

ME 225

HEAT

TRANSFER

DR. KOYLU

Compiled Mechanical Engineering Coursework by David Malawey
Class of 2011 December at Missouri University of Science & Technology
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Course Binder Part 1 of 2
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ME 225 HEAT TRANSFER
Spring 2011

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Office Hours:

TW: 13:00-14:00; Other times: open door or by appointment

Schedule:

TTh 9:30-10:45, 254 Toomey

Background:

Differential Equations, Thermodynamics (Fluid Mechanics +)

Textbook:

“Introduction to Heat Transfer” by Incropera et al., 5th Edition, Wiley

Suggested Reading:

Any heat transfer book, e.g., “Heat Transfer” by Cengel, McGraw-Hill

Objective:

To introduce students to the fundamentals of conduction, convection and radiation heat transfer, and to provide the necessary theoretical/practical tools to tackle real engineering applications involving energy systems

Work:

	<u>Chapters Covered</u>	<u>Tentative Date[†]</u>	<u>Weight</u>
Test 1	1, 2, 3, 5	02/15/11	25%
Test 2	6, 7, 8	03/24/11	25%
Test 3	All	05/06/11	25%
8 Problem Sets	All	Weekly	10%
2 Projects		To be announced	15%

General Outline:

<u>Book Chapter</u>	<u>Subject</u>
1	General Introduction
2	Introduction to Conduction
3	One-Dimensional Steady-State Conduction
5	Transient Conduction
6	Introduction to Convection
7	Forced Convection - External Flows
8	Forced Convection - Internal Flows
11	Heat Exchangers
12	Radiation: Processes and Properties
13	Radiation Exchange between Surfaces

[†] It is the student's responsibility to follow the most up-to-date announcements for all tests.

Course Policies

Grades: The grades will be treated on a relative basis, that is, the average score of the entire class, X , and the standard deviation, σ , will determine the final outcome. For a typical class, this approximately corresponds to a “B” for $X < \text{Grade} \leq X + \sigma$, and a “C” for $X - \sigma < \text{Grade} \leq X$ at the end of the semester. Any further positive or negative deviations from the mean will result in an “A” or a “D”, respectively. A 60% cut-off may be enforced for a passing grade when applicable. The instructor reserves minor discretion over grades for some critical cases.

Exams: There will be three tests, which will be related to the material covered in the lectures/textbook, problems solved in the class, and homework/project assignments. Students are allowed to bring the textbook, their own class notes, handouts, and assignments to each test. Because all exams are required, an unexcused absence will result in a zero for that exam. Therefore, a student, who cannot attend an exam because of illness or emergency, must contact the instructor and receive permission before the exam. After the test, only a last minute real emergency will be considered as a possible excused absence for which the student will be expected to notify the instructor about the situation as soon as possible. In any case, necessary official paperwork must be supplied to prove the emergency. The instructor has the final authority to determine whether a student will be given a make-up exam. Finally, academic dishonesty is a serious offense and will be treated accordingly.

Assignments: A total of eight homework assignments will be given on a regular basis. A problem set will be usually due in one week at the end of a class period after it is assigned. Solutions to homework will be e-mailed to students after due dates. It is extremely important that you first attempt to solve the problems on your own, and, if unsuccessful, contact the instructor with your specific questions. Group work is also encouraged. Late assignments are not accepted.

Projects: There will be two projects related to the practical and design aspects of heat transfer (details to be given during the semester). For each project, two students will team up to prepare a professional report. Projects are not to be discussed with other students/groups. Similar to assignments, late submissions of projects are strictly unacceptable.

Attendance: A regular attendance is expected from students to be successful in this course although it is not directly counted toward students’ final grades.

Expected Performance: Attend class regularly, participate in discussion and ask questions in the class, solve homework problems on a regular basis, submit assignments and projects on time, and not miss an exam.

Feedback: The instructor can provide feedback about your performance in this course and specifically what you need to do to improve it only per your request. He also would appreciate your feedback about how the course material and presentation could be further improved. Please see him confidentially for any suggestions/concerns you may have related to the course.

— Heat Exchanger —

$$G = \dot{m} C_p$$

$$\epsilon = \frac{q}{q_{\max}}$$

$$h_i = \frac{N_u k_f n_{\text{fluid}}}{D}$$

$$q_{\max} = G_{\min} (T_{hi} - T_{ci})$$

$$Re_D = \frac{V_m p D}{\mu}$$

$$\text{Thin-wall tube: } U = \left[\frac{1}{h_i} + \frac{1}{h_o} \right]^{-1}$$

$$\text{With Fouling & wall: } \frac{1}{U_i A_i} = \left[\frac{1}{h_i A_i} + \frac{R_{f,i}''}{A_i} + \frac{\ln \frac{D_o}{D_i}}{2\pi k L} + \frac{R_{f,o}''}{A_o} + \frac{1}{h_o A_o} \right]$$

$$NTU = \frac{UA}{G_{\min}}$$

— Ch. 12 Radiation —

- gray = α is independent of λ
- $\alpha = \epsilon$

$$\epsilon(T_s) = \frac{\int_0^{\infty} \epsilon_{\lambda} E_{\lambda b} d\lambda}{E_b}$$

- F factors, Pg 702

$$E_b = \sigma T^4 ; E = \epsilon \sigma T^4$$

- 2-surface enclosures pg 793

$$J_{\text{radiosity}} = E + G_{\text{reflected}}$$

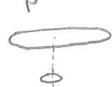


$$G_{\text{irradiation}} = \text{all incident rad.}$$

— Ch. 13 View Factors & Rad. —

Basic View factors: pg 775

$A_i \ll A_j$, pg 781



$$F_{12} = \frac{A_2}{A_1} F_{21}$$

$$F_{11} + F_{12} + F_{13} + F_{14} (\text{enclosed}) = 1$$

$$\text{net } q_{12, \text{general}} = \frac{\sigma (T_1^4 - T_2^4)}{\left[\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2} \right]}$$

Example Problems:

Flat Plates

Constant (T _{inf} -T _s) Re increase, find q ratio	Summer 2010 #3 p3
Boundary layer thickness, local	HW 7.2
Velocity(inf) and T(inf) given	HW 7.11
Convection & solar absorptivity, irradiation	HW 7.20
Cooling time with changed fluid	Spring 2010 #3 p7

Tubes

Circular

Constant T _s , find T _{so}	Spring 2010 #4 p8
Annular, find L to cool	HW 8.93
Cross flow, tube conduction	HW 8.50
Pump Power	HW 8.31
Pump power	Fall 2008 #1 p12
Air outside, fluid inside	example p11

Noncircular

Constant T _s , thin wall, find T _{si} , T _{so}	Summer 2011 #1 p4
Constant q'', find T _s , Toutlet	HW 8.74

Arbitrary surfaces

Velocity profile given, find average Nu	Summer 2010 #1 p1
Temp profile given, find average h	Spring 2011 #1 p9

Pin fin

Diameter doubles, find new q	
Diameter doubles, inf long, find new q	HW 7.45

Couette flow

Surface 1: Constant T _s , Surface 2: no HT	Summer 2010 #2 p6
Both Surfaces constant T _s . Find T _{max}	Spring 2010 #2 p10

Drag coefficient

Cooling spheres	HW 7.73
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Exam 3 Help Index

Chapter 11:

- (1) Overall U with fouling: 11.2 2008, #1 fine fouling factor given q
 (2) Shell and Tube HX: 11.22 , also 2008 #3 pg 7
 (3) Concentric Tube: 11.18
 (4) Cross-flow HX 11.34
 (5) Condensing steam 11.35

Chapter 12

- (22) G_{abs} as a function off F and alpha 12.46 (22) step function ϵ , 12.32
 (17) G with angle of approach, Q received by lens 12.47 Redo:
 (11) Absorptivity, $f(\lambda)$ and Emissive power 12.51
 (13) Transmitted through car window 2008, #2
 (13) Earth's radiation from sun Example 12.20
 (15) Find alpha from G_{abs}/G (step function) 2008 #2
 find detector incident radiation

Chapter 13:

- (18, 19) View factors, all sorts 13.1
 (8) Net radiation to A1 in enclosed surface 13.19
 or 19 -dot in cylinder
 (8) Net radiation of close parallel plates 13.41
notes Radiation shield, flat plates Ex 13.56
 (23) Radiation shield, q balance w/convection 13.62
 (14) Long oven view factors 2005 #3
 (21) Net HT between two enclosed surfaces A #3,ii
 -side in semicircle

Review for Exam 2

Air properties P.852

CH.7 EXTERNAL FLOW

C_f , average friction coefficient

δ , boundary layer thickness (where $\frac{u}{u_\infty} = 0.99$)

α = absorptivity

ε = emissivity

$$\text{Memorize } q'' = A \frac{(T_s - T_\infty)}{R_{tot}} \quad R_{tot} = \left(\frac{1}{h_1} + \frac{1}{h_2} + \frac{\zeta}{k_L} \right) \quad q = \frac{\delta T}{R_{tot}}$$

heat transfer correlations Table 7.9 p 431

T_F , flow temperature

C_D , drag coefficient p 401

$$Re_D = \text{cross flow} = \frac{V_\infty D}{\nu} \quad Re_{cr} = 5 \times 10^5$$

$$Re_x = \frac{V_\infty x}{\nu} = \frac{\rho V_\infty x}{\mu}$$

CH.8 INTERNAL FLOW

friction factor, MOODY CHART, p 461

$$\text{Circular pipe, } Re_D = \frac{U_m D}{\nu} \quad Re_{D,c} \approx 2300$$

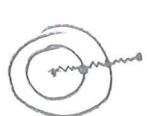
$$\dot{m}, \text{mass flow} = \rho U_m A_c \Rightarrow U_m = \frac{\dot{m}}{\rho A_c}$$

$$\bar{h}_i = \frac{Nu_p K_{fluid}}{D_n} \quad (\text{for local } \bar{h}_i \text{ avg } Nu) \quad p 489$$

$$D_h = \frac{4A_c}{\text{Perimeter}}$$

$$\text{Pumping Pwr} = \Delta P \frac{\dot{m}}{\rho} = \Delta P \dot{V} \quad \Delta P = f \left(\frac{E}{D} \right) \left(\frac{\rho U_m^2}{2} \right)$$

$$Re_D = \frac{\rho V_\infty D}{\mu} = \frac{V_\infty D}{\nu} = \boxed{\frac{4\dot{m}}{\pi D_i K}} \quad \text{circular only}$$



$$\frac{q}{L} = \frac{T_\infty - T_m}{\frac{1}{h_o A_o} + \frac{\ln \frac{D_o}{D_i}}{2\pi k L} + \frac{1}{\bar{h}_i A_i}}$$

$$q = \frac{\Delta T}{R_{tot}}$$

Reminder:

q heat trans. rate, W

q' heat trans. rate per length, W/m

q'' heat flux, W/m²

$$Nu = \frac{h x}{K}$$

Energy balance

$$T_m(x) = T_m + \frac{q'' p}{\dot{m} C_p} x$$

$$q'' = A \frac{(T_s - T_\infty)}{R_{tot}}$$

R_{ex} , reynolds by distance p 355 (6.11)

R_{ed} , Reynolds by diameter p 457 (8.1)

hydraulic diameter D_h 488. (8.66)

Relate $T_s \approx T_m$ $\boxed{T_m = T_s}$ p 463 (8.27)

Annular tubes • T_s const p 490

• q const p 491

$$h \text{ from } T(x) \text{ function (6.5)} \quad p 350 \quad h = \frac{-k_f \frac{\partial T}{\partial y} \Big|_{y=0}}{T_s - T_{\infty}}$$

$$\text{cooling time } t \text{ (5.5) (lumped capacitance)} \quad t = \frac{\rho V_c}{h A_s} \ln \left(\frac{T_i - T_{\infty}}{T - T_{\infty}} \right) \quad p 258$$

$$\text{Pipe flow } T_m \text{ & } T_{\infty, \text{outside}} \text{ NOT } q \text{ const} \quad \text{NOT } T_s \text{ const} \quad U \text{ (8.45a)} \quad p 472 \quad U = \left(\frac{1}{h_i} + \frac{1}{h_o} \right)^{-1}$$

cross flow relationships p 431 [chapter 7]

		hrs
ME 240	A	2
ME 279	A	3
EMGT 124	A	1
EE 281		3
ME 225		3

Notes Jan 11, 2011

$$q_x = -kA \frac{dT}{dx} \leftarrow \text{Fourier's Law}$$

Temperature gradient: slope of temperature.

(Within solid or non-moving fluid)



Conduction:

- Thermal diffusivity $\alpha = \frac{\text{heat conducted}}{\text{heat stored}} = \frac{k}{\rho c_p}$

$\alpha \uparrow \Rightarrow$ fast response to thermal environment

- In this class: Scott K.

- Zach Jennings

- Stephen Arnould

- Catzone

- Kayla Billadeau

Notes Jan 18]

Ch 2 Intro to Conduction

cylindrical coordinates, * radius is changing *

Analysis: determine T field in a solid/stationary fluid resulting from conditions imposed on system boundaries

- define a C.T.
 - identify relevant energy transfer processes } diff eq's & boundary int. conditions
 - write down governing eqs
- ↓
temp. field

governing eq for pure conduction problems = heat equation

$$\text{Energy conservation: } \dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g = \dot{E}_{\text{st}}$$

$$\dot{E}_g = \dot{g} dt = \dot{g} dx dy dz$$

↑ rate of energy gen. per unit volume (W/m^3)

$$\dot{E}_{\text{st}} = mc_p \frac{\partial T}{\partial t} = \rho dt c_p \frac{\partial T}{\partial t} = \rho dx dy dz c_p \frac{\partial T}{\partial t}$$

$$q_x = -k dy dz \frac{\partial T}{\partial x} \quad q_y = -k dz dx \frac{\partial T}{\partial y} \quad q_z = -k dx dy \frac{\partial T}{\partial z}$$

$$(q_x + q_y + q_z) - \left(q_x + \frac{dq_x}{dx} dx + q_y + \frac{dq_y}{dy} dy + q_z + \frac{dq_z}{dz} dz \right) + \dot{g} dx dy dz = \rho c_p \frac{\partial T}{\partial t} dx dy dz$$

Heat
Equation

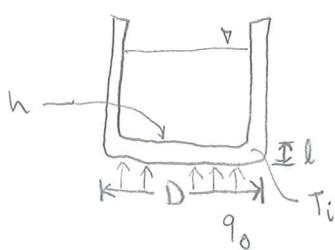
$$\boxed{\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{g} = \rho c_p \frac{\partial T}{\partial t}} \quad \leftarrow \text{Cartesian coords}$$

• also forms for cylindrical & spherical

Example (2.21) (assuming h.t. is in 1 direction) (no heat gen)

Stage 1:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{k} \frac{\partial T}{\partial t} \quad \text{boundary cond: } q_o = -k \left(\frac{\pi D^2}{4} \right) \frac{\partial T}{\partial x} \Big|_{x=0} \quad (1)$$



$$\text{b.c. } (2) \quad -k \frac{\partial T}{\partial x} \Big|_{x=L} = h [T(L, t) - T_{\infty}(t)]$$

$$\text{initial cond: } T(x, 0) = T_i$$

$$\text{Stage 2: } \frac{d^2 T}{dx^2} = 0 \quad \frac{\partial T}{\partial x} = C_1 \quad T = C_1 x + C_2 \quad \text{b.c.'s: } q_o = -k \left(\frac{\pi D^2}{4} \right) \frac{\partial T}{\partial x} \Big|_{x=0} \quad T(L) = T_b$$

* get Boundary conditions from Table 2.2.

Tan 20 notes

Ch. 3.1

- steady state $\frac{dT}{dt} = 0$
- 1-D $\Rightarrow \frac{\partial T}{\partial y} \approx \frac{\partial T}{\partial z} \ll \frac{\partial T}{\partial x}$
- no energy generation $\Rightarrow \dot{g} = 0$

$\left. \begin{array}{l} \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) = 0 \\ \frac{dq''_x}{dx} = 0 \Rightarrow q''_x = \text{const} \end{array} \right\}$

$k = \text{const} \Rightarrow \frac{d^2T}{dx^2} = 0 \Rightarrow \frac{dT}{dx} = \text{const.}$

$\Rightarrow T = C_1 x + C_2 \quad (\text{linear})$

Conduction thru wall

$$q = \frac{kA}{L} (T_{S_1} - T_{S_2}) = \frac{kA}{L} \Delta T = \frac{\Delta T}{(L/kA)} = \boxed{\frac{\Delta T}{R_{\text{cond}}}} = \frac{\text{driving potential for conduction}}{\text{thermal resistance " "}}$$

heat transfer $\rightarrow q = -\frac{kA}{L} (-\Delta T) = KA \frac{\Delta T}{L} = \frac{\Delta T}{L/kA}$

$$R_{\text{cond}} = \frac{L}{kA}$$

• the outside wall

$$\text{Similarly, } q_{\text{conv}} = hA (T_S - T_{\infty}) = \frac{T_S - T_{\infty}}{(1/hA)} = \frac{\Delta T}{R_{\text{conv}}}$$

$$R_{\text{convection}} = \frac{1}{hA}$$

Fig 3.1 $R_{\text{tot}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$ total thermal resistance $\Rightarrow q''_x = \frac{T_{\infty_1} - T_{\infty_2}}{R_{\text{tot}}}$

Multilayer Plane Walls

i) walls in a series : $R_{\text{tot}} = \sum R_i$

ii) walls of parallel layers $\frac{1}{R_{\text{tot}}} = \sum \frac{1}{R_i}$

— Ch 3 slide 6 —

case a)

$$q = \frac{\Delta T}{R_{\text{tot}}} = \frac{T_{\infty i} - T_{\infty o}}{\frac{1}{h_i A} + \frac{L_p}{K_p A} + \frac{L_f}{K_f A} + \frac{L_w}{K_w A} + \frac{1}{h_o A}} = \frac{20 - (-15)}{(0,2 + 0,059 + 1,92 + 0,083 + 0,067) / 1} = 15,0 \text{ W}$$

$$q = \frac{35}{2,33} = 15,0 \text{ W}$$

assume $A = 1 \text{ m}^2$

b) $q = \frac{T_{\infty i} - T_{\infty o}}{\frac{1}{h_i A} + \frac{L_g}{K_g A} + \frac{1}{h_o A}} = \frac{35}{0,269} = 130 \text{ W}$

c) $q = \frac{T_{\infty i} - T_{\infty o}}{\frac{1}{h_i A} + \frac{L_g}{K_g A} + \frac{L_a}{K_g A} + \frac{L_g}{K_g A} + \frac{1}{h_o A}} = \dots = 76 \text{ W}$

Notes Jan 25)

- for cylindrical wall, temperature gradient is logarithmic (not Linear)

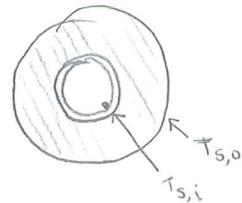
Overall H.T. coefficient

Emissivity is $0 < \epsilon < 1$, refers to how much radiation passes thru

Ex.] with insulated tube

$$\frac{\frac{1}{h_i A} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_{st} \cdot l} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_{ins} \cdot l}}{T_{S,i} - T_{S,0}} = \frac{\frac{T_{S,0} - T_{env}}{1}}{h_o 2\pi r_3 \cdot l} + \frac{(2\pi r_3 \cdot l)(\varepsilon \sigma)(T_{S,0}^4 - T_{env}^4)}$$

* using 'q' denotes "for 1 m length
of the pipe"

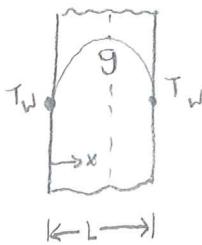


↑ would need this term had they not said "inside wall same temp as fluid"

- will need calculators to solve this

$$\Rightarrow f_3 = 394 \text{ m} \quad t_{ins} = f_3 - f_2 = 394 - 180 = 214 \text{ mm}$$

ex)



$$\text{B.C.'s } T(x=0) = T_W \Rightarrow T_W = (-\frac{g}{2k})0^2 + C_0 + C_1 \Rightarrow C_1 = T_W$$

$$T(x=L) = T_W \Rightarrow J_W = \left(\frac{-g}{2k}\right)L^2 + C_1 x + J_W \Rightarrow C_1 = \frac{gL}{2k}$$

$$T(x) = -\frac{\dot{g}}{2k}x^2 + \frac{\dot{g}}{2k}Lx + T_w$$

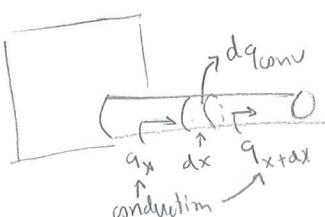
Pratik

$$\frac{1}{r} \frac{d}{dr} \left(kr \frac{dT}{dr} \right) + g = 0$$

Slide #18

Notes Jan 27]

(see cylindrical cooling fin slide 22
 (pin fin) assumed 1 dimensional temperature gradient - only depends on x



$$\text{Energy balance for } dx \quad q_{\text{und}, x} = q_{\text{cond}, x+dx} + dq_{\text{conv}}$$

$$q_x = -k A_c \frac{dT}{dx} \quad q_x + \frac{dn}{dx} dx = h(T-T_\infty) dA_s$$

$$= -kA_C \frac{dT}{dx} - \cancel{kA_C} \frac{d^2T}{dx^2} dx$$

$$0 = -kA_c \frac{d^2T}{dx^2} + h(T - T_\infty)P_{dx} \Rightarrow \frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

Notes Jan 27

$$\Theta = T - T_{\infty}, \quad m^2 \frac{hP}{kA_c} \Rightarrow \frac{d^2\Theta}{dx^2} - m^2 \Theta = 0$$

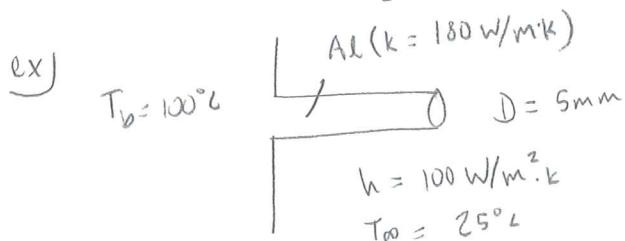
$$\Theta = C_1 e^{mx} + C_2 e^{-mx} \quad (1. b.c.): x=0 \Rightarrow \Theta = \Theta_b$$

$$\Theta_b = T_b - T_{\infty} = C_1 e^{m \cdot 0} + C_2 e^{-m \cdot 0}$$

$$\Theta_b = T_b - T_{\infty} = C_1 + C_2$$

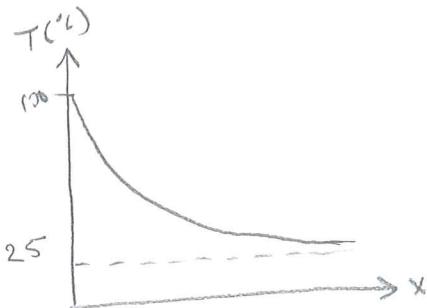
(2nd b.c.): 4 possibilities slide 24

$$\text{Note } \cosh x = \frac{e^x + e^{-x}}{2} \quad \sinh x = \frac{e^x - e^{-x}}{2}$$



Very long fin ($L \rightarrow \infty$) $\frac{\Theta}{\Theta_b} = e^{-mx}$
 $\Rightarrow T = T_{\infty} + (T_b - T_{\infty}) e^{-mx}$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{h}{k} \frac{\pi D}{\frac{\pi D^2}{4}}} = \sqrt{\frac{4h}{KD}} = 21 \Rightarrow T = 25 + 75 e^{-21x}$$



$$q_f = m = \sqrt{hPA_c} \Theta_b$$

$$q_f = (0.075)(75) = 5.6 \text{ W}$$

* Note; in general, $q_f = q_b$ heat lost by fin = heat entering fin @ base

$$= -k A_c \left. \frac{d\Theta}{dx} \right|_{x=0} \quad \text{OR} \quad q_f = \int_{A_f} h(T - T_{\infty}) dA_s$$

- What should be the length of a fin with negligible heat loss at the tip that can transfer 80% of $q_{f,L \rightarrow \infty}$?

$$\frac{q_{f,\text{adiabatic}}}{q_{f,L \rightarrow \infty}} = \frac{M \tanh(mL)}{M} = 0.8 \Rightarrow \tanh(mL) = 0.8 \Rightarrow mL = 1.1$$

$$\Rightarrow L = \frac{1.1}{21} = 0.05 \text{ m} = 5 \text{ cm}$$

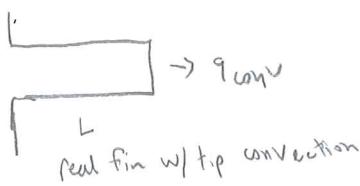
$$q_f = M \tanh(mL) = 4.8 \text{ W}$$

plug this
into

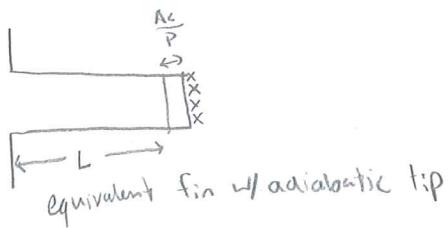
Tan 27 #3

Engineering Solution:

use corrected fin length $L_c = L + \frac{A_f}{P}$ instead of L in case B (Slide 22)



=



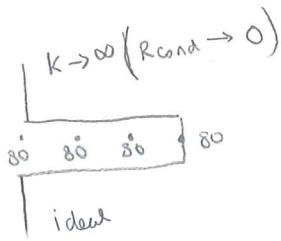
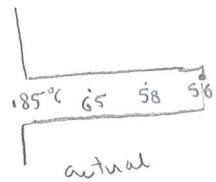
$$w + \frac{A_f}{P} \approx \frac{t}{2}$$

$$\textcircled{D} \quad \frac{A_f}{P} = \frac{D}{4}$$

— Fin Efficiency —

SLIDE 25 IMPORTANT

ex)



$$q_{f,\max} = h A_f \theta_b = h P L (T_b - T_\infty)$$

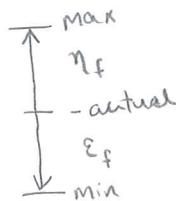
$$q_f = \eta_f q_{f,\max} \quad \text{AND} \quad \eta_f = \frac{q_f}{q_{f,\max}} = \frac{\sqrt{h P k A_c} \theta_b \cdot \tanh(m L_c)}{h P L \theta_b}$$

$$\eta_f = \frac{\tan(m L_c)}{m L_c}$$

* See table 3.5 for η_f
or figs. 3.18, 3.19

— Fin Effectiveness —

$$\varepsilon_f = \frac{q_{f,\text{actual}}}{q_{\text{no fin}}} \quad \varepsilon_f \geq 2 \text{ to justify use of fins}$$



$L \rightarrow \infty$ (upper limit for ε_f): $\varepsilon_f = \dots \left(\frac{K P}{h A_c} \right)^{1/2}$

to max. ε_f :
- $K \uparrow$ use metals

- $P/A_c \uparrow$ use thin fins

- $h \downarrow$ use fins on surfaces w/ low h

Feb 3, 2011] Notes

— Arrays of fins —

tot. h.t. rate from a surface of N fins

$$q_f = q_{\text{finned}} + q_{\text{unfinned}} = N q_f + h_{\text{unfinned}} A_{\text{unfinned}} \Theta_b$$

$$= N n_f h_{\text{finned}} A_f \Theta_b + h_{\text{unfinned}} A_{\text{unfinned}} \Theta_b$$

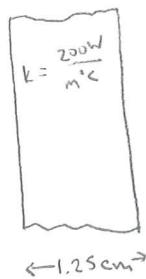
↑
calculate

to study for test 1, consider all configurations of fins on a surface

Ex)

$$T_{\infty 1} = 120^\circ C$$

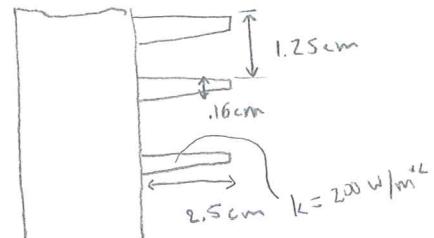
$$h_1 = 450 \frac{W}{m^2 \cdot K}$$



$$T_{\infty 2} = 20^\circ C$$

$$h_2 = 25 \frac{W}{m^2 \cdot K}$$

H.T. enhancement
due to fin addition?



$$\frac{q_{w/\text{fins}}}{q_{w/o \text{fins}}} = \frac{N \eta_f h A_f \Theta_b + h A_{\text{total unfinned}} \Theta_b}{h A_{\text{total base}} \cdot \Theta_b} = \frac{N n_f A_f + A_{\text{tot. unfinned}}}{A_{\text{total base}}}$$

$$mL_c = \sqrt{\frac{hP}{kA_c}} L_c = \sqrt{\frac{h \cdot 2W}{k \cdot 0.16}} (2.5 + \frac{0.16}{2})$$

$$mL_c = \sqrt{\frac{(L_c)^2}{(200)1.16 \times 10^{-2}}} (2.5 + \frac{0.16}{2}) \times 10^{-2} = .3225$$

$$\eta_f = \frac{\tanh(mL_c)}{mL_c} = 0.967 \quad A_f = 2WL_c, \quad A_{\text{total unfinned}} = W(H - Nt)$$

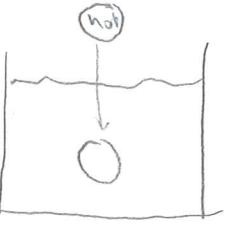
$$A_{\text{total base}} = HW$$

$$\frac{q_{w/\text{fin}}}{q_{w/o \text{fin}}} = \frac{2\eta_f L_c + \frac{H}{N} \cdot t}{\frac{H}{N}} = \frac{2\eta_f L_c + S \cdot t}{S} = \frac{2 \cdot (0.967)(2.38 \times 10^{-2}) + 1.25 \times 10^{-2} \cdot 0.16 \times 10^{-2}}{1.25 \times 10^{-2}} = 4.76$$

First, neglect spatial variations of temp within the body, $T = T(t)$

\Rightarrow lumped analysis

$$\Rightarrow R_{\text{cond}} \ll R_{\text{conv}} \Rightarrow \frac{q_{\text{conv}}}{q_{\text{cond}}} = \frac{R_{\text{cond}}}{R_{\text{conv}}} = \frac{\frac{h_e}{k A_s}}{\frac{1}{h A_s}} = \boxed{\frac{h L_c}{k} = \text{Biot number} = Bi}$$



* in this chapter, L_c means "characteristic length", not corrected length

$\Rightarrow Bi \ll 1 \quad Bi \leq 0.1 \Rightarrow$ use the following lump analysis

- Energy balance:

$$\text{Cof. } \dot{E}_{in} - \dot{E}_{out} + \dot{E}_S = \dot{E}_t$$

$$-h A_s (T - T_\infty) = \rho \cancel{A} C_p \frac{dT}{dt} \quad \Theta = T - T_\infty \Rightarrow \frac{dT}{dt} = \frac{d\Theta}{dt} \quad -h A_s \Theta = \rho \cancel{A} C_p \frac{d\Theta}{dt}$$

$$\int_0^t -dt = \int_{\Theta_i}^{\Theta} \frac{\rho \cancel{A} C_p}{h A_s} \frac{d\Theta}{\Theta} \Rightarrow t = \frac{\rho \cancel{A} C_p}{h A_s} \ln \left(\frac{T_i - T_\infty}{T - T_\infty} \right) \quad \text{or}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{h A_s t}{\rho \cancel{A} C_p}}$$

$$L_c = \frac{\cancel{A}}{A_s} = \begin{cases} L & \text{for plane wall of thickness } 2L \\ r_o/2 & \text{for very long cylinder of radius } r_o \\ r_o/3 & \text{for sphere of radius } r_o \end{cases} \quad \text{p 261}$$

$$\text{Now } \frac{h A_s t}{\rho \cancel{A} C_p} = \frac{h t}{\rho C_p L_c} = \frac{h L_c}{k} \frac{\cancel{A} t}{L_c^2} = Bi \quad \text{NON DIMENSIONAL}$$

$$\Theta^* = \frac{\Theta}{\Theta_i} = e^{-Bi \cdot F_o} \quad F_o = \text{Fourier number}$$

$$T(t) \Rightarrow q(t) = h A_s [T(t) - T_\infty] = h A_s \Theta(t)$$

$$\text{Total heat transfer upto time } Q = \rho \cancel{A} C_p [T_i - T(t)]$$

$$Q_{\max} = Q_o = \rho \cancel{A} C_p (T_i - T_\infty)$$

Notes

2-8-11

$$\beta_u > .1 \Rightarrow T(x, t)$$

$$\text{Heat equation: } \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

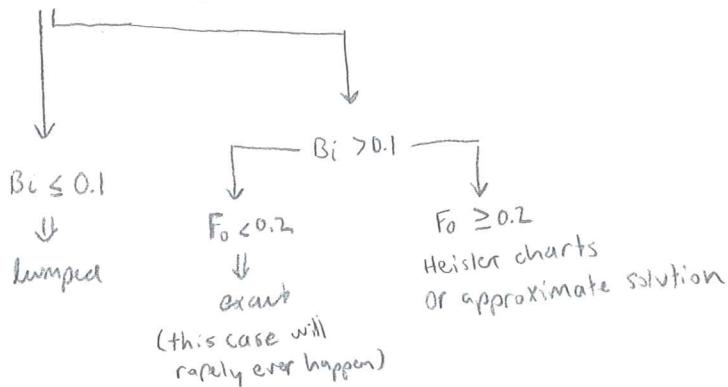
$$\text{B.C.'s } \left. \frac{\partial T}{\partial x} \right|_{x=0} = 0$$

(symmetry)

$$-k \left. \frac{\partial T}{\partial x} \right|_{x=L} = h [T(L, t) - T_\infty]$$

$$\text{Initial condition: } T(x, 0) = T_0$$

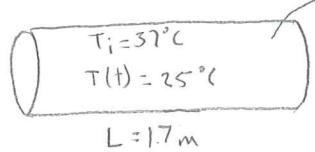
$$Bi \# = hL_c/k$$



Examples CSI Rolla

$$T_{\infty} = 20^\circ\text{C} \quad h = 8 \text{ W/m}^2\text{ °C}$$

$$D = 0.3 \text{ m}$$



Water, table A-6 ($\bar{T} \approx 30.5 \text{ K}$)

$$\rho = 995 \text{ kg/m}^3 \quad C_p = 4178 \text{ J/kg}\cdot\text{K}$$

$$k = 0.62 \text{ W/m}\cdot\text{K} \quad \alpha = \frac{k}{\rho C_p} = 1.5 \times 10^{-7} \frac{\text{m}^2}{\text{s}}$$

$$Bi_C = \frac{h}{k} = \frac{\pi D^2 L}{\pi D L + 2(\frac{\pi D^2}{4})} = \frac{1}{\frac{4}{D} + \frac{2}{L}} \quad \text{Note: long cyl. } L \gg D \Rightarrow h_C = \frac{D}{4} = \frac{R_o}{2}$$

$$Bi_C = \frac{h h_C}{k} = 0.89 > 0.1 \Rightarrow \text{heisler charts}$$

$$\Theta_0^* = \frac{T_o - T_{\infty}}{T_i - T_{\infty}} = \frac{25 - 20}{37 - 20} = 0.29$$

$$Bi^{-1} = \frac{k L}{h R_o} = 1.04 \quad \Rightarrow \quad Fo = \frac{\alpha t}{R_o^2} \approx 0.6 \quad \text{from Fig D.4}$$

$$t = \frac{(0.6)(0.15)^2}{1.5 \times 10^{-7}} = 90,000 \text{ s} = 25 \text{ hr.}$$

? Lumped analysis?

$$t = \frac{\rho C_p}{h} \ln \left(\frac{T_i - T_{\infty}}{T - T_{\infty}} \right)$$

$$t = 43815 \text{ s} = 12.2 \text{ h} \quad \text{Error} \cong 100\% \text{ Underestimate}$$

Feb 10 2011

Ex. oranges in cold weather

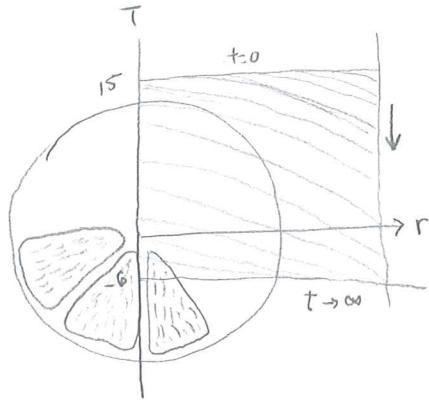
$$Bi = \frac{h(r_0/3)}{K} = \frac{30(4 \times 10^{-3})/3}{613 \times 10^{-3}}$$

Note:
use closest value
on tables, no need
to interpolate

$= 0.65 > 0.1 \Rightarrow$ Heisler

$$Bi^{-1} = \frac{K}{hr_0} = 0.51 \quad F_0 = \frac{\alpha t}{r_0^2} = \frac{(1.5 \times 10^{-7})(4 \times 3600)}{(4 \times 10^{-2})^2} = 1.35$$

$$\Theta_0^* = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.007 \xrightarrow{\text{from Fig. D.7}} = \frac{T_0 - (-6)}{15 - (-6)} \Rightarrow T_0 = -5.8^\circ\text{C}$$



Feb 17 2011

$$h = \frac{q_{\text{conv}}}{A_s(T_s - T_\infty)} = f(\rho, \nu, k, c_p, V, \text{type of flow, surface geometry})$$

use empirical correlations

solve boundary layer
eq's (for simple flows)

fluid viscosity momentum transport \Rightarrow relates gradients with b.l. \Rightarrow drag force

$$\text{shear stress } \tilde{\tau}_{s,x} = \frac{\text{drag force}}{\text{surface area}} = \frac{F_D}{A_s} = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} \text{ N/m}^2$$

↑
viscosity

If you know $u(y)$ w/in b.l. $\Rightarrow \left. \frac{\partial u}{\partial y} \right|_{y=0} \Rightarrow \tilde{\tau}_{s,x} \Rightarrow F_D$

$$\text{Practical approach: } \tilde{\tau}_{s,x} = C_{f,x} \frac{\rho V_\infty^2}{2};$$

↑
local friction coeff.

$$F_D = \int_{A_s} \tilde{\tau}_{s,x} \cdot dA_s = \int_{A_s} C_{f,x} \cdot \frac{\rho V_\infty^2}{2} dA_s \\ = \bar{C}_{f,L} A_s \frac{\rho V_\infty^2}{2} (N)$$

$$\bar{C}_{f,L} = \frac{1}{A_s} \int_{A_s} C_{f,x} \cdot dA_s = \frac{1}{L} \int_0^L C_{f,x} \cdot dx$$

Flow regime depends on Reynolds #

$$Re = \frac{V_\infty L}{\nu} = \frac{\text{inertial forces}}{\text{viscous forces}}$$

$$\text{Local } Re_x = \frac{V_\infty x}{\nu}$$

$$\text{nusselt } \# = \frac{q''_{\text{conv}}}{q''_{\text{cond}}} = \frac{h \Delta T}{k_f \frac{\Delta T}{L}} = \frac{h L}{k_f}$$

$Nu = 1$, pure conduction

$Nu \uparrow$, more enhanced h.t.

Notes 2-22-11) — Non-dimensional #'s —

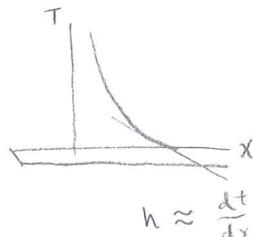
$$Nu = \frac{q_{\text{conv}}}{q_{\text{cond}}} = \frac{hb}{kf}$$

non-dimensional convection h.t. coeff.

$$Pr = \frac{\text{momentum transport}}{\text{heat transport}} = \frac{V}{\alpha} = \frac{\mu}{\rho k} = \frac{\mu c_p}{k}$$

$$\text{For laminar b.l.s. : } \frac{dv}{dx} \approx Pr^n$$

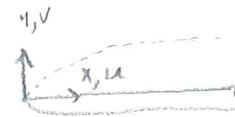
Convection H.T. coefficient is \approx temperature gradient right at surface



— Boundary layer approximation For incompressible ($\rho = \text{const}$), 2-D, steady flow over surfaces,

- B.L. thickness is typically very small so that

$$u \gg v, \quad \frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x} \Rightarrow \tau = \mu \frac{\partial u}{\partial y}$$



$$\frac{\partial T}{\partial y} \gg \frac{\partial T}{\partial x} \Rightarrow q_y'' \gg q_x''$$

$$\text{Then, mass: } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\underline{x\text{-momentum: }} \underbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}}_{\text{advection}} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \underbrace{v \frac{\partial^2 u}{\partial y^2}}_{\text{acceleration}}$$

$$y\text{-momentum: } \frac{\partial p}{\partial y} = 0$$

$$\underbrace{\frac{\partial^2 u}{\partial y^2}}_{\text{viscous forces}} \approx F/m$$

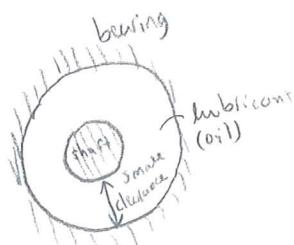
$$\underline{\text{Energy: }} \underbrace{u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}}_{\text{advection}} = \underbrace{\alpha \frac{\partial^2 T}{\partial y^2}}_{\text{conduction}} + \underbrace{\frac{\nu}{C_p} \left(\frac{\partial u}{\partial y} \right)^2}_{\text{viscous dissipation}}$$

(only important at high speeds)

advection: bulk motion of the fluid

Example of solving b.b. eqns for simple flows: Couette flow

$$\text{mass: } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{so} \quad \frac{\partial v}{\partial x} = 0 \quad \text{so} \quad u \neq f(x)$$



$$\text{momentum: } \frac{\partial u}{\partial x} = 0, \quad v = 0, \quad \frac{\partial P}{\partial x} = 0 \Rightarrow \frac{\partial^2 u}{\partial y^2} = 0 \quad \frac{\partial u}{\partial y} = C_1 \quad u(y) = C_1 y + C_2$$

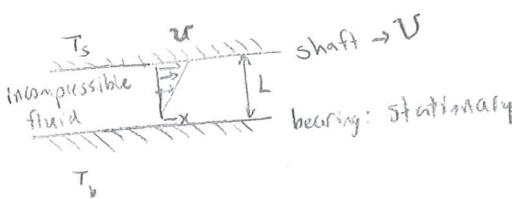
$$\text{B.C.'s } u(y=0) = 0 \Rightarrow C_2 = 0 \quad u(y=L) = U \Rightarrow C_1 = \frac{U}{L}$$

$$u = \frac{Uy}{L}$$

$$\text{Energy: } \frac{\partial T}{\partial x} = 0, \quad v = 0, \quad \dot{g} = 0 \Rightarrow 0 = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{1}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 \Rightarrow \frac{\partial^2 T}{\partial y^2} = -\frac{\mu}{k} \left(\frac{\partial u}{\partial y} \right)^2$$

$$= -\frac{\mu}{k} \left(\frac{U}{L} \right)^2 \Rightarrow \frac{\partial T}{\partial y} = -\frac{\mu}{k} \left(\frac{U}{L} \right)^2 y + C_3$$

$$T = -\frac{\mu}{2k} \left(\frac{U}{L} \right)^2 y^2 + C_3 y + C_4$$



2-22

B.C.'s $T(y=0) = T_b$ $T_b = a + b + c_4$ $T_b = c_4$

$$T(y=L) = T_s \quad \dots \quad c_3 = \frac{T_s - T_b}{L} + \frac{\mu}{2k} \left(\frac{U}{L} \right)^2$$

Thus $T = T_b + \frac{\mu}{2k} U^2 \left(\frac{y}{L} - \frac{y^2}{L^2} \right) + (T_s - T_b) \frac{y}{L}$

* Prandtl number is a property of the fluid

2-24-11 $\bar{C}_f L = f(R_{e_L})$

- don't worry about St # in book

$$\frac{\bar{C}_f}{2} = \frac{\bar{N}_{u_L}}{R_{e_L} \cdot \text{Pr}^{1/3}}$$

$$\bar{N}_{u_L} = \frac{0.025}{2} (1.23 \times 10^7) (0.72)^{1/3} = 13,780 = \frac{\bar{h}_L L}{k_f}$$

Prob. 6.43] $R_{e_L} = \frac{V_L L}{\nu} = \frac{100/2}{16.3 \times 10^{-6}} = 1.23 \times 10^7$; $\bar{h}_L = \frac{(13780)(0.022)}{2} = 152 \frac{W}{m^2 \cdot K}$

$\bar{q}_{\text{conv}} = \bar{h}_L (T_s - \bar{T}_w) = (152)(5 - (-12^3)) = 4260 \frac{W}{m^2}$

Prob 6.19] $q_1 = 1500 \text{ W}$
 Air \rightarrow
 $T_{\infty} = 35^\circ \text{C}$
 $L_1 = 0.15 \text{ m}$
 $V_1 = 100 \text{ m/s}$

$q_2 = ?$
 Air \rightarrow
 $T_{\infty} = 35^\circ \text{C}$
 $L_2 = 0.3 \text{ m}$
 $V_2 = 50 \text{ m/s}$

$$R_{e_{L1}} = \frac{V_1 L_1}{\nu} = \frac{15}{\nu} \quad R_{e_{L2}} = \frac{V_2 L_2}{\nu} = \frac{50 \times 0.3}{\nu} = \frac{15}{\nu} \quad \left. \begin{array}{l} \bar{N}_{u_{L1}} = \bar{N}_{u_{L2}} \\ \end{array} \right\}$$

$$\bar{h}_{L1} L_1 = \frac{\bar{h}_{L2} L_2}{k} \quad \bar{h}_{L2} = \frac{L_1}{L_2} \bar{h}_{L1} = \frac{L_1}{L_2} \frac{q_1}{A_1 (T_{s1} - T_{\infty})}$$

$$q_2 = \bar{h}_{L2} A_2 (T_{s2} - T_{\infty})$$

$$q_1 = \dots = \frac{T_{s2} - T_{\infty}}{T_{s1} - T_{\infty}} q_1 = 2066 \text{ W}$$

local Reynolds number

$$R_{e_L} = \frac{U L}{\nu} \quad R_{e_X} = \frac{U x}{\nu}$$

$$\frac{V_{\infty} L}{\nu}$$

March 1 2011

Reynolds analogy connects fluid dynamics to heat transfer

$$C_{f,x} = f(x^*, Re) \Rightarrow \bar{C}_{f,L} = f(Re_L)$$

$$Nu_x = f(x^*, Re, Pr) \Rightarrow \bar{Nu}_L = f(Re_L, Pr)$$

- be able to solve couette flow problem (using heat flux instead of temp gradient)

— FORCED CONVECTION: EXTERNAL FLOWS (Ch.7) —

The Empirical Method

$$\text{measure } h \text{ for a variety of conditions} \Rightarrow \bar{Nu}_L = C Re_L^m Pr^n$$

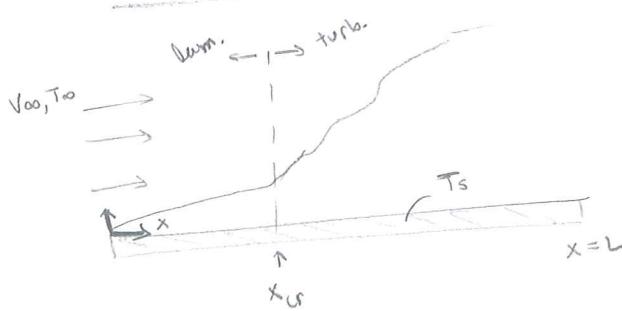
$m, n = f(\text{surface geometry, type of flow})$

$$\text{assume const. fluid properties at } T_f = \frac{T_s + T_\infty}{2}$$

Table 7.9 ← Print this on full page for exam

last 3 under geom WILL NOT USE

Flow over flat plates

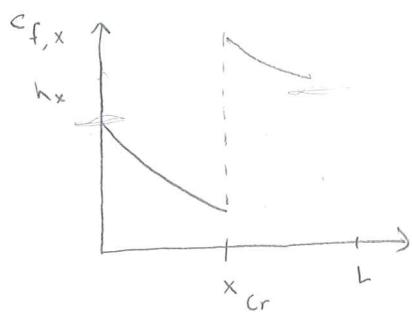


$$Re_{cr} = \frac{V_\infty x_{cr}}{\nu} = 5 \times 10^5$$

$$F_D = \cancel{F_{\text{pressure}}} + F_{\text{friction}}$$

$$F_D = \bar{C}_{f,L} \cdot A_S \cdot \rho \frac{V_\infty^2}{2}$$

- See table 7.9
for specific
correlations
p431



For laminar flow, $\delta_{v,t} \sim x^{1/2}$, $C_{f,x} \sim x^{-1/2}$, $Nu_x \sim x^{1/2}$, $h_x \sim x^{-1/2}$

For turbulent flow, $\delta_{v,t} \sim x^{4/5}$, $C_{f,x} \sim x^{-1/5}$, $Nu_x \sim x^{4/5}$, $h_x \sim x^{-1/5}$

$$\bar{C}_{f,L} = \frac{1}{L} \int_0^L C_{f,x} dx$$

$$\bar{h}_L = \frac{1}{L} \int_0^L h_x dx = \frac{1}{L} \left[\int_0^{x_{cr}} h_{x,\text{laminar}} dx + \int_{x_{cr}}^L h_{x,\text{turbulent}} dx \right]$$

March 1 2011

Flow over flat plates continued

Methology:

1. calculate Re_L using fluid properties at $T_f = \frac{T_s + T_\infty}{2}$

2. If $Re_L < 5 \times 10^5$ \Rightarrow entirely laminar flow, use laminar conditions

3. If $Re_L > 5 \times 10^5$ \Rightarrow mixed laminar & turbulent

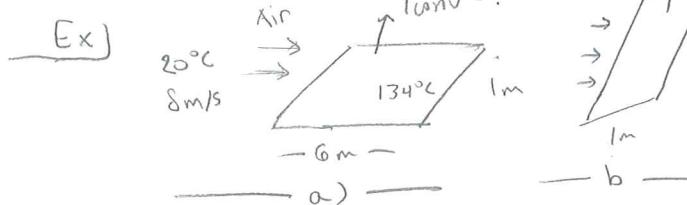
- for local $\delta_v, t, C_{f,x}, Nu_x$, use either laminar or turb. local correlations depending on Re_x

- for average $\bar{C}_{f,L}$ & \bar{Nu}_L use mixed avg. correlations

$$4. \bar{F}_p = \bar{C}_{f,L} \cdot A_s \cdot \rho \frac{V_\infty^2}{2}$$

$$\dot{q}_{conv} = \bar{h}_L \cdot A_s \cdot (T_s - T_\infty) \text{ where } \bar{h}_L = \frac{\bar{Nu}_L \cdot k_f}{L}$$

$$\bar{h}_L = \frac{\bar{Nu}_L \cdot k_f}{L}$$



$$T_f = \frac{T_s + T_\infty}{2} = \frac{134 + 20}{2} = 77^\circ\text{C} = 350^\circ\text{K}$$

$$\Rightarrow V = 20.92 \times 10^{-6} \text{ m}^2/\text{s} \quad k = 30 \times 10^{-3} \text{ W/m.K}$$

$$Pr = 0.7$$

a) $L = 6 \text{ m} \Rightarrow Re_L = \frac{8(6)}{20.92 \times 10^{-6}} = 2.29 \times 10^6 \quad Re_{cr} = 5 \times 10^5 \Rightarrow \text{mixed flow}$

flat plate ✓, mixed flow ✓ average \bar{Nu} ✓

p431 find Table 7.9 relationship

$$\bar{Nu}_L = (0.037 Re_L^{4/5} - 874) Pr^{1/3} = 3249 \Rightarrow \bar{h}_L = \frac{(3249)(30 \times 10^{-3})}{6} = 16.25 \text{ W/m}^2.\text{K}$$

$$\dot{q}_{conv} = (16.25)(6)(134 - 20) = 11,115 \text{ W}$$

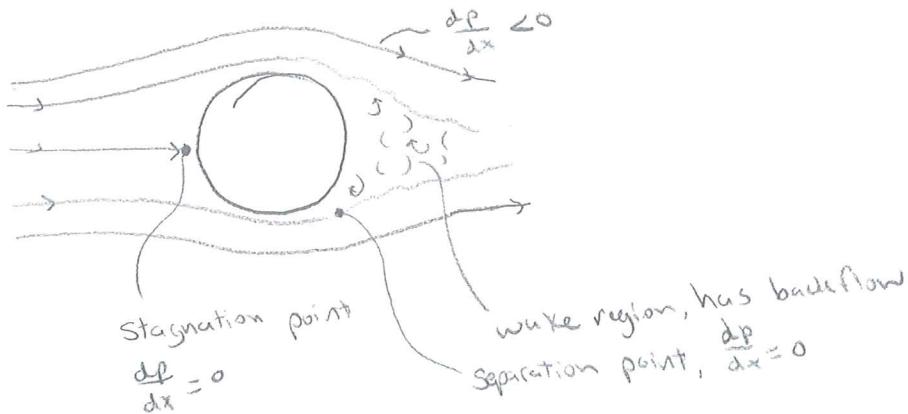
b) $Re_{cr} = \frac{8(1)}{20.92 \times 10^{-6}} = 3.82 \times 10^5 < Re_{cr} \Rightarrow \text{entirely laminar}$

$$\bar{Nu}_L = 0.664 \quad Re_L^{1/2} \cdot Pr^{1/2} = 364$$

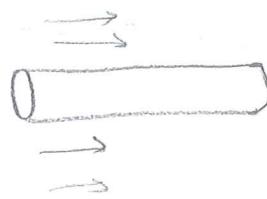
$$\bar{h}_L = \frac{(30 \times 10^{-3})(364)}{1} = 10.9 \text{ W/m}^2.\text{K}$$

$$\dot{q}_{conv} = 7469 \text{ W}$$

Cross flow



counter flow



$$Re = \frac{\rho V D}{\mu} = \frac{\text{inertia}}{\text{viscosity}}$$

location of separation

$$Re \approx 2 \times 10^5$$

$$\text{also } Re_D = \frac{4r}{\pi D \mu} = \frac{V D}{\nu}$$

$$Re \geq 2 \times 10^5$$

Fig 7.8, important P401

ex] 7.73

$$T_\infty = 280^\circ C$$

Water

$$\uparrow F_D = C_D A_f \frac{\rho V^2}{2}$$

$\leftarrow q_{\text{conv}}$

$\downarrow F_g = c$

$$\text{Cu sphere, } D = 20\text{ mm}$$

$$T_i(r, 0) = 360K$$

$$T_f(0, t_f) = 360K$$

Find: terminal velocity
tank height at point
when $T = T_f$

Balance forces:

$$C_D A_f \frac{\rho V^2}{2} = C_D \pi r^2 \frac{\rho V^2}{2} = g(\rho_{\text{Cu}} - \rho_{\text{H}_2\text{O}})$$

$$C_D V^2 = 2.07 \text{ m}^2/\text{s}$$

$$\text{Table 7.8} \quad Re_D = \frac{V D}{\nu}$$

$$\Rightarrow C_D V^2 = \frac{4}{3} D \frac{\rho_{\text{Cu}} - \rho}{\rho} g$$

make a guess and do iterations P401

$$\boxed{V = 2.1 \text{ m/s}} \quad Re_D = 21850 \quad \frac{1/d}{1/r} = \frac{1/0.02}{1/0.4} = 20 \quad Nu_D = 2 + (0.4 Re_D^{1/2} + 0.6 Re_D^{2/3}) \left(\frac{1/d}{1/r} \right)^{1/4} = 439$$

$$\bar{h} = 12775 \text{ W/m}^2 \cdot \text{K}$$

$$Bi = \bar{h} \left(\frac{r_0}{D} \right) k_{\text{Cu}} = 12775 \times \left(\frac{0.2}{0.02} \right) 398 = 0.11$$

$$\theta^* = C_1 e^{-3^2 F_0} \rightarrow 5.1$$

$$F_0 = 0.87 = \frac{\alpha t_f}{T_0} \quad H = 2.1 \times 0.76 \times 9890 =$$

$$= 1.6 \text{ m}$$

3-8-11] Notes

$$\overline{C}_{f,L} \rightarrow \text{---}$$

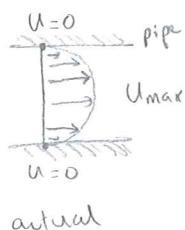
only friction drag

} summary for last class

$$\overline{C}_D \rightarrow \odot$$

friction and pressure drag

Momentum Considerations:



actual



- Idealized

Circular pipe with Diameter D $\Rightarrow D_h = D$

$$\dot{m} = \int_{A_c} \rho u(r) dA_c = \boxed{\rho u_m A_c}$$

$$Re = \frac{\rho U_m D_h}{\mu} \quad \text{where } D_h = \frac{4A_c}{P} \quad (\text{hydraulic diameter})$$

$$Re = \frac{4\dot{m}}{\rho \mu} = \frac{4\dot{m}}{\pi D \mu}$$

$Re_{cr} \approx 2,300$ (onset of turbulence)

Fully turbulent flow: $Re \approx 10,000$

hydrodynamic entry length: $\left(\frac{x_{fa,h}}{D}\right)_{turb} = .05 Re_D \quad 10 \approx \left(\frac{x_{fa,h}}{D}\right)_{turb} \leq 60$

— Fully developed conditions

pumping power = $\Delta P \cdot \dot{V} = \boxed{\Delta P \cdot \frac{\dot{m}}{\rho}}$

pressure drop = $\Delta P = P_1 - P_2 = f \left(\frac{L}{D}\right) \left(\frac{\rho u_m^2}{2}\right) N/m$

Friction factor = $f = \frac{-(dp/dx)D}{\left(\frac{\rho u_m^2}{2}\right)} = \frac{\Delta P_L \cdot D}{\left(\frac{\rho u_m^2}{2}\right)} = f(Re)$ for smooth surfaces

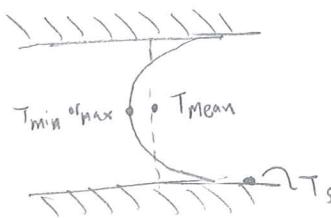
smooth surfaces { • laminar flow in circular tube: $f = \frac{64}{Re_D}$
• turbulent flow circular tube: $f = (0.79 \ln Re_D - 1.6)^{-2}$

• in general; $f = f(Re, e/D) \Leftarrow$ Moody chart (Fig 8.3) p 461
↑ relative pipe roughness

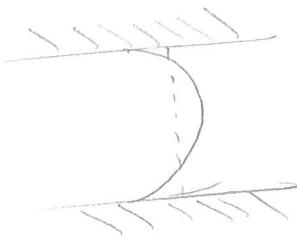
$\left.\frac{\partial u}{\partial r}\right|_{r=\frac{D}{2}} \neq f(x) \Rightarrow T_w \text{ constant} \Rightarrow f \text{ constant for } x > x_{fa,h} \leftarrow$ fully developed description

$$\frac{\partial u}{\partial x} = 0 \Rightarrow u_m = \text{const.}$$

Thermal considerations:



OR



$$\dot{E}_f = \int_{A_c} \rho u c_p T dA_c = \dot{m} c_p T_m \quad \text{Local heat flux: } q''_s = h(T_s - T_m)$$

$$\text{Thermal entry length: } \left(\frac{x_{fd,t}}{D} \right) \approx 0.05 Re_D Pr \quad \left(\frac{x_{fd,t}}{D} \right)_{turb} \approx 10$$

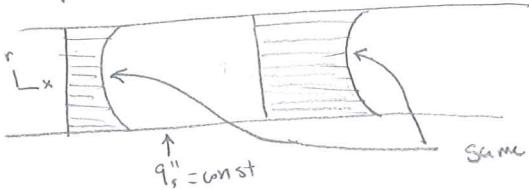
Note: $Pr \approx 1$,
then $x_{fd,t} \approx x_{fd,h}$

Fully developed conditions:

$$\frac{\partial v}{\partial x} = 0 \quad \text{but} \quad \frac{dT_m}{dx} \neq 0 \quad \text{and} \quad \frac{\partial T}{\partial x} \neq 0 \quad \text{as} \quad T_m = T_m(x)$$

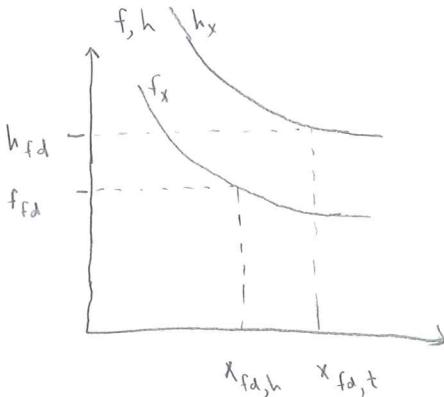
Although $T(r) = f(x)$, relative shape of the temperature profile does not change w/x,

$$(i.e.) \frac{d}{dx} \left(\frac{T_s - T}{T_s - T_m} \right)_{fd,t} = 0$$



same profile, greater T's

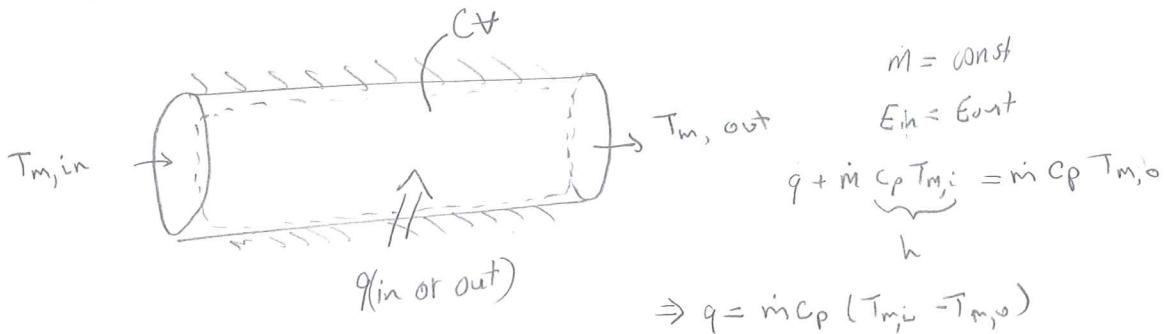
similar to f, $h = \text{const}$ for $x > x_{fd,t}$



— Fully developed internal flows —
 ↗ effect of friction felt all over x-section

$$\frac{du}{dx} = 0 \Rightarrow u_m \text{ const, but; } \frac{\partial T}{\partial x} \neq 0 \Rightarrow T_m = T_m(x) = ?$$

$f = \text{const}$ (at fully developed) $h = \text{const}$



$$\frac{d\dot{q}}{dx} = m c_p dT_m \quad \leftarrow q'' dx = m c_p dT_m \Rightarrow \frac{dT_m}{dx} = \frac{q'' P}{m c_p}$$

Solve this. D.E. for T_m

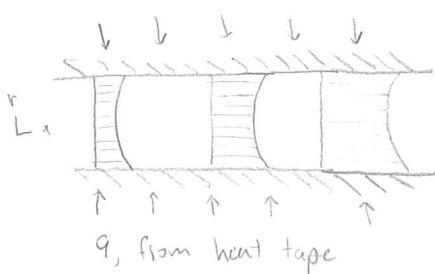
Surface conditions-

- 1) $q''_s = \text{const}$
 - 2) $T_s = \text{const}$
- heat tape ←

$$q''_s = h(T_s - T_m) = \text{const. } h = \text{const, for } x > x_{f_d,t} \Rightarrow T_s - T_m = \frac{q''_s}{h} = \text{const.}$$

i) $q_{\text{surface}} = \text{constant}$

$$\frac{dT_m}{dx} = \frac{q''_s P}{m c_p} = \text{const} \Rightarrow T_m(x) = T_{m,i} + \frac{q''_s P}{m c_p} x \quad (\text{linear}) \quad T_{m,o} = T_{m,i} + \frac{q''_s P \cdot L}{m c_p}$$



$$T_{m,o} = T_{m,i} + \frac{q''_s A_s}{m c_p}$$

ii) $T_s = \text{const}$ (phase change)

$$\frac{dT_m}{dx} = \frac{q'' P}{m c_p} = \frac{\overbrace{T_m}^h(T_s - T_m) P}{m c_p}$$

$$\Delta T \equiv T_s - T_m \Rightarrow \frac{d(\Delta T)}{dx} = -\frac{dT_m}{dx}$$

$$-\frac{d\Delta T}{dx} = \frac{h \Delta T P}{m c_p} \Rightarrow \frac{d\Delta T}{\Delta T} = -\frac{P h}{m c_p} dx$$

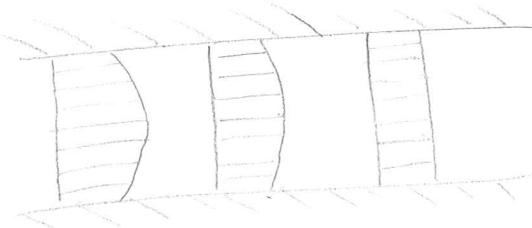
$$\text{integration} \Rightarrow \ln \frac{\Delta T}{\Delta T_i} = -\frac{P h}{m c_p} x$$

$$\boxed{\frac{\Delta T}{\Delta T_i} = \frac{T_s - T_{m,i}(x)}{T_s - T_{m,i}} = e^{-\frac{P h}{m c_p} x}}$$

$$\text{or } \frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = e^{-\frac{P h}{m c_p} L}$$

$$\text{or } \ln \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = -\frac{P h A_s}{m c_p}$$

$$\text{or } T_{m,o} = T_s - (T_s - T_{m,i}) e^{-\frac{P h A_s}{m c_p}}$$



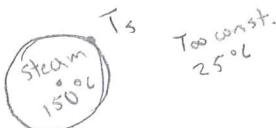
constant surface temp cooling

$$\frac{h A_s}{m c_p} = NTU \text{ (number of transfer units)}$$

$$L \rightarrow \infty, NTU \rightarrow \infty \Rightarrow T_{m,o} \rightarrow T_s$$

$$q = m c_p (T_{m,o} - T_{m,i}) = -\bar{h} A_s \left(\frac{T_s - T_{m,i}}{\ln \left(\frac{T_s - T_{m,o}}{T_s - T_{m,i}} \right)} \right) = -\Delta T_{lm}, \text{ log mean Temp. difference}$$

- If the external fluid temp T_∞ is constant (instead of pipe surface temp T_s const)
then replace T_s w/ T_∞ & \bar{h} with \bar{U}



$$\boxed{\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \left(\frac{-A_s \bar{U}}{m c_p} \right) L}$$

$$q = \bar{U} A_s \Delta T_{lm}$$

\uparrow
overall heat transfer coefficient

$$U A_s = \frac{1}{R_{total}}$$

$$\text{Review, ex } R_{tot} = R_{conv} + R_{cond} + R_{min, i}$$

$$= \frac{1}{h A} + \frac{\ln(D_o/D_i)}{2 \pi k L} + \frac{1}{h_o A}$$

$h = ?$ Table 8.4 summarizes Nu # correlations

For example i) fully developed laminar flow inside circular pipes:

$$Nu_D = \frac{h D}{k} = \begin{cases} 4.36 & \text{for } q'' \text{ s} = \text{const} \\ 3.66 & \text{for } T_s = \text{const} \end{cases}$$

for noncircular tubes use hydraulic diameter,

$$\Delta h = \frac{4 A_c}{P} \text{ w/ Table 8.1}$$

3-15-11

- i) fully-developed turbulent flow inside circular pipes
 $Nu_D = 0.023 Re_D^{0.8} Pr^N$ where $N = \begin{cases} 0.4 & \text{for heating } (T_s > T_m) \\ 0.3 & \text{for cooling} \end{cases}$

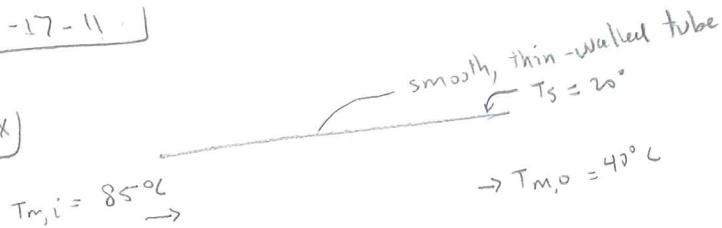
Turbulent correlations are valid for both surface conditions of

$$q''_s = \text{const} \quad \& \quad T_s = \text{const.}$$

- for noncircular pipes, use the correlations with $D_h = \frac{4A_c}{P}$

3-17-11

ex)



$$\begin{aligned} p &= 1000 \text{ kg/m}^3 \\ c_p &= 3000 \text{ J/kg·K} \\ \mu &= 0.04 \text{ Ns/m}^2 \\ k &= 0.26 \text{ W/mK} \end{aligned}$$

$$L = ? \quad i) D = 10 \text{ cm}, T_s = 20^\circ \text{C const} \quad \dot{m} = 1 \text{ kg/s}$$

$$(power) \quad P = ? \quad L = \frac{\dot{m} c_p}{\pi D h} \ln \left(\frac{T_s - T_{m,i}}{T_s - T_{m,o}} \right) \quad q = \dot{m} c_p (T_{m,o} - T_{m,i}) = \bar{h} A_s \Delta T_{\text{avg}} \quad P = \Delta P \dot{V} = f \frac{L}{D} \frac{\rho U_m^2}{2} \cdot \frac{\dot{m}}{p}$$

$$= 1(300)(40-85) = -135 \text{ kW}$$

$$Re = \frac{\rho U_m D}{\mu} ; \quad U_m = \frac{\dot{m}}{\rho \frac{\pi D^2}{4}} = \frac{1}{1000 \left(\frac{\pi}{4}\right)(0.1)^2} = 0.127 \text{ m/s}$$

$$Re = \frac{1000 (0.127)}{0.04}$$

$$Nu = 3.66 = \frac{\bar{h} D}{k_f} \Rightarrow \bar{h} = \frac{(3.66)(0.26)}{(0.1)} = 9.5 \text{ W/m}^2\text{K}$$

$$L = \frac{1(300)}{\pi (0.1)(9.5)} \ln \boxed{L = 1182 \text{ m}}$$

$$P = (0.2) \frac{1182}{(0.1)} \frac{(1000)(0.127)^2}{2} \frac{(1)}{(1000)} \Rightarrow \boxed{P = 19.2 \text{ NM/s}}$$

$$8.62 \quad Nu = \frac{f (Re - 1000) Pr}{1 + 12.7 \left(\frac{f}{8} \right)^{1/2} (Pr^{2/3} - 1)} = \bar{h} = 250 \text{ W/m}^2\text{K}$$

$$L = 450 \text{ m}$$

$$q = -1350$$

$$P = 1533 \text{ NM/s}$$

calculate this with
 R_c doubled from original -
still laminar

Table 8.4
p 499

3-17 - 2011

ex continued

(ii) a square duct of equal perimeter $P = \pi D = \pi(1) = 3.14m = 4a$ $a = 0.785$

$$D_h = \frac{4Ac}{P} = \frac{\pi(a^2)}{4\cdot\pi} = a = 0.785 \quad \text{See table 8.1}$$

$$U_m = 1.62 \text{ m/s} \Rightarrow Re_{dh} = \frac{\rho U_m D_h}{\mu} = 318, \text{ laminar}$$

$$f = \frac{S_7}{Re} = 0.179; N_w = 2.98 \Rightarrow h = 9.9 \text{ W/m}^2\text{K}$$

$$L = 1137 \text{ m} \quad q = -135 \text{ kW} \quad P = 34 \text{ Nm/s}$$

?

$$\rightarrow \text{iv) } D = 10 \text{ cm} \quad T_\infty = 20^\circ \text{C, const} \quad m = 1 \text{ kg/s} \quad \text{air, } V_\infty = 10 \text{ m/s}$$

$$L = \frac{mc_p}{\pi D k} \ln \left(\frac{T_\infty - T_{mi}}{T_\infty - T_{mo}} \right) \quad U_A = \frac{1}{R_{eff}} \Rightarrow U = \left(\frac{1}{h_i} + \frac{1}{h_o} \right)^{-1}$$

↑
9.5 at 46.2

$$h_o = 46.2 \text{ W/m}^2\text{K}$$

$$L = 1425 \text{ m}$$

Calculate this on your own, this is most common problem type

Project help - no need to draw air

3-22-11 Notes

Example test prob

Laminar
 $Pr = 4$ $Re_c = 2000$, circular pipe, $T_s = \text{const} > T_{mi,i}$ $m = \text{const}$ $D_2 = \frac{D_1}{5}$

$T_{mi,i} = \text{const}$ $T_{mo} = \text{const}$ $\frac{L_2}{L_1} = ?$ $\frac{f_2}{f_1} = ?$ $\frac{P_2}{P_1} = ?$ $Re_2 = 5Re_1 = 10,000 \rightarrow \text{turbulent}$

$$L = \frac{mc_p}{\pi D k} \ln \left(\frac{T_s - T_{mi}}{T_s - T_{mo}} \right) \quad \frac{L_2}{L_1} = \frac{\frac{1}{D_2 h_2}}{\frac{1}{D_1 h_1}} = \frac{D_1}{D_2} \frac{h_1}{h_2} = 5 \frac{h_1}{h_2}$$

$$Nu_2 = 0.023 (10,000)^{0.8} (4)^{0.4} = 63.5$$

$$Nu = \frac{h D}{k} \Rightarrow h = \frac{Nu k}{D} \quad \frac{L_2}{L_1} = 5 \frac{\frac{Nu_2}{Nu_1}}{\frac{D_2}{D_1}} = 5 \frac{D_1}{D_2} \frac{Nu_1}{Nu_2} = 5 \cdot \frac{1}{5} \frac{\frac{Nu_1}{Nu_2}}{\frac{D_1}{D_2}} = \frac{3.66}{63.5} = 0.06$$

$$\frac{f_2}{f_1} = \frac{0.316 \frac{Re_2}{Re_1}}{64/\frac{Re_2}{Re_1}} = \frac{0.316 (10,000)^{1/4}}{64/2000} = .9875$$

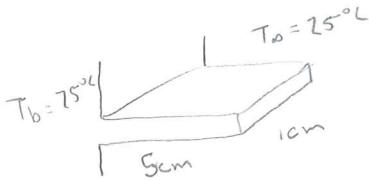
$$\frac{P_2}{P_1} = \frac{\frac{1}{P_1} \Delta P_2}{\frac{1}{P_1} \Delta P_1} = \frac{\frac{1}{P_1} f_2 \frac{L_2}{D_2} \frac{U_{mi}}{K}}{\frac{1}{P_1} f_1 \frac{L_1}{D_1} \frac{U_{mi}}{K}} = \frac{f_2 \frac{L_2}{D_2} \frac{D_1}{D_2} \left(\frac{U_{mi}}{U_{mi}} \right)^2}{f_1 \frac{L_1}{D_1} \frac{D_1}{D_2}} = \frac{f_2 \frac{L_2}{L_1} \frac{D_1}{D_2} \frac{D_1}{D_2}^4}{f_1 \frac{L_1}{D_1}}$$

$$\frac{\frac{m_2}{P_2 \frac{\pi D_2^2}{4}}}{\frac{m_1}{P_1 \frac{\pi D_1^2}{4}}} = \frac{D_1^2}{D_2^2} = \frac{(0.9875)(0.6)}{(5)} = 0.185$$

Notes 3-22-11

We Never use local Nu for cylinder or sphere, Sometimes with flat plate

2)



$$\eta_f = 0.9 \quad q = 2W = \eta_f \bar{h} A_f (T_b - T_\infty) = (0.9) \bar{h} (2 \times 5 \times 10^{-4}) (75 - 25) \quad (\text{dissipate})$$

$$\bar{h} = 44.5 \frac{V}{m^2 K}$$

$$\bar{N}_u \rightarrow Re \rightarrow V_\infty$$

what to use for fluid temp

$$T_f = \frac{T_s - T_\infty}{2} = 50^\circ C$$

\Rightarrow use air @ 300 K

$$V = 15.89 \times 10^{-6}$$

$$k = 26.3 \times 10^{-3}$$

$$Pr = .707$$

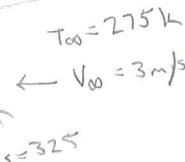
ii) Water with same velocity above (or use 20 m/s)

$$V = \frac{1}{\rho} = 1.011 \times 10^{-3} \frac{m^3}{kg} \quad \mu = 577 \times 10^{-6} \quad k = 640 \times 10^{-3} \text{ W/mK} \quad Pr = 3.71$$

$$Re_L = \frac{(6.5 \times 5 \times 10^{-2})}{(577 \times 10^{-6} \times 1.011 \times 10^{-3})} = 5.5 \times 10^{-5} \rightarrow Re_{cr} \Rightarrow \text{mixed flow}$$

$$\bar{N}_u = (\text{expression 7.31}) = 897 \quad \bar{h}_L = \frac{640 \times 10^{-3}}{(5 \times 10^{-2})} (897) = 11476 \text{ W/m}^2 \text{K}$$

$$q_f = (0.9)(11476)(2 \times 5 \times 10^{-4})(50) = 516 \text{ W}$$



2008

#3) arbitrary surface, $A_s = 4 \text{ m}^2 \quad T_s = 325 \text{ K}$

$$L = 2 \text{ m}$$

$$Re_{cr} = 10^6$$

Temp profile w/in bld: $T = 325 + 50e^{-\frac{y}{2}}$ when flow is locally laminar
 $T = 325 - 20000 \frac{y}{x^{1/5}}$ " " " turbulent

$$\text{Air at } \frac{325+275}{2} = 300 \text{ K} \quad V = 15.89 \times 10^{-6} \text{ m/s} \quad k = 26.3 \times 10^{-3} \quad Pr = .707 \quad \rho = 1.1616 \text{ kg/m}^3$$

$$\text{i) } Re_L = \frac{V_0 L}{V} = \frac{(3)(2)}{15.89 \times 10^{-6}} = 3.78 \times 10^6 < Re_{cr} = 10^6 \quad \text{laminar everywhere}$$

| know definition of Nusselt #

$$\text{ii) } h(x) = -k_f \frac{\partial T}{\partial y} \Big|_{y=0}$$

$$\text{iii) } \bar{h}_L = \frac{h \times k}{k}$$

$$\text{iv) } \bar{C}_{f,L} = \frac{2 N_u L}{Re_L Pr^{1/3}} = \dots \quad (\text{Reynolds analogy})$$

$$\text{v) } q = \bar{h}_L A_s (T_s - T_\infty)$$

$$F_D = \bar{C}_{f,L} \frac{\rho V_\infty^2}{2} A_s$$

Fouling Factor: fluid impurities & rust formation \Rightarrow deposition of a film layer on the surface
 \Rightarrow decrease heat transfer due to additional conduction resistance

$$R_f'' = f(T, V, t, \text{fluid properties})$$

for an unfinned H.X.: $\frac{1}{U_A} = \left(\frac{1}{U_i A_i} \right) + \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{R_{f,i}''}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,o}''}{A_o} + \frac{1}{h_o A_o}$

a) without fouling

$$\frac{1}{U_o} = \frac{1}{h_i} \frac{D_o}{D_i} + \frac{\ln(D_o/D_i)}{2k} + \frac{1}{h_o}$$

$$Re_{D_i} = \frac{4m_1}{\pi D_i \mu} = 21,220$$

See Ch. 11 slide 8

$$h_i = \frac{k_f}{D_i} (0.023 Re_{D_i}^{0.8} Pr^{0.4}) = 3400 \text{ W/m}^2\text{K}$$

$$U_o = \frac{1}{4.43 \times 10^{-4}} = 2255 \text{ W/m}^2\text{K}$$

b) with fouling $\left(4.43 \times 10^{-4} + \frac{D_o}{D_i} R_f'' \right)^{-1} = (5.55 \times 10^{-4})^{-1} = 1800 \text{ W/m}^2\text{K}$

c) $m_h h_{fg} = m_c c_p (T_{hi} - T_{ci}) \Rightarrow T_{ho} = T_{ci} + \frac{m_h h_{fg}}{m_c c_p} = 29.4^\circ\text{C}$

↑
condensation
rate

• note that example
gives mass flow of
water "per tube"

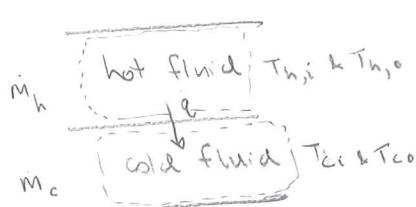
$$= 15 + \frac{10(2414 \times 10^3)}{400(4180)}$$

$$q = \dot{m}_{\text{cond/evap}} \cdot h_{fg}$$

— Analysis. —

- Assume
- steady state
 - $m_i = \text{const.}$
 - negligible ΔKE & ΔPE

- $C_p = \text{const.}$
- negligible axial heat conduction
- perfectly insulated H.X.



$$\dot{q} = \dot{m}_h C_{p,h} (T_{hi} - T_{ho}) = \dot{m}_h (T_{hi} - T_{ho})$$

$$\dot{q} = \dot{m}_c C_{p,c} (T_{co} - T_{ci}) = \dot{m}_c (T_{co} - T_{ci})$$

$C = \dot{m} C_p = \text{heat capacity rate}$

$$\dot{q} = C \Delta T$$

$$\dot{q} = U A \Delta T_{lm} \quad \text{where} \quad \Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\frac{\Delta T_1}{\Delta T_2})}$$

Parallel flow (PF): $\Delta T_1 = T_{hi} - T_{ci}$ $\Delta T_2 = T_{ho} - T_{co}$

Counter flow (CF): $\Delta T_1 = T_{hi} - T_{co}$ $\Delta T_2 = T_{ho} - T_{ci}$

- For the same in/outlet temps, $\Delta T_{lm, CF} > \Delta T_{lm, PF}$ Thus $A_{cf} < A_{pf}$
- for the same U , $A_{cf} < A_{pf}$ for a prescribed \dot{q}
- note that possibility of $T_{ci} > T_{ho}$ is only for a C.F.

Notes 4-12-11) continued

$$q = G_h (T_{h,i} - T_{h,o}) = C_v (T_{c,o} - T_{c,i})$$

Notes 4-14-11)

HX analysis: $\dot{q} = \dot{m} c_p, c (T_{c,o} - T_{c,i}) = \dot{m} c_{p,h} (T_{h,i} - T_{h,o}) = \underline{\underline{UA\Delta T_{lm}}}$
only for PF & CF HX's

ε -NTU method:

$$\varepsilon, \text{ effectiveness} = \frac{q}{q_{\max}} \quad (0 \leq \varepsilon \leq 1)$$

$$(0 \leq \varepsilon \leq 1)$$

$$q_{\max} = C_{\min} (T_{h,i} - T_{c,i})$$

Why?

The fluid with C_{\min} experiences the largest temp. difference

$$\varepsilon = f(\text{geometry, flow arrangement}) = f(\text{NTU}, C_r)$$

$$\text{where } \text{NTU} = \frac{UA}{C_{\min}}, \quad C_r = \frac{C_{\min}}{C_{\max}}$$

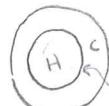
Tables 11.3 or Figs 11.10 - 11.15 give ε -NTU relationships

$C_i \rightarrow \infty$ for a condensing fluid or phase change

$$q = \varepsilon q_{\max} = \varepsilon C_{\min} (T_{h,i} - T_{c,i})$$

During phase change (Boiler or condenser): $C_r \rightarrow 0 \Rightarrow \varepsilon = 1 - e^{-\text{NTU}}$
or $\text{NTU} = -\ln(1 - \varepsilon)$

— Discuss prob 11.63 (slides)



surface area $\pi D_i L$

$$\varepsilon = \frac{q}{q_{\max}}$$

U , overall HT. coefficient

$$q = G_c (T_{c,o} - T_{c,i}) = C_v (T_{h,i} - T_{h,o}) = \varepsilon \underline{\underline{C_{\min} (T_{h,i} - T_{c,i})}} = \varepsilon q_{\max}$$

$$= UA \Delta T_{lm}$$

$$\varepsilon = 0.65$$

$$C_r = \frac{C_{\min}}{C_{\max}} = \frac{0.125(2100)}{125(4700)} = 0.5$$

$$\text{NTU} = \frac{\frac{UA}{C_{\min}}}{\sqrt{}} \Rightarrow$$

$$\frac{A_{CF}}{A_{PF}} = \frac{\text{NTU}_{CF}}{\text{NTU}_{PF}}$$

$$\stackrel{\text{fig 11.11}}{\cong} \frac{1.3}{2.6} \approx 0.5$$

fig 11.10

On slides they solve exactly using ΔT_{lm} , slide 15

Problem 11.38

Notes 4-14-2011

11.38] $A_s = 2\pi D L N$

q. for condenser: use Eq. 11.35 instead of figures

Notes 4-19-2011

Ch 12, - RADIATION -

Blackbody:

Stefan Boltzmann law: the radiation energy emitted by a blackbody over all λ 's per unit time and surface area is

$$\begin{array}{l} \text{total blackbody} \rightarrow E_b = \sigma T^4 \quad (\text{W/m}^2) \\ \text{emissive power} \quad \uparrow \\ \sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \end{array}$$

(must be in K!)

• the radiation emitted by a blackbody per unit wavelength is given by Planck's Law

$$E_{b\lambda} = \frac{C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)} \quad (\text{W/m}^2 \cdot \mu\text{m})$$

$$\begin{array}{l} \text{spectral blackbody} \quad C_1 = 3.742 \times 10^8 \text{ W } \mu\text{m}^4 / \text{m}^2 \\ \text{emissive power} \quad C_2 = 1.439 \times 10^4 \text{ } \mu\text{m} \cdot \text{K} \end{array}$$

As $T \uparrow$, $E_{b\max}$ shifts towards shorter λ : $\lambda_{\max} = f(T)$

$$\frac{\partial E_{b\lambda}}{\partial \lambda} = 0 \Rightarrow \lambda_{\max} = T = C_3 = 2897.8 \mu\text{m} \cdot \text{K} \quad (\text{Wien's law})$$

$$\text{Sun} \Rightarrow T = 5762 \text{ K} \quad \lambda_{\max} = \frac{2897.8}{5762} \approx 0.50 \mu\text{m} \quad (\text{Vis})$$

$$T = 298 \text{ K} \Rightarrow \lambda_{\max} = 9.72 \mu\text{m} \quad (\text{IR})$$

$$\text{Flame} \Rightarrow T \approx 2000 \text{ K} \Rightarrow \lambda_{\max} = 1.5 \mu\text{m}$$

• fraction of blackbody energy from 0 to λ

$$F_{(0 \rightarrow \lambda)} = \frac{\int_0^\lambda E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda} = \frac{\int_0^\lambda E_{b\lambda} d\lambda}{\sigma T^4} \quad 0 \leq F \leq 1$$

• fraction of blackbody energy between λ_1 & λ_2

$$F_{(\lambda_1 \rightarrow \lambda_2)} = \frac{\int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda - \int_0^{\lambda_1} E_{b\lambda} d\lambda}{\sigma T^4} = F_{(0 \rightarrow \lambda_2)} - F_{(0 \rightarrow \lambda_1)}$$



Notes 4-19-2011

(5800)

P. 12, 23: fraction of sun's radiation is emitted in visible range (VIS 0.4-0.7 μm)

$$F_{(0.4-0.7)} = F_{(0 \rightarrow 7)} - F_{(0 \rightarrow 4)}$$

$$\lambda_1 T = 0.4 (5800) = 2320 \text{ μm} \cdot \text{K}$$

$$\lambda_2 T = 0.4 (5800) = 4000 \text{ μm} \cdot \text{K}$$

$$= 0.48 - 0.14$$

$$= 0.34 = \boxed{34\%}$$

Radiation properties of Real Surfaces

(i) Emissivity: $\epsilon = \frac{\text{energy emitted by a surface} @ T}{\text{" " " blackbody" } @ T}$

$$0 \leq \epsilon \leq 1$$

$\epsilon = 1 \Rightarrow \text{blackbody}$

ϵ_λ = spectral emissivity

ϵ = total emissivity = average ϵ_λ over all λ 's

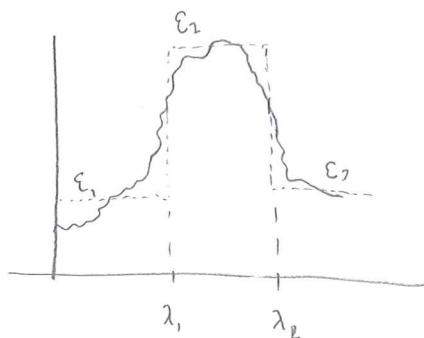
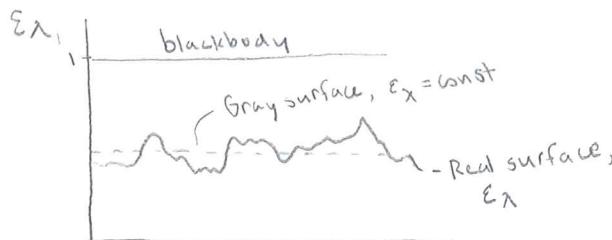
$$\epsilon_\lambda(T) = \frac{E_\lambda(T)}{E_{b\lambda}(T)}, \quad \epsilon(T) = \frac{E(T)}{E_b(T)} = \frac{E(T)}{\sigma T^4} = \frac{1}{\sigma T^4} \int_0^\infty \epsilon_\lambda E_{b\lambda} d\lambda$$

Real surface: $\epsilon_\lambda \neq \text{const}$ $\epsilon_\theta \neq \text{const}$

Diffuse surface: $\epsilon_\theta = \text{const}$ } diffuse & gray surface: $\epsilon = \epsilon_\theta = \epsilon_\lambda = \text{const}$

Gray surface: $\epsilon_\lambda = \text{const}$

Notes 4-21-11



Approximation:

$$\epsilon = \frac{1}{\sigma T^4} \left\{ \epsilon_{\lambda_1} \int_0^{\lambda_1} E_{b\lambda} d\lambda + \epsilon_{\lambda_2} \int_{\lambda_1}^{\lambda_2} \dots + \epsilon_{\lambda_3} \int_{\lambda_2}^{\infty} \dots \right\} \quad \leftarrow \quad \begin{cases} \epsilon_1 = \text{const. 1}, & 0 \leq \lambda \leq \lambda_1 \\ \epsilon_2 & 2 \quad \lambda_1 \leq \lambda \leq \lambda_2 \\ \epsilon_3 & 3 \quad \lambda_2 \leq \lambda < \infty \end{cases}$$

$$\epsilon = \epsilon_{\lambda_1} F_{(0 \rightarrow \lambda_1)} + \epsilon_{\lambda_2} F_{(\lambda_1 \rightarrow \lambda_2)} + \epsilon_{\lambda_3} F_{(\lambda_2 \rightarrow \infty)}$$

$$(F_{0 \rightarrow \lambda_2} - F_{0 \rightarrow \lambda_1})$$

Notes 4-21-11 Continued

ii) Absorptivity, Reflectivity, and transmissivity:

$$\alpha = \frac{\text{absorbed radiation}}{\text{Irradiation}} = \frac{G_{\text{abs}}}{G} \quad p = \frac{G_{\text{ref}}}{G} \quad T = \frac{G_{\text{tra}}}{G}$$

$$\sigma = 5.67 \times 10^{-8}$$

$$G = G_{\text{abs}} + G_{\text{ref}} + G_{\text{tra}}$$

$$1 = \alpha + p + T$$

• opaque surface: $T=0 \Rightarrow \alpha+p=1$ ($G_{\text{tra}}=0$)

• black surface: $p=T=0 \Rightarrow \alpha=1$

Spectral properties:

$$\alpha_\lambda = \frac{G_{\lambda, \text{abs}}}{G_\lambda} \quad p_\lambda = \frac{G_{\lambda, \text{ref}}}{G_\lambda} \quad T_\lambda = \frac{G_{\lambda, \text{tra}}}{G_\lambda} \quad \alpha_\lambda + p_\lambda + T_\lambda = 1$$

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}; \quad p = \frac{\int_0^\infty p_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} \quad T = \frac{\int_0^\infty T_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$$

Kirchhoff's law:

$$\alpha_\lambda = \epsilon_\lambda$$

Ex, 12.69] opaque $T_\lambda=0$ $\alpha_\lambda + p_\lambda + T_\lambda = 1$ $\alpha_\lambda = 1 - p_\lambda \Rightarrow p_\lambda = \begin{cases} 1.0 & 0 \leq \lambda \leq 3 \mu\text{m} \\ 1.2 & 3 \leq \lambda \leq \infty \mu\text{m} \end{cases}$

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{0.4 \int_1^3 G_\lambda d\lambda + 0.8 \int_3^6 G_\lambda d\lambda + 0.8 \int_6^\infty G_\lambda d\lambda}{\int_1^3 G_\lambda d\lambda + \int_3^6 G_\lambda d\lambda + \int_6^\infty G_\lambda d\lambda} = \frac{0.4 \frac{14(500)}{2} + 0.8(3)(500) + 0.8(2)(500)}{2500}$$

$$= \frac{1800}{2500} = 0.72 \quad \epsilon_\lambda = \alpha_\lambda$$

$$\epsilon = \epsilon_{\lambda_1} F_{(0 \rightarrow 3)} + \epsilon_{\lambda_2} F_{(3 \rightarrow \infty)} = \underbrace{\alpha_{\lambda_1} \cdot F_{(0 \rightarrow 3)}}_{0.4} + \underbrace{\alpha_{\lambda_2} (1 - F_{(0 \rightarrow 3)})}_{0.8}$$

$$\lambda T = 3(150) = 2250 \mu\text{m} \cdot \text{K} \Rightarrow F_{(0 \rightarrow 3)} = 0.1 \quad \epsilon = 0.756$$

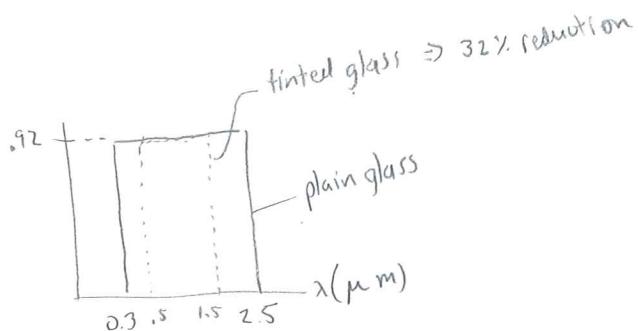
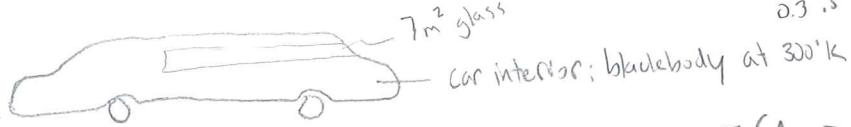
$$q''_{\text{radiation}} = G_{\text{absorbed}} - E = \alpha \cdot G - \epsilon \sigma T_s^4 = (0.72)(2500) - 0.756 (5.67 \times 10^{-8})$$

$$= -11,763 \text{ W/m}^2$$

Notes 4-21-11

continued

Greenhouse effect:



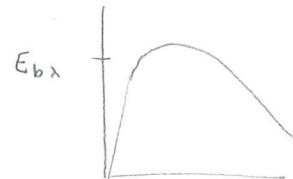
a) energy of sun transmitted through windows = $\tau G_{AS} = ?$

$$\tau = \frac{\int_0^\infty \tau_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{\int_0^\infty \tau_\lambda E_{b\lambda} d\lambda}{\int_0^\infty G_\lambda d\lambda} = \frac{.92 \int_0^{1.5} E_{b\lambda} d\lambda}{\int_0^\infty E_{b\lambda} d\lambda}$$

$$= (0.92)(F_{(0 \rightarrow 1.5)} - F_{(0 \rightarrow 0.3)}) = 0.85$$

$$\lambda_1 T = 2.5 (5800) = 14,500 \mu m \Rightarrow F_{(0 \rightarrow 2.5)} \cong 0.96$$

$$\lambda_1 T = 0.3 (5800) = 1740 " \Rightarrow F_{(0 \rightarrow 0.3)} \cong 0.04$$



$E_{b\lambda}$ "spectral radiation of the sun"

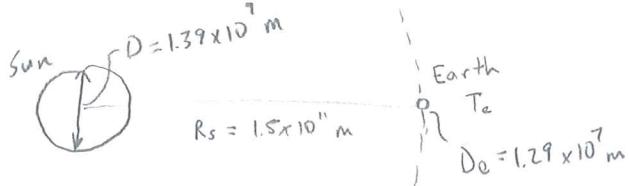
b) radiant energy from car's interior transmitted back to the surroundings thru windows = $\tau E_{b\lambda}^{(300K)} \cdot A_S = 1.35 \times 10^{-5} (5.67 \times 10^{-8})(300^4)(1) = .004 W$

$(2.5)(300) = 750 \mu m \cdot K \Rightarrow F_{(0 \rightarrow 2.5)} \cong .000015 \quad \left. \right\} \tau = 1.35 \times 10^{-5}$

$(.3)(300) = 90 \mu m \cdot K \Rightarrow F_{(0 \rightarrow 0.3)} \cong 0$

Notes 4-26-2011

Ex) Problem 12.20

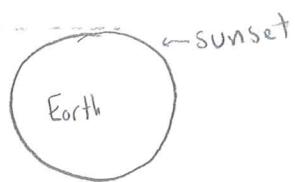


Solar flux at outer edge of Earth's atmosphere $\eta_s'' = 1353 \text{ W/m}^2$

$$E_s (\frac{\pi D_s^2}{4\pi R_s^2}) = \eta_s'' 4\pi (R_{s_e}^2) \Rightarrow E_s = 6.3 \times 10^7 \text{ W/m}^2 = \sigma T_s^4 \Rightarrow T_s = 5724 \text{ K}$$

gas molecules & small particles $C_{scat} \sim \frac{1}{\lambda^4}$; $C_{abs} \sim \frac{1}{\lambda}$

- more absorption can occur through a greater length & longer wavelengths will be preserved



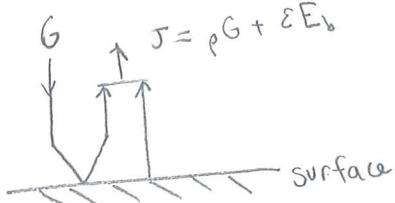
Radiation Exchange Between Surfaces (Ch.13)

Consider two surfaces separated by a non-participating medium (most gases)

Governing factors

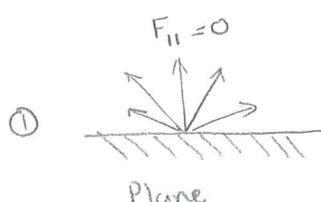
- Temperature
- Radiation Properties
- Geometries & orientations

- Radiosity: J



- View factor: fraction of radiation leaving a surface that is intercepted by the other surface

- J : emitted radiation + reflected radiation



F_{12} = how much does surface 1 "see" surface 2.

i) inspection

ii) Reciprocity rule: $A_1 F_{12} = A_2 F_{21}$

iii) Summation rule: $\sum_{j=1}^N F_{ij} = 1$ for an enclosure w/ N surfaces

iv) Symmetry rule: if surfaces 2 & 3 are symmetric about surface 1 $F_{12} = F_{13}$

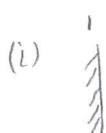
N surfaces $\Rightarrow N^2$ view factors but only $\frac{N(N+1)}{2}$ of them are needed directly

Notes 4-26 continued:

1. Integration:

See tables 13.1-13.2 Figures 13.4-13.6 for selected geometries

2. View factor Algebra

(i)  $\bar{F}_{11} = 0$ $F_{22} = 0$
 $F_{12} = 1 = \bar{F}_{21}$

(ii)



$$F_{11} = 0 \quad A_1 F_{12} = A_2 \bar{F}_{21}$$

$$F_{12} = 1 \quad \Rightarrow \quad F_{21} = \frac{A_1}{A_2} = \frac{D_1}{D_2}$$

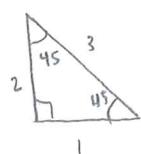
$$F_{21} + F_{22} = 1 \Rightarrow F_{22} = 1 - \frac{A_1}{A_2}$$



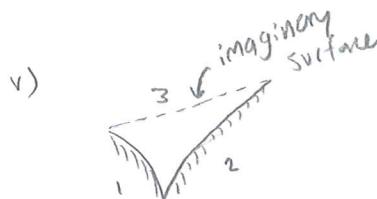
$$\bar{F}_{12} = \bar{F}_{13} = \frac{1}{2}$$

$$\bar{F}_{21} = \bar{F}_{23} = \frac{1}{2} \dots$$

(iv)



9 view factors?
find these



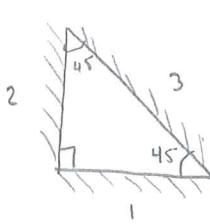
$$A_1 \bar{F}_{12} = A_2 \bar{F}_{21} = \frac{A_1 + A_2 - A_3}{2}$$



$$A_1 \bar{F}_{12} = A_2 \bar{F}_{21} = \frac{\sum (\text{crossed strings}) - \sum (\text{uncrossed strings})}{2}$$

$$= \frac{(A_5 + A_6) - (A_3 + A_4)}{2}$$

4-28-2011



$$\bar{F}_{11}^0 + \bar{F}_{12} + \bar{F}_{13} = 1$$

$$\bar{F}_{31} + \bar{F}_{32} + \bar{F}_{33}^0 = 1$$

$$\bar{F}_{31} = \bar{F}_{32} = 1/2$$

$$A_1 \bar{F}_{13} = A_3 \bar{F}_{31} = A_3 \left(\frac{1}{2}\right)$$

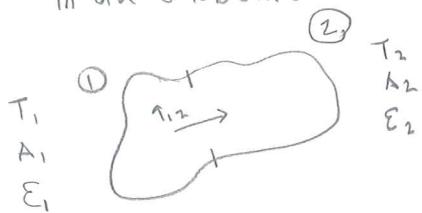
$$\bar{F}_{13} = \frac{A_3}{2A_1}$$

$$\nwarrow A_1 \bar{F}_{12} = \nearrow A_2 \bar{F}_{21} \Rightarrow \dots \text{ stopped taking notes}$$

$$\bar{F}_{21} = \frac{2A_1 - A_3}{A_2} \quad F$$

Radiation Exchange Between Two Diffuse, Gray, opaque surfaces

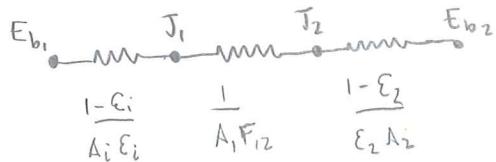
in an enclosure



$$q_{12} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1-E_1}{A_1 E_1} + \frac{1}{A_1 F_{12}} + \frac{1-E_2}{A_2 E_2}}$$

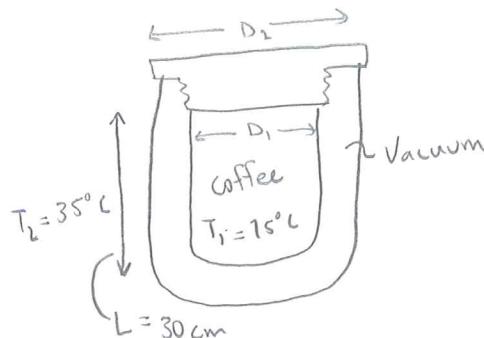
$R_i = \frac{1-E_i}{A_i E_i}$; surface resistance to radiation, a measure of departure from blackbody ($\varepsilon=1 \Rightarrow R=0$)

$R_{12} = \frac{1}{A_1 F_{12}}$: a measure of how two surfaces "see" each other



ex) Thermos bottle 13.06, but with vacuum
treat as long concentric cylinder

$$q_{12} = \frac{\sigma \pi D_1 L (T_1^4 - T_2^4)}{\frac{1}{E_1} + \frac{1-E_2}{E_2} \left(\frac{r_1}{r_2} \right)} = 3.2 \text{ W}$$



Thickness of an equivalent conduction insulation (say, $k_{\text{cork}} = 0.04 \frac{\text{W}}{\text{mK}}$)

$$q_{12} = 3.2 = \frac{T_1 - T_2}{\left(\frac{\ln r_2/r_1}{2\pi k L} \right)} \Rightarrow \ln \frac{D_2}{D_1} = \dots \quad D_2 = 18 \text{ cm}$$

— Radiation shields —

$$\varepsilon \downarrow \Rightarrow \frac{1-\varepsilon}{A_1 \varepsilon_1} \uparrow \Rightarrow q_{\text{rad}} \downarrow$$

special case: multiple shields w/ same emissivity: $(q_{12})_{N, \text{shields}} = \frac{1}{N+1} (q_{12})_{0, \text{shields}}$

WEE

Ex. 13.56]

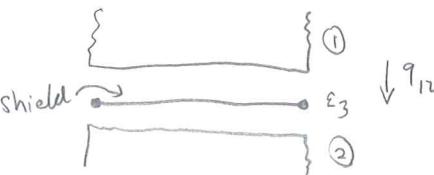
$$\varepsilon_1 = \varepsilon_2 = 0.8 \quad F_{13} = F_{12} = 1$$

$$\varepsilon_3 = ?$$

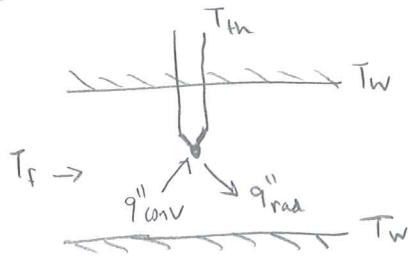
$$(q_{12})_{\text{w/o shield}} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$$

$$(q_{12})_{\text{w/shield}} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_3} - 1\right)} \quad (\text{pg 793})$$

$$\frac{q_{12\text{shield}}}{q_{12\text{no shield}}} = \frac{1}{10} = \frac{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2}{\varepsilon_3} - 2\right)} \Rightarrow \varepsilon_3 = 0.138$$



→ Radiation Correction → on temperature measurements



$$q_{\text{conv}} = q_{\text{rad}}$$

$$h(T_f - T_{th}) = \varepsilon_{th} \sigma (T_{th}^4 - T_f^4)$$

$$T_f = T_{th} + \varepsilon_{th} \sigma \frac{(T_{th}^4 - T_f^4)}{h}$$

try to minimize ε
radiation correction

Homework

11.2 (a,b), 18, 22, 34(a,b,c), 35(a)

11.2]

$$D_i = 22 \text{ mm} \quad R_{f,i}'' = .0004 \text{ m}^2 \text{ K/W}$$

$$D_o = 27 \text{ mm} \quad R_{f,o}'' = .0002 \text{ m}^2 \text{ K/W}$$

a) determine overall h.t. coefficient, U_o

$$\frac{1}{U_o} = A_o \left[\frac{1}{h_i A_i} + \frac{R_{f,i}''}{A_i} + \frac{\ln D_o/D_i}{2\pi k L} + \frac{R_{f,o}''}{A_o} + \frac{1}{h_o A_o} \right]^{-1}$$

$$\frac{\ln D_o/D_i}{2\pi k L} = .00216$$

 h_o :

$$Re_{D,o} = \frac{V_{o_0} D}{\nu} = \frac{20 \text{ m/s} (.027)}{1.027 \times 10^{-7}} = 33984$$

$$Nu_{D,o} = C = .26 \quad m = 0.6$$

pg. 431
(746)
why?
?

$$Nu_o = 0.3 + \frac{0.62 Re_{D,o}^{1/2} Pr_o^{1/3}}{\left[1 + \left(0.4 / Pr_o \right)^{2/3} \right]^{1/4}} \left\{ 1 + \left(\frac{Re_{D,o}}{282000} \right)^{4/5} \right\}$$

$$Nu_{D,o} = 103 \quad h_o = \frac{Nu_{D,o} k}{D} = 104 \text{ W/mK} \quad \checkmark$$

$$\frac{1}{U_o} = A_o \left[\frac{R_{cv,i}}{.00429} + \frac{R_{f,i}}{.00579} + \frac{R_w}{.00216} + \frac{R_{f,o}}{.00236} + \frac{R_{cv,o}}{.1134} \right]^{-1}$$

$$R_{tot} = 0.128$$

$$U_o = 92.1$$

57.8

$$R_{cv,o,2} = .007689$$

$$A_o = .0848$$

fouling contributes resistance as large as the wall and convection resistances. Convection resistance for air is the greatest. \checkmark

Part b) On back

(1)

David Malawey

11.2 b) cross flow water, 15°C , $V_0 = 1 \text{ m/s}$, $\Pr = 7.56$, $k = 0.598$, $D_o = 0.027 \text{ m}$

$$Nu_{D_o} : Re = \frac{V_0 D}{\nu \mu} = \frac{1(0.027)}{(1.001) \times 10^{-3} (1080) \times 10^{-6}} \Rightarrow Re = 24,975 \quad \boxed{31,xxx}$$

$$Nu_D = C Re^{\frac{m}{n}} \Pr^{\frac{1}{3}} = 198 \quad \text{Table 7.2}$$

$$\bar{h}_o = \frac{Nu_{D_o} k}{D_o} = 4385 \quad \Rightarrow \quad Re_{c,v,o} = \frac{1}{h_o A_o} = .002689 \quad \Rightarrow \boxed{U = 681} \quad \checkmark$$

<u>11.18</u>	concentric tube, $A_s = 50 \text{ m}^2$	Hot fluid	Cold	a) find $T_{h,o}$
heat capacity rate kJ/kgK	6	3		
$T_i, {}^\circ\text{C}$	60	30		
$T_o, {}^\circ\text{C}$	48	54		

$$q = C_h (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i}) \Rightarrow \boxed{T_{h,o} = 48^\circ\text{C}} \quad \checkmark$$

b) System is in counterflow because $T_{h,o} < T_{c,o}$

c) find U . $U = \frac{q}{A \Delta T_m}$ $\Delta T_m = \frac{\Delta T_1 - \Delta T_2}{\ln(\frac{\Delta T_1}{\Delta T_2})} = 19.4 \Rightarrow \boxed{U = 0.0742}$ check

d) effectiveness, $\varepsilon = \frac{q}{q_{\max}} = \frac{72}{90} = 0.80 \quad C_{\min}(T_{h,i} - T_{c,i}) = 3(60 - 30) = 90$

$$\boxed{\varepsilon = 0.80} \quad \checkmark$$

e) effectiveness if length were very long? $q = C_c (T_{c,o} - T_{c,i}) \quad \& \quad T_{c,o} \rightarrow T_{h,i}$

$$\Rightarrow q \rightarrow C_c (T_{h,i} - T_{c,i})$$

$$\Rightarrow \boxed{\varepsilon \rightarrow 1} \quad \checkmark$$

11.22, 34(a,b,c), 35(a)

11.22 — Shell & Tube HX. — water oil
 2.5 kg/s of water $T_{ci} = 15^\circ\text{C}$ $T_{hi} = 160^\circ\text{C}$
 $N=10$ $T_{co} = 85^\circ\text{C}$ $T_{ho} = 100^\circ\text{C}$

thin walled
 $D = 25\text{ mm}$
 tubes make 8 passes ea.

— what is flow rate of oil?

$$G_c(T_{co} - T_{ci}) = G_h(T_{ho} - T_{hi})$$

$$2.5(4.182)(85 - 15) = \dot{m}_{oil} (2.337)(160 - 100)$$

$$\boxed{\dot{m}_{oil} = 5.22 \text{ kg/sec}} \quad \checkmark$$

— How long must tubes be?

$$NTU = \frac{VA}{G_{min}}$$

$$\epsilon = \frac{q}{q_{max}} = \frac{C_o(T_{co} - T_{ci})}{C_{min}(T_{hi} - T_{ci})} = \frac{732}{151^\circ\text{C}}$$

$$\epsilon = .483$$

$$\frac{q_{min}}{q_{max}} = 0.857 \quad \left. \begin{array}{l} \\ NTU = 1 \end{array} \right\} \text{fig 11.12}$$

$$\frac{1}{VA} = \left[\frac{1}{h_i A_i} + \frac{1}{h_o A_o} \right]$$

$$\text{if } Re = \frac{V_m D}{\nu} = \frac{4m}{\pi D \mu} = 24114 \quad \overline{Nu} = .023 Re^{0.4} Pr^{0.4} = 121$$

$$h_i = \frac{Nu \cdot k}{D} = 3122 \quad \left. \begin{array}{l} \frac{1}{VA} = [] \Rightarrow V = 355 \checkmark \\ h_o = 400 \end{array} \right\}$$

$$\frac{NTU}{VA} \left(q_{min} \right) = 10 \pi (.025) L \quad \Rightarrow \frac{1}{355} \frac{(2.5)(4.182)}{(10 \pi (.025))} = \boxed{L = 37.5 \text{ m}} \quad \checkmark$$

$$\frac{Oil}{h_o} = 400 \text{ W/m}^2 \text{ outside tubes}$$

$$c_p @ 130^\circ\text{C} = 2.337 \quad (\rho 857)$$

11.34] a, b, c cross-flow
hx 5 liter/min

$$U = 750 \text{ W/m}^2\text{K}$$

blood

$$t_{hi} = 37^\circ\text{C}$$

ice bath

$$t_{li} = 0^\circ\text{C}$$

a) exchanger heat transfer
rate?

$$t_{lo} = 25^\circ\text{C}$$

$$t_{co} = 15^\circ\text{C}$$

$$\rho = 1050 \text{ kg/m}^3$$

$$c_p = 4.198 \text{ kJ/kg}$$

$$q = \dot{m}_b c_{p_b} (T_i - T_o)$$

$$c_p = 3740 \text{ J/kg}\cdot^\circ\text{C}$$

$$= .005 \text{ m}^3/\text{min} \left(\frac{1 \text{ min}}{60 \text{ sec}} \right) (1050 \text{ kg/m}^3) (3740) (37 - 25)$$

$$q = 3927 \text{ J/sec} = \boxed{3.927 \text{ kW}} \quad \checkmark$$

b) water flow rate $\dot{m}_w c_{p_w} (15) = 3.927 \text{ kW}$

$$\boxed{\dot{m}_w = .0624 \text{ kg/s}}$$

c) find A_s : $q = UA \Delta T_{am}$

$$\frac{G_{\min}}{G_{\max}} = \frac{\dot{m} c_{p_w}}{\dot{m} c_{p_b}} \frac{262}{327.3} = .800$$

$$\varepsilon = \frac{q}{q_{\max}} = \frac{3.927 \text{ kW}}{262 (37 - 0)} = 0.405$$

Fig 11.14

NTU = 0.7

$$NTU = \frac{UA}{G_{\min}} \Rightarrow A_s = \frac{0.7 (262)}{750}$$

$$\boxed{A_s = 0.245 \text{ m}^2}$$



11.35

Steam (sat) 0.14 bar, condensing $C_s \rightarrow \infty$

— Shell & tube —

- 1 shell pass, 2 tube passes

- 130 brass tubes, 2m per pass

find $U, T_{c,o}, m_{\text{condense}}$

$$D_i = 0.0134 \text{ m}$$

$$D_o = 0.0159 \text{ m}$$

Steam

$$h_{fg} = 2378 \text{ kJ/kg}$$

$$T = 52^\circ \text{C}$$

Cooling H_2O

$$T_{ci} = 20^\circ \text{C}$$

$$T_{co} =$$

$$\dot{m} =$$

$$V = 1.25 \text{ m/s}$$

$$h_o = 13.5 \text{ kW/m}^2\text{K}$$

Water guess $\sim 305^\circ \text{K}$

$$\rho = \frac{1}{1.005}$$

$$Pr = 5.2$$

$$k = .620$$

$$C_p = 4178$$

$$h_i: Re = \frac{\rho V_\infty D}{\mu} = \frac{1}{1.005 \times 10^3} \frac{1.25 (.0134)}{769 \times 10^{-6}} = 21,673 \checkmark$$

$$Nu_D = .023 Re^{0.8} Pr^{0.4} = 130.9$$

$$h_i = \frac{k_f Nu}{D} = 6057$$

$$\frac{1}{U_{A_i}} = \left[\frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A_o} \right]$$

$$\frac{1}{U_o} = \left[\frac{1}{h_i A_i} + \frac{\ln(D_o/D_i) \times D_o L}{2\pi k L} + \frac{1}{h_o} \right] = \left(\frac{1}{6057} \left(\frac{159}{134} \right) + \frac{\ln \left(\frac{159}{134} \right) \cdot 0.0159}{2 \cdot (114)} + \frac{1}{13.5 \times 10^{-3}} \right)$$

$$U_o = 3549 \text{ W/m}^2\text{K} \checkmark$$

$$T_{c,o}: q = \varepsilon q_{\max}$$

$$q_{\max} = C_{\min} (T_{hi} - T_{lo})$$

$$m_c = 1.25 \text{ m/s} (\pi \cdot 0.0134)(130 \text{ tubes}) \left(\frac{1}{1.005 \times 10^{-3} \text{ m}} \right) = 22.8 \text{ kg/sec}$$

$$\frac{C_{\min}}{C_{\max}} = 0$$

$$C_c = 22.8 \text{ kg/sec} (4178 \text{ J/kg}) = 95,258 = C_{\min} \checkmark$$

$$q = C_c (T_{c,o} - T_{ci}) =$$

$$NTU = \frac{U_o A_o}{C_{\min}}, A_o = \pi (0.0159)(2 \text{ m})(130)(2 \text{ pass}) = 26.0 \text{ m}^2 \quad \left. \right\}$$

$$NTU = 0.69 \Rightarrow \text{Fig 11.12} \Rightarrow \varepsilon = 0.61$$

$$\varepsilon = 1 - e^{-NTU} \Rightarrow \varepsilon = 0.621$$

$$T_{c,o} \Rightarrow \frac{q}{C_c} + T_{ci} =$$

$$T_{c,o} = 39.8^\circ \text{C}$$

$$q = \varepsilon C_c (T_{hi} - T_{ci}) \Rightarrow q = 1.89 \times 10^6$$

$$m_{\text{condense}} = \frac{\dot{m} C_p (T_{hi} - T_{lo})}{h_{fg}} = 0.793 \text{ kg/s}$$

(5)

EXAM 3 2008

#1) Shell & tube. 2 passes, 1 shell

condensing steam, $C = \infty$

$P = 1 \text{ bar}$

$T = 373 \text{ K}$

$h = 2000 \text{ W/m}^2\text{K}$

tube side:

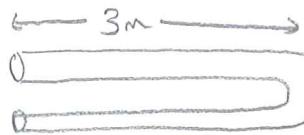
Water

$T_i = 270 \text{ K}$

$T_o = 320 \text{ K}$

$\dot{m} = .05 \text{ kg/s}$

$h = 500 \text{ W/m}^2\text{K}$



$D = 2.5 \text{ cm}$

find: fouling factor

$$\frac{1}{UA} = \left[\frac{1}{h_i A} + \frac{R''_f}{A} + \frac{1}{h_o A} \right] = \frac{1}{U} = \left[\frac{1}{h_i} + R''_f + \frac{1}{h_o} \right]$$

$$q_{\text{per tube}} = \frac{\dot{m} C_p (T_o - T_i)}{A}$$

$$= .05(4181)(320 - 270)$$

$$= 5774 \text{ W}$$

$$q_{\text{max}} = .05(4181)(373 - 270) = 41810 \text{ W}$$

$$G = 1 - e^{(-NTU)}$$

$$\epsilon = .485$$

$$NTU = .664$$

$$NTU = \frac{UA}{q_{\text{min}}} \Rightarrow .664 = \frac{U \cdot 6 \times \pi (0.025)}{.05(4181)} \quad U = 294.6$$

$$.00339 = \left(\frac{1}{500} + R''_f + \frac{1}{2000} \right) \quad R''_f = .00089$$

Homework

13.19] - 3 black surfaces



find F_{13} , net q'' from $A_1 \rightarrow A_3$ $A_1 = 0.05 m^2$

$$T_1 = 1000 K$$

$$T_3 = 500 K$$

$$F_{13} + F_{12} = 1 \quad F_{12} = \frac{D^2}{D^2 + 4L^2} \quad (P781) \quad F_{12} = 0.36 \quad \boxed{F_{13} = 0.64}$$

$$\text{net } q'': \quad q_{13} = \frac{\sigma(T_1^4 - T_3^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{13}} + \frac{1-\varepsilon_3}{\varepsilon_3 A_3}} = \frac{\sigma(1000^4 - 500^4)}{\frac{1}{.05 (.55)}}$$

$$\boxed{q_{13} = 1701 \text{ W}}$$

13.41] 2 large diffuse, gray surfaces



Find:

$$\bullet G_1 \text{ & } J_1$$

$$\bullet \text{net } q''_{\text{rad}}$$

$$G_1 = E_2 + G_1 p_2 = \varepsilon_2 \sigma T_2^4 + (\varepsilon_1 \sigma T_1^4) p_2$$

$$= \boxed{14175 \text{ W/m}^2}$$

$$J_1 = E_1 + \cancel{E_2 p_1} = \varepsilon_1 \sigma T_1^4 =$$

$$= \boxed{56700 \text{ W/m}^2}$$

$$q_{1 \rightarrow 2} = G_1 - J_1 = \boxed{42,525 \text{ to } A_1}$$

HW 9] — Exam Practice —

11.22)

shell-and-tube HX .

10 tubes
shell side

Water: $\dot{m} = 2.5 \text{ kg/s}$

engine oil $\dot{m} =$

$T = 320 \text{ K}$

$T_i = 15^\circ\text{C}$

$T_i = 160^\circ\text{C}$

$T_o = 85^\circ\text{C}$

$T_o = 100^\circ\text{C}$

$C_p = 4180$

$h_o = 400 \text{ W/m}^2\text{K}$

$$J = 1.011 \text{ m}^3/\text{kg} \times 10^{-3}$$

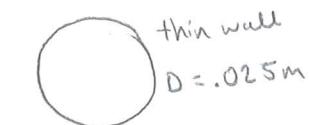
$C_p = 2340$

$$\mu = 577 \times 10^{-6}$$

$$k = .640$$

$$Pr = 3.77$$

$$\dot{m}_{\text{oil}} ? \quad 2.5 \text{ kg/s} (4180)(85 - 15) = \dot{m}_{\text{oil}} (2340)(160 - 100) \Rightarrow \boxed{\dot{m}_{\text{oil}} = 5.21 \text{ kg/s}}$$



$$h_i = \overline{Nu}_i k \quad V = \frac{2.5 \text{ kg/s} (.00101 \text{ m}^3/\text{kg})}{10 \times \pi \times \frac{(0.025)^2}{4}} = 0.514 \text{ m/sec}$$

$$Re_D = \frac{0.514 \rho D}{\mu} = 22,028 \quad \overline{Nu}_D = .023 Re_D^{0.8} Pr^{0.4} = 117$$

$$h = \frac{Nu K}{D} = 3000$$

$$NTU = \frac{UA}{C_{\min}} \quad \varepsilon = \frac{q}{q_{\max}} \quad q_{\max} = C_{\min} (T_{hi} - T_{co}) = 2.5 (4180) (160 - 15)$$
$$C_{\min} = .857$$
$$q_{\max} = 1.52 \times 10^6 \text{ W}$$
$$\varepsilon = .481$$
$$q = 731,500$$

$NTU \geq 1$ (Table 11.12)

$$U = \frac{1}{\frac{1}{h_c} + \frac{1}{h_n}} \approx 353$$

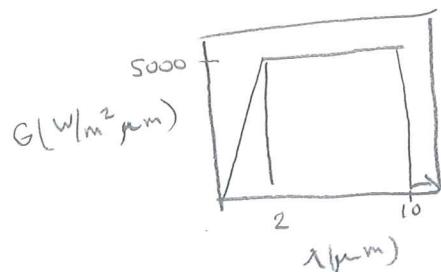
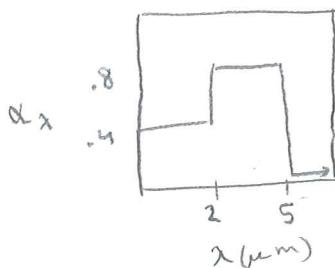
$$l = \frac{353 \times 10 \times \pi \times .025 \times L}{(2.5)(4180)}$$

$$\boxed{L = 37.7 \text{ m}}$$

HW 10

12.51) Find

- total absorptivity,
- Emissive Power
- surface temp going \uparrow or \downarrow ?



$$T_{\text{surface}} = 1250\text{K}$$

$$\chi = \frac{G_{\text{abs}}}{G} = \frac{0.4 \left(\frac{2 \times 5000}{2} \right) + 0.8 \left((5-2) \times 5000 \right)}{\frac{2 \times 5000}{2} + 8 \times 5000} = \frac{14,000}{45,000} = \boxed{0.311}$$

$$\epsilon = 0.4(F_{0 \rightarrow 2}) + 0.8(F_{2 \rightarrow 5}) = \\ = 0.4(0.161) + 0.8(0.754 - 0.161) = \boxed{0.5388}$$

$$E = \epsilon E_b = 0.54 \sigma T^4 = 74,750 \text{ W/m}^2$$

$$q''_{\text{net}} = G_{\text{abs}} - E = 14,000 - 74,750 = -60,750$$

Temperature is dropping

HW 10

$$G_{\text{solar}} = 900 \text{ W/m}^2$$

$$\alpha = 0.9$$

$$\epsilon = 0.1$$

$$h = 20 \text{ W/m}^2 \cdot \text{K}$$

$$T_{\text{sur}} = 290 \text{ K}$$

Find: T_s



$$\text{q-balance: } G_{\text{abs}} - E - q_{\text{conv}} = 0$$

$$G_{\text{abs}} = \alpha (G_{\text{solar}} + G_{\text{surroundings}})$$

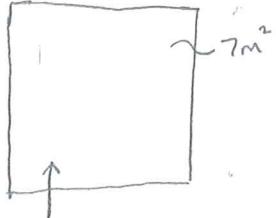
$$E = 0.1 \sigma T^4$$

$$q_{\text{conv}} = h(T_s - T_{\infty})$$

$$0.9(900 \text{ W/m}^2) + (0.1)(290)^4 = 0.1 \sigma T^4 + 20(T - 290 \text{ K})$$

$$\Rightarrow T = 329 \text{ K}$$

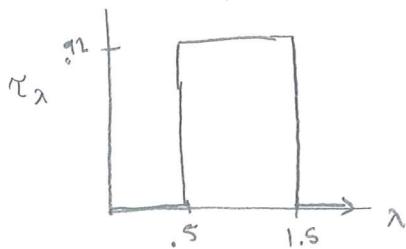
#2, car window



$$G_{\text{solar}} = 400 \text{ W/m}^2$$

find G_{trans}

- E_{interior} that is transmitted
- net rad trapped
in car in 1 hour



interior, blackbody, 300K

$$G_{\text{trans}} : \tau = 0.92 \quad (F_{0.5 \times 1.5}) = 0.92(0.880 - 0.250) = 0.580$$

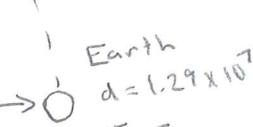
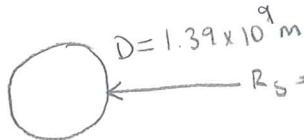
$$G_{\text{trans}} = \tau(400)(7\text{m}^2) = 162 \text{ W}$$

$$E_{\text{trans}} = 0.92 \left(F_{(1.5 \times 300)} - F_{(0.5 \times 300)} \right) = (0.92) \times 0$$

$$\text{net heat trapped in car} = 162 \text{ W} = 162 \text{ J/s} \times (3600 \text{ s}) = 583 \text{ kJ}$$

Example 12.20

SUN



Sphere Surface Area:

$$4\pi r^2 = \pi D^2$$

$$\text{solar flux @ edge of earth atmosphere} = 1353 \text{ W/m}^2$$

$$E_s(\pi D_s^2) = q_{\text{earth}}^2 \cdot 4\pi (R_s)^2$$

$$T_s = 5774$$

Exam 3 2005

#3) long oven $\sum \sum \sum \sum$
 $F_{12}, F_{21}, F_{13}, F_{23,34}$

$$F_{12} : F_{41} = F_{42} = \frac{1}{2} \text{ Symmetry}$$

$$F_{14} = \frac{A_4}{A_1} F_{41} = 1.73 \left(\frac{1}{2}\right) = .865$$

$$\cos(30) = \frac{L_u}{2L_1} = 1.73$$

$$F_{12} + F_{14} = 1 \Rightarrow$$

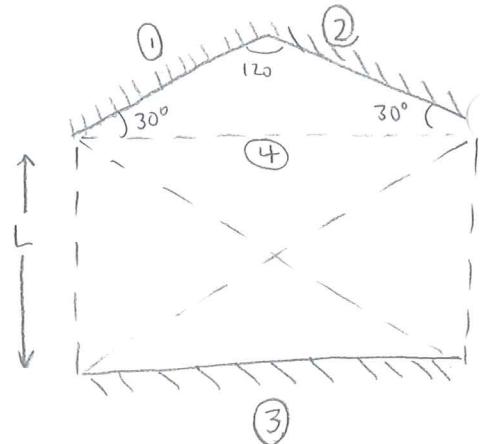
$$F_{21} = F_{12} = 0.135 \quad \checkmark$$

$$F_{43} = \frac{w_i = 1}{w_f = 1} \quad F_{ij} = \frac{(z^2 + 4)^{1/2} - [0^2 + 4]^{1/2}}{2(1)} \quad F_{43} = 0.414$$

$$F_{34} = .414 \quad |F_{31} = F_{32} = 0.207|$$

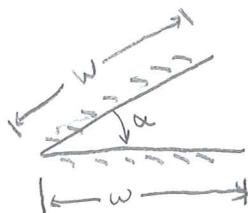
$$F_{13} = \frac{A_3}{A_1} F_{31} = 1.73 (.207) = 0.358 = F_{13}$$

$$F_{23} = .358$$



— Finding F_{14} —

Table 13.1 pg 775

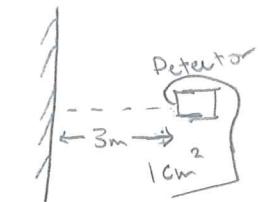
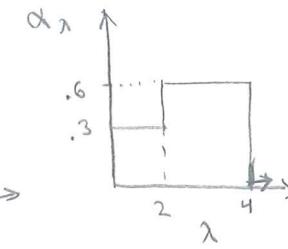
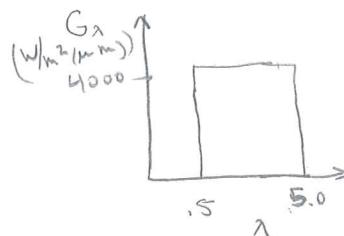
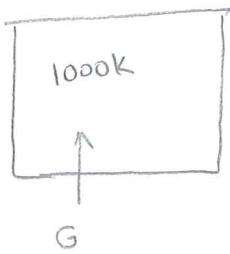


$$F_{ij} = 1 - \sin\left(\frac{\alpha}{2}\right)$$

MAY 2008

#2

$$2 \text{ m}^2 \quad T = 727^\circ\text{C}$$



— Find: α, ρ, ϵ

$$G_{abs} = 0.3(4000)(2-0.5) + 0.6(4000)(4-2) = 6600 \text{ W/m}^2$$

(W/m² μm)

$$\alpha = \frac{G_{abs}}{G} \Rightarrow \alpha = \frac{6600}{18000} = 0.367$$

$$\rho = 1 - \alpha \Rightarrow G_{ref.} = 0.7(4000)(2-0.5) + 0.4(4000)(4-2) + 1(4000)(5-4)$$

$$= 1140 \Rightarrow \rho = \frac{1140}{18000} = 0.0633$$

$$\rho = \frac{G_{ref.}}{G} \Rightarrow$$

— Find: Temp change over time:

$$\epsilon = \alpha = 0.367$$

$$E = \epsilon \sigma T^4: \quad \epsilon = 0.3(F_{0 \rightarrow 2}) + 0.6(F_{2 \rightarrow 4})$$

$$= 0.3(0.067) + 0.6(0.481 - 0.067) = .269$$

$$E = .269(5.67 \times 10^{-8})(1000)^4 = 15,252$$

$$T_{\text{entering plate}} = G_{abs} - E = 6600 - 15,252 = -8,650 \quad \text{plate is cooling}$$

$$J = ? \quad J = G_{ref.} + E = 11400 + 15,252 = 26,650$$

— Find: radiant power reaching detector

$$q_d = J A_s \frac{\Delta \lambda}{\pi L^2} = 26,650 \frac{2}{\pi} \frac{0.001}{9} = 0.188 \text{ W}$$

↑
heat flux to
the detector
 $\text{W/m}^2 (\text{m}^2)$ ($\frac{\text{detector area}}{\text{all area}}$)

$$\begin{aligned} & \text{Surface of sphere} \\ & = \pi D^2 \end{aligned}$$

Suggested problems for last material:

12.47, 13.1, 19, 41, 62

12.47] diffuse, opaque surface, 700 K
 radiant flux of 1000 W/m^2
 uniform $1 < \lambda < 6 \mu\text{m}$ is incident
 at angle 30° to normal

$$\varepsilon_\lambda = \begin{cases} 0.5 & 3 \mu\text{m} < \lambda \leq 10 \mu\text{m} \\ 0.9 & 10 \mu\text{m} < \lambda \leq \infty \end{cases}$$

total radiant Power = ? (received by detector)
 for 10^{-4} m^2 area
 aperture = 10^{-5} m^2 , $d = 1 \text{ m}$

$$q_d = I_e + r A_s \cos \theta_d - S \Delta W_d - S$$

$$q_d = \frac{\varepsilon \sigma T_s^4 + \rho G}{\pi} A_s \frac{\Delta d}{L^2}$$

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{h\nu}(700K) d\lambda}{E_b(700K)} = 0.5 (0.8081 - 0.0838) + 0.9 (0.8081) = 0.535$$

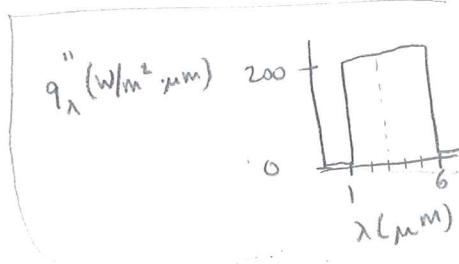
$$\rho = \frac{\int_0^\infty \rho_\lambda G_\lambda d\lambda}{G} = \frac{\int_0^\infty (1 - \varepsilon_\lambda) q''_\lambda d\lambda}{q''} = 1 \times F_{(0 \rightarrow 3 \mu\text{m})} + 0.5 \times F_{(3 \rightarrow 6 \mu\text{m})}$$

$$= 1 \times (0.4) + 0.5 \times (0.6) = 0.7 = \rho$$

$$G = q'' \cos \theta_i = 1000 \cos(30^\circ) = 866 \text{ W/m}^2$$

$$q_d = \frac{\varepsilon \sigma T_s^4 + \rho G}{\pi} A_s \frac{\Delta d}{L^2}$$

aperture (10^{-5} m^2)
 $\sqrt{(1 \text{ m})}$



$$\sigma = 5.67 \times 10^{-8}$$

$$q_d = \frac{7889}{\pi} (1 \times 10^{-9}) = 2.51 \times 10^{-6} \text{ W}$$

13.1

a) Long duct



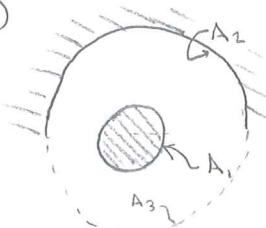
$$A_1 F_{12} = A_2 F_{21}$$

$$F_{12} = \frac{A_2}{A_1} F_{21} \quad \frac{A_2}{A_1} = \frac{\frac{3}{4} \pi D}{D} = \frac{3\pi}{4}$$

$$F_{11} + F_{12} = 1$$

$$F_{21} = \frac{4}{3\pi} (1) \Rightarrow F_{21} = \frac{4}{3\pi} = 0.424$$

b)



$$A_2 = 2A_1$$

$$\text{summation: } F_{11} + F_{12} + F_{21} = 1$$

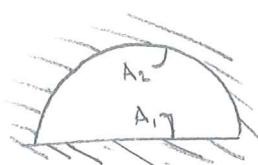
$$\text{symmetry} \quad F_{12} = F_{13} = \frac{1}{2}$$

$$A_1 F_{12} = A_2 F_{21} \quad F_{21} = \frac{A_1}{A_2} F_{12} = \frac{1}{2} \left(\frac{1}{2}\right) = 0.25$$

$$F_{12} = \frac{1}{2}$$

$$F_{21} = 0.25$$

c)



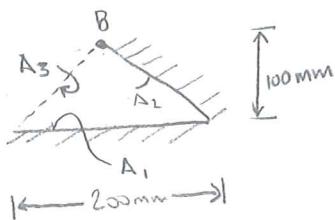
$$F_{22} = ?$$

$$F_{12} = 1 \quad F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\frac{1}{2} \pi R}{\frac{\pi R}{2}} (1) \Rightarrow F_{21} = \frac{2}{\pi}$$

$$F_{22} = 1 - \frac{2}{\pi}$$

$$F_{22} = 0.363$$

d)



$$F_{12} + F_{13} = 1 \Rightarrow F_{12} = F_{13} = \frac{1}{2}$$

$$A_1 F_{12} = A_2 F_{21} \quad F_{21} = \frac{A_1}{A_2} F_{12} = \frac{200}{\sqrt{100^2 + 100^2}} \left(\frac{1}{2}\right) = 0.707$$

$$F_{21} = 0.707$$

$$F_{12} = \frac{1}{2}$$

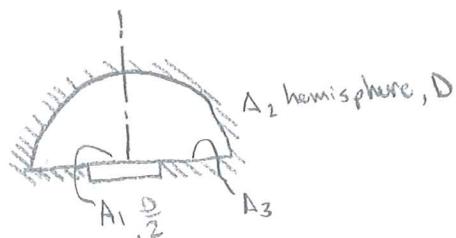
e)



$$F_{13} + F_{12} = 1 \Rightarrow F_{12} = F_{13} = \frac{1}{2}$$

$$F_{21} = \frac{A_1}{A_2} F_{12} \rightarrow 0 \quad F_{21} = 0$$

f)



$$F_{12} = 1 \text{ by obs.} \quad F_{32} = 1$$

$$F_{21} = \frac{A_1}{A_2} F_{12} \quad \frac{\frac{1}{2} (\frac{D}{2})}{\frac{\pi D^2}{2}} =$$

$$F_{21} = 0.125$$

$$F_{23} = \frac{A_3}{A_2} F_{32} = 0.375$$

$$F_{23} = 0.375$$

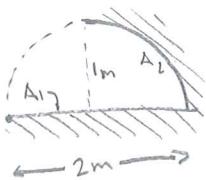
13.1

g)

long, open channel

$$F_{11} + F_{12} + F_{13} = 1$$

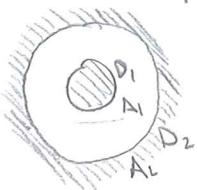
$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2 \times L}{(2\pi l) \times L} = \frac{4}{\pi} \times .50 = 0.637$$



$$\boxed{F_{12} = 0.5}$$

$$\boxed{F_{21} = 0.637}$$

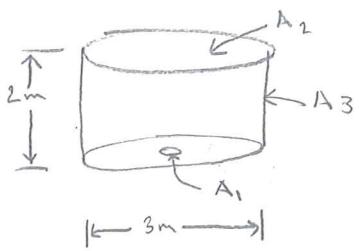
h) concentric cylinders (long)



$$F_{12} = 1$$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D_1}{\pi D_2} (1) \Rightarrow \boxed{F_{21} = \frac{D_1}{D_2}}$$

13.19



$$A_1 \ll A_2 \text{ & } A_3$$

$$F_{12} + F_{13} = 1$$

$$A_1 = .05 \text{ m}^2$$

$$r_1 = \left(\frac{A_1}{\pi} \right)^{1/2} = \left(\frac{0.05 \text{ m}^2}{\pi} \right)^{1/2} = .126$$

$$F_{12} = \frac{D^2}{D^2 + 4L^2} (E913.8) \rho g^{781} \quad F_{12} = 0.36$$

$$F_{13} = 1 - F_{12} = \boxed{0.64}$$

Net radiation H.T. from A_1 to A_3 $T_1 = 1000 \text{ K}$ $T_3 = 500 \text{ K}$ Blank surfaces, $\epsilon = 1$

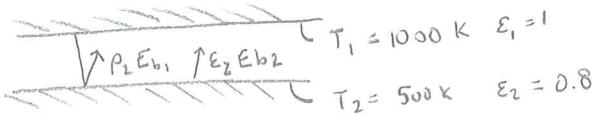
$$q_i = \frac{E_{bi} - \epsilon_i T^4}{(1 - \epsilon_i)/\epsilon_i A_i} \text{ Blank surfaces}$$

$$q_i = A_i F_{13} \sigma (T_1^4 - T_3^4)$$

$$= .05 \text{ m}^2 (0.64) (5.67 \times 10^{-8}) (1000^4 - 500^4) = \boxed{1700 \text{ W}}$$

13.41)

diffuse, gray surfaces



Find:

- irradiation & radiosity for upper plate?
- radiosity for lower plate? (J)
- net radiation exchange?

$$G_1 = \epsilon_2 E_{bb2} + \rho_2 E_{bb1} = \epsilon_2 \sigma T_2^4 + (1-\epsilon_2) \sigma T_1^4$$

$$G_1 = .8(5.67 \times 10^{-8})(500^4) + (0.2)(5.67 \times 10^{-8})(1000^4) = 14175 \text{ W/m}^2$$

G_1 , irradiation =
flux emitted by
surface 2 + reflected
flux emitted by
surface 1

Radiosity = radiant flux leaving the surface by emission & reflection

$$J_1 = E_{bb1} = \sigma T_1^4 = 56,700 \text{ W/m}^2$$

$$J_2 = \epsilon_2 E_{bb2} + \rho_2 G_2 = 0.8(\sigma T_2^4) + (0.2)(\sigma T_1^4) = 14,175 \text{ W/m}^2$$

$\downarrow \quad \downarrow \quad \downarrow$

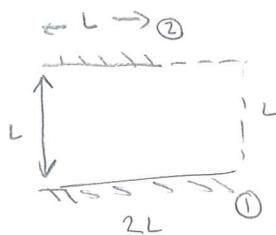
$(\rho = 1 - \epsilon)$

Net radiation exchange = $\frac{56,700 - 14,175}{J_1 - J_2} = 42,525 \text{ towards A}_2 \text{ (W/m}^2\text{)}$

$G_2 = E_{bb1}$ (blackbody)

Exam A

3)



$$F_{11} = \frac{\left((2+2)^2 + 4\right)^{1/2} - (4)^{1/2}}{2(2)} = 0.618$$

$$F_{12} = \frac{0.618}{2} = .309$$

$$F_{21} = \frac{A_2}{A_1} F_{12} = .618$$

ii)

$T_2 = 20^\circ C = 293$
 $\epsilon_2 = 0.1$

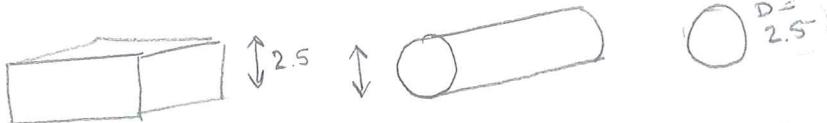
$T_1 = 0^\circ C = 273$
 $\epsilon_1 = 0.8$

$F_{12} = 1$

$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{1}{\frac{\pi(1)}{2}} (1) = .636$

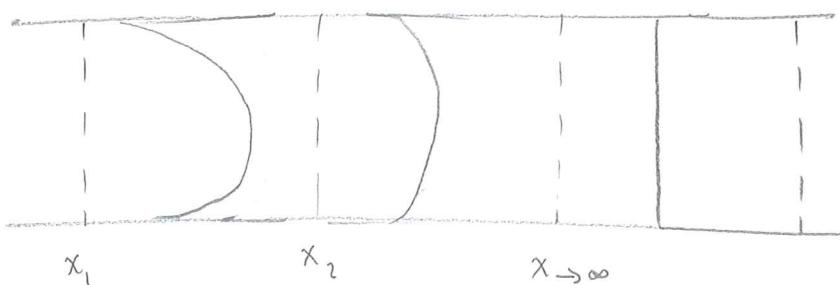
$q_{12} = \frac{\sigma (273^4 - 293^4)}{\frac{0.2}{30(0.8)} + \frac{1}{30} + \frac{0.9}{\pi 15(0.1)}} = -442 W$

4) i)



Sphere will cool fastest because it has the shortest distance for heat to escape in all directions. smallest volume too.

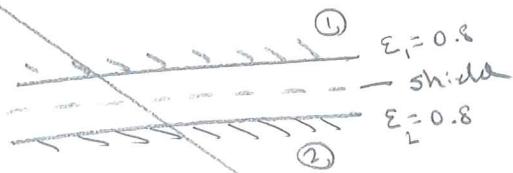
ii)



(2)

13.56

2 large, diffuse, gray surfaces

Find: $\varepsilon_{\text{shield}}$ to
reduce $q'' \times 10$ 

$$q \text{ w/o shield} = 10 q \text{ w/ shield}$$

(p792)

$$\frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon}{\varepsilon} + \frac{1-\varepsilon}{\varepsilon}} = 10 \frac{\sigma(T_1^4 - T_2^4)}{\frac{2}{\varepsilon} + \frac{2-2\varepsilon_s}{\varepsilon_s}}$$

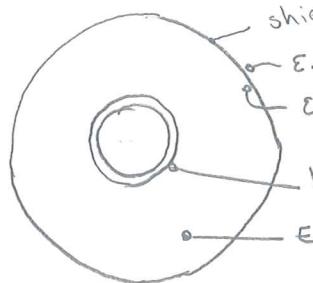
$$2 = \frac{10}{2.5} + \frac{\frac{10\varepsilon_s}{2-2\varepsilon_s}}{2-2\varepsilon_s}$$

$$\frac{1}{2-1.6} = \frac{10}{\frac{2}{.8} + \frac{2-2\varepsilon_s}{\varepsilon_s}}$$

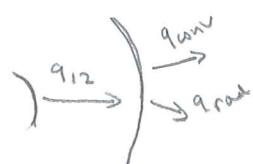
$$\Rightarrow \varepsilon = 0.444$$

13.62

diffuse, gray radiation shield 60mm diam

shield, $D_2 = 60\text{ mm}$ $\epsilon_{2,o} = 0.1$ $\epsilon_{2,i} = 0.01$ heated tube, $D_1 = 20\text{ mm}$ $\epsilon_1 = 1$ (black)

Evaluated

Walls 17°C convection, air $h = 10\text{ W/m}^2\text{K}$ Shield $T = 42^\circ\text{ constant}$ 

$$q_{\text{radiation}} + q_{\text{convection}} = q_{12}$$

$$q_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{1 + \frac{1 - \epsilon_2}{\epsilon_2} \left(\frac{r_1}{r_2} \right)^2} \quad (13.21)$$

$$q_{\text{convection}} = hA$$

$$q_{\text{rad}} = \epsilon_{2,o} A_2 \sigma (T_2^4 - T_{\text{wall}}^4) \quad (13.22), \text{ Pg 793}$$

$$\frac{\sigma D_1 (T_1^4 - T_2^4)}{1 + \frac{1 - \epsilon_2}{\epsilon_2} \left(\frac{D_1}{D_2} \right)^2} = \epsilon_{2,o} \sigma T_s^4 D_2 + h_o (T_s - T_{\text{amb}}) D - \alpha_{2,o} \sigma T_{\text{wall}}^4 D$$

$$3.34 \times 10^{-11} (T_1^4 - T_2^4) = 3.35 + 9 - 2.41$$

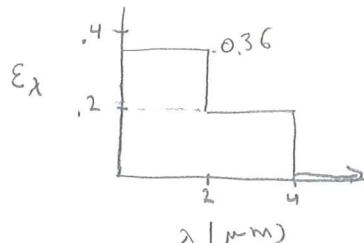
$$T_1 = 745\text{ K}$$

$$\Rightarrow T_1 = 472^\circ\text{C}$$

HW 10

12.32, 46, 51, 109,

12.32



a) find: total hemi emissivity ϵ & total emissive power E @ 2000 K

ϵ = average E_λ over all λ 's

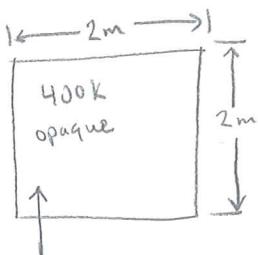
$$\epsilon(T) = F_{(0 \rightarrow 2)} + F_{(2 \rightarrow 4)} = 0.36(0.481) + 0.2(0.856 - 0.481)$$

$$\epsilon(T) = 0.248$$

$$\text{Total emissive power} @ 2000K = \epsilon \sigma T^4 = 0.248(5.67 \times 10^{-8})(2000K)^4$$

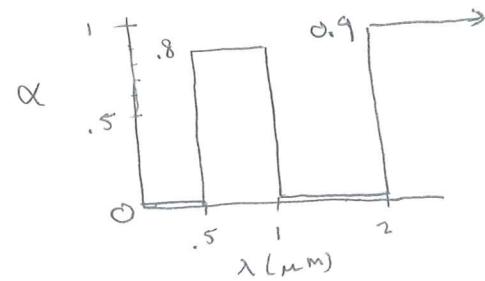
$$E = 225,000 \text{ W}$$

12.46



diffuse

- Find
- absorbed irradiation, G_{abs}
 - emissive power, E
 - radiosity
 - net h.t.



$$G_{solar} = 1200 \text{ W/m}^2 \quad T_{sun} = 5800 \text{ K}$$

$$G_{abs} = [F_{(1.5 \rightarrow 1)} \alpha + F_{(2 \rightarrow \infty)} \alpha] G_{solar} \Rightarrow \alpha = 0.43$$

$$= [0.8(0.720 - 0.250) + 0.9(1 - 0.940)](1200) * \text{Area}$$

$$= 516 \text{ W/m}^2 * 4 \text{ m}^2 = 2064 \text{ W}$$

$$\text{Emissivity } \epsilon = \frac{\int_0^\infty \epsilon_\lambda E_{\lambda b}(400) d\lambda}{E_b(400K)} = (0.8)(F_{0 \rightarrow 0.5}) + (0.9)(F_{2 \rightarrow \infty}) = 0.9(1 - 0) = 0.9$$

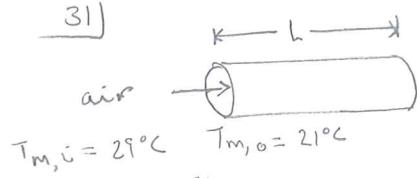
$$\text{Emissive Power, } E = \epsilon \sigma T^4 = 1306 \text{ W/m}^2 \times 4 = 5224 \text{ W}$$

$$J, \text{ radiosity} = E + \rho G = 5224 + (1 - 0.43)(1200)(4 \text{ m}^2) = 7960 \text{ W}$$

$$q_{\text{net, retrans}} = \frac{2064 - 5224}{(G_{abs} - J)} = \frac{3164 \text{ W}}{(0.2064 - 7960)} \text{ (heat leaving surface)}$$

8. 31, 50, 74, 93, 94

31)



Smooth surface

find L

$$Re_D = \frac{U_m D}{\nu} = \frac{141 (1.15)}{15.89} = 13,310 > 2300, \text{ Turbulent}$$

$$\frac{V_i}{A} = \frac{0.025 \text{ m}^3/\text{s}}{\pi (1.15)^2 / 4} = U_m = 1.41 \text{ m/s}$$

$$R_{\text{tot}} = \frac{1}{h_i A} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A}$$

$$(0.267 + 0.133 + 2.12 \times 10^{-5}) \frac{1}{L} = (0.40)$$

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.3} = 45.35 \quad (\bar{h}_i = 7.95 \text{ W/m}^2\text{K})$$

$$(\bar{U} A_s)^{-1} = \frac{1}{L} (R_{\text{tot}})$$

$$\frac{T_{\infty} - T_{m,o}}{T_{\infty} - T_{m,i}} = e^{\left(\frac{-\Delta S}{m c_p} \right) L} \quad \text{where} \quad \bar{U} A_s = \frac{1}{R_{\text{tot}}}$$

$$\ln \left(\frac{17-21}{17-29} \right) = - \frac{1}{R_{\text{tot}}} \left(\frac{1}{m c_p} \right) L$$

$$-1.099 = -(2.5 L) \left(\frac{1}{0.029 (1007)} \right)$$

$$L = 12.84 \text{ m}$$

$$k = 0.15 \text{ W/m}\cdot\text{K}$$

$$D_i = 0.15 \text{ m}$$

$$D_o = 0.17 \text{ m}$$

$$T_{\infty, \text{water}} = 17^\circ\text{C}$$

$$h_o \approx 15,000 \text{ W/m}^2\text{K} @ \text{outer pipe surface}$$

$$v_{\text{Air}, 25^\circ\text{C}} = 15.89 \times 10^{-6} \text{ m}^2/\text{s} \quad Pr = 0.707$$

$$\text{discharge temp } T_{m,o} = 21^\circ\text{C}$$

$$K_{\text{air}} = 26.3 \times 10^{-3} \text{ W/m}\cdot\text{K}$$

$$h_i = 7.95 \text{ W/m}^2\text{K}$$

$$A = \pi D_o L$$

$$Nu_D = 3.66 \Rightarrow h = \frac{Nu_D k_a}{D_o}$$

$$p'f = m R T$$

$$m = p'f = 0.0290 \text{ kg/s}$$

$$\Delta P \cdot f = \frac{f \rho_i U_{m,i}^2}{2 D_i} L \cdot f_i$$

$$f = .316 (13,310)^{-1/4} = .0294$$

$$p'f = .0290$$

$$U_{m,i} = 1.41$$

$$L = 12.84 \text{ m}$$

$$D_i = 0.15 \text{ m}$$

$$\boxed{\text{Power} = .073 \text{ W}}$$

8.50

$$\underline{134a}$$

$$\dot{m} = 1 \text{ kg/s}$$

$$T = 240 \text{ K}$$

$$\underline{\text{Teflon tube}}$$

$$D_i = 0.025 \text{ m}$$

$$D_o = 0.028 \text{ m}$$

$$\underline{\text{Air}}$$

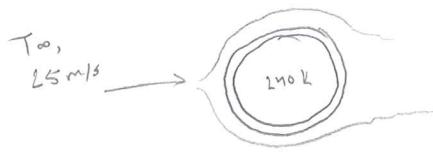
$$V = 25 \text{ m/s}$$

$$T = 300 \text{ K}$$

$$\rho_r = 0.707$$

$$k = 0.0263$$

finding per unit length



$$\dot{q}' = \frac{(T_\infty - T_m)}{R_{tot}}$$

$$Re_{D,i} = \frac{\rho V D}{\mu} \quad (\cancel{\text{Iterations}}) = \frac{4\dot{m}}{\pi D_i \mu} = 12126 \quad \text{Turbulent}$$

$$Nu = 0.23 Re^{0.4} Pr^{0.4} = 80.96$$

$$\bar{h}_i = \frac{Nu \cdot k}{D_i} = 347$$

$$\underline{\bar{h}_o}$$

$$\textcircled{2} \quad Nu_o = C R e^{m} P r^{1/3} \quad \text{pg 431 look up}$$

$$= 0.027 (44,050)^{0.805} (0.707)^{1/3} = 131.7$$

$$\textcircled{1} \quad R_{e,D} = \frac{V D}{\nu} = \frac{25 \text{ m/s} (0.028)}{15.89 \times 10^{-6}} = 44,050$$

$$\textcircled{3} \quad \bar{h}_o = \frac{k \cdot Nu_o}{D} = \frac{0.027 (131.7)}{0.028} = 124 \text{ W/m}^2 \text{K}$$

$$\dot{q}' = \frac{T_\infty - T_m}{\frac{1}{347 \pi \cdot 0.025} + \frac{1}{2 \pi \cdot \frac{0.028}{0.025}} + \frac{1}{124 \pi \cdot 0.028}} =$$

$$\frac{T_\infty - T_m}{\frac{1}{h_i A} + \frac{1}{2 \pi k} + \frac{1}{h_o A}} =$$

$$\underline{134a}$$

$$\underline{\text{Teflon}}$$

$$k = 0.35$$

$$\rho = 1398$$

$$C_p = 1.267$$

$$\mu = 0.042 \times 10^{-2}$$

$$\nu = 13006 \times 10^{-6}$$

$$L = 102$$

$$\rho_c = 5.0$$

$$\frac{T_\infty - T_m}{0.367 + 0.0515 + 0.0917} = \frac{T_\infty - T_m}{0.510} = \frac{300 - 240}{180}$$

$$\boxed{\dot{q}' = 333 \text{ W/m}} \quad \checkmark$$

8.74

Air

$$\dot{m} = 3 \times 10^{-4} \text{ kg/s}$$

$$T = 27^\circ\text{C} = 300\text{K}$$

$$C_p = 1007 \text{ J/kg}\cdot\text{K}$$

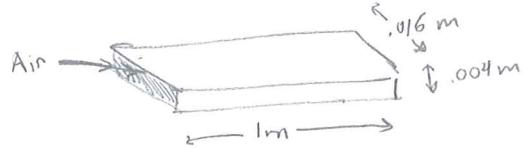
$$\mu = 184.6 \times 10^{-7}$$

$$k = 0.0263 \text{ W/mK}$$

$$\rho_r = 1.07$$

$$D_h = \frac{4A_s}{P} = \frac{4(0.004)(0.016)}{0.04} = 0.0064$$

$$Re_D = \frac{\rho u_m D_h}{\mu} = \frac{\dot{m} D_h}{A_c \mu} = 1625$$



$$q'' = 600 \text{ W/m}^2$$

Temp of air & Surface (at outlet)?

$$q = q'' \times A_s = 600 \times (0.004 \times 1 + 0.016 \times 1) / 2 = 24 \text{ W}$$

$$T_{m,0} = T_{mi} + \frac{q}{\dot{m} C_p} = 379 \text{ K} \quad \checkmark$$

$$T_{s,0} = T_{m,0} + q''/h \quad (8.27) \text{ pg 463}$$

$$h_{\text{local}} = \frac{k \cdot N_u}{D_h} = \frac{0.0263 \cdot 5.33}{0.0064} = 21.9$$

Laminar

$$T_{s,0} = 379 \text{ K} + \frac{600}{21.9} = 406 \text{ K} \quad \checkmark$$

8.93

$$\text{Water, } \dot{m} = 0.2 \text{ kg/s}$$

Tube

Steam

find length
of tube L

$$T_{m,i} = 20^\circ\text{C}$$

$$D_i = 0.025 \text{ m}$$

$$T_{s,i} = 100^\circ\text{C}$$

$$T_{m,o} = 75^\circ\text{C}$$

$$D_o = 0.100 \text{ m}$$

Constant $T_{s,i}$

$$L = -\frac{\dot{m} C_p}{\rho h_L} \ln \left(\frac{T_s - T_{g,m}}{T_s - T_{m,i}} \right)$$

(8.41) pg 471

H₂O
320°C

$$C_p = 4180$$

$$\mu = 577 \times 10^{-6}$$

$$\rho_r = 3.77$$

$$k = 0.640$$

$$\bar{h}_L = \frac{N_{u,i} k}{D_h}$$

$$N_{u,i} = 7.37$$

$$D_h = D_o - D_i = 0.075 \text{ m}$$

$$\bar{h}_L = 62.9$$

$$A_L = \pi \frac{D_o^2}{4} - \pi \frac{D_i^2}{4} = \frac{\pi}{4} (D_o^2 - D_i^2) = 0.00834 \text{ m}^2$$

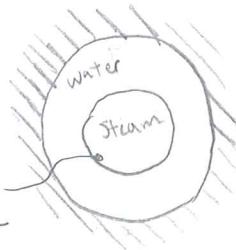
$$Re_D = \frac{\dot{m} D_h}{A_c \mu} =$$

$$Re_D = 353$$

$$L = 19.7 \text{ m}$$



Table 8.2 annular tubes pg 490



8.94] how long must annulus be if $m = 0.3 \text{ kg/s}$?

Water 320°

$$Re_D = 5296 > 2300 \text{ turbulent}$$
$$Pr = 3.77 \quad Nu_D = .023 Re_D^{4/5} Pr^{0.4} = 33.8$$
$$K = .640$$

$$\bar{h} = \frac{K}{D_h} Nu = 318$$

$$L = -\frac{m C_p}{\rho h_f} \ln \left(\frac{T_s - T_{m,i}}{T_s - T_{m,o}} \right)$$

$$\frac{L_2}{L_1} = \frac{.3}{.02} \frac{62.9}{318} \Rightarrow L_2 = 58.5 \text{ m}$$

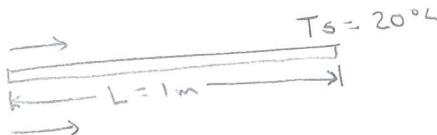
✓

7.2 a, b, c, 11, 20, 45, 73,

$$V_{0.1, 330K} = 96.6 \times 10^{-6} \text{ m}^2/\text{s}$$

Table A.5

7.2) oil 100°C
 0.1 m/s



$$Re_L = \frac{uL}{v} = \frac{0.1 \text{ m/s} (1 \text{ m})}{22 \times 10^{-6} \text{ m}^2/\text{s}} = \frac{1035}{4546} < 5 \times 10^5$$

laminar flow velocity & thermal boundary layer thickness?

a) $\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5(1)}{\sqrt{4546}} = 0.1554 \text{ m}$ $\delta_t = \delta^{1/3} = 0.1554 (1205)^{-1/3}$
 $\delta_t = 0.0146 \text{ m}$

b) local heat flux & surface shear stress @ the trailing edge

$$C_{f,x} = \frac{T_{s,x}}{\rho u_\infty^2/2} = 0.664 Re_x^{-1/2} \quad T_{s,x} = 0.664 \sqrt{\frac{1}{1035}} \left(865.8 \frac{\text{kg}}{\text{m}^2} \right) \left(0.1 \frac{\text{m}}{\text{s}} \right)^2 \frac{1}{2}$$

$$T_s = 0.0894 \text{ N/m}^2$$

$$\Pr = 1205 \text{ (Table A.5)}$$

$$k = 141 \times 10^{-3} \text{ W/m}$$

heat flux $Nu_x = \frac{h_x x}{k}$ $Nu_x = 0.332 Re_x^{1/2} \Pr^{1/3} = 114$

$$h_x = \frac{(141 \times 10^{-3} \text{ W/m})(114)}{(1 \text{ m})} = 16.08 \text{ W/m}^2 \cdot \text{K}$$

$$q'' = h(T_s - T_\infty) = 16.08 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} (20 - 100) =$$

$$-1.286 \frac{\text{kW}}{\text{m}^2}$$

c) Total avg h.t. of plate

$$\bar{N}_{u,x} = 0.664 Re_x^{1/2} \Pr^{1/3} = \frac{\bar{h}_x x}{k} \Rightarrow \bar{h} = 32.05 \text{ W/m}^2 \cdot \text{K}$$

$$\bar{C}_f = 1.328 Re_x^{-1/2}$$

$$q'' = -80/32.05 = 2564 \text{ sides of } 2 \text{ plates}$$

$$\bar{D}' = 2L \left(\frac{\rho u_\infty^2}{2} \right)^{1/2} 1.328 Re_L^{-1/2} = 0.337 \text{ N/m}$$

$$D' = 2L \bar{C}_f$$

✓

a) find velocity & BL thickness @ trailing edge



a) find \bar{h} , q , D

$$L = 2 \text{ m} \times \text{BOTH SIDES}$$

$$W = 2 \text{ m}$$

Air flow, $T_s = 50^\circ \text{C}, 80^\circ \text{C}$

$$\frac{20+50}{2} = 35^\circ$$

$$\frac{20+80}{2} = 40^\circ$$

$$313 \text{ K}$$

$$308 \text{ K}$$

$$\bar{C}_f = 0.074 Re_L^{-1/5} - 1742 Re_L^{-1}$$

$$= 2.35 \times 10^{-3}$$

$$F_{D,1} = \bar{C}_f \frac{\rho U_\infty^2}{2} A_s \Rightarrow F_D = 0.273 \text{ N}$$

For BOTH TEMPERATURES

$$\bar{h}_L = \frac{\bar{N}_{u,L} \cdot k_f}{L}$$

$$\bar{N}_{u,L} = (0.037 Re_L)^{4/5} - 871 \Pr^{1/3} = 660.2$$

$$= \frac{660(0.0263)}{2} \quad \bar{h}_L = 8.679$$

$$q_{\text{conv}} = \bar{h}_L \cdot A_s (T_s - T_\infty)$$

$$q_{50^\circ \text{C}} = 2083 \text{ W} \quad q_{80^\circ \text{C}} = 3471 \text{ W}$$

Air, 300K 1ATM

$$\rho = 1.161 \text{ kg/m}^3$$

$$\nu = 15.89 \times 10^{-6} \text{ m}^2/\text{s}$$

$$k = 0.0263 \text{ W/mK}$$

$$\Pr = 0.706$$

$\text{H}_2\text{O, liquid } 315 \text{ K}$

$$\rho = 991 \text{ kg/m}^3$$

$$\nu = 1584 \text{ m}^2/\text{s}$$

310 .993

1439

$$k = 0.634$$

$$\Pr = 5.83$$

b) $L = 0.1 \text{ m}$ $W = 0.1 \text{ m}$ $A_s = 0.01 \text{ m}^2$

$$Re_1 = \frac{U_\infty L}{\nu} = 6.86 \times 10^5 \quad Re_2 = 9.02 \times 10^5$$

$$\bar{C}_{f,1} = 0.074 Re_1^{-1/5} - 1742 Re_1^{-1} = 2.49 \times 10^{-3} \quad C_{f,2} = 2.84 \times 10^{-3}$$

$$F_{D,1} = \bar{C}_f \frac{\rho U_\infty^2}{2} A_s = 0.620 \text{ N}$$

$$F_{D,2} = 0.700 \text{ N}$$

$$\bar{N}_{u,1} = (0.037 Re_1^{4/5} - 871) \Pr^{1/3}$$

$$\bar{N}_{u,2} = 1952$$

$$\bar{h}_1 = \frac{\bar{N}_{u,1} \cdot k_f}{L}$$

$$\bar{h}_1 = 9050 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \quad q_1 = 5430 \text{ W}$$

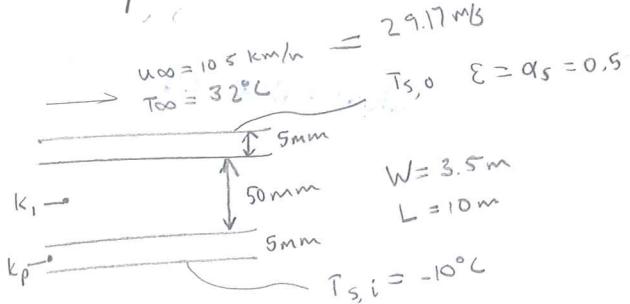
$$\bar{h}_2 = 12,600 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}$$

$$q_2 = 15100 \text{ W}$$

$$q = \bar{h}_1 A_s (T_s - T_\infty)$$

David Mafawey

7.20]



a) find T_{so}

$$\alpha_s G_s + q''_{\text{env}} - E = q''_{\text{cond}} = \frac{T_{so} - T_{si}}{R_{\text{tot}}}$$

$$\alpha_s G_s + \bar{h}(T_{oo} - T_{so}) - \varepsilon \sigma T_{so}^4 = \frac{T_{so} - T_{si}}{2R_p'' + R_i''} \quad R_{\text{tot}} = \frac{h_{oo}L}{\sqrt{h_{oo}}} = 1.84 \times 10^7$$

$$\bar{h} = \frac{h_{oo}}{L} \cdot 0.037 R_{\text{tot}} = 56.2 \text{ W/m}^2\text{K}$$

calculator

$$T_{so} = 307 \text{ K} = 34^\circ C$$

$$q = A \frac{(T_{so} - T_{oo})}{R_{\text{tot}}} \Rightarrow q = 780 \text{ W}$$

b) $\alpha = 0.15 \quad \varepsilon = 0.8$

$$\alpha_s G_s + \bar{h}(T_{oo} - T_{so}) - \varepsilon \sigma T_{so}^4 = \frac{T_{so} - T_{si}}{2R_p'' + R_i''} \Rightarrow T_{so} = 300 \text{ K} = 27^\circ C$$

$$q = 675 \text{ W}$$

c) no insulation

$$R_{\text{tot}}'' = 2 \times (2.78 \times 10^{-5}) = 5.56 \times 10^{-5}$$

$$\bar{h} = 56.2 \text{ W/m}^2\text{K}$$

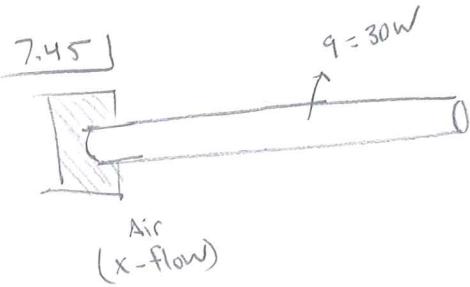
$$\alpha_s = \varepsilon = 0.5$$

$$T_{so} = 263.1 \text{ K}$$

$$q = 90.63 \text{ kW}$$



$$J = 15.89 \times 10^{-6} \frac{\text{m}^2}{\text{s}}$$



$$D = 10\text{mm}$$

$$k = 240 \text{ W/m}^{\circ}\text{C}$$

$$\rho = 2730 \text{ kg/m}^3$$

$$\zeta_p = 900 \text{ J/kg}^{\circ}\text{C}$$

$$R_e = 4000$$

infinitely long pin
Ch 3 slide 24

$$\text{Cylinder, } Re_D = 4 \times 10^5 \quad Nu_D = CR_{ed}^{m} Pr^{1/3}$$

$$\bar{h} \sim D^{-1} D^{0.966} = D^{-0.534} \quad q_{conv} \sim (D^{0.534} \cdot D \cdot D^2)^{1/2}$$

$$q_{conv} = (\bar{h} \cdot k \Delta L)^{1/2} \theta_b \Rightarrow q_{conv} \sim D^{(-0.534 + D \cdot D^2)^{1/2}}$$

$$q_{conv} \sim D^{1.23}$$

$$q_2 = q_1 \left(\frac{D_2}{D_1} \right)^{1.23} = 30W^{(2)}$$

$$q_2 = 70.4W \quad \checkmark$$

7.73

$\uparrow F_D = C_D A_f \frac{\rho V^2}{2}$ (7.42) p401

Cu Sphere

$$\downarrow -F_D (\rho_{Cu} - \rho_{H2O}) g \frac{\pi D^3}{6}$$

$$Cu, 350^{\circ}\text{C}$$

$$\rho = 8933$$

$$k = 398$$

$$\zeta_p = 387$$

$$H_2O 230^{\circ}\text{C}$$

$$\rho = 1000$$

$$\nu = 1422 \times 10^{-6}$$

$$k = 581$$

$$\rho_f = 10.26$$

$$T_s \approx 340$$

$$N_s = \frac{420 \times 10^{-5}}{420 \times 10^{-5}} \text{ N.S/m}^2$$

$$C_D V^2 = \frac{4D}{3} \frac{\rho_{Cu} - \rho_{H2O} g}{\rho} = 2.07 \text{ m}^2/\text{s}^2$$

Fig 7.8 $\rightarrow \sqrt{2-1} \approx 0.46$ $Nu_D = 2 + (0.4 (29.85^0)^{1/8} + 0.06 (29.85^0)^{2/3}) (10.26)^{1/4} \left(\frac{1422}{420} \right)^{1/4} = 439$

$$Re_D = 29580 \quad C_D \approx 0.46$$

$$\bar{h} = \overline{Nu_D} \frac{k}{D} = 12,775 \text{ W/m}^2\text{K}$$

$$Bi = \bar{h} \left(\frac{r_0}{3} \right) / k_{Cu} = 0.11 \quad \text{No lumped method, single term approximation}$$

$$\theta_i^* = C_{1,2} \left(1 - \zeta_i^2 \bar{F}_0 \right) \quad C_{1,2} = 1.0937 \quad \zeta_i = 0.972 \quad 0.5 = 1.094 e^{-0.472 \bar{F}_0} \Rightarrow \bar{F}_0 = 0.87 = \frac{\alpha t_f}{r_0^2}$$

$$\alpha_{Cu} = \frac{k}{\rho c_p} = 1.15 \times 10^{-4} \text{ m}^2/\text{s}$$

$$t_f = 0.76 \text{ s}$$

$$H = t_f \cdot V = 1.6 \text{ m} \quad \checkmark$$

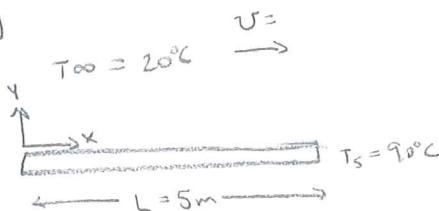
ME225 HW6

David Malawey due 3/1/11

6.8, 15, 18, 32, 35

100

6.8]



$$T(^\circ\text{C})(x) = 20 + 70 e^{-600x/5}$$

find $h(x)$, \bar{h}

$$\frac{dT}{dy} = -70(600) \times e^{-600x/5} \Big|_{y=0} \Rightarrow -42000x$$

$$h = \frac{-k_f \left(\frac{dT}{dy}\right)|_{y=0}}{T_s - T_\infty}$$

$$\frac{dT}{dy} = -70(600) \times e^{-600x/5} \Big|_{y=0} \Rightarrow -42000x$$

$$\bar{T} = \frac{90 - 20}{2} + 273 = 328\text{K} \Rightarrow k_f \approx .03 \text{ W/m}^\circ\text{K}$$

$$h = \frac{-k_f (-42000x)}{90 - 20} = 18 \text{ W/m}^2\text{K} = h$$

$$(300\text{K})$$

$$\text{average coefficient, } \bar{h}: \quad = \frac{1}{L} \int_0^L h dx = \frac{18}{5} \int_0^5 x dx = \frac{18}{5} \left[\frac{x^2}{2} \right]_0^5 = 45 \text{ W/m}^2\text{K} = \bar{h}$$

6.15] $v_{max} = 50 \text{ m/s}$ a) find L_{min}

$$P = 1 \text{ ATM}$$

$$T = 25^\circ\text{C}$$

$$Re_x = 10^3$$

$$Re_{x,c} = \frac{\rho u_{\infty} x_c}{\mu} = \frac{u_{\infty} x_c}{V}$$

$$x_c = \frac{108 (15.89 \times 10^{-6})}{50 \text{ m/s}}$$

$$L_{min} = 31.8 \text{ m}$$

$$PV = RT$$

$$P = \frac{P}{RT} = \frac{101 \text{ kPa}}{8.314 (293)} = 0.408 \text{ kg/m}^3$$

$$b) Re_{x,c} = 5 \times 10^5 \Rightarrow 31.8 \left(\frac{5 \times 10^5}{10^8} \right) = 0.159 \text{ m} \quad x_c \checkmark$$

$$S(20) = 1(100) \Rightarrow Re_{L1} = Re_{L2}^2$$

6.18]

$$L = 1 \text{ m}$$

$$T_s = 400 \text{ K}$$

$$T_\infty = 300 \text{ K}$$

$$V = 100 \text{ m/s}$$

$$\bar{q}'' = 20,000 \text{ W/m}^2$$

$$L_2 = 5 \text{ m}$$

$$T_s = 400 \text{ K}$$

$$T_\infty = 300 \text{ K}$$

$$\sqrt{= 20 \text{ m/s}}$$

$$Re_L = \frac{VL}{\nu}$$

$$h_1 = \frac{q_{conv}}{A_s (T_s - T_\infty)} = \frac{20,000}{100} = 200 \text{ W/m}^2\text{K}$$

$$\bar{h}_2 = \frac{1}{A_2} \left(\frac{1}{1} \right) = (200 \text{ W/m}^2\text{K}) \cdot \frac{q_{conv}}{(T_s - 5A_2)} \Rightarrow 40 \text{ W/m}^2\text{K} \quad \bar{h}_2$$

6.32]

$$L = 800 \text{ mm}$$

$$t = 6 \text{ mm}$$

$$\sqrt{20 \text{ mph}} = 31.29$$

$$T_{\infty} = -15^{\circ}\text{C}$$

$$\text{Table A3: } k_f = 1.4 \text{ W/m}\cdot\text{K}$$

$$\text{Model: } \overline{Nu}_L = 0.030 Re_L^{0.8} Pr^{1/3}$$

$$Re_L = \frac{VL}{\nu}$$

$$k_{air} = 0.023 \text{ W/m}\cdot\text{K}$$

$$\nu = 12.5 \times 10^{-6} \text{ m}^2/\text{s}$$

$$Pr = 0.71$$

$$T_{DP} = 10^{\circ}\text{C}$$

$$T_{\infty,i} = 50^{\circ}\text{C}$$

$$\overline{h}_i \text{ minimum} = ?$$

$$R_{tot} = \frac{1}{\overline{h}_i} + \frac{t}{k_f} + \frac{1}{\overline{h}_o}$$

$$\frac{T_{\infty,i} - T_{\infty}}{\overline{h}_i} = \frac{T_i - T_{\infty,i}}{\frac{t}{k_f} + \frac{1}{\overline{h}_o}}$$

$$\frac{50 - 10}{\overline{h}_i} = \frac{10 - (-15)}{\frac{0.006}{1.4} + \frac{1}{\overline{h}_o}}$$

$$Re_L = \frac{VL}{\nu} = \frac{31.29 \text{ m/s} (0.8)}{12.5 \times 10^{-6} \text{ m}^2/\text{s}}$$
$$= 2.00 \times 10^6$$

$$\overline{h}_o = ? \quad Nu = \frac{hL}{k_f} = 0.030 (Re_L)^{0.8} Pr^{1/3}$$
$$= 2940$$

$$\Rightarrow \overline{h}_o = 84.5$$

$$q_o(\overline{h}_i) = \frac{25}{0.00429 + 0.018}$$

$$\boxed{\overline{h}_{i,\min} = 388 \text{ W/m}^2\cdot\text{K}}$$

$$Pr = 0.71$$

6.35]

$$V_{\infty} = 15 \text{ m/s}$$

$$T_{\infty} = 15^{\circ}\text{C}$$

$$T_s = 140^{\circ}\text{C}$$

$$A_s = 0.25 \text{ m}^2$$

$$\text{drag force } 0.25 \text{ N}$$

$$\text{power needed} = ?$$

$$Pr = 0.707$$

$$q_{in} = q_{out} = h A_s (T_s - T_{\infty})$$

$$\overline{h}_o = \frac{C_F}{2} Pr^{-2/3} \rho u_{\infty} C_P$$

$$\overline{h}_o = \frac{T_s - T_{\infty}}{\frac{C_F \rho u_{\infty} C_P}{2} Pr^{-2/3}} = \frac{T_s}{u_{\infty}} C_P Pr^{-2/3}$$

$$\overline{h}_o = \frac{F_D C_P}{A u_{\infty} Pr^{2/3}} = \frac{0.25}{0.25} \frac{1009}{15} (0.707)^{-2/3}$$

$$\overline{h}_o = 84.8 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}$$

$$q = \frac{T - T_{\infty}}{\frac{1}{\overline{h}}} (A) = 85 ((40 - 15)(0.25)) = \boxed{2.66 \text{ kW}}$$

David Malawey 225 HW#4

100

3.116, 129, 134b, 145 adiabatic tip

$$3.116) \quad k = 20 \text{ W/m}\cdot\text{K} \quad T_{\infty} = 1200^\circ\text{C}$$

$$L = 50 \text{ mm} \quad h = 250 \text{ W/m}^2\cdot\text{K}$$

$$A_c = 6 \times 10^{-4} \text{ m}^2 \quad \text{Perimeter } P = 110 \text{ mm} \quad T_{\text{base}} = 300^\circ\text{C} \quad \text{find } T_{\text{tip}}$$



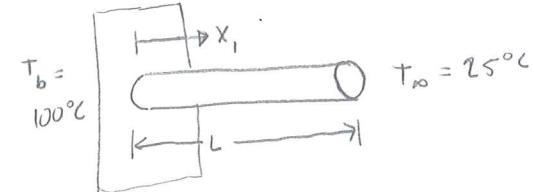
$$\frac{\Theta}{\Theta_b} = \frac{T - T_{\infty}}{T_b - T_{\infty}} = \frac{\cosh m(L_c - x)}{\cosh m L_c} = \frac{1}{5.52} = .1811 \Rightarrow T = 1037^\circ\text{C}$$

a) $M = \sqrt{hP/kA_c} = 47.87$ 1037°C < 1050. Yes, cooking is satisfactory

b) $q_f = M \tanh(mL_c) =$

$$M = \sqrt{hPKA_c\Theta_b} = -517.0$$

$$q_f = -508 \text{ W}$$



3.129) $k_A = 200 \text{ W/m}\cdot\text{K}$ find k_B

$$\left. \begin{array}{l} T_A = 75^\circ\text{C} \\ T_B = 60^\circ\text{C} \end{array} \right\} \text{at } x_1$$

$$\text{Temp. dist.} \quad \frac{T - T_{\infty}}{T_b - T_{\infty}} = e^{-mx, A} = 0.667 \quad -mx, A = -0.055$$

$$e^{-mx, B} = 0.533 \quad -mx, B = -.7621$$

$$\frac{-0.055}{-.7621} = \frac{-m_A L}{-m_B L} = \sqrt{\frac{k_B}{k_A}} \Rightarrow k_B = 2831 \text{ K} \quad \boxed{k_B = 56.62 \text{ W/m}\cdot\text{K}}$$

225 HW 4

134) b)

$$T_{max} = 75^\circ C$$

4x4 array, Cu pins

$$q_c = ?$$

$$T_c = 75^\circ C$$

$$D_p = .0015 \text{ m}$$

$$L_p = 15 \text{ mm}$$

Conditions, 3.27

$$h_o = 1000 \text{ W/m}^2 \cdot K$$

$$k_{Cu} = 400 \text{ W/m.K}$$

$$T_{\infty,o} = 20^\circ C$$

$$r_{chip/board} = 10^{-4} \text{ m}^2 \cdot K/W$$

$$L_{board} = 5 \text{ mm}$$

$$k_b = 1 \text{ W/m.K}$$

$$\text{and } h_i = 40 \text{ W/m}^2 \cdot K \quad T_{\infty,i} = 20^\circ C$$

$$q = N \eta_f h_{fined} A_f \theta_b + h_{unfined} A_{unfined} \theta_b$$

$$(16)(.6768)(1000)(1.77 \times 10^{-6})(75 - 20) \\ 7.25 \times 10^{-5}$$

$$= 43.2 \text{ W}$$

$$+ (1000)(1.33 \times 10^{-4})(75 - 20) = 7.3 \text{ W}$$

$$q_{total} = 50.5 \text{ W}$$



$$145) (L_c)^{\eta_f} h / k A_p = 0.11$$

$$q_f = \eta_f h A_f \theta_b =$$

$$= 2\pi(0.11)(40 \text{ W/m}^2 \cdot K)[0.042^2 - 0.025^2](180)$$

$$a) q_f = 50 \text{ W}$$

$$q_f = \frac{q_f}{h A_{c,b} \theta_b} = 11.05$$

$$b) \bar{q} = N' q_f + h(1-N') (2\pi r_c) \theta_b$$

$$\bar{q} = 6820 \text{ W/m}$$



Conditions, 3.27

$$h_o = 1000 \text{ W/m}^2 \cdot K$$

$$k_{Cu} = 400 \text{ W/m.K}$$

$$T_{\infty,o} = 20^\circ C$$

$$r_{chip/board} = 10^{-4} \text{ m}^2 \cdot K/W$$

$$L_{board} = 5 \text{ mm}$$

$$k_b = 1 \text{ W/m.K}$$

$$\text{and } h_i = 40 \text{ W/m}^2 \cdot K \quad T_{\infty,i} = 20^\circ C$$

$$\eta_f = \frac{\tanh ml_c}{ml_c} = .6768$$

$$m = \sqrt{h_p/k_{Ac}} = 81.65 \text{ m}^{-1}$$

$$L_c = L + \frac{D}{4} = .01538$$

$$A_f = \frac{\pi}{4} D^2 (N)$$

$$A_{unfined} = \left[(12.7 \times 10^{-3})^2 - 16 \left(\frac{\pi}{4} \right) (0.015)^2 \right] \\ = 1.33 \times 10^{-4} \text{ m}^2$$

$$r_{2c} = r_2 + \frac{t}{2} = .042 \text{ m}$$

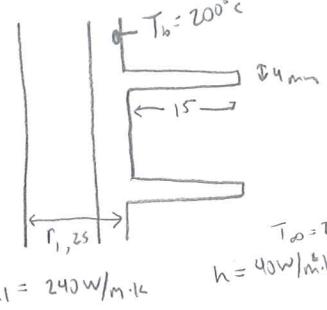
$$r_{2c}/r_1 = 1.68$$

$$L_c = L + t/2 = .017 \text{ m}$$

$$A_p = L_c t = 6.8 \times 10^{-5} \text{ m}^2$$

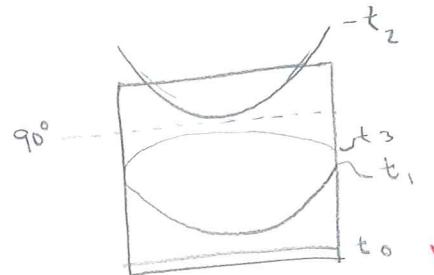
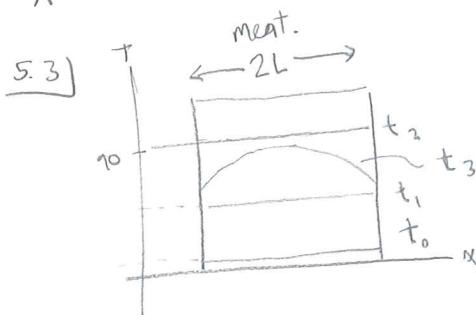
$$k_{Al} = 240 \text{ W/m.K}$$

$$h = 40 \text{ W/m}^2 \cdot K$$



100

David Malawey 225 HW 5

5
5.3, 11, 43(a,b) 57,

assuming constant specific heat

$$T_{75\%} = 456^\circ C \quad \checkmark$$

microwave

$$5.11) T_\infty = 600^\circ C$$

$$h = 100 \text{ W/m}^2 \cdot K$$

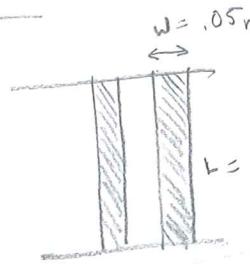
- Time to achieve 75% of capacity?

- Temp of aluminum @ 75%?

$$R_{\text{conduction}} = \frac{L}{kA} = \frac{0.025}{237 \text{ W/m}\cdot\text{K}}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hAt}{\rho + c_p}}$$

$$t = 96.8 \text{ s}$$



$$k = 237 \text{ W/m}\cdot\text{K}$$

$$T_i = 25^\circ C$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{100 \text{ W/m}^2 \cdot \text{K}}$$

$$Bi = \frac{\frac{0.025}{237 \text{ W/m}\cdot\text{K}}}{\frac{1}{100 \text{ W/m}^2 \cdot \text{K}}} = 0.011$$

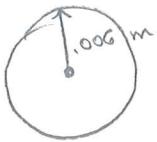
$$t = \frac{\rho + c_p}{hA_s} \ln(0.75) = \frac{2702(1033)^{1/2}}{100 \text{ W/m}^2 \cdot \text{K} \cdot A} \ln(0.25)$$

$$\frac{A}{A_s} = \frac{L}{w} = \frac{0.025}{0.05} = 0.025 \text{ m}$$

$$\frac{456 - 600}{25 - 600} = 0.25$$

5.5)

sphere



$$T_i = 1150 \text{ K}$$

$$T_f = 400 \text{ K}$$

$$T_\infty = 325 \text{ K}$$

$$h = 20 \text{ W/m}^2\text{K}$$

$$\rho = 7800 \text{ kg/m}^3$$

$$C = 600 \text{ J/kg.K}$$

$$k = 40 \text{ W/m.K}$$

$$\text{find } Bi \# = \frac{hL}{k} = \frac{20 \frac{\text{W}}{\text{m}^2\text{K}} \left(\frac{0.006}{3}\right)^2}{40 \frac{\text{W}}{\text{m.K}}} = .001$$

Time required for cooling?

$$t = \frac{\rho \pi C p}{h A s} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

$$= \frac{7800 \left(\pi \left(\frac{0.006}{3}\right)^2\right) 600}{20 \pi \left(0.012\right)^2} \ln \left(\frac{1150 - 325}{400 - 325}\right)$$

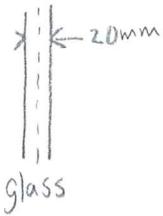
$$t = 1122 \text{ seconds}$$

5.43)

$$\text{diffusivity } \alpha = 6 \times 10^{-7} \text{ m}^2/\text{s}$$

$T_i \rightarrow T_s$ suddenly

$$L = .01 \text{ m}$$

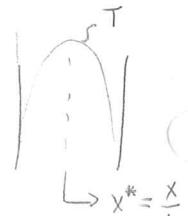


$$Bi = \frac{hL}{k} \Big|_{h \rightarrow \infty} \rightarrow Bi = 0$$

$$-k \frac{\partial T}{\partial x} \Big|_{x=L} = h [T(L,t) - T_\infty]$$

Weisler charts, $\theta^* = \frac{T_\infty - T(x)}{T_i - T_\infty} = 0.5$ (to reach 50% of temp. reduction)

$$F_0 = 0.375 = \frac{\alpha t}{L^2} \Rightarrow t = \frac{37.5 (0.01)^2}{6 \times 10^{-7}} \Rightarrow t = 62.5 \text{ s}$$



b) $T_i - T_s = 300^\circ\text{C}$, find max temp gradient

$$\theta^* = C_1 e^{-\zeta_1 x^*} \cos(\zeta_1 x^*) \quad \frac{\partial \theta^*}{\partial x^*} = C_1 e^{-\zeta_1 x^*} \left(-\zeta_1^2 F_0 \right) - \zeta_1 \sin(\zeta_1 x^*)$$

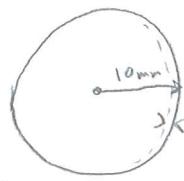
$$\zeta_1 = (\text{append. B3}) 1.5708 \quad C_1 = 1.2733 \quad (\text{table 5.1}) \quad x^* = 1 \quad \frac{\partial \theta^*}{\partial x^*} \Big|_{x^*=1} = -0.793$$

$$\frac{\partial \theta^*}{\partial x^*} = \frac{\partial}{\partial \left(\frac{x}{L}\right)} \left(\frac{T - T_\infty}{T_i - T_\infty} \right) = \frac{L}{T_i - T_\infty} \frac{\partial T}{\partial x} \Rightarrow \frac{\partial T}{\partial x} \Big|_{x=L} = \frac{T_i - T_\infty}{L} \frac{\partial \theta^*}{\partial x^*} \Big|_{x^*=1} = \frac{300}{0.01} (-0.793)$$

$$\frac{\partial T}{\partial x} = -23560 \frac{^\circ\text{C}}{\text{m}}$$

David Malawey

5.57/



$$\alpha = \frac{k}{\rho h} = 1.28 \times 10^{-5}$$

$$c = 600 \text{ J/kg}\cdot\text{K}$$

$$\rho = 7830 \text{ kg/m}^3$$

$$k = 50 \text{ W/m}\cdot\text{K}$$

$$T_{\infty} = 1300 \text{ K}$$

$$h = 5000 \text{ W/m}^2$$

$$T_i = 300 \text{ K}$$

$$\text{surface } \frac{T - T_{\infty}}{T_0 - T_{\infty}} = 0.6 = \frac{1000 - 1300}{T_0 - 1300} \Rightarrow T_0 = 800$$

$$\text{Chart: } Bi^{-1} = 1.0 \quad \theta_o^* = 0.5 \quad F_o = 0.4 \Rightarrow t = \frac{F_o r_o^2}{\alpha} = \frac{0.4 (0.01)^2}{1.28 \times 10^{-5}} = 3.12 \text{ sec}$$

find t for outer 1 mm to reach 1000 K

$$Bi = \frac{k}{hr} = \frac{50}{5000(0.01)} = 1.0$$

$$\frac{r}{r_o} = \frac{q}{10} = 0.90 \Rightarrow \frac{\theta}{\theta_o} = 0.475$$

chart D.8

$$\theta_o^* = \frac{T_0 - T_{\infty}}{T_i - T_{\infty}} = \frac{800 - 1300}{300 - 1300} = 0.5$$

✓

① find Bi , Bi^{-1}

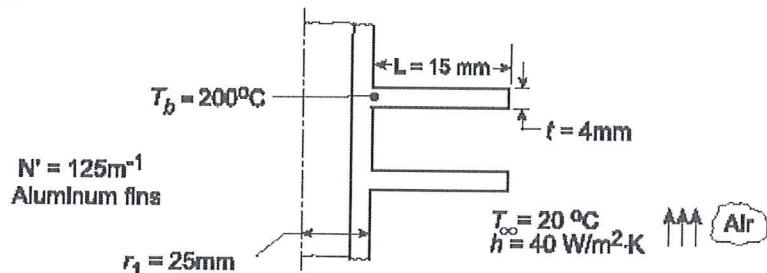
② use chart for surface to find $\frac{\theta}{\theta_o} = 0.6$

PROBLEM 3.145

KNOWN: Dimensions and base temperature of aluminum fins of rectangular profile. Ambient air conditions.

FIND: (a) Fin efficiency and effectiveness, (b) Rate of heat transfer per unit length of tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) Negligible base contact resistance, (6) Uniform convection coefficient.

PROPERTIES: Table A-1, Aluminum, pure ($T \approx 400 \text{ K}$); $k = 240 \text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) The fin parameters for use with Figure 3.19 are

$$r_{2c} = r_2 + \frac{t}{2} = 40 \text{ mm} + 2 \text{ mm} = 0.042 \text{ m} \quad L_c = L + \frac{t}{2} = 15 \text{ mm} + 2 \text{ mm} = 0.017 \text{ m}$$

$$r_{2c}/r_1 = 0.042 \text{ m}/0.025 \text{ m} = 1.68 \quad A_p = L_c t = 0.017 \text{ m} \times 0.004 \text{ m} = 6.8 \times 10^{-5} \text{ m}^2$$

$$\Gamma_c^{3/2} \left(\frac{h}{k A_p} \right)^{1/2} = (0.017 \text{ m})^{3/2} \left[40 \text{ W/m}^2 \cdot \text{K} / 240 \text{ W/m} \cdot \text{K} \times 6.8 \times 10^{-5} \text{ m}^2 \right]^{1/2} = 0.11$$

The fin efficiency is $\eta_f \approx 0.97$. From Eq. 3.86,

$$\eta_f q_{\text{max}} = \eta_f h A_f (\text{ann}) \theta_b = 2\pi \eta_f h \left[\frac{2}{r_{2c}} - \frac{1}{r_1} \right] \theta_b$$

$$\eta_f = 2\pi \times 0.97 \times 40 \text{ W/m}^2 \cdot \text{K} \left[(0.042)^2 - (0.025)^2 \right] \text{m}^2 \times 180^\circ\text{C} = 50 \text{ W} \quad <$$

From Eq. 3.81, the fin effectiveness is

$$\epsilon_f = \frac{q_f}{h A_{e,b} \theta_b} = \frac{50 \text{ W}}{40 \text{ W/m}^2 \cdot \text{K} \cdot 2\pi (0.025 \text{ m})(0.004 \text{ m}) 180^\circ\text{C}} = 11.05 \quad <$$

(b) The rate of heat transfer per unit length is

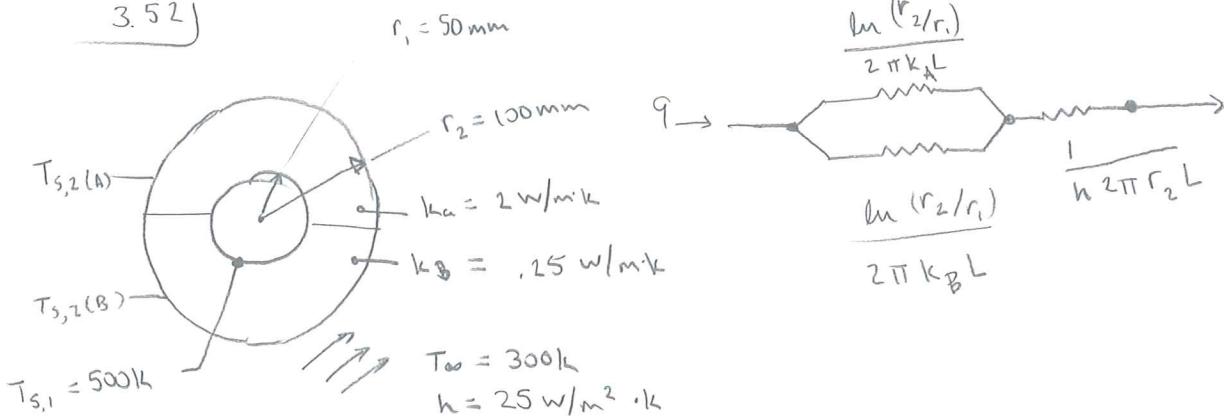
$$q' = N' \eta_f + h(1 - N') (2\pi r_1) \theta_b$$

$$q' = 125 \times 50 \text{ W/m} + 40 \text{ W/m}^2 \cdot \text{K} (1 - 125 \times 0.004) (2\pi \times 0.025 \text{ m}) \times 180^\circ\text{C}$$

$$q' = (6250 + 565) \text{ W/m} = 6.82 \text{ kW/m} \quad <$$

COMMENTS: Note the dominant contribution made by the fins to the total heat transfer.

3.52)



$$\ln \frac{100}{50} = .6931$$

$$\dot{Q}_r = \frac{\Delta T}{R_{\text{tot}}}$$

$$\begin{aligned} \frac{1}{R_{AB}} &= \frac{\frac{1}{\pi k_A L} + \frac{1}{\ln(r_2/r_1)}}{\ln(r_2/r_1)} \\ \frac{1}{R_{AB}} &= \frac{\frac{1}{\pi L (k_A + k_B)}}{\ln(r_2/r_1)} \Rightarrow R_{AB} = \frac{\ln(r_2/r_1)}{\pi L (k_A + k_B)} \end{aligned}$$

$$R_{\text{total}} = \frac{\ln(r_2/r_1)}{\pi L (k_A + k_B)} + \frac{1}{h 2\pi r_2 L} = .1617$$

b)

$$\dot{Q} = \frac{500 - 300}{.1617} = 1236 \text{ Watts/meter}$$

find $T_{s2,A}$ $T_{s2,B}$ *

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{\dot{q}}{k} = 0$$

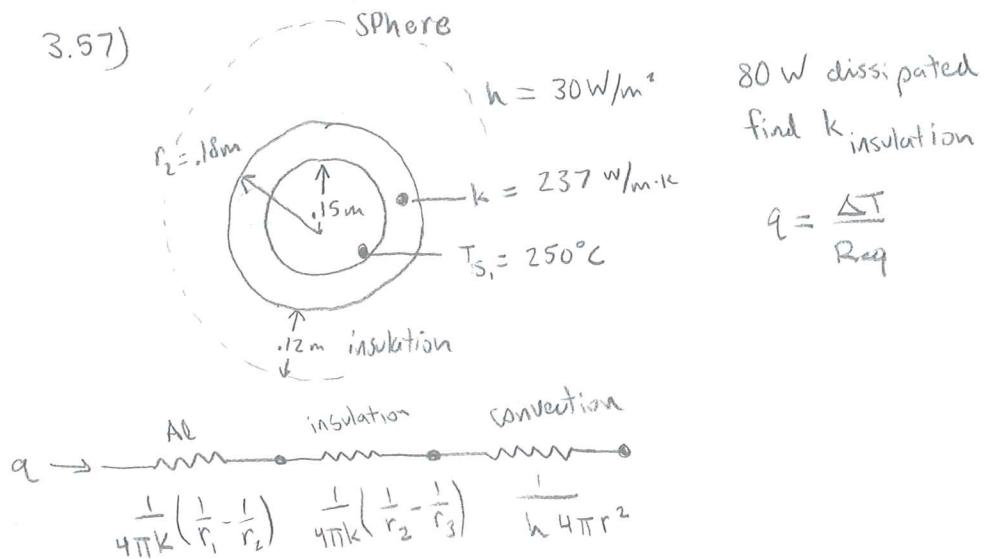
above calculations assume
uniform temperature outer surface.
not realistic.

$$q_{\text{outside surface}} = \dot{q}_{\text{inside}}$$

$$\dot{q} = \frac{T_s - T_\infty}{R} = h 2\pi r L (T_s - T_\infty) = 2\pi (.100) 25 \text{ W/m}^2 (T_s - 300) = 1236 \text{ W/m}$$

$$T_s = 379 \text{ K}$$

3.57)



$$R_{\text{AL}} = 3.731 \times 10^{-4}$$

$$R_{\text{ins.}} = \frac{1}{k} (0.1768)$$

$$R_{\text{air}} = .0295$$

$$80 \text{ W} = \frac{250 - 20}{.0295 + 3.731 \times 10^{-4} + \frac{0.1768}{k}}$$

$$\frac{14.14}{k} = 227.61 \Rightarrow k = .0621 \text{ W/mK}$$

✓

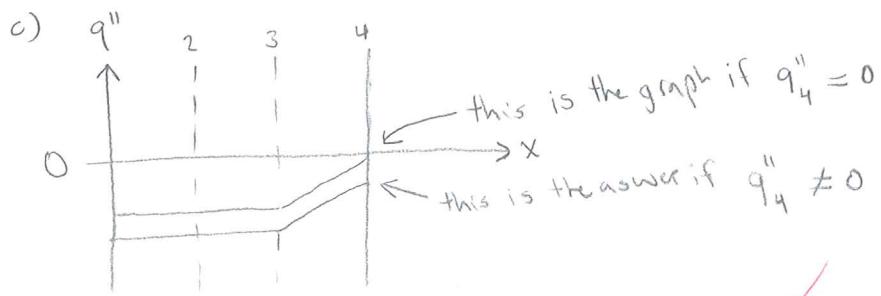
1

3.71) a) $q''_2 = q''_3$: because temp gradients are linear

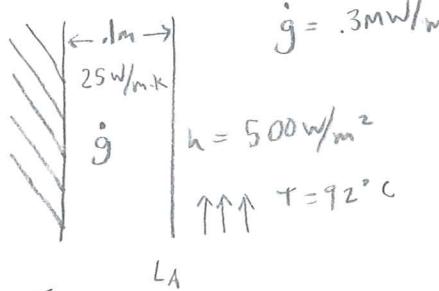
$q''_3 > q''_4$ because the curved temp indicates heat generation in layer "2"

b) $K_A < K_B$ because slope of temp grad. for A is larger

$K_B > K_C$ because slope of temp gradient is less but q'' is equal



3.72



$$\dot{q} = .3 \text{ MW/m}^3$$

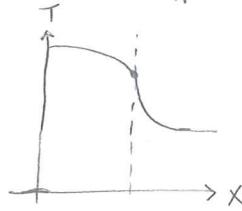
$$h = 500 \text{ W/m}^2$$

determine T_{\max} in wall

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) = -\dot{q}$$

$$k \frac{d^2 T}{dx^2} = -\dot{q} \quad \frac{d^2 T}{dx^2} = -\frac{\dot{q}}{k}$$

$$T = -\frac{\dot{q}}{2k} x^2 + C_1 x + C_2$$



$$\underbrace{\text{B.C.'s}}_{x(0)} \quad \frac{dT}{dx} = 0 = -\frac{\dot{q}}{k} x + C_1 \Rightarrow C_1 = 0$$

at surface $-\dot{q} = \frac{T_{\infty} - T_L}{R}$ $\dot{q}_R - T_{\infty} = -T_L$ $R = \frac{L}{h}$ $T_L = 92 + \frac{3 \times 10^6 (0.1)}{500} =$

$$T_L = 152^\circ\text{C} = -\frac{\dot{q}}{2k} x^2 + C_1 = -\frac{.3 \times 10^6}{2(25)} (.1)^2 + (C_1)$$

$$152^\circ\text{C} + 60 = C_1 = 212$$

$$[T_o = 212^\circ\text{C}]$$

✓ Find $T(r)$

3.94 $\dot{q} = \dot{q}_o [1 - (r/r_o)^2]$

↑ ↑ ↑
local gen. const container radius
rate



use eqn on slide 18 X No.

$$3.94] \quad \dot{g} = \dot{g}_o \left[1 - \left(\frac{r}{r_o} \right)^2 \right] \quad \text{AND} \quad \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) + \frac{\dot{g}}{k} = 0$$

$$\frac{d}{dr} \left(r^2 \frac{dT}{dr} \right) = - \frac{\dot{g}_o}{k} \left[1 - \left(\frac{r}{r_o} \right)^2 \right] r^2 \quad \text{INTEGRATE}$$

$$r^2 \frac{dT}{dr} = \int \left[-\frac{\dot{g}_o r^2}{k} + \frac{\dot{g}_o r^4}{kr_o} \right] = -\frac{\dot{g}_o r^3}{3k} + \frac{\dot{g}_o r^5}{5kr_o} + C_1 \quad \text{DIVIDE BY } r^2, \text{ INTEGRATE}$$

$$T = \int \left[-\frac{\dot{g}_o r}{3k} + \frac{\dot{g}_o r^3}{5kr_o} + \frac{C_1}{r^2} \right] = -\frac{\dot{g}_o r^2}{6k} + \frac{\dot{g}_o r^4}{20kr_o} - \frac{C_1}{r} + C_2$$

BOUNDARY CONDITIONS

$$\textcircled{1} \quad \left. \frac{dT}{dr} \right|_{r=0} = 0$$

$$0 = -\frac{\dot{g}_o r}{3k} + \frac{\dot{g}_o r^3}{5kr_o} + C_1 \Rightarrow C_1 = 0$$

\textcircled{2}

$$-k \left. \frac{dT}{dr} \right|_{r=r_o} = h [T(r_o) - T_\infty]$$

$$h [T(r_o) - T_\infty] = \frac{k \dot{g}_o r_o}{3k} - \frac{\dot{g}_o r_o^3}{5kr_o} + C_0 = \frac{\dot{g}_o r_o}{3} - \frac{\dot{g}_o r_o^2}{5}$$

$$\dot{g}_o \left[\frac{r_o}{3} - \frac{r_o^2}{5} \right] = h \left[-\frac{\dot{g}_o}{k} \left(\frac{r_o^2}{6} - \frac{r_o^2}{20} \right) + C_2 - T_\infty \right]$$

PLUG IN
T(r) & r_o for r's

$$\Rightarrow C_2 = \frac{2r_o \dot{g}_o}{15h} + \frac{7\dot{g}_o r_o^2}{60k} + T_\infty$$

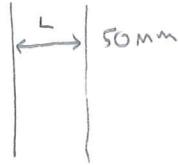
$$T(r) = T_\infty + \frac{2r_o \dot{g}_o}{15h} + \frac{\dot{g}_o r_o^2}{k} \left[\frac{7}{60} - \frac{1}{6} \left(\frac{r}{r_o} \right)^2 + \frac{1}{20} \left(\frac{r}{r_o} \right)^4 \right]$$

2.23, 2.5, 46 3.2, 3.13

2.23) therm. conductivity $K = \frac{50W}{m \cdot K}$

$$T = a + b x^2 \quad a = 200^\circ C \quad b = -2000^\circ C/m^2$$

x (meters)



a) $\dot{q} = ?$ heat eq. $\frac{\partial}{\partial x} (k \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y} (k \frac{\partial T}{\partial y}) + \frac{\partial}{\partial z} (k \frac{\partial T}{\partial z}) + \dot{g} = \rho C_p \frac{\partial T}{\partial t}$

$$\dot{g} = - \frac{\partial}{\partial x} (k \frac{\partial T}{\partial x})$$

$$\dot{g} = - \frac{\partial}{\partial x} (2b k x) = -2bk$$

$$\dot{g}(L) - \dot{g}(0) = -2b k(L) + 2b k(0) = -2(-\frac{2000^\circ C}{m^2})(0.05m)$$

$$\boxed{\dot{q} = 101 kW \frac{m}{m^2}}$$



2.25) $2L = 10 \text{ mm}$ (thickness)

$$K = 5 \frac{W}{m \cdot K}$$

\dot{q} volumetric heat gen rate

What is \dot{q} in wall?

$$\frac{\partial}{\partial x} (k \frac{\partial T}{\partial x}) + \dot{g} = 0$$

$$\frac{\partial}{\partial x} (kb + 2kcx) + \dot{g} = 0$$

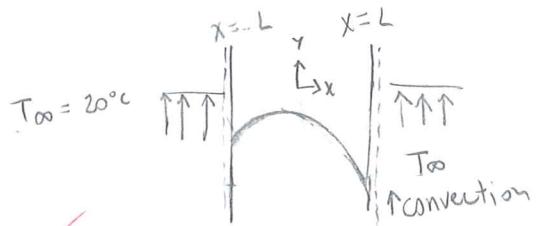
b) $\dot{g} = -2kc = -2 \frac{5W}{m \cdot K} \left(-2 \times 10^4 \frac{^\circ C}{m^2} \right) = \boxed{2 \times 10^5 \frac{W}{m^3}}$

c) $\dot{q}_x(x) = -k \left(\frac{\partial T}{\partial x} \right) = -kb - 2kcx = -[5(-210) + 2(5)(-2 \times 10^4)(0.02m)]$

$$\boxed{\dot{q}_x(L) = +5050 \frac{W}{m^2}}$$

$$\boxed{\dot{q}_x(-L) = -2950 \frac{W}{m^2}}$$

a)



\checkmark

$$\dot{q}_x = a + bx + cx^2 \quad a = 82.0^\circ C \quad b = -210^\circ C/m$$

x -meters

$$T = {}^\circ C \quad c = -2 \times 10^4 {}^\circ C/m^2$$

a) convection coefficients for surfaces?

$$\dot{q}_{\text{conv}, L}'' = \dot{q}_x''(-L) =$$

$$h_1 [T_{\infty} - T(-L)] = h_1 (20 - 78.2)k = -2950 \frac{W}{m^2}$$

$$\boxed{h_1 = \frac{51W}{m^2 \cdot K}}$$

$$h_1 [T(+L) - T_{\infty}] = h_R (69.81 - 20)k = 5050 \frac{W}{m^2}$$

$$\boxed{h_R = 101 \frac{W}{m^2 \cdot K}}$$

$$\begin{aligned} T(-L) &= 82 - 210(-0.02) - 20000(0.02)^2 \\ &= 78.2 \\ T(L) &= 82 - 210(0.02) - 20000(0.02)^2 \\ &= 69.8 \end{aligned}$$

2.25 continued

c) heat flux distribution $\dot{q}_x''(x)$

$$\dot{q}_x''(x) = -k \left(\frac{\partial T}{\partial x} \right) = -k(b + 2cx) = -5(-210 + 2(-2 \times 10^4)x)$$

$$= 1050 + 2 \times 10^5 x$$

$$-1050 = -2(10^5)x \quad x = -\frac{1050}{2(10^5)}$$

flux = 0 @ $x = -5.25 \text{ mm}$

t_{\max} is here ✓

f) $\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \dot{q}'' = \rho C_p \frac{\partial T}{\partial t}$

$$k(2c) = 5 \frac{W}{m \cdot K} \left(-2(10^4) \frac{\text{°C}}{m^2} \right) = \boxed{-2 \times 10^5 \frac{W}{m^3}}$$

g) $\rho = 2600 \text{ kg/m}^3$ will eventually reach $\boxed{20^\circ\text{C}}$

$$C_p = 800 \frac{J}{kg \cdot K} \quad E_{in}'' - E_{out}'' = \Delta E_{st}'' = E_f'' - E_i''$$

$$E_{out}'' = \rho C_p \int_{-L}^L (T_i - T_\infty) dx$$

$$= \rho C_p \int_{-L}^L [a + bx + cx^2 - T_\infty] dx = \rho C_p \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3} - T_\infty x \right]_{-L}^L$$

$$= \rho C_p \left[aL(2) + 2 \frac{CL^3}{3} + -2T_\infty L \right]$$

$$= 2(2600) \frac{kg}{m^3} \frac{800 J}{kg \cdot K} \left[82^\circ\text{C}(.02 \text{ m}) + -2 \times 10^4 \frac{^\circ\text{C}}{m^2} (.02)^3 - 20^\circ\text{C}(.02 \text{ m}) \right]$$

$$= \boxed{4.94 \times 10^6 \frac{J}{m^2}}$$

✓

Daniel Mulvaney

2.46]

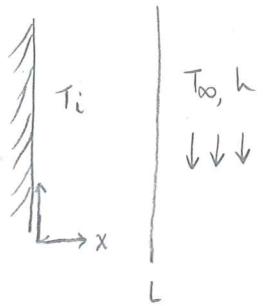
a) $\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) = \rho C_p \frac{\partial T}{\partial t}$

$$k \frac{\partial^2 T}{\partial x^2} = \rho C_p \frac{\partial T}{\partial t}$$

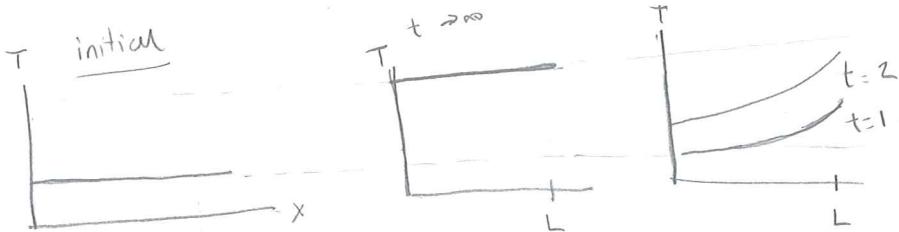
initial conditions $t \leq 0, T_x = T_i$

boundary cond. at $x=0, q''_x(0) = 0 \quad \frac{\partial T}{\partial x} = 0$

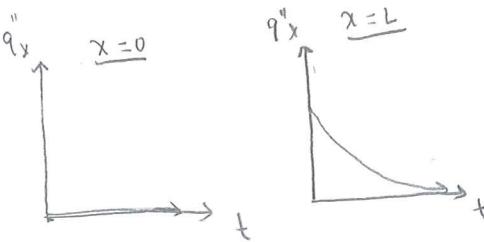
$$\text{at } x=L \quad -k \frac{\partial T}{\partial x} = h [T(L,t) - T_{\infty}]$$



b)



c)



d) Total energy transferred to the wall (J/m^3)

$$E_{in} = \int_0^{\infty} q''_{conv} A_s dt = h A_s \int_0^{\infty} (T_{\infty} - T(L,t)) dt$$

$$\boxed{\frac{E_{in}}{V} = \frac{h}{L} \int_0^{\infty} (T_{\infty} - T(L,t)) dt}$$

✓

3.2]

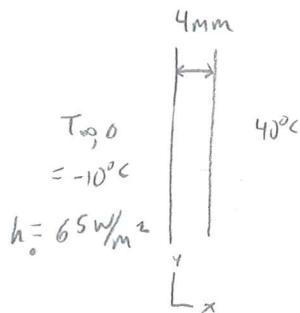
$$T_{\infty,i} = 40^\circ C$$

$$h_i = 30 \text{ W/m}^2 \cdot \text{K}$$

find T of glass surfaces

$$k_{\text{glass}} = 1.4 \text{ W/m} \cdot \text{K}$$

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_o} + \frac{L}{h} + \frac{1}{h_i}} = \frac{50^\circ C}{\frac{1}{65} + \frac{0.04}{30} + \frac{1}{30}} = 969 \frac{\text{W}}{\text{m}^2}$$



$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40 - \frac{969}{30} = 7.7^\circ C \quad \text{inside}$$



$$T_{s,o} = T_{\infty,o} + \frac{q''}{h_o} = -10 + \frac{969}{65} = 4.9^\circ C \quad \text{outside}$$

b)? -5

3.13)

$$\text{a)} R_{\text{tot}} = \sum R_i = \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_i A} + \frac{1}{h_o A}$$

b))

$$q = \frac{20 - (-15)}{R_{\text{tot}}} = \frac{35}{831 \times 10^{-5}} = 4.21 \text{ kW}$$

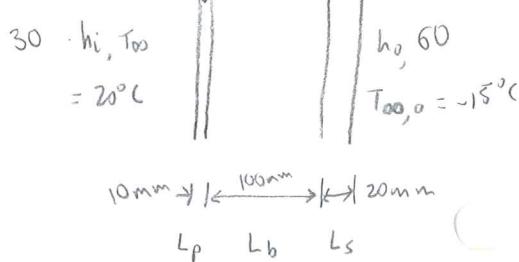
c))

$$R_{\text{tot}} \Rightarrow 826 \times 10^{-5} \Rightarrow q = \uparrow 0.6\% \quad \text{with wind blowing}$$

d))

glass fiber blanket is the controlling resistance,

$$\frac{L_b}{k_b A} = \frac{1}{0.38} = 2.63$$



$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{\dot{a}}{k} = 0$$

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) = - g_0 \left(1 - \left(\frac{r}{r_0} \right)^2 \right)$$

$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) = - \frac{r g_0}{k} + \frac{r g_0}{k} \left(\frac{r^2}{r_0^2} \right) \quad \text{integrate}$$

$$r \frac{dT}{dr} = - \frac{r^2}{2} \frac{g_0}{k} + \frac{g_0}{kr_0^2} \left(\frac{r^4}{4} \right) + C_1 \quad \text{divide by } r \text{ & integrate}$$

$$T(r) = - \frac{r^2}{4} \frac{g_0}{k} + \frac{g_0}{kr_0^2} \frac{r^4}{16} + C_1 \ln r + C_2$$

B.C.'s: $\frac{dT}{dr} \Big|_{r=0} = 0$

$$-\frac{r}{2} \frac{g_0}{k} + \frac{g_0}{kr_0^2} \left(\frac{1}{4} \right) + \frac{C_1}{r} = 0 \Rightarrow C_1 = 0$$

(2) $T(r=r_0) = T_s$

$$-\frac{r^2}{4} \frac{g_0}{k} + \frac{g_0 r^4}{kr_0^2 16} + C_2 = T_s$$

$$-\frac{r_0^2 g_0}{4k} + \frac{g_0 r_0^2}{k 16} + C_2 = T_s \Rightarrow C_2 = T_s + \left(\frac{3}{16} \right) \frac{g_0 r_0^2}{k}$$

$$T(r) = \frac{r^4 g_0}{16 k r_0^2} - \frac{r^2 g_0}{4k} + \left(\frac{3}{16} \right) \frac{g_0 r_0^2}{k} + T_s$$

$$\text{mesh 1} \quad 150 + 20i_1 + 10(i_1 - i_2) = 0$$

$$\text{mesh 2: } 15i_2 - 100 + 10(i_2 - i_1) = 0$$

$$25i_2 - 10i_1 = 100$$

$$2.5i_2 = 10 + i_1$$

heat flux q''_x is the heat transfer rate in x-direction W/m^2

per unit area perpendicular to direction of transfer

q'_x = heat transfer rate per unit length W/m

\dot{q} = rate of energy generation per unit volume W/m^3

Pg 133

OK

Repeated

integration

(correctly)

$$\int \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) \right] dr \Rightarrow r \left(\frac{dT}{dr} \right)$$

$$\int -\dot{q}r = \int -\frac{\dot{q}r^2}{2} + C = -\frac{\dot{q}r^3}{6} + Cr + C_2$$

$$\frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{\dot{q}}{k} = 0$$

$$\frac{1}{r} \left(-\frac{\dot{q}r^3}{6} + Cr + C_2 \right)$$

$$\int \frac{d}{dr} \left(r \frac{dT}{dr} \right) = \int -\frac{\dot{q}r}{k}$$

$$= -\frac{\dot{q}r^2}{6} + C_1 + \frac{C_2}{r}$$

$$\int \left(\frac{dT}{dr} \right) = \left(-\frac{\dot{q}r}{2} + \frac{C_1}{r} \right)$$

$$T(r) = -\frac{\dot{q}r^2}{4} + C_1 \ln r + C_2$$

225 math prob

$$\frac{1}{r} \frac{d}{dr} \left(kr \frac{dT}{dr} \right) + \dot{q} = 0$$

$$u = \frac{1}{r} \quad dv = \frac{d}{dr} \left(kr \frac{dT}{dr} \right)$$

$$du = -r^{-2} \quad v = kr \left(\frac{dT}{dr} \right)$$

$$\int uv - \int v du = \frac{1}{r} \left(kr \left(\frac{dT}{dr} \right) \right) - \int \underbrace{\frac{kr \left(\frac{dT}{dr} \right)}{r^2} dr}_{①}$$

$$① \int \frac{kr \left(\frac{dT}{dr} \right)}{r^2} dr = k \int \frac{1}{r} \left(\frac{dT}{dr} \right) dr$$

$$u = \frac{1}{r} \quad dv = \left(\frac{dT}{dr} \right) dr$$

$$du = -r^{-2} \quad v = T(r) = \left(\frac{1}{r} \right) T(r) - \int \underbrace{\frac{T(r)}{-r^2} dr}_{②}$$

$$② \int \frac{T(r)}{-r^2} dr = \frac{T}{r} + C$$

$$\Rightarrow \frac{1}{r} \left(kr \left(\frac{dT}{dr} \right) \right) - \left[\left(\frac{1}{r} \right) T - \left(\frac{T}{r} + C \right) \right]$$

$$= k \frac{dT}{dr} + \frac{T}{r} - \frac{T}{r} + C$$

$$= k \frac{dT}{dr} + q' r + C$$

David Malawey

100

225 HW #1

P. 1.6, 22, 24, 55, 57

1.6 $w = 1m$

$h = 2m$

5mm thik $k_g = 1.4 \frac{W}{m \cdot K}$

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{(-20 - 15)}{0.005m} \quad q''_x = -k \frac{dT}{dx} = -1.4 \frac{W}{m \cdot K} \left(\frac{-35K}{0.005m} \right)$$

$$q''_x = 9800 \frac{W}{m^2} \quad q_x = 9800 (2m^2) = 19600 W \quad \boxed{19.6 kW}$$

10mm air: $10^\circ C, -15^\circ C \quad 1 \times 2 \text{ m window} \quad k_a = .024 \frac{W}{m \cdot K}$

$$q_x = (.024) \frac{(-15 - 10)}{(0.01)} (2m^2) = \boxed{120 W} \quad \checkmark$$

1.22 $T = 225^\circ C \quad \frac{dT}{dt} = -\frac{0.22K}{s} \quad T_{air} = 25^\circ C \quad \text{plate } 3 \times 3m \quad w = 3.75 \text{ kg}$

find convection coefficient $h (W/m^2 \cdot K)$

$$\dot{Q} = -228.5 \text{ W}$$

$$c_p = 2770 \frac{J}{kg \cdot K}$$

$$\dot{Q} = c_p \frac{dT}{dt} = -\frac{0.22K}{s} (2770) = -60.9 \frac{J}{kg \cdot s} (3.75 \text{ kg})$$

$$q'' = h(T_s - T_\infty) \quad T_s = 225^\circ C \quad T_\infty = 25^\circ C$$

$$h = -\frac{228.5 \text{ W}}{2 \times (3)^2 \text{ m}^2} \left(\frac{1}{200 \text{ K}} \right) = \boxed{6.35 \frac{W}{m^2 \cdot K}} = h \quad \checkmark$$

1.24 $T_{air} = 20^\circ C$ $T_{walls} = 27^\circ C \text{ summer}$ $T_{person} = 32^\circ C$
 $14^\circ C \text{ winter}$

$$h = 2 \frac{W}{m^2 K}$$

$$\dot{q}_{rad}'' = \epsilon \sigma (T_s^4 - T_{sur}^4)$$

$$\dot{q}_{summer}'' = (.9)(5.67 \times 10^{-8}) \frac{W}{m^2 K^4} (305^4 - 300^4) = 28.3 \text{ W/m}^2$$

$$\dot{q}_{winter}'' = (.9)(5.67 \times 10^{-8}) \frac{W}{m^2 K^4} (305^4 - 287^4) = 95.4 \text{ W/m}^2 \checkmark$$

heat is lost from the body at 3.4 times faster in the winter

1.55 $\text{absorb from } 800 \text{ W/m}^2$ $h = 12 \frac{W}{m^2 K}$ $\dot{E}_{in} = \dot{E}_{out}$

a) $T_{air} = 20^\circ C$ $T_{roof} = ? \text{ (steady state)}$ $\dot{q}_{rad}'' - \dot{q}_{cond}'' - \dot{q}_{conv}'' = 0$

$$800 \frac{W}{m^2} - h(T_s - 20^\circ C) = 0$$

$$800 = 12(T_{roof} - 20) \quad \boxed{T_{roof} = 86^\circ C}$$

b) surface emissivity = 0.8

$$\dot{q}_{s,abs}'' A_s - \dot{q}_{conv}'' - \dot{E} \cdot A_s = 0$$

$$\dot{q}_{s,abs}'' A_s - h A_s (T_s - T_{\infty}) - \epsilon A_s \sigma T_s^4 = 0$$

$$800 \frac{W}{m^2} - 12 \frac{W}{m^2 K} (T_s - 293K) - 0.8(5.67 \times 10^{-8}) = 0$$

$$12T_s + 4.536 \times 10^{-8} T_s^4 = 4316 \quad \boxed{T_s = 320K = 47^\circ C} \checkmark$$

c) ?

1.57 $A = 1.8 \text{ m}^2$ 2100 kcal 2000 kcal thermal energy $T_{air} = 20^\circ C$ $h = 3 \frac{W}{m^2 K}$

a) $T_{skin} = ?$ $\epsilon = 0.95$ $\frac{2000 \times 10^3 \text{ cal}}{0.239 \frac{\text{cal}}{\text{J}} (86400 \text{ s/day})} = 96.9 \text{ W}$ $\dot{E}_{in} = \dot{E}_{out}$

$$96.9 \text{ W} = h A (T_s - T_{\infty}) + \epsilon \sigma A (T_s^4 - T_{sur}^4) = 3(1.8)(T_s - 293) + .95(5.67 \times 10^{-8})(1.8)(T_s^4 - 293^4)$$

$$\boxed{T_s = 299K = 26^\circ C}$$

b) $\dot{E}_{out} = m h_f g$ $m = \frac{\dot{E}_{out}}{h_f g} = \frac{96.9 \text{ W}}{2421 \text{ kJ/kg}} = \boxed{4.0 \times 10^{-5} \text{ kg/s}}$
 $= \boxed{4 \times 10^{-5} \text{ kg/s}}$ \checkmark

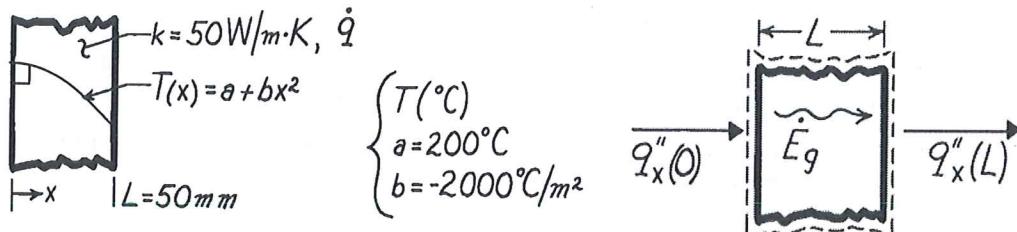
HW 2 Solutions

PROBLEM 2.23

KNOWN: Temperature distribution in a one-dimensional wall with prescribed thickness and thermal conductivity.

FIND: (a) The heat generation rate, \dot{q} , in the wall, (b) Heat fluxes at the wall faces and relation to \dot{q} .

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.19 re-written as

$$\dot{q} = -k \frac{d}{dx} \left[\frac{dT}{dx} \right]$$

Substituting the prescribed temperature distribution,

$$\dot{q} = -k \frac{d}{dx} \left[\frac{d}{dx} (a + bx^2) \right] = -k \frac{d}{dx} [2bx] = -2bk$$

$$\dot{q} = -2(-2000^\circ\text{C}/\text{m}^2) \times 50 \text{ W/m}\cdot\text{K} = 2.0 \times 10^5 \text{ W/m}^3.$$

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x(x) = -k \left. \frac{dT}{dx} \right|_x$$

Using the temperature distribution $T(x)$ to evaluate the gradient, find

$$q''_x(x) = -k \frac{d}{dx} [a + bx^2] = -2kbx.$$

The fluxes at $x = 0$ and $x = L$ are then

$$q''_x(0) = 0$$

$$q''_x(L) = -2kbL = -2 \times 50 \text{ W/m}\cdot\text{K} \left(-2000^\circ\text{C}/\text{m}^2 \right) \times 0.050 \text{ m}$$

$$q''_x(L) = 10,000 \text{ W/m}^2.$$

COMMENTS: From an overall energy balance on the wall, it follows that, for a unit area,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \quad q''_x(0) - q''_x(L) + \dot{q}L = 0$$

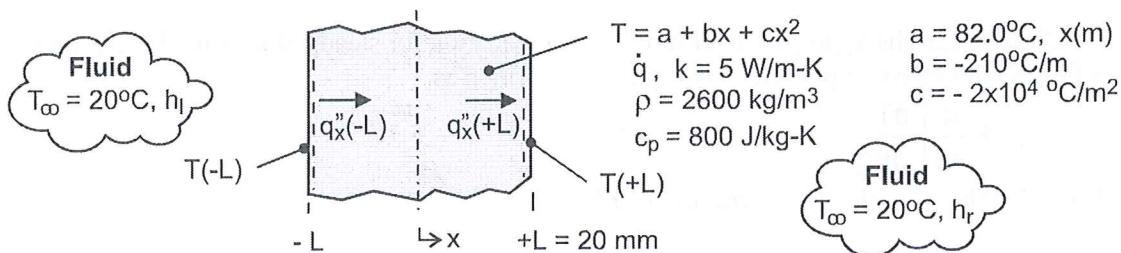
$$\dot{q} = \frac{q''_x(L) - q''_x(0)}{L} = \frac{10,000 \text{ W/m}^2 - 0}{0.050 \text{ m}} = 2.0 \times 10^5 \text{ W/m}^3.$$

PROBLEM 2.25

KNOWN: Analytical expression for the steady-state temperature distribution of a plane wall experiencing uniform volumetric heat generation \dot{q} while convection occurs at both of its surfaces.

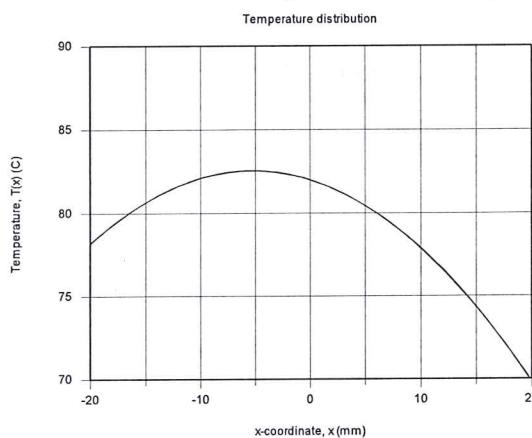
FIND: (a) Sketch the temperature distribution, $T(x)$, and identify significant physical features, (b) Determine \dot{q} , (c) Determine the surface heat fluxes, $q_x''(-L)$ and $q_x''(+L)$; how are these fluxes related to the generation rate; (d) Calculate the convection coefficients at the surfaces $x = L$ and $x = +L$, (e) Obtain an expression for the heat flux distribution, $q_x''(x)$; explain significant features of the distribution; (f) If the source of heat generation is suddenly deactivated ($\dot{q} = 0$), what is the rate of change of energy stored at this instant; (g) Determine the temperature that the wall will reach eventually with $\dot{q} = 0$; determine the energy that must be removed by the fluid per unit area of the wall to reach this state.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform volumetric heat generation, (3) Constant properties.

ANALYSIS: (a) Using the analytical expression in the Workspace of IHT, the temperature distribution appears as shown below. The significant features include (1) parabolic shape, (2) maximum does not occur at the mid-plane, $T(-5.25 \text{ mm}) = 83.3^\circ\text{C}$, (3) the gradient at the $x = +L$ surface is greater than at $x = -L$. Find also that $T(-L) = 78.2^\circ\text{C}$ and $T(+L) = 69.8^\circ\text{C}$ for use in part (d).



(b) Substituting the temperature distribution expression into the appropriate form of the heat diffusion equation, Eq. 2.19, the rate of volumetric heat generation can be determined.

$$\frac{d}{dx} \left(\frac{dT}{dx} \right) + \frac{\dot{q}}{k} = 0 \quad \text{where} \quad T(x) = a + bx + cx^2$$

$$\frac{d}{dx} (0 + b + 2cx) + \frac{\dot{q}}{k} = (0 + 2c) + \frac{\dot{q}}{k} = 0$$

Continued

PROBLEM 2.25 (Cont.)

$$\dot{q} = -2ck = -2 \left(-2 \times 10^4 \text{ } ^\circ\text{C/m}^2 \right) 5 \text{ W/m}\cdot\text{K} = 2 \times 10^5 \text{ W/m}^3 <$$

(c) The heat fluxes at the two boundaries can be determined using Fourier's law and the temperature distribution expression.

$$q''_x(x) = -k \frac{dT}{dx} \quad \text{where} \quad T(x) = a + bx + cx^2$$

$$q''_x(-L) = -k [0 + b + 2cx]_{x=-L} = -[b - 2cL]k$$

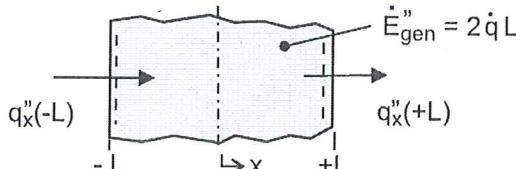
$$q''_x(-L) = - \left[-210 \text{ } ^\circ\text{C/m} - 2 \left(-2 \times 10^4 \text{ } ^\circ\text{C/m}^2 \right) 0.020 \text{ m} \right] \times 5 \text{ W/m}\cdot\text{K} = -2950 \text{ W/m}^2 <$$

$$q''_x(+L) = -(b + 2cL)k = +5050 \text{ W/m}^2 <$$

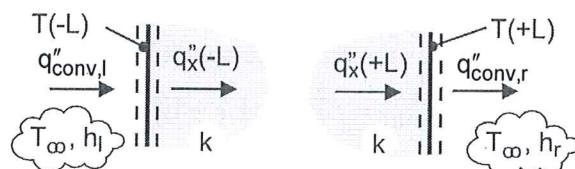
From an overall energy balance on the wall as shown in the sketch below, $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$,

$$+q''_x(-L) - q''_x(+L) + 2\dot{q}L = 0 \quad \text{or} \quad -2950 \text{ W/m}^2 - 5050 \text{ W/m}^2 + 8000 \text{ W/m}^2 = 0$$

where $2\dot{q}L = 2 \times 2 \times 10^5 \text{ W/m}^3 \times 0.020 \text{ m} = 8000 \text{ W/m}^2$, so the equality is satisfied



Part (c) Overall energy balance



Part (d) Surface energy balances

(d) The convection coefficients, h_l and h_r , for the left- and right-hand boundaries ($x = -L$ and $x = +L$, respectively), can be determined from the convection heat fluxes that are equal to the conduction fluxes at the boundaries. See the surface energy balances in the sketch above. See also part (a) result for $T(-L)$ and $T(+L)$.

$$q''_{conv,l} = q''_x(-L)$$

$$h_l [T_\infty - T(-L)] = h_l [20 - 78.2] \text{ K} = -2950 \text{ W/m}^2 \quad h_l = 51 \text{ W/m}^2 \cdot \text{K} <$$

$$q''_{conv,r} = q''_x(+L)$$

$$h_r [T(+L) - T_\infty] = h_r [69.8 - 20] \text{ K} = +5050 \text{ W/m}^2 \quad h_r = 101 \text{ W/m}^2 \cdot \text{K} <$$

(e) The expression for the heat flux distribution can be obtained from Fourier's law with the temperature distribution

$$q''_x(x) = -k \frac{dT}{dx} = -k [0 + b + 2cx]$$

$$q''_x(x) = -5 \text{ W/m}\cdot\text{K} \left[-210 \text{ } ^\circ\text{C/m} + 2 \left(-2 \times 10^4 \text{ } ^\circ\text{C/m}^2 \right) x \right] x = 1050 + 2 \times 10^5 x <$$

Continued

PROBLEM 2.25 (Cont.)

The distribution is linear with the x-coordinate. The maximum temperature will occur at the location where $q''_x(x_{\max}) = 0$,

$$x_{\max} = -\frac{1050 \text{ W/m}^2}{2 \times 10^5 \text{ W/m}^3} = -5.25 \times 10^{-3} \text{ m} = -5.25 \text{ mm} \quad <$$

(f) If the source of the heat generation is suddenly deactivated so that $\dot{q} = 0$, the appropriate form of the heat diffusion equation for the ensuing transient conduction is

$$k \frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) = \rho c_p \frac{\partial T}{\partial t}$$

At the instant this occurs, the temperature distribution is still $T(x) = a + bx + cx^2$. The right-hand term represents the rate of energy storage per unit volume,

$$\dot{E}_{st}'' = k \frac{\partial}{\partial x} [0 + b + 2cx] = k[0 + 2c] = 5 \text{ W/m} \cdot \text{K} \times 2(-2 \times 10^4 \text{ }^\circ\text{C/m}^2) = -2 \times 10^5 \text{ W/m}^3 \quad <$$

(g) With no heat generation, the wall will eventually ($t \rightarrow \infty$) come to equilibrium with the fluid, $T(x, \infty) = T_\infty = 20^\circ\text{C}$. To determine the energy that must be removed from the wall to reach this state, apply the conservation of energy requirement over an interval basis, Eq. 1.11b. The "initial" state is that corresponding to the steady-state temperature distribution, T_i , and the "final" state has $T_f = 20^\circ\text{C}$. We've used T_∞ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \quad \text{with} \quad E''_{in} = 0.$$

$$E''_{out} = c_p \int_{-L}^{+L} (T_i - T_\infty) dx$$

$$E''_{out} = \rho c_p \int_{-L}^{+L} [a + bx + cx^2 - T_\infty] dx = \rho c_p \left[ax + bx^2/2 + cx^3/3 - T_\infty x \right]_{-L}^{+L}$$

$$E''_{out} = \rho c_p \left[2aL + 0 + 2cL^3/3 - 2T_\infty L \right]$$

$$E''_{out} = 2600 \text{ kg/m}^3 \times 800 \text{ J/kg} \cdot \text{K} \left[2 \times 82^\circ\text{C} \times 0.020 \text{ m} + 2(-2 \times 10^4 \text{ }^\circ\text{C/m}^2) (0.020 \text{ m})^3/3 - 2(20^\circ\text{C}) 0.020 \text{ m} \right]$$

$$E''_{out} = 4.94 \times 10^6 \text{ J/m}^2 \quad <$$

COMMENTS: (1) In part (a), note that the temperature gradient is larger at $x = +L$ than at $x = -L$. This is consistent with the results of part (c) in which the conduction heat fluxes are evaluated.

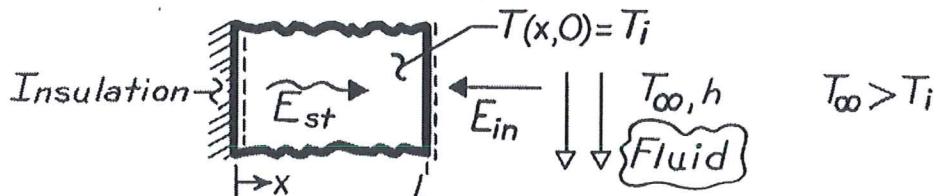
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PROBLEM 2.46

KNOWN: Plane wall, initially at a uniform temperature, is suddenly exposed to convective heating.

FIND: (a) Differential equation and initial and boundary conditions which may be used to find the temperature distribution, $T(x,t)$; (b) Sketch $T(x,t)$ for these conditions: initial ($t \leq 0$), steady-state, $t \rightarrow \infty$, and two intermediate times; (c) Sketch heat fluxes as a function of time for surface locations; (d) Expression for total energy transferred to wall per unit volume (J/m^3).

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

ANALYSIS: (a) For one-dimensional conduction with constant properties, the heat equation has the form,

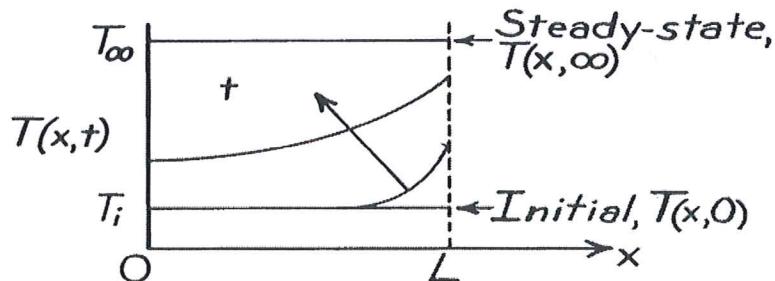
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

and the conditions are:

$$\begin{cases} \text{Initial, } t \leq 0: & T(x,0) = T_i \\ \text{Boundaries: } & x=0 \quad \frac{\partial T}{\partial x}|_0 = 0 \\ & x=L \quad -k \frac{\partial T}{\partial x}|_L = h[T(L,t) - T_{\infty}] \end{cases}$$

uniform
adiabatic
convection

(b) The temperature distributions are shown on the sketch.

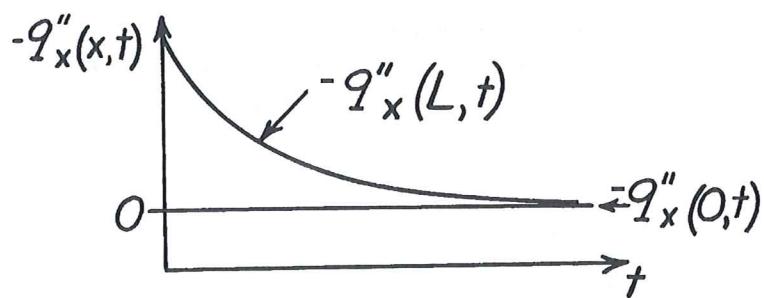


Note that the gradient at $x = 0$ is always zero, since this boundary is adiabatic. Note also that the gradient at $x = L$ decreases with time.

(c) The heat flux, $q''_x(x,t)$, as a function of time, is shown on the sketch for the surfaces $x = 0$ and $x = L$.

Continued

PROBLEM 2.46 (Cont.)



For the surface at $x = 0$, $q''_x(0, t) = 0$ since it is adiabatic. At $x = L$ and $t = 0$, $q''_x(L, 0)$ is a maximum (in magnitude)

$$|q''_x(L, 0)| = h |T(L, 0) - T_{\infty}|$$

where $T(L, 0) = T_i$. The temperature difference, and hence the flux, decreases with time.

(d) The total energy transferred to the wall may be expressed as

$$\begin{aligned} E_{in} &= \int_0^{\infty} q''_{conv} A_S dt \\ E_{in} &= h A_S \int_0^{\infty} (T_{\infty} - T(L, t)) dt \end{aligned}$$

Dividing both sides by $A_S L$, the energy transferred per unit volume is

$$\frac{E_{in}}{V} = \frac{h}{L} \int_0^{\infty} [T_{\infty} - T(L, t)] dt \quad [J/m^3]$$

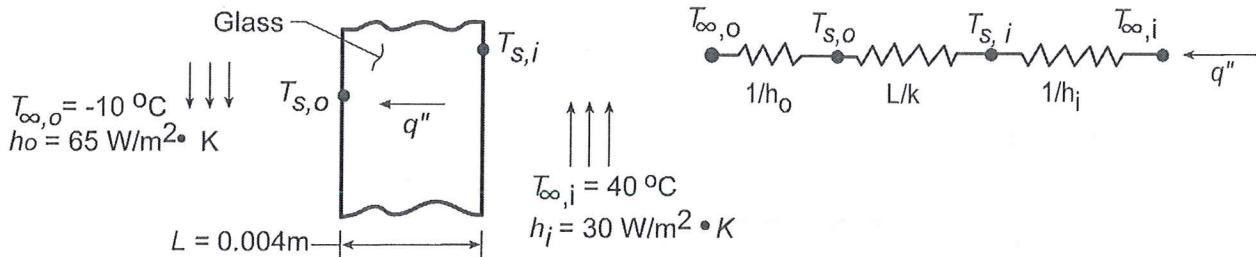
COMMENTS: Note that the heat flux at $x = L$ is into the wall and is hence in the negative x direction.

PROBLEM 3.2

KNOWN: Temperatures and convection coefficients associated with air at the inner and outer surfaces of a rear window.

FIND: (a) Inner and outer window surface temperatures, $T_{s,i}$ and $T_{s,o}$, and (b) $T_{s,i}$ and $T_{s,o}$ as a function of the outside air temperature $T_{\infty,o}$ and for selected values of outer convection coefficient, h_o .

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation effects, (4) Constant properties.

PROPERTIES: Table A-3, Glass (300 K): $k = 1.4 \text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) The heat flux may be obtained from Eqs. 3.11 and 3.12,

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}} = \frac{40^\circ\text{C} - (-10^\circ\text{C})}{\frac{1}{65 \text{ W/m}^2 \cdot \text{K}} + \frac{0.004 \text{ m}}{1.4 \text{ W/m}\cdot\text{K}} + \frac{1}{30 \text{ W/m}^2 \cdot \text{K}}}$$

$$q'' = \frac{50^\circ\text{C}}{(0.0154 + 0.0029 + 0.0333) \text{ m}^2 \cdot \text{K/W}} = 969 \text{ W/m}^2.$$

Hence, with $q'' = h_i (T_{\infty,i} - T_{\infty,o})$, the inner surface temperature is

$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40^\circ\text{C} - \frac{969 \text{ W/m}^2}{30 \text{ W/m}^2 \cdot \text{K}} = 7.7^\circ\text{C}$$

Similarly for the outer surface temperature with $q'' = h_o (T_{s,o} - T_{\infty,o})$ find

$$T_{s,o} = T_{\infty,o} + \frac{q''}{h_o} = -10^\circ\text{C} + \frac{969 \text{ W/m}^2}{65 \text{ W/m}^2 \cdot \text{K}} = 4.9^\circ\text{C}$$

(b) Using the same analysis, $T_{s,i}$ and $T_{s,o}$ have been computed and plotted as a function of the outside air temperature, $T_{\infty,o}$, for outer convection coefficients of $h_o = 2, 65$, and $100 \text{ W/m}^2 \cdot \text{K}$. As expected, $T_{s,i}$ and $T_{s,o}$ are linear with changes in the outside air temperature. The difference between $T_{s,i}$ and $T_{s,o}$ increases with increasing convection coefficient, since the heat flux through the window likewise increases. This difference is larger at lower outside air temperatures for the same reason. Note that with $h_o = 2 \text{ W/m}^2 \cdot \text{K}$, $T_{s,i} - T_{s,o}$ is too small to show on the plot.

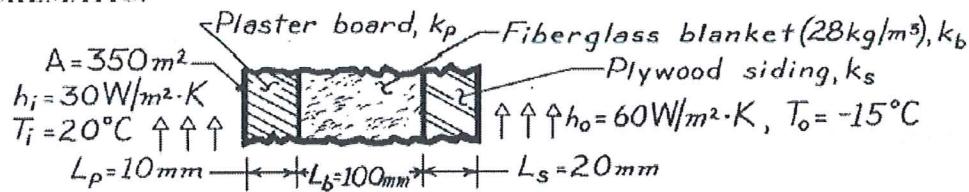
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PROBLEM 3.13

KNOWN: Composite wall of a house with prescribed convection processes at inner and outer surfaces.

FIND: (a) Expression for thermal resistance of house wall, R_{tot} ; (b) Total heat loss, $q(\text{W})$; (c) Effect on heat loss due to increase in outside heat transfer convection coefficient, h_o ; and (d) Controlling resistance for heat loss from house.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Negligible contact resistance.

PROPERTIES: Table 4-3, $(\bar{T} = (T_i + T_o)/2 = (20 - 15)^\circ \text{C}/2 = 2.5^\circ \text{C} \approx 300\text{K})$: Fiberglass blanket, 28 kg/m^3 , $k_b = 0.038 \text{ W/m}\cdot\text{K}$; Plywood siding, $k_s = 0.12 \text{ W/m}\cdot\text{K}$; Plasterboard, $k_p = 0.17 \text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) The expression for the total thermal resistance of the house wall follows from Eq. 3.18.

$$R_{\text{tot}} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_o A}, \quad <$$

(b) The total heat loss through the house wall is

$$q = \Delta T / R_{\text{tot}} = (T_i - T_o) / R_{\text{tot}}.$$

Substituting numerical values, find

$$\begin{aligned} R_{\text{tot}} &= \frac{1}{30 \text{ W/m}^2 \cdot \text{K} \times 350 \text{ m}^2} + \frac{0.01 \text{ m}}{0.17 \text{ W/m} \cdot \text{K} \times 350 \text{ m}^2} + \frac{0.10 \text{ m}}{0.038 \text{ W/m} \cdot \text{K} \times 350 \text{ m}^2} \\ &\quad + \frac{1}{0.12 \text{ W/m} \cdot \text{K} \times 350 \text{ m}^2} + \frac{1}{60 \text{ W/m}^2 \cdot \text{K} \times 350 \text{ m}^2} \\ R_{\text{tot}} &= [9.52 + 16.8 + 752 + 47.6 + 4.76] \times 10^{-5} \text{ }^\circ\text{C/W} = 831 \times 10^{-5} \text{ }^\circ\text{C/W} \end{aligned}$$

The heat loss is then,

$$q = [20 - (-15)] \times 10^{-5} \text{ }^\circ\text{C/W} = 4.21 \text{ kW}, \quad <$$

(c) If h_o changes from 60 to 300 $\text{W/m}^2 \cdot \text{K}$, $R_o = 1/h_o A$ changes from $4.76 \times 10^{-5} \text{ }^\circ\text{C/W}$ to $0.95 \times 10^{-5} \text{ }^\circ\text{C/W}$. This reduces R_{tot} to $826 \times 10^{-5} \text{ }^\circ\text{C/W}$, which is a 0.6% decrease and hence a 0.6% increase in q .

(d) From the expression for R_{tot} in part (b), note that the insulation resistance, $L_b/k_b A$, is $752/830 \approx 90\%$ of the total resistance. Hence, this material layer controls the resistance of the wall. From part (c) note that a 5-fold decrease in the outer convection resistance due to an increase in the wind velocity has a negligible effect on the heat loss.