FS 2010

7% of problems in FE exam

ME 231 Thermo-Fluid Mechanics I (Section 1A)
Fall 2010: Mechanical & Aerospace Engineering Department

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Instructor: Professor Arindam Banerjee
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Jeff has Alofs

Class schedule: Tuesday-Thursday: 9.30 a.m. – 10.45 a.m @ Toomey Hall 295.

Office hours : Wednesday – 4-5 PM.
There will also be a 2 hour recitation period every week (TBD) Mon 6-8

Grading Outline: **Home-Work: 25%, Mid-Term Exams (4): 75%**
(the lowest score will be dropped, i.e. 3 @25% each)

Grading Policy: Final grades will be curved. Tentative breakdown:
B-C: Cutoff at class mean.
A : Above 1 standard deviation from mean
D : Below 1 standard deviation from mean
F : Below 2 standard deviations from mean
Class statistics and grades will be provided ONLY after each exam.

Catalog Description: Principles of viscous and inviscid flow in ducts, nozzles, diffusers, blade passages and application to design; dimensional analysis and laws of similarity; external flows; compressible flows.

Prerequisites: A grade of "C" or better in ME 219

Course objectives: Provide a one-semester treatment of the fluid mechanics topics, which are judged to be most useful to the average mechanical engineer.

187.20 New

Textbook: **White, Frank, Fluid Mechanics, 7th Ed., McGraw-Hill, Inc, 2010** 9780073529349
6th edition will be fine, but find the right prob. #'s

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Administrative Issues:

- All tests will be open notes, open text book.
- Home-works are due at the **start** of class – no late home-works will be accepted.
- There will be 4 mid-term exams. **Tentative** dates are: 9/14, 10/19, 11/11 and 12/3 or 12/7.
- The midterm with the lowest score will be dropped and will not be counted towards the final grade.
- There will NO make-up exam.
- All exams are open book only. You are not allowed your home-works or class notes.

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Date: 8/16/10
Prepared by: Dr. A. Banerjee

8-24-10

ME 231

Absolute derivatives $\quad f(x)$ at $P=(x=x_0)$

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

Partial derivatives : $\quad f(x,y)$ at $P=(x_0, y_0)$

$$\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x, y_0) - f(x_0, y_0)}{\Delta x} = f_x$$

$$\frac{\partial f}{\partial y} = \lim_{\Delta y \to 0} \frac{f(x_0, y_0+\Delta y) - f(x_0, y_0)}{\Delta y} = f_y$$

Mixed derivatives

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) \qquad f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) \qquad f_{yx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

$$f_{xyx} = \frac{\partial}{\partial x}\left[\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\right]$$

☆ note f_{xy} may or may not $= f_{yx}$

ex ① $f(x,y) = x\sin(xy^2)$ Prove $f_{xy} = f_{yx}$

② $f(x,y) = \dfrac{xy(x^2-y^2)}{x^2+y^2}$ if $x,y \neq (0,0)$ & $f(x,y)=0$ if $f(x,y)=(0,0)$

$$\boxed{f_{xy} = -1 \quad f_{yx} = +1}$$

Composite function : chain differentiation

① Let $f[x(t)] = F(t)$ $\quad \dfrac{dF}{dt} = \dfrac{df}{dx}\cdot\dfrac{dx}{dt}$

② $f(x,y) = f(t)$ where $x = x(t)$ & $y = y(t)$

$$\frac{dF}{dt} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$$

③ $r = r(x,y) = \sqrt{x^2+y^2}$ $\quad x = 3t^2 \quad y = \sin t$ $\quad$ only valid in $1 < t < 4$ $\quad$ find dr/dt

$$= \frac{\partial r}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial r}{\partial y}\frac{\partial y}{\partial t} = \text{? (take partial deriv's)}$$

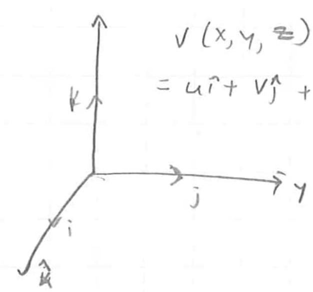
Notes 8-14-10

Vector algebra / calculus in 3d space

Vector: quantity with mag. & direction

- notes on operations, addition, scalar multiplication, dot product
  if $\vec{u}$ & $\vec{v} \neq 0$ & $\vec{u}\cdot\vec{v} = 0$, $\theta = \frac{\pi}{2}$

Coord. systems
- cartesian (x, y, z)
- cylindrical (r, θ, z)
- spherical (r, ϕ, θ)



$V(x,y,z) = u\hat{i} + v\hat{j} + w\hat{k}$

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

① <u>gradient operator</u>: scalar → vector

$$\underbrace{\nabla}_{\text{nabla}}(\ ) = \hat{i}\frac{\partial}{\partial x}(\ ) + \hat{j}\frac{\partial}{\partial y}(\ ) + \hat{k}\frac{\partial}{\partial z}(\ )$$

↑ scalar

② <u>divergence operator</u>: vector → scalar

$$\nabla\cdot\vec{u} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot(u\hat{i} + u\hat{j} + u\hat{k})$$
$$= \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$$

③ <u>curl operator</u>: vector → vector

$$\nabla\times\vec{V} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} - \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}$$

$F(x,y,z) = 2e^x\sin y\,\hat{i} + 3e^x\cos y\,\hat{j} + (4z^2 + x + y)\hat{k}$

Find ⓐ curl (F) ⓑ divergence (F)

} homework due thurs

Summary

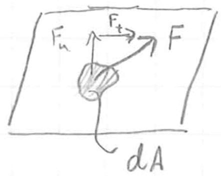
operator	input	output
gradient	scalar	vector
divergence	vector	scalar
curl	vector	vector

10-mins late to class

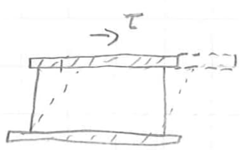
Hydrodynamics (mostly H_2O) — Incompressible fluids ($\rho = const$)
↳ hydraulics – flow of H_2O in pipes & open channels

Gas dynamics — compressible fluids ($\rho \neq const$)
↳ aerodynamics: flow of gases over aircraft, rockets, autos

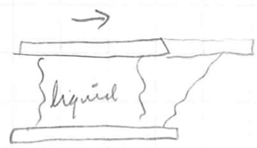
Fluids → liquids
Fluids → gases



normal stress $\sigma = F_n/dA$ (Pressure)
tangential stress $\tau = F_t/dA$ (shear stress)



stress & strain, Solid (proportional)



stress & strain rate (proportional)

solid: can resist an applied load
liquid: cannot resist an applied load

Dimensions & Units

dimension: a measure of physical variable
unit: a way of classifying a # to a dimension

Primary dimensions:

	SI	BG	Conv. factors
Mass [M]	kg	Slug	1 slug = 14.594
Length [L]	m	ft	1 ft = .3048 m
Time [T]	s	s	1s = 1s
Temperature [Θ]	K	R	1 K = 1.8 R

SI basic force unit $1N = 1kg \cdot \frac{1m}{s^2}$

BG $1 lbf = 1 slug \cdot 1 ft/s^2$
$= 32.171\ lbm \cdot 1 ft/s^2$

$\vec{F} = \frac{m \cdot a}{g_c}$; $g_c = 32.174 \ \frac{ft \cdot lbm}{s^2 \cdot lbf}$

in-class problems

1) body 1000 lbf @ $g = 32.174\ ft/s^2$

$1000 = m(32.174)$ $m = 31.08$ slugs

b) 31.08 slugs = 453.58 kg

Principle of dimensional homogeneity: classical definition: every additive term in an eq. must have the same dimension. Layman def: can't add apples & oranges

Bernoulli's eq'n $P + \frac{1}{2}\rho V^2 + \rho g z = \text{constant}$

$[P]$ = pressure, = normal force/area $= \left[\frac{\text{Mass} \times \text{accl'n}}{\text{area}}\right] = \left[\frac{M \times \frac{L}{T^2}}{L^2}\right] = [M L^{-1} T^{-2}]$

$[\frac{1}{2}\rho V^2] = [\text{density} * \text{velocity}^2] = \frac{\text{Mass}}{\forall}(V^2)$ $\left[\frac{M}{L^3} * \frac{L^2}{T^2}\right]$ $[M L^{-1} T^{-2}]$

$[\rho g z] = [\text{density} * \text{accl'n} * \text{Length}] = [M L^{-1} T^{-2}]$

Set #1, p2. $V = \frac{1.49}{n} R^{2/3} S^{1/2}$: R = hydraulic radius (ft, [L]) S = channel slope ($\tan\theta$) [.]

$V = [M^0 L T^{-1}]$ N = roughness factor [.]

$= \frac{1.49}{M^0 L^0 T^0} \cdot [L]^{2/3} \cdot [M^0 L^0 T^0]^{\frac{1}{2}}$ $\Rightarrow [M^0 L^0 T^{-1}] = 1.49 [M^0 L^{2/3} T^0]$

$\Rightarrow [1.49] = L^{1/3} T^{-1}$ $\frac{ft^{1/3}}{s}$

— Overview & governing eq'ns —

5 basic cons. eqn's :
- (a) cons. of mass
- (b) " linear momentum (Newton's 2nd law)
- (c) " energy (1st law of thermo)
- × (d) 2nd law of thermo (not in this class)
- × (e) cons. of momentum of momentum

Variables:
- scalars – pressure (P), density, (ρ), Temperature (Θ) 3
- vectors – velocity ($\vec{V}$) accel'n ($\vec{a}$) Force ($\vec{F}$) 3
- tensors – shear stresses (τ) 1

(P) pressure: compression stress @ a pt. units Pa, lbf/ft², 1 atm = 101.3 kPa

(Θ) temp: measure of internal energy of a liquid

(ρ) density: Mass/volume units: kg/m³, slugs/ft³

(γ) Specific weight: weight/volume $= \frac{Mg}{\forall} = \rho g$

(SG) specific gravity: ratio of density of a fluid to a standard reference fluid liquids: H_2O @ 4°C

notes 8-26-10

continued...

$SG\ (gas) = \dfrac{\rho_{gas}}{\rho_{air}\ @STP}$

- encouraged to print out recitation homework & bring to recitation

Notes 8-31-10

2ndary fluid properties

a) Viscosity: quantitative measure of fluid's resistance to flow (analog of friction)

element ABCD has undergone shear

$ABCD \xrightarrow[\tau]{\text{shear}} \text{New element } AB'C'D$

$\angle BAB' = \delta\theta \qquad \overline{AB} = \delta y \qquad$ "common" fluids $\tau \propto \dfrac{\delta B}{\delta t}$

$\tan \delta\theta = \dfrac{\overline{BB'}}{\overline{AB}} = \dfrac{U\,\delta t}{\delta y}$ if $U = \delta u$, $\tan \delta\theta \approx \delta\theta = \dfrac{\delta u \cdot \delta t}{\delta y}$

$\Rightarrow \dfrac{\delta\theta}{\delta t} = \dfrac{\delta u}{\delta y} \qquad \tau \propto \dfrac{d\theta}{dt} = \dfrac{du}{dy}$ Hence, $\boxed{\tau = \mu \dfrac{du}{dy}}$ Newton's law of viscosity

μ = coefficient of viscosity. Common fluids ≡ Newtonian Fluids

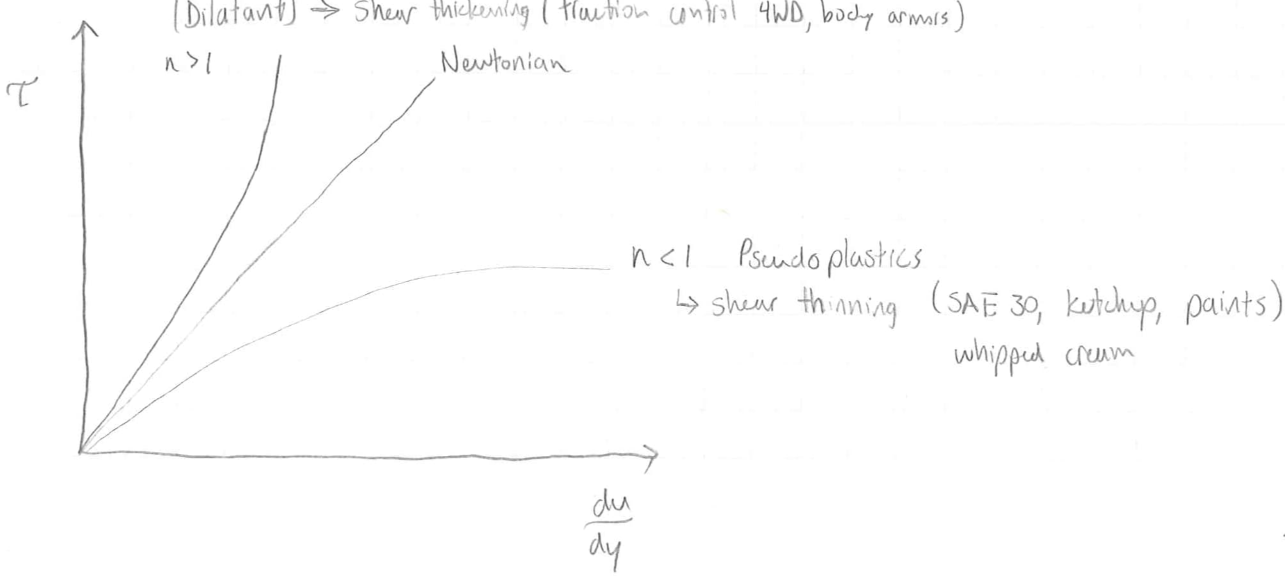
μ, dynamic viscosity: SI $\dfrac{kg}{m \cdot s}$ BG $\dfrac{slugs}{ft \cdot s}$ only when ρ = constant

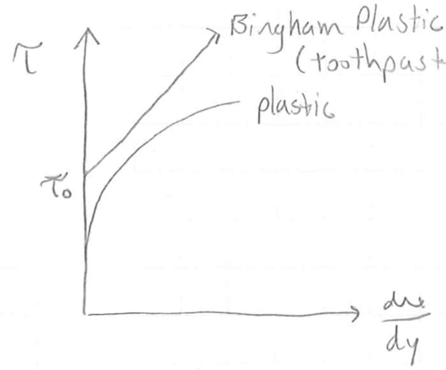
Kinematic Viscosity $\nu = \dfrac{\mu}{\rho}$ (independent of mass) Units SI: m^2/s BG: ft^2/s

Non-newtonian Fluids

$\tau = k\left(\dfrac{du}{dy}\right)^n$ Newtonian: $n = 1$, $k = \mu$

(Dilatant) → Shear thickening (traction control 4WD, body armors)

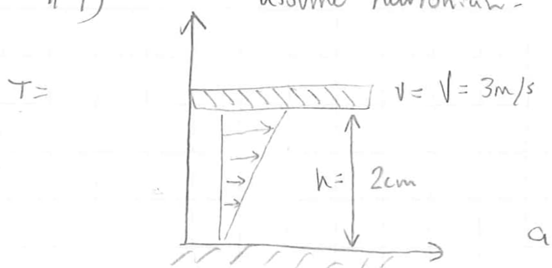




Bingham Plastic $\tau = \tau_0 + \mu_{BP}\left(\frac{du}{dy}\right)$
(toothpaste, clay suspensions)

#1)

assume newtonian?

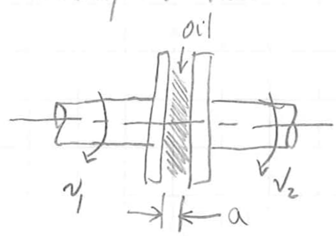


$\mu = .29 \frac{kg}{m \cdot s}$ $\quad \tau = \mu \frac{du}{dy} = \mu \frac{V-0}{h}$

$= \mu \frac{V}{h} = 43.5 \frac{N}{m^2} = 43.5 \ Pa$

assumption: the velocity profile is linear → only true if "h" is small

Set 2 #2) Viscous clutch → transmits power from the drive shaft to the front diff mostly in 4wd

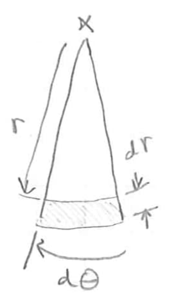


Basic eq: $\tau = \mu \frac{du}{dy} = \mu \frac{\Delta u}{\alpha} = \mu \cdot \frac{r(\omega_i - \omega_o)}{\alpha}$

torque

$dT = R\,dF = R \cdot \tau \cdot dA$ $\qquad \boxed{\tau = \frac{dF_t}{dA}}$

incremental F



$dA = (r\,dr)\,d\theta \qquad r \to 0 \to R \qquad \theta \to 0 \to 2\pi$

$dT = \tau r^2 dr\, d\theta = \mu \frac{r \Delta\omega}{a} r^2 dr\, d\theta$

$dT = \frac{\mu \Delta\omega}{a} r^3 dr\, d\theta \qquad$ Torque $T = \int_{r=0}^{R} \int_{\theta=0}^{2\pi} dT = \iint_{r=0,\ \theta=0} \frac{\mu \Delta\omega}{a} r^3 dr\, d\theta$

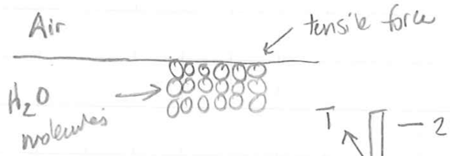
$$\boxed{T = \frac{\pi \mu \Delta\omega R^4}{2a}}$$

Power transmitted $= T\omega_o \qquad \pi \mu \Delta\omega\, \omega_o \frac{R^4}{2\alpha}$

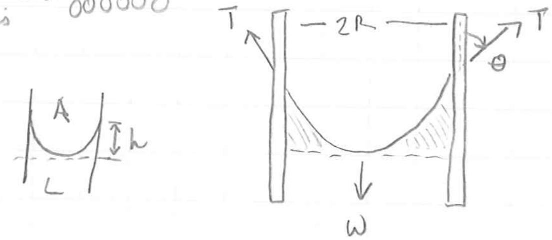
slip ratio : $S = \frac{\Delta\omega}{\omega_i} = \frac{2\alpha T}{\pi \mu R^4 \omega_i}$

Notes 8-31-10

② Surface tension (σ) force developed on a surface : the intensity of an attraction per unit length along a line



Units: SI : $\frac{N}{m}$ BG $\frac{lbf}{ft}$



weight $W = T\cos\theta$
$= 2\pi R\,\sigma)\cos\theta$

$\pi R^2 h \cdot \gamma$

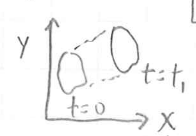
$\Rightarrow h = ?$ reading assignment

pg 18-33 + Ex 1.8

Note: Case I $\theta < 90° \rightarrow$ wetting liquid (Hg) *smiling*
Case II $\theta > 90° \rightarrow$ non-wetting (Water) *frowning*

Descriptions of fluid motion

2 view points
Eulerian: → stay fixed & the fluid moves past
Lagrangian → you move with the fluid



Lagrangian, $\vec{V} = \vec{V}(t)$
Eulerian $\vec{V} = \vec{V}(x,y,z,t)$

$u(x,y,z,t)\hat{i}\;v(x,y,z,t)\hat{j}\;w(x,y,z,t)\hat{k}$

Notes 9-2-10

Description of fluid motion continued

~~Velocity~~ vector Eulerian $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{v}(x,y,z,t)}{dt} = \frac{du}{dt}\hat{i} + \frac{dv}{dt}\hat{j} + \frac{dw}{dt}\hat{k}$

x-component: $\frac{du(x,y,z,t)}{dt}$ $\frac{\partial u}{\partial y}\cdot\underbrace{\frac{dx}{dt}}_{u} + \frac{\partial u}{\partial y}\cdot\underbrace{\frac{dy}{dt}}_{v} + \frac{\partial u}{\partial z}\underbrace{\frac{dz}{dt}}_{w} + \frac{\partial u}{\partial t}\cdot\cancel{\frac{\partial t}{\partial t}}$

$= \underbrace{\frac{\partial u}{\partial t}}_{\text{local accel}} + \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\text{convective accel}}$

there are all 4 of these terms for each component $\hat{i}, \hat{j}, \hat{k}$

$\vec{a} = \underbrace{\frac{\partial v}{\partial t}}_{\text{local derivative}} + \underbrace{(\vec{v}\cdot\nabla)\vec{v}}_{\text{convective derivative}} = \frac{D\vec{v}}{Dt}$

$\vec{V} = 3t\hat{i} + xz\hat{j} + ty^2\hat{k}$ $\quad \vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$ compare

$u = 3t \quad v = xz \quad w = ty^2$ $\qquad \frac{D\vec{V}}{dt} = \frac{Du}{dt}\hat{i} + \frac{Dv}{dt}\hat{j} + \frac{Dw}{Dt}\hat{k}$

$\frac{Du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 3 + 3t\cdot 0 + xz\cdot 0 + ty^2\cdot 0 = 3 + 0$
(local) (convective)

$\frac{Dv}{dt} = \frac{\partial v}{\partial t} \quad \frac{\partial v}{} \quad \frac{\partial v}{} \quad — = 0 + 3t\cdot z + xz\, 0 + ty^2\cdot x = 0 + 3tz + xy^2t$

$\partial w \quad \partial w$

$= y^2 + 3t\cdot 0 + xz\cdot 0 + ty^2\cdot 0 = y^2 + 2xyzt$

$\vec{a}_{total} = 3\hat{i} + (3tz + xy^2t)\hat{j} + (y^2 + 2xyzt)\hat{k}$

$\vec{a}_{local} = 3\hat{i} + y^2\hat{k}$ $\qquad \vec{a}_{convective} = \vec{a}_{total} - \vec{a}_{local}$

Properties of the flow field: Steady: properties do not change with time

$\phi(x,y,z)$ is steady. Transient: properties change w/ time $\phi(x,y,z,t)$

1D, 2D, or 3D $\qquad \vec{V} = ae^{-bx}\hat{i} + cx^2\hat{j} = \vec{V}(x)$ so it is 1D

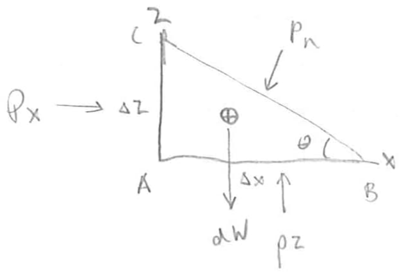
transient 3D looks like $\vec{V}(x,y,z,t)$, ~~t~~x, $\vec{V} = xyz^{-t}\hat{i}$

assign - look at the web links on Ch. 1 review sheet

9-14 HW 2 due, $\quad$ 9-16 Midterm #1 $\quad$ previous tests will be →
→ on blackboard

Chapter 2, Motivation: (a) Solve fluid problems that DONT involve fluid motion
(b) pressure distribution in a static fluid → its effects on submerged bodies

(1) Pressure at a point: $P = \frac{dF_n}{dA} = \frac{\text{Normal force}}{\text{Area}}$ $\quad$ Width = b (into the board)



$\overline{AB} = \Delta x \quad \overline{AC} = \Delta z \quad \overline{BC} = \Delta s \qquad \Sigma F_x = 0 = P_x\,\Delta z\cdot b$

$- P_n b\,\Delta s \sin\theta = P_x\,\Delta z\cdot b - p_n\cdot\Delta z\cdot b \quad \Rightarrow \boxed{P_x = P_n}$

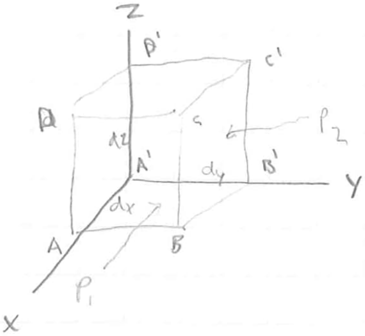
$\Sigma F_z = P_z b\Delta x - dw - p_n \cancel{b\Delta s\cos\theta} = 0.$ $\qquad P_z b\Delta x = \frac{1}{2}\rho\,\Delta z\,\Delta x\, b\, g$
$dw = \rho\cdot d\forall = \rho\cdot\frac{1}{2}\Delta z\cdot\Delta x\cdot b$ $\qquad + p_n b\Delta x$

$\boxed{p_z = P_n + \frac{1}{2}\rho g\,\Delta z}$

to make this a point, $\Delta x \to 0 \quad \Delta z \to 0 \quad \Rightarrow \quad \boxed{P_x = P_n = P_z}$

Note: Pressure in a static fluid at any given pt. is independent of direction

② Pressure on a fluid element



$d\forall = dx \cdot dy \cdot dz$

① front surface ABCD & ② BB'CC'

Taylor series expansion $P_2 = P_1 + \frac{\partial P}{\partial x} \cdot dx + \left(\frac{\partial p}{\partial x}\right)^2 \frac{(dx)^2}{2} + \ldots$

neglect 2nd & higher order terms $\boxed{P_2 = P_1 + \frac{\partial p}{\partial x} \cdot dx}$

Net force x-direction $dF_x = P_1 \cdot dydz - P_2\, dydz = (P_1 - P_2)\, dydz$

$dF_x = P_1 \cdot dydz - P_2\, dydz = -\frac{\partial p}{\partial x} \cdot dxdydz = -\frac{\partial p}{\partial x} \cdot d\forall$

$dF_y = -\frac{\partial p}{\partial y} \cdot d\forall$ $\quad dF_z = -\frac{\partial p}{\partial z}\, d\forall$ $\quad dF_{pressure} = dF_x \hat{i} + dF_y \hat{j} + dF_z \hat{k}$

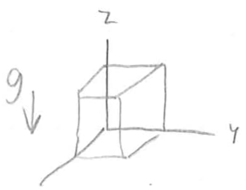
$d\vec{F}_p = \left[\frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} \hat{k}\right] dV = -\nabla p\, d\forall$ (gradient)

$\vec{f}_{pressure} = \frac{\text{Total force}}{\text{Volume}} = \frac{d\vec{F}_p}{d\forall} = \nabla p$

Apply Newton's law: $\Sigma f = f_{surface} + f_{body} = \frac{m\vec{a}}{\forall} = \rho\vec{a}$

$-\nabla p + \rho g = \rho\vec{a} \Rightarrow \boxed{\nabla p = \rho(\vec{g} - \vec{a})}$ pressure gradient acts in the direction $\vec{g} - \vec{a}$

Case 1: Hydrostatics Fluid at rest: $\vec{a} = 0$



$\nabla P = \rho g$ Sign convention: z is "up"

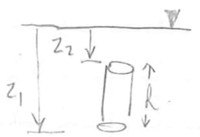
$g = -g\hat{k}$ $\quad \nabla P\ \frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} k = -\rho g \hat{k}$

$\frac{\partial p}{\partial x} = 0 \quad \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = -g$ replace Partial D.E. $\frac{dp}{dz} = -\rho g = -\gamma$ (specific wt.)

$\boxed{\frac{dp}{dz} = -\gamma}$ or, $\int_{P_1}^{P_2} dp = -\gamma \int_{z_1}^{z_2} dz$ or, $P_2 - P_1 = -\gamma(z_2 - z_1)$

start with this eqn and define situation

— Case 1a. liquids



$p_2 - p_1 = -\gamma(z_2 - z_1)$ or, $z_1 - z_2 = \frac{p_2}{\gamma} - \frac{p_1}{\gamma} = \frac{p_2 - p_1}{\gamma}$

Note: pressure increases as depth increases

— Case 1b: gases: $\frac{dp}{dz} = -\gamma = -\rho g$ compressible, ρ not constant

Ideal gas law: $P = \rho RT$ $\quad \frac{dp}{dz} = \frac{-P}{RT} \cdot g$ or $\int_{P_1}^{P_2} \frac{dp}{p} = -\int_{z_1}^{z_2} \frac{g}{RT} dz \Rightarrow \ln\left(\frac{P_2}{P_1}\right)$

Isothermal assumption – $T = T_0 = \text{constant} \Rightarrow \ln\left(\frac{P_2}{P_1}\right) = \frac{-g}{RT_0}(z_2 - z_1)$

$$\boxed{p_2 = p_1 \exp\left[\frac{-g(z_2 - z_1)}{RT_0}\right]}$$

$T(z) = T_0 - Bz$ $\quad B$ = lapse rate = $.0065 \frac{k}{m}$

T_0 = sea level temp $\simeq 15°C$ $\quad$ using lapse rate (non isothermal) $\quad \int \frac{dP}{P} = -\int_{z_1}^{z_2} \frac{g\,dz}{R(T_0 - Bz)}$

$$P_2 = P_{atm}\left[1 - \frac{Bz}{T_0}\right]^{\left(\frac{g}{RB}\right)}$$

$\frac{g}{RB} = 5.26$ (air)

reference pressure: absolute pressure – the actual value of P (no reference)

gage/vaccum pressure $\quad P(\text{gage}) = P - P_{atm} \qquad P(\text{vacuum}) = P_{atm} - P$

P_{atm} = 100 – 101.3 kPa → look up local P_{atm}!

Measurement of pressure: "Manometry" → liquid columns (vertical/inclined)

for measuring pressure

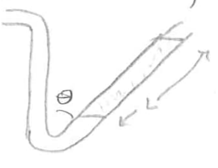
ⓐ V-tube monometer

$P_A = ?$

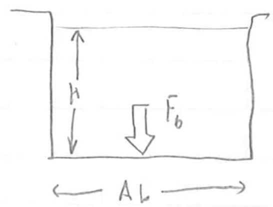
Advantages: ① can be used w/ gases ② if any liquid is a gas, $\rho_1 g \Delta z$ would be small

Problem set 4 #2 $\quad$ Start with one end. going up, subtract γh, going down, add it.

on an incline, use $L \cos\theta\, \gamma$

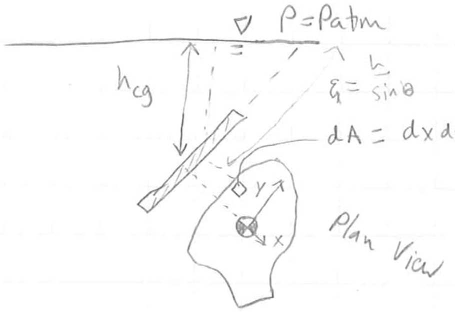


Problem: find the hydrostatic force on a "plane" or "curved" surface that is submerged



$F_b = P \cdot A_b = \gamma H A_b$ (easy!)

Free surface (Plane)



$\xi_{CG} = \frac{1}{A}\int_A \xi\, dA$

At any depth $P = P_{atm} + \gamma h$ total hydrostatic force on one side of the plate

$$F = \int_A P\,dA = \int_A (P_{atm} + \gamma h)\,dA = P_{atm} \cdot A + \gamma \int_A \xi \sin\theta\, dA$$

$$F = P_{atm} \cdot A + \gamma \sin\theta \underbrace{\int_A \xi\, dA} = P_{atm} \cdot A + \gamma \underbrace{\sin\theta \cdot \xi_{CG}}\, A$$

$$= P_{atm} A + \gamma h_{CG} \cdot A = (P_{atm} + \gamma h_{CG}) A = P_{CG} \cdot A$$

(h_{CG} = depth of the CG of the plate)

Combined Pressure & bending force!

Resultant force acts on "Center of pressure"

find X_{cp} & Y_{cp}

$$X_{cp} = -\gamma \sin\theta \frac{I_{xx}}{P_{CG} A}$$

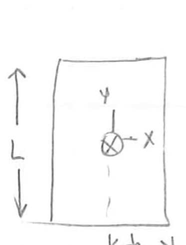
$$Y_{cp} = -\gamma \sin\theta \frac{I_{xy}}{P_{CG} A}$$

these distances are measured from CG

I_{xx} = area moment of inertia $= \int y^2\, dA$

I_{xy} = product moment of inertia

Review



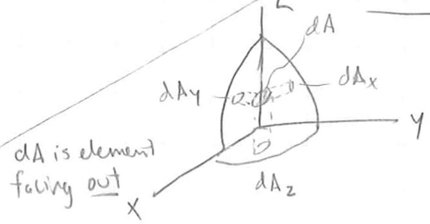
$A = bL$

$I_{xx} = \frac{bL^3}{12}$

$I_{xy} = 0$

circle, semicircle, triangle

Hydrostatic force of a curved surface



$d\vec{F} = -p\, d\vec{A}$

Hydrostatic forces on a curved surface continued

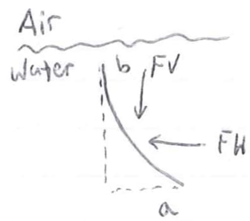
$\vec{dF} = -p\,d\vec{A}$ - sign indicates $\vec{dF}$ & $\vec{dA}$ are opposite directions

F_r = resultant force on surface $\widehat{ABC}$ $\vec{F_R} = F_{Rx}\hat{i} + F_{Ry}\hat{j} + F_{Rz}\hat{k}$

$F_R \cdot i = \cancel{F_{Rx} i \cdot i}^{\,1} + \cancel{F_{Ry} j \cdot i}^{\,0} + \cancel{F_{Rz} k \cdot i}^{\,0} = F_{R,x} = \int dF_x = \int -p \cdot dA_x$

dA_x = projection of dA onto x-axis (Y-Z plane)

Ex/



FH
Fv

h_{cg} = depth of center of gravity of surface ab

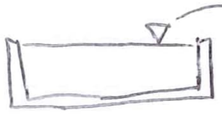
$F_H = \Sigma F_x$ $F_{R,x} = p \cdot A_x = \gamma\, h_{CG} \cdot (\overline{bc} \times w)$

$F_V = \Sigma F_y$ = weight of the liquid columns

Reading assignment 86-89 in book, Example 2.8   Study set 2 problem 6

homework due tuesday

-Set 5 Problem 1 in-class-



this means free surface : atmospheric pressure acting on it.

in Air, Centroid = center of gravity

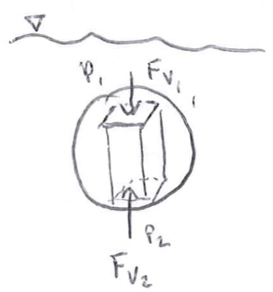
in Water, centroid ≠ center of gravity

Recitation tuesday 5pm

Exam I next thursday

Buoyancy: Archimedes Laws (320 BC)

-A body immersed in a fluid experiences a vertical force, (buoyant force) equal to the weight of the fluid it displaces



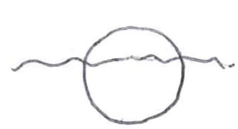
for column: $F_B = F_{V2} - F_{V1} = \int (P_2 - P_1)\,dA = (P_2 - P_1) A$

$(P_2 - P_1) = -\gamma(Z_2 - Z_1) = -\gamma(Z_2 - Z_1)A$

$|F_B| = \gamma(Z_2 - Z_1)A = \gamma\, \forall_{body}$

$$\boxed{F_{Buoyant} = \rho g \forall_{body}}$$

Center of Buoyancy



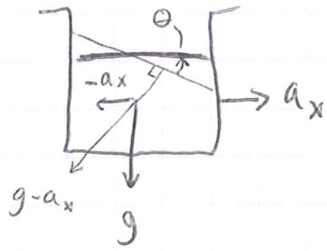
$F_B = \gamma \times \forall_{bod\ displaced}$

Notes 9-9-10

$\nabla P = \rho(\vec{g} - \vec{a})$

Pressure distribution in rigid body motion

No relative motion between particles



$\theta = \tan^{-1}\left(\frac{a_x}{g}\right)$

Notes Recitation
9-14-10

Problem 5

Figure 5 $\quad T = \int_0^{2\pi} dT = \tau \cdot 2\pi R_i^2 \cdot l$

$\tau = \mu \frac{du}{dy} = \mu \frac{\omega R_i}{(R_o - R_i)} = \boxed{\mu \frac{2\pi R_i^3 \omega l}{R_o - R_i} = \text{Torque}}$

spec volume	=	$v = \frac{1}{\rho}$
spec weight	=	$\gamma = \rho g$
spec. gravity	=	$\frac{\rho}{\rho_{H_2O} @ 4°C}$

Problem 3 $\quad P_1 + .4ft(8)\gamma_{H_2O} - 1.6(1)\gamma_{H_2O} = 0$

$P_1 = -99.8 \text{ lb}/ft^2$ (gage)

Prob. 4

$\Sigma M_o = 0 \quad F_{R_1} \times l_1 = F_{R_2} \times l_2$

$h = 1.88$ ft

$F_{R_1} = \gamma h_{c_1} A_1$
$= \gamma\left(\frac{h}{2}\right) \times (4ft \times h)$

$l_1 = h/3$

$F_{R_2} = P \cdot A = \gamma h \times \frac{\pi}{4}(1)^2$

$l_2 = 3'$

HW due by 12pm tomorrow, in mailbox, 196 Toomey

Jane
Casey Pezold
Stephen arnold
Katie Clark

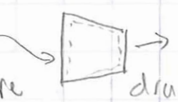
} got good HW scores

Ch. 3 Integral relations for a Control volume

"Closed" system → corresponds to an entity of "constant" mass
will not have a constant volume

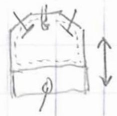
"control" volume → corresponds to an entity which allows for mass flow through its boundary → $\boxed{\text{INDEPENDENT OF MASS}}$

1. Fixed C$\forall$: Flow through a nozzle



2. Moving C$\forall$: Flow analysis to measure drag on a body

3. Deforming/Expanding C$\forall$: IC Engines, Balloons

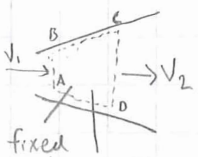


Reynolds Transport theorem (RTT)

Define System: Extensive Property: dependent on mass ⇒ ammount of the fluid Ex; mass, density, momentum, total E., KE, P.E, (B)
C$\forall$: Intensive property – independent of mass, ex; velocity, internal energy, specific KE., specific P.E. (B/m)

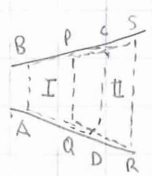
RTT

time = t



C$\forall$, fixed
system & C$\forall$ are same for no flow

time t + Δt



area I = ABPQ
area II = CDRS

system = C$\forall$ − I + II
= C$\forall$ − inflow + outflow

"Ext. Property"

$$B_{sys}(t) = B_{C\forall}(t)$$

$$B_{sys}(t+\Delta t) = B_{C\forall}(t+\Delta t) - B_{I}(t+\Delta t) + B_{II}(t+\Delta t)$$

time rate of change $\dfrac{dB_{sys}}{dt} = \dfrac{B_{sys}(t+\Delta t) - B_{sys}(t)}{\Delta t}$ } at $\lim_{\Delta t \to 0}$ continued...

$$\frac{dB_{sys}}{dt} = \lim_{\Delta t \to 0}\left[\frac{B_{cv}(t+\Delta t) - B_{cv}(t)}{\Delta t} - \frac{B_{I}(t+\Delta t)}{\Delta t} + \frac{B_{II}(t+\Delta t)}{\Delta t}\right]$$

$$= \frac{\partial B_{cv}}{\partial t} - \dot{B}_{in} + \dot{B}_{out} \qquad \dot{B}_{in} = \dot{B}_{I} = \lim_{\Delta t \to 0} \frac{B_I(t+\Delta t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{b_I \cdot m(t+\Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{b_I V_1 \rho_1 A_1 \cancel{\Delta t}}{\cancel{\Delta t}} = b_I V_1 \rho_1 A_1$$

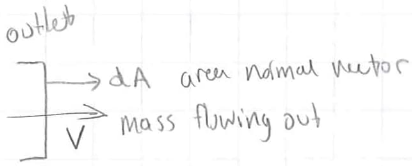
Mass flow rate: $V_1 \cdot \rho_1 \cdot A_1$ $\quad$ mass $= V_1 \rho_1 A_1 dt$

$\dot{B}_{out} = B_2 = b_{II} \rho_2 V_2 A_2$ $\qquad$ $B_{net} = \dot{B}_{out} - \dot{B}_{in} = b_1 V_1 \rho_1 A_1 - b_{II} V_2 \rho_2 A_2$

$$= \int_1^2 b\rho\vec{V}\cdot d\vec{A} \qquad \boxed{\frac{dB_{sys}}{dt} = \frac{\partial}{\partial t}\int_{cv} b\rho d\forall + \int_1^2 b\rho\vec{V}\cdot d\vec{A}} \leftarrow \text{RTT}$$

$$B_{cv} = b \cdot m \quad (m = \rho\forall) = \int_\forall b\rho d\forall$$

$\dot{B}_{net} = \dot{B}_{out} - \dot{B}_{in}$ corresponds to



$\rho(\vec{V}\cdot d\vec{A})$ means (+) $\dot{B}_{out}$

Skip derivation of Conservation of mass ...

$$\boxed{0 = \frac{\partial}{\partial t}\int_\forall \rho d\forall + \int_A \rho\vec{V}\cdot dA}$$

special cases ① incompressible flow, ρ constant $\Rightarrow$ $\boxed{0 = \frac{\partial \forall}{\partial t} + \int_A \vec{V}\, d\vec{A}}$

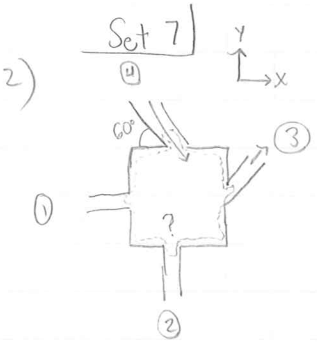
② ρ const, $C\forall$ is fixed $\frac{\partial \forall}{\partial t} = 0$ $\quad \boxed{\int_{CS} \vec{V}\cdot d\vec{A} = 0}$ $\quad \vec{V} = \vec{V}(t)$

Conservation of momentum $\quad B = mV \quad b = \frac{mV}{m} = V$

note $\quad \frac{dP}{dt}(m\vec{V}) = m\vec{a} = F \qquad \underset{\uparrow \text{ surface}}{\vec{F}_S} + \underset{\uparrow \text{ body}}{\vec{F}_B} = \frac{\partial}{\partial t}\int \vec{V}\rho d\forall + \int \vec{V}\rho\vec{V}\cdot d\vec{A}$

Notes 9-21-10

2) Set 7



assumptions
(1) steady flow
(2) incompressible flow
(3) uniform velocity at each ctrl surface

$$\frac{d}{dt}\int_{\forall} d\forall + \int_{cs} \rho \vec{V}\cdot d\vec{A} = 0$$

$$\Rightarrow \int_{cs} \rho \vec{V}\cdot d\vec{A} = 0$$

$$\Rightarrow \int_{cs} \vec{V}\cdot d\vec{A} = 0$$

$$\int_1 \vec{V}_1 d\vec{A}_1 + \int_2 \vec{V}_2 d\vec{A}_2 + \int_3 \vec{V}_3 d\vec{A}_3 + \int_4 \vec{V}_4 d\vec{A}_4 = 0$$ (all $d\vec{A}$'s point outwards)

surface 1: $\vec{V}_1 = 10\hat{i}$ ft/s $\quad d\vec{A}_1 = (A_1)-\hat{i}$ ft^2 $\quad \int_1 V_1 dA_1 = -V_1 A_1$ ft^3/s

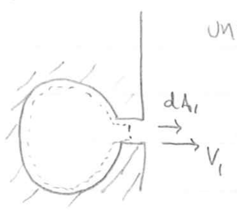
Surface 3: $\int_3 \vec{V}_3\cdot d\vec{A}_3 = +\frac{\dot{m}_3}{\rho}$ (out)

surface 4: $\int_4 \vec{V}_4 \cdot dA_4 = -Q_4$ (in) — flow rate

$$-10A_1 + \int_2 \vec{V}_2 d\vec{A} + \frac{\dot{m}_3}{\rho} - Q_4 \Rightarrow \rho\int_2 \vec{V}_2\cdot dA_2 = 1.4 \text{ slugs}$$

$V_2 = \frac{1.94}{\rho A_2} = 2$ ft/s $\quad \boxed{\vec{V}_2 = -2\hat{j} \text{ ft/s}}$

3)



unsteady flow, compressible, uniform flow at each control surface

$$\underbrace{\overset{\text{I}}{\frac{d}{dt}\int_{\forall} \rho d\forall} + \overset{\text{II}}{\int_1 \rho \vec{V}_1 dA}}_{} = 0$$

term I. $= \frac{d}{dt}(\rho\forall) = \forall\frac{d\rho}{dt}$

Term II. $\int_1 \rho\vec{V}\cdot d\vec{A} = +|\rho V_1 A_1|$

$$\frac{d\rho}{dt} + |\rho V_1 A_1| = 0$$

$$\frac{d\rho}{dt} = \frac{-|\rho V_1 A_1|}{\forall} = \frac{-6.13\frac{kg}{m^3} \times 311\text{ m/s} \times \frac{65\text{ mm}^2}{10^6\text{ mm}^2}\times\text{m}^2}{.05\text{ m}^2}$$

$$= -2.48\,(\text{kg/m}^3)/\text{s}$$

Homework due next thursday

F_S = Surface force
F_B = Body force

Conservation of Linear Momentum (Newton's 2^{nd} law)

RTT $\left.\frac{dB}{dt}\right)_{System} = \frac{d}{dt}\int_{\forall} \vec{V}\rho\, d\forall + \int_{CS} \vec{V}\rho \vec{V}\cdot d\vec{A}$

$F = m\cdot\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d(m\vec{V})}{dt} = \frac{dP}{dt}$

$B =$ linear momentum $= m\vec{V} = P \qquad b = \frac{B}{m} = \frac{m\cdot\vec{V}}{m} = \vec{V}$

$\boxed{F_S + F_B = \frac{d}{dt}\int_{\forall}\vec{V}\rho\, d\forall + \int_{CS}\vec{V}\rho\vec{V}\cdot d\vec{A}}$ Valid for non-accelerating $C\forall$

momentum flux

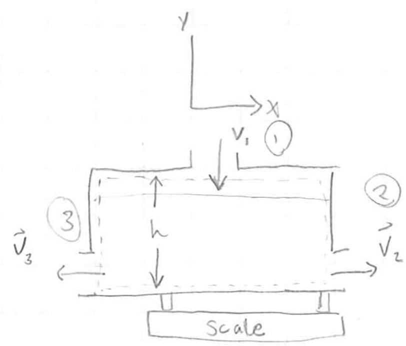
$x: F_{S,x} + F_{B,x} = \frac{d}{dt}\int_{\forall} u\rho\, d\forall + \int_{CS} u\rho\vec{V}\cdot d\vec{A}$

$y: F_{S,y} + F_{B,y} = \frac{d}{dt}\int_{\forall} v\rho\, d\forall + \int_{CS} v\rho\vec{V}\cdot d\vec{A}$

$z: F_{S,z} = F_{B,z} = \frac{d}{dt}\int_{\forall} w\rho\, d\forall + \int_{CS} w\rho\vec{V}\cdot d\vec{A}$

Set 8 problem 1

$F_S + F_B = \cancel{\frac{d}{dt}}^{\,0}\int_{\forall}\vec{V}\rho\, d\forall + \int_{CS}\vec{V}\rho\vec{V}\, d\vec{A}$



$F_S = R_y$

$F_{B,y} = -W_{tank} - W_{H_2O\,column} = -W_{tank} - \gamma A h$

(?) ✓ ✓

$R_y - W_{tank} - \gamma A h = \int_{CS} v\rho\vec{V}\cdot d\vec{A}$ ✮

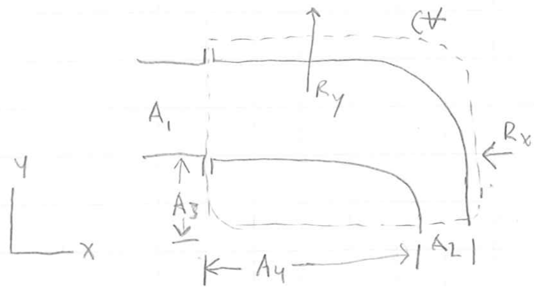
Given: $A_1 = A_2 = A_3 = 0.1\, ft^2 \quad \vec{V}_1 = -5\hat{j}$ ft/s

✮ $= \int_1 v_1\rho\vec{V}_1\cdot d\vec{A} + \int \cancel{v_2}^{\,0}\rho\vec{V}_2\cdot d\vec{A}_2 + \int \cancel{v_3}^{\,0}\rho \vec{V}_3\, d\vec{A}_3$

$= \int_1 v_1\rho\vec{V}_1\cdot d\vec{A} = -\rho v_1 |\vec{V}_1\cdot\vec{A}_1| \qquad \vec{V}_1 = -5\hat{j}$ ft/s $\quad \vec{A}_1 = 0.1\hat{j}\, ft^2 \quad v_1 = -5$ ft/s

$R_y = 128$ lbf

Problem set 8, #2



$A_1 = .01\,m^2$ $P_1 = 221\,kPa$ $P_{atm} = 101.3\,kPa$

$A_2 = .0025\,m^2$ $V_2 = 16\,m/s$

$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

Total: vertical area $= A_1 + A_3$

horiz area $= A_2 + A_4$

cons. mass $\cancel{\frac{d}{dt}}\int \rho\, d\forall^{\,0} + \int_{cs} \rho \vec{V}\cdot d\vec{A}$

Assumptions ① steady flow ② incompressible ③ uniform properties @ inlet/outlet

$\int_1 \vec{V}\cdot d\vec{A} + \int_2 \vec{V}_2\cdot d\vec{A}_2$

$-|V_1||A_1| + 16|A_2| = 0$

$V_1 = 4\,m/s$

$\vec{V}_1 = |V_1|\hat{i}\ m/s$ $\vec{A}_1 = -|A_1|\hat{i}\ m^2$

$\vec{V}_2 = 16\hat{j}\ m/s$ $A_2 = -|A_2|\hat{j}\ m^2$

x momentum) $F_{s,x} + \cancel{F_{B,x}}^{\,0} = \int_1 u_1 \rho_1 \vec{V}_1\cdot d\vec{A}_1 + \cancel{\int_2 u_2 \rho \vec{V}_2\cdot d\vec{A}_2}^{\,0}$

$F_{s,x} = \int_1 u_1 \rho \vec{V}_1 \cdot d\vec{A}_1$

Surface Pressure $P_{abs}\cdot A_1 + P_{atm}\cdot A_2 - P_{atm}(A_1 + A_3) - R_x = -u_1 \cdot \rho\,|V_1||dA_1|$

$(P_{gage} + P_{atm}) \Rightarrow A_1 P_{1\,gage} - R_x = -u\cdot\rho\,|V_1||dA_1|$

$\boxed{R_x = 1.36\,kN}$

y-momentum) $\cancel{F_{s,y}}^{\,0} + F_{B,y} = \int_2 v_2 \rho \vec{V}_2 \cdot d\vec{A}_2 \Rightarrow R_y = v_2 \rho\,|\vec{V}_2||d\vec{A}_2| = \underbrace{\underset{m/s}{(-16)} \times \underset{kg/m^3}{999}}_{\text{momentum}} \times \underbrace{\underset{m/s}{(16)} \times .0025\,m^2}_{\text{Flux}}$

$\boxed{R_y = -639\,N}$

— All info for HW 3 covered now. —

Cons. momentum: C∀ moves with constant velocity

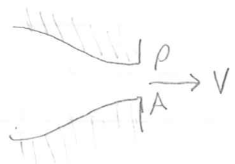
$$F_s + F_B = \frac{d}{dt}\int \vec{V}_{rel}\,\rho\, d\forall + \int_{cs} \vec{V}_{rel}\,\rho\,\vec{V}_{rel}\cdot d\vec{A}$$

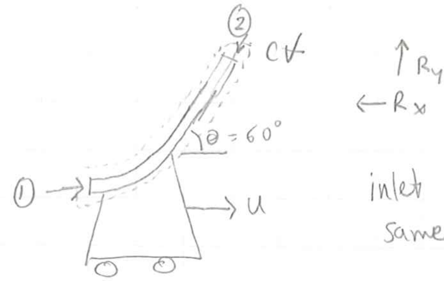
relative velocity of the fluid at the control

surface force

Body force

Notes 9-23-10





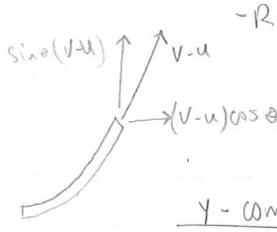
inlet & outlet areas are the same

x-component: $F_{S,x} + \cancel{F_{B,x}}^{\,0} = \int_{CS} u_{rel}\, \rho \vec{V}_{rel} \cdot d\vec{A}$

$-R_x = \int_1 u_{1rel}\, \rho \vec{V}_{1rel} \cdot d\vec{A}_1 + \int_2 u_{2rel}\, \rho \vec{V}_{2rel} \cdot d\vec{A}_2$

$-R_x = -(V-u)\rho(V-u) \cdot A + (V-u)\cos\theta\,(V-u)\,A$

$R_x = (V-u)^2 \rho A (1-\cos\theta) = \boxed{-599\text{ N to the left}}$



y-component: $F_{S,y} + F_{B,y} = \int_{CS} v_{rel}\, \rho \vec{V}_{rel} \cdot dA$

$R_y = \int_1 \cancel{v_{rel}}^{\,0} \rho \vec{V}_{rel} \cdot dA + \int_2 v_{2rel}\, \rho \vec{V}_{rel_2} \cdot dA_2$

$R_y = (V-u)\sin\theta\, \rho\, (V-u) A = (V-u)^2 \sin\theta\, \rho A = \boxed{R_y = 1.04\text{ kN}}$

Force required to hold elbow $\boxed{R = -599\hat{i} + 1040\hat{j}\text{ N}}$

RTT: $\left.\frac{dB}{dt}\right)_{system} = \frac{d}{dt}\int_{\forall} b\rho\, d\forall + \int_{CS} b\rho \vec{V} \cdot d\vec{A}$ B: extensive property
b: intensive property $= \frac{B}{m}$

cons. mass: $B = M \Rightarrow b = 1$

cons. momentum: $B = m\vec{V} \Rightarrow b = \frac{m\vec{V}}{m} = \vec{V}$

cons. energy: $B = E$ = total energy of system $b = \frac{E_t}{m} = e_t$ = total energy / mass

- $\left.\frac{dE_t}{dt}\right)_{system} = \overbrace{\frac{d}{dt}\int_{\forall} e_t \rho\, d\forall + \int_A e_t \rho \vec{V} \cdot dA}^{\text{ME 231}}$ ← First law of thermo

$\left.\frac{dE}{dt}\right)_{system} = \overbrace{\frac{dQ}{dt}}^{\text{ME225}} - \underbrace{\frac{dW}{dt}}_{\text{ME 219}}$

(time rate of change total energy of system) = (Heat trans to the system) − (Time rate of work done by system)

Notes 9-28-10

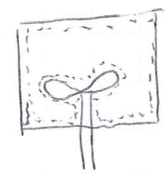
Cons. energy continued

$e_t = u + \frac{V^2}{2} + gz$ + other $\rightarrow$ ignore

RTT: $\left.\frac{dE_t}{dt}\right)_{system} = \frac{d}{dt}\int_{\forall} e_t \rho \, d\forall + \int_A e_t \rho \vec{V} \cdot d\vec{A} = \frac{dQ}{dt} - \frac{dW}{dt}$

rate of work $\quad \frac{dW}{dt} = \dot{W} = \dot{W}_{shaft} + \dot{W}_p + \dot{W}_{viscous}$

$\dot{W}_{shaft}$ = shaft work $\equiv$ Isolate portion of the work that is done by a M/c (pump impeller, fan blade, pistons) $\rightarrow$ protrude through the controll surface into C$\forall$



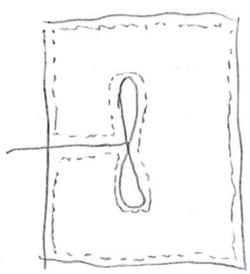
$\dot{W}_p$ = pressure work $\equiv \int_A p \vec{V} \cdot d\vec{A}$

$\dot{W}_v$ = shear work due to viscous forces $= -\int_A \vec{\tau} \cdot \vec{V} \, dA$

$\dot{W} = \dot{W}_{shaft} + \int_A p \vec{V} \cdot d\vec{A} - \int_A \vec{\tau} \cdot \vec{V} \, dA$

$q - \dot{W}_{shaft} - \int_A p\vec{V} \cdot d\vec{A} + \int_A \vec{\tau} \cdot \vec{V} \, dA = \frac{d}{dt}\int e_t \rho \, d\forall + \int_A e_t \rho \vec{V} \cdot d\vec{A}$

or $\dot{Q}_1 - \dot{W}_{shaft} + \int_A \vec{\tau} \cdot \vec{V} \, dA = \frac{d}{dt}\int_{\forall} e_t \rho \, d\forall + \int_A \left(e_t + \frac{P}{\rho}\right) \rho \vec{V} \cdot d\vec{A}$



$\int \vec{\tau} \cdot \cancel{\vec{V}} \, dA$ will ignore the viscous term most of the time because shaft work accounts for energy in.

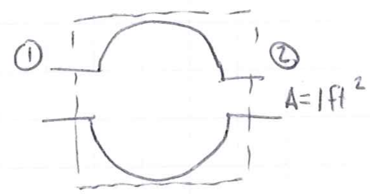
$\dot{Q} - \dot{W}_{shaft} = \frac{d}{dt}\int_{\forall} e_t \rho \, d\forall + \int_A \left(e_t + \frac{P}{\rho}\right) \rho \vec{V} \cdot d\vec{A}$

$e_t + \frac{P}{\rho} = \left(u_i + \frac{P}{\rho}\right) + \frac{V^2}{2} + gz$

$$\boxed{e_t + \frac{P}{\rho} = h + \frac{V^2}{2} + gz}$$

$$\boxed{\dot{Q} - \dot{W}_{shaft} = \frac{d}{dt}\int_{\forall}\left(u_i + \frac{V^2}{2} + gz\right)\rho \, d\forall + \int_A \left(h + \frac{V^2}{2} + gz\right) \rho \vec{V} \cdot d\vec{A}}$$

set 9 #1



① $P = 14.7\,psia$, $T = 70°F$, $V_1 \simeq 0$

② $T = 700°F$, $P = 50\,psia$

$\dot{W}_{shaft} = -600\,hp$

$$\dot{Q} - \dot{W}_{shaft} = \cancel{\frac{d}{dt}\int_{\forall} (\ \ )\rho\, dV} + \int \left(h + \frac{V^2}{2} + gz\right)\rho \vec{V}\cdot d\vec{A}$$

assume: ① Steady flow ② Since $V_1 \approx 0$ inlet K.E. is negligible, ③ Air → ideal gas ④ ignore change in elevation

$$\dot{Q} = \dot{W}_{shaft} - \left(h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right)\rho V_1 A_1 + \left(h_2 + \frac{V_2^2}{2} + \cancel{gz_2}\right)\rho V_2 A_2$$

$\rho V_1 A_1 = \rho V_2 A_2 = \dot{m} = 20\ \frac{lbm}{s}$

$$\dot{Q} = \dot{W}_{shaft} + \dot{m}\left[(h_2 - h_1) + \frac{V_2^2}{2}\right]$$
$$= \dot{W}_{shaft} + C_p(T_2 - T_1) + \dot{m}\frac{V_2^2}{2}$$

$\dot{m} = \rho_2 V_2 A_2 \Rightarrow V_2 = \frac{\dot{m}}{\rho_2 A_2}$

$$\boxed{\dot{Q} = -277\left(\frac{Btu}{s}\right)}$$

$\rho_2 = \frac{P_2}{RT_2}$ $\qquad V_2 = 32.9\ ft/s$

Steady Flow energy equation

Assumptions: steady flow, 1D flow w/ 1 inlet & 1 outlet

$$\dot{Q} - \dot{W}_{shaft} = -\dot{m}_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right)$$

$\frac{\dot{Q}}{\dot{m}} = q \qquad \frac{\dot{W}_{shaft}}{\dot{m}} = \cancel{ws}\ w_s$

$$q - w_s = \left(h_2 + \frac{V_2^2}{2} + gz_2\right) - \left(h_1 + \frac{V_1^2}{2} + gz_1\right)$$

$\boxed{\&\ h = u + \frac{P_2}{\rho}}$

divide everything by g

this gives units of length for all terms

$h_q = \frac{q}{g} \qquad h_s = \frac{w_s}{g}$

heights

$$\boxed{\left(\frac{u_1}{g} + \frac{P_1}{g} + \frac{V_1^2}{2g} + z_1\right) = \left(\frac{u_2}{g} + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2\right) - h_q + h_s}$$

$\frac{P}{\gamma} \equiv$ Pressure head $\qquad \frac{V^2}{2g} \equiv$ velocity head/ kinetic head $\qquad z \equiv$ elevation head

Low speed flows:

$$\underbrace{\left(\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1\right)}_{\text{total head ①}} = \underbrace{\left(\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2\right)}_{②} + \underbrace{\frac{u_2-u_1}{g} - h_q}_{h_f} + h_s$$

$\frac{u_2-u_1}{g} - h_q = h_f \equiv$ frictional head loss $\qquad h_s =$ shaft work

$h_s = +h_{pump}$ (adding energy to the system) $= -h_{turbine}$ (extracting energy from the system)

$\alpha_1, \alpha_2 =$ kinetic energy correction factor, goes in $\frac{V^2}{2g}$ term

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Set 10 #1 $\quad h_p + \left(\frac{P_1}{\rho g} + \alpha_1\frac{V_1^2}{2g} + \cancel{z_1}\right) = \frac{P_2}{\rho g} + \alpha_2\frac{V_2^2}{2g} + \cancel{z_2} + \cancel{h_f}^{\,0}$

$h_p = \frac{P_2 - P_1}{\rho g} + \alpha\frac{V_2^2 - V_1^2}{2g} \qquad V_1 = \frac{Q}{A_1} = 19.9\,m/s \qquad V_2 = \frac{Q}{A_2} = 8.84\,m/s$ (flow rate)

$h_p = 12.6\,m$

$\dot{W}_{pump} = \rho g Q h_p = 860\,(9.81)(0.1)(12.6) = 10.6\,kW$

$\dot{W}_{shaft} = \eta_{motor} \cdot \dot{W}_{motor} = .9\,(25\,kW) \qquad \eta_{pump} = \frac{10.6\,kW}{.9 \times 25} = \boxed{47.1\,\%}$

Notes 9-30-10

HW #4 due Thursday

Bernoulli Equation: "Frictionless" Flow

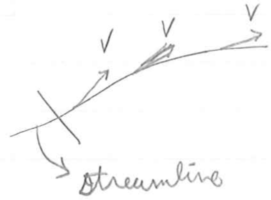
① Flow is inviscid ($\mu = 0$) ie no friction

② Incompressible Flow

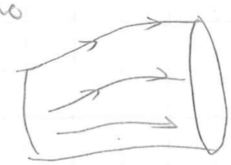
③ Flow is along a "streamline"

- most widely used eq'n
- derivation by Daniel Bernoulli (1738)

"streamline" a line of flow such that velocity vector is always tangential to that line

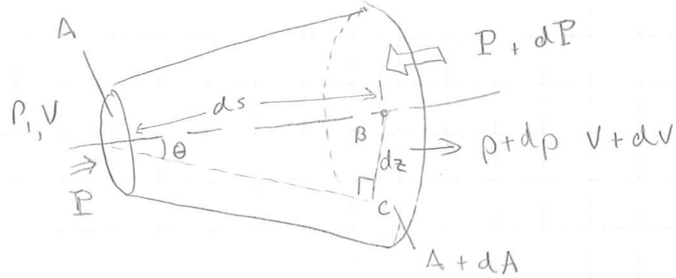


→ no flow across a streamline



stream-tube

conservation of mass: $\frac{d}{dt}\int_{\forall} \rho \, d\forall + \int_A \rho \vec{V} \cdot d\vec{A} = 0$



cons. momentum $\Sigma dF = \underbrace{\frac{d}{dt}\int_{\forall} \vec{V}\rho \, d\forall}_{\text{changes in momentum}} + \underbrace{\int_A \vec{V}\rho\vec{V}\cdot d\vec{A}}_{\text{momentum flux}}$

$$-\rho g d\forall - A dP \simeq \frac{d}{dt}(\rho\vec{V}) \cdot A \, ds + d(\dot{m}\vec{V})$$

$$\Sigma dF = dF_B + dF_s = -\rho g d\forall$$

Notes 9-30-10

$$\boxed{\int_1^2 \frac{d\vec{V}}{dt}\cdot ds + \int_1^2 \frac{dP}{\rho} + \frac{1}{2}\left(V_2^2 - V_1^2\right) + g(z_2 - z_1) = 0} \quad (2)$$

- Bernoulli eqn for unsteady, incompressible flow (frictionless) along a streamline

$$\boxed{\frac{P_1}{\rho} + \frac{V_1^2}{2} + g z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + g z_2} \quad (3)$$

- Bernoulli for steady incompressible flow

Form A: divide eq. (3) by "g"

$$\boxed{\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2}$$ dimension is "Length"

$\frac{P}{\gamma}$ Pressure head, represents height of a liquid column to produce pressure P

$\frac{V^2}{2g}$ velocity head, vertical dist of fluid needed to travel to reach a velocity "V" from rest

z elevation head, represents the PE of a fluid particle

Form B: multiply eqn (3) by 'ρ'

$$\boxed{P + \frac{1}{2}\rho V^2 + \gamma z = \text{constant}}$$ dimension of terms "$\frac{\text{Force}}{\text{Area}}$"

P static pressure, actual thermodynamic pressure of the liquid *

$\frac{1}{2}\rho V^2$ dynamic pressure

Above definitions to be used often

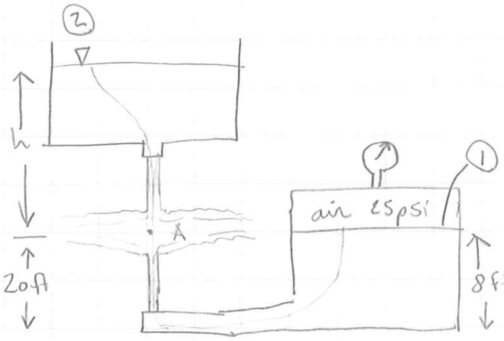
γz "hydrostatic" pressure

* to measure this, you must travel wit the same velocity of fluid

Notes 9-30-10

EX 1 In class problem set 11

$V_A = 0$ A: stagnation pt. → dynam. pressure = 0



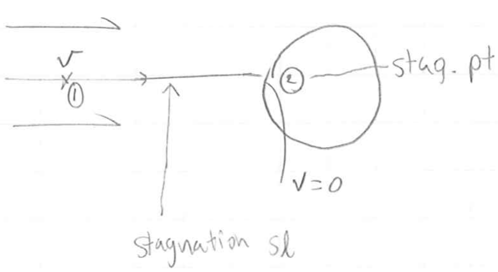
$$\frac{\cancel{P_2}}{\gamma} + \frac{\cancel{V_2^2}}{2g} + z_2 = \frac{P_A}{\gamma} + \frac{\cancel{V_A^2}}{2g} + z_A$$

$$\frac{P_A}{\gamma} = z_2 - z_A = (20+h)\,(-20)$$

$$\boxed{P_A = \gamma h}$$

$$\frac{P_A}{\gamma} + \frac{\cancel{V_A^2}}{2g} + z_A = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 \qquad h + 0 + 20 = \frac{P_1}{\gamma} + 0 + 8$$

$$h = \frac{P_1}{\gamma} - 12 \qquad \frac{P_1}{\gamma} = \frac{25\ \frac{lb}{in^2} \times 144\ \frac{in^2}{ft^2}}{62.4\ lb/ft^3} \qquad \boxed{h = 45.7\ ft}$$



$$P_1 + \frac{1}{2}\rho V_1^2 + \cancel{\gamma z_1} = P_2 + \frac{1}{2}\cancel{\rho V_2^2} + \cancel{\gamma z_2}$$

$$\Rightarrow P_2 = P_1 + \frac{1}{2}\rho V_1^2$$

(Stagnation pressure) = (static pressure) + (dynamic pressure)

— Talked about P. 2 —

$$P_2 = -\frac{1}{2}\rho V_2^2$$

$$P_2 + \gamma_{H_2O}\, h - \gamma_{oil} \cdot \frac{1}{12} = 0$$

$h = .223$ ft

9-30-10

Basic Flow Analysis

ⓐ control volume → integral equations (ch 3)

↳ approximate approach

ⓑ Exact approach → "fluid elements" → differential approach

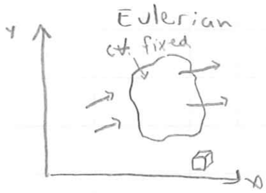
(chap. 4) ↳ PDE's govern fluid flow

↳ solve PDE's

fluid elements ↳ simplification PDE → ODE (Few basic soln's)

Viewpoints for fluid flow motion

Eulerian



"on a bridge"

Lagrangian

observer "on the boat"

Velocity field $\vec{V}(x,y,z,t) = u\hat{i} + v\hat{j} + w\hat{k}$ $u(x,y,z,t)$

$v(x,y,z,t)$

accel field: $\frac{D\vec{V}}{dt} = \frac{Du}{Dt}\hat{i} + \frac{DV}{dt}\hat{j} + \frac{dW}{dt}\hat{k}$ $w(x,y,z,t)$

$$\frac{Du}{dt} = \underbrace{\frac{\partial u}{\partial t}}_{} + \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{}$$

$\vec{a} = \frac{D\vec{V}}{dt} = \frac{\partial V}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}$

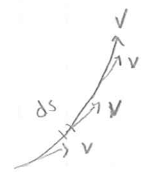
(total component) = (local comp) + (convective component)

$$V\cdot\nabla = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Example 4.1 (White) $\vec{V} = 3t\hat{i} + xz\hat{j} + ty^2\hat{k}$ ans: $\vec{a} = 3\hat{i} + t(3z + xy^2)\hat{j} + y(y + 2t + xz)\hat{k}$

Flow pattern visualization:

⊙ streamline: always tangent to velocity



$\vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$$V \times ds = 0 = \begin{vmatrix} i & j & k \\ u & v & w \\ dx & dy & dz \end{vmatrix}$$

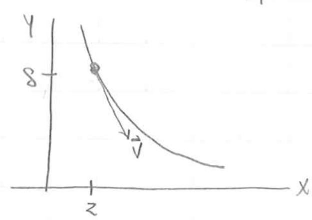
$$\left.\begin{aligned} i&: \; v\,dz - w\,dy = 0 \;\rightarrow\; v\,dz = w\,dy \\ j&: \; u\,dz - w\,dx = 0 \;\ldots \\ k&: \; u\,dy - v\,dx = 0 \;\ldots \end{aligned}\right\} \underbrace{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}_{\text{eqn of a streamline}}$$

Set 13 ① $\vec{V} = ax\hat{\imath} - ay\hat{\jmath}$; $a = .1\left(\frac{1}{s}\right)$ $x, y = m$ $\frac{dx}{u} = \frac{dy}{v}$

eqn of SL $u = ax$, $v = -ay$ | $\frac{dx}{\cancel{a}x} = \frac{-dy}{\cancel{a}y}$ on integration $\Rightarrow \int \frac{dx}{x} = -\int \frac{dy}{y}$

$\ln x = -\ln y + \ln c$ $\ln(xy) = \ln c$ $\Rightarrow xy = c$

b) $(x, y) = 2(8)$ $xy = 16$

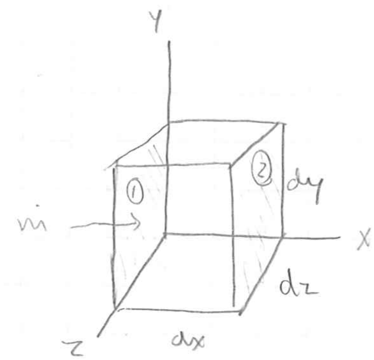


c) $\vec{V} = -.2\hat{\imath} - .8\hat{\jmath}$

(b) Path line - a path traced out by moving particles (not dependent on time)

Note: ⓐ SL & PL are the same for steady flows ⓑ for unsteady flows, we will have "instantaneous SL" and SL ≠ PL

Conservation of mass (differential approach)



$d\forall = dx\,dy\,dz$ 6-faces → 3 inflows, 3 outflows

$\dot{m}_1 = \rho AV = \rho\,dy\,dz \cdot u$ $\dot{m}_2 = \left[\rho u + \frac{\partial}{\partial x}(\rho u)\cdot dx\right] dy\,dz$

Face	$\dot{m}$ in	$\dot{m}$ out	differential mass flow
x	$\rho u\,dy\,dz$	$[\rho u\ \frac{\partial}{\partial x}\rho u\,dx]\,dy\,dz$	$\frac{\partial}{\partial x}(\rho u)\,dx\,dy\,dz$
y	$\rho v\,dx\,dz$	$[\rho v\ \frac{\partial}{\partial y}\rho v\,dy]\,dx\,dz$	$\frac{\partial}{\partial y}(\rho v)\,dy\,dx\,dz$
z	$\rho w\,dx\,dy$	$[\rho w\ \frac{\partial}{\partial z}\rho w\,dz]\,dx\,dy$	$\frac{\partial}{\partial z}(\rho w)\,dx\,dy\,dz$

previously used integrals

$\int$ $\int$

or $\frac{\partial}{\partial t}\int_V \rho\,d\forall + \sum_i(\rho_i A_i V_i)_{out} - \sum_i(\rho_i A_i V_i)_{in} = 0$

or $\left(\frac{\partial \rho}{\partial t}\right)\cancel{d\forall} + \left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]\cancel{d\forall} = 0$

$\boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0}$ true for

- steady / unsteady
- compressible / incompressible
- viscous / inviscid

Cons. of mass, DE approach (continued)

Cylindrical coordinates: $\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho V_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho V_\theta) + \frac{\partial}{\partial t}(\rho V_z) = 0$

continuity equation (continuous functions)

Special cases: ① steady compressible flow $\frac{\partial}{\partial t} = 0 \Rightarrow \boxed{\nabla \cdot (\rho \vec{v}) = 0}$

$\Rightarrow \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho v) = 0$

② steady incompressible $\rho =$ constant $\Rightarrow \rho \vec{v} \cdot \vec{v} = 0$

$\Rightarrow \boxed{\nabla \cdot \vec{v} = 0}$ $\quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{d w}{\partial z} = 0$

Q: when is fluid incompressible? $\frac{\partial}{\partial x}(\rho u) = \underbrace{u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x}}_{\text{chain rule}}$

~~if~~ ρ const: $\frac{\partial}{\partial x}(\rho u) = \rho \frac{\partial u}{\partial x}$, So $\left|u\frac{\partial \rho}{\partial y}\right| << \rho \frac{\partial u}{\partial x}$ or $\frac{\delta \rho}{\rho} << \frac{\delta u}{u}$

- speed of sound $a^2 = \frac{\partial P}{\partial \rho} \Rightarrow \delta \rho = \frac{\partial P}{a^2}$ from bernoulli's: $\delta p = -\rho u \, \delta u$

$\left|\frac{\delta P}{\rho a^2}\right| << \frac{\delta P}{\delta u^2} \Rightarrow \frac{u^2}{a^2} << 1$

$Ma = \frac{\text{local velocity}}{\text{speed of sound}}$ $\quad Ma^2 << 1 \Rightarrow \overset{\text{incomp.}}{\boxed{Ma >> 0.3}}$

↑ $\quad$ ↑ if $Ma^2 \approx 0.01$

$\left(\frac{\text{velocity of fluid}}{\text{Velocity of sound in the fluid}}\right)$

Stream function: valid for 2D flow only

continuity (2D) $\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$ + incompressible $\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0$ $\quad$ Solving for (u, v)

$(u, v) \longrightarrow \psi$ (streamfunction) $\quad$ Define: (2d, incompressible flow)

$\boxed{u = \frac{\partial \psi}{\partial y} ; \; v = \frac{\partial \psi}{\partial x}}$ $\quad$ ψ must satisfy continuity eqn $\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \ldots 0$ ok

Sign convention: Sign is on 'v' and NOT 'u', convention is selected so that ψ increases in the +ve y direction when flow is from left to right

$d\psi = 0$

Note: curves of constant ψ are streamline of the flow

$\psi = \psi(x, y)$ $\quad d\psi = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} dy$

$-v \cdot dx + (u) dy = 0$ $\quad \boxed{\frac{dx}{u} = \frac{dy}{v}} \rightarrow$ eqn of SL

Set 13

(2)

$u = x^3 - 3xy^2$ → 2d, steady

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = \boxed{-3x^2 + 3y^2}$ integrate

$v = v(x,y)$ $\quad v(x,y) = \int(-3x^2+3y^2)\,dy = -3x^2y + y^3 + f(x)$

a) $\boxed{v(x,y) = y^3 - 3x^2y + f(x)}$

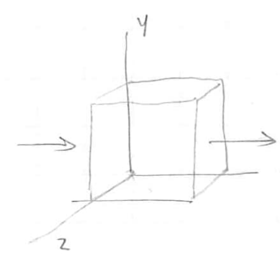
$\boxed{v(x,y) = y^3 - 3x^2y + g(x,t)}$ so, Valid for unsteady flow also

Conservation of momentum · Navier stokes equation (1840)

(1785-1836) (1819-1903)

Nonlinear PDE → all flows. No closed form solution of this eq.
simplifying assumptions → solutions for specific cases

$$\Sigma \vec{F} = \frac{\partial}{\partial t}\int_{\forall} \vec{V}\rho\, d\forall + \int_A \vec{V}\rho\, \vec{V}\cdot d\vec{A}$$



$$\boxed{\rho\frac{D\vec{V}}{Dt} = -\nabla p + \rho\vec{g} + \mu\nabla^2\vec{V}}$$

like surface force · body force · viscous force

inertial force

Pg 238-243 derivation

Boundary & initial conditions, ex.

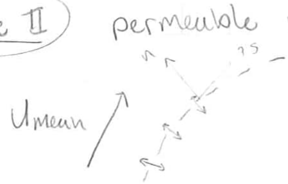
in. cond.: unsteady flows a @ $t=0$; $\rho\,\vec{V}\,P = \rho(x,y,z)\ \vec{V}(x,y,z),\ P(x,y,z)$

boundary cond: Type I: rigid wall

no slip b.c. —  $V_{fluid} = V_{wall} = 0$ (for special cases)

no temp jump b.c. — $T_{fluid} = T_{wall}$

Type II permeable wall



V_s → no slip b.c.

$V_{normal} \neq 0$ → +ve/-ve

$T_{fluid} = T_{wall}$ (suction)

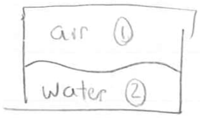
$$k\left.\frac{\partial T}{\partial N}\right|_{wall} = \rho_{wall} V_n C_p (T_w - T_{fluid})$$

conduction · convection

Notes 10-7-10

Boundary & initial conditions continued

Type III   liquid/liquid or liquid/vapor interface



$S=0$

$V_1 = V_2 \quad T_1 = T_2 \quad \tau_1 = \tau_2 \quad q_1 = q_2 \quad \dot{m}_1 = \dot{m}_2$

Type IV: infinite domain as $x \to \pm\infty$; $u \to U_\infty$ ← free stream velocity (finite value)

Type V Symmetry: cylindrical coord's $\frac{\partial}{\partial \theta} = 0$

Notes 10-12-10

Exam 2 Oct. 21 Rec. tonight 6-8 (cannot go, EWB ice breaker)

Set 13

(3) Steady incomp. velocity field $\vec{V} = (ax+b)\hat{i} + (-ay+c)\hat{j}$

Calculate $P = P(x,y)$

cons. mass: $\cancel{\frac{\partial \rho}{\partial t}}^{0} + \nabla\cdot(\rho\vec{V}) = 0 \Rightarrow \boxed{\nabla\cdot\vec{V} = 0}$ or

Steady

$\rightarrow \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad \nabla \equiv \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\cancel{\frac{\partial}{\partial z}}^{0} \qquad \vec{V} = u\hat{i} + v\hat{j}$

$u = ax+b \Rightarrow \frac{\partial u}{\partial x} = a \qquad v = -ay+c \Rightarrow \frac{\partial v}{\partial y} = -a \qquad$ so $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ ✓

X momentum: Mom. eq'n: $\rho\frac{d\vec{V}}{dt} = -\nabla p + \rho\vec{g} + \mu\nabla^2\vec{V}$

$\rho\left[\cancel{\frac{\partial u}{\partial t}}^{0} + u\frac{\partial u}{\partial x} + \cancel{v\frac{\partial u}{\partial y}}^{0}\right] = -\frac{\partial P}{\partial x} + \cancel{\rho g_x}^{0} + \mu\left[\cancel{\frac{\partial^2 u}{\partial t^2}}^{0} + \cancel{\frac{\partial^2 u}{\partial y^2}}^{0}\right]$ should be dx?

$\rho u\frac{\partial u}{\partial x} = -\frac{\partial P}{\partial x} \Rightarrow \rho(ax+b)a = -\frac{\partial P}{\partial x} \qquad \frac{\partial P}{\partial x} = \rho(-a^2x - ab)$ integrate

$P(x,y) = -\rho a^2\frac{x^2}{2} - \rho abx + g(y)$ how do we find what $g(y)$ is?

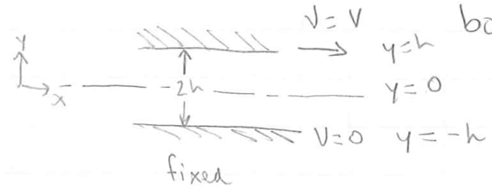
Y-momentum: $\rho\left[\cancel{\frac{\partial \rho}{\partial t}}^{0} + \cancel{u\frac{\partial v}{\partial x}}^{0} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial P}{\partial y} + \cancel{\rho g_y}^{0} + \mu\left[\cancel{\frac{\partial^2 v}{\partial x^2}}^{0} + \cancel{\frac{\partial^2 v}{\partial y^2}}\right]$

$\rho v\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} \qquad \frac{\partial P}{\partial y} = \rho(-a^2y + ac) = g'y$ (w/r/t x-mom. eq'n above)

$g(y) = \rho(-a^2\frac{y^2}{2} + acy) + k \qquad \boxed{P(x,y) = \rho\left[-\frac{a^2x^2}{2} - \frac{a^2y^2}{2} - abx + acy\right] + k}$

① wall driven flow → (gravity & pressure gradients are neglected)
② gravity driven flow → pressure gradients are neglected
③ Pressure driven flow → gravity is neglected
1) Couette flow 3) Poiseuille Flow

① **Wall-driven:**



boundary conditions: $\underbrace{@y = +h,\ u = V \quad @y = -h,\ u = 0}$

no slip bc's: valid for 99.99% of situations

assumptions: a) incompressible b) steady flow c) 2d flow: $\frac{\partial}{\partial z} = 0$
d) Flow is axial $v = w = 0$

continuity eq'n $\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{0} = 0 \Rightarrow u \neq u(x)$, only $u(y)$

basic eqn memorize if you have the time,

x momentum: $\rho\left[\cancel{\frac{\partial u}{\partial t}}^{0} + u\cancel{\frac{\partial u}{\partial x}}^{0} + v\cancel{\frac{\partial u}{\partial y}}^{0}\right] = -\cancel{\frac{\partial p}{\partial x}}^{0} + \rho g_x^{\,0} + \mu\left[\cancel{\frac{\partial^2 u}{\partial x^2}}^{0} + \frac{\partial^2 u}{\partial y^2}\right]$

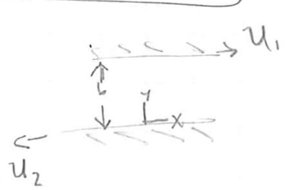
$\mu \frac{d^2u}{dy^2} = 0$ $\frac{d^2u}{dy^2} = 0$ integrate x2 $\Rightarrow$ $\boxed{u(y) = C_1 y + C_2}$

profile 

$@y = +h \quad V = C_1 h + C_2$
$@y = -h \quad 0 = C_1 h + C_2$
$\Big\}\ C_1 = \frac{V}{2h} \quad C_2 = \frac{V}{2}$

$u(y) = \frac{V}{2}\left[1 + \frac{y}{h}\right]$

Set 14 #1)

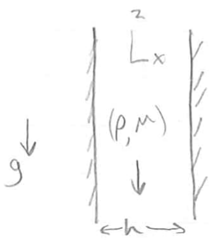


BC's)
$y = 0,\ u = -U_2 \quad y = b,\ u = U_2$. $u(y) = C_1 y + C_2$

$U(y) = (U_1 + U_2)\frac{y}{b} - U_2$

profile 

② gravity driven flow, ex prob. set 14 # 2



fixed walls

assumptions: incompressible, steady flow, 2D flow $\partial/\partial y = 0$

a) flow is axial $u, v = 0$ ← show this

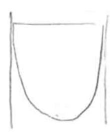
continuity eq'n $\cancel{\frac{\partial u}{\partial x}}^{0} + \cancel{\frac{\partial v}{\partial y}}^{0} + \frac{\partial w}{\partial z} = 0$ $w = w(x)$ only

z-momentum $\rho\left[\cancel{\frac{\partial w}{\partial t}}^{0} + u\cancel{\frac{\partial w}{\partial x}}^{0} + v\cancel{\frac{\partial w}{\partial y}}^{0} + w\cancel{\frac{\partial w}{\partial z}}\right] = -\cancel{\frac{\partial P}{\partial z}}^{0} + \rho g_z + \mu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right]$

or $\boxed{-\rho g + \mu \frac{d^2 w}{dx^2} = 0}$

$\frac{\partial^2 w}{\partial x^2} = \frac{\rho g}{\mu}$ $w(x) = \frac{\rho g}{2\mu} x^2 + C_1 x + C_2$ (parabolic)

@ $x = \frac{h}{2}$; $w = 0$ @ $x = \frac{-h}{2}$, $w = 0$ $\boxed{w(x) = \frac{\rho g}{2\mu}\left[x^2 - \left(\frac{h}{2}\right)^2\right]}$



Only last 2 hw probs not yet covered
due next tues

10-14-10

③ due to pressure gradient

assume
a) incomp.
b) steady $\frac{\partial}{\partial t} = 0$
c) axial flow $v = w = 0$
d) 2D flow $\frac{\partial}{\partial z} = 0$

boundary cond: @ $y = \pm h$, $u = 0$

cons: mass/continuity $\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{0} = 0 \Rightarrow \frac{\partial u}{\partial x} = 0$

$\therefore u \neq u(x) \Rightarrow u = u(y)$ only

x-momentum

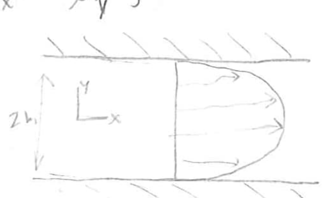
$\rho[\ /^{0}\ \ /^{0}\ \ /^{0}\ ] = -\frac{\partial P}{\partial x} + /^{0} + \mu\left[\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2}\right]$

steady continuity   continuity

or, $\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial P}{\partial x}$ Since $u = u(y)$ $\frac{d^2 u}{dy^2} = \frac{1}{\mu}\frac{\partial P}{\partial x}$ (need to know if $\partial P/\partial x$ is a function of y)

y-momentum $\rho\left[\cancel{\frac{\partial v}{\partial t}}^{0} + \cancel{\frac{\partial v}{\partial x}}^{0} + \cancel{\frac{\partial v}{\partial y}}^{0}\right] = -\frac{\partial P}{\partial y} + \rho g_y + \mu\left[\cancel{\frac{\partial^2 v}{\partial x^2}}^{0} + \cancel{\frac{\partial^2 v}{\partial y^2}}^{0}\right]$

$(v = 0)$ $\boxed{\frac{\partial P}{\partial y} = -\rho g} \rightarrow \boxed{P(x,y) = -\rho g y + f(x)}$



So, $\frac{\partial P}{\partial x} = \frac{df(x)}{dx}$ not a $f(y)$ Now, $\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{dP}{dx}$

$\frac{du}{dy} = \frac{1}{\mu}\frac{dP}{dx}(y) + C_1$ $u(y) = \frac{1}{\mu}\frac{dP}{dx}\frac{y^2}{2} + C_1 y + C_2$

continued...

with boundary conditions:

$$u(y) = 0 = \frac{1}{\mu}\frac{dP}{dx}\frac{h^2}{2} + c_1 h + c_2 \quad @\ y=h$$

$$0 = \frac{1}{\mu}\frac{dP}{dx}\frac{(-h)^2}{2} + c_1(-h) + c_2 \quad @\ y=-h \qquad c_2 = 0,\ C_1 = -\frac{1}{2\mu}\frac{dP}{dx}h^2$$

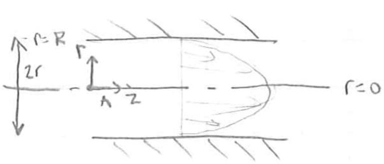
$$u(y) = -\frac{dP}{dx}\frac{h^2}{2\mu}\left[1 - \frac{y^2}{h^2}\right]$$

Summary: wall-driven $\frac{d^2u}{dy^2} = 0$ $u(y) = $ linear

gravity driven $\frac{d^2u}{dy^2} = \frac{\rho g}{\mu}$ $u(y) = $ parabolic

pressure driven $\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dP}{dx}$ $u(y) = $ parabolic pressure grad.

Fully developed Pipe flow, cylindrical coordinates → hagen-Poiseuilli Flow



assumptions: a) incomp. b) steady $\frac{\partial}{\partial t} = 0$ c) Axial Flow $V_r = V_\theta = 0$
d) rotationally symmetric $\frac{\partial}{\partial \theta} = 0$

continuity eq^n $\frac{1}{r}\frac{\partial}{\partial r}(r\cancel{V_r})^0 + \frac{1}{r}\frac{\partial}{\partial \theta}(\cancel{V_\theta})^0 + \frac{\partial}{\partial z}(V_z) = 0 \Rightarrow \frac{\partial V_z}{\partial z} = 0$

$\therefore V_z \neq V_z(z)$ V_z only $V_z(r)$

r-momentum: $\cancel{\frac{\partial V_r}{\partial t}}^0 + \cancel{(V\cdot\nabla)V_r}^0 - \cancel{\frac{1}{r}V_\theta^2}^0 = -\frac{1}{r}\frac{1}{\rho}\frac{\partial P}{\partial r} + \cancel{g_r} + \nu\left[\cancel{\nabla^2 V_r}^0 - \cancel{\frac{V_r}{r^2}}^0 - \frac{2}{r^2}\cancel{\frac{\partial V_\theta}{\partial \theta}}^0\right]$

steady ignore grav.

so $\frac{1}{\rho}\frac{\partial P}{\partial r} = 0 \Rightarrow P \neq P(r)$ ∂ $P = P(z)$ only

z momentum. $\cancel{\frac{\partial V_z}{\partial t}} + \underbrace{(V\cdot\nabla)V_z}_{\text{convective accel}} = \frac{1}{\rho}\frac{\partial P}{\partial z} + \underbrace{\cancel{g_z}}_{\text{ignore}} + \underbrace{\nu\nabla^2 V_z}_{\text{viscous dissipation}}$

steady

convective accel $(\vec{V}\cdot\nabla)V_z = \cancel{V_r}^0\frac{\partial V_z}{\partial r} + \frac{1}{r}\cancel{V_\theta}^0\frac{\partial V_z}{\partial \theta} + V_z\cancel{\frac{\partial V_z}{\partial z}}^{0,\ V_z(r)}$

Viscous dissip, laplacian operator $\nu\nabla^2 V_z = \nu\cdot\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 V_z}{\partial \theta^2}}^0 + \cancel{\frac{\partial^2 V_z}{\partial z^2}}^0$

$$\boxed{\frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = \frac{dP}{dz}} < 0 \qquad \frac{1}{\rho}\frac{dP}{dz} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right)$$

integrating last eq'n twice : $V_z(r) = \frac{1}{4\mu}\frac{dP}{dz}r^2 + C_1 \ln r + C_2$

@ $r=R$, $V_z = 0 = \frac{1}{4\mu}\frac{dP}{dz}r^2 + C_1 \ln r + C_2$

@ $r=0$, V_z = finite $\Rightarrow \frac{\partial V_z}{\partial r} = 0 \quad \therefore \underline{C_1 = 0}$

$C_2 = -r^2/_{4\mu}\frac{dP}{dz} \qquad V_z(r) = \left(-\frac{dP}{dz}\right)\frac{1}{4\mu}(R^2 - r^2)$

$V_{max} = V_z(r=0) = -\frac{dP}{dz}\cdot\frac{R^2}{4\mu} \qquad V_{avg} = \frac{1}{A}\int V_z(r)\,dA$

$V_{avg} = \frac{1}{\pi R^2}\int V_z(r)(2\pi r dr) \qquad V_{avg} = \frac{V_{max}}{2}$

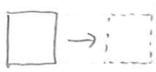
$Q = V_z\,dA = V_{avg}(A) = \frac{\pi r^4}{8\mu}\left(-\frac{dP}{dz}\right) = \frac{\pi d^4}{128\mu}\left(-\frac{dP}{dz}\right)$

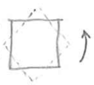
$Q \propto D^4$ double the radius, flow ↑ × 16

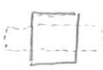
Notes 10-19-10

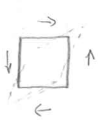
Kinematic description of fluid Motion -Not on Exam 2

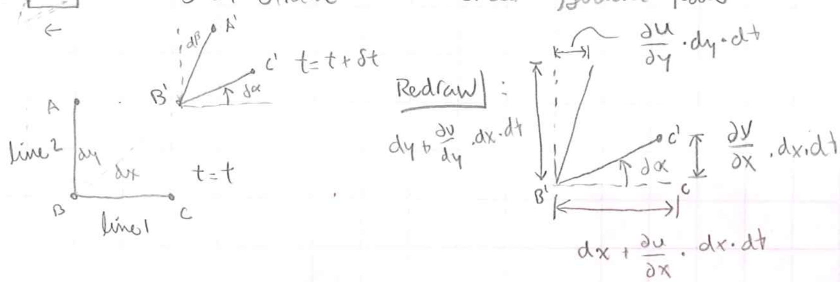
- 4 fundamental types of motion → motion of a fluid element

(a)  translation - velocity, linear $V = u\hat{i} + v\hat{j} + w\hat{k}$

(b)  rotation - angular velocity ?

(c)  linear strain (extensional strain) - linear strain rate

(d)  shear strain - shear strain rate

b) 

$d\alpha = \tan^{-1}\left(\frac{CC'}{B'C}\right)$ or, $\delta\alpha$ small $\delta\alpha = \frac{CC'}{B'C}$ $\boxed{\frac{\partial\beta}{\partial t} = \frac{\partial u}{\partial y}}$ $\boxed{\frac{dx}{dt} = \frac{\partial u}{\partial x'}}$

$\boxed{\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}$

Angular velocity (ω)

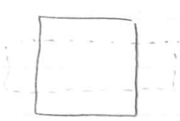
$\omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$

$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right] \qquad \omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right] \qquad \omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

Bonus Parameter: Vorticity: $\vec{\zeta} = 2\vec{\omega} = \nabla \times \vec{V}$ "curl of $\vec{V}$"

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \qquad \text{so } \vec{\omega} = \frac{1}{2}\begin{vmatrix} & \end{vmatrix}$$

if $\nabla \times \vec{V} = 0$, $\vec{\omega}, \vec{\zeta} = 0$, "IRROTATIONAL"



$$\epsilon_{xx} = \frac{\partial u}{\partial x} \qquad \epsilon_{yy} = \frac{\partial v}{\partial y} \qquad \epsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\overbrace{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}}^{\text{gradient of the velocity vector}} = \nabla \cdot \vec{V}$$

incompressible $\Rightarrow \nabla \cdot \vec{V} = 0$

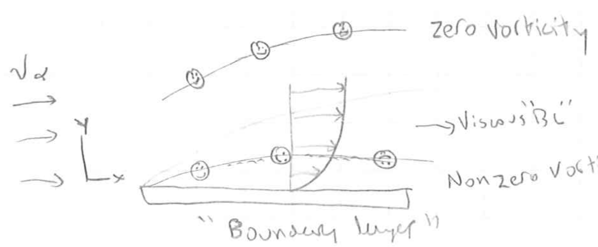
- volumetric dilatation rate $= \frac{1}{V}\frac{dV}{dt} = \nabla \cdot \vec{V} = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}$
  - Vol dilatation rate $= 0$ for incompressible flow

Shear strain rate: avg of decrease of angle between 2 mutually $\perp$ lines

$$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \epsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \qquad \epsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \epsilon_{zy}$$

correct eq. 4.113 p262

Strain rate tensor: $\epsilon_{ij} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix}$ shear strain rate; normal strain rate, principal strain



For zero vorticity $\mu \nabla^2 \vec{V} \approx 0$

$\Rightarrow \rho \frac{D\vec{V}}{Dt} = -\nabla P + \rho \vec{g}$ Euler's eqn

Velocity potential - irrotationality gives rise to a new "scalar" function called velocity potential function $\phi = \phi(x, y, z, t)$

$\vec{\zeta} = \nabla \times \vec{V} = 0 \Rightarrow \vec{V} = \vec{\nabla}\phi$ ← scalar function

$\nabla \times \nabla\phi = 0$ if $\nabla \times \vec{V} = 0$

Notes 10-19-10

Velocity pot., cartesian

$u = \frac{\partial \phi}{\partial x} \quad V = \frac{\partial \phi}{\partial y} \quad w = \frac{\partial \phi}{\partial z}$

Stream function $\psi(x,y)$ $u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \quad V = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$

$\left(\frac{dy}{dx}\right)_{\phi=c} = \frac{\partial \phi}{\partial x} \cdot \frac{\partial \psi}{\partial \phi} = -\frac{u}{V}$

$\left(\frac{\partial y}{\partial x}\right)_{\psi=c} = \frac{u}{V}$

$\frac{du}{u} = \frac{dy}{y}$ (

$\boxed{\left(\frac{dy}{dx}\right)_{\phi=c} \times \left(\frac{dy}{dx}\right)_{\psi=c} = -1}$ SL & Vpot are ⊥

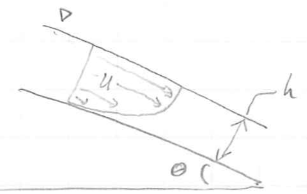
1x bernoulli
1x cons. m, mom, energy
1x diffeq's

know control vol approach CV fixed, CV moving

Cartesian coords only, not cylind.
↓
know how to simplify for each flow type
boundary eqns

Recitation 6pm
10-19-10

Prob #3



gravity driven flow

assumptions: incomp., steady, 2-d flow ($w=0$, $\frac{\partial}{\partial z}=0$)
fully developed $u \neq u(x)$ $\frac{\partial u}{\partial x}=0$

b.c.'s @ $y=0$, $u=0$ @ $y=h$, $\tau=0=\mu\frac{du}{dy}$, so $\frac{\partial u}{\partial y}=0$

continuity) $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \Rightarrow \frac{\partial v}{\partial y}=0, \Rightarrow v \neq f(y)$ $v \neq f(x)$, fully developed,
$\Rightarrow \underline{v=0}$

lose points if you assume $v=0$, must show it

navier stokes)

x-mom. $\rho\left[\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{steady}+\underbrace{u\cancel{\frac{\partial u}{\partial x}}}_{FD}+v\cancel{\frac{\partial u}{\partial y}}\right]=-\cancel{\frac{\partial p}{\partial x}}+\rho g_x+\mu\left[\underbrace{\cancel{\frac{\partial^2 u}{\partial x^2}}}_{FD}+\frac{\partial^2 u}{\partial y^2}\right]$

$\mu\frac{\partial^2 u}{dy^2}+\rho g\sin\theta=0$ $\frac{\partial^2 u}{\partial y^2}=-\rho g\frac{\sin\theta}{\mu}$ $u(y)=-\frac{\rho g\sin\theta}{\mu}\frac{y^2}{2}+C_1y+C_2$

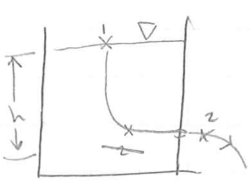
use b.c.'s $y=0 \Rightarrow C_2=0$ $y=h \Rightarrow C_1=\frac{\rho g\sin\theta h}{\mu}$ a $\boxed{}$

b) $\tau_{xy}=\mu\frac{du}{dy}=\rho g\sin\theta(h-y)$ Shear stress distribution

c) vol. flow rate = $Q=\int u\cdot dA=\int_0^h u\,b\,dy$ $\boxed{Q=\frac{\rho g\sin\theta}{\mu}\frac{bh^3}{3}}$

Nav. Stokes: x mom: d) avg. velocity $\bar{v}=\frac{Q}{A}=\frac{\frac{\rho g\sin\theta\, bh^3}{3\mu}}{bh}$ $\bar{v}=\frac{\rho g\sin\theta h^2}{3\mu}$

#4)



assumptions: (a) steady flow (b) incompressible (c)

$\frac{\cancel{P_1}}{\gamma}+\frac{\cancel{V_1^2}}{2g}+z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2$ $\frac{\cancel{P_2}}{\gamma}+\frac{V_2^2}{2g}=z_1-z_2=h$

$V_2=\sqrt{2gh}$

Notes 10-26-10

Benefits of dimensional analysis

- Save time & money
- helps with physical insight to the problem
- develop "scaling laws" (simple laws)

$s = s_0 + ut + \frac{1}{2}at^2$ Vars s_0, u, t, a, s

dimensional analysis

non-dimensionalize by s_0

$\frac{s}{s_0} = s^* \rightarrow s^* = 1 + \frac{ut}{s_0} + \frac{at^2}{2s_0}$

define $t^* = \frac{ut}{s_0}$ $= 1 + t^* +$ t^* is a non-dimensional time

so $t = \frac{s_0 t^*}{u} \Rightarrow s^* = 1 + t^* + \left(\frac{a s_0}{2u^2}\right)t^{*2}$

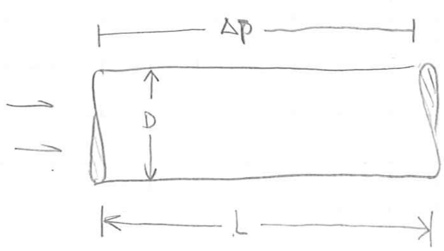
$u = LT^{-1}$ / $\frac{u^2}{s_0} = LT^{-2}$ $\boxed{s^* = 1 + t^* + \alpha t^{*2}}$

$a = LT^{-2}$

$s_0 = L$

(total # of dimensional vars) − (# principal dimensions in prob) = (Total # of non-dimen. variables)

Example Set 18 #1



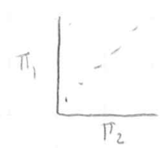
Pr. gradient $= \frac{\Delta P}{L}$

Vars: viscosity, length, diameter, pressure drop, density

→ 3 principal dimensions

Vars can be reduced to 2

$\frac{D\Delta P}{\rho V^2} = f\left(\frac{\rho V D}{\mu}\right)$ $\Pi_1 = f(\Pi_2)$



Work Example next pg.

Notes

① parameters:

② list primary dims.

$\Delta P/L$	D	V	ρ	μ
$ML^{-1}T^{-2}$	L	LT^{-1}	ML^{-3}	$ML^{-1}T^{-1}$

$N=5$
$j=3$

③ $K = n - j = 5 - 3 = 2$

④ classify variables into 3 general groups

a) geometry D

b) material properties ρ

c) External effects: V

⑤ $\Pi_1 = \Delta P \, \rho^a V^b D^c$ → solve for $a, b, c,$ & e, f, g

$\Pi_2 = \mu \, \rho^e V^f D^g$

$\Delta P \rho^a V^b D^c = M^0 L^0 T^0$

$[ML^{-2}T^{-2}][ML^{-3}]^a[LT^{-1}]^b[L]^c$

$$\begin{cases} M: 1 + a = 0 \Rightarrow a = -1 \\ L: -1 - 3a + b + c = 0 \quad c = 1 \\ T: -2 - b = 0 \Rightarrow b = -2 \end{cases}$$

so $\Pi_1 = \frac{\Delta P}{L} \rho^{-1} V^{-2} D$

for $\Pi_2 \Rightarrow$ $\Pi_2 = \frac{\mu}{\rho V D}$ now $\Pi_1 = f(\Pi_2)$

Set 17 Notes 10-28

#2)

Step 1 - $n = 5$

Step 2: primary dimensions

Step 3

F	V	D	ρ	μ
MLT^{-2}	LT^{-1}	L	ML^{-3}	$ML^{-1}T^{-1}$

Step 4 repeating parameters $\Rightarrow j = 3$

5) choose repeating parameters Geometry D; Mat. Properties ρ; Ext. effects, V

Step 6: choose Π groups $5 - 3 = 2$ $\Pi_1 = f(\Pi_2)$

$\Pi_1 = F \rho^a v^b D^c = [MLT^{-2}][ML^{-3}]^a[LT^{-1}]^b[L]^c = M^0 L^0 T^0$

$\Pi_2 = \mu \rho^d v^e D^f$

M: $1 + a = 0$

L: $1 - 3a + b + c = 0$

T $-2 - b$

$a = -1$

$b = -2$

$c = -2$

$$\boxed{\Pi_1 = \frac{F}{\rho V^2 D^2}}$$

$$\boxed{\frac{F}{\rho V^2 D^2} = f\left(\frac{\mu}{\rho V D}\right)}$$

hint of the day try to choose: Velocity, ρ, L or Mass

Notes 10-28-10

Theory of models

$\Pi_1 = \phi(\Pi_2, \Pi_3, \Pi_4, \Pi_k)$

$\Pi_{1,m} = \phi(\Pi_{2m}, \Pi_{3m}, \Pi_{4m}\ \Pi_{km})$

- geometric similarity
- Kinematic similarity – velocities @ corresponding pts differ by a const. scale factor
- dynamic similarity - forces differ by a const. scale factor

Set 18 #1)

using $\frac{F}{\rho V^2 D^2} = f\left(\frac{\mu}{\rho V D}\right)$ dynamic similarity $\underbrace{\left(\frac{\rho V D}{\mu}\right)_m}_{\text{reynolds \#}} = \left(\frac{\rho V D}{\mu}\right)_p$

$V_p = \frac{5\ n\text{-}mi}{hr}\left(\frac{6080\ ft}{n\text{-}mi}\right)\left(\frac{hr}{3600\ sec}\right) = 8.44\ ft/s$

$\rho_p =$ (Sea-water 5°C) $= 1.99\ slug/ft^3$

$\nu_p = \frac{\mu}{\rho} = 1.68 \times 10^{-5}\ ft^2/s$

$\rho_{air} = .00238\ slug/ft^3$

$Re_p = \frac{\rho_p V_p D_p}{\mu_p} = \boxed{5.02 \times 10^5}$

so Re_m must $= 5.02 \times 10^5$

b) if $\Pi_{2p} = \Pi_{2m}$ then $\Pi_{1p} = \Pi_{1m}$

a) $\Pi_{1p} = f(\Pi_{2p})$

$\Pi_{1m} = f(\Pi_{2m})$

You know some variables: ⇒ found V_m to be used

$\frac{F_m}{\rho_m V_m^2 D_m^2} = \frac{F_p}{\rho_p V_p^2 D_p^2}$ $F_p = F_m\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p D_p}{V_m D_m}\right)$

This material on blackboard, read it.

Try to do #2 before tuesday

Multiple pi groups

<u>example : Centrifugal pump</u> Set 18 #2

notes:

pump head $h = g_1(\rho, V, D, \mu)$

reynolds #, $Re = \frac{\rho V D}{\mu} = \frac{\rho(\omega D) D}{\mu} = \frac{\rho \omega D^2}{\mu}$

$\Delta P = f(D, \omega, \rho, Q)$

step 1: $n = 5$ variables

step 2: Primary dimensions MLT ; $j = 3$

step 3: Π groups, $n - j = 2$

step 4: D, ρ, ω $\Rightarrow$ $\Pi_1 = \Delta P \rho^a \omega^b D^c = M^0 L^0 T^0$

$\Pi_2 = Q \rho^d \omega^e D^f = M^0 L^0 T^0$

$\Pi_1 = \frac{\Delta P}{\rho \omega^2 D^2}$ $\quad \Pi_2 = \frac{Q}{\omega D^3}$ $\quad \frac{\Delta P}{\rho \omega^2 D^2} = f\left(\frac{Q}{\omega D^3}\right)$

<u>Theory of models</u>: $\Pi_{1m} = f(\Pi_{2m})$, $\Pi_{1p} = f(\Pi_{2p})$

then if $\Pi_{2m} = \Pi_{2p} \Rightarrow \Pi_{1m} = \Pi_{1p}$

<u>note</u> $\left(\frac{Q}{\omega D^3}\right)_p = \left(\frac{Q}{\omega D^3}\right)_m$ ← kinematic similarity

$Q_m = Q_p \left(\frac{\omega_m}{\omega_p}\right)\left(\frac{D_m^3}{D_p}\right) = 1.185 \text{ ft}^3/\text{s}$ $\quad \left(\frac{\Delta P}{\rho \omega^2 D^2}\right)_m = \left(\frac{\Delta P}{\rho \omega^2 D^2}\right)_p$

$\Delta P_p = \Delta P_M \frac{(\rho \omega^2 D^2)_p}{(\rho \omega^2 D^2)_m}$ find ΔP_m, use graph @ $Q_m = 1.185$, $\Delta P = 5.5$ psi

$\boxed{\Delta P_p = 27.84 \text{ psi}}$

— From now on, banerjee follows the book closely

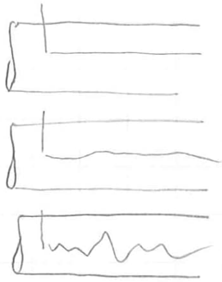
laminar or turbulent flow Reynolds #: ratio of $\frac{\text{inertial force}}{\text{viscous force}}$

$Re = \frac{\rho V D}{\mu}$ - increase the pressure difference induces greater turbulence

11-2-2010

Re

< 2100



smooth pipe, rounded entrance,

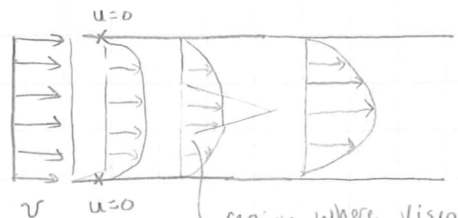
$2100 < Re < 4000$

Critical reynolds # is function of geometry of situation

> 4000

Large Re: viscous force not important at large scales of flow

☆ Entrance region effects

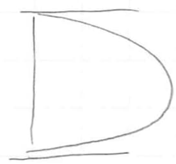


region where viscosity slows down the flow

how long is entrance profile?

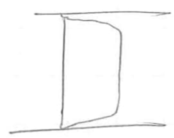
$\frac{l}{D} = 0.06 Re$

laminar



$\frac{l}{D} =$ ______

turbulent



head loss: H_L

major losses - friction effects

minor losses - fittings, area changes

Notes 11-4-2010 (Copied from Zach J.)

9) $u = f\left(h, \frac{dP}{dx}, \mu, y\right)$ 5 var, -3 Pd = 2 Pi groups

$h, \frac{dP}{dx}, y, \mu$ (No other choice) $\quad L \quad \frac{dP}{dx}\ \frac{M}{L^2T^2} \quad \frac{M}{LT}(\mu)$

$uh^A\left(\frac{dP}{dx}\right)^B \mu^C$

$\frac{L}{T} L^A \left(\frac{M}{L^2T^2}\right)^B \left(\frac{M}{LT}\right)^C = 1$

M: $B + C = 0$

L: $1 + A - 2B = 0$

T: $-1 - 2B - C = 0$

$\Pi_1 = \frac{u}{h^2\left(\frac{dP}{dx}\right)}$

$C = 1$

$B = -1$

$A = -2$

$yh^A\left(\frac{dP}{dx}\right)^B \mu^C = 1 \ldots$ $C = 0$, $B = 0$, $A = -1$

$\Pi_2 = \frac{y}{h}$

Notes 11-4-2010 (copied)

<u>headloss coefficient</u>   <u>minor loss</u>   <u>major loss</u>

minor loss: fittings, area changes, entrance

major loss: friction

<u>major losses</u> $\Delta P = f(v, D, l, \rho, \mu, \varepsilon)$ roughness factor

-7 variables   4 dim-less groups

one rep is: $\dfrac{\Delta P}{\frac{1}{2}\rho v^2} = \Theta\left(\dfrac{\rho v D}{\mu}, \dfrac{l}{D}, \dfrac{\varepsilon}{D}\right)$

$$\frac{\Delta P}{\frac{1}{2}\rho v^2} = \frac{l}{D}\cdot\phi\left(Re, \frac{\varepsilon}{D}\right) \qquad f = \frac{l}{D}\cdot\phi\left(Re, \frac{\varepsilon}{D}\right)$$

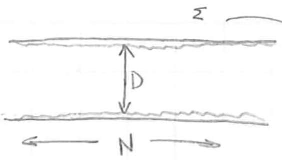
f = friction factor — moody diagram, easiest way (back cover)

$\Delta P = f\dfrac{l}{D}\left(\tfrac{1}{2}\rho v^2\right)$ if laminar $f = \dfrac{64}{Re}$

if you don't like moody diagram $\dfrac{1}{\sqrt{f}} = -2.0\log\left(\dfrac{\varepsilon/D}{3.7} + \dfrac{2.51}{Re\sqrt{f}}\right)$

$f\dfrac{l}{D}\dfrac{v^2}{2}$

$\boxed{\dfrac{\Delta P}{\gamma}}$ — $H_L = H_{L\,maj} + H_{L\,min}$



$P_1 - P_2 = f\dfrac{l}{D}\left(\tfrac{1}{2}\rho v^2\right)$

$\Delta P = (\rho g)h_L^{maj}$

γ — $\gamma h_{L\,maj}$

BBex. - read & use moody chart

$- \dfrac{dP}{dz} = P_2 - P_1$

P. Set 19-11

$\dot{m} = .25$ kg/s

$D = 40$ mm

$P_1 = 690$ kPa

$P_2 = 650$ kPa

$L = ?$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{gz_1} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2}$$ — perfect world, no losses

no Δelevation

$\therefore$ constants, so $V = C$

$\dot{m} = \rho V A$

but $+ h^L_{maj} + \cancel{h^L_{min}}$ (≈ 0)

$$\therefore \quad \frac{P_1 - P_2}{\rho} = h_{maj, L}$$

ρ use IDGL to get ρ $\boxed{\rho = \frac{P}{RT} = 8.81 \text{ kg/m}^3}$

$$h^L_{maj} = f \frac{L}{D} \frac{\bar{V}^2}{2} = \frac{\Delta P}{\rho} \qquad L = \frac{\Delta P}{\rho} \cdot \frac{2D}{f \bar{V}^2}$$

$\Delta P = 40$ kPa

$D = 40$ mm

$\vec{V} = \frac{\dot{m}}{\rho A} = 22.6$ m/s

$f =$

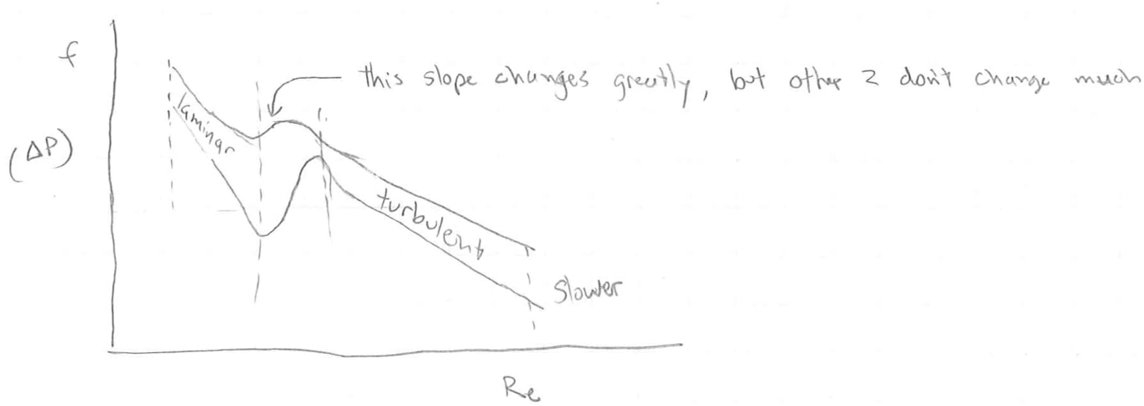
need Re for the flow

$Re = \frac{\rho \bar{V} D}{\mu}$ @ 40°C $\mu = 1.8E{-5}$ $\frac{kg}{ms}$ $\therefore = 4.42E5 > 2300 \rightarrow$ turbulent

ε not given $\rightarrow$ smooth walls

f from moody chart $\rightarrow 0.0135 \rightarrow L = 53.1$ m $\qquad L = \frac{2D}{f\bar{V}^2} \rightarrow \boxed{53.1 \text{ m}}$

but how can we neglect minor losses?



Notes 11-4
(copied)

Minor losses $\rightarrow H_{L,min} = k \frac{V^2}{2g} \rightarrow$ find for ea section

$H_L = H_{L\,maj} + H_{L\,min} = f \frac{L}{D} \frac{\vec{V}^2}{2g} + k_L \frac{\vec{V}^2}{2g}$ find based on # of fittings

Slides buy a valve, — know what loss coefficient is for specific ke #

Notes 11-9-10

Hydraulic diameter $D_h = \frac{4 \times \text{area}}{\text{wetted perimeter}} = \frac{4A}{P}$

Noncircular pipes, - approximate by using "hydraulic diameter"

$h_{f,major} = f \left(\frac{L}{D}\right) \frac{V^2}{D}$

using multiple pipes, read 408-429, fluid meters

Measuring flow - orifice pipes use $\beta = b = \frac{d}{D}$ d: diameter of obstruction
D: diameter of pipe

then find C_d, discharge coefficient
use bernoulli's eq., $Q_{actual} = C_d Q_{ideal}$
↳ get Q_{ideal} using cons. of energy. then take losses into account w/ C_d

——— All test material covered by now ———

Ch 11- TURBOMACHINERY : device used to move fluid

- most common practical application of fluid mechanics is design/analysis of turbomachinery
  - Pump - adds energy
  - turbine - extracts energy

Notes 11-9-10

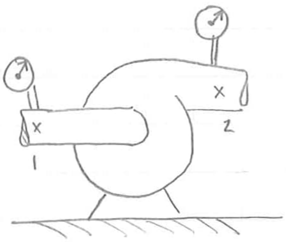
— Pumps —

- Oldest fluid energy transfer device
  250 B.C. Screw pump, archimedes
- Name of device based on type of fluid
  - liquids - pumps
  - blower - $\Delta P \approx 1 atm$,
  - compressor $\Delta P > 1 atm$
- 2 broad classifications
  1) Positive displacement
  2) dynamic or momentum change pumps * our focus

Screw pump

Note: use reading guide on slideshow

centrifugal pumps



Apply mod. Bernoulli's Eq'n

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L - h_p$$

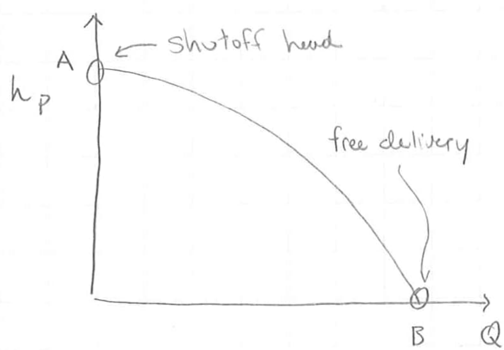
$$h_p - \cancel{h_L} = \left(\frac{P_2 - P_1}{\gamma}\right) + \left(\cancel{\frac{V_2^2 - V_1^2}{2g}}\right) + (\cancel{z_2 - z_1})^{\approx 0}$$

ignore $\quad\quad \approx 0 \quad\quad z_1 \approx z_2$

$h_p = \dfrac{\Delta P}{\gamma}$ power delivered by pump: $P_f = \gamma Q h_p = \Delta P Q$

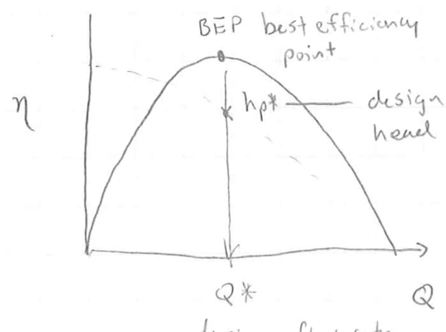
Pump efficiency (overall) $= \eta = \dfrac{P_f}{P_{in}} = \dfrac{P_f}{\dot{W}_{shaft}}$

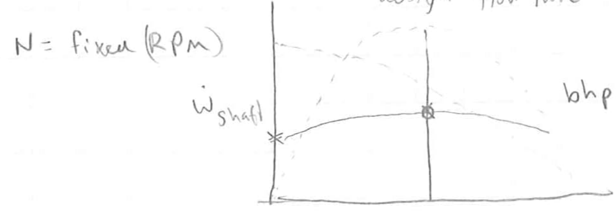
info needed for testing / selecting a pump
h_p, η, $\dot{W}_{shaft}$



A: increase in head @ zero discharge → shutoff head
all energy is converted to heat - may damage the pump
B: free delivery: no head is added to fluid - no work

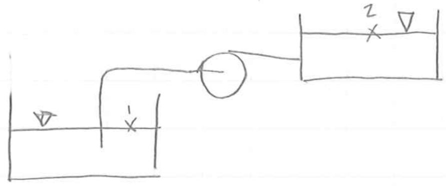
PUMP CURVES (on right)





11-9-2010

— System Characteristics / Pump selection —



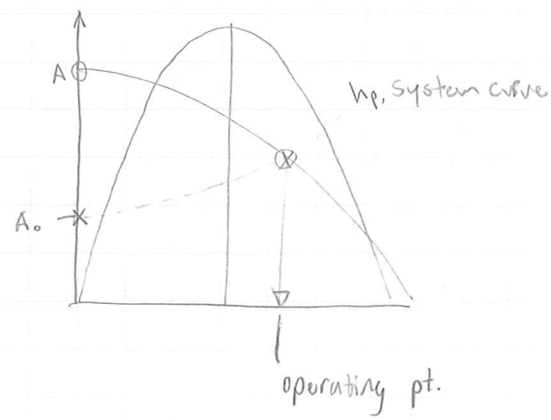
$V_1 = V_2 = 0$ (free surface)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{L}{D}\frac{\bar{V}^2}{2g} + \Sigma K_L \frac{\bar{V}^2}{2g} \qquad \bar{V} = \frac{Q}{A}$$

or,

$$h_p = \left(\frac{P_2 - P_1}{\gamma}\right) + \left(\frac{V_2^2 - V_1^2}{2g}\right) + (z_2 - z_1) + f\frac{L}{D}\frac{\bar{V}^2}{2g} + \Sigma K_L \frac{\bar{V}^2}{2g}$$

$\boxed{h_p = A_o + kQ^2}$ system curve



$$A_o = \frac{P_2 - P_1}{\gamma} + z_2 - z_1 \qquad k = \left(f\frac{L}{D}\frac{\bar{V}^2}{2g} + \Sigma K_L \frac{1}{2g}\right)\frac{1}{A^2}$$

not dependent on Q. this is system head @ Q = 0

Notes 11-11-10

Exam III thursday

Topics

1. kinematic description of fluid motion (ch. 4 - sections 4.7, 4.8, 4.9)
2. dimensional analysis & similitude (ch 5 sections 5.1, 2, 3, 5)
3. internal viscous flow (ch 6 section Sections 6.1, 2, 3, 4, 6, 7, 8, 9, 12 pg 419-429)

Only need moody chart.

Coverage for test
- hw set 6 & 7
- study prob. sets 6 & 7
- in-class p. sets 16-19

Breakup. 10 pts → T/F questions
90 pts → 3 problems, 30 pts each on topics 1, 2, & 3

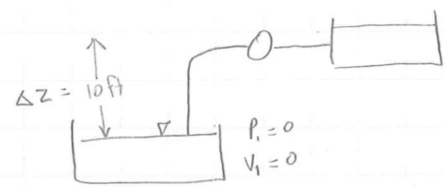
Book & dimension sheet allowed.

Notes 11-11-10

Ex) P.set 19 supplement #1

$$\frac{P}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{L}{D}\frac{V^2}{2g} + \Sigma k_L \frac{V^2}{2g}$$

$$h_p = (z_2 - z) + f\frac{L}{D}\frac{\bar{V}^2}{2g} + \Sigma k_L \frac{\bar{V}^2}{2g}$$



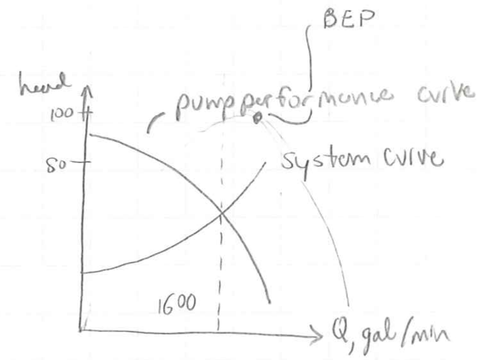
$f = .02$ $\qquad \Sigma k_L = .5 + 1.5 + 1.0 = 3.0$

$L = 200$ ft

$D = 6$ in $= .5$ ft

$\bar{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2}$

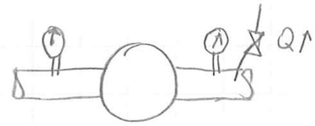
$$h_p = 10 + .02\left(\frac{200}{.5}\right)\frac{Q^2}{\left[\pi(.5)^2\right]^2(2)(32.2)} + \frac{3 \cdot Q^2}{\left[\frac{\pi}{4}(.5)^2\right]^2(2)(32.2)}$$



$h_p = 10 + 4.43\,Q^2$ where $Q = ft^3/s$

system curve $\rightarrow$ $h_p = 10 + 2.2(10^{-5})\,Q^2$ where $Q = $ gal/min

$\eta = .84 = \frac{P_f}{\dot{W}_{shaft}}$



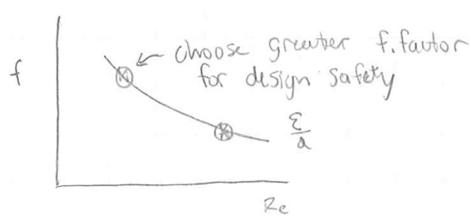
$h_p = \frac{\Delta P}{\gamma}$

Ex) P set 19, supplement #2

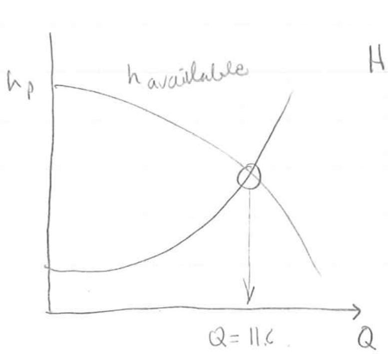
$h_p = (\Delta z) + f\frac{L}{D}\frac{\bar{V}^2}{2g} + \Sigma k_L \frac{\bar{V}^2}{2g}$

$\Sigma k_L = 0.5 + 17.5 + .92 \times 5 + 1.05$
$= 23.65$

$\varepsilon = .25$ mm $\qquad \varepsilon/d = 0.0123$



$Re = \frac{\rho \bar{V} D}{\mu}$ $\qquad Q = 11.6$ Lpm



$$H_o - aQ^2 = \Delta z + f\frac{L}{D}\frac{Q^2}{2A^2 g} + \Sigma k_L \frac{Q^2}{2A^2 g}$$

system curve $\quad h_p = \Delta z + f\frac{L}{D}\frac{\bar{V}^2}{2g} + \Sigma k_L \frac{\bar{V}^2}{2g} = a_0 + kQ^2$

performance curve $\quad h_{available} = h_{shutoff} - aQ^2$

☆ solve for intersection

Ex, P#3

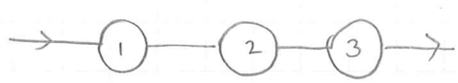
8.25" dia. impeller   req'd bhp $= \dot{w}_{shaft} = \dfrac{\gamma Q h_p}{\eta} = \dfrac{62.3(32.2)(370)(24)}{0.7}$

$\eta = \dfrac{P_f}{\dot{w}_{shaft}} = \dfrac{\Delta P Q}{\dot{w}_{shaft}} = \dfrac{\gamma h_p Q}{\dot{w}_{shaft}}$ $= 3.20$

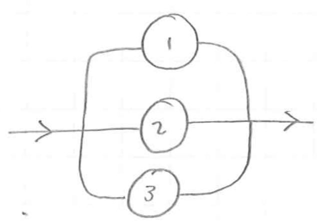
9.75" diam impeller $\eta = .765$ $h_p = 46$ ft req'd bhp $= 5.00$ hp

- download Powerpoints for pumps

Pumps in Series / Parallel



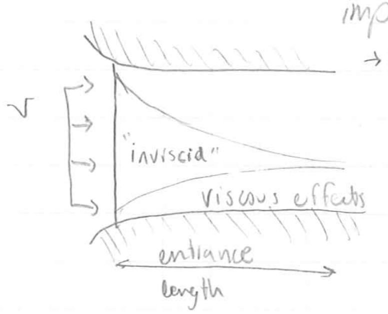
dissimilar pumps: should shut off a pump if Q rises above free delivery



dissimilar pumps: should shut off a pump if H

P 4 - mistakes, change p #1 to 3 in description

Boundary layer : thin region close to a body in which viscous forces are important.



in-class set 20

1) $\left.\dfrac{\delta}{x}\right|_{laminar} = \dfrac{5.0}{Re_x^{1/2}} = \dfrac{5.0}{\left(\frac{Vx}{\nu}\right)^{1/2}}$ Table A-1, H_2O 68°F → $\nu = 1.082(10^{-5})\ ft^2/s$

$$\frac{\frac{1}{12}\ ft}{x} = \frac{5}{\left(\frac{20x}{1.082(10^{-5})}\right)^{1/2}} \qquad x \approx 513\ ft \qquad Re_x = 9.5(10^8)$$

try turbulent flow formula $\frac{\delta}{x} = .16/Re_x^{(1/7)}$ $x \approx 5.17\ ft$ $Re_x = 9.6(10^6)$ → OK

2) Transitional reynolds #, $Re_{x,t} = 5\times10^5$ $Re_{x,t} = \dfrac{V x_t}{\nu} = 5\times10^5$

$\dfrac{x_t}{L} = 5\times10^5 \cdot \dfrac{\nu}{VL}$ Re_{xt} → local reynolds #, (varies)

Re_L → global reynolds #, (constant)

$\boxed{\text{Case A}}$ $L = 20\ ft$ $V = 20\ mph = 36.675\ ft/s$ $Re_L = \dfrac{VL}{\nu} = 4.53(10^6)$

$\dfrac{x_t}{L} = \dfrac{5(10^5)}{4.53(10^6)} = \underline{0.11}$

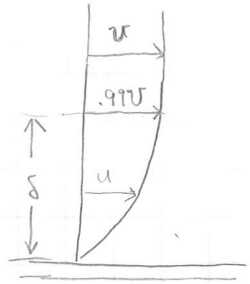
$\boxed{\text{Case B}}$ $L = 20\ ft$ $V = 65\ mph = 95.36\ ft$ $Re_L = \dfrac{VL}{\nu} = 1.177(10^7)$

$\dfrac{x_t}{L} = \dfrac{5(10^5)}{1.177(10^7)} = 0.042$

$\boxed{\text{Case C}}$ $L = 8\ ft$ $V = 150\ mph = 220.05\ ft/s$ $Re_L = \dfrac{VL}{\nu} = 1.054(10^7)$

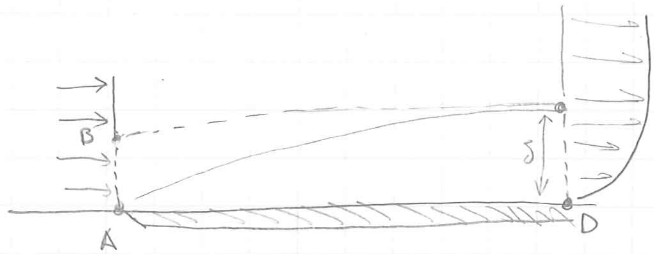
$\dfrac{x_t}{L} = \dfrac{5(10^5)}{1.054(10^7)} = 0.047$

$\boxed{\text{Case d}}$ $L = 30\ ft$, $V = 836\ ft/s$ $Re_L = \dfrac{VL}{\nu} = 5.879(10^7)$ $\dfrac{x_t}{L} = \dfrac{5(10^5)}{5.879(10^7)} = .009$



$\delta = y$ where $u = 0.99\,U$

Von Karman Momentum Integral eq'n ; steady flow



BC = flow streamline

cons. mass

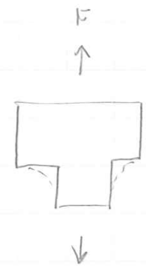
$$\int_{CS} \vec{V} \cdot d\vec{A} = 0 \Rightarrow -\int_0^H U \cdot 1 \cdot dy + \int u(y) \cdot 1 \cdot dy = 0$$

or $\boxed{U H = \int_0^\delta u(y)\, dy} = \int_0^\delta (U + u(y) - U)\, dy = U \cdot \delta + \int_0^\delta (u(y) - U)\, dy$

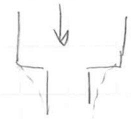
$U(\delta - H) = \int_0^\delta [U - u(y)]\, dy \qquad \delta - H = \delta^* =$ displacement thickness $= \int_0^\delta \left[1 - \dfrac{u(y)}{U}\right] dy$

Exam III statistics

	3	test 1	test 2
Avg -	75	74	72
Max -	98	94	95
Min -	45	44	31
SD -	12	11.7	15.7



is stress concentration related to flow?



How to get an "A" in Fluids

- answers in 2 decimal places
- 1 recitation & ______ will take place before homework is due
- I share this class with Amber Carver, Jim Holtgreven (20/20 HW 3)
- exams are OPEN BOOK

Homework

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee David Malavey

****

Home –Work 0

Objective: Revision of Basic Calculus for ME231

Assigned: 8/24/2010 Due: 8/26/2010

1. For a function: $f(x,y) = x\sin(xy^2)$, prove that the mixed derivatives are equal, i.e. $f_{xy} = f_{yx}$

[Note: $f_{xy} = \dfrac{\partial}{\partial y}\left(\dfrac{\partial f}{\partial x}\right)$; $f_{yx} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial f}{\partial y}\right)$]

2. For a function: $f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{(x^2+y^2)} & if\ (x,y) \neq (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases}$, prove that the mixed derivatives are not equal. Hence prove that: $f_{xy} = -1$ and $f_{yx} = +1$.

3. Let $R(t) = r(x,y)$ where $r = \sqrt{x^2+y^2}$, $x = 3t^2, y = \sin t$ for $1 < t < 4$. Evaluate $\dfrac{dR}{dt}$ in the interval.

4. For a function: $F(x,y,z) = 2e^x \sin y\ \hat{i} + 3e^x \cos y\ \hat{j} + (4z^2 + x + y)\ \hat{k}$. Evaluate:

a. Curl **F**
b. Divergence **F**

****

1)

$f_{xy}\left[x\sin(xy^2)\right] \overset{\frac{d}{dx}\Rightarrow}{=} 1(\sin xy^2) + x\left[y^2\cos(xy^2)\right]$

$\frac{d}{dy} \Rightarrow (x)(2y)\cos(xy^2) + 2yx\cos(xy^2) - 2y^3x^2\sin(xy^2) = \boxed{4yx\cos(xy^2) - 2x^2y^3\sin(xy^2)}$

$f_{yx}\left[x\sin(xy^2)\right] = \frac{d}{dy} \Rightarrow 2yx^2\cos(xy^2)$

$\frac{d}{dx} \Rightarrow \boxed{4xy\cos(xy^2) - \cancel{2}yx^2(y^2)\sin(xy^2)}$ same

Dance Malawey

18/20

#3 $\frac{dR}{dt} = \frac{\partial R}{\partial x}\frac{dx}{dt} + \frac{\partial R}{\partial y}\frac{dy}{dt}$

$= \frac{x}{\sqrt{x^2+y^2}}(6t) + \frac{y}{\sqrt{x^2+y^2}}(\cos t)$ Substitute x & y with given values

-2

#4 curl F = $\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ F\hat{i} & F\hat{j} & F\hat{k} \end{bmatrix} = \left(\frac{d}{dy}(F\hat{k}) - \frac{d}{dz}F\hat{j}\right)i - \ldots(\quad)j + (\quad)k$

a)

$= \frac{d}{dy}(4z^2+x+y) - \frac{d}{dz}(3e^x\cos y)\ i - \left[\frac{d}{dx}(4z^2+x+y) - \frac{d}{dz}2e^x\sin y\right]j + \left[\frac{d}{dx}(3e^x\cos y) - \frac{d}{dy}2e^x\sin y\right]K$

$= (1-0)i - (1-0)j + (3e^x\cos y - 2e^x\cos y)k$

$= \boxed{i - j + (e^x\cos y)k}$

b) divergence $F = \frac{\partial}{\partial x}2e^x\sin y + \frac{\partial}{\partial y}3e^x\cos y + \frac{\partial}{\partial z}4z^2$

$= 2e^x\sin y + (-3e^x\sin y) + 8z$

$= \boxed{-e^x\sin y + 8z}$

David Malaway

RM{303}

② find f_{xy}

quotient rule

$$f = \frac{xy(x^2-y^2)}{(x^2+y^2)} = \frac{g(x)}{h(x)} \quad \frac{df}{dx} = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$$

$$g'(x) = y(x^2-y^2) + 2x(xy) \qquad h'(x) = 2x$$

$$= yx^2 - y^3 + 2x^2y$$

$$= 3x^2y - y^3$$

$$\frac{df}{dx} = \frac{[3x^2y - y^3](x^2+y^2) - xy(x^2-y^2)(2x)}{(x^2+y^2)^2} =$$

$$\frac{df}{dx} = \frac{(3x^4y + 3x^2y^3 - y^3x^2 - y^5) - \cancel{(x^3y - xy^3)2x}^{\,2x^4y - 2x^2y^3}}{(x^2+y^2)^2}$$

$$= \frac{x^4y + 5x^2y^3 - y^3x^2 - y^5}{(x^2+y^2)^2} = \frac{x^4y + 4x^2y^3 - y^5}{(x^2+y^2)^2}$$

$$\frac{d}{dy}\left(\frac{df}{dx}\right) = \frac{(1 + 12x^2y^2 - 5y^4)(x^2y^2)^2 - (x^4y + 4x^2y^3 - y^5)(4y^3 + 4yx^2)}{(x^2+y^2)^4}$$

???

Incomplete

−1

back →

2) $$\frac{d}{dx}\left(\frac{d}{dy}\left(\frac{xy(x^2-y^2)}{x^2+y^2}\right)\right) = \frac{x^6+9x^4y^2-9x^2y^4-y^6}{(x^2+y^2)^3}$$

$$\frac{d}{dy}\left(\frac{d}{dx}\left(\frac{xy(x^2-y^2)}{x^2+y^2}\right)\right) = \frac{x^6+9x^4y^2-9x^2y^4-y^6}{(x^2+y^2)^3}$$ ✓

calculator attempt

show it by hand calculation

David Malawey

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee

office 4-5pm Friday or email over the weekend

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Home –Work 1

1. The pressure difference, Δp, across a partial blockage in an artery (called a *stenosis*) is approximated by the equation:

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1 \right)^2 \rho V^2$$

where, V is the blood velocity, μ is the blood viscosity, ρ the blood density, D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of stenosis. Determine the dimensions of the constants K_v and K_u if the formula is dimensionally homogeneous. Would this equation be valid in any system of units?

2. The pressure inside an automobile tire depends on the temperature of the air inside the tire (see figure 1). When the air temperature is 25°C, the pressure gauge reads 210 *kPa*. If the volume of the tire is 0.025 m^3, determine the pressure rise in the tire when the air temperature in the tire rises to 50°C. Also, determine the amount of air that must be bled off to restore pressure to its original value at this temperature. Assume that the atmospheric pressure is 100 *kPa*.

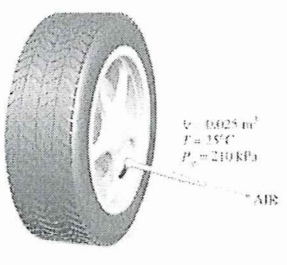


Figure 1

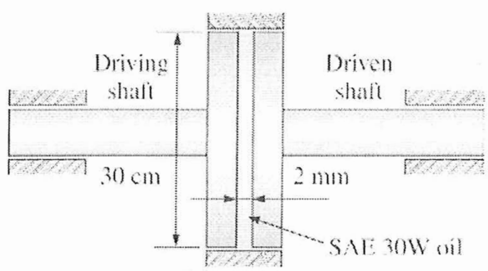


Figure 2

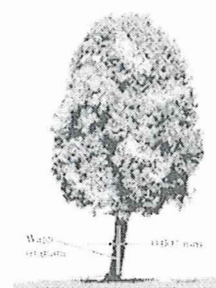


Figure 3

3. The clutch system shown in figure 2 is used to transmit torque through a 2-mm-thick oil film with μ = 0.38 Ns/m2 between two identical 30-cm diameter disks. When the drive shaft rotates at 1450 rpm, the driven shaft is observed to rotate at 1398 rpm. Assuming a linear velocity profile for the oil film, determine the transmitted torque.
4. Problem 1.51 from White 7th Edition.
5. Problem 1.59 from White 7th Edition.
6. Nutrients dissolved in water are carried to upper parts of plants by tiny tubes, partly because of capillary effects (see figure 3). Determine how high the water solution will rise in a tree in a 0.002 mm diameter tube as a result of capillary effect. Treat the solutions as water at 20°C with a contact angle of 15°.
7. For the velocity fields (i) – (viii) given below, determine: (a) whether the flow is one-, two- or three-dimensional and why?; (b) whether the flow is steady or unsteady, and why?
(i) $\vec{V} = ae^{-bx}\hat{i}$, (ii) $\vec{V} = ax^2\hat{i} + bx\hat{j}$, (iii) $\vec{V} = ax^2e^{-bt}\hat{i}$; (iv) $\vec{V} = ax\hat{i} - by\hat{j}$; (v) $\vec{V} = (ax+t)\hat{i} - by^2\hat{j}$;
(vi) $\vec{V} = ax^2\hat{i} + bxz\hat{j}$; (vii) $\vec{V} = a\left(x^2+y^2\right)^{\frac{1}{2}}\left(\frac{1}{z^3}\right)\hat{k}$; (viii) $\vec{V} = axy\hat{i} - byzt\hat{j}$
8. Given the Eulerian velocity field: $\vec{V} = xt\,\hat{i} + yt\,\hat{j} + xy\,\hat{k}$, find the total acceleration, local acceleration and the convective acceleration of the particle.

****

David Malawey

Homework 1 Due 9-7-10

$\frac{18}{20}$

D diameter = L

A_0 = area L^2

A_1 = area L^2

ρ = blood density = ML^{-3}

μ = blood viscosity $ML^{-1}T^{-1}$

V = blood velocity LT^{-1}

P = pressure = $ML^{-1}T^{-2}$

#1

$$\Delta p = k_v \frac{\mu V}{D} + k_u \left(\frac{A_0}{A_1} - 1\right)^2 \rho V^2$$

$$\mu \frac{V}{D} = \frac{\cancel{L} T^{-1}}{\cancel{L}} (ML^{-1}T^{-1})$$

$$ML^{-1}T^{-2} = k_v \, ML^{-1}T^{-2}$$

$$\boxed{k_v = M^0L^0T^0}$$

$$\left(\frac{A_0}{A_1} - 1\right)^2 \rho V^2$$

$$= \left(\frac{\cancel{L^2}}{\cancel{L^2}}\right)^{\cancel{2}} ML^{-1}T^{-2} (L^2T^{-2})$$

$$ML^{-1}T^{-2} = k_u (M^1L^1T^{-4})$$

$$\boxed{k_u = M^0L^{-2}T^2}$$

The units will have to include absolute temperature ✓

#2

$\forall = .025 m^3$ $P_{atm} = 100kPa$

$T_1 = 25°C$ $P_1 = 210kPa$

$T_2 = 50°C$ $\boxed{P_2 = 227.6kPa}$

$$Pv = RT$$

$$\frac{P}{T} = \frac{R}{v} \text{ const} \Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{210}{(25+273)} = \frac{P_2}{(50+273)}$$

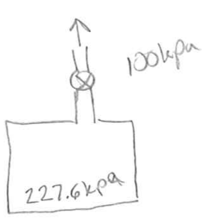
$(210 + 100) = 310$

$Pv = mRT$ $P_1\forall_1 = mRT_3$

$P_3 = 210kPa$

$$M_1 = \frac{P_1\forall_1}{RT_1} = \frac{210(.025)}{.2869(298)} = .0614 kg$$

$$\frac{P_2}{P_1} = \frac{m_2}{m_1} = \frac{210}{227.6} = .9226$$



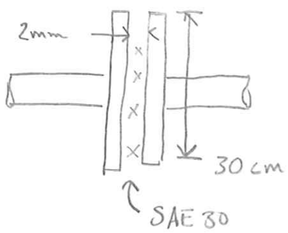
.00475 kg of air must be released to reduce pressure to 210 kPa

-1

David Malawey

ME231 Homework 1

#3



$\mu = .38 \ Ns/m^2$ $\omega_1 = 1450$ $\omega_2 = 1398$

$$T = \frac{\pi \mu \Delta\omega R^4}{2a} = \frac{\pi (.38) \frac{Ns}{m^2} (52 \frac{rad}{s})(.15m)^4}{2(.002m)}$$

$$\boxed{T = 7.857 \ N \cdot m} \ (\text{rad})$$

-0.5

#4 1.51) Helium, 20°C $P = 1atm$ $U = 10.8 \ m/s$ $\delta = .03 \ m$ $\mu(He) = 1.97 \times 10^{-5} \ \frac{N \cdot s}{m^2}$

$u(y) = U \sin\left(\frac{\pi y}{2\delta}\right)$ $0 \le y \le \delta$ $\tau_w = \mu \frac{V}{h} =$

a) wall shear stress τ_w ?

$u(y) = 10.8 \ m/s \ \sin\left(\frac{\pi y}{2(.03)}\right)$

$= 10.8 \ m/s \ \sin(52.36 \ y)$

b) Find position where $\tau = .5 \tau_w$

$$\tau = \mu \frac{du}{dy} = \mu \frac{d}{dy}\left[10.8 \ m/s \ \sin 52.36y\right]$$

$\tau = \mu \ 565.488 \cos 52.36y$

$= .0111 \cos 52.36y$

$\tau_w = \lim \tau$ as $x \to 0$ $= \boxed{.0111 \ Pa}$

$\tau = .5 \tau_w$ when $\cos 52.36y = .5$

$$\frac{\cos^{-1} .5}{52.36} = \boxed{y = .02 \ m}$$

David Malaney

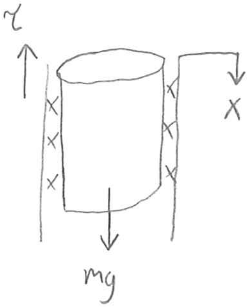
+5 | 1.59)

cylinder diam D, L, ρ_s density,

tube diam D_o filled with fluid with ρ & μ

Clearance $D_o - D << D$

$\Sigma F = m\cancel{a}^{0}$



steel $D = 2cm$

$D_o = 2.04cm$

$L = 15cm$

SAE 30 oil 20°C

$\Sigma F_y = 0$

$-mg + drag = 0$

$mg = \mu \frac{V}{h} A$

Terminal fall velocity?

$\tau = \mu \frac{V}{h}$, h is small

~~$\forall = \pi(.01m)^2(.15m) = 4.712E^{-5} m^3$~~

~~$M = \rho\forall = 7850 \, kg/m^3 (4.712E{-5}m^3) = .370 kg$~~

$mg = V_{terminal}\left(\frac{\mu A}{h}\right)$

$V_{terminal} = \frac{mgh}{\mu A} = \cancel{\frac{.370 kg (9.81 m/s^2)(.0002m)}{(.29 \frac{kg}{ms})(.0628 m^2)}} =$

~~$A = \pi D = .0628 m^2$~~

Reworked symbolically

$$V = \frac{mgh}{\mu A} = \frac{\rho\left(\frac{D}{2}\right)^2 \cancel{\pi} L \, g \left(\frac{D_o - D}{2}\right)}{\mu(\cancel{\pi} D \cancel{L})} = \frac{\rho D^{\cancel{2}} g(D_o - D)}{4\mu(2)\cancel{D}} = \frac{\rho D g (D_o - D)}{8\mu}$$

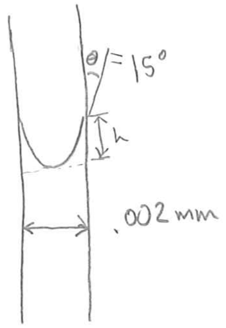
$$= \frac{7850 \frac{kg}{m^3}(9.81 m/s^2)(.02m)(.0204 - .02 m)}{8(.29) \, kg/m \cdot s}$$

$$\boxed{V_{terminal} = .265 \, m/s}$$

David Mallaway ME231

HW 1

#6



H_2O, 20°C

how high will it rise?

$$h = \frac{2\Upsilon\cos\theta}{\gamma R} = \frac{2\,(.0728\ \frac{N}{m})\cos 20^\circ}{1000\,kg/m^3\,(9.81\,m/s^2)(.001\times10^{-3}\,m)}$$

$\Upsilon = .0728\ N/m$

$\boxed{h = 6.06\ m}$ ✗ −0.5

#7

i) $\vec{V} = ae^{-bx}\,\hat{i}$ — 1D, steady * ✓

ii) $\vec{V} = ax^2\hat{i} + bx\hat{j}$ — 1D, steady * ✓

iii) $\vec{V} = ax^2e^{-bt}\,\hat{i}$ — 1D, transient ✓

iv) $\vec{V} = ax\hat{i} - by\hat{j}$ — 2D, steady * ✓

v) $\vec{V} = (ax+t)\hat{i} - by^2\hat{j}$ — 2D, transient ✓

vi) $\vec{V} = ax^2 i + bxzj$ — 2D, steady * ✓

vii) $\vec{V} = a(x^2+y^2)^{.5}\left(\frac{1}{z^3}\right)\hat{k}$ — 3D, steady * ✓

viii) $\vec{V} = axy\hat{i} - byzt\hat{j}$ — 3D, transient ✓

* has no "t" term

David Malawey 231 Fluids HW #1

#8 $\vec{V} = xt\,\hat{i} + yt\,\hat{j} + xy\,\hat{k}$

total accelleration : $u = xt$
$v = yt$
$w = xy$

(local)

$$\frac{Du}{dt} = \frac{du}{dt} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = x + xt(t) + yt(0) + xy(0) = x + xt^2$$

$$\frac{Dv}{dt} = \frac{dv}{dt} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = y + 0 + t(yt) + 0 = y + yt^2$$

$$\frac{Dw}{\partial t} = \frac{dw}{dt} + u \quad v \quad w = 0 + y(xt) + x(yt) + 0 = 0 + 2xyt$$

Total accell $= (x + xt^2)\,\hat{i} + (y + yt^2)\,\hat{j} + (2xyt)\,\hat{k}$

$\vec{a}_{local} = (x)\,\hat{i} + (y)\,\hat{j}$

$\vec{a}_{convective} = (xt^2)\,\hat{i} + (yt^2)\,\hat{j} + (2xyt)\,\hat{k}$

ME 231 Thermo-Fluid Mechanics I

Prof. A. Banerjee

****

Home – Work 2

1. Problem 2.31 from White
2. A teapot with a brewer at the top is used to brew tea as shown in figure 1. The brewer may partially block the vapor from escaping, causing the pressure in the teapot to rise and overflow from the service tube to occur. Disregarding thermal expansion and the variation in the amount of water in the service tube to be negligible to the amount of water in the teapot, determine the maximum cold-water height that would not cause an overflow at gage pressures of up to 0.32 kPa for the vapor.

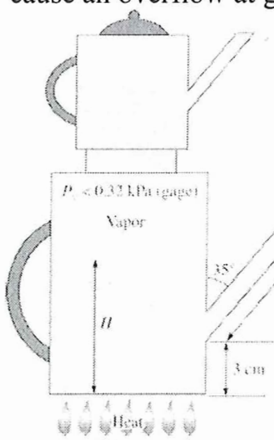


Figure 1

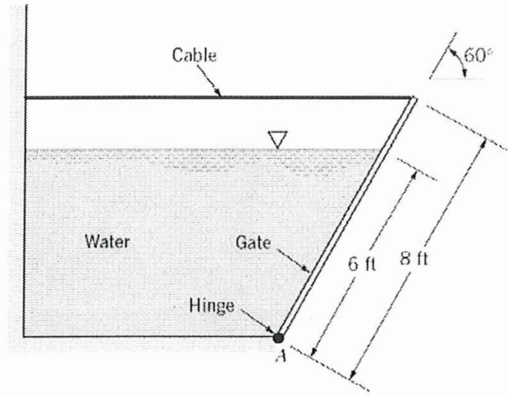


Figure 2

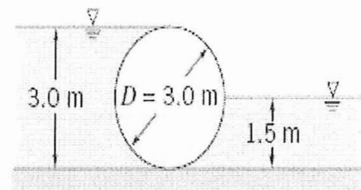


Figure 3

3. A homogeneous, 4-ft-wide, 8-ft-long rectangular gate (see Figure 2) weighing 800 lb is held in place by a horizontal flexible cable as shown in the figure. Water acts against the gate, which is hinged at point A. Friction in the hinge is negligible. Determine the tension in the cable.
4. A cylindrical weir shown in figure 3 has a diameter of 3 m and a length of 6 m. If the fluid on the left has a SG of 1.6 and the fluid on right has a SG of 0.8, find the magnitude and direction of the resultant force acting on the weir from the water.
5. A semicircular 40-ft diameter tunnel is to be built under a 150-ft deep, 800-ft-long lake, as shown in figure 4. Determine the total hydrostatic force acting on the roof of the tunnel.

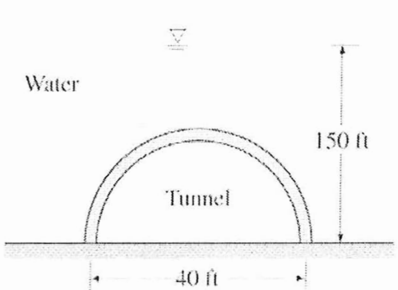


Figure 4

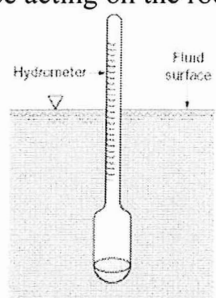


Figure 5

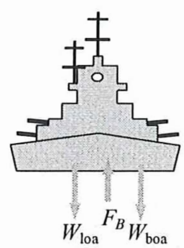


Figure 6

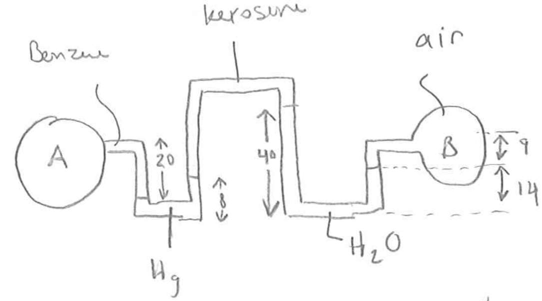
6. The hydrometer shown in the figure 5 has a mass of 0.045 kg, and the cross-sectional area of its stem is 290 mm^2. Determine the distance between the graduations (on the stem) for specific gravities of 1.0 & 0.9.
7. The hull of a boat as shown in figure 6 has a volume of 180 m^3, and the total mass of the boat when empty is 8560 kg. Determine how much load this boat can carry without sinking (a) in a lake, and (b) in seawater with a specific gravity of 1.03
8. A water tank is being towed on an uphill road that makes and angle of 20° with the horizontal with a constant acceleration of 5 m/s^2. Determine the angle that the free surface of water makes with the horizontal. What would your answer be if the direction of motion were downward on the same road with the same acceleration?

David Malaway ME231 HW # 2

9-13-10

20/20

#1



2.31) 20°C

Units: cm

P_a = pressure difference a-b = ?

$\gamma_{H_2O} = 9790\ N/m^3$

$\gamma_{Benzene} = SG \cdot 9790 = 8642\ N/m^3$

$\gamma_{kerosene} = 7886\ N/m^3$

$\gamma_{Hg} = 132920\ N/m^3$

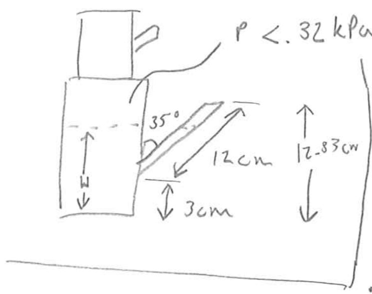
$A + .2\, SG_{Benz}\gamma_{H_2O} - .08\, SG_{Hg}\gamma_{H_2O} - .32\, SG_k\gamma_{H_2O} + .26\, SG_{H_2O}\gamma_{H_2O} = B$

$A + .20m(8642) - .08(132920) - .32(7886) + (+.40 - .14)(9790)$

$A - 8883\ N/m^2 = B$

$\boxed{P_{a-b} = 8.88\ kPa}$

#2



P < .32 kPa (gage) determine H

5/5

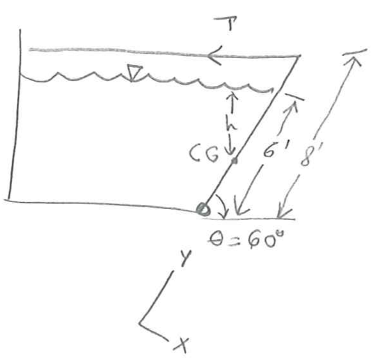
$3 + 12\cos 35° = 12.83\ cm$

$.32\ kPa + H\gamma_{H_2O} = \gamma_{H_2O}(12.83)$

$.32 = (12.83 - H)\gamma_{H_2O}$

$\boxed{H = .09561\ m = 9.561\ cm}$

#3



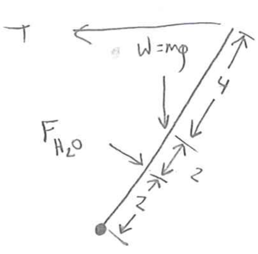
$A_{gate\ under\ water} = 4ft \times 6ft$

$F = P_{CG}A = \gamma h_{CG} A = \gamma\, 3'\sin\theta\,(4' \times 6')$

$F = (64\,lbf/ft^3)(3'\sin 60)(24ft^2) = 3991\ lbf$

$Y_{cp} = \dfrac{I_{xx}\sin\theta}{h_{CG}A} = \dfrac{\frac{1}{12}(4)(6)^3\sin\theta}{3(\sin\theta)(4 \cdot 6)} = 1'$

$l = -Y_{cp} = -1$



$\Sigma M = 0 \quad F_{H_2O}(2) + W(4\cos 60) - T(8\sin 60) = 0$

$3991(2) + 8000lb(4\cos 60) - T(8\sin 60) = 0$

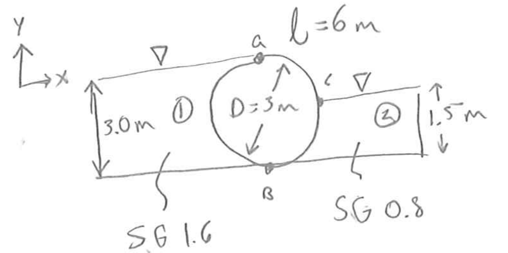
$\boxed{T = 1383\ lb}$

5/5

David Malawey HW #2, 231

Need vertical force

#4



$F_y = F_{y_1} - F_{y_2}$

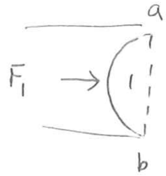
$= \rho_1 g \left(\frac{\pi}{2} R^2\right) l + \rho_2 g \left(\frac{\pi}{4} R^2\right) l$

$= \left(\frac{\rho_1}{2} + \frac{\rho_2}{4}\right)(g \pi R^2) l$

$\quad kg/m^3 \quad m/s^2 \cdot m^2 \cdot m$

$= \left(\frac{1.6}{2} + \frac{0.8}{4}\right)(998\ kg/m^2)(9.81 \cdot \pi \cdot 1.5^2)(6m)$

$\boxed{+\uparrow F_y = 415{,}225\ kN}$

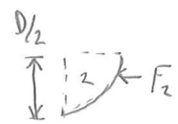


Projection of $\widehat{AB} = \overline{AB}$

$F_1 = \gamma_1 h_{cG} (A_p)$

$= SG_1 \gamma_{H_2O} \left(\frac{D}{2}\right)(D \cdot 6m)$

$\widehat{BPC} = \overline{BC}$



$F_2 = \gamma_2 h_{cG} (A_p)$

$= SG_2 \gamma_{H_2O} \left(\frac{D}{4}\right)\left(\frac{D}{2} \cdot 6m\right) =$

$\boxed{F_{total} = (370{,}062\,\hat{i} + 415{,}225\,\hat{j})\ kN}$

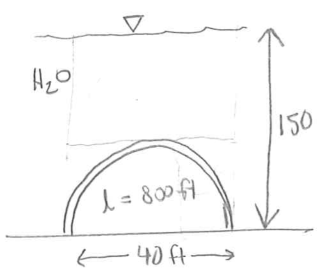
$\xrightarrow{+} F_{total} = F_1 - F_2 \quad 1.6\,\gamma_{H_2O} D^2 (3) - 0.8\,\gamma_{H_2O} D^2 \left(\frac{3}{4}\right)$

$\gamma_{H_2O} D^2 (4.8 - .6) m$

$= 9790\ kN/m^3\ (37.8) m^3$

$F_x = \boxed{370{,}062\ kN}$

#5



~~$A_p = (40 \cdot 800)\ ft^2$~~

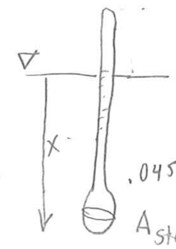
$F = \gamma_{H_2O} V_{H_2O} = 64\ \frac{lbf}{ft^3} \left[(150 \cdot 40) - \left(\frac{\pi}{2} 20^2\right)\right] (800\ ft)$

$\boxed{+\downarrow F = 275 \times 10^6\ lbf}$

5/5

David Malawey HW#2

#6)



$.045\,kg$

$A_{stem} = 290\,mm^2$

specific gravities 1.0, 0.9 $\Delta x = ?$

$F_{buoyant} = V_{displaced}\,\rho_{fluid}\,g$

$\Delta F = \Delta V_{disp}\,g\,\rho_{fluid}$

$\Sigma F = 0$

$W = \forall_{disp}\,g\,\rho_{fluid}$ $\quad m\cancel{g} = \forall_{disp}\,\cancel{g}\,\rho_{fluid}$ $\quad .045\,kg = 290\times10^{-6}\,m^2\,(x)(1-0.9)(998\,\frac{kg}{m^3})$

↑ plug in ΔSG

$$x = \frac{.045\,kg}{290E^{-6}\,m^2\,(SG)(998\,\frac{kg}{m^3})}$$

$\boxed{\Delta x = 17.28\,mm}$

#7)

$\forall_{hull} = 180\,m^3$ $\quad mass_{boat} = 8{,}560$

a) lake

b) ocean $SG = 1.3$

$m\cancel{g} = \forall_{hull}\,\rho_{water}\,(\cancel{g})$

$(total)\ m = 180\,m^3\,(998\,kg/m^3)$

$m_{total} = M_{hull} + m_{load} = 179640$

a) $\boxed{m_{load} = 171{,}080\,kg}$

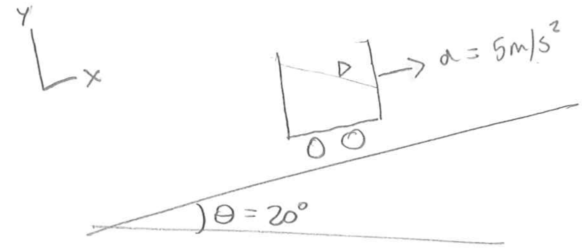
b) $m_{total} = 180\,m^3\,(1.03)(998\,kg/m^3)$

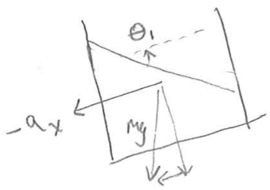
$\boxed{M_{load} = 176{,}469\,kg}$

David Mallaway

ME 237 HW #2

#8)





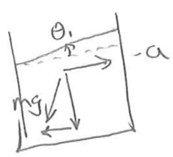
$\Sigma \ddot{y} = g\cos 20° = 9.2183 \ m/s^2$

$\Sigma \ddot{x} = -5 m/s^2 - g\sin 20° = 8.355 \ m/s^2$

$\theta_1 = \tan^{-1} \frac{9.2183}{8.355} = 47.81°$

$\theta_{horizontal} = \theta_1 - 20° = \boxed{27.81°}$

Downward



$\ddot{y} = 9.2183$

$\ddot{x} = -5 m/s^2 + g\sin 20 = 1.645$

$\theta_1 = 10.12°$

$\theta_{horizontal} = \theta_1 + 20° = \boxed{-30.12°}$

5/5

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee

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Home –Work 3 due Thursday 9-30-10

Section 1: The Equation of Conservation of Mass

1. In the incompressible flow through the device shown in figure 1, velocities may be considered to be uniform over inlet and outlet sections. If the fluid flowing is water, obtain an expression for the mass flow rate at section 3. The following conditions are known: A_1=0.1 m^2, A_2=0.2 m^2, and A_3=0.15 m^2, V_1= 5 m/s and V_2= 10 +5 cos(4πt) m/s.

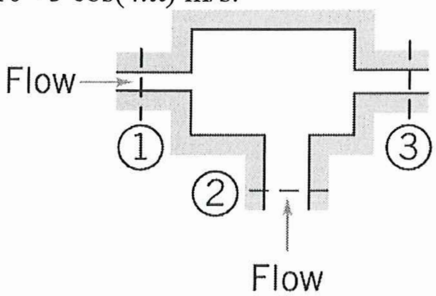


Figure 1

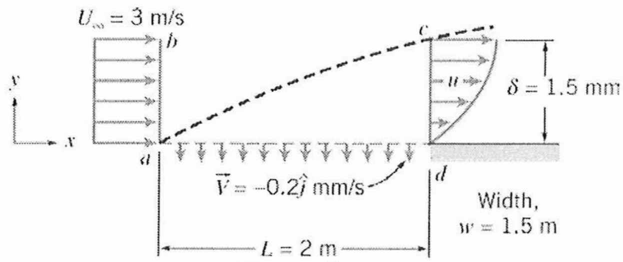


Figure 2

2. Water flows steadily past a porous flat plate as shown in figure 2. Constant suction is applied along the porous section. If the velocity profile at section cd is: $\frac{u}{U_\infty} = 3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{3/2}$, evaluate the mass flow rate at section bc.

3. Problem 3.35 White 7th Edition

Section 2: The Equation of Conservation of Momentum

4. A vertical plate has a sharp-edged orifice at its center (see figure 3). A water jet of speed V strikes the plate concentrically. Obtain an expression for the external force needed to hold the plate in place, if the jet leaving the orifice also has speed V. Evaluate the force for V = 5 m/s, D = 100 mm, and d = 25 mm.

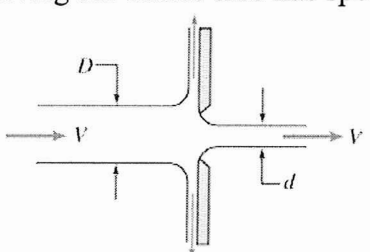


Figure 3

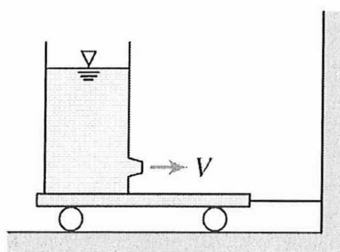


Figure 4

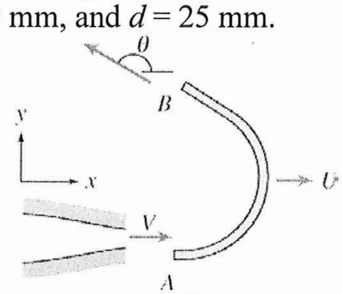


Figure 5

5. A large tank of height h = 1 m, D = 0.75 m is affixed to a cart as shown. Water issues from the tank through a nozzle of diameter d = 15 mm. The speed of the liquid leaving the tank is approximately $V = \sqrt{2gy}$ where y is the height from the nozzle to the free surface. Determine the tension in the wire when y = 0.9 m. Plot the tension as a function of water depth for $0 \le y \le 0.9$ m (use a plotting software).

6. Problem 3.50 White 7th Edition

7. A jet of water as shown in figure 5 is directed against a vane, which could be a blade in a turbine or in any other piece of hydraulic machinery. The water leaves the stationary 40-mm diameter nozzle with a speed of 25 m/s and enters the vane tangent to the surface at A. The inside surface of the vane at B makes angle θ = 150° with the x- direction. Compute the force that must be applied to maintain the vane speed at U = 5 m/s.

David Malawey

231 HW 3

18.5/20

Cons. Mass Sep. 14

1) incompressible, uniform flow Velocity, H_2O

① $A = .1 m^2$ $V = 5 m/s$

② $A = .2 m^2$ $V = 10 + 5\cos(4\pi t)\, m/s$

③ $A = .15 m^2$

4/4

$\dot{m} = \rho \vec{V} \cdot d\vec{A}$

$\dot{m}_1 = 998\, kg/m^3 \left(\frac{5m}{s}\right)(0.1 m^2) = 499\, kg/s$

$\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = 0$
$(-)\ (-)\ (+)$

$\dot{m}_2 = 998\, kg/m^3 \left(\frac{10 + 5\cos(4\pi t)\, m}{s}\right)(.2 m^2) = (1996 + 998\cos(4\pi t))\, kg/s$

$\boxed{\dot{m}_3 = (2495 + 998\cos(4\pi t))\, kg/s^{\cancel{2}}}$

2) $\cancel{\frac{d}{dt}\int_{\forall} \rho\, d\forall}_{0} + \int_A \rho \vec{V} \cdot d\vec{A} = 0$

assume: ρ constant

$\frac{u}{U_\infty} = 3\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^{3/2}$

$\int_a^b \vec{V} \cdot d\vec{A} + \int_a^d \vec{V} \cdot d\vec{A} + \int_c^d \vec{V} \cdot d\vec{A} + \int_b^c \vec{V} \cdot d\vec{A} = 0$

$(-) U_\infty \delta w\ (+)\ \vec{V} L w\ (+) \int_c^d \vec{V} \cdot d\vec{A} + \int_b^c \vec{V} \cdot d\vec{A} = 0$

volume flow rate $\int u \cdot dA = \int_0^\delta u\, w\, dy$

$= U_\infty w \int_0^\delta 3\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^{3/2} dy =$

$= U_\infty w \left[\frac{3y}{2\delta} - \frac{2}{\delta^{3/2}} \frac{y^{5/2}\, 2}{5} \right]_0^\delta$

$= U_\infty w \left[\frac{3\delta^2}{2\delta} - \frac{4}{5} \frac{\delta^{5/2}}{\delta^{3/2}} \right]$

$= U_\infty w \left[\frac{3}{2}\delta - \frac{4}{5}\delta^{5/3} \right]$

$= 3 m/s (1.5 m) \left(\frac{3}{2}(.0015) - \frac{4}{5}(.0015)^{5/3} \right) m$

$= .01005\, m^3/s$

$\Rightarrow -3\frac{m}{s}(.0015 m)(1.5 m) + .0002\frac{m}{s}(1.5 m)(2 m)$

$+ .01005\, m^3/s = -\text{flow}_{bc}$

$\boxed{\text{flow}_{bc} = -.003904\, m^3/s} = \boxed{-3.896\, kg/s}$

flow is in

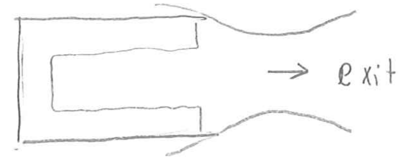
David Malawey

231 HW 3

#3) Norton 3.35

CV analysis

find rate of mass loss of propellant, exit gas has molecular weight of 28



Combustion 1500k
950 kPa

Exit $D_e = 18$ cm
$P = 90$ kPa
$V = 1150$ m/s
$T = 750$ k

4/4

$$R = \frac{8313}{28} = 297 \frac{m^2}{s^2 K}$$

$$P = \rho RT \qquad \rho = \frac{P}{RT} = \frac{90{,}000}{297\,(750)}$$

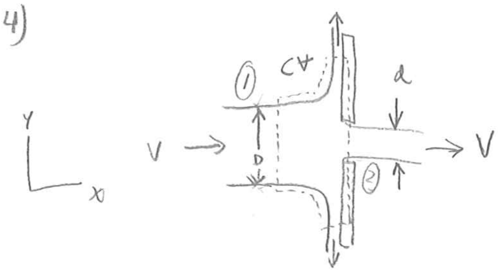
$$\rho_{exit} = .404 \text{ kg/m}^3$$

$$\frac{d}{dt}(m_{propellant}) + \dot{m}_{out} = 0$$

$$\frac{d}{dt} m_{propellant} = -\dot{m}_{exit} = \rho_e A_e V_e = -(.404)(\pi/4)(.18)^2(1150) = \boxed{-11.8 \frac{kg}{s}}$$

231 HW #3

4)



F on plate = ?

D = .10 m

d = .025 m

V = 5 m/s

water jet   x-momentum

$$F_x = \int_{cs} \vec{V} \rho \vec{V} \cdot d\vec{A}$$

$$= V \rho_{water} -\left(V \pi \frac{D^2}{4}\right) + V \rho_{water} +\left(V \pi \frac{d^2}{4}\right)$$

760 5041

$$F_x = V^2 \rho_{water} \frac{\pi}{4} \left(d^2 - D^2\right)$$

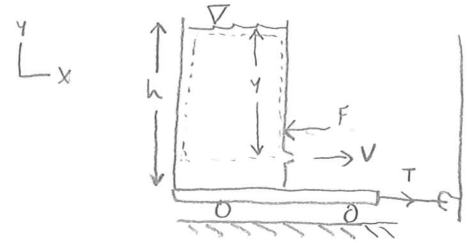
$$= 25 \frac{m^2}{s^2} \, 998 \frac{kg}{m^3} \, \frac{\pi}{4} \left(.025^2 - .1^2\right) m^2$$

$$= -183.7 \frac{kg\,m}{s^2} = \boxed{-183.7 \hat{i} \text{ N}} = F$$

4/4

David Malawey

231 HW 4



$V = \sqrt{2gy}$ $h = 1m$

$Y = 0.9m$ $D = .75m$

5) $\Sigma F_x = \cancel{ma_x}$ 0

$T = F_{s_x}$

$T = \int \vec{V}_\rho \vec{V} \cdot d\vec{A}$

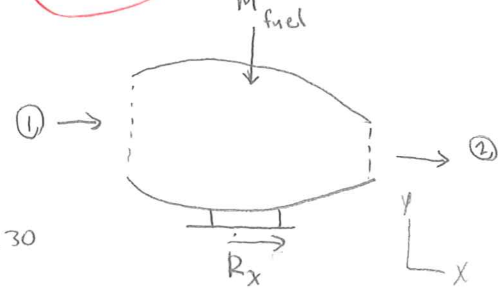
$= \sqrt{2gy}\,\rho\left(\sqrt{2gy} \cdot \frac{\pi}{4} .0015m^2\right)$

$= \frac{2gy\rho\pi}{4}(.0015m)^2$ $T = .03464$

$\boxed{T = .0341\ N}$

$T = 3.12\ N$ $-1/2$

$\frac{3.5}{4}$



6) P 3.50 $A_1 = .5m^2$ 20°C 1atm $V_1 = 250\,m/s$

F/A ratio = 1:30 $A_2 = .4m^2$ $V_2 = 900\,m/s$

$F_s + F_B = \cancel{\frac{d}{dt}}\int V_{rel}\,\rho\,dV^0 + \int_{cs} V\rho\vec{V}\cdot d\vec{A}$

x-momentum

$F = (-)\int_1 V_1^2\rho_1 A \quad (+) \int_2 V_2^2$

$\rho_1 = \frac{1}{v} = \frac{P_1}{RT_1} = \frac{101.3kPa}{.2870(293K)} = 1.205\ kg/m^3$

$\dot{m}_1 = 250\frac{m}{s}(.5m^2)(1.205\ kg/m^3)$

$= 150.625\ kg/s$

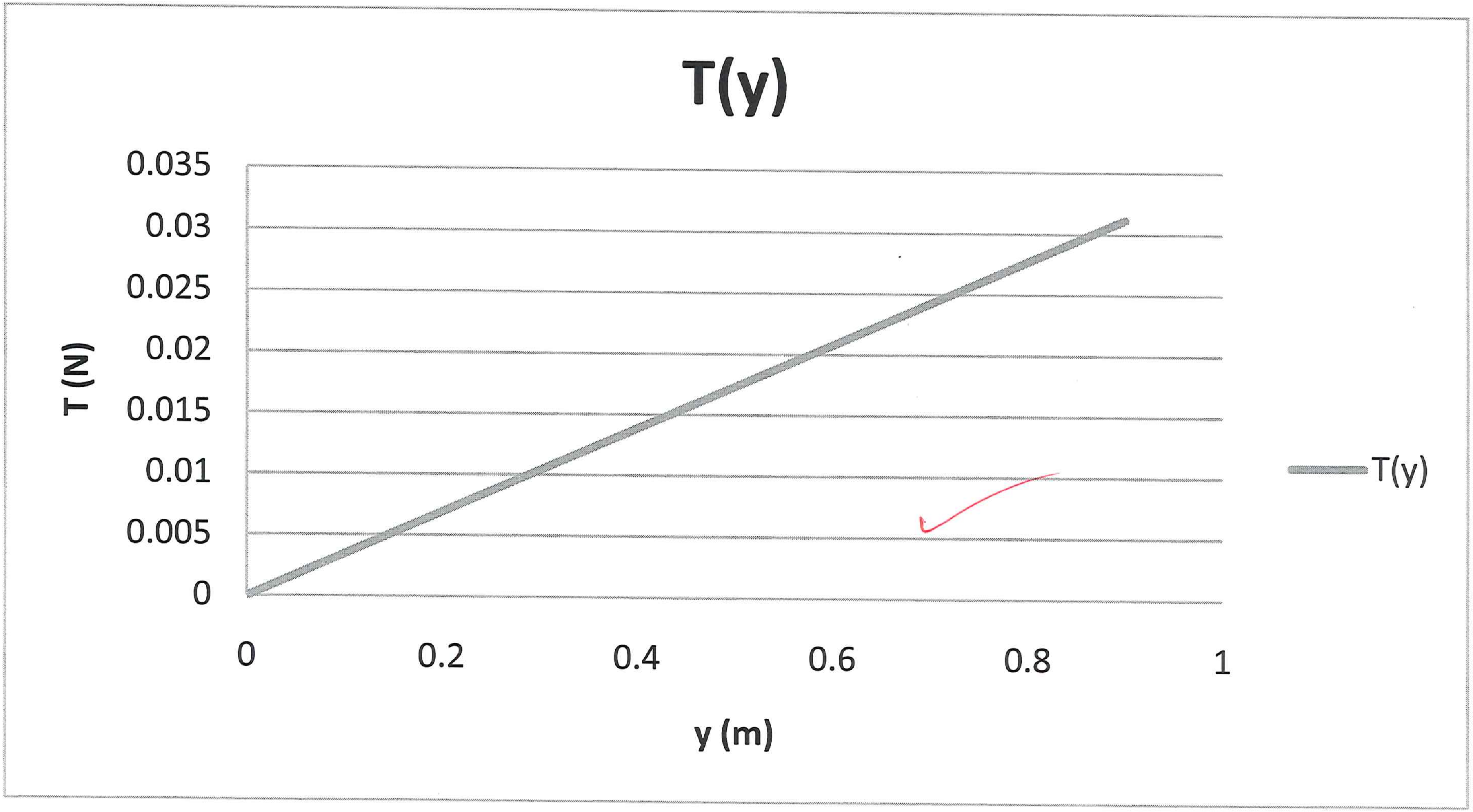
$\dot{m}_2 = 150.625\ kg/s$

$\dot{m}_3 = \frac{1}{30}(150.63)$

$F = -150.625\left(250\frac{m}{s}\right) + 150.625(900\,m/s) + \frac{150.63}{30}(900\,m/s)$

$= 102425\ N$

$\boxed{F_x = 102.4\ kN}$



T(y)
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
T (N)
0
0.2
0.4
0.6
0.8
1
y (m)
T(y)

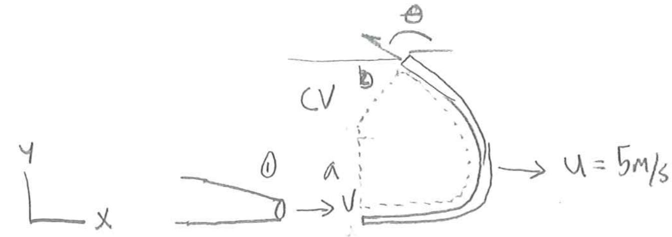
Homework # 3   231

David Malawey

7) ① $d = .04\,m$

$V = 25\,m/s$

$\theta = 150$



$F_s + F_B = \cancel{\frac{\partial}{\partial t}}\int \vec{V}_{rel}\,\rho\, dV + \int_{cs} \vec{V}_{rel}\,\rho\,\vec{V}_{rel}\cdot dA$

$= -\int_a \vec{V}_{rel}^{\,2}\rho\, dA + \int_b \vec{V}_{rel_b}\,\rho\,\vec{V}_{rel_b}\cdot dA$

$= -\dot{m}u_1 + \dot{m}u_2$

$u_{1rel}\ 20\,m/s \quad u_{2\,rel} = 20\,m/s\,[\cos(\pi-\theta)]$

$F = \dot{m}[-u - u\cos(\pi-\theta)]$

$= 31.35\,\frac{kg}{s}[20 + 20\cos(180-150)]$

$\boxed{F = 1170\,N}$

-1

$f_x = -940\,N$

$f_y = 252\,N$

$V = u\hat{i} + v\hat{j} + w\hat{k}$

only looking @ $u\hat{i}$

$V_{rel_1} = (25-5)\,m/s$

$\dot{m} = 25\frac{m}{s}\,\frac{\pi}{4}(.04\,m)^2\left(998\,\frac{kg}{m^3}\right)$

$\dot{m} = 31.35\,kg/s$

$\frac{3}{4}$

David Malaskey

Good

20/20

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee

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Home –Work 4

Section 1: The Equation of Conservation of Energy

1. A fire-truck shown in figure 1 is to deliver 1.5 ft^3/s of water to a maximum elevation of 60 ft above the hydrant. The pressure at the 4-in diameter outlet of the hydrant is 10 psi. If head losses are negligibly small, determine the power that the pump must add to the water.
2. Water is supplied at 150 ft^3/s and 60 psi to a hydraulic turbine through a 3-ft inside diameter inlet pipe as indicated in figure 2. The turbine discharge pipe has 4-ft inside diameter. The static pressure at section (2), 10 ft below the turbine inlet, is 10-in. Hg vacuum. If the turbine develops 2500 hp, determine the power lost between sections 1 and 2.

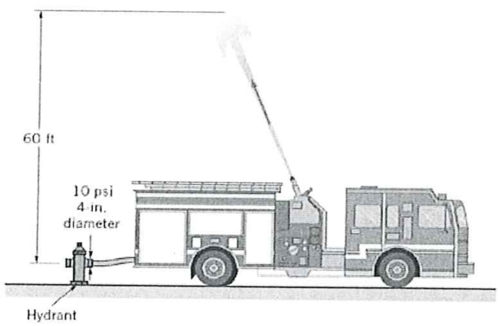


Figure 1

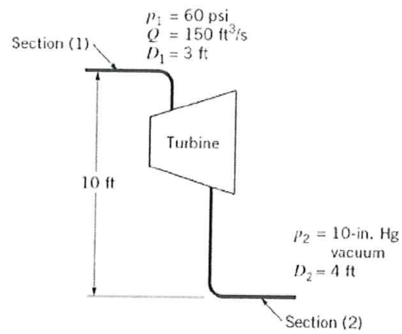


Figure 2

Section 2: Bernoulli's Equation

3. Problem 3.124 (White 7E)
4. Problem 3.128 (White 7E)
5. Problem 3.139 (White 7E)
6. Problem 3.142 (White 7E)

Joule $\frac{Kg\,m}{s^2}$ $M L S^{-2}$

Joule/g $L S^{-2}$

power $= \frac{F \cdot d}{t} =$

$\frac{M L}{T^2} \frac{L}{T}$

Power $= M L^2 T^{-3}$

David Malawey 10-07-10

1)

cons. energy

$A_1 = \pi\left(\frac{4}{12}ft\right)^2 \frac{1}{4} = .0873$

$V_1 = \frac{\dot{V}}{A_1} = 17.19\ ft/s$

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + \cancel{Z_1}^{0} + h_{pump} = \cancel{\frac{P_2}{\rho g}}^{0} + \cancel{\alpha_2 \frac{V_2^2}{2g}}^{0} + Z_2$$

$$h_{pump} = Z_2 - \frac{P_1}{\rho g} - \cancel{\alpha_1}\frac{V_1^2}{2}$$

$$h_p = 60\,ft - \frac{10\,lbf}{in^2}\left(\frac{144\,in^2}{ft^2}\right)\frac{1\,ft^3}{62.4\,lbf} - \left(17.19\,\frac{ft}{s}\right)^2 \frac{1}{2(32.2)}$$

$$h_p = 60 - 23.08 - 4.589 = 32.33\,ft$$

$$\dot{W}_{pump} = \dot{V}\rho g h = \left(1.5\,\frac{ft^3}{s}\right)62.4\frac{lbm}{ft^3}(32.33\,ft) = 3026 \frac{1\,hp}{550}$$

$$\boxed{\dot{W}_p = 5.502\,hp}$$

5/5

2)

$\dot{V} = 150\,\frac{ft^3}{s}$ $\qquad \rho g = 62.4\,\frac{lbf}{ft^3}$

① P= 60 psi
- d = 3 ft
- z = 10 ft
- A = 7.069 ft²
- V = 21.22 ft/s
- P = 8640 lbf/ft²

② P = 10 in Hg vacuum
- D = 4 ft
- A = 12.57 ft²
- V = 11.94 ft/s
- P = -4.912 psi = -707 lbf/ft²
- Z = 0

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = h_{turbine} + \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$-h_{turb} = \frac{P_2 - P_1}{\rho g} + \frac{V_2^2 - V_1^2}{2g} + Z_2 - Z_1$$

$$= \frac{-707 - 8640}{62.4} + \frac{11.94^2 - 21.22^2}{2(32.2)} + -10$$

$$h_{turb} = 149.8 + 4.778 + 10 = 164.58\,ft$$

2500 hp

$$164.6\,ft\left(9360\,\frac{lbm}{s}\right) = 181{,}900 / 550\,lbf\,ft/s$$

$$= 2801\,hp$$

5/5

Power lost = 2801 - 2500 = $\boxed{301\,hp}$

David Malawey

3) 3.124

$\Delta z = 0 \qquad P_1 + \frac{1}{2}\rho v_1^2 = P_{atm} + \frac{1}{2}\rho V_2^2$

$P_{atm} - P_1 = \frac{1}{2}\rho(V_1^2 - V_2^2) = \left(\frac{\rho}{2}\right)V_2^2\left[\left(\frac{D_2}{D_1}\right)^4 - 1\right] \qquad P_{atm} - P_1 = \rho g h$

5/5

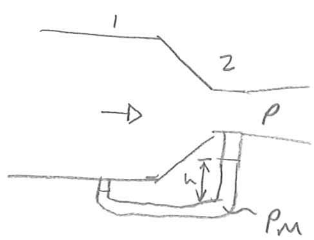
$V_1 = V_2\left(\frac{D_2}{D_1}\right)^2 \qquad V_1^2 = V_2^2\left(\frac{D_2}{D_1}\right)^4$

$\rho g h = \frac{\rho}{2} V_2^2\left[\left(\frac{D_2}{D_1}\right)^4 - 1\right] \qquad V^2 = \sqrt{\frac{2gh}{\left(\frac{D_2}{D_1}\right)^4 - 1}} \qquad \boxed{V_1 = \sqrt{\frac{2gh}{1-(D_1/D_2)^4}}}$

4) 3.128

steady, incompressible

Show $Q = \frac{A_2}{\sqrt{1-(D_2/D_1)^4}}\sqrt{\frac{2gh(\rho_M-\rho)}{\rho}}$



$P_1 = P_2 + (\rho_m - \rho)gh \qquad P_1 - P_2 = (\rho_M - \rho)gh$

$\Delta z = 0, \quad P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2 \Rightarrow (\rho_m - \rho)gh = \frac{\rho}{2}(V_2^2 - V_1^2) = \frac{\rho}{2}V_1^2\left(\frac{D_1}{D_2}\right)^4 - V_1^2$

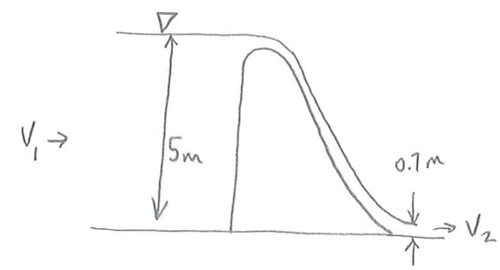
$= \frac{\rho}{2}V_1^2\left(\left(\frac{D_1}{D_2}\right)^4 - 1\right)$

$V_2 = V_1\left(\frac{D_1}{D_2}\right)^2 \qquad Q = A_2V_2 \qquad Q = A_2V_1\left(\frac{D_1}{D_2}\right)^2 \quad (1)$

$V_1^2 = \frac{(\rho_M - \rho)2gh}{\rho}\left[1 - \left(\frac{D_2}{D_1}\right)^{-4}\right]$

$$\boxed{Q = \frac{A_2\sqrt{\frac{2gh(\rho_M-\rho)}{\rho}}}{\sqrt{1-\left(\frac{D_2}{D_1}\right)^4}}}$$

David Malawey



5) $\underline{3.139}$ cons. mass $V_2 = \frac{V_1 h_1}{h_2} = 7.14 V_1$

a) $\cancel{\frac{P_1}{\rho g}}^{0} + \frac{V_1^2}{2g} + h_1 = \cancel{\frac{P_2}{\rho g}}^{0} + \frac{V_2^2}{2g} + h_2$

$V_1^2 = \cancel{2g}\frac{V_2^2}{\cancel{2g}}(h_2 - h_1)2g = (7.14 V_1)^2 + (0.7-5)\,2(9.81\,m/s^2)$

$V_1 = 1.3\,m/s$ $\boxed{V_2 = 9.28\,m/s}$

$\Sigma F_x = -F + \frac{\gamma}{2}h_1^2 - \frac{\gamma}{2}h_2^2 = \dot{m}(V_2 - V_1)$

$F = \frac{1}{2}(9790)[5^2 - .7^2) - 998(1.3)(5)(9.28-1.3) = \boxed{68\,300\ \frac{N}{m}}$

6) $\underline{3.142}$ $\cancel{\int_1^2 \frac{\partial V}{\partial t} ds}^{\text{small}} + \cancel{\frac{P_2}{\rho}} + \frac{V_2^2}{2} + g z_2 = \cancel{\frac{P_1}{\rho}} + \frac{V_1^2}{\rho} + g z_1$

Continuity $V_2 = V_1 \frac{A_1}{A_2} = V_1 \left(\frac{D}{d}\right)^2$ $V_1 = \left[\frac{2gh}{\left(\frac{D}{d}\right)^4 - 1}\right]^{1/2}$, $V_1 = \frac{dh}{dt}$

$\int_{h_0}^{h} \frac{dh}{h^{1/2}} = -\left[\frac{2g}{\left(\frac{D}{d}\right)^4 - 1}\right]^{1/2} \int_0^t dt$

$\boxed{h = \left[h_0^{1/2} - \left(\frac{g}{2\left(\left(\frac{D}{d}\right)^4 - 1\right)}\right)^{1/2} t\right]^2}$

$h = 0 \Rightarrow (h_0^{1/2})^2 = \frac{g}{2\left[\left(\frac{D}{d}\right)^4 - 1\right]} t^2$

$\boxed{t = \left[\frac{2g\left[\left(\frac{D}{d}\right)^4 - 1\right]}{h_0}\right]^{1/2}}$

$\frac{5}{5}$

19/20

David Malawey

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee Due 10-19-2010

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Home –Work 5

Section 1: Velocity Field Description and Flow Visualization

√ **1.** A steady, incompressible, two-dimensional velocity field is given by the following components in the xy-plane: $u = 1.85 + 2.33x + 0.656y$; $v = 0.754 - 2.18x - 2.33y$. Calculate the acceleration field. Also calculate the acceleration at the point $(x,y) = (1,2)$.

√ **2.** The equation for a velocity field is given as: $\vec{V} = \left(ax\,\hat{i} - ay\,\hat{j}\right)(z + \cos\omega t)$, where: $a = 3s^{-1}$ and $\omega = \pi\ s^{-1}$. Find:
(a) Algebraic equation for streamline at t = 0; (b) A plot of streamline through point (2,4) at t = 0.
(c) Will the streamline change with time? Explain. (d) Show velocity vector at point (2,4)

Section 2: Navier Stokes Equation (Conservation of linear momentum for a viscous fluid)

√ **3.** Consider the following steady, two dimensional, incompressible velocity field (where a, b and c are constants): $V = (ax + b)\,\hat{i} + (-ay + cx^2)\,\hat{j}$. Calculate the pressure as a function of x & y.

4. Consider liquid in a cylindrical tank in figure 1. Both the tank and the liquid rotate as a rigid body. The free surface of the liquid is exposed to ambient air in the room. Surface tension effects are negligible. Discuss the boundary conditions required to solve the problem. Specifically, what are the velocity boundary conditions in terms of cylindrical co-ordinate system (r,θ,z) and the velocity components (u_r, u_θ, u_z) at all surfaces including the tank walls and the free-surface? What pressure boundary conditions are appropriate for this flow field? Write mathematical equations for each boundary condition and discuss.

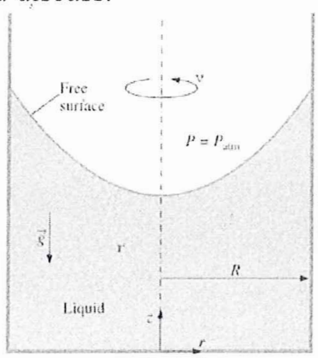


Figure 1

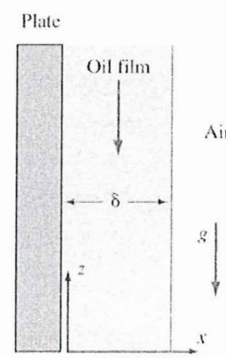


Figure 2

√ **5.** Oil of density ρ and viscosity μ, drains steadily down the side of a vertical plate as shown in figure 2. After a development region near the top of the plate, the oil film becomes independent of z and of constant thickness δ. Assume that $w = w(x)$ only and that the atmosphere offers no shear resistance to the surface of the film. Solve Navier Stokes equation to evaluate $w(x)$.

√ **6.** Problem 4.83 from White (7th Edition).

√ **7.** Problem 4.91 from White (7th Edition).

8. An incompressible Newtonian liquid is confined between two concentric cylinders of infinite length – a solid inner cylinder of radius R_i and a hollow outer cylinder of radius R_o (the z-axis is out of the page). The inner cylinder rotates at angular velocity ω_i while the outer cylinder rotates with angular velocity ω_o The flow is steady, laminar and 2-dimensional in the $r\theta$-plane. The flow is also rotationally symmetric, meaning that nothing is a function of coordinate θ. The flow is also circular, meaning that velocity component u_r=0 everywhere. Generate an exact expression for velocity component u_θ as a function of the radius r and other parameters in the problem. You may ignore gravity.

David Malawey

231 HW #5

1) steady, incompressible, 2 dimensional velocity field given

$u = 1.85 + 2.33x + 0.656y$; $v = .754 - 2.18x - 2.33y$ find accelleration field

$$\vec{a} = \frac{D\vec{V}}{dt} = \cancel{\frac{\partial V}{\partial t}}^{0} + (\vec{V}\cdot\nabla)\vec{V} = u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z}$$

$$u\frac{\partial V}{\partial x} = (1.85 + 2.33x + .656y)(2.33\,\hat{i} + -2.18\,\hat{j} + 0\hat{k})$$

$$v\frac{\partial V}{\partial y} = (.754 - 2.18x - 2.33y)(.656\,\hat{i} - 2.33\hat{j} + 0\hat{k})$$

$$\vec{a} = (4.805 + 4.00x)\hat{i} + (-5.790 + 0x + 4.00y)\hat{j}$$

$$\boxed{\vec{a}(1,2) = 8.81\hat{i} + 2.21\hat{j}}$$

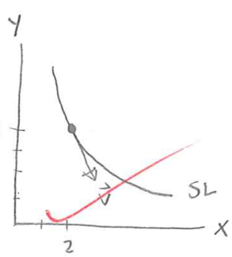
2) $\vec{V} = (ax\hat{i} - ay\hat{j})(z + \cos\omega t)$ $a = 3s^{-1}$ $\omega = \pi s^{-1}$

a) $u = ax(z + \cos\omega t)$
$v = -ay(z + \cos\omega t)$

$$\frac{dx}{ax\cancel{(z+\cos\omega t)}} = \frac{dy}{-ay\cancel{(z+\cos\omega t)}} \Rightarrow \frac{dx}{x} = \frac{dy}{-y}$$

$$\int\frac{dx}{x} = -\int\frac{dy}{y} \Rightarrow \ln x = -\ln y + \ln c \Rightarrow \ln(xy) = \ln(c) \Rightarrow \boxed{xy = c} \text{ or } \boxed{xy = 8}$$

b)



c) the streamline will not change with time, there is no t variable in the equation

d) $\vec{V}$ is shown. $\vec{V} = [3s^{-1}(2)\hat{i} - (3s^{-1})(4)\hat{j}](z + \cos(\pi s^{-1}\cdot 0))$

$$\vec{V} = \left(\frac{6}{s}\,i - \frac{12}{s}\,\hat{j}\right)(z+1)$$

5/5

David Malawey

231 HW #5

3) steady, 2-D, incompressible. find $P(x,y)$

$V = (ax+b)\hat{\imath} + (-ay+cx^2)\hat{\jmath}$

conservation of mass $\cancel{\frac{d\rho}{dt}}^0 + \nabla(\rho\vec{v}) = 0$ $\quad \nabla\cdot\vec{v} = 0,$ $\quad \underbrace{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}}_{✓} = 0$

$\nabla = i\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y}$ $\quad V = u\hat{\imath} + v\hat{\jmath}$ $\quad \frac{\partial u}{\partial x} = a$ $\quad \frac{\partial v}{\partial y} = -a$

1

1 X momentum: $\rho\frac{d\vec{V}}{dt} = \rho g_x - \nabla P + \mu\nabla^2\vec{V}$

$$\rho\left[\cancel{\frac{du}{dt}}^0 + u\frac{\partial u}{\partial x} + v\cancel{\frac{\partial u}{\partial y}}^0\right] = \cancel{\rho g_x}^0 - \frac{\partial P}{\partial x} + \mu\left[\cancel{\frac{\partial^2 u}{\partial x^2}}^0 + \cancel{\frac{\partial^2 u}{\partial y^2}}^0\right]$$

$\rho u\frac{\partial u}{\partial x} = -\frac{\partial P}{\partial x}, \quad -\rho(a^2x + ab) = \frac{\partial P}{\partial x}$ integrate

1 $P(x,y) = -\frac{\rho a^2x^2}{2} - \rho abx + g(y)$

1 Y momentum: $\rho\left[\cancel{\frac{\partial v}{\partial t}}^0 + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = \cancel{\rho g_y}^0 - \frac{\partial P}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \cancel{\frac{\partial^2 v}{\partial y^2}}^0\right]$

$\rho\left[(ax+b)(2cx) + (-ay+cx^2)(-a)\right] = -\frac{\partial P}{\partial y} + \mu(2c)$

$\rho\left[2acx^2 + 2bcx - a^2y - acx^2\right] - \mu(2c) = -\frac{\partial P}{\partial y}$

$\frac{\partial P}{\partial y} = \rho\left[-2acx^2 - 2bcx + a^2y + acx^2\right] + \mu 2c = g'(y)$

$g(y) = \rho\left[-2acx^2y - 2bcxy + \frac{a^2y^2}{2} + acx^2y\right] + 2\mu cy + k$

$$\boxed{P(x,y) = \rho\left[-\frac{a^2x^2}{2} - abx - 2acx^2y - 2bcxy + \frac{a^2y^2}{2} + acx^2y\right] + 2\mu cy + k}$$

-1 THE Pr. CANNOT BE FOUND.

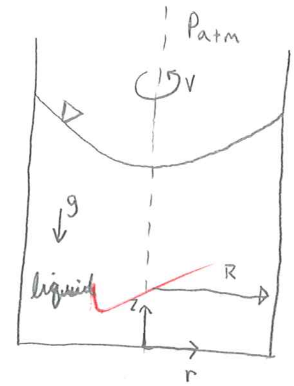
$\frac{4}{5}$

David Malawey

231 HW #5

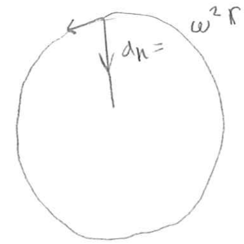
4) coordinate system (r, θ, z)

give boundary conditions for velocity components (u_r, u_θ, u_z)



- because liquid & container move as a rigid body, $u_r = u_z = 0$ everywhere in the liquid
- at the tank walls, and all throughout liquid, $u_\theta = r\omega$
- at the free surface, $P = P_{atm}$, $u_z = 0$
- Mathematically $\frac{\partial u_r}{\partial z} = \frac{\partial u_z}{\partial z} \neq 0$ at the free surface

5 / 5



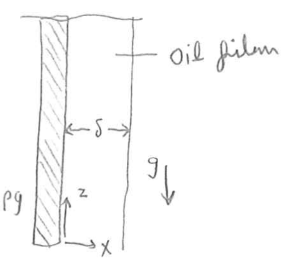
5) Given ρ, μ - No atmospheric shear resistance

- constant thickness δ

$w = w(x)$ ← only

Navier Stokes eqn:

pressure gradients neglected



$$\text{Z momentum: } \rho\left[\cancel{\frac{\partial w}{\partial t}}^{0} + \cancel{u}\frac{\partial w}{\partial x}^{0} + \cancel{v}\frac{\partial w}{\partial y}^{0} + w\cancel{\frac{\partial w}{\partial z}}\right] = \cancel{\frac{\partial p}{\partial z}}^{0} + \mu\left[\frac{\partial^2 w}{\partial x^2} + \cancel{\frac{\partial^2 w}{\partial y^2}}^{0} + \cancel{\frac{\partial^2 w}{\partial z^2}}^{0}\right] + \rho g$$

$$\rho g - \mu\frac{\partial^2 w}{\partial x^2} = 0 \qquad \frac{\partial^2 w}{\partial x^2} = \frac{\rho g}{\mu} \Rightarrow \frac{dw}{dx} = \frac{\rho g x}{\mu} + c_1 \quad w(x) = \frac{\rho g x^2}{2\mu} + c_1 x + c_2$$

boundary values

$$\frac{\partial w}{\partial x} = 0 \text{ @ } x = \delta \Rightarrow \frac{\rho g \delta}{\mu} + c_1 = 0 \Rightarrow c_1 = -\frac{\rho g \delta}{\mu}$$

$$w(x=0) = 0 = \frac{\rho g \cancel{x^2}^{0}}{2\mu} + \frac{\rho g \delta \cancel{x}^{0}}{\cancel{\mu}} + c_2 \qquad c_2 = 0$$

$$w(x) = \frac{\rho g}{\mu}\left(\frac{x^2}{2} - \delta x\right) \qquad \boxed{w(x) = \frac{\rho g x}{\mu}\left(\frac{x}{2} - \delta\right)}$$

Dana Malawry

231 HW #5

6) 4.83)

$$\underbrace{\cancel{\rho g_x}}_{\text{ignore}} - \frac{\partial P}{\partial x} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{\,0} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}^{\,0}\right) = \cancel{\rho \frac{du}{dt}}^{\,0}$$

u is $u(y)$ only, $P = P(x)$ only

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial P}{\partial x}\frac{1}{\mu} = \text{constant} \qquad u(y) = \frac{\partial P}{\partial x}\frac{y^2}{2\mu} + C_1 y + C_2$$

??why??

boundary eq's $u(0) = U \qquad u(h) = 0$

$$0 = \frac{\partial P}{\partial x}\frac{h^2}{2\mu} + C_1 h + C_2$$

$$U = 0 + C_1(0) + C_2$$

$$C_2 = U \qquad C_1 = \frac{1}{h}\left(\frac{\partial P}{\partial x}\frac{h^2}{2\mu} - U\right) = \frac{\partial P}{\partial x}\frac{1}{2\mu} - \frac{U}{h}$$

$$\boxed{u(y) = \frac{\partial P}{\partial x}\frac{1}{2\mu}\left(y^2 + yh\right) + U\left(1 - \frac{y}{h}\right)}$$

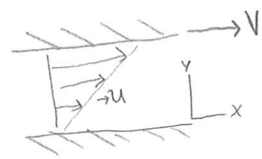
7) 4.91)

$$\tau_{xx} = a\left(\frac{\partial u}{\partial x}\right)^c \quad \tau_{yy} = a\left(\frac{\partial v}{\partial y}\right)^c \quad \tau_{zz} = a\left(\frac{\partial w}{\partial z}\right)^c$$

$$\tau_{xy} = \tau_{yx} = \frac{1}{2}a\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^c \quad \tau_{xz} = \tau_{zx} = \frac{1}{2}a\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^c \quad \tau_{yz} = \tau_{zy} = \frac{1}{2}a\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^c$$

find velocity profile $u(y)$ fully developed, steady flow, $v = w = 0$

x momentum: $$\rho\left[\cancel{\frac{\partial u}{\partial t}}^{\,0} + \cancel{\frac{\partial u}{\partial x}} + \frac{\partial u}{\partial y} + \cancel{\frac{\partial u}{\partial z}}^{\,0}\right] = \cancel{-\frac{\partial P}{\partial x}}^{\,0} + \cancel{\rho g_x}^{\,0} + \mu\left[\cancel{\frac{\partial^2 u}{\partial x^2}}^{\,0} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}^{\,0}\right]$$



— continued next page —

David Malawey

231 HW 5

$\boxed{7,4.91}$ $\cancel{\rho g_x}^{0} - \cancel{\frac{\partial p}{\partial x}}^{0} + \cancel{\frac{\partial \tau_{xx}}{\partial x}}^{0} + \frac{\partial \tau_{yx}}{\partial y} + \cancel{\frac{\partial \tau_{zx}}{\partial z}}^{0} = \rho\left(\cancel{\frac{\partial u}{\partial t}}^{0} + u\cancel{\frac{\partial u}{\partial x}}^{0} + v\frac{\partial u}{\partial y} + w\cancel{\frac{\partial u}{\partial z}}^{0} \right)$

$$\cancel{\rho v \frac{\partial u}{\partial y}}^{0} = \frac{\partial \tau_{yx}}{\partial y} = \frac{\partial}{\partial y}\left[\frac{a}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\right)^c\right]$$

$$\frac{\partial}{\partial y}\left[\frac{a}{2}\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}^{0}\right)^c\right] = 0$$

$\frac{d}{dy}\left[\frac{a}{2}\left(\frac{du}{dy}\right)^c\right] = 0$ $\quad \frac{\partial u}{\partial y}$ is a constant with couette flow

$$u(y) = C_1 y + C_2$$

Boundary conditions

$u(y=-h) = C_1(-h) + C_2 = 0$ $\quad V - 0 = C_1 h + \cancel{C_2} + C_1 h - \cancel{C_2}$

$u(y=h) = C_1(h) + C_2 = V$ $\quad V = 2C_1 h \quad C_1 = \frac{V}{2h}$

$$\frac{V}{2} + C_2 = V \Rightarrow C_2 = \frac{V}{2}$$

a) $\boxed{u(y) = \frac{V}{2h}(y) + \frac{V}{2}}$

b) velocity profile is the same for a newtonian fluid & this fluid

David Malaway

231 HW #5

8)

Cylindrical continuity

$$\frac{1}{r}\frac{\partial}{\partial r}(r u_r)^{0} + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta)^{0} + \frac{\partial}{\partial z}v_z^{0} = 0$$

θ momentum :

$$\rho(\vec{V}\cdot\nabla)v_\theta^{0} + \frac{\rho v_r v_\theta}{r}^{0} = \frac{1}{r}\frac{\partial p}{\partial \theta}^{0} + \rho g_\theta^{0} + \mu\left(\nabla^2 v_\theta - \frac{v_\theta}{r^2}\right)$$

steady, incomp.

$$0 = \mu\left(\nabla^2 v_\theta - \frac{v_\theta}{r}\right)$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r u_\theta)\right) = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r u_\theta) = c_1 \Rightarrow \frac{\partial}{\partial r}(r u_\theta) = c_1 r \Rightarrow r u_\theta = \frac{c_1 r^2}{2} + c_2$$

boundary conditions

$r = r_o \quad U = R_o \omega_o$

$r = r_i \quad U = R_i \omega_i$

$$\Rightarrow U_\theta = c_1 \frac{r}{2} + \frac{c_2}{r} \quad \text{solve for } c\text{'s}$$

$$r_o \omega_o = c_1 \frac{r_o}{2} + \frac{c_2}{r_o} \qquad c_1 = \left(r_o\omega_o - \frac{c_2}{r_o}\right)\frac{2}{r_o} = 2\omega_o - \frac{2c_2}{r_o^2}$$

$$r_i \omega_i = c_1 \frac{r_i}{2} + \frac{c_2}{r_i} \qquad c_1 = 2\omega_i - \frac{2c_2}{r_i^2}$$

$\omega_o =$

$r_o =$

$\omega_i =$

$r_i =$

$$\omega_o - \frac{c_2}{r_o^2} = \omega_i - \frac{c_2}{r_i^2}$$

$$c_2(r_o^{-2} - r_i^{-2}) = \omega_o - \omega_i$$

$$c_2 = \frac{\omega_o - \omega_i}{(r_o^{-2} - r_i^{-2})} \Rightarrow c_1 = 2\omega_o - \frac{2}{r_o^2}\left(\frac{\omega_o - \omega_i}{r_o^{-2} - r_i^{-2}}\right)$$

$$u_\theta = r\left[\omega_o - \frac{1}{r_o^2}\left(\frac{\omega_o - \omega_i}{r_o^{-2} - r_i^{-2}}\right)\right] + \frac{1}{r}\left(\frac{\omega_o - \omega_i}{r_o^{-2} - r_i^{-2}}\right)$$

$$\boxed{u_\theta = \left(-\frac{r^2\omega_o}{r_o^2} + \frac{r\omega_i}{r}\right)\left(\frac{\omega_o - \omega_i}{r_o^{-2} - r_i^{-2}}\right)}$$

David Malawey

17.5
20

17.5 – check

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee

Due Thurs, 11-4-2010

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Home –Work 6

Section 1: Kinematic Descriptions of Fluid Motion

1. Consider fully developed two-dimensional Poiseuilli flow –flow between two infinite parallel plates separated by a distance h, with both the top and the bottom plate stationary, and a forced pressure gradient dP/dx driving the flow. The flow is steady, incompressible, and two-dimensional in the x-y plane. The velocity components are given by: $u = \frac{1}{2\mu}\frac{dP}{dx}\left(y^2 - hy\right);\quad v = 0$ where μ is viscosity. (a) Is this flow rotational or irrotational? (b) If it is irrotational, calculate the vorticity component in the z-direction. (c) Do fluid particles in this flow rotate clockwise or counter-clockwise? (d) Calculate the linear strain rates in the x- and y- directions. (e) Calculate the shear strain rate ε_{xy}. (f) Combine your results to form the 2D strain rate tensor ε_{ij} - are the x- and y-axes principal axes?
2. Consider the flow field represented by the stream function: $\psi = 10xy + 17$. Is this a possible two-dimensional, incompressible flow? Is the flow irrotational?
3. Consider a steady, 2D, incompressible, irrotational velocity field specified by its velocity potential function: $\phi = 4(x^2 - y^2) + 3x - 2y$. (a) Calculate the velocity components u and v. (b) verify that the velocity field is irrotational in the region where ϕ applies. (c) Generate an expression for the stream function in that region.

Section 2: Dimensional Analysis and Similitude

4. Spin plays an important role in the flight trajectory of golf, Ping-Pong, and tennis balls. Therefore, it is important to know the rate at which spin decreases for a ball in flight. The aerodynamic torque, *T*, acting on the ball in flight, is thought to depend on the flight speed; *V*, air density; ρ, air viscosity; μ, ball diameter; *D*, spin rate (angular speed); ω, and diameter of the dimples on the ball; *d*. Determine the dimensionless parameters that result.
5. A model hydrofoil is to be tested at 1:20 scale. The test speed is chosen to duplicate the Froude number corresponding to the 60 knot prototype speed. To model cavitation correctly, the cavitation number must also be duplicated. At what ambient pressure must the test be run? Water in the model test basin can be heated to 130F, compared to 45F for the prototype. (Hint: Froude Number: $Fr = V/\sqrt{gL}$ and Cavitation Number: $Ca = \dfrac{p - p_v}{\frac{1}{2}\rho v^2}$. Given p_v = 2.23 psia at 130˚F and 0.15 psia at 45˚F.
6. A 1:50 scale model of a submarine is tested in a water tunnel. The drag force, F_D, depends on the water speed, *V*, density, ρ, viscosity, μ, and on model volume, Ω. Find a set of dimensionless parameters suitable to organize the resulting test data. Estimate the drag of the full-scale submarine at 27 knots if a model test at 10 knots yielded a drag force of 13 N.

$F_r = \dfrac{V}{\sqrt{gL}}$

HW#6

constant pressure gradient

1) $u = \left(\frac{1}{2\mu}\right)\left(\frac{dP}{dx}\right)(y^2 - hy)$ $V = 0$ Steady, incompressible, 2-D in x-y gradient

pressure-driven

a) $\nabla \times \vec{V} = 0i + \cancel{\frac{\partial u}{\partial z}}^{0} \hat{j} + \left(\frac{1}{2}\right) -\frac{\partial u}{\partial y} \hat{k}$ $\quad -\frac{1}{2}\frac{du}{dy} = -\frac{1}{2}\left[\frac{1}{2\mu}\frac{dP}{dx}(2y - h)\right]$ - rotational, $\omega \neq 0$

b) it is not irrotational

c) for $\frac{h}{2} > y \geq -\frac{h}{2}$ $\quad \omega > 0 \Rightarrow$ Vorticity is $\boxed{CCW}$

d) $\epsilon_{x \& y}$ $\quad \epsilon_{xx} = \frac{\partial u}{\partial x} = \boxed{0}$

$\epsilon_{yy} = \boxed{\frac{\partial v}{\partial y} = 0}$

e) $\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$ $\quad \boxed{\frac{1}{2}\left[\frac{1}{2\mu}\left(\frac{dP}{dx}\right)(2y - h)\right]}$

f) strain rate tensor ε_{ij}

2) stream function $\psi = 10xy + 17$ $\quad \frac{du}{dx} + \frac{dv}{dy} + \cancel{\frac{dw}{dz}}^{0} = 0$

$u = \frac{\partial \psi}{\partial y} = 10x$ $\quad V = \frac{\partial \psi}{\partial x} = 10y$ $\quad \frac{du}{\partial x} = 10$ $\quad \frac{\partial v}{dy} = 10$ $\quad 10 - 10 = 0$ ✓

a) only x & y are present $\Rightarrow$ $\boxed{2\text{-D}}$

b) $\psi = f(x, y)$ $\boxed{\text{incompressible}}$ ✓

c) $\omega = \frac{1}{2}\nabla \times \vec{V} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{\partial z} \\ 10x & -10y & 0 \end{vmatrix} = \frac{1}{2}[0 + 0 + 0] = 0$

$\boxed{\text{irrotational}}$ ✓

$\frac{5}{5}$

HW 6

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

#3 $\phi = 4(x^2 - y^2) + 3x - 2y$ velocity potential function

a) $u = \frac{\partial \phi}{\partial x} = \boxed{8x + 3}$ $v = \frac{\partial \phi}{\partial y} = \boxed{-8y - 2}$

b) Verify irrotational $\omega = \nabla \times V = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 8x+3 & -8y-2 & 0 \end{vmatrix} = [0 + 0 + 0]$ $\boxed{\omega = 0, \text{ irrotational}}$

"in the region where ϕ applies"

c) Generate an expression for stream function

$u = \frac{\partial \phi}{\partial x} = 8x + 3$ integrate w.r.t. y $8xy + 3y + f(x) = \psi$

$\frac{\partial \psi}{\partial x} = 8y + f'(x) = -(-8y - 2)$

$\cancel{8y} + f'(x) = \cancel{8y} + 2 \Rightarrow f'(x) = 2 \Rightarrow f(x) = 2x \Rightarrow$ $\boxed{\psi = 8xy + 3y + 2x}$

5/5

HW #6

4) $T, V, \rho, \mu, D, \omega, d$

Step 2

T	V	ρ	μ	D	ω	d
ML^2T^{-2}	L^1T^{-1}	ML^{-3}	$ML^{-1}T^{-1}$	L	T^{-1}	L

Step 1 $n = 7$ variables

Step 4: repeating parameters $j = 3$ $7-3=4$ $K=4$

Step 5: choose parameters Geom. D Mat. properties ρ external effects V

3 π groups

$\Pi_1 = T D^a \rho^b V^c \Rightarrow$
M $1 + 0 + 1b + 0 = 0$ $b = -1$
L $2 + a + -3b + c = 0 \Rightarrow a = -3$
T $-2 + 0 + 0 + -c = 0 \Rightarrow c = -2$

$$\boxed{\Pi_1 = \frac{T}{D^3 \rho V}}$$

$\Pi_2 = \mu D^a \rho^b V^c \Rightarrow$
M $1 + 0 + b + 0 = 0$ $b = -1$
L $-1 + a + -3b + c = 0$ $a = -1$
T $-1 + 0 + 0 + -1c = 0$ $c = -1$

$$\boxed{\Pi_2 = \frac{\mu}{D \rho V}}$$

$\Pi_3 = \omega D^a \rho^b V^c =$
M $0 + 0 + b + 0 = 0$ $b = 0$
L $0 + a - 3b + c = 0$ $a = 1$
T $-1 + 0 + 0 - c = 0$ $c = -1$

$$\boxed{\Pi_3 = \frac{\omega D}{V}}$$

$$\boxed{\Pi_4 = \frac{d}{D}}$$ -1

$\frac{4}{5}$

HW #6

5]

$$Ca = \left(\frac{P - P_v}{\frac{1}{2}\rho V^2}\right)_{model} = \left(\frac{P - P_v}{\frac{1}{2}\rho V^2}\right)_{prototype}$$

	Prototype	Model	
P	14.7 psia		
P_v	0.15 psia	2.23 psia	
ρ	1.94	1.903	slug/ft³
V^2	10270 $\frac{ft^2}{s^2}$	6373 $(\frac{ft}{s})^2$	

$$V_{proto} = 60 \text{ Knots} = 60 \frac{n\text{-mi}}{hr}\left(\frac{6080 \text{ ft}}{n\text{-mi}}\right)\left(\frac{hr}{3600 \text{ sec}}\right) = 101.3 \frac{ft}{sec}$$

$$Fr = \frac{V}{\sqrt{gL}} = \frac{101.3}{\sqrt{32.2(\frac{1}{20})}} = 79.84 \text{ ft/sec}$$

P_{model}
3.672 psia

$$\frac{P_{pro} - 0.15}{\cancel{\frac{1}{2}}(1.94)(10270)} = \frac{P_{mod} - 2.23}{\cancel{\frac{1}{2}}(1.903)(6373)}$$

$\boxed{P_{model} = 11.087 \text{ psi}}$

6]

F_D	V,	ρ	μ	Ω
MLT^{-2}	LT^{-1}	ML^{-3}	$ML^{-1}T^{-1}$	L^3

$n = 5 \quad j = 3$

$$\Pi_1 = F_D V^a \rho^b \Omega^c = \begin{array}{llllll} M & 1 + & 0 & + b & + 0 & = 0 \\ L & 1 + & a & + -3b & + 3c & = 0 \\ T & -2 + & -c & + 0 & + 0 & = 0 \end{array} \quad \begin{array}{l} b = -1 \\ a = 2 \\ c = -2 \end{array}$$

$$\Pi_1 = \frac{F_D V^2}{\rho \Omega^2}$$ $-\frac{1}{2}$

$$\Pi_2 = \mu V^a \rho^b \Omega^c = \begin{array}{llllll} M & 1 & + 0 & + b & + 0 & = 0 \\ L & -1 & + a & - 3b & + 3c & = 0 \\ T & -1 & - c & + 0 & + 0 & = 0 \end{array} \quad \begin{array}{l} b = -1 \\ a = 1 \\ c = -1 \end{array}$$

$$\Pi_2 = \frac{\mu V}{\rho \Omega}$$ $-\frac{1}{2}$

	model	prototype
F_D	13N	
V	10knot	27 knots
ρ	1.94	1.94
μ	H_2O	H_2O
Ω	8×10^{-6}	1

$$\frac{13(10)^2}{\cancel{1.94}(8 \times 10^{-6})^2} = \frac{F_D (27)^2}{\cancel{1.94}(1)}$$

2.786×10^{10} N

$\boxed{2.786 \times 10^{4} \text{ kN}}$ $-\frac{1}{2}$

237 kN.

=

$\frac{3.5}{5}$

David Malawey ME 231

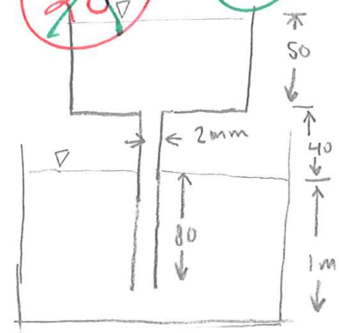
✓ ✓ ✓ ✓ ✓ ✓ ✓
25, 49, 56, 62, 76, 109, 149

~~18/20~~ 20/20

1.) P 6.25 (ethanol) ethyl alcohol 20°c

Tanks very wide

find Q (m^3/hr) laminar?



units: cm

neglect minor losses

$\rho = 789\ kg/m^3$

$\mu = 1.2\text{E-}3\ \frac{kg}{m \cdot s}$

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}}^{0} + z_1 = \cancel{\frac{P_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}}^{0} + z_2 + h_{maj} + \cancel{h_{min}}^{0}$$

no change, Patm

$1.9m = 1m + h_{maj}$ $\boxed{h_f = 0.9m}$

$h_f = f \frac{LV^2}{d\,2g}$ laminar $\rightarrow f = \frac{64}{Re}$ $Re = \frac{\rho V L}{\mu} =$ d

$$= \frac{32\,\cancel{64}\mu}{\rho \cancel{V} d}\left(\frac{L V^{\cancel{2}}}{d\,\cancel{2}g}\right) = \frac{32(1.2\text{E-}3)}{789\ kg/m^3}\ \frac{(1.2m)(V)}{(.002)^2(9.81)} = h_f$$

$V = .6047\ m/s$ $Q = .6047\ m/s\left(\frac{(.002)^2\pi}{4}\right) = 1.900\text{E}^{-6}\ m^3/s\left(\frac{3600\ sec}{1hr}\right) = \boxed{.006839\ m^3/hr}$

$Re = \frac{4\rho Q}{\pi \mu d} \approx \frac{\rho V d}{\mu} \approx 795 < 2300$ Ok, laminar

p369

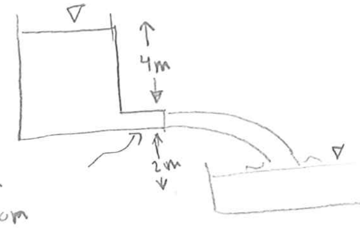
Re < 2300 for laminar

5/5

HOMEWORK 6

2) $\underline{6.49}$

$Q = 11\ m^3/hr \quad V = \frac{Q}{A} \quad A = \frac{(.03)^2}{4}\pi \Rightarrow V = 4.323\ m/s$



$\rho = 998\ kg/m^3$

$\mu = .001\ \frac{kg}{m \cdot s}$

L = 5m
d = 3cm

find ϵ_{max}

$\frac{V_1}{2g} \quad z_1 = z_2 + h_{maj} \Rightarrow h_{maj} = 4\ m - \frac{V_2^2}{2g}$ (From free surface to spout)

$h_f = f\frac{LV^2}{2dg} \qquad h_f = 4 - \frac{(4.323)^{(2)}}{2(9.81)} = 3.047$

$Re = \frac{\rho V d}{\mu} = \frac{998(4.32)(.03)}{.001} = 129{,}000 = \underline{1.3 \times 10^5}$

moody chart

$\frac{\epsilon}{d} \approx .0003$

$f = h_f\frac{2dg}{LV^2} = \frac{3.047(2)(.03)(9.81)}{5m(4.32)^2} = \underline{.0192}$

$\epsilon = 9\ E{-}6m = \boxed{.009mm}$

3) $\underline{6.56}$

$Q = 4.47\ E^7$ gallons/day $(.1337\ ft^3/gal)(\frac{1 day}{x\ Seconds}) = 69.17\ ft^3/s$

find ΔP, a) 2680 lbf/in^2

find h_p, pumps b) 5300 hp

L = 800 mi = 4.224 E6 ft $\quad \rho = .86(9790)\ N/m^3 = 53.60\ lbf/ft^3$

9 pumps, equal spacing $\quad \mu = 4\ \frac{N \cdot s}{m^2} = .0835\ \frac{lbf\ s}{ft^2}$

Oil, 60°C $\quad$ SG = .86

galv Fe wall $\quad \nu = 3.75(10^{-6})\ m^2/s = 4.035(10^{-5})\ ft^2/s$

$\epsilon = .0005$ ft

assume $P_1 = P_2$, $z_1 = z_2$, $V_1 = V_2$ $\quad$ 1st assume laminar flow

Mean velocity $= \frac{Q}{A} = \frac{69.17\ ft^3/s}{\pi\ 2^2\ ft^2} = 5.504\ ft/s \qquad Re = \frac{Vd}{\nu} = \frac{5.504(4)}{4.035(10^{-5})} = 5.456\ E5$

$\frac{\epsilon}{d} = \frac{.0005}{4} = .000125 \qquad Re = \frac{\rho V d}{\mu} = \frac{858.3(5.504)(4)}{.0835} = 2.263\ E5$

$f = .015$ OR $.017$

a $\quad \frac{P_1}{\rho g} + \cancel{h_{pump}} = \frac{P_2}{\rho g} + h_L \qquad h_L = \frac{V^2}{2g} f \frac{L}{D}$

$P_2 - P_1 = \rho g(-\frac{V^2}{2g}(f)\frac{L}{D}) = 53.60\left(-\frac{(5.504)^2}{2(32.2)}(.017)\frac{4.224(10^6)}{4}\right) \Rightarrow \Delta P = 4.526\ E5\ lb/ft^2$

$\Delta P = 3143$ psi

$\dot{W} = Q\Delta P = \frac{69.17\ \frac{ft^3}{s}(4.526\ E5)}{9\ pumps} = \boxed{6325\ hp}$

L = 117 miles $\quad \cancel{-1}$

P = 26.5 MW $\quad -1$

$\frac{3}{5}$

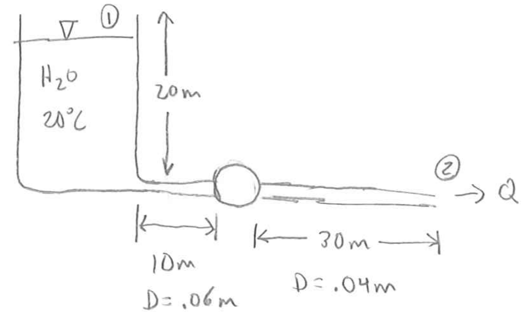
6.76

$\dot{W}_s = 400W$ wrought iron

$\varepsilon = .046\,mm$

$\frac{\varepsilon}{D_1} = 7.67(10^{-4})$

$\frac{\varepsilon}{D_2} = 11.5\ (10^{-4})$



$$\cancel{\frac{P_1}{\rho g}}^0 - \cancel{\frac{V_1^2}{2g}}^0 + z_1 + \cancel{h_{pump}}^0 = \cancel{\frac{P_2}{\rho g}}^0 + \frac{V_2^2}{2g} + z_2 + h_{turb} + h_{L_1} + h_{L_2}$$

$$\frac{V_2^2}{2g}\ z_2 - z_1 + h_{turb} + h_L = 0$$

$$Re_1 = \frac{VD\rho}{\mu} = V_1(59900)$$

$$Re_2 = \quad = V_2(39900)$$

$$h_{L_1} = f\frac{l}{D}\frac{V^2}{2g}$$

$$h_t = \frac{\dot{W}_s}{\rho g Q} = \frac{400\,W}{998(9.81)\,Q} = \frac{.0409}{Q}$$

$$= f_1\frac{10}{.06}\frac{V_1^2}{2(9.81)}$$

$$h_{L_1} = f_1(8.495)V_1^2$$

$$Q = V_1A_1 = V_2A_2 = V_1(\tfrac{1}{354}) = V_2(\tfrac{1}{796})$$

$$h_{L_2} = f_2(38.23)V_2^2$$

$$h_{L_1} = f_1Q^2(6.78\times10^{-5})$$

$$h_{L_2} = f_2Q^2(6.03\times10^{-5})$$

$$20m = \frac{.0409}{Q} + Q^2(1.063(10^6)f_1 + 2.36(10^7)f_2 + 32300)$$

$$\frac{.0409}{Q} = +20 - Q^2(21{,}800 + 496245 + 32{,}300)\ (\#1)$$

$\Downarrow$

$$\frac{V_2^2}{2g} = \frac{Q^2}{A_2^2\,2g}$$

$$= \frac{Q^2}{.0247}$$

Guess $Q = .01\,m^3/s \Rightarrow$

V_1	V_2	f_1
3.54		

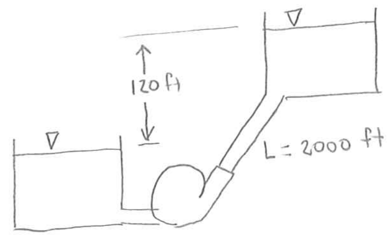
	V_1	V_2	R_1	R_2	f_1	f_2	$\Rightarrow Q =$
$Q = .01\,m^3/s$	3.54	7.96	$2.12(10^5)$	$3.18(10^5)$	.0205	.021	.00434
$Q = .00434$	1.54	3.45	$9.22(10^4)$	$1.38(10^5)$	.0215	.0225	$\boxed{.00416}$

I only got 1 answer

$$.0409 = 20Q - Q^3(22300 + 531000 + 32300)$$

5/5

6.62



H_2O 20°C

L = 200 ft

$Q = 3 ft^3/s$

d = 6 in, cast iron

$\eta_p = 75\%$

find h_p req'd

$\varepsilon = .00085$ ft

$\rho = 998 = 62.30$ lbm/ft³

$= 1.936$ slug/ft³

$\mu = .001$ N S/m²

$= 2.089(10^{-5})$ lbf·s/ft²

$\frac{\varepsilon}{d} = .0017$

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}}^{0} + z_1 + h_{pump} = \cancel{\frac{P_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}}^{0} + z_2 + h_L$$

$$\eta_p h_{pump} = z_2 - z_1 + h_L$$

$$= 120 + \left(\frac{.0227(2000)}{.5}\right)\left(\frac{(15.28)^2}{2(32.2)}\right)$$

$$h_p = 120 + 329.0 = 449$$

$$h_L = \left(f\frac{l}{D} + \cancel{\Sigma k_i}^{0}\right)\frac{V^2}{2g}$$

$$\bar{V} = \frac{3 ft^3/s(4)}{\pi(.5)^2} = 15.28 \text{ ft/s}$$

$$Re = \frac{VD\rho}{\mu} = \frac{15.28 \frac{ft}{s}(.5 ft)(1.936 \frac{slug}{ft^3})}{2.089(10^{-5}) \text{ lbf}\cdot s/ft^2}$$

$$Re = 7.09(10^5)$$

$$f = .0227$$

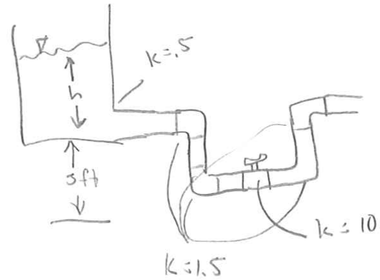
$$\dot{W} = \frac{\rho g Q h_p}{\eta_p} = \frac{1.936(32.2)(3)(449)}{0.75}$$

$$\dot{W}_{shaft} = 112000 \frac{ft\cdot lbf}{s}\left(\frac{1 hp}{550 \text{ ft lb/s}}\right) = \boxed{204 hp}$$

#4

$L = 20\,ft$ $\quad f = 0.03 \quad \Delta z = 4.5$ chosen height h

$D = .6\,in$



$Q = V_T A_T = (VA)_{pipes}$

$V_1 = \frac{dh}{dt}(\text{graph}) = \frac{-1}{250} = .004\ ft/s$

we could have

You can assume $V_1 = 0$

$$\frac{V_1^2}{2g} + (\Delta z) = \frac{V_2^2}{2g} + \frac{V_2^2}{2g}\left(f\frac{l}{d} + \sum_i k_i\right)$$

$\Sigma k_i' = .5 + 5(1.5) + 10 = 18$

$$V_2^2\left(1 + f\frac{l}{d} + \sum_i k_i\right) = V_1^2 + \Delta z(2g)$$

$$V_2^2 = \frac{(.004)^2 + 4.5(2)(32.2)}{1 + (.03)\left(\frac{20(12)}{.6}\right) + 18} \Rightarrow V_2 = 3.06$$

$$A_1 = \frac{V_2 A_2}{V_1} = \frac{3.06\left(\frac{\pi}{4}\left(\frac{.6}{12}\right)^2\right)}{.004} \Rightarrow A_1 = 1.5\ ft^2$$

$Re = 62179\,V$

$\cancel{D = .009\ m}$

$\cancel{L = .35\ m}$

$\cancel{\mu = 4 \times 10^{-7}\ \frac{Ns}{m^2}}$

$SG = 1$

Eqn's

$$\boxed{\frac{P_1}{\rho g} - \frac{V_1^2}{2g} + Z_1 + h_{pump} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_{turb} + h_L}$$

$$\boxed{h_L = \left(f\frac{l}{D} + \sum_i K_i\right)\frac{V^2}{2g}}$$

reynolds # based on diam: $\boxed{Re_D = \frac{VD\rho}{\mu}}$

$$\boxed{\dot{W}_s = \rho g Q h_p}$$

(laminar) friction factor $\boxed{f = \frac{64}{Re} =}$

$$Re = \frac{\rho Q D}{\mu \left(\frac{\pi}{4}\right) D^2}$$

---

finding V, Re, & f $\quad V = \sqrt{\frac{\dots}{\dots f + \dots}}$ $\quad Re = \dots V$ } guess on f, plug into V, plug into Re, read

Table 6.3 laminar friction factors p 385

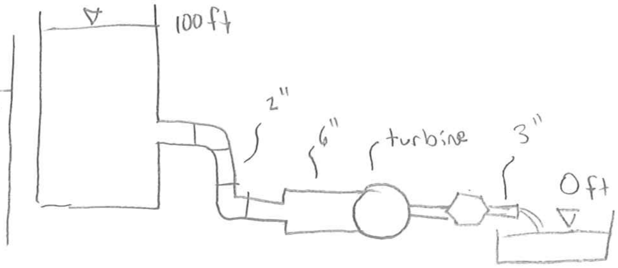
discharge coefficient p 421

$C_d =$

$$Q = C_d A_T \left[\frac{2(P_1 - P_2)/\rho}{1-\beta^4}\right]^{1/2}$$

6.109

	L	D	ε	ε/D	V (ft/s)	Re
1)	125 ft	2 in	.00085 ft	.0051	7.33	1.13×10^5
2)	75 ft	6 in	"	.0017	0.815	3.78×10^4
3)	150 ft	3 in	"	.0034	3.26	7.55×10^4



90° elbow (x3), open globe valve, all flanged

$\dot{W}_{turbine} = ?$ $Q = .16\ ft^3/s$ $20°C$ H_2O

$V_2 = \frac{Q}{A_2} = \frac{.16}{\pi\left(\frac{1.5}{12}\right)^2} = 3.26$ ft/s

$z_1 = \frac{V_2^2}{2g} + z_2 + h_t + h_L$

$Re_1 = \frac{VD\rho}{\mu} = \frac{VD\ 1.937}{2.09E\text{-}5}$

Major

$h_{L_1} = f_1 \frac{l_1}{D_1}\frac{V_1^2}{2g} = 19.40$

$h_{L_2} = f_2 \frac{l_2}{D_2}\frac{V_2^2}{2g} = .042$

$h_{L_3} = f_3 \frac{l_3}{D_3}\frac{V_3^2}{2g} = 2.77$

$= 22.21$

$f_1 = .031$ $f_2 = .027$ $f_3 = .028$

$k_{elbow} = .39$ $k_{entrance} = 0.5$

$k_{globe} = 7.25$

Minor losses $= \sum_i k_i \frac{V_i^2}{2g}$

$= 3(0.39)\frac{7.33^2}{2(32.2)} = .976$ elbow

$+ 1(7.25)\frac{3.26^2}{2(32.2)} = 1.20$ valve

$+ 1(.5)\frac{7.33^2}{2(32.2)} = .417$ entrance

$(.8)\frac{.815^2}{2(32.2)} = .00825$ s.e.

$= 2.601$

$h_{L,total} = 24.8$

$100 = \frac{3.26^2}{2(32.2)} + 24.8 + h_t \Rightarrow h_t = 75.04$

$\dot{W}_s = \rho g Q h_t = \frac{1.937(32.2).16(75.04)}{550\ \frac{ft \cdot lbs/s}{hp}} \Rightarrow \boxed{\dot{W}_s = 1.36\ hp}$

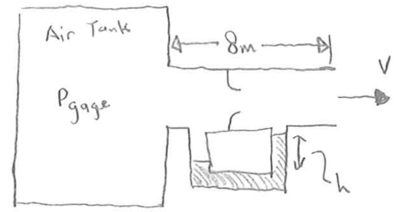
$\frac{5}{5}$

6.149

2 cm smooth pipe

long-radius nozzle of 1 cm diam $\frac{d}{D} = \frac{1}{2}$

Meriam red oil (SG = .827)



P_{tank} Pa (gage)	60	320	1200	2050	2470	3500	4900
h_{mano} mm	6	38	160	295	380	575	820
V, m/s							

$K_{entrance} = 0.5$

$K_{nozzle} = 0.7$

$\rho = 1.2\ kg/m^3$

$\mu = 1.5(10^{-5})\ kg/m\cdot s$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_L$$

$$P_1 = P_2 + \frac{\rho}{2}V_2^2 + h_L \rho g \qquad h_L = \left(f\frac{l}{D} + \sum_i k_i\right)\frac{V^2}{2g}$$

$$P_{tank} - P_{atm} = \frac{\rho}{2}V^2\left(1 + f\frac{L}{D} + k_{ent.} + k_{noz}\right)$$

$$V_{throat} = C_d\sqrt{\frac{2\,\Delta P_{mano}}{\rho(1-\beta^4)}} \qquad \Delta P_{mano} = (\rho_{oil} - \rho_{air})gh$$

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee

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Home –Work 7

Section 1: Internal Viscous Flow in a Duct

1. Problem 6.25 (Note: Please neglect minor losses for your calculations).
2. Problem 6.49
3. Problem 6.56
4. Problem 6.62
5. Problem 6.76
6. Problem 6.109
7. Problem 6.149

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee
Solutions to Home-Work 7

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Solution 1:

6.25 For the configuration shown in Fig. P6.25, the fluid is ethyl alcohol at 20°C, and the tanks are very wide. Find the flow rate which occurs in m^3/h. Is the flow laminar?

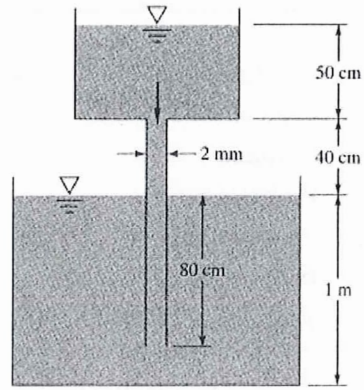

Fig. P6.25

Solution: For ethanol, take ρ = 789 kg/m^3 and μ = 0.0012 kg/m·s. Write the energy equation from upper free surface (1) to lower free surface (2):

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_f,\quad \text{with } p_1=p_2 \text{ and } V_1\approx V_2\approx 0$$

$$\text{Then}\quad h_f=z_1-z_2=0.9\text{ m}=\frac{128\mu LQ}{\pi\rho g d^4}=\frac{128(0.0012)(1.2\text{ m})Q}{\pi(789)(9.81)(0.002)^4}$$

Solve for $Q \approx 1.90\text{E–}6\ \text{m}^3/\text{s} = \mathbf{0.00684\ m^3/h}$. *Ans.*

Check the Reynolds number $Re = 4\rho Q/(\pi\mu d) \approx 795$ – **OK, laminar flow.**

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee
Solutions to Home-Work 7

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Solution 2:

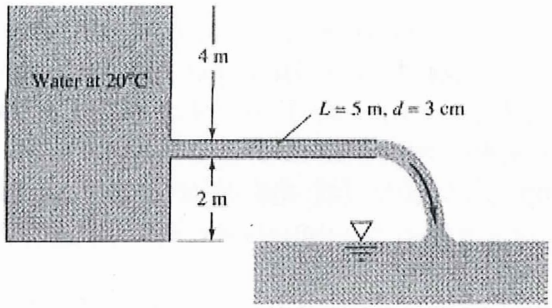

Fig. P6.49

Solution: For water at 20°C, take ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. Evaluate V and Re for the expected flow rate:

$$V = \frac{Q}{A} = \frac{11/3600}{(\pi/4)(0.03)^2} = 4.32\ \frac{m}{s};\quad Re = \frac{\rho V d}{\mu} = \frac{998(4.32)(0.03)}{0.001} = 129000$$

The energy equation yields the value of the head loss:

$$\cancel{\frac{p_{atm}}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{p_{atm}}{\rho g}} + \frac{V_2^2}{2g} + z_2 + h_f \quad \text{or} \quad h_f = 4 - \frac{(4.32)^2}{2(9.81)} = 3.05\ m$$

$$\text{But also}\quad h_f = f\frac{L}{d}\frac{V^2}{2g},\quad \text{or:}\quad 3.05 = f\left(\frac{5.0}{0.03}\right)\frac{(4.32)^2}{2(9.81)},\quad \text{solve for}\ f \approx 0.0192$$

With f and Re known, we can find ε/d from the Moody chart or from Eq. (6.48):

$$\frac{1}{(0.0192)^{1/2}} = -2.0\log_{10}\left[\frac{\varepsilon/d}{3.7} + \frac{2.51}{129000(0.0192)^{1/2}}\right],\quad \text{solve for}\ \frac{\varepsilon}{d} \approx 0.000394$$

Then $\varepsilon = 0.000394(0.03) \approx 1.2E{-}5\ m \approx$ **0.012 mm** (very smooth) *Ans.*

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee
Solutions to Home-Work 7

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Solution 3:

6.56 Consider a horizontal 4-ft-diameter galvanized-iron pipe simulating the Alaska Pipeline. The oil flow is 70 million U.S. gallons per day, at a density of 910 kg/m^3 and viscosity of 0.01 kg/m·s (see Fig. A.1 for SAE 30 oil at 100°C). Each pump along the line raises the oil pressure to 8 MPa, which then drops, due to head loss, to 400 kPa at the entrance to the next pump. Estimate (a) the appropriate distance between pumping stations; and (b) the power required if the pumps are 88% efficient.

Solution: For galvanized iron take $\varepsilon = 0.15$ mm. Convert $d = 4$ ft $= 1.22$ m. Convert $Q =$ 7E7 gal/day $= 3.07$ m^3/s. The flow rate gives the velocity and Reynolds number:

$$V = \frac{Q}{A} = \frac{3.07}{\pi(1.22)^2/4} = 2.63\ \frac{m}{s};\quad Re_d = \frac{\rho V d}{\mu} = \frac{910(2.63)(1.22)}{0.01} = 292{,}500$$

$$\frac{\varepsilon}{d} = \frac{0.15\ mm}{1220\ mm} = 0.000123,\quad f_{Moody} \approx 0.0157$$

Relating the known pressure drop to friction factor yields the unknown pipe length:

$$\Delta p = 8{,}000{,}00 - 400{,}000\ Pa = f\frac{L}{d}\frac{\rho}{2}V^2 = 0.0157\frac{L}{1.22}\left(\frac{910}{2}\right)(2.63)^2,$$

Solve $L = \mathbf{188{,}000\ m} = 117$ miles *Ans.* (a)

The pumping power required follows from the pressure drop and flow rate:

$$Power = \frac{Q\,\Delta p}{Efficiency} = \frac{3.07(8E6 - 4E5)}{0.88} = 2.65E7\ \text{watts}$$

$= \mathbf{26.5\ MW}$ (35,500 hp) *Ans.* (b)

****

Solution 4:

6.62 Water at 20°C is to be pumped through 2000 ft of pipe from reservoir 1 to 2 at a rate of 3 ft^3/s, as shown in Fig. P6.62. If the pipe is cast iron of diameter 6 in and the pump is 75 percent efficient, what horsepower pump is needed?

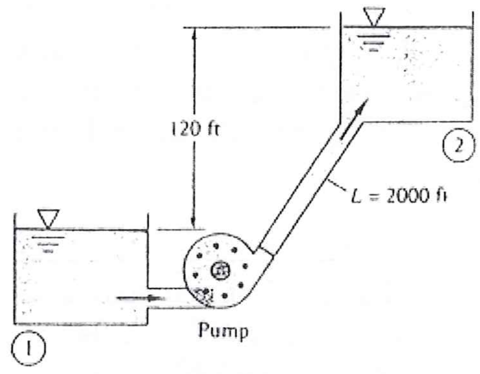

Fig. P6.62

Solution: For water at 20°C, take $\rho = 1.94\ slug/ft^3$ and $\mu = 2.09E{-}5$ slug/ft·s. For cast iron, take $\varepsilon \approx 0.00085$ ft, or $\varepsilon/d = 0.00085/(6/12) \approx 0.0017$. Compute V, Re, and *f*:

$$V = \frac{Q}{A} = \frac{3}{(\pi/4)(6/12)^2} = 15.3\ \frac{ft}{s};$$

$$Re = \frac{\rho V d}{\mu} = \frac{1.94(15.3)(6/12)}{2.09E{-}5} \approx 709000 \quad \varepsilon/d = 0.0017, \quad f_{Moody} \approx 0.0227$$

The energy equation, with p1 = p2 and V1 ≈ V2 ≈ 0, yields an expression for pump head:

$$h_{pump} = \Delta z + f\frac{L}{d}\frac{V^2}{2g} = 120\text{ ft} + 0.0227\left(\frac{2000}{6/12}\right)\frac{(15.3)^2}{2(32.2)} = 120 + 330 \approx 450\text{ ft}$$

$$\text{Power:} \quad P = \frac{\rho g Q h_p}{\eta} = \frac{1.94(32.2)(3.0)(450)}{0.75} = 112200 \div 550 \approx \mathbf{204\ hp} \quad \textit{Ans.}$$

$\mu = 2.09(10^{-5})\ \frac{slug}{ft \cdot s}$

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee
Solutions to Home-Work 7

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Solution 5:

6.76 The small turbine in Fig. P6.76 extracts 400 W of power from the water flow. Both pipes are wrought iron. Compute the flow rate Q m^3/h. Sketch the EGL and HGL accurately.

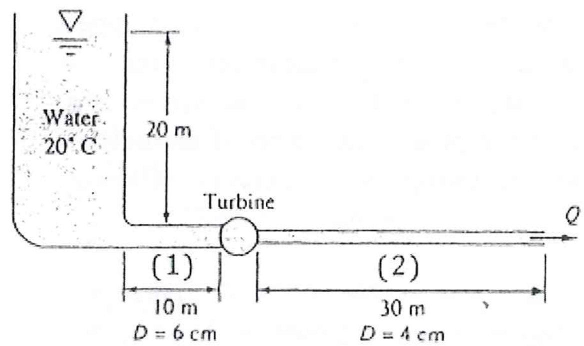

Fig. P6.76

Solution: For water, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For wrought iron, take $\varepsilon \approx 0.046$ mm, hence $\varepsilon/d1 = 0.046/60 \approx 0.000767$ and $\varepsilon/d2 = 0.046/40 \approx 0.00115$. The energy equation, with $V1 \approx 0$ and $p1 = p2$, gives

$$z_1 - z_2 = 20 \text{ m} = \frac{V_2^2}{2g} + h_{f2} + h_{f1} + h_{turbine}, \quad h_{f1} = f_1 \frac{L_1}{d_1}\frac{V_1^2}{2g} \text{ and } h_{f2} = f_2 \frac{L_2}{d_2}\frac{V_2^2}{2g}$$

$$\text{Also, } h_{turbine} = \frac{P}{\rho g Q} = \frac{400 \text{ W}}{998(9.81)Q} \text{ and } Q = \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2$$

The only unknown is Q, which we may determine by iteration after an initial guess:

$$h_{turb} = \frac{400}{998(9.81)Q} = 20 - \frac{8f_1L_1Q^2}{\pi^2 g d_1^5} - \frac{8f_2L_2Q^2}{\pi^2 g d_2^5} - \frac{8Q^2}{\pi^2 g d_2^4}$$

$$\text{Guess } Q = 0.003 \frac{\text{m}^3}{\text{s}}, \quad \text{then } Re_1 = \frac{4\rho Q}{\pi \mu d_1} = 63500. \quad f_{1,Moody} \approx 0.0226.$$

$$Re_2 = 95300, \quad f_2 \approx 0.0228.$$

But, for this guess, hturb(left hand side) ≈ 13.62 m, hturb(right hand side) ≈ 14.53 m (wrong). Other guesses converge to hturb ≈ 9.9 meters. For $Q \approx 0.00413$ m^3/s ≈ **15 m^3/h**. *Ans.*

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee
Solutions to Home-Work 7

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Solution 6:

6.109 In Fig. P6.109 there are 125 ft of 2-in pipe, 75 ft of 6-in pipe, and 150 ft of 3-in pipe, all cast iron. There are three 90° elbows and an open globe valve, all flanged. If the exit elevation is zero, what horsepower is extracted by the turbine when the flow rate is 0.16 ft^3/s of water at 20°C?

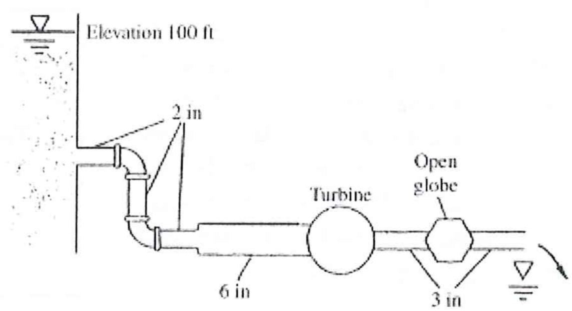

Fig. P6.109

Solution: For water at 20°C, take $\rho = 1.94$ slug/ft^3 and $\mu = 2.09\text{E}{-5}$ slug/ft·s. For cast iron, $\varepsilon \approx 0.00085$ ft. The 2″, 6″, and 3″ pipes have, respectively,

$$\text{(a)}\quad L/d = 750,\ \varepsilon/d = 0.0051;\quad \text{(b)}\quad L/d = 150,\ \varepsilon/d = 0.0017;$$

$$\text{(c)}\quad L/d = 600,\ \varepsilon/d = 0.0034$$

The flow rate is known, so each velocity, Reynolds number, and f can be calculated:

$$V_a = \frac{0.16}{\pi(2/12)^2/4} = 7.33\ \frac{\text{ft}}{\text{s}};\quad Re_a = \frac{1.94(7.33)(2/12)}{2.09\text{E}{-5}} = 113500,\quad f_a \approx 0.0314$$

Also, $V_b = 0.82$ ft/s, $Re_b = 37800$, $f_c \approx 0.0266$; $V_c = 3.26$, $Re_c = 75600$, $f_c \approx 0.0287$

Finally, the minor loss coefficients may be tabulated:

sharp 2″ entrance: $K = 0.5$; three 2″ 90° elbows: $K = 3(0.95)$ ×

2″ sudden expansion: $K \approx 0.79$; 3″ open globe valve: $K \approx 6.3$

The turbine head equals the elevation difference minus losses and the exit velocity head:

$$h_t = \Delta z - \sum h_f - \sum h_m - V_c^2/(2g)$$

$$= 100 - \frac{(7.33)^2}{2(32.2)}[0.0314(750) + 0.5 + 3(0.95) + 0.79]$$

$$- \frac{(0.82)^2}{2(32.2)}(0.0266)(150) - \frac{(3.26)^2}{2(32.2)}[0.0287(600) + 6.3 + 1] \approx \mathbf{72.8\ ft}$$

The resulting turbine power $= \rho g Q h_t = (62.4)(0.16)(72.8) \div 550 \approx$ **1.32 hp**. *Ans.*

ME 231 Thermo-Fluid Mechanics I
Prof. A. Banerjee
Solutions to Home-Work 7

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Solution 7:

6.149 In a laboratory experiment, air at 20°C flows from a large tank through a 2-cm-diameter smooth pipe into a sea-level atmosphere, as in Fig. P6.149. The flow is metered by a long-radius nozzle of 1-cm diameter, using a manometer with Meriam red oil (SG = 0.827). The pipe is 8 m long. The measurements of tank pressure and manometer height are as follows:

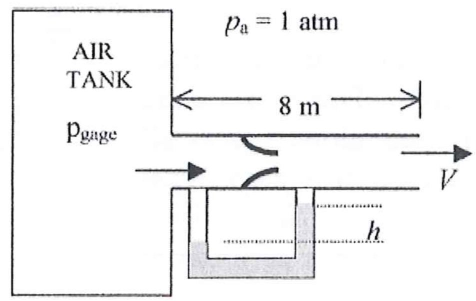


Fig. P6.149

p_{tank}, Pa (gage):	60	320	1200	2050	2470	3500	4900
h_{mano}, mm:	6	38	160	295	380	575	820

Use this data to calculate the flow rates Q and Reynolds numbers ReD and make a plot of measured flow rate versus tank pressure. Is the flow laminar or turbulent? Compare the data with theoretical results obtained from the Moody chart, including minor losses. Discuss.

Solution: For air take ρ=1.2 kg/m^3 and μ = 0.000015 kg/m·s. With no elevation change and negligible tank velocity, the energy equation would yield

$$p_{tank} - p_{atm} = \frac{\rho V^2}{2}\left(1 + f\frac{L}{D} + K_{entrance} + K_{nozzle}\right), \quad K_{ent} \approx 0.5 \text{ and } K_{noz} \approx 0.7$$

Since Δp is given, we can use this expression plus the Moody chart to predict V and $Q = AV$ and compare with the flow-nozzle measurements. The flow nozzle formula is:

$$V_{throat} = C_d\sqrt{\frac{2\Delta p_{mano}}{\rho(1-\beta^4)}} \quad \text{where } \Delta p = (\rho_{oil} - \rho_{air})gh,\ C_d \text{ from Fig. 6.42 and } \beta = 0.5$$

The friction factor is given by the smooth-pipe Moody formula, Eq. (6.48) for ε = 0. The results may be tabulated as follows, and the plot on the next page shows excellent (too good?) agreement with theory.

ptank, Pa:	60	320	1200	2050	2470	3500	4900
V, m/s (nozzle data):	2.32	5.82	11.9	16.1	18.2	22.3	26.4
Q, m^3/h (nozzle data):	2.39	6.22	12.9	17.6	19.9	24.5	29.1
Q, m^3/h (theory):	2.31	6.25	13.3	18.0	20.0	24.2	28.9
fMoody:	0.0444	0.0331	0.0271	0.0252	0.0245	0.0234	0.0225

ME 231 Thermo-Fluid Mechanics I (Section 1A)
Prof. A. Banerjee
Recitation for Mid-term 3

Problem 1: (30 points) The pressure rise, Δp, of a liquid flowing steadily through a centrifugal pump depends on pump diameter, D, angular speed of the motor, ω, volumetric flow rate, Q, and density, ρ. The table gives data for the prototype and for a geometrically similar pump. For conditions corresponding to dynamic similarity between model and prototype pumps, calculate the missing values in the table. (Estimated time: 20 minutes)

choose these, all parameters are given

Variable	Prototype	Model
Δp		29.3 kPa
Q	1.25 m^3/min	
ρ	800 kg/m^3	999 kg/m^3
ω	183 rad/s	367 rad/s
D	150 mm	50 mm

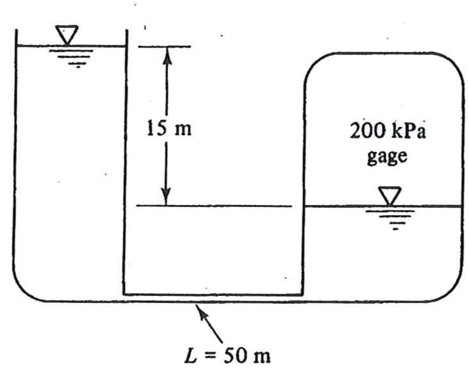
Problem 2. (30 points) Consider the flow field of water given by $\vec{V} = Ax^2y^2\hat{i} - Bxy^3\hat{j}$, where A = 3 /($m^3$.sec) and B = 2 /($m^3$.sec). Determine: (Estimated time: 20 minutes)
(a) The corresponding stream function for this flow
(b) The velocity potential for this flow
(c) Neglecting gravity, calculate the pressure difference between points (0,0,0) and (1,1,1).

Problem 3: (20 marks) A large artery in a person's body can be approximated by a tube of diameter 9 mm and length 0.35 m. Also, assume that blood has a viscosity of approximately 4×10^{-3} N-s/m^2, a specific gravity of 1.0, and that the pressure at the beginning of the artery is equivalent to 120 mm of Hg. If the flow were steady with V = 0.2 m/s, determine the pressure at the end of the artery if it is oriented vertically upwards. Sp. Wt of Hg = 133 kN/m^3. (Estimated time: 10 minutes)

Problem 4: (20 marks) Consider the flow represented by the stream function $\psi = Ax^2y$, where A is a dimensional constant equal to 2.5 $ft^{-1}s^{-1}$. The density is 2.45 slug/ft^3. Find the following: (a) The Velocity field; (b) Is the flow rotational? (c) Can the pressure difference between the points (x,y) = (1,4) and (2,1) be evaluated? If so, calculate it, and if not, explain why. (Estimate Time: 15 minutes)

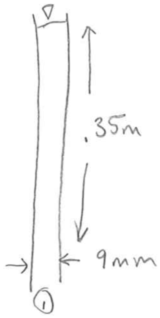
Problem 5: Problem#2. (15 marks). In the figure, the connecting pipe is smooth and 6 cm in diameter. Compute the flow rate in cubic meters per hour if the fluid is SAE 30 oil at 20°C. Which way is the flow? (For SAE 30 oil, ρ= 917 kg/m^3, and μ= 0.29 kg/m.s)

(Estimate Time: 15 minutes)



Recitation III

P. 3)



$\mu = 4(10^{-3})\ \frac{N \cdot s}{m^2}$ $\quad$ $H_g = 133\,kN/m^3$

$\rho = 998$ $\quad$ $SG = 1$

$P_1 = 120\ mm\ Hg$ $\quad$ $P_1 = 15.96\ kN/m^2 = 15.96\,kPa$

$V = .2\,m/s$

$P_2 = ?$

$$\frac{P_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + z_1 - h_L = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + z_2$$

$$\frac{P_2 - P_1}{\rho g} = z_1 - z_2 - h_L \qquad P_2 = \rho g(-L \quad - h_L) + P_1$$

$$h_L = f\,\frac{L}{D}\,\frac{V^2}{2g} \qquad Re = \frac{\rho V d}{\mu} = \frac{998(.2)(.009)}{4(10^{-3})}\ \ 449 \Rightarrow \text{laminar} \Rightarrow f = \frac{64}{Re} = .1425$$

$$= .1425$$

$$P_2 = 15.96 - 9790\left(.35 + .1425\ \frac{.35}{.009}\left(\frac{(.2)^2}{2(9.81)}\right)\right)$$

$$P_2 = 15.96 - 3537\ Pa$$

$$\boxed{P_2 = 12.42\,kPa}$$

#4) $\psi = Ax^2y \quad A = 2.5\ ft^{-1}s^{-1}$

Assumption ① steady flow
② 2D - flow
property - must satisfy continuity

$u = \frac{\partial \psi}{\partial y} = Ax^2$

$v = -\frac{\partial \psi}{\partial x} = -2Ayx$

Check for continuity $\quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2Ax + (-2Ax) = 0$

Hence $\boxed{\vec{V} = Ax^2 \hat{i} - 2Ayx \hat{j}}$

b) is flow rotational? $\omega_z = \nabla \times V = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$

Only in k direction, so

$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \left[\frac{\partial}{\partial x}(-2Axy) - 0\right]\hat{k} = \boxed{\omega_z = -2Ay\,\hat{k}, \text{ rotational}}$

assumptions	stream function:	Velocity potential ϕ
		a) steady flow (b) can be 3D Property: Has to be irrotational $\nabla \times \vec{V} = 0,\ \vec{\omega} = 0$

∴, this problem you cannot define a velocity potential

bernoulli's

So for points (1,4), & (2,1) ψ is constant,
so pts share a single streamline, so bernoulli's is valid, ΔP can be found

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2} \qquad \frac{P_1 - P_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g}$$

$V_1 = 2.5\hat{i} - 20\hat{j} \quad |V_1|^2 = 406\ ft^2/s^2$

$V_2 = 10\hat{i} - 10\hat{j} \quad |V_2|^2 = 200\ ft^2/s^2$

$\Delta P = -252.7 \ \frac{slugs}{ft^2}$

Recitation III

P #2 check satisfies continuity, then you know if

$\vec{V} = Ax^2y^2\hat{i} - Bxy^3\hat{j}$ $A = 3/m^3 sec$ $B = 2/m^3 sec$

a) need to find SF! $u =$ $v =$ $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2Axy^2 - 3Bxy^2$

$= 6xy^2 - 6xy^2 = 0 \Rightarrow$ continuity ✓!

definition of sf: $u = \frac{\partial \psi}{\partial y}$; $v = -\frac{\partial \psi}{\partial x}$ or or $\frac{\partial \psi}{\partial y} = 3x^2y^2 \Rightarrow \psi = \frac{3x^2y^3}{3} + f(x)$

review this method

$-Bxy^3 = -\frac{\partial}{\partial x}[x^2y^3 + f(x)]$ $-2xy^3 = -2xy^3 + f'(x)$ so $f(x) =$ Constant

$\psi = x^2y^3 + C$

$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(-\overset{2}{B}y^3 - 2\overset{3}{A}x^2y) \neq 0$ rotational!

ϕ is not defined for the given $\vec{V}$

find ΔP. $(0,0,0) \Rightarrow \psi = const$ $(1,1,1) \Rightarrow \psi = 1 + const$

$\boxed{\text{- Cannot use bernoulli's, must use N. Stokes}}$

✗ Know where we can use S.F. and where we can use Veloc Potential

Start with problem on Pi theorem

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① $m = 5$ $n = 3$ $\Pi = m - n = 2$

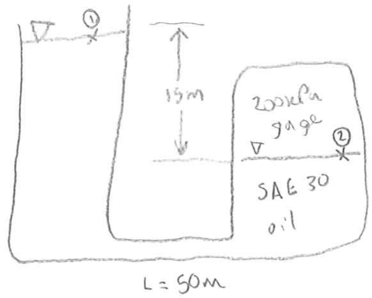
ΔP: $ML^{-1}T^{-2}$ D: L Q: L^3T^{-1} ρ: ML^{-3} ω: T^{-1}

selection $\underbrace{D, \rho, \omega}_{\text{values all given}}$ Π groups $\Pi_1 =$

$\Delta P_{proto} = 82.5\ kPa$

$Q_{model} = .0928\ m^3/min$

(5)



$$\cancel{\frac{p_1}{\gamma}}^{0} + \cancel{\frac{V_1^2}{2g}}^{0} + z_1 = \frac{p_2}{\gamma} + \cancel{\frac{V_2^2}{2g}}^{0} + z_2 + h_f$$

$$h_f = z_1 - z_2 - \frac{p_2}{\gamma} = 15 - \frac{200 \times 10^3}{917 \times 9.81}$$

$$= 15 - 22.23 = -7.23\,m$$ $\therefore$ flow is from 2 to 1

$$h_f = \frac{128\, \mu L Q}{\pi \rho g d^4} \Rightarrow Q = 1.427\,(10^{-3})\ m^3/s$$

(3)