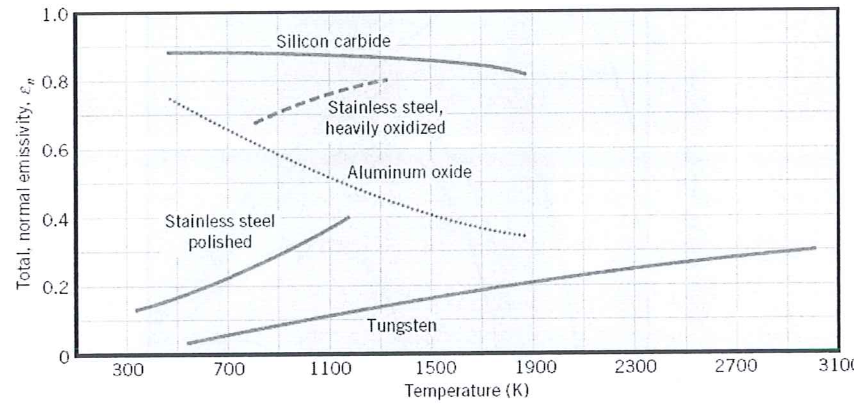
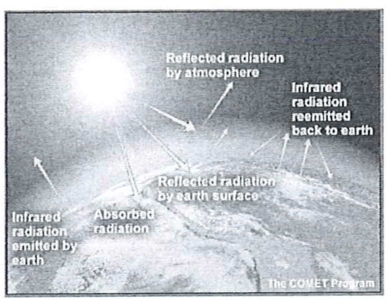
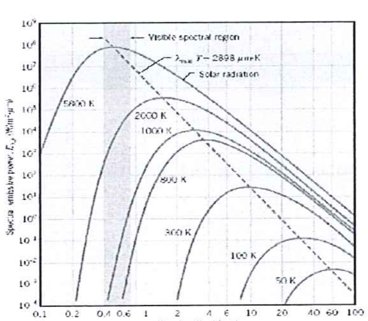


FIGURE 12.18 Temperature dependence of the total, normal emissivity ε_n of selected materials.





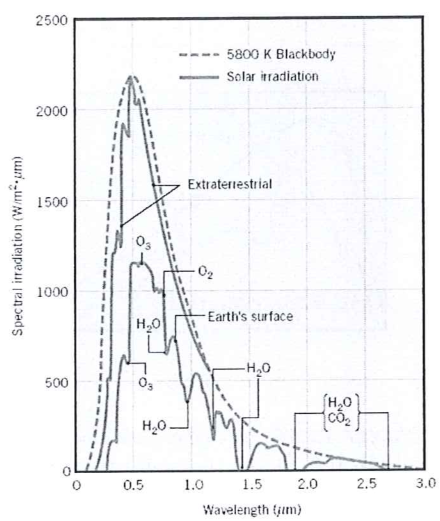


FIGURE 12.28 Spectral distribution of solar radiation.

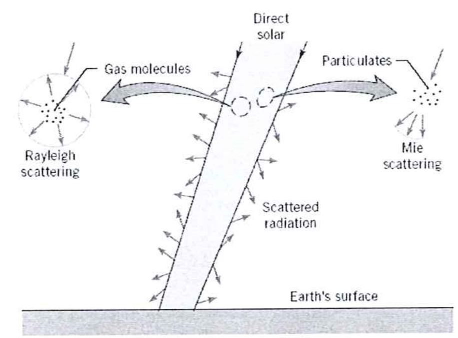


FIGURE 12.29 Scattering of solar radiation in the earth's atmosphere.

- Interaction of solar radiation with earth's atmosphere:
  - Absorption by aerosols over the entire spectrum.
  - Absorption by gases (CO_2, H_2O, O_3) in discrete wavelength bands.
  - Scattering by gas molecules and aerosols.

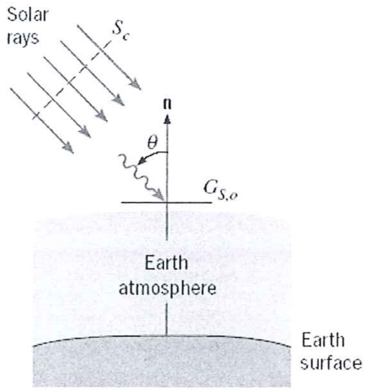
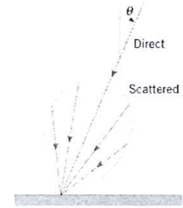


FIGURE 12.27
Directional nature of solar radiation outside the earth's atmosphere.

$S_c \rightarrow$ the solar constant or heat flux $\left(1353\ \mathrm{W/m^2}\right)$
when the earth is at its mean distance from the sun.

$$G_{S,o} = f \times S_c \times \cos\theta$$

- Effect of Atmosphere on Directional Distribution of Solar Radiation:
  - Rayleigh scattering is approximately uniform in all directions (isotropic scattering), while Mie scattering is primarily in the direction of the sun's rays (forward peaked).
  - Directional distribution of radiation at the earth's surface has two components.
    - Direct radiation: Unscattered and in the direction θ of the sun's rays.
    - Diffuse radiation: Scattered radiation strongly peaked in the forward direction.

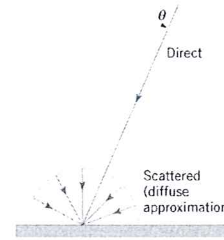


  - Calculation of solar irradiation for a horizontal surface often presumes the scattered component to be isotropic.

$$G_S = G_{S,dir} + G_{S,dif} = q''_{dir}\cos\theta + \pi I_{dif}$$

$$0.1 < \left(G_{S,dif} / G_S\right) < 1.0$$

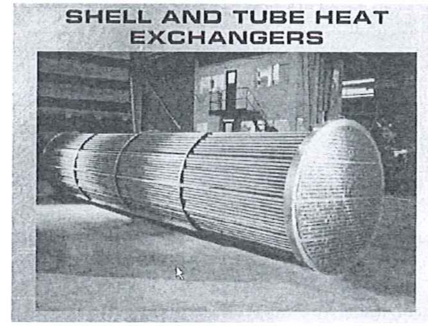
↳ Clear skies ↳ Completely overcast



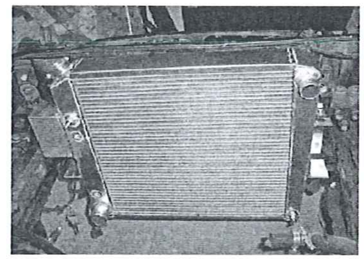
1

Chapter 11
Heat Exchangers

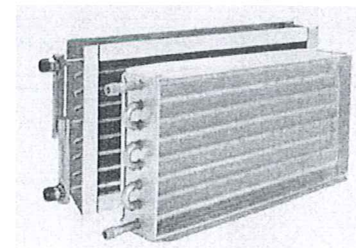
2



Car Radiator



Air conditioning



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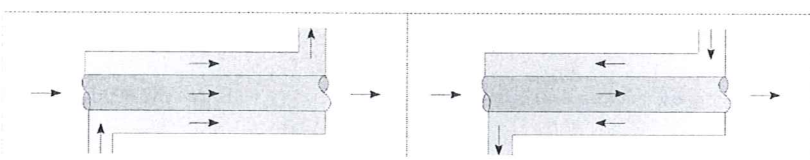
Heat Exchanger Types

Heat exchangers are ubiquitous to energy conversion and utilization. They involve heat exchange between two fluids separated by a solid and encompass a wide range of flow configurations.

- Concentric-Tube Heat Exchangers

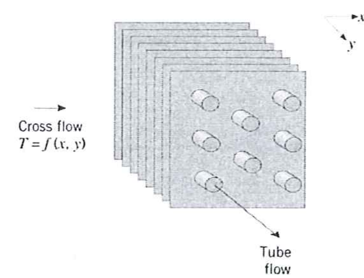


Parallel Flow — Counterflow

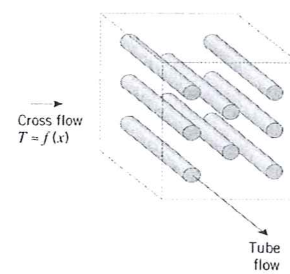
- ➢ Simplest configuration
- ➢ Superior performance associated with counter flow

4

- Cross-flow Heat Exchangers



Finned-Both Fluids Unmixed



Unfinned-One Fluid Mixed the Other Unmixed

- ➢ For cross-flow over the tubes, fluid motion, and hence mixing, in the transverse direction (y) is prevented for the finned tubes, but occurs for the unfinned condition.
- ➢ Heat exchanger performance is influenced by mixing.

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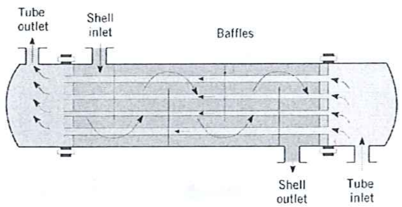
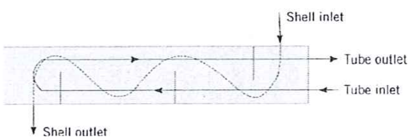
- Shell-and-Tube Heat Exchangers



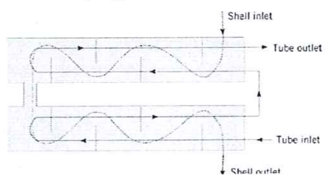
One Shell Pass and One Tube Pass

➢ Baffles are used to establish a cross-flow and to induce turbulent mixing of the shell-side fluid, both of which enhance convection.

➢ The number of tube and shell passes may be varied, e.g.:



One Shell Pass,
Two Tube Passes



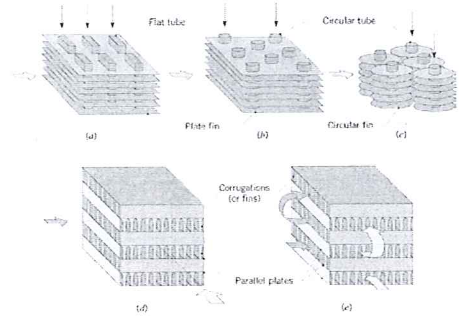
Two Shell Passes,
Four Tube Passes

6

- Compact Heat Exchangers

➢ Widely used to achieve large heat rates per unit volume, particularly when one or both fluids is a gas.

➢ Characterized by large heat transfer surface areas per unit volume, small flow passages, and laminar flow.



(a) Fin-tube (flat tubes, continuous plate fins)
(b) Fin-tube (circular tubes, continuous plate fins)
(c) Fin-tube (circular tubes, circular fins)
(d) Plate-fin (single pass)
(e) Plate-fin (multipass)

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TABLE 11.1 Representative Fouling Factors [1]

Fluid	R''_f ($m^2 \cdot K/W$)
Seawater and treated boiler feedwater (below 50°C)	0.0001
Seawater and treated boiler feedwater (above 50°C)	0.0002
River water (below 50°C)	0.0002–0.001
Fuel oil	0.0009
Refrigerating liquids	0.0002
Steam (nonoil bearing)	0.0001

TABLE 11.2 Representative Values of the Overall Heat Transfer Coefficient

Fluid Combination	U ($W/m^2 \cdot K$)
Water to water	850–1700
Water to oil	110–350
Steam condenser (water in tubes)	1000–6000
Ammonia condenser (water in tubes)	800–1400
Alcohol condenser (water in tubes)	250–700
Finned-tube heat exchanger (water in tubes, air in cross flow)	25–50

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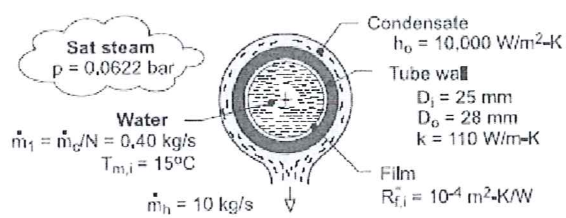
PROBLEM 11.7

KNOWN: Number, inner and outer diameters, and thermal conductivity of condenser tubes. Convection coefficient at outer surface. Overall flow rate, inlet temperature and properties of water flow through the tubes. Flow rate and pressure of condensing steam. Fouling factor for inner surface.

FIND: (a) Overall coefficient based on outer surface area, U_o, without fouling, (b) Overall coefficient with fouling, (c) Temperature of water leaving the condenser.

SCHEMATIC:



ASSUMPTIONS: (1) Water is incompressible with negligible viscous dissipation, (2) Fully-developed flow in tubes, (3) Negligible effect of fouling on D_i.

PROPERTIES: Water (Given): $c_p = 4180$ J/kg·K, $\mu = 9.6 \times 10^{-4}$ N·s/m^2, k = 0.60 W/m·K, Pr = 6.6. *Table A-6*, Water, saturated vapor (p = 0.0622 bars): $T_{sat} = 310$ K, $h_{fg} = 2.414 \times 10^6$ J/kg.

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ANALYSIS: (a) Without fouling, Eq. 11.5 yields

$$\frac{1}{U_o} = \frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{D_o \ln(Do/Di)}{2k_t} + \frac{1}{h_o}$$

With $Re_{D_i} = 4\dot{m}_1/\pi D_i \mu = 1.60\,kg/s/\left(\pi \times 0.025m \times 9.6 \times 10^{-4}\,N\cdot s/m^2\right) = 21,220$, flow in the tubes is turbulent, and from Eq. 8.60

$$h_i = \left(\frac{k}{D_i}\right) 0.023 Re_{D_i}^{4/5} Pr^{0.4} = \left(\frac{0.60\,W/m\cdot K}{0.025m}\right) 0.023(21,200)^{4/5}(6.6)^{0.4} = 3400\,W/m^2\cdot K$$

$$U_o = \left[\frac{1}{3400}\left(\frac{28}{25}\right) + \frac{0.028\ln(28/25)}{2\times110} + \frac{1}{10,000}\right]^{-1} W/m^2\cdot K =$$

$$\left(3.29\times10^{-4} + 1.44\times10^{-5} + 10^{-4}\right)^{-1} W/m^2\cdot K = 2255\,W/m^2\cdot K$$

(b) With fouling, Eq. 11.5 yields

$$U_o = \left[4.43\times10^{-4} + (Do/Di)R''_{f,i}\right]^{-1} = \left(5.55\times10^{-4}\right)^{-1} = 1800\,W/m^2\cdot K$$

(c) The rate at which energy is extracted from the steam equals the rate of heat transfer to the water, $\dot{m}_h h_{fg} = \dot{m}_c c_p (T_{m,o} - T_{m,i})$, in which case

$$T_{m,o} = T_{m,i} + \frac{\dot{m}_h h_{fg}}{\dot{m}_c c_p} = 15°C + \frac{10\,kg/s \times 2.414\times10^6\,J/kg}{400\,kg/s \times 4180\,J/kg\cdot K} = 29.4°C \qquad <$$

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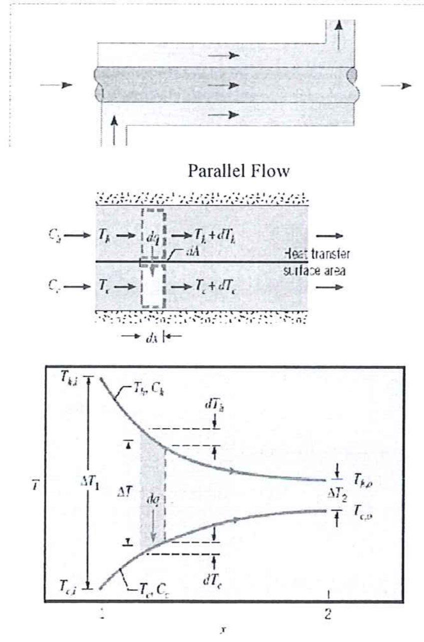

FIGURE 11.7 Temperature distributions for a parallel-flow heat exchanger.

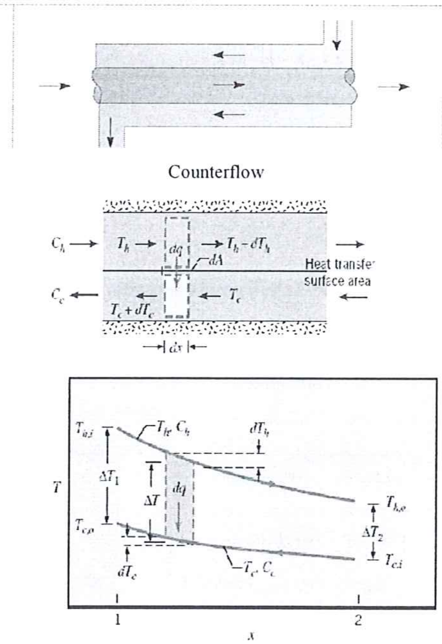

FIGURE 11.8 Temperature distributions for a counterflow heat exchanger.

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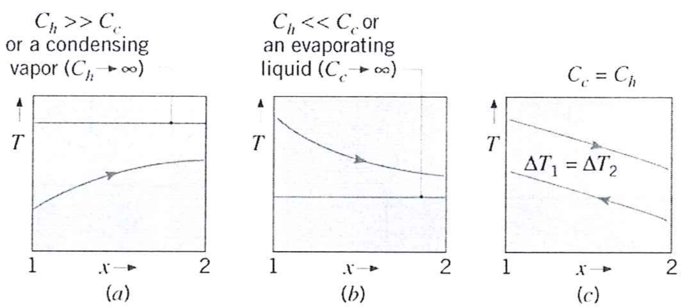
Special Operating Conditions



- Case (a): $C_h >> C_c$ or h is a condensing vapor $(C_h \to \infty)$.
  - Negligible or no change in T_h $(T_{h,o} = T_{h,i})$.
- Case (b): $C_c >> C_h$ or c is an evaporating liquid $(C_c \to \infty)$.
  - Negligible or no change in T_c $(T_{c,o} = T_{c,i})$.
- Case (c): $C_h = C_c$.
  - $\Delta T_1 = \Delta T_2 = \Delta T_{lm}$

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TABLE 11.3 Heat Exchanger Effectiveness Relations [5]

Flow Arrangement	Relation		
Concentric tube			
Parallel flow	$\varepsilon = \dfrac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r}$		(11.28a)
Counterflow	$\varepsilon = \dfrac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]}$	$(C_r < 1)$	
	$\varepsilon = \dfrac{NTU}{1 + NTU}$	$(C_r = 1)$	(11.29a)
Shell-and-tube			
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \dfrac{1 + \exp[-(NTU)_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(NTU)_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$		(11.30a)
n Shell passes ($2n$, $4n$, . . . tube passes)	$\varepsilon = \left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$		(11.31a)
Cross-flow (single pass)			
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(NTU)^{0.22}\{\exp[-C_r(NTU)^{0.78}] - 1\}\right]$		(11.32)
C_{max} (mixed), C_{min} (unmixed)	$\varepsilon = \left(\dfrac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-NTU)]\})$		(11.33a)
C_{min} (mixed), C_{max} (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(NTU)]\})$		(11.34a)
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-NTU)$		(11.35a)

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TABLE 11.4 Heat Exchanger NTU Relations

Flow Arrangement	Relation	
Concentric tube		
Parallel flow	$\mathrm{NTU} = -\dfrac{\ln[1-\varepsilon(1+C_r)]}{1+C_r}$	(11.28b)
Counterflow	$\mathrm{NTU} = \dfrac{1}{C_r - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \quad (C_r < 1)$	
	$\mathrm{NTU} = \dfrac{\varepsilon}{1-\varepsilon} \quad (C_r = 1)$	(11.29b)
Shell-and-tube		
One shell pass (2, 4, . . . tube passes)	$(\mathrm{NTU})_1 = -(1+C_r^2)^{-1/2}\ln\left(\dfrac{E-1}{E+1}\right)$	(11.30b)
	$E = \dfrac{2/\varepsilon_1 - (1+C_r)}{(1+C_r^2)^{1/2}}$	(11.30c)
n Shell passes ($2n$, $4n$, . . . tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \dfrac{F-1}{F-C_r} \quad F = \left(\dfrac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad \mathrm{NTU} = n(\mathrm{NTU})_1$	(11.31b, c, d)
Cross-flow (single pass)		
C_{max} (mixed), C_{min} (unmixed)	$\mathrm{NTU} = -\ln\left[1 + \left(\dfrac{1}{C_r}\right)\ln(1-\varepsilon C_r)\right]$	(11.33b)
C_{min} (mixed), C_{max} (unmixed)	$\mathrm{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r\ln(1-\varepsilon)+1]$	(11.34b)
All exchangers ($C_r = 0$)	$\mathrm{NTU} = -\ln(1-\varepsilon)$	(11.35b)

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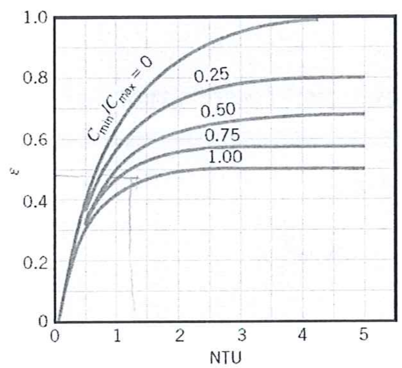

FIGURE 11.10 Effectiveness of a parallel-flow heat exchanger (Equation 11.28).

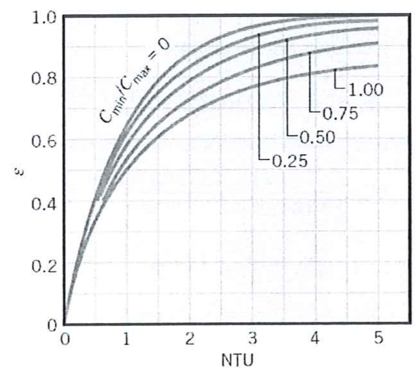

FIGURE 11.11 Effectiveness of a counterflow heat exchanger (Equation 11.29).

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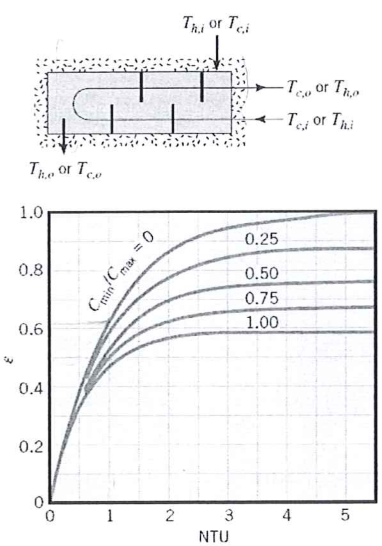

FIGURE 11.12 Effectiveness of a shell-and-tube heat exchanger with one shell and any multiple of two tube passes (two, four, etc. tube passes) (Equation 11.30).

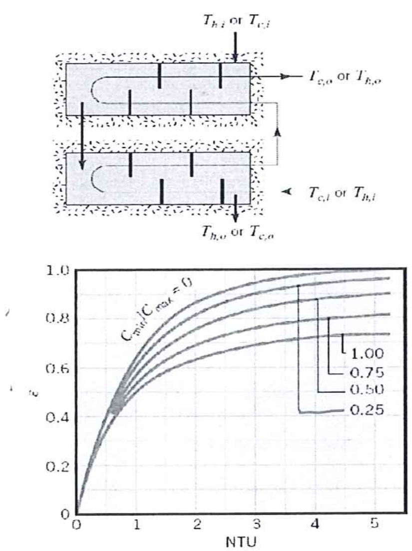

FIGURE 11.13 Effectiveness of a shell-and-tube heat exchanger with two shell passes and any multiple of four tube passes (four, eight, etc. tube passes) (Equation 11.31 with $n = 2$).

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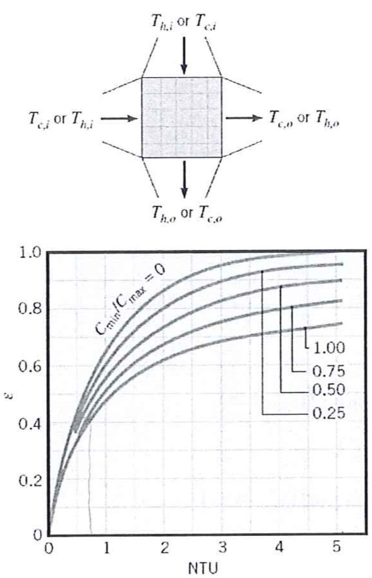


FIGURE 11.14 Effectiveness of a single-pass, cross-flow heat exchanger with both fluids unmixed (Equation 11.32).

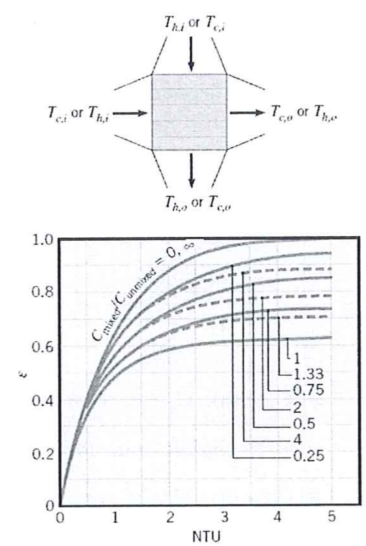


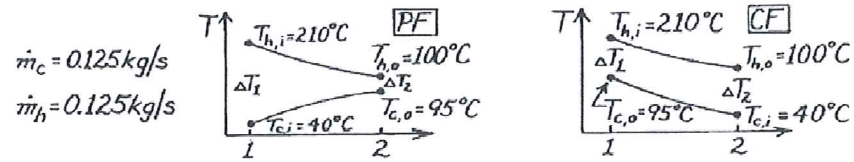
FIGURE 11.15 Effectiveness of a single-pass, cross-flow heat exchanger with one fluid mixed and the other unmixed (Equations 11.33, 11.34).

PROBLEM 11.63

KNOWN: Concentric tube heat exchanger with prescribed conditions.

FIND: (a) Maximum possible heat transfer, (b) Effectiveness, (c) Whether heat exchanger should be run in PF or CF to minimize size or weight; determine ratio of required areas for the two flow conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Constant properties, (3) Overall heat transfer coefficient remains unchanged for PF or CF conditions.

PROPERTIES: Hot fluid (given): c = 2100 J/kg·K; Cold fluid (given): c = 4200 J/kg·K.

ANALYSIS: (a) The maximum possible heat transfer rate is given by Eq. 11.18,

$$q_{max} = C_{min}\left(T_{h,i} - T_{c,o}\right).$$

The minimum capacity fluid is the hot fluid with $C_{min} = \dot{m}_h c_h$, giving

$$q_{max} = \dot{m}_h c_h\left(T_{h,i} - T_{c,o}\right) = 0.125\frac{\text{kg}}{\text{s}} \times 2100\frac{\text{J}}{\text{kg}\cdot\text{K}}(210-40)\text{K} = 44,625\ \text{W}.$$

(b) The effectiveness is defined by Eq. 11.19 and the heat rate, q, can be determined from an energy balance on the cold fluid.

$$\varepsilon = q/q_{max} = \dot{m}_c c_c\left(T_{c,o} - T_{c,i}\right)/q_{max}$$

$$\varepsilon = 0.125\ \text{kg/s} \times 4200\ \text{J/kg}\cdot\text{K}(95-40)\text{K}/44,625\ \text{W} = 0.65. \qquad <$$

(c) Operating the heat exchanger under CF conditions will require a smaller heat transfer area than for PF conditions. The ratio of the areas is

$$\frac{A_{CF}}{A_{PF}} = \frac{q/U\Delta T_{\ell m,CF}}{q/U\Delta T_{\ell m,PF}} = \frac{\Delta T_{\ell m,PF}}{\Delta T_{\ell m,CF}}.$$

To calculate the LMTD, first find $T_{h,o}$ from overall energy balances on the two fluids.

$$T_{h,o} = T_{h,i} - \frac{\dot{m}_c c_c}{\dot{m}_h c_h}\left(T_{c,o} - T_{c,i}\right) = 210°\text{C} - \frac{0.125 \times 4200}{0.125 \times 2100}(95-40)°\text{C} = 100°\text{C}.$$

Using Eq. 11.15 with ΔT_1 and ΔT_2 as shown below, find $\Delta T_{\ell m} = (\Delta T_1 - \Delta T_2)/\ell n(\Delta T_1/\Delta T_2)$.

Substituting values, find

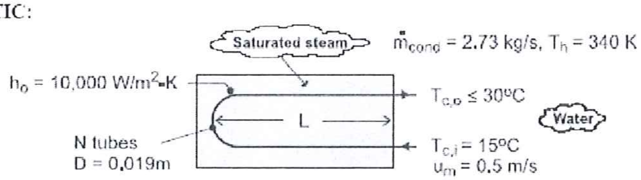
$$\frac{A_{CF}}{A_{PF}} = \frac{\left[(210-40)-(100-95)\right]/\ell n(170/5)}{\left[(210-95)-(100-40)\right]/\ell n(115/60)} = \frac{46.8°\text{C}}{84.5°\text{C}} = 0.55. \qquad <$$

PROBLEM 11.38

KNOWN: Temperature, convection coefficient and condensation rate of saturated steam. Tube diameter for shell-and-tube heat exchanger with one shell pass and two tube passes. Velocity and inlet and maximum allowable exit temperatures of cooling water.

FIND: (a) Minimum number of tubes and tube length per pass, (b) Effect of tube-side heat transfer enhancement on tube length.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat exchange with surroundings, (2) Negligible tube wall conduction and fouling resistance, (3) Constant properties, (4) Fully developed internal flow throughout.

PROPERTIES: *Table A-6*, Sat. water (340 K): $h_{fg} = 2.342 \times 10^6$ J/kg; Sat. water ($\overline{T}_c = 22.5°\text{C} \approx 295$ K): ρ=998 kg/m^3, c_p = 4181 J/kg·K, μ = 959 × 10^{-6} N·s/m^2, k = 0.606 W/m·K, Pr = 6.62.

ANALYSIS: (a) The required heat rate and the maximum allowable temperature rise of the water determine the minimum allowable flow rate. That is, with

$$q = q_{cond} = \dot{m}_{cond} h_{fg} = 2.73\ \text{kg/s} \times 2.342 \times 10^6\ \text{J/kg} = 6.39 \times 10^6\ \text{W}$$

$$\dot{m}_{c,min} = \frac{q}{c_{p,c}\left(T_{c,o} - T_{c,i}\right)} = \frac{6.39 \times 10^6\ \text{W}}{4181\ \text{J/kg}\cdot\text{K}(15°\text{C})} = 101.9\ \text{kg/s}$$

With a specified flow rate per tube of $\dot{m}_{c,1} = \rho u_m \pi D^2/4$ = 998 kg/m^3 × 0.5 m/s × π (0.019m)^2/4 = 0.141 kg/s, the minimum number of tubes is

$$N_{min} = \frac{\dot{m}_{c,min}}{\dot{m}_{c,1}} = \frac{101.9\ \text{kg/s}}{0.141\ \text{kg/s}} = 720 \qquad <$$

To determine the corresponding tube length, we must first find the required heat transfer surface area. With $Re_D = \rho u_m D/\mu$ = 998 kg/m^3 (0.5 m/s) 0.019m/959 × 10^{-6} N·s/m^2 = 9,886, the Dittus-Boelter equation yields

$$\overline{h}_i = (k/D)0.023\,Re_D^{4/5}\,Pr^{0.4} = (0.606\ \text{W/m}\cdot\text{K}/0.019\text{m})0.023(9886)^{4/5}(6.62)^{0.4} = 2454\ \text{W/m}^2\cdot\text{K}$$

$$\text{Hence, } U = \left[\overline{h}_i^{-1} + h_o^{-1}\right]^{-1} = 1970\ \text{W/m}^2\cdot\text{K}$$

With $C_r = 0$, $C_{min} = \dot{m}_c\ c_{p,c} = 101.9$ kg/s $\times$ 4181 J/kg·K = 4.26 $\times 10^5$ W/K, $q_{max} = C_{min}\ (T_{h,i} - T_{c,i})$ = 4.26 $\times 10^5$ W/K (340 – 288) K = 2.215 $\times 10^7$ W and $\varepsilon = q/q_{max} = 0.289$, Eq. 11.35b yields NTU = – ln (1 – ε) = – ln (1 – 0.289) = 0.341. Hence the tube length per pass is

$$L = \frac{A}{2N\pi D} = \frac{NTU \times C_{min}}{2N\pi DU} = \frac{0.341 \times 4.26 \times 10^5 \text{ W/K}}{2 \times 720 \times \pi (0.019\text{m}) 1970 \text{ W/m}^2 \cdot \text{K}} = 0.858\text{m} \quad <$$

(b) If the tube-side convection coefficient is doubled, $\overline{h}_i = 4908 \text{ W/m}^2 \cdot \text{K}$ and U = 3292 W/m^2·K. Since q, C_r, C_{min}, q_{max} and hence ε are unchanged, the number of transfer units is still NTU = 0.341. Hence, the tube length per pass is now

$$L = \frac{NTU \times C_{min}}{2N\pi DU} = \frac{0.341 \times 4.26 \times 10^5 \text{ W/K}}{2 \times 720 \times \pi (0.019\text{m}) 3292 \text{ W/m}^2 \cdot \text{K}} = 0.513\text{m} \quad <$$

Chapter 8
Internal Flows

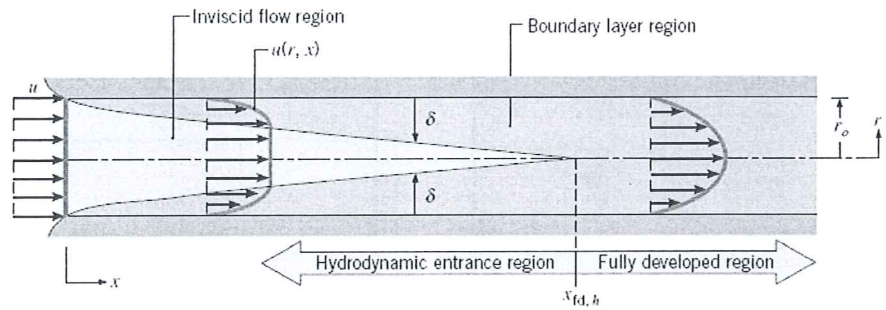


FIGURE 8.1 Laminar, hydrodynamic boundary layer development in a circular tube.

- Velocity boundary layer develops on surface of tube and thickens with increasing x.
- Inviscid region of uniform velocity shrinks as boundary layer grows.
- Subsequent to boundary layer merger at the centerline, the velocity profile becomes parabolic and invariant with x (hydrodynamically fully-developed flow).

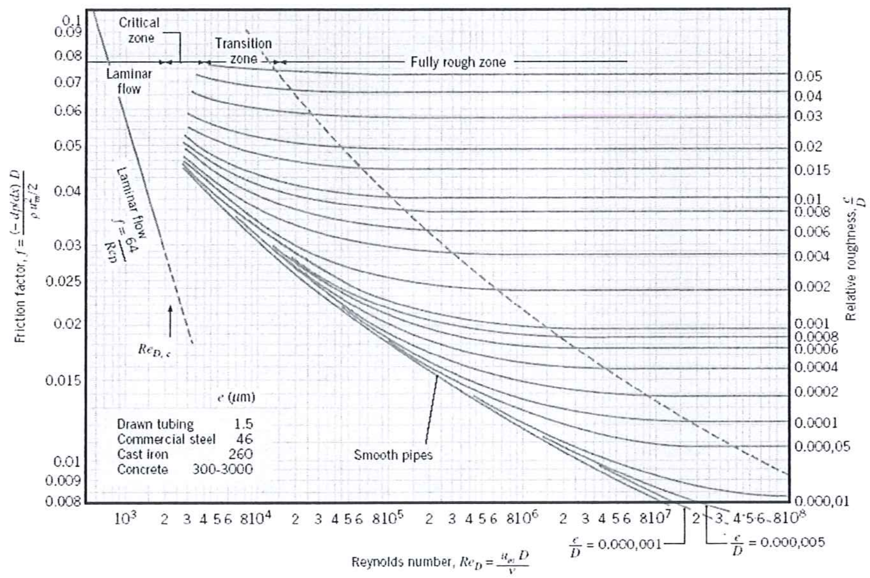


FIGURE 8.3 Friction factor for fully developed flow in a circular tube [3]. Used with permission.

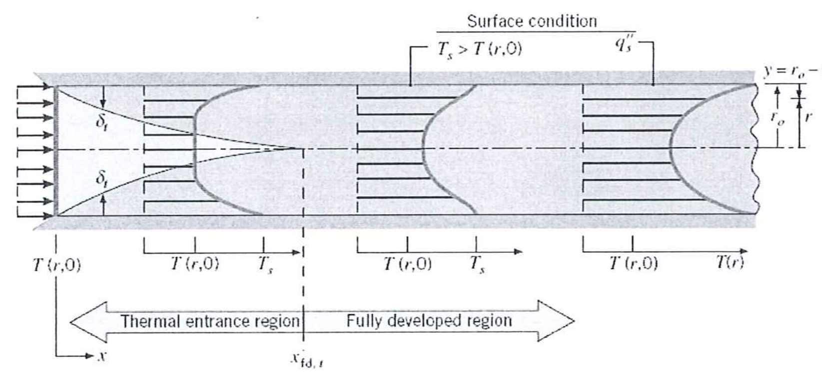


FIGURE 8.4 Thermal boundary layer development in a heated circular tube.

- Thermal boundary layer develops on surface of tube and thickens with increasing x.
- Isothermal core shrinks as boundary layer grows.
- Subsequent to boundary layer merger, dimensionless forms of the temperature profile become independent of x (thermally-fully developed flow).

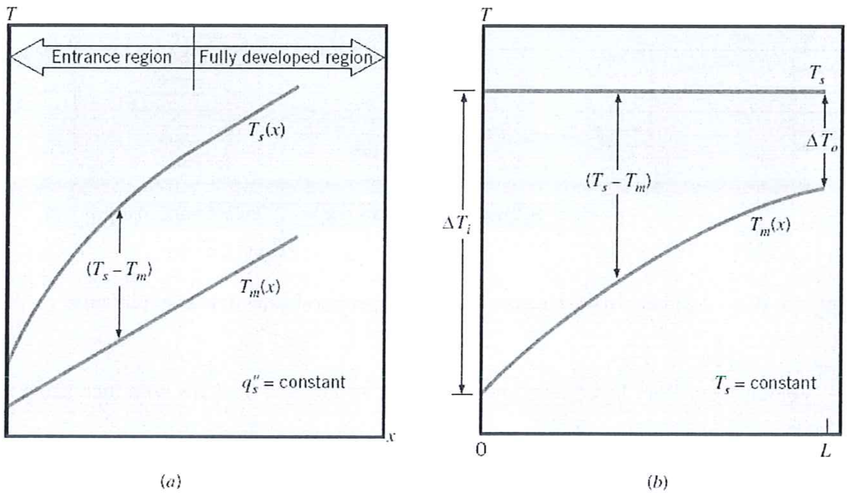
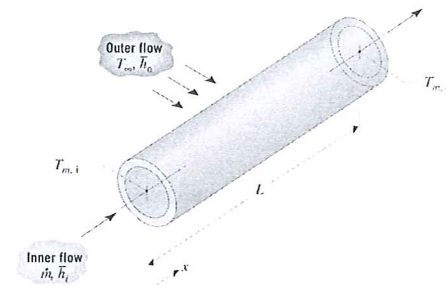


FIGURE 8.7 Axial temperature variations for heat transfer in a tube. (*a*) Constant surface heat flux. (*b*) Constant surface temperature.

- Special Case: Uniform External Fluid Temperature



$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\bar{U}A_s}{\dot{m}c_p}\right) = \exp\left(-\frac{1}{\dot{m}c_p R_{tot}}\right)$$

$$q = \bar{U}A_s\,\Delta T_{\ell m} = \frac{\Delta T_{\ell m}}{R_{tot}}$$

$\Delta T_{\ell m} \rightarrow T_s$ replaced by T_∞.

Note: Replacement of T_∞ by $T_{s,o}$ if outer surface temperature is uniform.

TABLE 8.4 Summary of convection correlations for flow in a circular tube[a,d]

Correlation		Conditions
$f = 64/Re_D$	(8.19)	Laminar, fully developed
$Nu_D = 4.36$	(8.53)	Laminar, fully developed, uniform q_s''
$Nu_D = 3.66$	(8.55)	Laminar, fully developed, uniform T_s
$\overline{Nu}_D = 3.66 + \dfrac{0.0668(D/L)Re_D Pr}{1 + 0.04[(D/L)Re_D Pr]^{2/3}}$	(8.56)	Laminar, thermal entry (or combined entry with $Pr \gtrsim 5$), uniform T_s
or		
$\overline{Nu}_D = 1.86\left(\dfrac{Re_D Pr}{L/D}\right)^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8.57)	Laminar, combined entry, $0.6 \lesssim Pr \lesssim 5$, $0.0044 \lesssim (\mu/\mu_s) \lesssim 9.75$, uniform T_s
$f = 0.316 Re_D^{-1/4}$	(8.20a)[b]	Turbulent, fully developed, $Re_D \lesssim 2 \times 10^4$
$f = 0.184 Re_D^{-1/5}$	(8.20b)[b]	Turbulent, fully developed, $Re_D \gtrsim 2 \times 10^4$
or		
$f = (0.790 \ln Re_D - 1.64)^{-2}$	(8.21)[b]	Turbulent, fully developed, $3000 \lesssim Re_D \lesssim 5 \times 10^6$
$Nu_D = 0.023 Re_D^{4/5} Pr^n$	(8.60)[c]	Turbulent, fully developed, $0.6 \lesssim Pr \lesssim 160$, $Re_D \gtrsim 10{,}000$, $(L/D) \gtrsim 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
or		
$Nu_D = 0.027 Re_D^{4/5} Pr^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8.61)[c]	Turbulent, fully developed, $0.7 \lesssim Pr \lesssim 16{,}700$, $Re_D \gtrsim 10{,}000$, $L/D \gtrsim 10$
or		
$Nu_D = \dfrac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$	(8.62)[c]	Turbulent, fully developed, $0.5 \lesssim Pr \lesssim 2000$, $3000 \lesssim Re_D \lesssim 5 \times 10^6$, $(L/D) \gtrsim 10$
$Nu_D = 4.82 + 0.0185(Re_D Pr)^{0.827}$	(8.64)	Liquid metals, turbulent, fully developed, uniform q_s'', $3.6 \times 10^3 \lesssim Re_D \lesssim 9.05 \times 10^5$, $10^2 \lesssim Pe_D \lesssim 10^4$
$Nu_D = 5.0 + 0.025(Re_D Pr)^{0.8}$	(8.65)	Liquid metals, turbulent, fully developed, uniform T_s, $Pe_D \gtrsim 100$

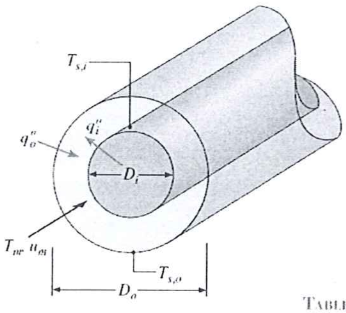
[a] Properties in Equations 8.53, 8.55, 8.60, 8.61, 8.62, 8.64, and 8.65 are based on T_m; properties in Equations 8.19, 8.20, and 8.21 are based on $T_f \equiv (T_s + T_m)/2$; properties in Equations 8.56 and 8.57 are based on $\overline{T}_m \equiv (T_{m,i} + T_{m,o})/2$.
[b] Equations 8.20 and 8.21 pertain to smooth tubes. For rough tubes, Equation 8.62 should be used with the results of Figure 8.3.
[c] As a first approximation, Equations 8.60, 8.61, or 8.62 may be used to evaluate the average Nusselt number $\overline{Nu}_D$ over the entire tube length, if $(L/D) \gtrsim 10$. The properties should then be evaluated at the average of the mean temperature, $\overline{T}_m \equiv (T_{m,i} + T_{m,o})/2$.
[d] For tubes of noncircular cross section, $Re_D \equiv D_h u_m/\nu$, $D_h \equiv 4A_c/P$, and $u_m = \dot{m}/\rho A_c$. Results for fully developed laminar flow are provided in Table 8.1. For turbulent flow, Equation 8.60 may be used as a first approximation.

TABLE 8.1 Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$ (Uniform q_s'')	(Uniform T_s)	$f\,Re_{D_h}$
	—	4.36	3.66	64
a, b	1.0	3.61	2.98	57
a, b	1.43	3.73	3.08	59
a, b	2.0	4.12	3.39	62
a, b	3.0	4.79	3.96	69
a, b	4.0	5.33	4.44	73
b	8.0	6.49	5.60	82
Heated	∞	8.23	7.54	96
Insulated	∞	5.39	4.86	96
	—	3.11	2.49	53

Used with permission from W. M. Kays and M. E. Crawford, *Convection Heat and Mass Transfer*, 3rd ed. McGraw-Hill, New York, 1993.

Concentric Tube Annulus



$$q_i'' = h_i\left(T_{s,i} - T_m\right)$$
$$q_o'' = h_o\left(T_{s,o} - T_m\right)$$

$$Nu_i \equiv \frac{h_i D_h}{k} \qquad Nu_o \equiv \frac{h_o D_h}{k}$$

FIGURE 8.11

TABLE 8.2 Nusselt number for fully developed laminar flow in a circular tube annulus with one surface insulated and the other at constant temperature

D_i/D_o	Nu_i	Nu_o	*Comments*
0	—	3.66	See Equation 8.55
0.05	17.46	4.06	
0.10	11.56	4.11	
0.25	7.37	4.23	
0.50	5.74	4.43	
≈1.00	4.86	4.86	See Table 8.1, $b/a \rightarrow \infty$

Used with permission from W. M. Kays and H. C. Perkins, in W. M. Rohsenow and J. P. Hartnett, Eds., *Handbook of Heat Transfer*, Chap. 7, McGraw-Hill, New York, 1972.

Chapter 7
External Flows

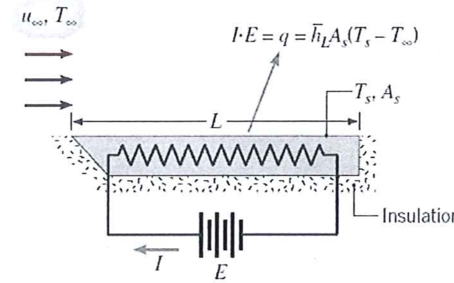


FIGURE 7.1
Experiment for measuring the average convection heat transfer coefficient $\bar{h}_L$.

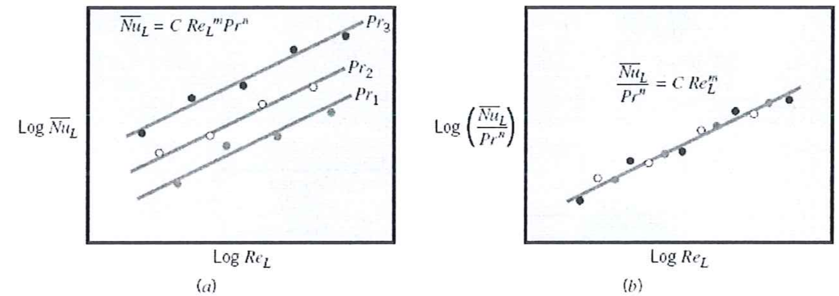
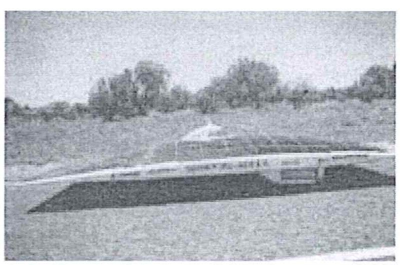
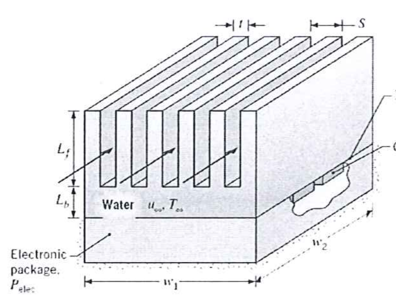
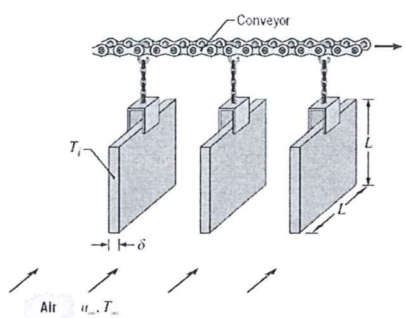


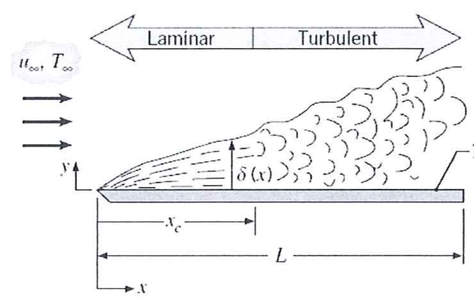
FIGURE 7.2 Dimensionless representation of convection heat transfer measurements.

Flow Over Flat Plate









$$Re_{x,c} \approx 5 \times 10^5$$

FIGURE 7.3
The flat plate in parallel flow.

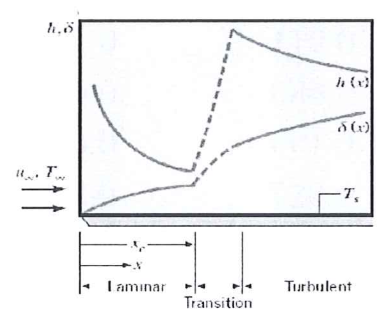
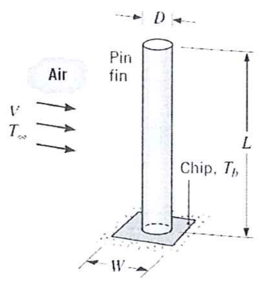
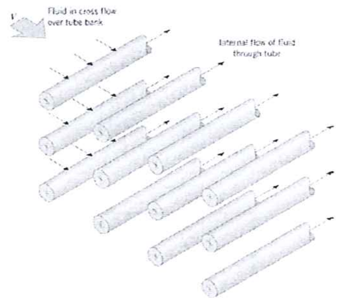


FIGURE 6.6
Variation of velocity boundary layer thickness δ and the local heat transfer coefficient h for flow over an isothermal flat plate.

Cylinder/Sphere in Cross Flow





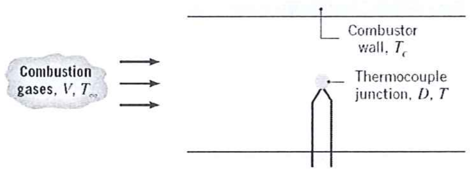


TABLE 7.9 Summary of convection heat transfer correlations for external flow[a]

Correlation		Geometry	Conditions[b]
$\delta = 5x\,Re_x^{-1/2}$	(7.17)	Flat plate	Laminar, T_f
$C_{f,x} = 0.664 Re_x^{-1/2}$	(7.18)	Flat plate	Laminar, local, T_f
$Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$	(7.21)	Flat plate	Laminar, local, T_f, $Pr \gtrsim 0.6$
$\delta_t = \delta Pr^{-1/3}$	(7.22)	Flat plate	Laminar, T_f
$\overline{C}_{f,x} = 1.328 Re_x^{-1/2}$	(7.24)	Flat plate	Laminar, average, T_f
$\overline{Nu}_x = 0.664 Re_x^{1/2} Pr^{1/3}$	(7.25)	Flat plate	Laminar, average, T_f, $Pr \gtrsim 0.6$
$Nu_x = 0.565 Pe_x^{1/2}$	(7.26)	Flat plate	Laminar, local, T_f, $Pr \lesssim 0.05$, $Pe_x \gtrsim 100$
$C_{f,x} = 0.0592 Re_x^{-1/5}$	(7.28)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$
$\delta = 0.37x\,Re_x^{-1/5}$	(7.29)	Flat plate	Turbulent, T_f, $Re_x \lesssim 10^8$
$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$	(7.30)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{C}_{f,L} = 0.074 Re_L^{-1/5} - 1742 Re_L^{-1}$	(7.33)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$
$\overline{Nu}_L = (0.037 Re_L^{4/5} - 871) Pr^{1/3}$	(7.31)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{Nu}_D = C\,Re_D^m Pr^{1/3}$ (Table 7.2)	(7.44)	Cylinder	Average, T_f, $0.4 \lesssim Re_D \lesssim 4 \times 10^5$, $Pr \gtrsim 0.7$
$\overline{Nu}_D = C\,Re_D^m Pr^n (Pr/Pr_s)^{1/4}$ (Table 7.4)	(7.45)	Cylinder	Average, T_∞, $1 \lesssim Re_D \lesssim 10^6$, $0.7 \lesssim Pr \lesssim 500$
$\overline{Nu}_D = 0.3 + [0.62 Re_D^{1/2} Pr^{1/3} \times [1 + (0.4/Pr)^{2/3}]^{-1/4}] \times [1 + (Re_D/282{,}000)^{5/8}]^{4/5}$	(7.46)	Cylinder	Average, T_f, $Re_D Pr \gtrsim 0.2$
$\overline{Nu}_D = 2 + (0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}) Pr^{0.4} \times (\mu/\mu_s)^{1/4}$	(7.48)	Sphere	Average, T_∞, $3.5 \lesssim Re_D \lesssim 7.6 \times 10^4$, $0.71 \lesssim Pr \lesssim 380$
$\overline{Nu}_D = 2 + 0.6 Re_D^{1/2} Pr^{1/3}$	(7.49)	Falling drop	Average, T_∞
$\overline{Nu}_D = 1.13 C_1 C_2 Re_{D,max}^m Pr^{1/3}$ (Tables 7.5, 7.6)	(7.52), (7.53)	Tube bank[c]	Average, $\overline{T}_f$, $2000 \lesssim Re_{D,max} \lesssim 4 \times 10^4$, $Pr \gtrsim 0.7$
$\overline{Nu}_D = C C_2 Re_{D,max}^m Pr^{0.36} (Pr/Pr_s)^{1/4}$ (Tables 7.7, 7.8)	(7.56), (7.57)	Tube bank[c]	Average, $\overline{T}$, $1000 \lesssim Re_D \lesssim 2 \times 10^6$, $0.7 \lesssim Pr \lesssim 500$

TABLE 7.2 Constants of Equation 7.44 for the circular cylinder in cross flow [11, 12]

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

TABLE 7.3 Constants of Equation 7.44 for noncircular cylinders in cross flow of a gas [13]

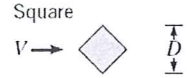
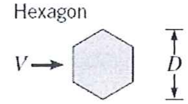
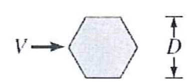
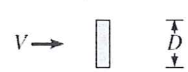
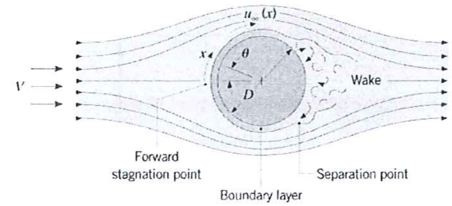
Geometry	Re_D	C	m
Square			
	5×10^3–10^5	0.246	0.588
	5×10^3–10^5	0.102	0.675
Hexagon			
	5×10^3–1.95×10^4	0.160	0.638
	1.95×10^4–10^5	0.0385	0.782
	5×10^3–10^5	0.153	0.638
Vertical plate			
	4×10^3–1.5×10^4	0.228	0.731

TABLE 7.4 Constants of Equation 7.45 for the circular cylinder in cross flow [16]

Re_D	C	m
1–40	0.75	0.4
40–1000	0.51	0.5
10^3–2×10^5	0.26	0.6
2×10^5–10^6	0.076	0.7

- Conditions depend on special features of boundary layer development, including onset at a stagnation point and separation, as well as transition to turbulence.

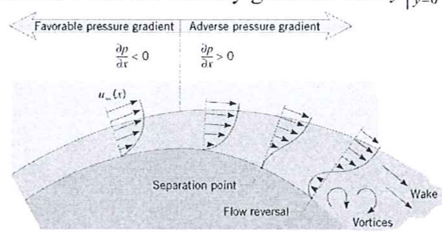


– Stagnation point: Location of zero velocity $(u_\infty = 0)$ and maximum pressure.

– Followed by boundary layer development under a favorable pressure gradient $(dp/dx < 0)$ and hence acceleration of the free stream flow $(du_\infty/dx > 0)$.

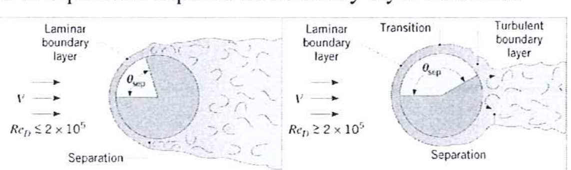
– As the rear of the cylinder is approached, the pressure must begin to increase. Hence, there is a minimum in the pressure distribution, *p(x)*, after which boundary layer development occurs under the influence of an adverse pressure gradient $(dp/dx > 0,\ du_\infty/dx < 0)$.

– Separation occurs when the velocity gradient $du/dy|_{y=0}$ reduces to zero.



and is accompanied by flow reversal and a downstream wake.

– Location of separation depends on boundary layer transition.



$$Re_D \equiv \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

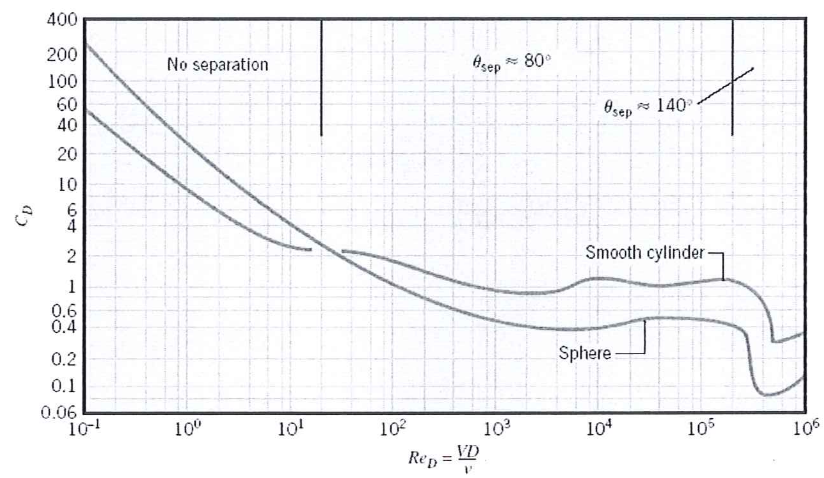


FIGURE 7.8 Drag coefficients for a smooth circular cylinder in cross flow and for a sphere [2]. Boundary layer separation angles are for a cylinder. Adapted with permission.

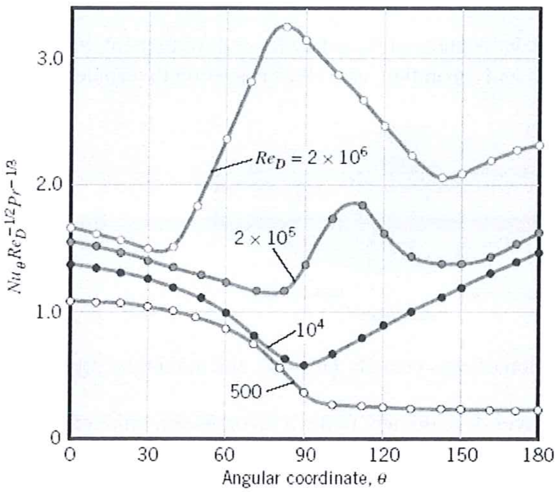


FIGURE 7.9 Local Nusselt number for airflow normal to a circular cylinder. Adapted with permission from Zukauskas, A., "Convective Heat Transfer in Cross Flow," in S. Kakac, R. K. Shah, and W. Aung, Eds., *Handbook of Single-Phase Convective Heat Transfer*, Wiley, New York, 1987.

①

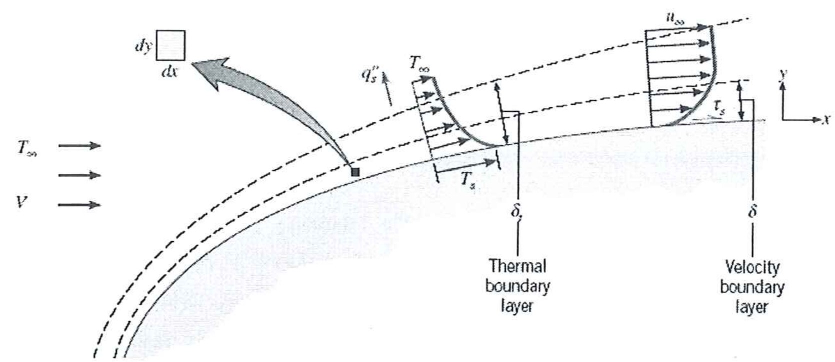


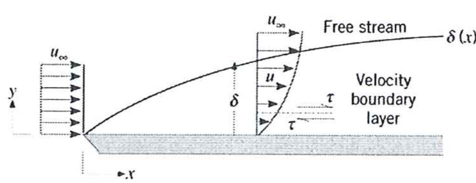
FIGURE 6.7 Development of the velocity and thermal boundary layers for an arbitrary surface.

Chapter 6
Introduction to Convection

Boundary Layers: Physical Features

- Velocity Boundary Layer

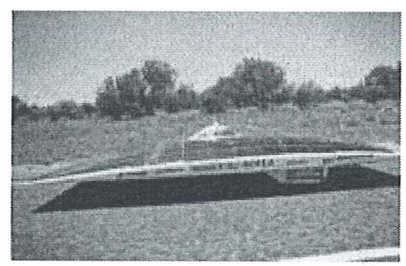
  - A consequence of viscous effects associated with relative motion between a fluid and a surface.
  - A flow region characterized by shear stresses and velocity gradients.
  - A region between the surface and the free stream whose thickness δ increases in the flow direction.
  - Manifested by a surface shear stress τ_s that provides a drag force, F_D

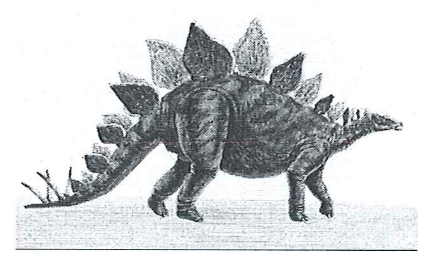


$$\delta \rightarrow \frac{u(y)}{u_\infty} = 0.99$$

$$\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$F_D = \int_{A_s} \tau_s dA_s$$





PROBLEM 6.25

KNOWN: Air, water, engine oil or mercury at 300K in laminar, parallel flow over a flat plate

FIND: Sketch of velocity and thermal boundary layer thickness

ASSUMPTIONS: (1) Laminar flow.

PROPERTIES: For the fluids at 300K:

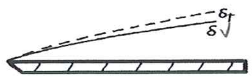
Fluid	Table	Pr
Air	A.4	0.71
Water	A.6	5.83
Engine Oil	A.5	6400
Mercury	A.5	0.025

ANALYSIS: For laminar, boundary layer flow over a flat plate.

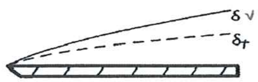
$$\frac{\delta}{\delta_t} \sim Pr^n$$

where n > 0. Hence, the boundary layers appear as shown below

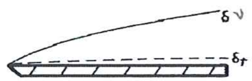
Air:



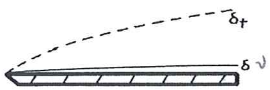
Water:



Engine Oil:



Mercury:



COMMENTS: Although Pr strongly influences relative boundary layer development in laminar flow, its influence is weak for turbulent flow.

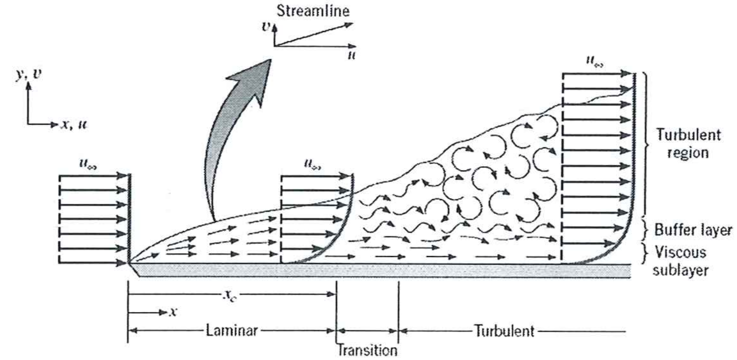


FIGURE 6.4 Velocity boundary layer development on a flat plate.

$$Re_{x,c} \equiv \frac{\rho u_\infty x_c}{\mu} \rightarrow \text{critical Reynolds number}$$

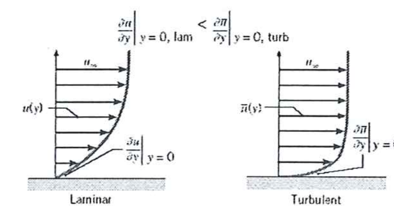


FIGURE 6.5 Comparison of laminar and turbulent velocity boundary layer profiles for the same free stream velocity.[1]

Boundary Layer Similarity

- Dependent boundary layer variables of interest are: τ_s and q'' or h

- For a prescribed geometry, the corresponding independent variables are:

Geometrical: Size (L), Location (x,y)

Hydrodynamic: Velocity (V)

Fluid Properties: Hydrodynamic: ρ, μ

Thermal: c_p, k

Hence,

$$u = f(x, y, L, V, \rho, \mu)$$

$$\tau_s = f(x, L, V, \rho, \mu)$$

and

$$T = f(x, y, L, V, \rho, \mu, c_p, k)$$

$$h = f(x, L, V, \rho, \mu, c_p, k)$$

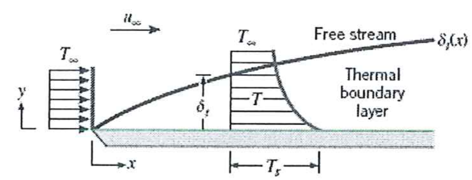
- Thermal Boundary Layer

  - A consequence of heat transfer between the surface and fluid.

  - A region of the flow characterized by temperature gradients and heat fluxes.

  - A region between the surface and the free stream whose thickness δ_t increases in the flow direction.

  - Manifested by a surface heat flux q_s'' and a convection heat transfer coefficient h .



$$\delta_t \rightarrow \frac{T_s - T(y)}{T_s - T_\infty} = 0.99$$

$$q_s'' = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$h \equiv \frac{-k_f \partial T / \partial y|_{y=0}}{T_s - T_\infty}$$

The Reynolds Analogy

- Equivalence of dimensionless momentum and energy equations for negligible pressure gradient (dp*/dx*~0) and Pr~1:

$$\underbrace{u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}}_{\text{Advection terms}}=\underbrace{\frac{1}{\mathrm{Re}}\frac{\partial^2 u^*}{\partial y^{*2}}}_{\text{Diffusion}}$$

$$\overbrace{u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}}=\overbrace{\frac{1}{\mathrm{Re}}\frac{\partial^2 T^*}{\partial y^{*2}}}$$

- Hence, for equivalent boundary conditions, the solutions are of the same form:

$$u^*=T^*$$

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}=\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

$$C_f\frac{\mathrm{Re}}{2}=Nu$$

or, with the Stanton number defined as,

$$St\equiv\frac{h}{\rho V c_p}=\frac{Nu}{Re\,Pr}$$

With Pr = 1, the Reynolds analogy, which relates important parameters of the velocity and thermal boundary layers, is

$$\cancel{\frac{C_f}{2}=St}$$

- Modified Reynolds (Chilton-Colburn) Analogy:
  - An empirical result that extends applicability of the Reynolds analogy:

$$\frac{C_f}{2}=\cancel{St\,Pr^{2/3}}=\frac{Nu}{Re\,Pr^{1/3}}\quad 0.6<\mathrm{Pr}<60$$

  - Applicable to laminar flow if dp*/dx* ~ 0.
  - Generally applicable to turbulent flow without restriction on dp*/dx*.

- Recast the boundary layer equations by introducing dimensionless forms of the independent and dependent variables.

$$x^*\equiv\frac{x}{L}\qquad y^*\equiv\frac{y}{L}$$

$$u^*\equiv\frac{u}{V}\qquad v^*\equiv\frac{v}{V}$$

$$T^*\equiv\frac{T-T_s}{T_\infty-T_s}$$

- Neglecting viscous dissipation, the following normalized forms of the x-momentum and energy equations are obtained:

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{dp^*}{dx^*}+\frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{1}{\mathrm{Re}_L\,\mathrm{Pr}}\frac{\partial^2 T^*}{\partial y^{*2}}$$

- For a prescribed geometry (dP/dx is known):

$$u^*=f\left(x^*,y^*,\mathrm{Re}_L\right)\qquad T^*=f\left(x^*,y^*,\mathrm{Re}_L,\mathrm{Pr}\right)$$

$$\tau_s=\mu\left.\frac{\partial u}{\partial y}\right|_{y=0}=\left(\frac{\mu V}{L}\right)\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

$$h=\frac{-k_f\,\partial T/\partial y|_{y=0}}{T_s-T_\infty}=-\frac{k_f}{L}\frac{(T_\infty-T_s)}{(T_s-T_\infty)}\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}=+\frac{k_f}{L}\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

$$C_f\equiv\frac{\tau_s}{\rho V^2/2}=\frac{2}{\mathrm{Re}_L}\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}\qquad Nu\equiv\frac{hL}{k_f}=\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}=f\left(x^*,\mathrm{Re}_L\right)\qquad \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}=f\left(x^*,\mathrm{Re}_L,\mathrm{Pr}\right)$$

$$C_{f,x}=\frac{2}{\mathrm{Re}_L}f\left(x^*,\mathrm{Re}_L\right)=f\left(x^*,\mathrm{Re}_L\right)\qquad Nu_x=f\left(x^*,\mathrm{Re}_L,\mathrm{Pr}\right)$$

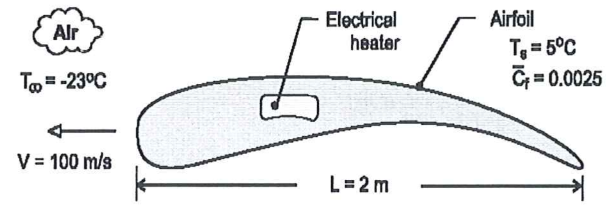
$$\overline{C_f}=f\left(\mathrm{Re}_L\right)\qquad \overline{Nu}=f\left(\mathrm{Re}_L,\mathrm{Pr}\right)$$

PROBLEM 6.43

KNOWN: Nominal operating conditions of aircraft and characteristic length and average friction coefficient of wing.

FIND: Average heat flux needed to maintain prescribed surface temperature of wing.

SCHEMATIC:



ASSUMPTIONS: (1) Applicability of modified Reynolds analogy, (2) Constant properties.

PROPERTIES: Prescribed. Air: $\nu = 16.3 \times 10^{-6}$ m^2/s, k = 0.022 W/m·K, Pr = 0.72.

(1)

Chapter 5
Transient Conduction

(2)

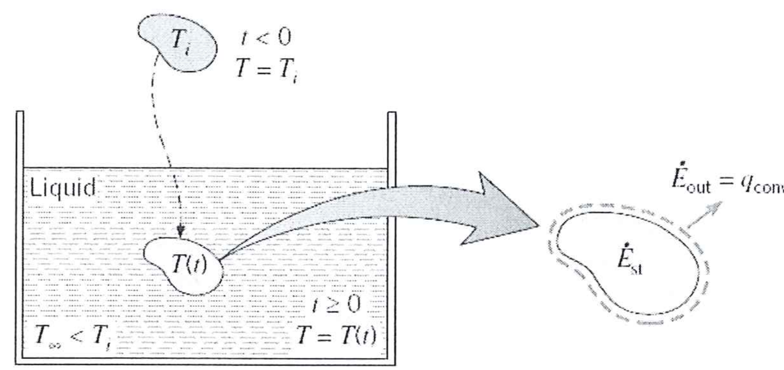


FIGURE 5.1 Cooling of a hot metal forging.

(3)

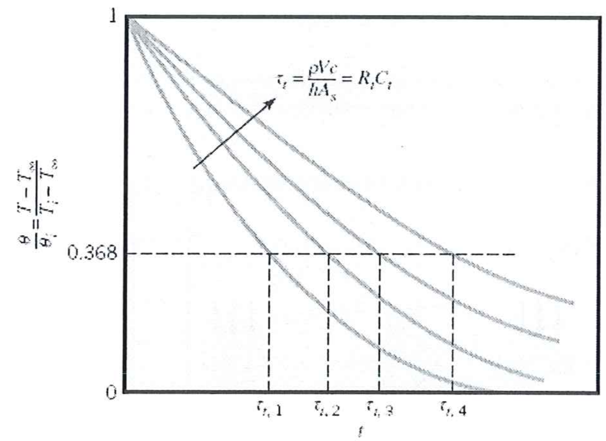


FIGURE 5.2 Transient temperature response of lumped capacitance solids for different thermal time constants τ_t.

(4)

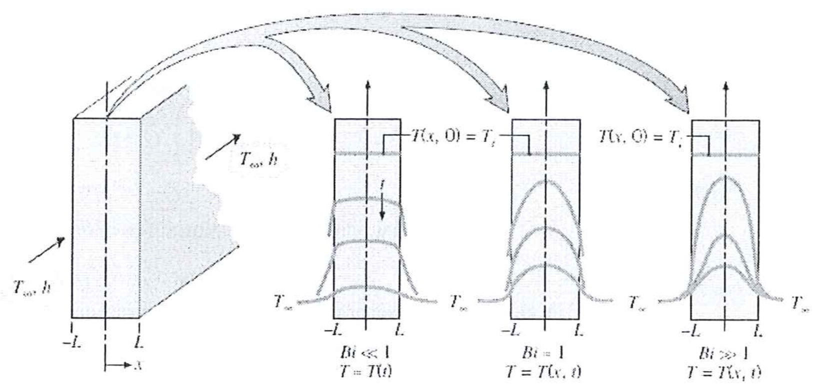


FIGURE 5.4 Transient temperature distributions for different Biot numbers in a plane wall symmetrically cooled by convection.

5

Solution to the Heat Equation for a Plane Wall with Symmetrical Convection Conditions

- If the lumped capacitance approximation can not be made, consideration must be given to spatial, as well as temporal, variations in temperature during the transient process.

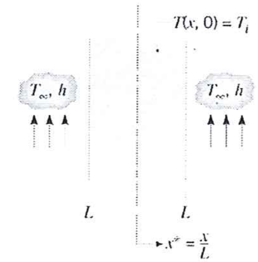
  - For a plane wall with symmetrical convection conditions and constant properties, the heat equation and initial/boundary conditions are:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (5.26)$$

$$T(x,0) = T_i \quad (5.27)$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \quad (5.28)$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty\right] \quad (5.29)$$



  - Existence of seven independent variables:

$$T = T(x, t, T_i, T_\infty, k, \alpha, h) \quad (5.30)$$

  How may the functional dependence be simplified?

6

- Non-dimensionalization of Heat Equation and Initial/Boundary Conditions:

  Dimensionless temperature difference: $\theta^* \equiv \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty}$

  Dimensionless coordinate: $x^* \equiv \frac{x}{L_c}$

  Dimensionless time: $t^* \equiv \frac{\alpha t}{L_c^2} \equiv Fo$ The Fourier Number

  The Biot Number: $Bi \equiv \frac{hL}{k_{solid}}$

$$\theta^* = f(x^*, Fo, Bi)$$

- Exact Solution:

$$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 Fo)\cos(\zeta_n x^*) \quad (5.39a)$$

$$C_n = \frac{4\sin\zeta_n}{2\zeta_n + \sin(2\zeta_n)} \qquad \zeta_n \tan\zeta_n = Bi \quad (5.39b,c)$$

See Appendix B.3 for first four roots (Eigenvalues $\zeta_1, \ldots, \zeta_4$) of Eq. (5.39c)

7

- The One-Term Approximation $(Fo > 0.2)$:
  - ➢ Variation of midplane temperature ($x^* = 0$) with time(Fo) :

$$\theta_o^* \equiv \frac{(T_o - T_\infty)}{(T_i - T_\infty)} \approx C_1 \exp(-\zeta_1^2 Fo) \quad (5.41)$$

  Table 5.1 $\rightarrow$ C_1 and ζ_1 as a function of Bi Page 274

  - ➢ Variation of temperature with location (x^*) and time(Fo):

$$\theta^* = \theta_o^* \cos(\zeta_1 x^*) \quad (5.40b)$$

  - ➢ Change in thermal energy storage with time:

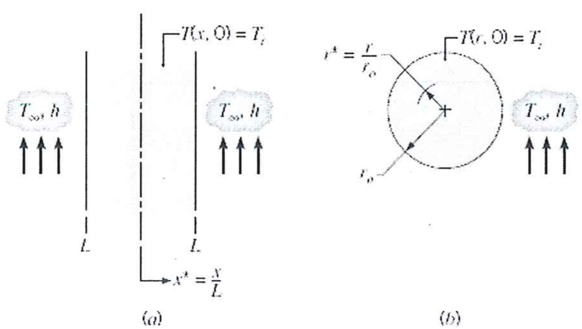
$$\Delta E_{st} = -Q \quad (5.43a)$$

$$Q = Q_o\left(1 - \frac{\sin\zeta_1}{\zeta_1}\theta_o^*\right) \quad (5.46)$$

$$Q_o = \rho c V(T_i - T_\infty) \quad (5.44)$$

Can the foregoing results be used for a plane wall that is well insulated on one side and convectively heated or cooled on the other?

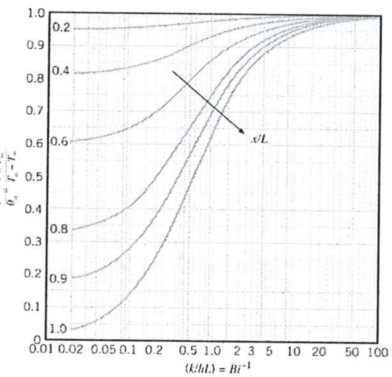
8



$L_c = \frac{V}{A_s} = \begin{cases} L & \text{plane wall thickness } 2L \\ r_o/2 & \text{long cyl} \\ r_o/3 & \text{sphere} \end{cases}$

FIGURE 5.6 One-dimensional systems with an initial uniform temperature subjected to sudden convection conditions. (a) Plane wall. (b) Infinite cylinder or sphere.

- Temperature Distribution:



- Change in Thermal Energy Storage:

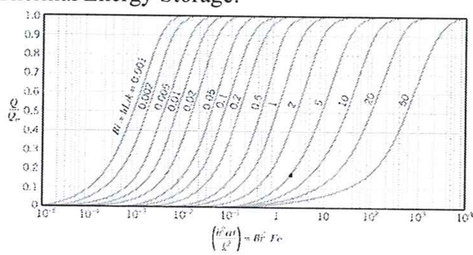


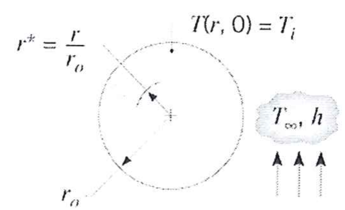
Table 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

	Plane Wall		Infinite Cylinder		Sphere	
Bi^a	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.05	0.2218	1.0082	0.3143	1.0124	0.3854	1.0149
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.07	0.2615	1.0114	0.3709	1.0173	0.4551	1.0209
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.09	0.2956	1.0145	0.4195	1.0222	0.5150	1.0268
0.10	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.15	0.3779	1.0237	0.5376	1.0365	0.6609	1.0445
0.20	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.25	0.4801	1.0382	0.6856	1.0598	0.8447	1.0737
0.30	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0932	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5994	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2402	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2733	2.4050	1.6018	3.1415	2.0000

$^a Bi = hL/k$ for the plane wall and hr_o/k for the infinite cylinder and sphere. See Figure 5.6.

Radial Systems

- Long Rods or Spheres Heated or Cooled by Convection.

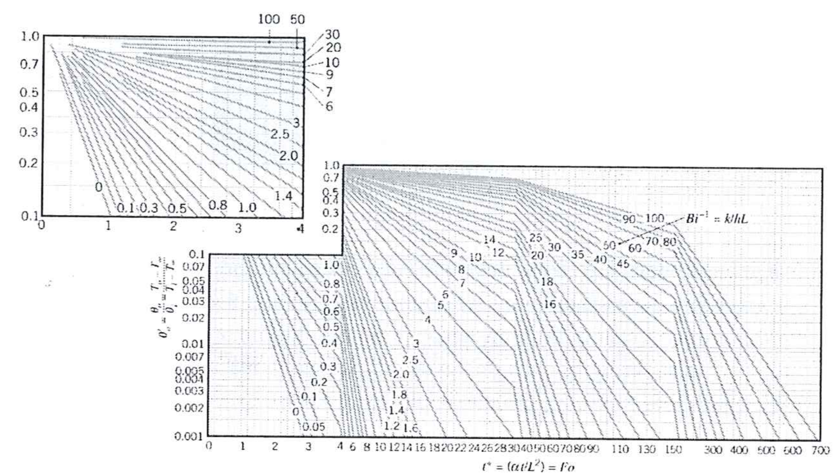


- One-Term Approximations:
  Long Rod: Eqs. (5.49) and (5.51)
  Sphere: Eqs. (5.50) and (5.52)

- Graphical Representations:
  Long Rod: Figs. 5 S.4 – 5 S.6
  Sphere: Figs. 5 S.7 – 5 S.9

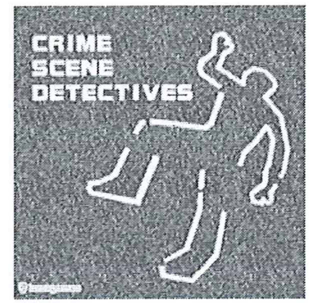
Graphical Representation of the One-Term Approximation The Heisler Charts, Section 5 S.1

- Midplane Temperature:



CSI Rolla!

A person is found dead with a body center temperature of 25 °C in a room at 20 °C. The heat transfer coefficient is estimated to be h = 8 W/m^2. °C. Rolla Police Department asked you to predict the time of death. Modeling the body as a 30-cm-diameter and 1.70-m-long cylinder, and assuming the body to have the properties of water, estimate how long the person has been dead. Note: The temperature of a healthy body is 37 °C.



1

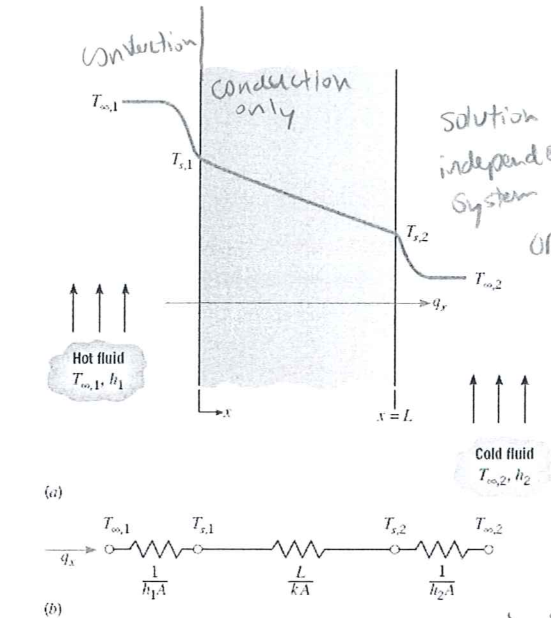


FIGURE 3.1 Heat transfer through a plane wall. (a) Temperature distribution. (b) Equivalent thermal circuit.

solution will be independent of coord. System (we can place origin wherever)

$\boxed{T = C_1 x + C_2}$

BC.'s

1) $T(x=0) = T_{s1}$

$T_{s1} = C_1 \cdot 0 + C_2 \Rightarrow C_2 = T_{s1}$

2) $T(x=L) = T_{s2}$

$T_{s2} = C_1 \cdot L + C_2 \Rightarrow C_1 = \frac{T_{s2}-T_{s1}}{L}$

$T = \frac{T_{s2}-T_{s1}}{L} x + T_1$

$\Rightarrow q''_x = -k\frac{dT}{dx} = -kC_1 = -k\frac{T_{s2}-T_{s1}}{L} = \frac{k}{L}(T_{s1}-T_{s2}) \neq f(x) = \text{const}$

continued in notes Jan 20

Chapter 3
One-Dimensional, Steady-State Conduction

-Without Thermal Energy Generation (Thermal Resistance)

-With Thermal Energy Generation

-Extended Surfaces

2

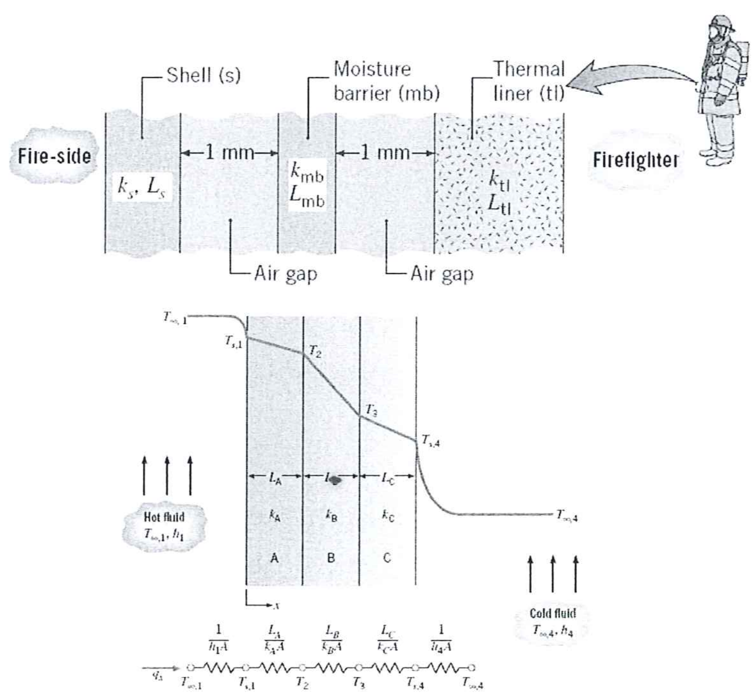


FIGURE 3.2 Equivalent thermal circuit for a series composite wall.

One-Dimensional, Steady-State Conduction without Thermal Energy Generation

Chapters 3.1 - 3.4

7

Radial Systems

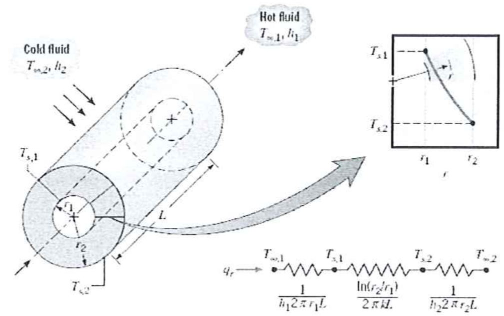


FIGURE 3.6 Hollow cylinder with convective surface conditions.

- For a long cylinder, the temperature gradient in the radial direction is large enough that $T = T(r)$, i.e., 1-D case.
- The fluid temperatures inside and outside the pipe remain constant such that $T \neq T(t)$, i.e., steady conditions.
- Constant properties.

5

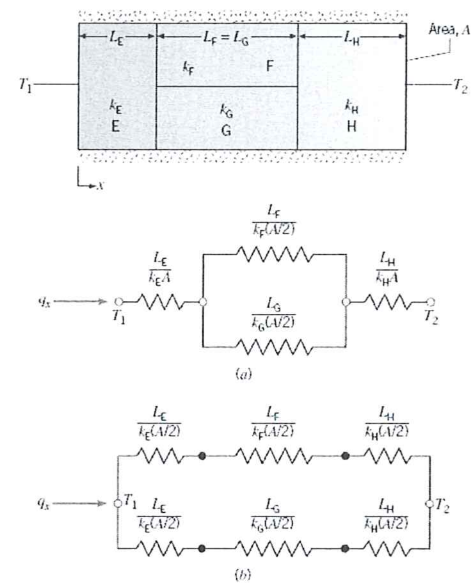


FIGURE 3.3 Equivalent thermal circuits for a series–parallel composite wall.

8

Governing Equation and Simplifications

- The general heat equation in cylindrical coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$

- Because $\partial T/\partial \phi \equiv \partial T/\partial z \equiv \partial T/\partial t = 0$

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)=0$$

know how to integrate this

- Integrating twice,

$$T(r) = C_1 \ln r + C_2$$

- Boundary conditions: $T(r_1) = T_{s,1}$ and $T(r_2) = T_{s,2}$

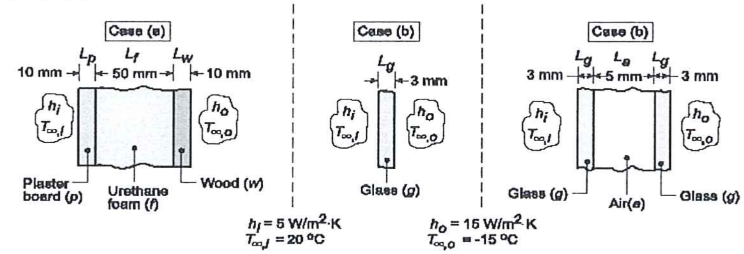
  2 equations and 2 unknowns ⇒ Solve for C_1 and C_2.

6

KNOWN: Configurations of exterior wall. Inner and outer surface conditions.

FIND: Heating load for each of the three cases.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation effects.

PROPERTIES: (T = 300 K): *Table A.3*: plaster board, $k_p = 0.17$ W/m·K; urethane, $k_f = 0.026$ W/m·K; wood, $k_w = 0.12$ W/m·K; glass, $k_g = 1.4$ W/m·K. *Table A.4*: air, $k_a = 0.0263$ W/m·K.

foam governs b/c because high thermal resistance

(9)

Thermal Resistance

- Thermal resistance for conduction is defined as

$$R_{t,cond} = \frac{\Delta T}{\dot{Q}_r} = \frac{\text{Driving temperature potential}}{\text{rate of heat transfer}}$$

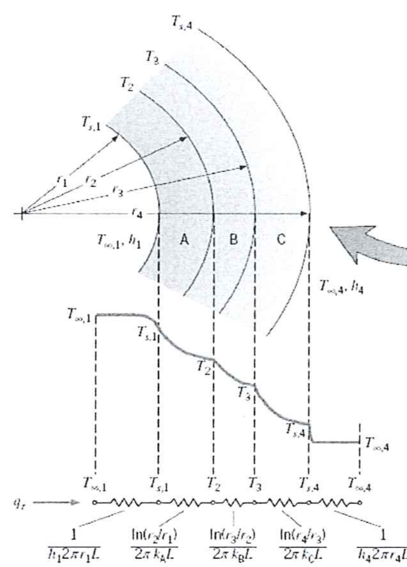
- Then, for a cylinder,

$$R_{t,cond} = \frac{\left(T_{s,1} - T_{s,2}\right)}{\dot{Q}_r} = \frac{\ln\left(r_2/r_1\right)}{2\pi kL}$$

- A similar analysis for a sphere yields,

$$R_{t,cond} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

(10)



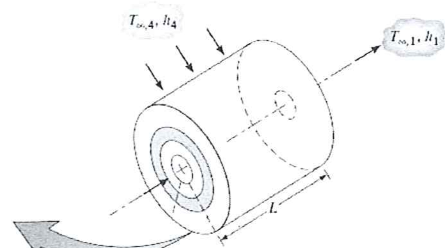


FIGURE 3.7 Temperature distribution for a composite cylindrical wall.

- The heat transfer rate can be found from

$$\dot{Q}_r = \frac{\Delta T}{R_{tot}} = \frac{T_{\infty,1} - T_{\infty,4}}{\sum_i R_{t,i}}$$

where

$$R_{tot} = \frac{1}{h_1 2\pi L r_1} + \frac{\ln(r_2/r_1)}{2\pi k_A L} + \frac{\ln(r_3/r_2)}{2\pi k_B L} + \frac{\ln(r_4/r_3)}{2\pi k_C L} + \frac{1}{h_4 2\pi L r_4}$$

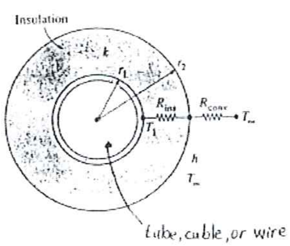
- The overall heat transfer coefficient is

$$UA = \frac{1}{R_{tot}} = \frac{1}{\sum R_{t,i}}$$

(11)

Cylindrical Insulation

- Consider a cylindrical pipe surrounded by an insulation material.



$$\dot{Q} = \frac{T_1 - T_\infty}{R_{tot}}$$

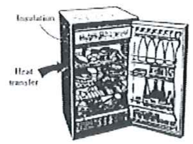
where $R_{tot} = R_{t,ins} + R_{t,conv}$

$$R_{tot} = \frac{\ln(r_2/r_1)}{2\pi kL} + \frac{1}{h 2\pi r_2 L}$$

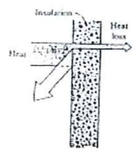
- Question: Is there an insulation radius that minimizes heat loss by maximizing the total resistance to heat heat transfer?

(12)

Thermal Insulation

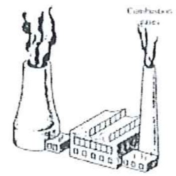


The insulation layers in the walls of a refrigerator reduce the amount of heat flow into the refrigerator and thus the running time of the refrigerator, saving electricity.



Thermal insulation retards heat transfer by acting as a barrier in the path of heat flow.

This reduces not only energy cost but also pollutant emission.



Insulation also helps the environment by reducing the amount of fuel burned and the air pollutants released.

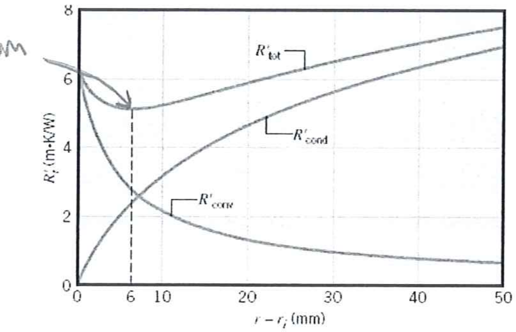
In cold weather, we minimize heat loss from our bodies by putting on thick layers of insulation (coats or furs).

The hood of the engine compartment of a car is insulated to reduce its temperature and to protect people from burning themselves.

(13)

Exhaust wrap?

Minimum



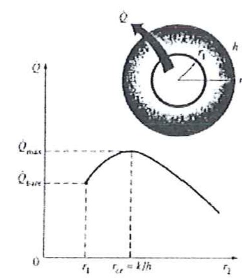
$$\frac{dR_{tot}}{dr_2} = \frac{d}{dr_2}\left[\frac{\ln(r_2/r_1)}{2\pi kL} + \frac{1}{h2\pi r_2 L}\right] = 0$$

$r_{cr} = \dfrac{k}{h}$ for a cylindrical insulation

$r_{cr} = \dfrac{2k}{h}$ for a spherical insulation

- Answer: No, an insulation thickness that minimizes heat loss does *not* exist!

(14)

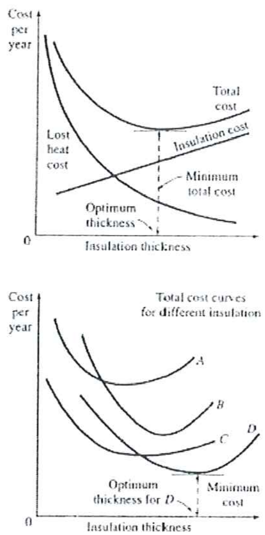


- There is a critical insulation radius, r_{cr}, such that at $r_2 = r_{cr}$, $\dot{Q} = \dot{Q}_{max} > \dot{Q}_{bare}$.
- For $r_2 > r_{cr}$, $\dot{Q}$ decreases,
  For $r_2 < r_{cr}$, $\dot{Q}$ increases (heat loss from a pipe may be increased by insulating it!).
- r_{cr} is only a function of k and h.
- Should we check and make sure that $r_{outer} > r_{cr}$ before installing an insulation?
  Probably not, because r_{cr} is generally on the order of mm.

- For an electric wire, the insulation radius is typically small enough that $r_2 < r_{cr}$.
- In this small-size range, our heat transfer analysis showed that insulation enhances the heat loss from an electric wire.
- Thus, plastic electrical insulation does not only protect us from an electric shock but also increases cooling to keep operating temperature at lower (safer) levels!

(15)

Economic Analysis of Insulations



- For typical engineering applications, $r_2 > r_{cr}$ such that the thicker the insulation, the lower the heat loss. However, this is only based on heat transfer considerations.
- As the insulation radius is increased to reduce the energy cost, the initial investment on insulation material increases at the same time.
- Therefore, we can find an optimum insulation thickness that corresponds to a minimum combined cost of insulation and heat lost.

(16)

One-Dimensional, Steady-State Conduction with Thermal Energy Generation

Chapter 3.5

(19)

Sphere

$$\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)+\dot{g}=0$$

$$\int \frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=\int -\frac{\dot{g}r^2}{k}$$

$$r^2\left(\frac{dT}{dr}\right)=-\frac{\dot{g}r^3}{3k}+C_1$$

Extended Surfaces

$$\frac{dT}{dr}=-\frac{\dot{g}r}{3k}+\frac{C_1}{r^2}$$

$$T(r)=-\frac{\dot{g}r^2}{6k}+\frac{C_1}{r}+C_2$$

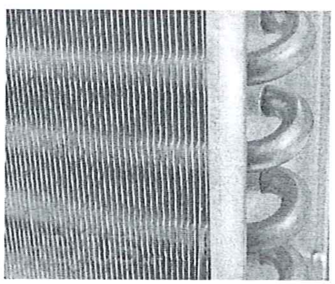
Chapter 3.6

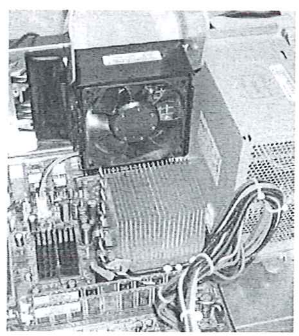
B.C's: $\left.\frac{dT}{dr}\right|_{r=0}=0 \Rightarrow C_1$

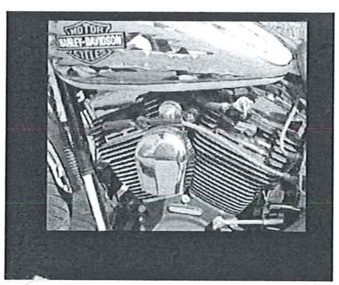
cons. of energy:

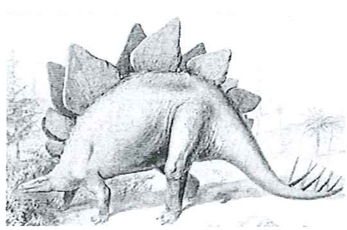
(20) $-k\left.\frac{dT}{dr}\right|_{r=r_s}=h\left(T|_{r=r_s}-T_\infty\right)$ from $-k\frac{dT}{dr}=h(T-T_\infty)$

Examples





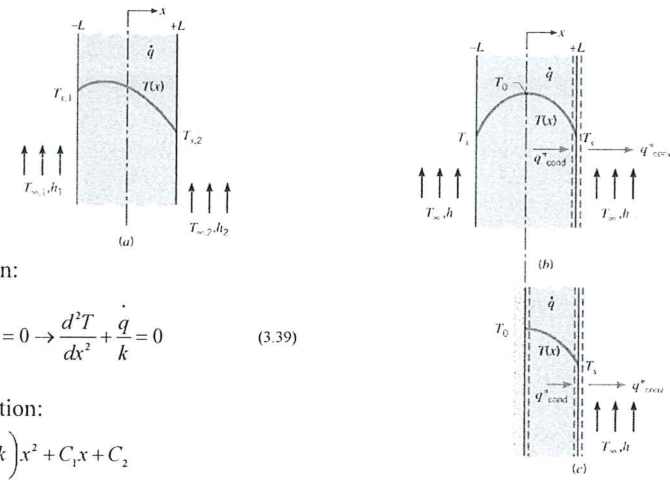




(17)

The Plane Wall

- Consider one-dimensional, steady-state conduction in a plane wall of constant k, uniform generation, and asymmetric surface conditions:



- Heat Equation:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+\dot{q}=0\rightarrow\frac{d^2T}{dx^2}+\frac{\dot{q}}{k}=0 \qquad (3.39)$$

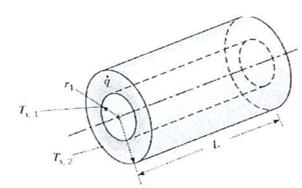
- General Solution:

$$T(x)=-\left(\dot{q}/2k\right)x^2+C_1x+C_2$$

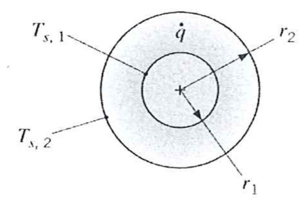
(18)

Radial Systems

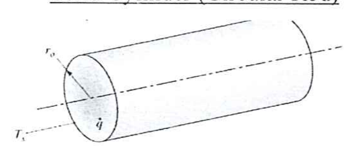
Cylindrical (Tube) Wall



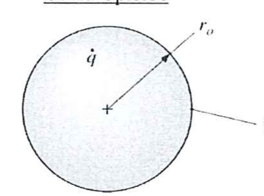
Spherical Wall (Shell)



Solid Cylinder (Circular Rod)



Solid Sphere



- Heat Equations:

Cylindrical

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)+\dot{g}=0$$

integrate wrt r

Spherical

$$\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)+\dot{g}=0$$

23

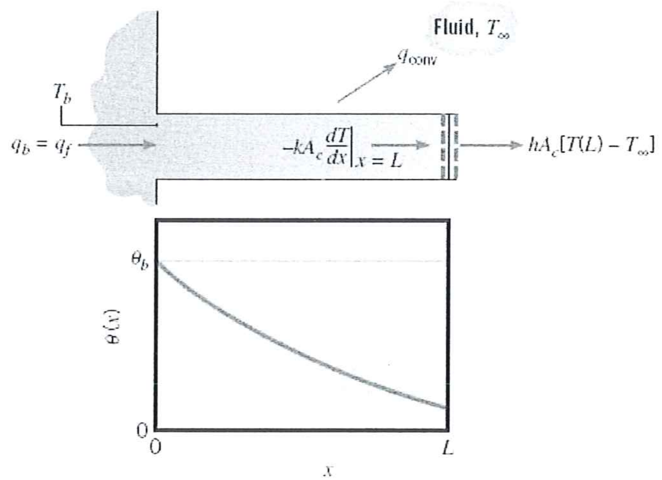


FIGURE 3.17 Conduction and convection in a fin of uniform cross section.

21

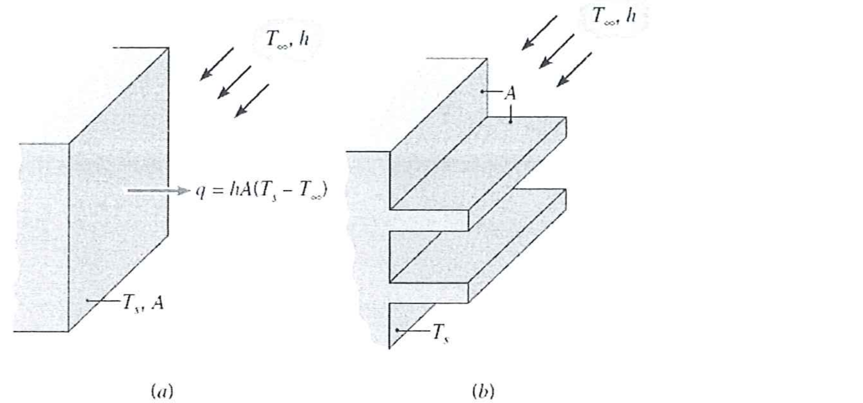


FIGURE 3.12 Use of fins to enhance heat transfer from a plane wall. (a) Bare surface. (b) Finned surface.

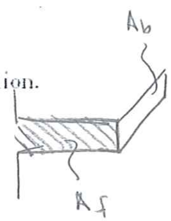
24

$\varepsilon_f A_b = \eta_f A_f$

$\varepsilon = \frac{q_{fins}}{q_{w/o}}$

$A_f = 2WL_c$

$A_b = Wt$



22

$A_f = PL$

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.70)	$M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.72)
B	Adiabatic $d\theta/dx\vert_{x=L} = 0$	$\frac{\cosh m(L_c - x)}{\cosh mL}$ (3.75)	$M\tanh(mL_c)$ (3.76)
C	Prescribed temperature: $\theta(L) = \theta_L$ (fix the tip temp.)	$\frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.77)	$M\frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.78)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx} (3.79)	M (3.80)

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

Won't use really

← will use, good approx

cosh & sinh

replace L with L_c, next page

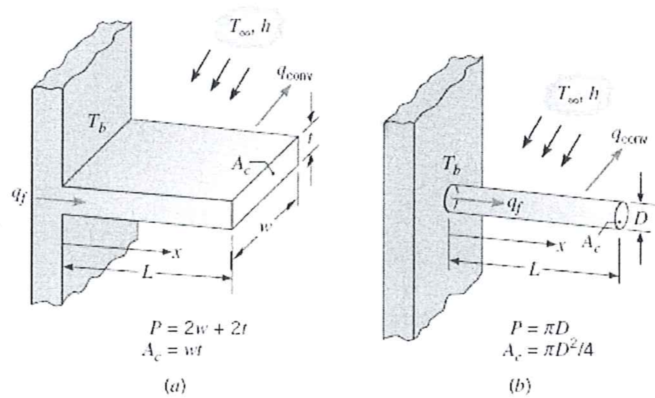


FIGURE 3.16 Straight fins of uniform cross section. (a) Rectangular fin. (b) Pin fin.

27

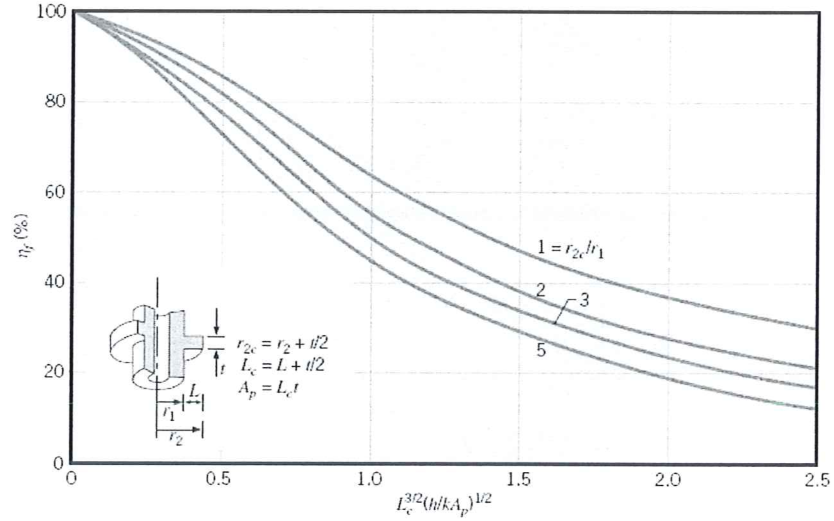


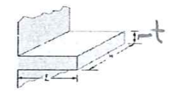
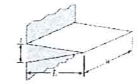
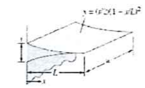
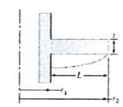
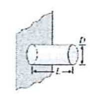
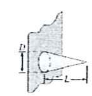
FIGURE 3.19 Efficiency of annular fins of rectangular profile.

Lc corrected Length

25

$L_c = L + \frac{t}{2}$

TABLE 3.5 Efficiency of common fin shapes

Straight Fins			
Rectangular[a] $A_f = 2wL_c$ $L_c = L + (t/2)$ $A_p = tL$	 t	$\eta_f = \frac{\tanh mL_c}{mL_c}$	(3.89)
Triangular[a] $A_f = 2w[L^2 + (t/2)^2]^{1/2}$ $A_p = (t/2)L$		$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$	(3.93)
Parabolic[a] $A_f = w[C_1L + (L^2/t)\ln(t/L + C_1)]$ $C_1 = [1 + (t/L)^2]^{1/2}$ $A_p = (t/3)L$	 $y = (t/2)(1 - x/L)^2$	$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$	(3.94)
Circular Fin			
Rectangular[a] $A_f = 2\pi(r_{2c}^2 - r_1^2)$ $r_{2c} = r_2 + (t/2)$ $V = \pi(r_2^2 - r_1^2)t$		$\eta_f = C_2\frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$ $C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$	(3.91)
Pin Fins			
Rectangular[b] $A_f = \pi DL_c$ $L_c = L + (D/4)$ $V = (\pi D^2/4)L$	 $L_c = L + \frac{D}{4}$	$\eta_f = \frac{\tanh mL_c}{mL_c}$	(3.95)
Triangular[b] $A_f = \frac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$ $V = (\pi/12)D^2L$		$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$	(3.96)

28

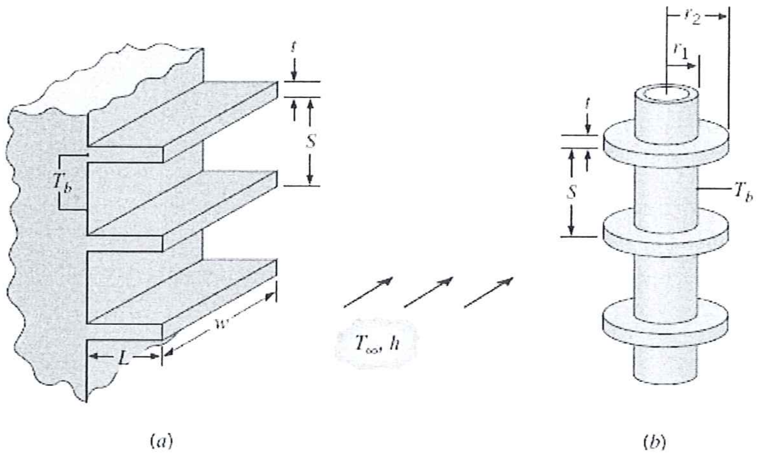


FIGURE 3.20 Representative fin arrays. (*a*) Rectangular fins. (*b*) Annular fins.

26

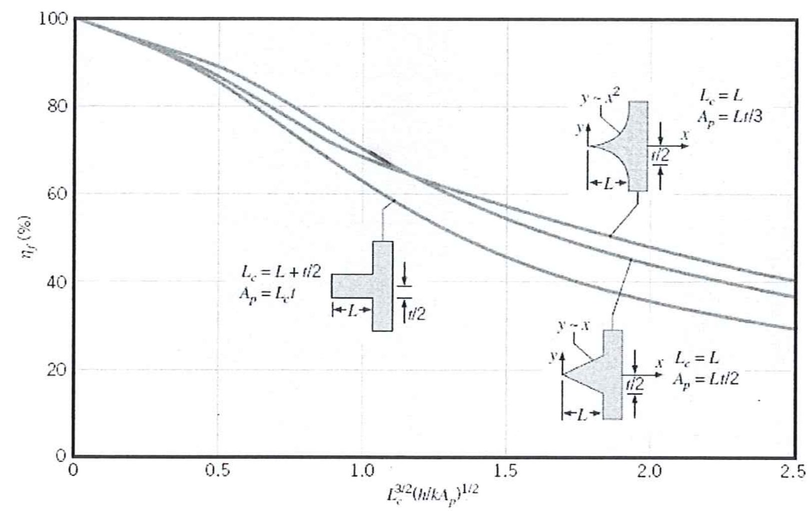


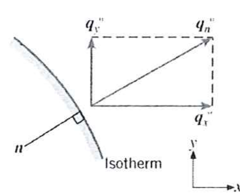
FIGURE 3.18 Efficiency of straight fins (rectangular, triangular, and parabolic profiles).

Chapter 2
Introduction to Conduction

Fourier's Law

- A rate equation that allows determination of the conduction heat flux from knowledge of the temperature distribution in a medium
- Its most general (vector) form for multidimensional conduction is:

$$\vec{q}'' = -k\vec{\nabla} T$$



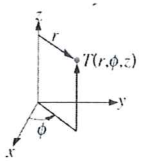
Implications:

- Heat transfer is in the direction of decreasing temperature (basis for minus sign).
- Direction of heat transfer is perpendicular to lines of constant temperature (isotherms).
- Heat flux vector may be resolved into orthogonal components

---

- Cartesian Coordinates: $T(x, y, z)$

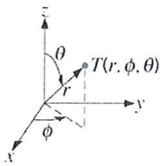
$$\vec{q}'' = \underbrace{-k\frac{\partial T}{\partial x}\vec{i}}_{q_x''} \underbrace{-k\frac{\partial T}{\partial y}\vec{j}}_{q_y''} \underbrace{-k\frac{\partial T}{\partial z}\vec{k}}_{q_z''} \qquad (2.3)$$



- Cylindrical Coordinates: $T(r, \phi, z)$

$$\vec{q}'' = \underbrace{-k\frac{\partial T}{\partial r}\vec{i}}_{q_r''} \underbrace{-k\frac{\partial T}{r\partial \phi}\vec{j}}_{q_\phi''} \underbrace{-k\frac{\partial T}{\partial z}\vec{k}}_{q_z''} \qquad (2.22)$$

$$q_r = A_r q_r'' = 2\pi r L q_r''$$



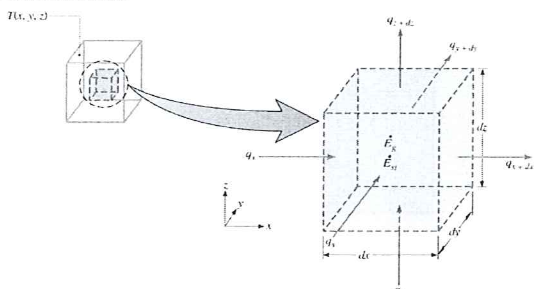
- Spherical Coordinates: $T(r, \phi, \theta)$

$$\vec{q}'' = \underbrace{-k\frac{\partial T}{\partial r}\vec{i}}_{q_r''} \underbrace{-k\frac{\partial T}{r\partial \theta}\vec{j}}_{q_\theta''} \underbrace{-k\frac{\partial T}{r\sin\theta\,\partial \phi}\vec{k}}_{q_\phi''} \qquad (2.25)$$

$$q_r = A_r q_r'' = 4\pi r^2 q_r''$$

The Heat Equation

- A differential equation whose solution provides the temperature distribution in a stationary medium.
- Based on applying conservation of energy to a differential control volume through which energy transfer is exclusively by conduction.
- Cartesian Coordinates:



$$\underbrace{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}_{\text{Net transfer of thermal energy into the control volume (inflow-outflow)}} + \underbrace{\dot{q}}_{\text{Thermal energy generation}} = \underbrace{\rho c_p \frac{\partial T}{\partial t}}_{\text{Change in thermal energy storage}} \qquad (2.17)$$

Thermophysical Properties

Thermal conductivity, k: A measure of a material's ability to transfer thermal energy by conduction.

Thermal diffusivity, α: A measure of a material's ability to respond to changes in its thermal environment.

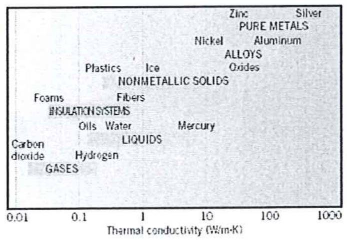


FIGURE 2.4 Range of thermal conductivity for various states of matter at normal temperatures and pressure.

Property Tables:

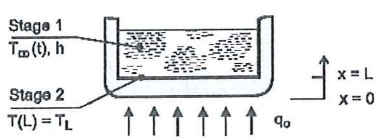
- Solids: Tables A.1 – A.3
- Gases: Table A.4
- Liquids: Tables A.5 – A.7

PROBLEM 2.21

KNOWN: Diameter D, thickness L and initial temperature T_i of pan. Heat rate from stove to bottom of pan. Convection coefficient h and variation of water temperature $T_\infty(t)$ during Stage 1. Temperature T_L of pan surface in contact with water during Stage 2.

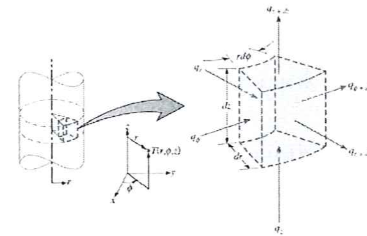
FIND: Form of heat equation and boundary conditions associated with the two stages.

SCHEMATIC:



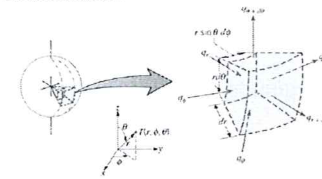
ASSUMPTIONS: (1) One-dimensional conduction in pan bottom, (2) Heat transfer from stove is uniformly distributed over surface of pan in contact with the stove, (3) Constant properties.

- Cylindrical Coordinates:



$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \quad (2.24)$$

- Spherical Coordinates:

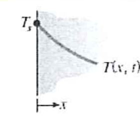


$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \quad (2.27)$$

TABLE 2.2 Boundary conditions for the heat diffusion equation at the surface ($x = 0$)

1. Constant surface temperature

$$T(0,t)=T_s \quad (2.29)$$

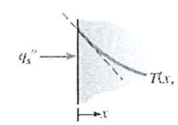


Example: Phase change

2. Constant surface heat flux

(a) Finite heat flux

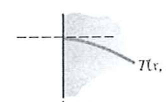
$$-k\frac{\partial T}{\partial x}\Big|_{x=0}=q_s'' \quad (2.30)$$



Example: Heat tape

(b) Adiabatic or insulated surface

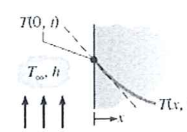
$$\frac{\partial T}{\partial x}\Big|_{x=0}=0 \quad (2.31)$$



Example: Insulation

3. Convection surface condition

$$-k\frac{\partial T}{\partial x}\Big|_{x=0}=h[T_\infty - T(0,t)] \quad (2.32)$$



Example: Car window

Chapter 1
Introduction

Basic Concepts

Heat: Energy that can be transferred from one system to another as a result of temp. difference.

Thermodynamics is concerned with the amount of heat transfer as a system undergoes a process from one *equilibrium* state to another. However, in practice we are more concerned about the rate of heat transfer (heat transfer per unit time) than we are with the amount of it.

Example: How long will it take for hot coffee to cool down from 90 °C to 80 °C?

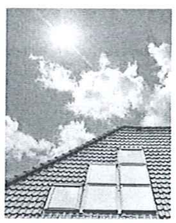
Heat Transfer

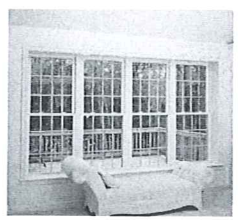
- Determination of rates of heat transfer (cooling/heat time)
- Variation of temperature

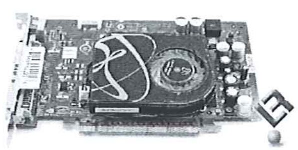
In contrast to thermodynamics, which deals with equilibrium states, *heat transfer* deals with the systems that lack thermal equilibrium and thus it is a *non-equilibrium* phenomenon.

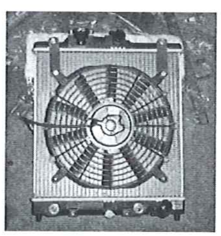
Some Applications of Heat Transfer


Daily life


Solar energy

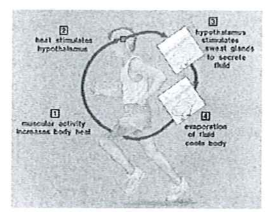

Buildings

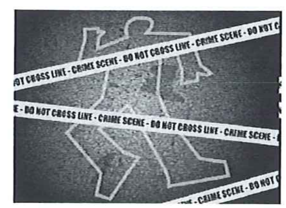

Electronics

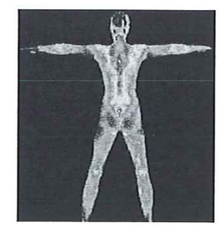

Heat exchangers

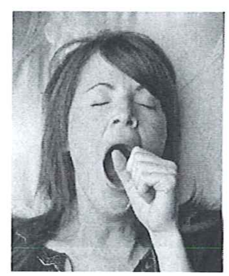

Combustion

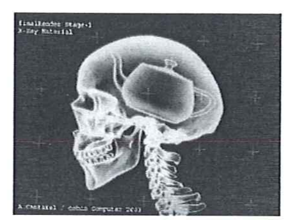
Human Body



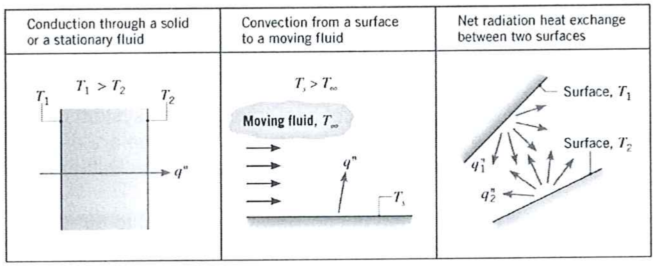








Modes of Heat Transfer



Conduction: Heat transfer in a solid or a stationary fluid (gas or liquid) due to the random motion of its constituent atoms, molecules and /or electrons.

Convection: Heat transfer due to the combined influence of bulk and random motion for fluid flow over a surface.

Radiation: Energy that is emitted by matter due to changes in the electron configurations of its atoms or molecules and is transported as electromagnetic waves (or photons).

- Conduction and convection require the presence of temperature variations in a material medium.
- Although radiation originates from matter, its transport does not require a material medium and occurs most efficiently in a vacuum.

Conduction

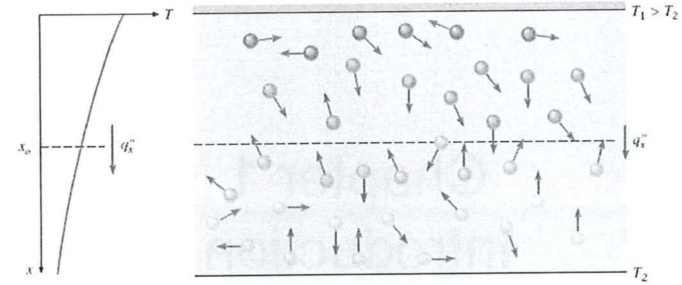
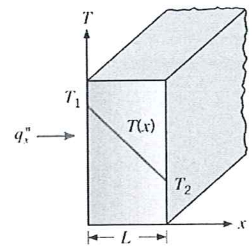


FIGURE 1.2 Association of conduction heat transfer with diffusion of energy due to molecular activity.



Fourier's Law:

$$q''_x = -k\frac{dT}{dx} = -k\frac{T_2 - T_1}{L}$$

Convection

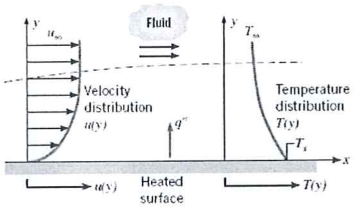


FIGURE 1.4 Boundary layer development in convection heat transfer.

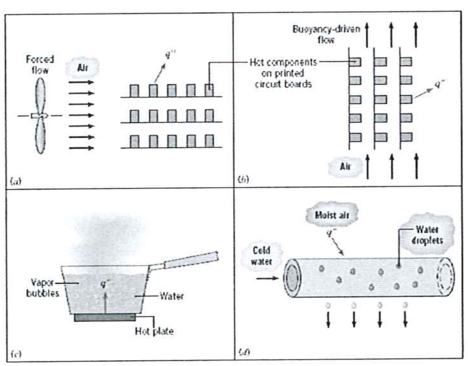


FIGURE 1.5 Convection heat transfer processes. (a) Forced convection. (b) Natural convection. (c) Boili… (d) Condensation.

Newton's law: $q'' = h(T_s - T_\infty)$

TABLE 1.1 Typical values of the convection heat transfer coefficient

Process	h (W/m$^2\cdot$K)
Free convection	
Gases	2–25
Liquids	50–1000
Forced convection	
Gases	25–250
Liquids	100–20,000
Convection with phase change	
Boiling or condensation	2500–100,000

Radiation

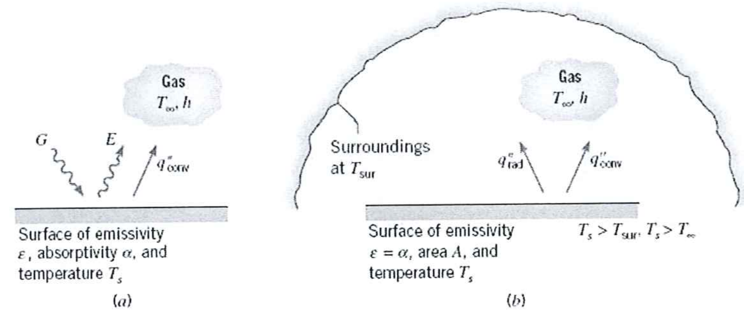


FIGURE 1.6 Radiation exchange: (a) at a surface and (b) between a surface and large surroundings.

Stefan-Boltzmann Law:

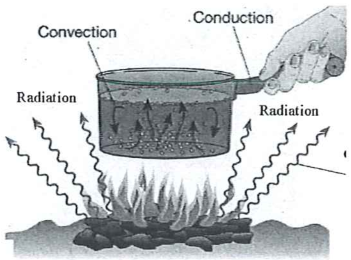
$$E = \varepsilon E_b = \varepsilon\sigma T_s^4$$

If $\alpha = \varepsilon$, the net radiation heat flux from the surface due to exchange with the surroundings is:

$$q''_{rad} = \varepsilon E_b(T_s) - \alpha G = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)$$

TABLE 1.5 Summary of heat transfer processes

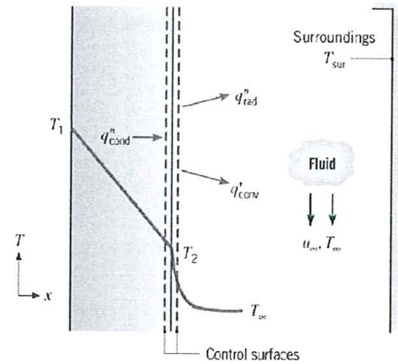
Mode	Mechanism(s)	Rate Equation	Equation Number	Transport Property or Coefficient
Conduction	Diffusion of energy due to random molecular motion	$q''_x\,(\mathrm{W/m^2}) = -k\frac{dT}{dx}$	(1.1)	k (W/m · K)
Convection	Diffusion of energy due to random molecular motion plus energy transfer due to bulk motion (advection)	$q''(\mathrm{W/m^2}) = h(T_s - T_\infty)$	(1.3a)	h (W/m² · K)
Radiation	Energy transfer by electromagnetic waves	$q''(\mathrm{W/m^2}) = \varepsilon\sigma(T_s^4 - T_{sur}^4)$ or $q(\mathrm{W}) = h_r A(T_s - T_{sur})$	(1.7) (1.8)	ε h_r (W/m² · K)



Surface Energy Balance

- With no mass and volume, energy storage and generation are not pertinent to the energy balance of surface.

Consider the surface of a wall with heat transfer by conduction, convection and radiation.



Conservation of Energy:

$$\dot{E}_{in} = \dot{E}_{out}$$

$$k\frac{T_1 - T_2}{L} = h(T_2 - T_\infty) + \varepsilon_2\sigma\left(T_2^4 - T_{sur}^4\right)$$

cond conv radiation

for radial

$$k\left.\frac{dT}{dr}\right|_{r=r_s} = \frac{-\dot{q}\,r_s}{2k}$$

σ & α p67, 68

Projects

ME 225 HEAT TRANSFER

PROJECT 2

Due on April 28, 2011

In a small engineering company, you were given the task of designing a heat exchanger. Like many real-world situations, the customer did not describe, or does not know, additional requirements that would allow you to proceed directly to a final configuration. The only information given were that the heat exchanger was to be used to cool hot water with a mass flow rate of 10 kg/s from 80 °C to 35 °C using cold lake water entering at 25 °C. The main criteria for the design are to minimize size and pumping power.

Initially, you decide to make a first-cut tentative design to meet the above specifications and therefore start with a simple counter-flow configuration of two concentric tubes. List and explain your assumptions, provide equations and models used, explain the heat transfer and other analyses. Your calculations, which should include fouling estimates and material selection, should yield the overall size (necessary surface area), that is, length, tube diameters, number of tubes, as well as the pumping power. The effects of these parameters on the design should be considered by varying the length and tube sizes in an iterative manner in order to minimize heat exchanger size as well as the pump power. After this initial stage, your design should be evaluated by identifying what features and configurations could be explored in order to develop more complete specifications. In doing so, you should propose a more complex configuration such as shell-and-tube. Again, determine the overall size of the final design and suggest your own improvements. You also should report general drawings of your final design as well as cost (initial and operation) estimates. Additionally, you should conclude your report with lessons learned and references used.

Two students are to be teamed up and submit only one professional report of less than 5 total pages entirely prepared on a computer.

100

David Malawey, Zachary Jennings
ME 225 Project 2

Due April 28, 2011

Part 1: concentric-tube heat exchanger

To start part 1, we chose arbitrary pipe diameters and N=10 pipes. We calculated Reynolds number and heat transfer using Matlab. A mass flow rate for cooling water was chosen such that heat capacity rates for cold and hot water were equal. This makes for simpler calculations with Cr=1, and yields a specific cooling water outlet temperature. Fouling factors were estimated from Table 11.1 (Incropera) and the equation for overall heat transfer coefficient was written with respect to chosen pipe diameters. The following **assumptions** were made:

®Steady state, ®constant mass flow rate, ®constant heat capacity, ®negligible change in kinetic and potential energies, ®perfectly insulated heat exchanger ✓

All of the **equations** for this part of the project are shown in the Matlab code in Attachment 1. This attachment is supplemented with comments that describe each part of the calculations. The main equations that were used include (8.60) for Nusselt Number, (8.66 for hydraulic diameter) ,(11.5) for overall h.t. coefficient, and the system was modeled from an annular tube design shown in chapter 11.

For the calculations to produce the table below, N was held constant at 10 tubes, a cooling-water exit temperature was held constant, cold-water mass flow rate was 10.01 kg/s, and the tubes were AISI302 stainless steel with wall thicknesses of 1mm. In the table, L shows the length per tube, A_s is the resulting total outer surface area, V represents total system volume, and P_{total} represents the sum of pumping power for cold and hot water.

Input Values					Resulting Values				
N	di(mm)	do(mm)	Di(mm)	Do(mm)	L (m)	U(W/m^{2}K)	V(m^3)	P_{total}(kW)	
10	20	22	28	30	171	1756	1.21	64.1	
10	25	27	38	40	162	1484	2.03	7.90	
10	50	52	75	77	152	790	7.06	0.257	
10	75	77	100	102	146	547	12.0	0.106	

Table 1: Annular Tubes ~~*as small is small tube outer surface area~~

Part 2: Improved Design

The optimal inner diameter for part 1 seems to be 25mm because the flow is turbulent enough to offer high heat transfer but not so much that excessive pumping power is needed. Now if more, smaller ✓tubes are used then heat transfer can increase. In a shell-and- tube heat exchanger, this can take place.

The proposed design uses two shell passes as shown in **figures 1 & 2**. The new U was desired to be equal to Part 1 or greater. Therefore, we aimed for similar h's on inside and outside of the hot tube. For the inside, N was chosen at 50 and the new diameter was adjusted until inside convection coefficient, h_i, was improved from part 1. Pumping power was cautiously considered throughout this process. The cold water convection coefficient was approximated by external cylindrical cross flow. This assumption was made based on an article published by AICE. This article on shell-and-tube heat exchangers breaks down shell flow into 5 streams and says the main crossflow stream is the main means of heat exchange. Area for this crossflow stream was manipulated by changing baffle spacing to yield a respectable velocity, Reynolds numbers, and h_o. Next, a new U was calculated. The new U was required to be greater than the old U, so the process of changing N and diameters was iterated a number of times.

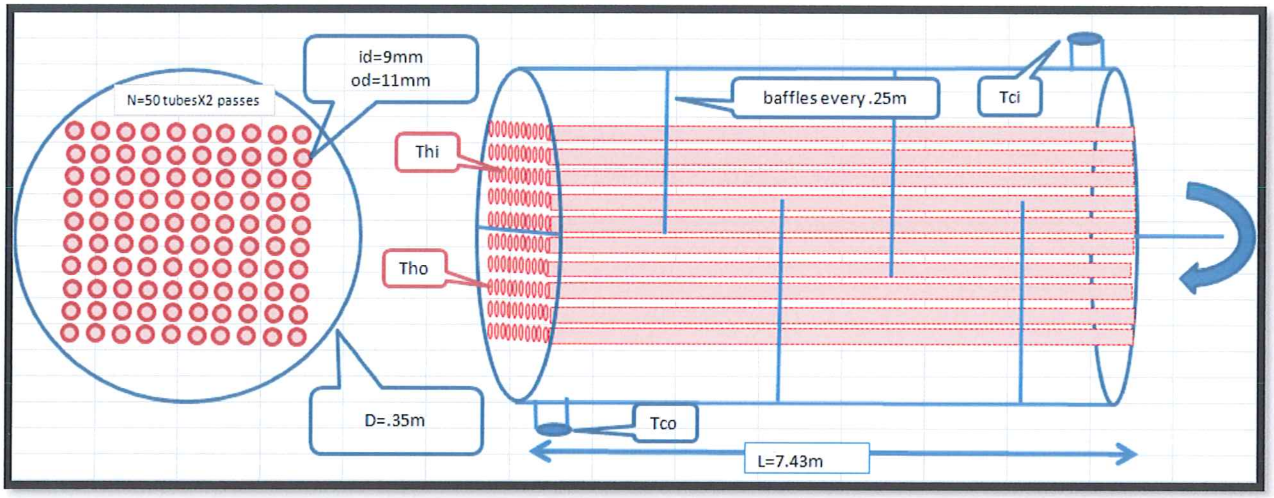


Figure 1: Shell Design (drawn without end caps)

Finally, NTU method was used to find the necessary heat transfer surface area for the shell-and-tube. Calculations were made using mostly Table 11.4 and Figure 11.13 (two shell passes & 4 tube passes) as the basis for design. **Attachment 2** shows the calculations that are *unique of the new design*, not including revised water properties and simple calculations repeated with different numbers. The resulting performance characteristics are summarized in **Table 2**. Besides better heat transfer, an additional benefit to the new design is easier fabrication and disassembly, if needed.

Performance Characteristic	Annular	Final design
U (overall heat transfer coefficient, W/m^2K)	1484	1680
Required heat transfer surface (inside surface of hot flow streams) (m^3)	127 m^2	51.3 m^2
System volume (m^3) (total large pipe or shells)	2.03 m^3	1.42 m^3
Approximate cost of materials for system ($)	$35,400	$21,800
Pumping power (kW)	7.90 kW	8.2kW
Monthly Power cost ($, based on 7 cents per kWh)	$398	$413

Table 2, overall design comparison

The most difficult estimation for this design was the pumping power. This was done using tube bank relationships from section 7.6 of Incopera, and possibly has the greatest room for error. This calculation is included in **Attachment 2.**

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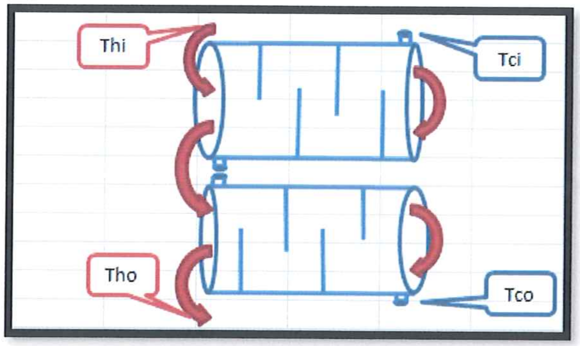

Figure 2: Overall 2-shell layout

Conclusions:

The project showed what major characteristics of a heat exchanger most directly benefits performance and least increase cost. Basicallly, improvement follows any instance that surface area can be increased without exessive velocity rise, mixing of flows can be added, or greater temperature differences can be introduced. For this design, our greatest suggestion for improvement would be to dump the T_{co} from the first shell back into the lake, and introduce fresh cold water into the second shell. This would improve performance without sacrificing any power or cost. Trying to calculate pumping power for the shell-side flow taught us that the only practical way to do this is with computer simulation. We also found that the natural progression of our own ideas on improvements to a simple heat exchanger led to designs similar to those commonly used in industry.

References:

Introduction to Heat Transfer, 5E. 2007, Incropera

Effectively Design Shell-and-Tube Heat Exchangers. 1997, American Institute of Chemical Engineers
http://www-unix.ecs.umass.edu/~rlaurenc/Courses/che333/Reference/exchanger.pdf

Heat Exchangers, Types and Primary Components. 2011, Rotounds Inc.
http://rotunds.com/Heat%20Exchanger.html

Shell-and-tube design:
http://engineering.wikia.com/wiki/Shell_and_tube_heat_exchanger

Material prices:
www.mcmaster.com, & http://www.onlinemetals.com/index.cfm

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Attachment 1, Annular Tube Calculations

```
% Heat Exchanger Requirements
Thi=80; %degrees Celsius
Tho=35; %degrees Celsius
Tci=25; %degrees Celsius
Tco=70; % required in special case Cr=1 (p642)

%Water Properties for hot water (using average 57.5)
Mdoth=10; %kilograms per second
Muh=489*10^(-6); % N*s/m^2
Cph=4184;    %J/kg*K
Kh=.650;     % W/m*K
Prh=3.15;    % Prandtl number
rhoh=993;    %kg/m^3
Ch=Mdoth*Cph; % heat capacity rate
% Water Properties for Cold water (using average 47.5 C)
Muc=577*10^(-6); %N*s/m^2
Cpc=4180;  % J/kg*K
Kc=.640;   % W/m*K
Prc=3.77;  %Prandtl number
rhoc=989;  %kg/m^3
Mdotc=Ch/Cpc;  %Set cooling water flow rate such that Cr=1

%Specify Pipes sizes
di=.025; % small pip inside, meters
do=.027; % small pipe outside, meters
Di=.038; % large pipe inside, meters
Do=.040; % large pipe outside, meters
Dh=4*(pi*Di^2/4-pi*do^2/4)/(pi*(Di+do)); % Eq 8.66 hydraulic diameter
N=10; % number of pipes
Ai=pi*di^2/4;                  % m^2, flow area per pipe hot water
Ao=(pi*Di^2/4-pi*do^2/4);      % m^2, flow area per pipe cold water
Ah=N*pi*di^2/4;                % m^2, total flow area for hot water
Ac=N*(pi*Di^2/4-pi*do^2/4); % m^2, total flow area for cold water
k=15.1; %Stainless steel AISI302 Table A1

% Calculate Nusselt Number of inside fluid
Vdoth=Mdoth/(N*rhoh); % m^3/second per pipe
Umh=Vdoth/Ai; % m/second, Average Velocity
Rei=Umh*di*rhoh/Muh;    % Reynolds Number
Nui=.023*Rei^(.8)*Prh^(.3);% equation 8.60 for Nusselt Number
hi=Nui*Kh/di;
% Calculate Nusselt Number of outside fluid
Vdotc=Mdotc/(N*rhoc); % m^3/second per pipe
Umc=Vdotc/Ao; % m/second, Average velocity
Reo=Umc*Dh*rhoc/Muc; % Reynolds Number eq
Nuo=.023*Reo^(.8)*Prc^(.3);%equation 8.60 for Nusselt Number
ho=Nuo*Kc/Dh;

%equation for overall heat transfer coefficient
Rfo=.0002; % (m^2*K/W) conservative estimate, table 11.1, river water
Rfi=.0002; % (m^2*K/W)
Ui=1/(1/(hi)+Rfi+log(do/di)*di/(2*k) + Rfo*di/do + di/(ho*do)); %Eq 11.5

% calculate effectiveness and surface area
q=Ch*(Thi-Tho);     % Watts, required heat transfer
qmax=Ch*(Thi-Tci); % Watts, max possible heat transfer
effect=q/qmax;      % effectiveness
NTU=effect/(1-effect); %for Cr=1, counterflow, Table 11.4
dTln=(Tho-Tci);    %((Thi-Tco)-(Tho-Tci))/log((Thi-Tco)/(Tho-Tci)); % log mean temp. diff.
Asi=q/(Ui*dTln);   % Eq. 11.4 small tube total inside surface area
L=Asi/(pi*di*N);   %length of each pipe
Aso=N*L*pi*Do; %total outside surface area, m^2
Volume=L*N*pi*Do^2/4; % m^3

% calculate pumping power
fo=(0.790*log(Reo)-1.64)^(-2); % outer friction factor (assume smooth)
fi=(0.790*log(Rei)-1.64)^(-2); % inner friction factor (assume smooth)
Po=fo*rhoc*Umc^2*L*N/(2*Dh)*Vdotc; % W, cold fluid eq 8.22b pg 461
Pi=fi*rhoh*Umh^2*L*N/(2*di)*Vdoth; % W, hot fluid eq 8.22b pg 461
Ptotal=Pi+Po; %total pump power
```

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ME 225 Project 2

Attachment 2, Shell-and-tube design

```
% Heat Exchanger Requirements
Declare Thi,Tho, Tci, Tco; %degrees Celsius
%Tco=35; % estimation before calcuation, degrees celcius
% Water Properties for hot water and cold water (cold estimated at Taverage=30)

Mdotc=4*Ch/Cpc;  %Set cooling water flow rate such that Cr=.25
% Specify Pipes sizes
% Calculate Nusselt Number of inside fluid
% Calculate Nusselt Number of outside fluid
Ac1=.1375; % m^2 crossflow per 1 meter baffle spacing
Ac=.25*Ac1; % Specify baffle spacing to be .25m
Vdotc=Mdotc/rhoc;
Umc= Vdotc/Ac; % average velocity m/s
Rec=Umc*do*rhoc/Muc; % Eq 7.51
Nuo=.193*Rec^.618*Prc^(1/3); % Eq 7.44 external cylinder cross flow
ho=Nuo*Kc/do; % Eq 7.44 cross flow average nusselt

%equation for overall heat transfer coefficient
Ui=1/(1/(hi)+Rfi+log(do/di)*di/(2*k) + Rfo*di/do + di/(ho*do));

% calculate effectiveness and surface area using NTU method
q=Ch*(Thi-Tho);    % Watts, required heat transfer
qmax=Ch*(Thi-Tci); %Watts, max heat transfer
epsilon=q/qmax;
Cr=.25;
F=((epsilon*Cr-1)/(epsilon-1))^(1/2);    % eq. 11.31 c
epsilon1=(F-1)/(F-Cr);                   % eq. 11.31 b
E=(2/epsilon1-(1+Cr))/(1+Cr^2)^.5;       % eq. 11.30 c
NTU=2*(-(1+Cr^2)^(-.5)*log((E-1)/(E+1)));% eq. 11.31 d & 11.30 b combined
Cmin=Mdoth*Cph;
A=NTU*Cmin/Ui; % Equation 11.24, heat transfer surface area required
L=A/(N*pi*do); % find length of each pipe
Cc=Cpc*Mdotc;
Tco=Tci + epsilon*Cmin*(Thi-Tci)/Cc; %this yielded an outlet temperature of 36.25, validating
average Tc assumption used for fluid properties

% calculate pumping power
% hot pumping power
fi=(0.790*log(Rei)-1.64)^(-2); % inner friction factor
Ph=fi*rhoh*Umh^2*L*N/(2*di)*Vdoth; %hot pumping power eq 8.22b pg 461 (w)
% cold pumping power
Pp=.0137;    % pipe pitch
do=.011;     % pipe outside diameter
St=Pp+do;    % center-to-center distance for tube bank calculations, pg 413
Vmax=St/(St+do)*Umc;  % max velocity pg 415
ReDmax=rhoc*Vmax*do/Muc;    % calculate reynolds number
f=.6;        % from Fig 7.14 pg 419
X=1.025;     % from Fig 7.14 pg 419
NL=10;       % number of rows
delP = NL*X*rhoc*Vmax^2/2*f; % delta p
%calculate power per "baffle section"
Pb=delP*Vdotc;     % pumping power per baffle (W)
% total pumping power= hot pump power + cold power per baffle * 60 baffles

% cost analysis: total monthly cost =7 cents per kWh * hours per month

% construction cost calculations
% Piping prices
Pl=8*(.3048);    % Length of each pipe; converting from feet to meters
Ppr=26.52;       % pipe price each in $
Pn=L*N/Pl;       % number of pipes needed
Pc=Pn*Ppr;       % pipe cost for total piping
% Tubing prices for Alloy Steel Sheet 4130 ANNEALED, custom cut w=43.3" L=587" T=0.063"
Sc=2796*2;          % shell cost for 2 shells
% end cap based on Alloy Steel Sheet 4130 ANNEALED w=12" L=12" T=0.063"
Ec=4*13.89; % end cap cost 2x
% total cost for heat exchanger
CstC = Pc+Sc+Ec;     % cost for construction
```

Cross flow

[illegible]	0.005615597
[illegible]	0.004908739
Ai	4.91E-04
Ao	5.62E-04
Asi	1.27E+02
Aso	2.03E+02
Ch	41840
Cpc	4180
Cph	4184
Dh	0.011
Di	0.038
Do	0.04
Kc	0.64
Kh	0.65
L	1.62E+02
Mdotc	10.00956938
Mdoth	10
Muc	5.77E-04
Muh	4.89E-04
N	10
NTU	4.5
Nui	3.35E+02
Nuo	1.44E+02
Pi	2.43E+03
Po	5.48E+03
Prc	3.77
Prh	3.15
al	7.90E+03
Rei	1.04E+05
Reo	3.40E+04
Rfi	2.00E-04
Rfo	2.00E-04
Tci	25
Tco	70
Thi	80
Tho	35
Ui	1.48E+03
Umc	1.802283801
Umh	2.05154408
Vdotc	0.00101209
Vdoth	0.001007049
Volume	2.029873889
cst	3.98E+02
dTln	10
di	0.025
do	0.027
effect	0.818181818
fi	0.017837959
fo	0.022939388
hi	8.72E+03
ho	8.40E+03
hrs	720
k	15.1
[illegible]	1882800
qmax	2301200
rhoc	989
rhoh	993

	Shell and Tube; di=.015, do-.017	Shell and Tube; di=.009, do-.011	2 shell
A	78.4327689	63.16947438	51.30232126
Ac	0.034375	0.034375	0.034375
Ac1	0.1375	0.1375	0.1375
Ah	0.008835729	0.003180863	0.003180863
Ai	1.77E-04	6.36E-05	6.36E-05
Cc	167360	167360	167360
Ch	41840	41840	41840
Cmin	41840	41840	41840
Cpc	4178	4178	4178
Cph	4184	4184	4184
Cr	0.25	0.25	0.25
E	1.15878132	1.15878132	2.060647375
F	4.375	4.375	2.091650066
Kc	0.62	0.62	0.62
Kh	0.65	0.65	0.65
L	29.37167734	36.55903309	29.69097462
Mdotc	40.05744375	40.05744375	40.05744375
Mdoth	10	10	10
Muc	7.69E-04	7.69E-04	7.69E-04
Muh	4.89E-04	4.89E-04	4.89E-04
N	50	50	50
NTU	2.531849986	2.531849986	2.05621121
Nui	1.39E+02	2.09E+02	2.09E+02
Nuo	1.78E+02	1.36E+02	1.36E+02
Pi	2.90E+02	4.13E+03	3.35E+03
Prc	5.2	5.2	5.2
Prh	3.15	3.15	3.15
Rec	2.58E+04	1.67E+04	1.67E+04
Rei	3.47E+04	5.79E+04	5.79E+04
Rfi	2.00E-04	2.00E-04	2.00E-04
Rfo	2.00E-04	2.00E-04	2.00E-04
Tavgc	30	30	30
Tci	25	25	25
Tco	36.25	36.25	36.25
Thi	80	80	80
Tho	35	35	35
Ui	1.35E+03	1.68E+03	1.68E+03
Umc	1.171163271	1.171163271	1.171163271
Umh	1.139746711	3.165963087	3.165963087
Vdotc	0.040258737	0.040258737	0.040258737
Vdoth	2.01E-04	2.01E-04	2.01E-04
di	**0.015**	**0.009**	0.009
do	**0.017**	**0.011**	0.011
epsilon	0.818181818	0.818181818	0.818181818
epsilon1	0.818181818	0.818181818	0.59275651
fi	0.022822241	0.02027479	0.02027479
fo	0.024538682	0.02741265	0.02741265
hi	6.03E+03	1.51E+04	1.51E+04
ho	6.49E+03	7.66E+03	7.66E+03
k	15.1	15.1	15.1
q	1882800	1882800	1882800
qmax	2301200	2301200	2301200
rhoc	995	995	995
rhoh	993	993	993

L tot = 14.8

$\times$ 2 shells

4 - pass

L per shell = $\frac{29.7\,m}{4}$ = 7.4

2 pass

L per shell = $\frac{36.5}{2}$ = 18.25

Ltot = 9.125

David Malawey

ME 225 HEAT TRANSFER

Project 1

Due on March 17, 2011

An aluminum extended surface of uniform rectangular cross section has a thickness of 5 cm, a length of 25 cm, and a very long width. The base of the fin is at 240 °C when the surrounding air is at 25 °C with a convection heat transfer coefficient of 20 $W/m^2.K$.

√ 1. Assuming one-dimensional heat transfer in the fin, plot the temperature distribution, and calculate the temperature at the tip and the fin heat transfer rate per unit width.

2. Using the finite element heat transfer (FEHT) software of the textbook, perform a two-dimensional analysis on the fin. Specifically, follow the examples given in the software to set up/scale, draw/outline, and specify boundary conditions/properties. Using an initial mesh size of 2.5 cm by 2.5 cm, show the nodal network. Then, show the temperature contours (isotherms) as well as temperature gradients from the software. With the help of the output table within FEHT, plot the temperature distribution, determine the tip temperature and fin heat rate from output file, and discuss key features.

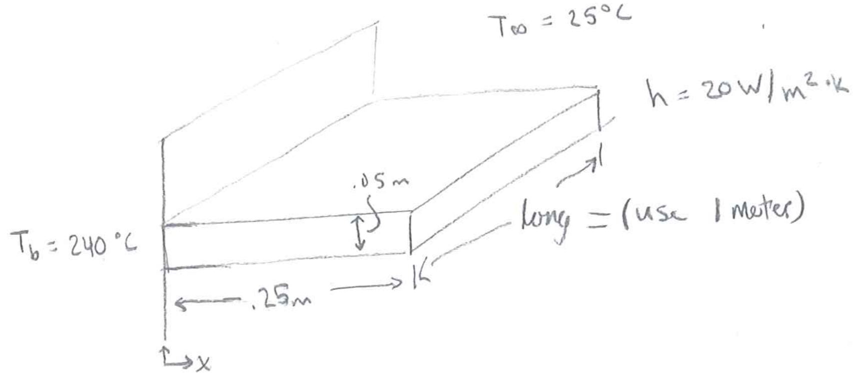
3. Compare the computational results to the one-dimensional analytical solution in step 1. How good is the 1-D solution (quantify)?

4. Consider the effect of mesh size by repeating step 2 for reduced mesh sizes of 1.25 cm and 0.625 cm. For each case, plot temperature distribution and calculate fin heat rate. Compare the computational results from three mesh sizes.

5. Consider the effect of convection conditions by repeating computations with a mesh size of 0.625 cm for convection heat transfer coefficients of 60 and 180 $W/m^2.K$. For each case, plot temperature distribution and calculate fin heat rate. Compare the computational results from three convection heat transfer coefficients.

6. Consider the effect of material type (thermal conductivity) by repeating computations with the smallest mesh size and a convection heat transfer coefficient of 20 $W/m^2.K$ for copper and steel. For each case, plot temperature distribution and calculate fin heat rate. Compare the results from three materials.

Two students are to be teamed up for this project, submitting only one professional report of less than 5 total pages using a computer.



1 dimensional

find $T(x)$ 9" per width

Tip Condition: Convection heat transfer: $h\Theta(L) = -k \left.\frac{d\theta}{dx}\right|_{x=L}$

Temp Distribution: $\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + \left(\frac{h}{mk}\right)\sinh mL} = \dfrac{\Theta}{\Theta_b}$

$$m = \sqrt{\frac{hP}{kA_c}} = \left[\left(20\,W/m^2k \cdot 2.1m\right)/\left(237\,W/mk \cdot 0.05m^2\right)\right]^{1/2} \Rightarrow m = 1.883$$

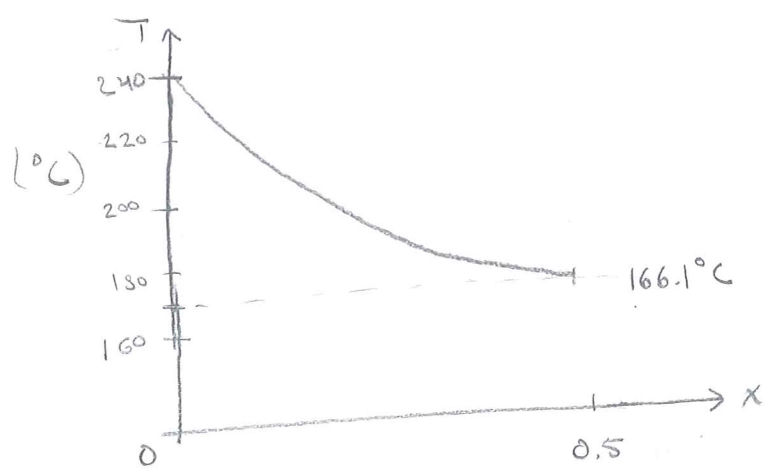
$P = 2(.05+1) = 2.1m$

$A_c = .05(1) = .05m^2$

$k_{Al} = 237\,W/mk$

$$\frac{\Theta}{\Theta_b} = \frac{\cosh 1.88(.5-x) + .0448 \sinh 1.88(.5-x)}{1.524} = \frac{T_x - T_\infty}{T_b - T_\infty} = \frac{T_x - 25}{240-25}$$

$$T(x) = 25 + 215\left(\frac{\cosh 1.88(.5-x) + 0.0448 \sinh(1.88(.5-x))}{1.524}\right)$$



$\boxed{T_{tip} = 166.1\,°C}$ used matlab to plug numbers in *

$$\frac{\Theta}{\Theta_b} = \frac{T - T_\infty}{T_b - T_\infty} = \frac{T-25}{240-25}$$

$$\frac{\Theta}{\Theta_b}(215) + 25 = T$$

Exams

90

ME 225 HEAT TRANSFER
EXAM 2

S

24 March 2011

David Malawey
NAME

You may use the textbook, handouts, homework solutions, and your *own* notes.
Please briefly show/explain your solutions for full credit. All questions are worth 25%.

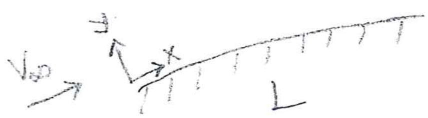
25

1. The velocity of a fluid with a Prandtl number of 4 within a boundary layer over an unknown surface of 2 m length varies as:

$$\frac{u}{V_\infty} = \frac{100y}{\sqrt{x}} - 5e^{-10y^2}$$

$u = V_\infty \frac{100y}{\sqrt{x}} - 5e^{-10y^2}$

Calculate the average Nusselt number over the entire length.



$\bar{Nu} = \frac{\bar{h}_x X}{k}$

$$\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = \mu \left[\frac{100 V_\infty}{\sqrt{x}} + \cancel{100 y e^{-10y^2}}^{\,0}\right]_{y=0} = \frac{100 \mu V_\infty}{\sqrt{x}}$$

$$C_f = \frac{2}{\rho V_\infty^2}\tau_s = \frac{200 \mu V_\infty}{\rho V_\infty^2 \sqrt{x}} = \frac{200\mu}{\rho \sqrt{x}\, V_\infty}$$

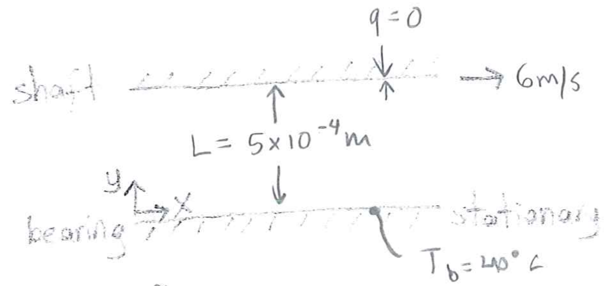
$$\bar{C_f} = \frac{1}{L}\int_0^L C_{f_x}\,dx = \frac{1}{L}\left[(2)\frac{200\mu}{\rho V_\infty} x^{1/2}\right]_0^L = \frac{1}{L}\,\frac{400\mu L^{1/2}}{\rho V_\infty} = \frac{400\mu}{\rho\sqrt{2}\, V_\infty}$$

$$\bar{Nu} = \frac{\bar{C_f}}{2} Re\, Pr^{1/3} = \left(\frac{1}{2}\frac{400}{\sqrt{2}}\frac{\mu}{\rho V_\infty}\right)\underbrace{\left(\frac{V_\infty L}{\nu}\right)}_{Re}(4)^{1/3} = 224(2) = 449$$

$\boxed{\bar{Nu} = 449}$

24

2. A shaft moves at 6 m/s 0.5 mm away from a stationary bearing when a lubricant (viscosity of $0.7\ N.s/m^2$ and thermal conductivity of 0.2 W/m.K) exists between them. At steady operating conditions, the bearing surface is maintained at 40 °C while the heat transfer to the shaft is negligible. Using the assumptions and results of the Couette flow discussed in the classroom, determine the maximum temperature of the lubricant.



$\mu = 0.7$

$k = 0.2$

Couette flow

$$T = -\frac{\mu}{2k}\left(\frac{U}{L}\right)^2 y^2 + C_1 y + C_2$$ ✓

B.C. 1: $T(y=0) = 40°C = C_2$ ✓

B.C. 2: @ $y = L$ $\quad q'' = 0 = -k_f \left.\frac{dT}{dy}\right|_{y=L}$ ✓

$$\left.\frac{\partial T}{\partial y}\right|_{y=L} = -\frac{\mu}{k}\left(\frac{U^2}{L^{\cancel{2}}}\right)\cancel{y} + C_1\Big|_{y=L} = 0 \Rightarrow C_1 = \frac{\mu}{k}\frac{U^2}{L}$$

$$T = -\frac{\mu}{2k}\frac{u^2}{L}y^2 + \frac{\mu}{k}U^2 y + 40°C$$

T_{MAX} @ $y = L$ ✓ $= \dfrac{+(.7)36}{2(.2)}(5\times10^{-4}) + 40 = \boxed{40.00126\ °C}$

Seems too small for Max temp

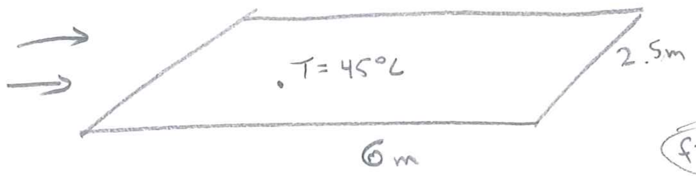
$$-\frac{\mu}{2k}\frac{u^2}{\cancel{L}}L^{\cancel{2}} + \frac{\mu}{k}U^2 L + 40 =$$

$$= \frac{\mu U^2 L}{2k} + 40$$

double checking

14

3. A solar car's main body profile is such that it can be approximated to a first order as a flat plate with a length of 6 m and a width of 2.5 m. In a typical race when the air temperature is 25 °C, the solar car's speed is 18 m/s in parallel to its length. Estimate the total drag force acting on the car and the total rate of convection heat loss from its surface to the surrounding air assuming a surface temperature of 45 °C.



$V_\infty = 18 \text{ m/s}$

$T_\infty = 25\,°C$

find F_D, $\bar{h}$

Air, 300K

$\rho = 1.161$

$c_p = 1{,}007$

$\nu = 15.89 \times 10^{-6}$

$k = 26.3$

$Pr = 0.707$

$$Re_x = \frac{V_\infty x}{\nu} = \frac{18(6)}{15.89 \times 10^{-6}} = 6.80 \times 10^6 \quad \text{Turbulent}$$

$$F_D = \bar{C}_f A_s \left(\frac{\rho V^2}{2}\right)$$

$$\bar{C}_f = .074\, Re^{-.2} - 1742\, Re^{-1} = 0.00213$$

$$F_D = .00213\,(6 \times 2.5)(1.161)\frac{(18)^2}{2} = \boxed{8.27 \text{ N}}$$

$\times$ 2 surface, top & bottom $= \boxed{16.54 \text{ N}}$

$$\bar{h} = (.037\, Re^{0.8} - 871)\, Pr^{1/3} = \boxed{8863 \text{ W/m}^2\text{K}}$$

$\uparrow$ $\overline{Nu}$

$\uparrow$ seems high

24

4. A fluid with a Prandtl number of 5 flows in a rectangular duct ($b/a = 3$) whose surface is kept at a constant temperature that is colder than the fluid. If only the mean fluid velocity is increased by 10 times such that the Reynolds number increases from 1,500 to 15,000, calculate the ratio of the pump powers for these two fully-developed conditions. Also, assuming that the rate of heat transfer is nearly proportional to the convection heat transfer coefficient, calculate the ratio of the heat transfer rates for the same conditions.

$Pr = 5$ $\quad T_s < T_m$ (const) $\quad V \uparrow$ $\quad Re_1 = 1500$, $Re_2 = 15{,}000$

$\dot{P} = \Delta P \dot{\forall} = f\left(\frac{L}{D}\right)\left(\frac{\rho u_m^2}{2}\right)\dot{\forall}$

$1500 < 2300$, Laminar

$Nu_1 = 3.96$

$$\frac{\dot{P}_1}{\dot{P}_2} = \frac{f_1 \left(\frac{L}{D_1}\right) \frac{\rho_1 u_{m1}^2}{2} \dot{\forall}_1}{f_2 \left(\frac{L}{D_2}\right) \frac{\rho_2 u_{m2}^2}{2} \dot{\forall}_2} = \frac{f_1}{f_2} \frac{u_{m1}^2}{u_{m2}^2} \frac{\dot{\forall}_1}{\dot{\forall}_2}$$

AND $\left\{ \begin{array}{l} \dot{\forall} \sim V \\ u_m \sim V \end{array} \right.$

$$\frac{P_2}{P_1} = \frac{f_2}{f_1} \frac{V_2^3}{V_1^3}$$ ✓

$$f_1 = \frac{69}{Re_{D_1}} = .046$$ ✓

$$f_2 \text{ (turbulent)} = .031\, Re_D^{-1/4} = .0028$$ ✓ ✓

$$\boxed{\frac{P_2}{P_1} = 60.87}$$ X

$q \sim \bar{h}$ $\quad \frac{\bar{h}_2}{\bar{h}_1} = \frac{Nu_{D_2} k / D_h}{Nu_{D_1} k / D_h}$

where $Nu_{D_1} = 3.96$

$Nu_{D_2} = .023\, Re_D^{0.8}\, Pr^{0.3} = 81.71$

$$\boxed{\frac{q_2}{q_1} = 20.63}$$ ✓

ME 225 HEAT TRANSFER
EXAM 1

15 February 2011

68 well above average.

David Malawey
NAME

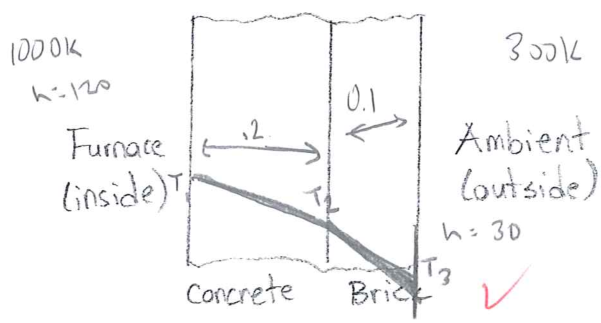
You may use the textbook, handouts, homework solutions, and your *own* notes.
Please briefly show/explain your solutions for full credit. All questions are worth 25%.

20

1. A furnace has a composite wall consisting of a 0.2 m-thick inner layer of concrete stone mix (properties at 300 K) and a 0.1 m-thick outer layer of fire clay brick (properties at 478 K). The furnace gases are at 1000 K and the ambient air is at 300 K. The inside and outside convection heat transfer coefficients are given as 120 and 30 W/m^2.K, respectively.

Steady State

(i) Draw the expected temperature variations within the wall in the sketch below and clearly compare the relative temperature slopes in both layers.
(ii) Calculate the rate of heat loss through this composite wall per unit surface area.
(iii) An insulation layer is added to the outer wall to reduce the heat transfer rate through the entire wall by 80% of the above value. Determine the temperature at the interface between the concrete and brick layers corresponding to this case.



i) Slope $T_1 - T_2$ is > $T_2 - T_3$ because k is > for concrete

ρ 2300
k 1.4

ρ 2645
k 1.0 W/m·K

$$\dot{q} = \frac{\Delta T}{R_{total}}$$

$$R_{total} = \frac{L_2}{kA} + \frac{L_1}{k_A} = \frac{0.2}{1.4\,A} + \frac{0.1}{1\,A} = \frac{.2429}{A}$$

$$\dot{q} = \frac{1000 - 300\,A}{.2429} = \boxed{2881\ W/m^2}\ \text{out}$$

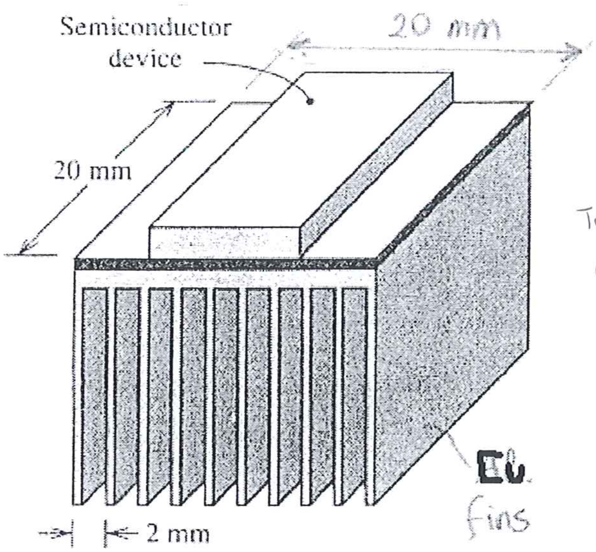
$\dot{q}_{new} = 0.2\,\dot{q}$

$R_{tot\,new} =$

$$\frac{1000 - T_2}{\frac{0.2}{1.4}} = 0.2\,(2881) \qquad \boxed{T_2 = 918\ K}$$

7

2. A semiconductor device is mounted on a 2×2 cm copper plate with ten 0.2 mm-thick rectangular fins. Cooling air at $\boxed{25\ °C}$ flows through the fin array and gives a convection heat transfer coefficient of $\boxed{40\ W/m^2.K}$. Using the properties of pure copper at 300 K:
(i) Find the fin length that can transfer 70% of the heat by an infinitely long fin.
(ii) For this fin length, calculate the temperature at the base of the fins if the total heat dissipation is to be 8 W.



$T_\infty = 25\ °C$

Cu (300) k =

$\theta_b = T_b - 25$

$q_{f,\ infinite} = M = \sqrt{hPkA_c}\,\theta_b$

q_f

$L_c = L + \left(\frac{0.0002}{2}\right)$

$A_c =$ base area

$q_{f\ 70\%} = \cancel{M}\tanh(mL_c) = 0.7\left(\cancel{\sqrt{MPkA_c}\,\theta_b}\right)$

$\tanh(mL_c) = 0.7$ ✓

$\tanh\left(\sqrt{\frac{hP}{kA_c}}\,(L + .0001\,m)\right) = 0.7$

Cu properties page 840 ⇒ calculator, ⇒ L

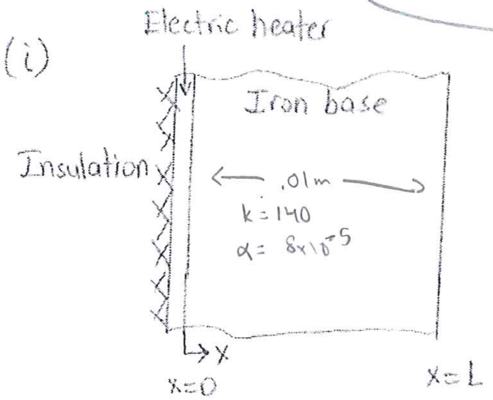
Tbase: $N\eta_f A_f + A_{unfinned}$ = $(N\varepsilon_f A_b + A_{unfinned})h$

19

3. (i) The base plate of an iron has a thin electrical heater at the inner surface ($x = 0$) backed by insulation. A constant heat flux of q_0'' is provided at the inner surface under steady-state conditions when the outer surface ($x = L$) is exposed to ambient air at T_∞. The iron is suddenly unplugged from the electricity for cooling. For this general case, sketch and label, on T-x coordinates, the temperature distributions within the base plate just before it is unplugged ($t = 0$), after a long period of time ($t \rightarrow \infty$), and two intermediate times (t_1 and t_2). Also, sketch the heat flux at the outer surface, $x = L$, as a function of time.

(ii) Now, consider the same problem with the following specific values: the base plate has a thickness of ~~1 cm~~ with a thermal conductivity of 140 W/m.K and a thermal diffusivity of 8×10^{-5} m^2/s. The effective convection heat transfer coefficient between the iron and ambient air at 25 °C is 100 W/m^2.K. Determine how many minutes will it take for the base plate to cool from 160 °C down to 40 °C.

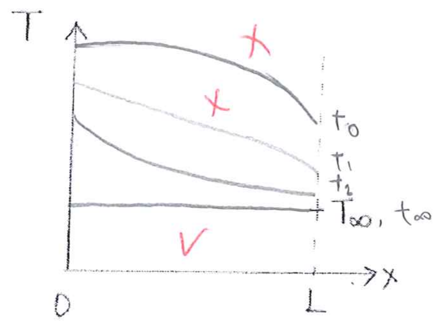
(i)

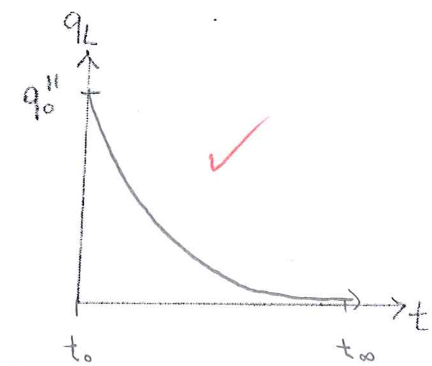


$\alpha = 8\times10^{-5}$ m^2/s

T_∞, h

25°C, 100





$$k\frac{d^2T}{dx^2} + \cancel{\dot{q}}^{\,0} = 0$$

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

$$Bi = \frac{hL_c}{k} = \frac{\overset{100}{(25)}(.01)/2}{140} = .0017 \rightarrow \text{lumped approx}$$

$$t = \frac{\rho V c_p}{hA_s}\ln\frac{T_i - T_\infty}{T - T_\infty}$$

$$t = \frac{7870(447)}{100}(.01\,m)\ln\left(\frac{160-25}{40-25}\right)$$

$$t = 773.4 \text{ sec}$$

I did not account for diffusivity α

22

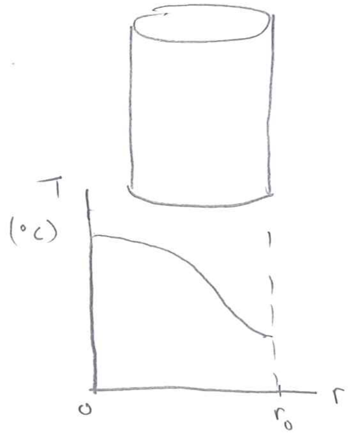
4. A cylinder is subject to solar radiation of 500 W/m^2 when then surrounding air is at 30 °C. The cylinder has a radius of 0.2 m, a volume of 0.15 m^3, thermal conductivity of 1.0 W/m.K, a total heat generation of 135 W over the entire volume, a total absorptivity of 0.7, and a total emissivity of 0.9. The cylinder surface temperature is kept at 50 °C. Under such steady-state conditions:
(i) Express the temperature distribution (in °C) within the cylinder as a function of radial location.
(ii) What is the temperature at the cylinder center?
(iii) Calculate the convection heat transfer coefficient between the cylinder surface and the surrounding air.

$E_{in} = E_{out}$

find h

$$\underbrace{-k\frac{dT}{dr} + \alpha G}_{E_{in}} = \underbrace{\varepsilon\sigma(T_s^4 - T_{surr}^4) + h(T_s - T_\infty)}_{E_{out}}$$

multiply necessary terms by A_s, surface area



$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\dot{g}$$

$$\int\left(\frac{d}{dr}\left(r\frac{dT}{dr}\right)\right) = \int\left(\frac{-\dot{g}r}{k}\right)$$

$$r\left(\frac{dT}{dr}\right) = \frac{-\dot{g}r^2}{2k} + C_1$$

$$\frac{dT}{dr} = \frac{-\dot{g}r}{2k} + \frac{C_1}{r}$$

$$T(r) = \frac{-\dot{g}r^2}{4k} + \cancel{C_1 \ln r} + C_2$$

B.C.'s

$$\left.\frac{dT}{dr}\right|_{r=0} = \text{finite} \Rightarrow C_1 = 0$$

$$T_r\Big|_{r=r_0} = \frac{-\dot{g}r^2}{4k} + C_2 = 50°C$$

$$-\frac{135(.2)}{4(1)} + C_2 = 50°C$$

$$C_2 = 56.75\ °C$$

$$\boxed{T_{center} = 56.75°C}$$

$\pi r^2 L = 0.15$

$\pi(.2)^2 L = 0.15$

$L = 1.19$

$A_s = 1.75\ m^2$

$$135\,W + 0.7(500)\left(\underbrace{2\pi(.2)(1.19) + 2\pi(.2)^2}_{\text{area}}\right) = 0.9\,\sigma(50^4 - 30^4)A_s + h(50-30)A_s$$

0.15

$\sigma = 5.67\times10^{-8}$

ok now

$$0.9(323^4 - 303^4)(5.67\times10^{-8})A_s = 125\,A_s$$

$746 = 125A_s + 20h\,A_s$

$$\boxed{h = 15.06\ W/m^2}$$

Practice exams

ME 225 HEAT TRANSFER
EXAM 2

25 March 2010

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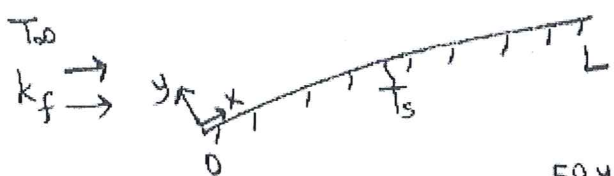
NAME

You may use the textbook, handouts, homework solutions, and your *own* notes.
Please briefly show/explain your solutions for full credit. All questions are worth 25%.

1. A boundary layer develops over an arbitrary surface in which the temperature, T, changes as

$$\frac{T-T_\infty}{T_s-T_\infty}=\exp\left(-\frac{50y}{k_f x^{1/4}}\right)$$

Express the variation of local convection heat transfer coefficient with x and then calculate the average convection heat transfer coefficient over the entire length $L = 2$ m.



$$T = T_\infty + (T_s - T_\infty)\, e^{-\frac{50y}{k_f . x^{1/4}}}$$

$$\frac{\partial T}{\partial y} = (T_s - T_\infty)\left(\frac{-50}{k_f . x^{1/4}}\right) e^{-\frac{50y}{k_f . x^{1/4}}} \Rightarrow \left.\frac{\partial T}{\partial y}\right|_{y=0} = (T_s - T_\infty)\left(-\frac{50}{k_f . x^{1/4}}\right)$$

$$h_x = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty} = \frac{50}{x^{1/4}}$$

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x . dx = \frac{1}{L}\int_0^L \frac{50}{x^{1/4}} . dx = \frac{50}{L} . \frac{4}{3} x^{3/4}\Big|_0^L$$

$$\bar{h}_L = \frac{50}{L} \frac{4}{3} L^{3/4} = \frac{200}{3 L^{1/4}} = \frac{200}{(3)(2)^{1/4}} = 56.06 \text{ W/m}^2 . \text{K}$$

2. A 6-cm-diameter shaft rotates at 3000 rpm in a bearing with a clearance of 0.2 mm filled with lubricating oil (viscosity of 0.5 N.s/m^2 and thermal conductivity of 0.15 W/m.K). At steady operating conditions, both the shaft and the bearing surfaces around the oil gap are at 50 °C. Using the assumptions and results of the Couette flow discussed in the classroom, determine the maximum temperature of the oil and the heat flux from the oil to the shaft.

Class notes for Couette flow: $T = T_b + \frac{\mu}{2k} V^2 \left(\frac{y}{L} - \frac{y^2}{L^2}\right) + (T_s - T_b)\frac{y}{L}$

$T_s = T_b = 50°C$, $V = w \cdot r = \left(\frac{3000}{60} \frac{rev}{s}\right)\left(6 \times 10^{-2}\ m\right) = 3\ m/s$

.06 m

diameter

$$T = 50 + \frac{(0.5)}{(2)(0.15)}(3)^2\left(\frac{y}{L} - \frac{y^2}{L^2}\right) = 50 + 15\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

Max. temperature $\Rightarrow y = \frac{L}{2} = \frac{0.2mm}{2} = 0.1\ mm \Rightarrow \frac{y}{L} = \frac{1}{2} = 0.5$

$$T_{max} = 50 + 15\left(0.5 - 0.5^2\right) = 53.75\ °C$$

Heat flux from oil to shaft: $q_s'' = -k \left.\frac{dT}{dy}\right|_{y=L}$

$$q_s'' = -\cancel{k} \frac{\mu V^2}{2\cancel{k}}\left[\left(\frac{1}{L} - \frac{2y}{L^2}\right)\right]_{y=L} = -\frac{\mu V^2}{2L}(1-2) = \frac{\mu V^2}{2L}$$

$$q_s'' = \frac{(0.5)(3)^2}{(2)(2 \times 10^{-3})} = 1125\ W/m^2$$

↑

2×10^{-4} m

3. A cold fluid with a Prandtl number of 6, a viscosity of 10^{-6} m^2/s and a thermal conductivity of 0.5 W/m.K suddenly starts running parallel to a flat plate with a velocity such that the Reynolds number at the end of the plate is 10^6. If a second fluid with a Prandtl number of 60, a viscosity of 10^{-5} m^2/s and a thermal conductivity of 5 W/m.K runs over the same plate with the same velocity, estimate how fast relative to the first case (ratio) the plate will cool from the same initial temperature to the same desired temperature under the same conditions, assuming that the cooling time is inversely proportional to the average convection heat transfer coefficient.

Cooling time: $t \sim \frac{1}{\bar{h}_L} \Rightarrow \frac{t_2}{t_1} = \frac{\bar{h}_{L_1}}{\bar{h}_{L_2}}$

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k_f} \Rightarrow \bar{h}_L = \frac{\overline{Nu}_L \cdot k_f}{L} \Rightarrow \frac{t_2}{t_1} = \frac{\overline{Nu}_{L_1} \cdot k_{f1}}{\overline{Nu}_{L_2} \cdot k_{f2}}$$

$$Re_{L_1} = \frac{V_\infty . L}{\nu_1} = 10^6 > 5\times10^5 \Rightarrow \text{turbulent flow (mixed)}$$

$$Re_{L_2} = \frac{V_\infty L}{\nu_2} = \frac{Re_{L_1} \cdot \nu_1}{\nu_2} = 10^6 . \frac{10^{-6}}{10^{-5}} = 10^5 < 5\times10^5 \Rightarrow \text{laminar flow}$$

From Table 7.9:

$$\overline{Nu}_{L_1} = (0.037 Re_{L_1}^{4/5} - 871) Pr_1 = [(0.037)(10^6)^{0.8} - 871](6)^{1/3} = 2660$$

$$\overline{Nu}_{L_2} = 0.664 . Re_{L_2}^{1/2} . Pr_2^{1/3} = (0.664)(10^5)^{0.5}(60)^{1/3} = 822$$

$$\frac{t_2}{t_1} = \left(\frac{2660}{822}\right)\left(\frac{0.5}{5}\right) = 0.32$$

4. A fluid (with a specific heat of 1 kJ/kg.K, a viscosity of 0.05 N.s/m^2, a thermal conductivity of 0.5 W/m.K, a density of 800 kg/m^3, and a Prandtl number of 50) enters a pipe (with a diameter of 5 cm and a length of 50 m) with a mass flow rate of 1 kg/s and a mean inlet temperature of 25 °C. (i) If the surface temperature of the pipe is kept constant at 100 °C, determine the mean outlet temperature of the fluid. (ii) While keeping all the parameters the same, it is desired to change the pipe surface condition from constant temperature to constant heat flux. Calculate the heat flux need to be applied on pipe's surface that will yield the same mean fluid outlet temperature as part (i). Also, calculate the corresponding pipe surface temperature right at the outlet under the same constant heat flux surface condition.

(i) $T_s = \text{const.} = 100\,°C$

$$Re = \frac{V_m D}{\nu} = \frac{4\dot{m}}{\pi \mu D} = \frac{(4)(1)}{(\pi)(0.05)(0.05)} = 509 < 2300 \Rightarrow \text{laminar}$$

$$\text{Table 8.1} \Rightarrow Nu = \frac{hD}{k} = 3.66 \Rightarrow h = \frac{(3.66)(0.5)}{(0.05)} = 36.6\ W/m^2.K$$

$$T_{m,o} = T_s - (T_s - T_{m,i})\, e^{-\frac{h\pi D L}{\dot{m} c_p}} = 100 - (100-25)\, e^{-\frac{(36.6)(\pi)(0.05)(50)}{(1)(1000)}} = 43.7\,°C$$

(ii) $q_s'' = \text{const.}$

$$T_{m,o} = T_{m,i} + \frac{q_s'' \pi D L}{\dot{m} c_p} \Rightarrow q_s'' = (T_{m,o} - T_{m,i}) \frac{\dot{m} c_p}{\pi D L}$$

$$q_s'' = (43.7-25)\frac{(1)(1000)}{(\pi)(0.05)(50)} = 2381\ W/m^2$$

$$q_s'' = h(T_s - T_m) = \text{const} \Rightarrow T_s - T_m = \frac{q_s''}{h}$$

$$\text{Table 8.1} \Rightarrow Nu = 4.36 \Rightarrow h = \frac{(4.36)(0.5)}{(0.05)} = 43.6\ W/m^2.K$$

$$T_{s,o} = T_{m,o} + \frac{q_s''}{h} = 43.7 + \frac{2381}{43.6} = 98.3\,°C$$

S

ME 225 HEAT TRANSFER
EXAM 2

10 April 2008

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NAME

You may use the textbook, handouts, HW solutions, and your *own* notes.
Please briefly show/explain your solutions for full credit. Q1 = 30%, Q2 = 35%, Q3 = 35%.

1. A fluid with a Prandtl number of 4 and a Reynolds number of 2000 flows in a cylindrical pipe whose surface is kept at a constant temperature warmer than the fluid. Without changing other parameters, including the mass flow rate through the pipe, only the pipe diameter is reduced by 5 times such that the Reynolds number changes accordingly. For the same mean fluid inlet and outlet temperatures and assuming fully-developed conditions, calculate the relative change (ratio) in the required pipe length when the diameter is decreased 5 times. Also, calculate the corresponding relative changes (ratios) in the friction factor and the pumping power if the pipe inside surfaces can be considered smooth.

2. A computer chip is to be cooled by attaching several rectangular cross section fins. A fan blows air at 25 °C parallel to the fins (flow over flat plate) in the direction shown in the figure below. Each fin with 90% efficiency is to dissipate 2 W by convection.
   (i) Assuming first laminar flow over the entire fin surface, calculate the free stream air velocity needed for a maximum base temperature of 75 °C.
   (ii) Instead of air, if water with the same velocity is used as cooling fluid (if you cannot find it from part (i) above, you may assume a water velocity of 20 m/s), what will be the new heat loss rate from each fin (no need to account for the unfinned surface area)?

3. An arbitrary surface (not a flat plate!) with a surface area of 4 m^2 is kept at a temperature of 325 K. Air with a temperature of 275 K and a velocity of 3 m/s flows over this external surface parallel to its length of 2 m. The critical Reynolds number at which the flow turns from laminar to turbulent over this surface is given to be 10^6. It is also given that the temperature profile (in Kelvin) within the boundary layer varies as a function of downstream location, x, and cross stream location, y:

$$T = 275 + 50\exp\left(\frac{-500y}{x^{1/2}}\right) \text{ when the flow is locally laminar}$$

$$T = 325 - 20{,}000\frac{y}{x^{1/5}} \text{ when the flow is locally turbulent}$$

Determine:

(i) the type of flow over the surface, i.e., laminar, turbulent, or mixed,

(ii) how the local convection heat transfer coefficient varies with x,

(iii) the average convection heat transfer coefficient over the entire surface,

(iv) the average friction coefficient over the entire surface, and

(v) the total heat loss rate from the surface and the total drag force on the surface.

Summer 2010

#1) Given,

Velocity : $\frac{u}{V_\infty} = \frac{50y}{x^{1/3}} - 3e^{(-10y^2)}$

$\boxed{\overline{Nu} = ?}$

Arbitrary surface

$\longrightarrow V_\infty$

Length = 2 m

$P_r = 3$

$\overline{Nu}_x \equiv \frac{\bar{h}_x x}{k}$

$\boxed{\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}}$ $\boxed{C_f = \frac{2}{\rho V_\infty^2} \tau_s}$

$\tau_s = \mu \left[\frac{50}{x^{1/3}} + \cancel{60ye^{-10y^2}}\right] V_\infty \Rightarrow C_f = \frac{2\nu}{V_\infty}\left[50x^{-1/3}\right] = \frac{100\nu x^{-1/3}}{V_\infty}$

$\boxed{\overline{C_f} = \frac{1}{L}\int_0^L C_{f,x} \cdot dx} = \frac{1}{L}\int_0^L \frac{100\nu x^{-1/3}}{V_\infty} dx = \left[\left(\frac{3}{2}\right)\frac{100\nu x^{2/3}}{L V_\infty}\right]_0^L$

$\overline{C_f} = \frac{150\nu L^{-1/3}}{V_\infty}$

$\overline{Nu} = \frac{C_f}{2} Re P_r^{1/3} = \frac{150}{\cancel{2}} \frac{\cancel{\nu} L^{2/3}}{\cancel{V_\infty}} \frac{\cancel{V_\infty L}}{\cancel{\nu}} P_r^{1/3} = 150 L^{-1/3} P_r^{1/3} = \boxed{172}$

$\boxed{Re = \frac{V_\infty L}{\nu}}$

formulas

Reynolds Analogy

$\boxed{\frac{C_f}{2} = \frac{Nu}{Re P_r^{1/3}}}$

Ch 6 slides, pg 3

Problem 7.45

Pin fin

$D = .01mm$

$q = 30W$

$L = \infty$

$Re = 4000$

$\boxed{\frac{D_2}{D_1} = 2 \quad \text{find } q_2}$

$q_{fin} = (hPk_f A_c)^{1/2} \theta$

Ch 3
Slide 24

$\overline{Nu}_{D,1} = C Re_D^m Pr^{1/3} = .193(4000)^{.618} Pr^{1/3}$

$= .193(8000)^{.618} Pr^{1/3}$

$\frac{Re_2}{Re_1} = \frac{\frac{V D_2}{\nu}}{\frac{V D_1}{\nu}} = \frac{D_2}{D_1} = 2 \Rightarrow Re_2 = 8000$

$\frac{\overline{Nu}_{D,2}}{\overline{Nu}_{D,1}} = 1.53$

$\frac{A_2}{A_1} \approx \frac{D_2^2}{D_1^2}$

$\bar{h} \approx \frac{Nu}{D}$

$\frac{\bar{h}_2}{\bar{h}_1} = 1.53$

$\frac{P_2}{P_1} = \frac{D_2}{D_1}$

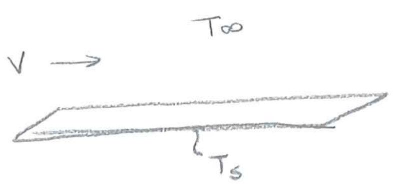
$\frac{q_2}{q_1} = \left(\frac{Nu_2}{Nu_1} \frac{\cancel{D_1}}{\cancel{D_2}} \frac{\cancel{D_2}}{D_1} \frac{D_2^2}{D_1^2}\right)^{1/2} = 2.47$

$\boxed{q_2 = 74.2W}$

2

Summer 2010 Exam 2

#3



$(T_\infty - T_s) = \text{const}$

given: $Re_1 = 10^5 \Downarrow Re_2 = 10^6$

find: i) $\frac{F_{D_2}}{F_{D_1}}$ (%) ii) $\frac{\bar{h}_2}{\bar{h}_1}$ (%)

$$F_D = \bar{C}_{f,L} A_s \frac{\rho V_\infty^2}{2}$$

$$Re = \frac{\rho V D}{\mu}$$

(p431) $C_{f_1} = 1.328 Re_x^{-1/2} = .0042$

$C_{f_2} = .074 Re_L^{-.2} - 1742 Re_L^{-1} = .00293$

$Re \approx V$

$$\frac{V_2}{V_1} = \frac{Re_2}{Re_1}$$

$$\frac{F_{D_2}}{F_{D_1}} = \frac{C_{f_2}}{C_{f_1}} \frac{Re_2^2}{Re_1^2} = \boxed{69.8}$$

$q_{conv} = \bar{h}_L A_s (T_s - T_\infty)$ $\bar{h} \approx \bar{Nu}$

$$\frac{\bar{Nu}_1 = .664 Re^{.5} \cancel{Pr^{1/3}}}{\bar{Nu}_2 = (.037 Re^{.8} - 871) \cancel{Pr^{1/3}}} = .143$$

Laminar

Turb

$$\boxed{\frac{q_2}{q_1} = 6.97}$$

$$\frac{Nu_2}{Nu_1} = 6.97$$

#4 for external flow over a flat plate, why does $h_{local} \downarrow$ with downstream location for laminar (or turbulent) flow?

a)

boundary layer increases, then $\frac{\partial T}{\partial y} \downarrow$ because $dy \uparrow$ (6.5) p 350

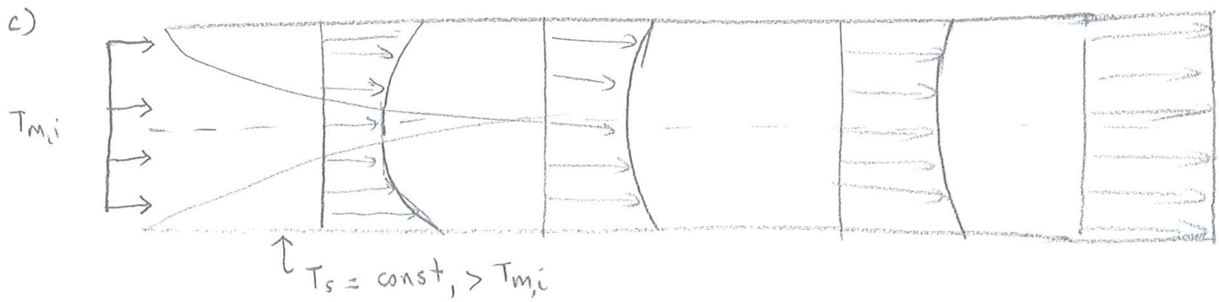
$h \approx \frac{dT}{dy}$ so $h \downarrow$.

b) flow $\rightarrow$ how will drag force & convection h.t. rate change?

drag = $F_{friction} + F_{pressure} \uparrow$ so $F_{drag} \uparrow$

h depends on C_D, but we cannot relate C_D to Nu. h. remains

c)

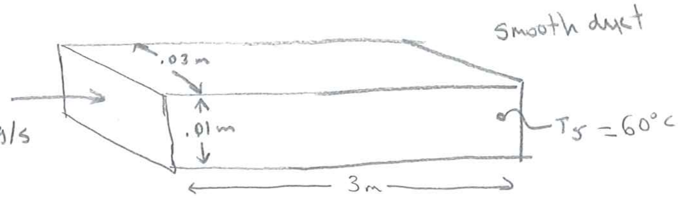


summer

Exam 3 2011

1)

Water @315K



find $\bar{h}$, $T_{m,o}$

$\dot{m} = .01\ kg/s,\ .2\ kg/s$

$T = 20°C$

$\rho = \frac{1}{1.009}$

$c_p = 4179$

$\mu = 631 \times 10^{-6}$

$k_f = .634$

$P_r = 4.16$

$\boxed{\bar{h}_1 = \frac{Nu_D k}{D_h}} = \frac{3.96(.634)}{.015} = \boxed{167}$

$\boxed{D_h = \frac{4A_c}{P} \quad pg\ 489}$

$D_h = \frac{4(.01)(.03)}{2(.04)} = .015$

$\bar{h}_2 = \frac{Nu\, k}{D_h} \qquad Nu_2 = .023\, Re_D^{4/5}\, P_r^{.4} = 93.1$

$Re_1: \quad U_m = \frac{.01 kg/s\ (1.009\ m^3/kg)}{(.01)(.03)\ m^2} = 33.6$

$Re_1 = \frac{\rho V D_h}{\mu} = 792$ laminar

$Re_2 = \quad = 15832$ turbulent

$\bar{h}_2 = \frac{93.1(.634)}{.015} = \boxed{3935}$

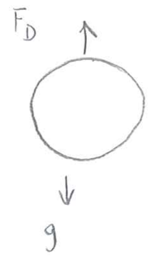
$\boxed{T_{m,o_1} = T_s - (T_s - T_{m,i})\, e^{-\left(\frac{\bar{h} A_s}{\dot{m} c_p}\right)}} = 60 - (40) e^{-\frac{167(.24)}{(.01)(4179)}}$

$A_s = (.02 + .06)(3) = .24$

$$\boxed{\begin{aligned} T_{si,1} &= 44.7°C \\ T_{si,2} &= 47.1°C \end{aligned}}$$

4

8.74



Copper spheres

D = .02 m

ν

engine oil

T = 300°K

$\nu = 550 \times 10^{-6}$

drop @ terminal Velocity

drag vs. gravity

height of oil to drop from 360 to 320?

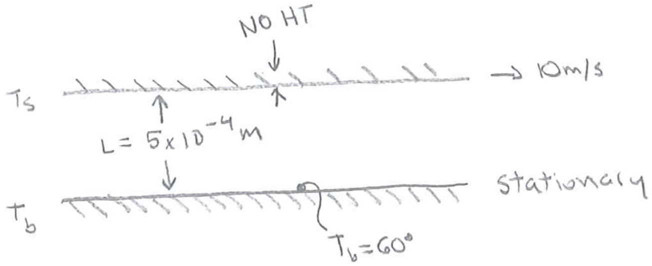
$F_D = C_D A_f \frac{\rho V^2}{2}$

$\frac{D}{\nu} = \frac{.02}{\nu} = 36.4$

$f_g = 9.81\,(8920\ \text{kg/m}^3)(3.35\times10^{-5}\ \text{m}^3) = 2.60\ \text{N}$

$-1000\ \text{kg/m}^3$

Summer 2010 Exam 2 #2



Oil
350 K

$\rho = 853.9$
$C_p = 2118$
$\mu = 3.56\times10^{-2}$
$\nu = 41.7\times10^{-6}$
$k = .138$
$Pr = 546$

Assume no H.T to upper plate
find temp of upper plate

— Couette flow —

$$T = -\frac{\mu}{2k}\left(\frac{U}{L}\right)^2 y^2 + C_1 y + C_2$$

BC @ y = L

$$q'' = -k_f \frac{dT}{dy}\Big|_{y=L}$$

$$-k_f \frac{dT}{dy} = 0$$

BC @ y = 0

$$T_0 = 0 + 0 + C_2 = 60°C$$

$$\frac{\partial T}{\partial y} = -\frac{\mu}{k} y \left(\frac{U}{L}\right)^2 + C_1 = 0 \text{ @ } y = L$$

$$-\frac{\mu}{k}\frac{U^2}{L} + C_1 = 0 \Rightarrow C_1 = \frac{\mu}{k}\frac{U^2}{L}$$

$$T_{y=L} = -\frac{\mu}{2k}\frac{U^2}{\cancel{L^2}}(\cancel{L^2}) + \frac{\mu}{k}\frac{U^2}{\cancel{L}}(\cancel{L}) + 60$$

$$= \frac{\mu}{2k}U^2 + 60 = \boxed{72.9°C}$$

(6)

Exam 2 SS'10

3.

fluid 1	fluid 2
$P_r = 6$	$P_r = 60$
$\mu = 10^{-6}\ m^2/s$	$\mu = 10^{-5}\ m^2/s$
$k = 0.5\ W/m.K$	$k = 5\ W/mK$

use $\frac{q_1}{q_2} \Rightarrow$ $\boxed{\text{find: } \frac{\text{Cooling time 2}}{\text{Cooling time 1}} \text{ when fluid has changed}}$

$Re_{L,1} = 10^6$ $\quad Re_{cr} = 5\times10^5$ $\quad$ mixed flow $\qquad 10^6 = \frac{\rho u_\infty x}{\mu}$

$$\frac{Re_2}{Re_1} = \frac{\frac{\cancel{\rho u_\infty x}}{\mu}}{\frac{\cancel{\rho u_\infty x}}{\mu}} = \frac{\mu_1}{\mu_2} = \frac{10^{-6}}{10^{-5}} = 0.1 \qquad Re_2 = 0.1\,Re_1 = 10^5 < 5\times10^5 \text{ laminar flow}$$

$\boxed{\text{cooling time } t \sim \frac{1}{\bar{h}_L}}$ eq (5.5) p 258 [cooling spheres example 411] actually relationship was stated in problem

$$\bar{h}_L = \frac{\overline{Nu}_L\, k_f}{L} = \frac{t_2}{t_1} = \frac{\overline{Nu}_1\, k_{f_1}}{\overline{Nu}_2\, k_{f_2}}$$

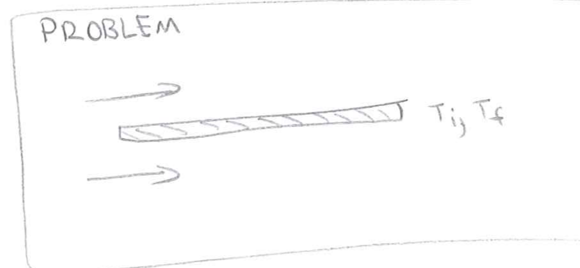


Table 7.9 (p 431)

$$\overline{Nu}_{L_1} = (.037\,Re_L^{4/5} - 871)\,Pr^{1/3} = 2659$$

$$\overline{Nu}_{L_2} = .664\,Re_x^{1/2}\,Pr^{1/3} = 822$$

$$\frac{t_2}{t_1} = \frac{2660\,(0.5)}{822\,(5)} = \boxed{.324}$$

Exam 2 SS10

4

Fluid

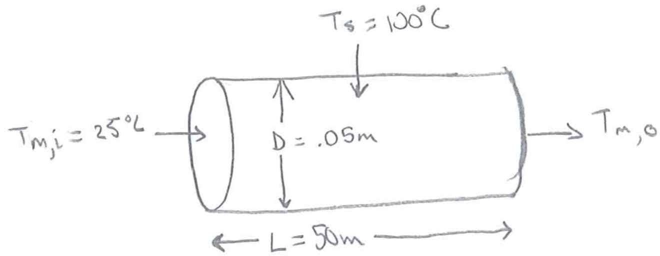
$C_p = 1 \frac{kJ}{kg \cdot K}$ $\dot{m} = 1 kg/s$

$\mu = .05 \, Ns/m^2$

$k = 0.5 \, W/m^2$

$\rho = 800 \, kg/m^3$

$P_r = 50$



$\boxed{\text{i) Find } T_{m,o}}$

$$T_{m,o} = T_s - (T_s - T_{m,i}) e^{-\left(\frac{\bar{h} A_s}{\dot{m} c_p}\right)} \qquad \bar{h} = \frac{Nu_D k_f}{D_h} \qquad Nu_D =$$

$$\boxed{Re_L = \frac{U_m D}{\nu}} = \frac{\rho U_m D}{\mu} \qquad U_m = \frac{\dot{m} \nu}{A} = \frac{\dot{m}}{\rho A} \qquad \frac{\frac{\cancel{kg}}{s} \frac{m^3}{\cancel{kg}}}{\cancel{m^2}} = \frac{m}{s}$$

$$= \frac{\cancel{\rho} \dot{m} D}{\cancel{\rho} \mu \frac{\pi D^2}{4}} = \frac{\dot{m} 4}{\mu \pi D} = \frac{1(4)}{.05(\pi)(.05)} = 509 \Rightarrow \text{laminar}$$

$Nu_D = 3.66$

$$\bar{h} = \frac{3.66 \, k_f}{D_h} = 36.6 \Rightarrow T_{m,o} = 100° - 75 e^{-\left(\frac{36.6 \, \pi (.05)(50)}{1000 (1)}\right)}$$

$$\boxed{T_{m,o} = 43.7 \, °C}$$

$\boxed{\text{ii) change } T_s \text{ const to } q'' \text{ const. to get same } T_{m,o} \quad \text{find } q''}$

$$Nu_D = 4.36 \qquad T_{m,o} = T_{m,i} + \frac{q'' A_s}{\dot{m} C_p} \qquad q'' = \frac{(43.7 - 25)(1)(1000)}{\pi (.05)(50)}$$

$$\boxed{q'' = 2381 \, W/m^2}$$

iii) $\boxed{\text{find } T_{s,o} \text{ for } q''}$

$$q''_s = h(T_s - T_m) \qquad T_s = \frac{2381}{h} + 43.7°C \qquad h = \frac{Nu \, k}{D_h} = \frac{4.36(.5)}{.05}$$

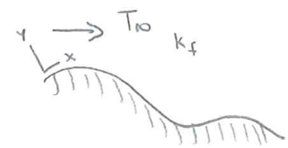
$$\boxed{T_s = 98.3°C}$$

8

SS11 EXAM 2

1)

$T(x)$, arbitrary surface $\qquad \dfrac{T - T_\infty}{T_s - T_\infty} = e^{\left(-\frac{50y}{k_f} x^{1/4}\right)}$ $\qquad L = 2m$



convection coefficient h, $\boxed{h = \dfrac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty}}$ (6.5) p 350

$$T = T_\infty + (T_s - T_\infty) e^{-\left(\frac{50y}{k_f x^{1/4}}\right)}$$

$$\frac{\partial T}{\partial y} = (T_s - T_\infty) \frac{-50}{k_f x^{1/4}} e^{-\left(\frac{50y}{k_f x^{1/4}}\right)}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = (T_s - T_\infty) \frac{-50}{k_f x^{1/4}} \Rightarrow \boxed{h = \frac{50}{x^{1/4}}}$$

average $\bar{h}$ across length $L = 2m$

$\boxed{\bar{h} = \dfrac{1}{A_s}\displaystyle\int_{A_s} h\, dA_s}$ $\qquad A_s = L(W=1) \Rightarrow \bar{h} = \dfrac{1}{2}\displaystyle\int_0^2 50x^{-1/4}\, dx = \dfrac{1}{2}\left[\left(\dfrac{4}{3}\right)50x^{3/4}\right]_0^2$

(6.9) p351 $\qquad \boxed{\bar{h} = 56.\ W/m^2 K}$

9

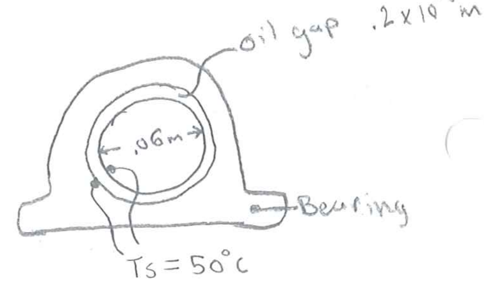
Exam 2 SS'10

2)

Couette flow:

Max oil temp ?

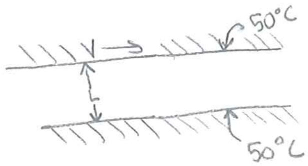
heat flux from oil to shaft ?



oil

$\mu = 0.5 \; Ns/m^2$

$K = 0.15 \; W/mK$



— Couette flow —

$$T = T_b + \frac{\mu}{2k} U^2 \left(\frac{y}{L} - \frac{y^2}{L^2} \right) + \cancel{(T_s - T_b)}^{\,0} \frac{y}{L}$$

$T_s = T_b = 50°C$

$$U = \omega r = 3000 \text{ rpm} \left(\frac{2\pi}{\text{rev}} \right) \left(\frac{1 \text{min}}{60 \text{sec}} \right) \times (.03 \text{m}) = 9.42 \text{ m/s}$$

[Solution uses 3 m/s]

$$T = 50 + \frac{0.5}{2(.15)} (3)^2 \left(\frac{y}{L} - \frac{y^2}{L^2} \right)$$ Max temp at $y = \frac{L}{2}$

$$T_{max} = 50 + 15 \left(.5 - (.5)^2 \right) = \boxed{53.75 \text{ °celcius}}$$

heat flux q' fluid to shaft = $\boxed{q'' = -k_f \left. \frac{dT}{dy} \right|_{y=L}}$ eq (6.3) pg 350

$$q_s'' = -\cancel{k_f} \frac{\mu}{2\cancel{k_f}} U^2 \left[\frac{1}{L} - \frac{2y}{L^2} \right]_{y=L} = -\frac{\mu V^2}{2} \left[\frac{1}{L} - \frac{2\cancel{L}}{L^{\cancel{2}}} \right] = -\frac{\mu V^2}{2} \left(-\frac{1}{L} \right)$$

$$= \frac{\mu V^2}{2L} = \frac{0.5 \, (3)^2}{2 \, (2 \times 10^{-4})} = 11250 \; W/m^2$$ Solution

(10)

Example from Class

$\boxed{\text{find Length}}$

air x-flow

$V_\infty = 10\,m/s$

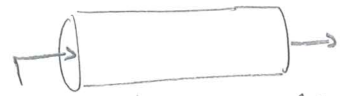
$P_r = .707$

$D = .10m$

$T_\infty = 20°C$

$\dot{m} = 1 kg/s$

T_∞, external fluid



$T_{m,o} = 40°C$

$T_{m,i} = 85°C$

$\rho = 1000$

$C_p = 3000$ J/kg·K

$\mu = 0.041$

$k = .26$

internal fluid

eq (8.45a) p472

$$\boxed{\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = e^{-\left(\frac{\bar{U} A_s}{\dot{m} c_p}\right)}} \Rightarrow \ln\left(\frac{20-40}{20-85}\right) = -\frac{\bar{U} PL}{1(3000)}$$

Air

$k = 26.3 \times 10^{-3}$

$$\bar{U} = \left(\frac{1}{h_i} + \frac{1}{h_o}\right)^{-1}$$

$$\cancel{Re_i = \frac{\rho V D}{\mu} = \frac{4\dot{m}}{\pi D \mu} = 318} \quad \text{~~laminar~~}$$

$$\cancel{h_i = \frac{Nu_D k_f}{D} =}$$

$$\cancel{f = \frac{64}{Re} = 0.201}$$

$h_i = 9.5$, given

(7.44) p 431 x-flow

$$Re_{,o} = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{10(.10)}{15.89 \times 10^{-6}} = 63{,}000 < 5 \times 10^5 \text{ laminar}$$

$$\longrightarrow Nu_D = C Re_D^m Pr^{1/3} = .027(63000)^{.805}(.707)^{1/3} = 176 \qquad h_o = \frac{Nu\,k}{D} = 46.3$$

$\bar{U} = 7.9 \qquad P = \pi(0.1) \Rightarrow \boxed{L = 1425}$

Exam SS 08

1)

fluid

$Pr_1 = 4$

$Re_1 = 2000$

$T_s = \text{const} \;, > T_m$

Given

$$\boxed{\frac{D_2}{D_1} = \frac{1}{5}}$$

find,

$$\boxed{\frac{L_2}{L_1} \quad \text{friction factor } \frac{2}{1}}$$

$$\ln\left(\frac{\Delta T_o}{\Delta T_i}\right) = -\frac{\bar{h} P L}{\dot{m} c_p} \qquad \bar{h} \approx \frac{Nu\, k_f}{D_h} \qquad \frac{\bar{h}_2}{\bar{h}_1} = \frac{Nu_2}{Nu_1}\frac{D_1}{D_2} = \frac{63.5}{3.66}(5) = 86.7$$

$$\frac{Re_2}{Re_1} = \frac{\frac{4\dot{m}}{\pi D_{i_2} \mu}}{\frac{4\dot{m}}{\pi D_{i_1} \mu}} = \frac{D_{i_1}}{D_{i_2}} \Rightarrow Re_2 = 5(2000) = 10{,}000 \quad \text{turbulent}$$

$$Nu_2 = .023(10{,}000)^{4/5}(4)^{0.4} = 63.5$$

$$Nu_1 = 3.66$$

$$L \approx -\frac{1}{h}\frac{1}{P} \qquad P \approx D$$

$$\frac{L_2}{L_1} = \frac{\frac{1}{h_2}\frac{1}{P_2}}{\frac{1}{h_1}\frac{1}{P_1}} = \frac{h_1 D_1}{h_2 D_2} = \boxed{.058}$$

ii) friction factor

$$f_1 = \frac{64}{Re_1} = .032 \qquad f_2 = .316(10{,}000)^{-1/4} = .0316 \qquad \boxed{\frac{f_2}{f_1} = .9875}$$

iii) pump power

$$\frac{P_2}{P_1} = \frac{f_2\left(\frac{L_2}{D_2}\right)\left(\frac{\cancel{\rho} u_{m_2}^2}{\cancel{2}}\right)\left(\frac{\cancel{\dot{m}}}{\cancel{\rho}}\right)}{f_1\left(\frac{L_1}{D_1}\right)\left(\frac{\cancel{\rho} u_{m_1}^2}{\cancel{2}}\right)\left(\frac{\cancel{\dot{m}}}{\cancel{\rho}}\right)} = \left(\frac{f_2}{f_1}\right)\frac{L_2 D_1}{L_1 D_2}\frac{u_{m_2}^2}{u_{m_1}^2}$$

$$= (.9875)(.058)(5)\left(\frac{D_1^4}{D_2^4}\right)$$

$$\boxed{\begin{aligned} \text{Power} &= \frac{\Delta P\, \dot{m}}{\rho} \\ &= \frac{\dot{m}}{\rho} f \frac{L}{D} \frac{\rho u_m^2}{2} \end{aligned}}$$

$$u \approx \frac{\dot{m} v}{A} \approx \frac{1}{D^2}$$

$$\frac{u_2}{u_1} = \frac{D_1^2}{D_2^2}$$

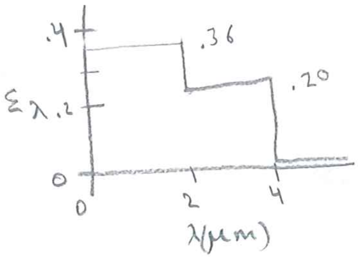
$$\boxed{\frac{P_2}{P_1} = 179}$$

David Malawey 225 HW #10

100

12.32a, 46, 51, 109

32) find total hemispherical emissivity ε & total emissive power E @ 2000K



$$\varepsilon = \frac{1}{\sigma T^4}\left[\varepsilon_{\lambda_1}\int_0^{\lambda_1}\frac{E_{b\lambda_1}\,d\lambda}{E_b(T)} + \int_{\lambda_1}^{\lambda_2}\frac{E_{b\lambda_2}\,d\lambda}{E_b(T)}\right]$$

$$= \varepsilon_{\lambda_1}F_{(0\to\lambda_1)} + \varepsilon_{\lambda_2}F_{(\lambda_1\to\lambda_2)} \quad \text{(Table 12.1)}$$

$$(.36).4809 + .20(.8563-.4809) = \boxed{0.248 = \varepsilon}$$

$$E = \varepsilon(2000k)\cdot E_b(2000k)$$

$$= 0.25(5.67\times10^{-8})\times 2000k^4 = 226.8\,kW/m^2$$

46) opaque
$2m \times 2m$ $400K = T$
$\tau = 0$, $\alpha + \rho = 1$
$T_s = 5800$

Solar Irradiation
$G_s = 1200\,W/m^2$
$\alpha_\lambda = 0, 0.8, 0, .9$
$\lambda \Rightarrow 0, .5, 1, 2, >2$

find absorbed radiation, $(G\alpha)$ emissive pwr $\boxed{E}$
radiosity, (J)
net radiation h.t.

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda\,d\lambda}{\int G_\lambda\,d\lambda} = \frac{\int_0^\infty \alpha_\lambda E_{\lambda b}(5800k)\,d\lambda}{E_b} = \alpha_1 F_{(0.5\to1)} + \alpha_2 F_{(2\to\infty)}$$

$$\alpha_s = 0.8(.720-.250) + 0.9(1-.941) = .429$$

$$G_{abs} = \alpha_s G_{(s)} = 0.429(1200\,W/m^2) = \boxed{515\,W/m^2}$$

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda b}(400k)\,d\lambda}{E_b} = \varepsilon_1 F_{(0.5\to1)} + \varepsilon_2 F_{(2\to\infty)}$$

$$E = \varepsilon\sigma T_s^4 = 0.9\times5.67\times10^{-8}(400)^4 = \boxed{1306\,W/m^2}$$

$$J = E + \rho_s G_s = E(1-\alpha_s)G_{(s)} = (1306 + .571(1200)) = \boxed{1991\,W/m^2}$$

$$q_{net} = (E - \alpha_s G_s)A_s = 1306 - 515\,W/m^2 \times 4m^2 = \boxed{3164\,W}$$

12.51 opaque, diffuse surface

$T = 1250K$

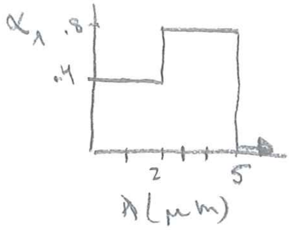
Find:

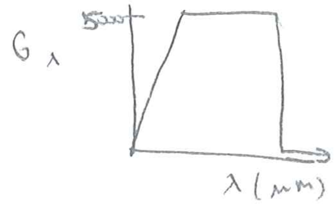
Total absorptivity $\frac{G_{abs}}{G}$ or α

$E(1250K)$

how will surface temp change?

$\boxed{\text{decreasing}}$





$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} \qquad G_{abs} = \frac{0.4(5000\ W/m^2(2))}{0.8 \times 5000(5-2)} + 0 = 14{,}000\ W/m^2$$

$$G = \frac{2\mu m \times 5000}{2} + (10-2) \times 5000 = 45{,}000$$

$$\alpha = \frac{14000}{45{,}000} = \boxed{0.311} = \alpha$$

Eq 12.36 —
$$\varepsilon = .4\int_0^2 \frac{E_{\lambda b}\, d\lambda}{E_b} + 0.8\int_2^5 \frac{E_{\lambda b}\, d\lambda}{E_b}$$

$$= 0.4(.162) + 0.8(.757 - .162) = 0.54$$

$$E = \varepsilon E_b = \varepsilon \sigma T^4 = 0.54(5.67 \times 10^{-8})(1250) = \boxed{74{,}751\ W/m^2}$$

$$q''_{net} = \alpha G - E = (14000 - 74{,}751) = \boxed{-60{,}751\ W/m^2}$$

12.109 $T_{sur} = 17°C$

$T_\infty = 17°C$

$\bar{h} = 20\ W/m^2 K$

$\alpha_s = 0.9$

$\varepsilon = 0.1$

Kirchoff's law $\alpha = \varepsilon$

$\dot{E}_{in} - \dot{E}_{out} = 0$

$\alpha_s G(s) + \alpha G_{sur} - q''_{conv} - \varepsilon E_b(T_s) = 0$

$$\alpha_s G_s + \varepsilon \sigma T_{sur}^4 - \bar{h}(T_s - T_\infty) - \varepsilon \sigma T_s^4 = 0$$

$$.9(900) + 0.1\sigma(17+273)^4 - 20(T_s - 290) - 0.1\sigma T_s^4 = 0$$

$$6850 = 20T_s + 5.67 \times 10^{-9} T_s^4 \Rightarrow \boxed{T_s = 329.2\ K}$$

Steady-state Temp

Handouts

HEISLER CHARTS

1. Plane Wall

$$\theta_o^* = \frac{\theta_o}{\theta_i} = \frac{T_o - T_\infty}{T_i - T_\infty}$$

$$\theta^* = \frac{T_s - T_\infty}{T_i - T_\infty}$$ T_s or at other given x-value

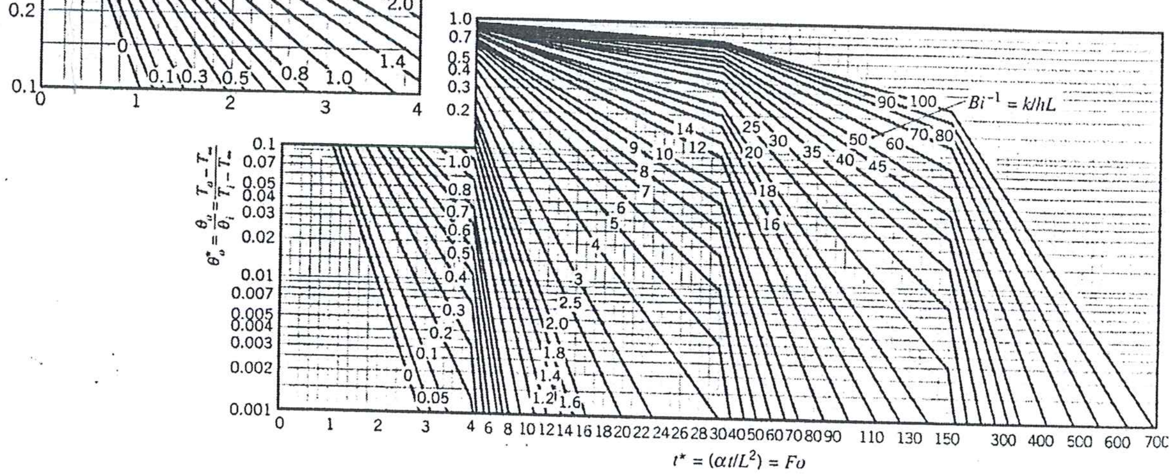


FIGURE D.1 Midplane temperature as a function of time for a plane wall of thickness $2L$ [1]. Used with permission.

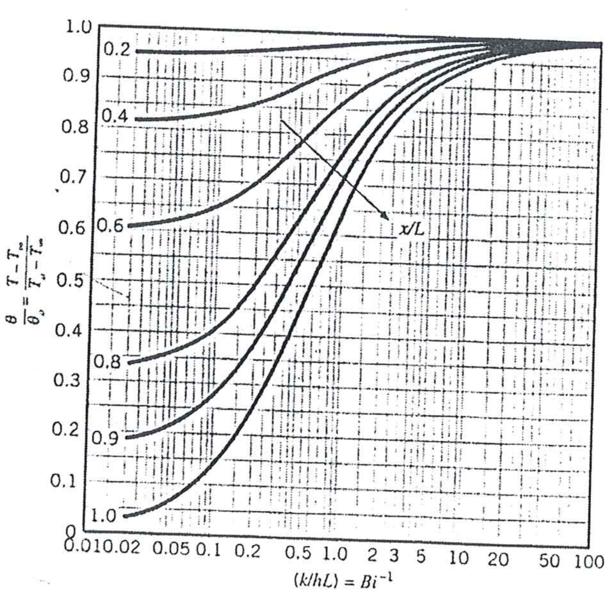
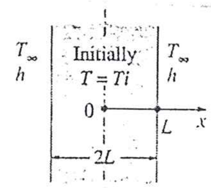


FIGURE D.2 Temperature distribution in a plane wall of thickness $2L$ [1]. Used with permission.



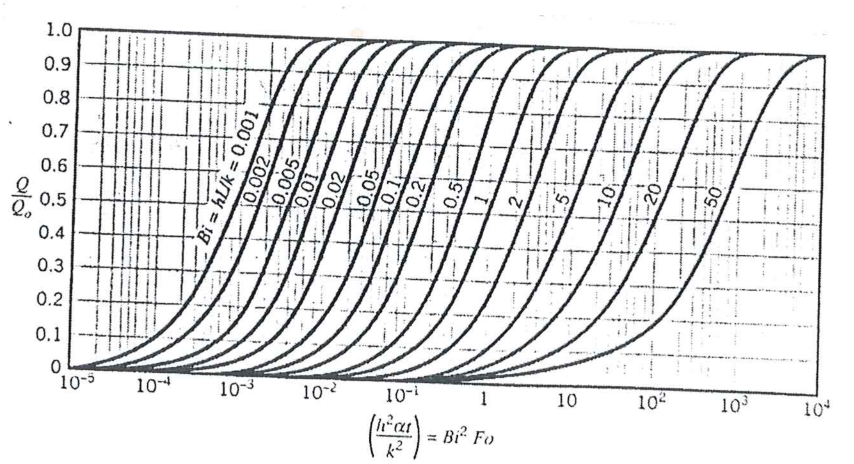


FIGURE D.3 Internal energy change as a function of time for a plane wall of thickness $2L$ [2]. Adapted with permission.

2. Long Cylinder

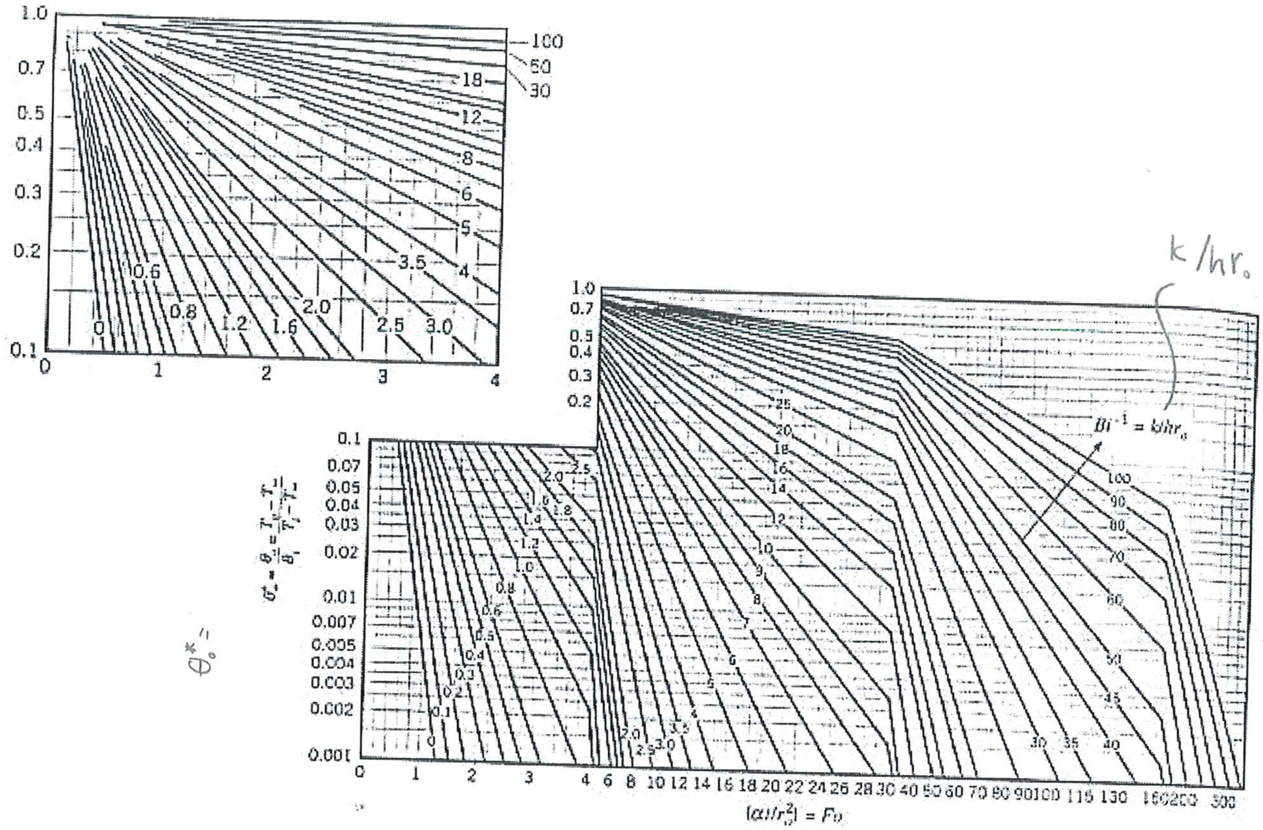


FIGURE D.4 Centerline temperature as a function of time for an infinite cylinder of radius r_o [1]. Used with permission.

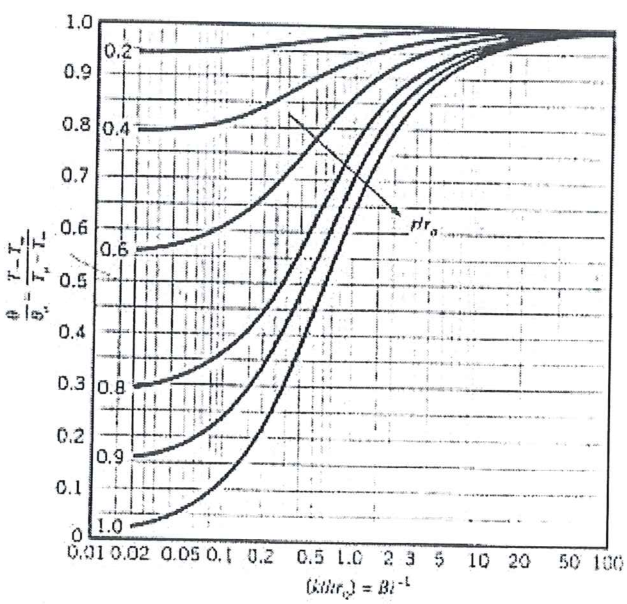


FIGURE D.5 Temperature distribution in an infinite cylinder of radius r_o [1]. Used with permission.

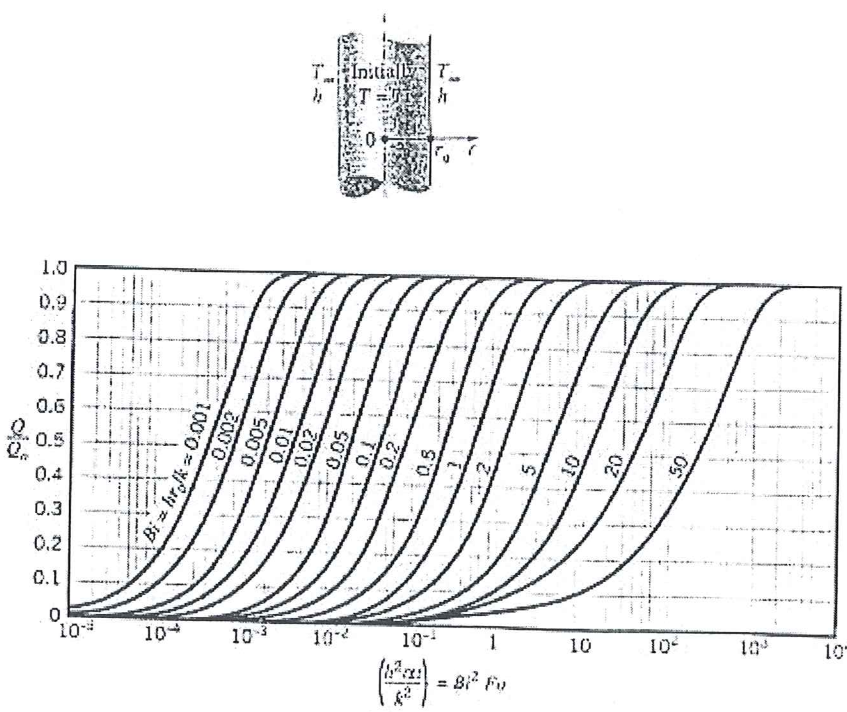


FIGURE D.6 Internal energy change as a function of time for an infinite cylinder of radius r_o [2]. Adapted with permission.

3. Sphere

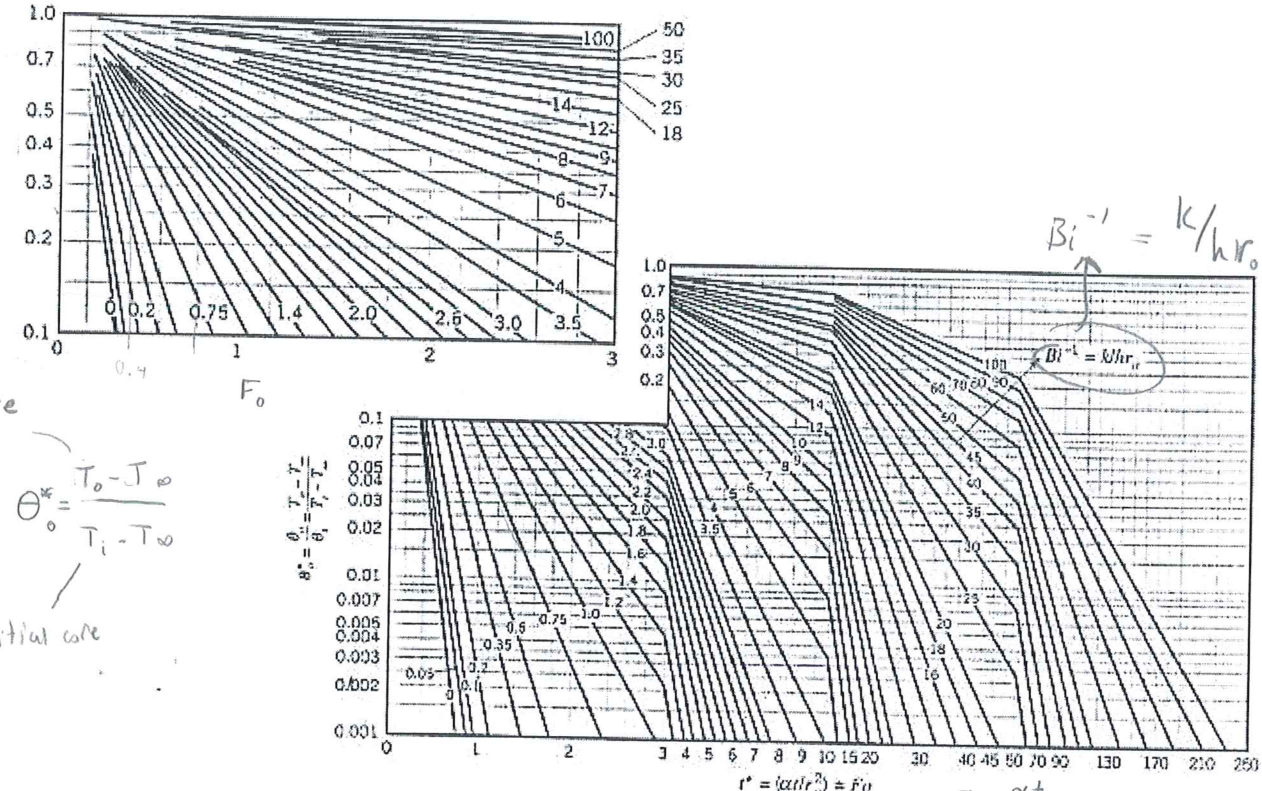
θ^*

at core

$$\theta^*_o = \frac{T_o - T_\infty}{T_i - T_\infty}$$

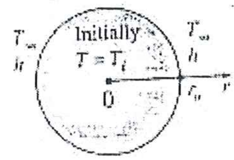
initial core

$Bi^{-1} = k/hr_o$



$F_o = \frac{\alpha t}{r_o^2}$

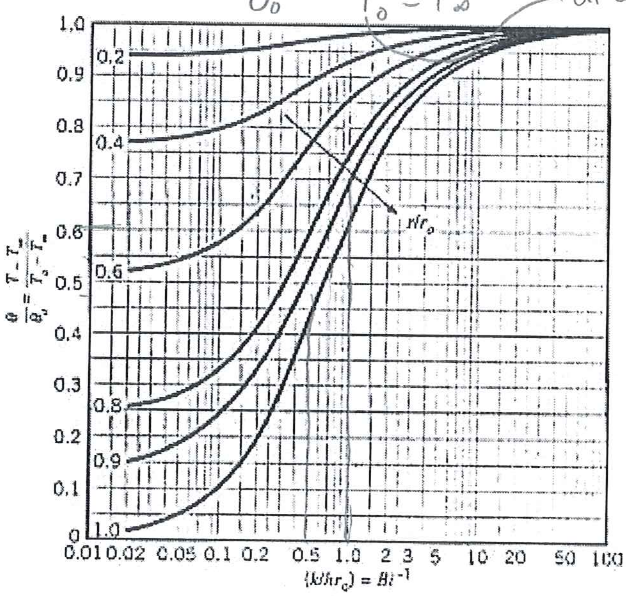
FIGURE D.7 Center temperature as a function of time in a sphere of radius r_o [1]. Used with permission.



at spec'd radius

$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty}$$

at core



...URE D.8 Temperature distribution in a sphere of radius r_o [1]. Used with permission.

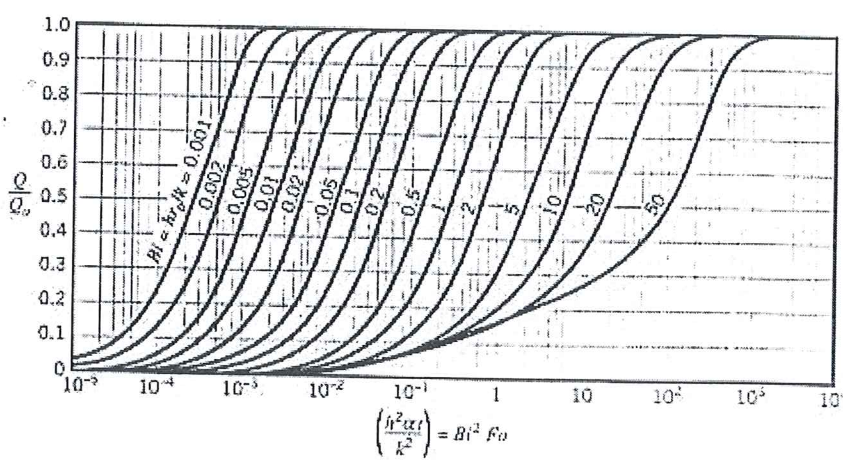


FIGURE D.9 Internal energy change as a function of time for a sphere of radius r_o [2]. Adapted with permission.