Mead®

IDE 110
Mechanics of Materials

FIVE STAR®
★★★★★

Dates	Day	Classes	Chapters	Problems
Jan. 11	M	Introduction		
13	W	Discussion 1	1.1-4 Stress 2.1-4 Strain	
15	F	Problems 1a		P1.2, 3, 8, 10, 11, 12, 13, 16
18	M	Martin Luther King, Jr Day		
20	W	Problems 1b		P1.17, 20, 21, 23, 24; P2.1, 3, 4
22	F	Problems 1c		P2.9, 11, 12, 14, 15, 16, 17
25	M	Quiz 1		
27	W	Discussion 2	3.1-4 Mechanical Properties of Materials 4.1-4 Design Concepts 5.1-6 Axial Deformation	
29	F	Problems 2a		P3.2, 4, 8, 10, 12, 17, 18; P4.2, 5, 6, 8, 9, 11, 12, 16
Feb. 1	M	Problems 2b		P5.1, 4, 8, 9, 15, 17, 21, 22
3	W	Problems 2c		P5.27, 29, 30, 36, 38, 40, 43
5	F	Problems 2d		P5.44, 48, 49, 55, 56, 58, 60
8	M	Quiz 2		
10	W	Discussion 3	6.1-9 Torsion 7.1-3 Equilibrium of Beams	
12	F	Problems 3a		~~P6.5, 10, 16, 18, 22, 27, 28, 33~~ X
15	M	Problems 3b	did not complete	~~P6.34, 35, 38, 42, 48, 50, 58, 59~~ X
17	W	Problems 3c		~~P6.63, 68, 69, 74, 78, 81, 90~~ X
19	F	Problems 3d		~~P7.24, 27, 30, 32, 34, 36, 46~~ X
22	M	Quiz 3 (last day to drop w/o WD)		91%
24	W	Discussion 4	8.1-7 Bending 9.1-7 Transverse Shear	
26	F	Problems 4a		P8.6, 8, 10, 18, 20, 22, 29, 32
Mar. 1	M	Problems 4b		P8.34, 38, 45, 47, 51, 52, 57, 60
3	W	Problems 4c		P9.12, 14, 16, 20, 22
5	F	Problems 4d (mid-semester)		P9.26, 28, 29, 34, 37
8	M	Quiz 4		87%
10	W	Discussion 5	9.8 Transverse Shear 10.1-6 Beam Deflections	
12	F	Spring Recess		
15	M	Problems 5a		$\boxed{\text{P9.40}}$, 42, 44, 47; P10.3, 6
17	W	Problems 5b		P10.9, 19, 31, 34, 38
19	F	Problems 5c		P10.42, 44, 48, 52, 56, 58
22	M	Quiz 5 (advising week)		90%
24	W	Discussion 6	11.1-2,4 Statically Indeterminate Beams 12.1-4,6-9 Stress Transformations	
26	F	Problems 6a		P11.19, 21, 24, 36, 37, 40, 42
29	M	Spring Break		
31	W			
Apr. 2	F			
5	M	Problems 6b		P11.44, 46, 49, 50, 52, 56
7	W	Problems 6c		P12.9, 13, 14, 19, 24, 29, 30, 31, 32, 46
9	F	Problems 6d		P12.48, 52, 57, 60, 69
12	M	Quiz 6		107%
14	W	Discussion 7	13.1-8 Strain Transformations 14.1-4 Pressure Vessels	
16	F	Problems 7a (last day to drop)		P13.2, 9, 11, 18, 23, 27, 31, 39
19	M	Problems 7b		P13.42, 45, 49, 52, 57, 70, 76; P14.2, 4
21	W	Problems 7c		P14.5, 6, 8, 10, 12
23	F	Problems 7d		P14. 14, 16, 18
26	M	Quiz 7		
28	W	Discussion 8	15.1-4 Combined Loads	
30	F	Problems 8a		P15.4, 6, 10, 14, 16, 18
May 3	M	Problems 8b		P15.21, 22, 23, 26, 27, 32
5	W	Problems 8c		P15.38, 40, 43, 46, 51, 56
7	F	Quiz 8		
13	R	Final Exam		

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IDE 110
Mechanics of Materials
Sections 1B and 1C
Spring Semester 2010
Missouri S&T

Description: Application of the principles of mechanics to engineering problems of strength and stiffness. Topics include stress, strain, thin cylinders, torsion, beams, columns, and combined stresses at a point. The course is intended to prepare students for the rigorous details of problem solving in the academic and professional engineering environments. Format: 3 hour, on-campus, blended face-to-face and online instruction. Prerequisites: IDE 50 with a grade of "C" or better and Math 22

Pre-requisite Requirement Verification: All students are required to meet the prerequisite requirements for this course. If you have not met the requirements, you must either drop this course immediately or send the instructor a written explanation for why you should be allowed to remain in this course. If you have not met the pre-requisite requirements for this course and have not received a waiver, the instructor will drop you from the course.

Objectives: Traditionally, the purpose of this class has been to teach some of the analytical techniques used to insure that a particular structural design is safe and durable. The most effective way to teach these techniques is to have the students work lots of short drill problems like the ones in the textbook. The drill problems represent only a small part of the structural analysis (one joint or member in the structure), and the structural analysis represents only one part of the overall design process. The analytical techniques should be learned or understood in the context of the overall design process. Specific course objectives are listed below.

1. To understand the axial, shear and bearing stresses associated with simple truss design and analysis.
2. To understand the stress-strain and load-displacement relationships for axial force members.
3. To learn to calculate the stresses, strains and angular displacements for torsion members (shafts), and to understand how power is transmitted through a gearbox.
4. To learn to calculate the stresses, strains and displacements for beams under various loading configurations.
5. To learn to calculate the stresses, strains and displacements for pressure vessels.
6. To understand the concepts of stress and strain as second order tensors.
7. To learn how to calculate the principal stresses, and how they are related to the failure of various materials.
8. To use the mechanics of materials technique to analyze a few structures.

Schedule: Section 1A: 9:00-9:50 MWF in TMH 199; Section 1B: 10:00-10:50 MWF in TMH 199; Section 1C: 11:00-11:50 MWF in TMH 199

Textbook: Mechanics of Materials, An Integrated Learning System, by T.A. Philpot, Wiley, 2008. You will need either a hardcopy of the text ($136 @ Amazon , $183 @ Wiley, $189 @ bookstore) or the online version ($74 @ Wiley, $89 @ bookstore) but not both. The draft, soft-back version from 2007 is acceptable. To access the online version, purchase the registration code (alt), register for a WileyPLUS account, access account and enter code.

7. remote, anytime problem-working sessions with your peers using Live Classrooms in Blackboard

Attendance: Attendance is not required, but students missing two quizzes may be dropped by the instructor.

Homework: Textbook problems will be assigned but not collected. Proper learning of the course material can only be achieved by spending a few hours (typically 2-5) per class period on these problems. (problem suggestions)

Quizzes and Exams: A multiple-choice quiz will be given approximately every other week, and there will be a two-hour cumulative exam at the end of the semester. Quiz and final-exam problems will be similar to the textbook and MecMovie problems. Your lowest quiz score will be dropped. If your entire section does sufficiently well on the final exam, the instructor may curve grades by dropping your second lowest quiz score. A formula sheet will be provided with the quizzes and final exam. Multiple versions of each quiz will be given, and graded quizzes will not be returned. Communication devices must be turned off and put away, and hats must be turned backwards. Make-up quizzes will only be given for off-campus university-sponsored activities. A memo from your sports-team coach or competition-team advisor is required. Sickness, job interviews, etc. do not count, so plan accordingly. It is the student's responsibility to schedule the make-up quiz with the Testing Center at least 7 business days prior to taking the quiz.

Academic Dishonesty: Teamwork is encouraged for studying course topics, but giving aid to another student or taking information from another student during a quiz or the final exam constitutes academic dishonesty. Academic dishonesty will not be tolerated and will be handled according to the Conduct of Students section of the Student Academic Regulations. The first occurrence will result in zero credit on that quiz or final exam and a notification sent to the Vice-Provost for Undergraduate Studies, your academic advisor, and your department chair. A second occurrence may result in more severe penalties. A quiz assigned zero credit due to academic dishonesty may not be dropped.

Grades: Your grade for this course will be determined on the following basis, and the standard university grading policy will be followed. However, the instructor reserves the right to curve grades as he deems necessary. Scores will be available in Blackboard. A: 90-100, B: 80-89, C: 70-79, D: 60-69, F: 0-59 (Evaluating Student Work in Engineering). Seven in-class quizzes: 11% each. Final exam: 23%.

Safety: Egress maps for all areas of instruction on campus are available on the web at registrar.mst.edu/links/egress.html. You are encouraged to review this site and be aware of the emergency exit signs near your classrooms.

Disability Support: If you have a documented disability and anticipate needing accommodations in this course, you are strongly encouraged to meet with the instructor early in the semester. You will need to request that the Disability Services staff (204 Norwood Hall) send a letter verifying your disability and specifying the accommodation you will need before the instructor can arrange your accommodation.

Communication: If you have a problem that cannot be resolved by the instructor, feel free to contact the department chair, Dr. William Schonberg (BCH 211). The course schedule, content, and assignments are subject to modification when circumstances dictate and as the course progresses and matures. If changes are made, you will be given due notice.

Schedule:

Notes 1-13-10

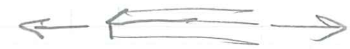
σ_1 bending = 4266 lb/in^2

1 

σ_2 bend = 21.33 lb/in^2

2 

σ_3 tension = 44.4 psi

3 

τ_1 torsion = 130 psi

4 

stress = $\frac{N}{A}$, $\frac{V}{A}$, Tc/J My/I VQ/B force per area

normal stress/strain

$\sigma_{axial} = N/A$

$\varepsilon_{axial} = \Delta L / L_0$

$\varepsilon_{transverse} = \Delta d / d$

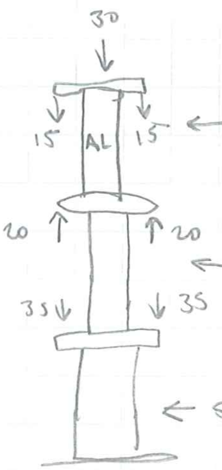
1,2,3, 8, 10, 11, 12, 13, 16

1.2) .06 27? in

for sure

1.3) (c) 176.8 MPa (T)

8)



aluminum 25 ksi

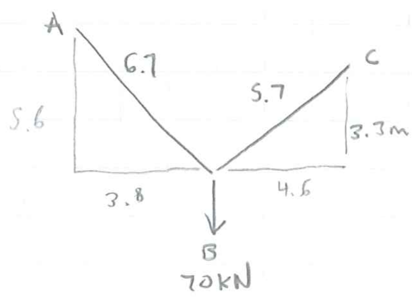
$\frac{60 \text{ kip}}{x} = 25\text{ksi}$ $x = 2.4 \text{ in}^2$

$\pi r^2 = 2.4$ $25 = \frac{60}{\frac{\pi}{4}d^2}$ $d = 1.74$ in

brass 15ksi $= \frac{20 \text{ kN}}{\frac{\pi}{4}d^2}$ $d = 1.30$ in

steel rod 10ksi $= \frac{90}{\frac{\pi}{4}d^2}$ $d = 3.39$ in

10) 165MPa



$\frac{3.8}{6.7} A = \frac{4.6}{5.7} C$

$A = 1.42\, C$

$\frac{5.6}{6.7} A + \frac{3.3}{5.7} C = 70$

$C = 39.55$ kN

$A = 56.16$ kN

$165000 \text{ kPa} = \frac{39.55 \text{ kN}}{\frac{\pi}{4}d^2}$ $d = .0174 \text{ m} = \boxed{17.4\text{mm}}$ C

$165{,}000 \text{ kPa} = \frac{56.16}{\frac{\pi}{4}d^2}$ $d = .0208 \text{ m} = \boxed{20.8\text{mm}}$ A

11, 12, 13, 16

11) $\tau_d = \frac{10}{6} P$ $\quad 0.75 in^2 \ (30 ksi) = 22.5$ kip $\quad \boxed{13.5 \text{ kip}}$

12) $\tau = 120$ psi find L $P = 10{,}000$ $\quad \cancel{2}(L - 0.5)(6)(120) = 10{,}000$ (L W psi) $\quad$ ~~L = 7.44~~

Shear: $\tau = \frac{V}{A}$ force (shear) / area $\quad 120 = \frac{10{,}000}{(6)(L - .5)}$ $\quad \boxed{L = 14.39 \text{ in}}$

13) diam 24mm $\quad P = 175$ kN $\quad \tau = \frac{V}{A} = \frac{175/2}{\frac{\pi}{4}(24 mm)^2}$ $\quad \tau = 0.1934 \times 1000 = 193$ MPa

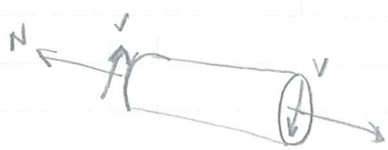
16) $P = 550$ kN $\quad \tau_{average}$ 270 MPa $\quad$ min. bolt diameter

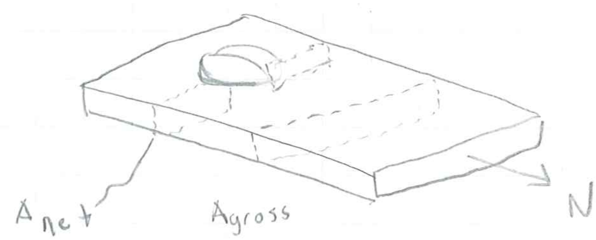
$270 = 550/A$ $\quad A = \frac{2.037}{5} = 0.407 = \frac{\pi}{4}d^2$ $\quad d = 22.76$

$407 mm^2$

$\sigma = \frac{N}{A}$

$\tau = \frac{V}{A}$

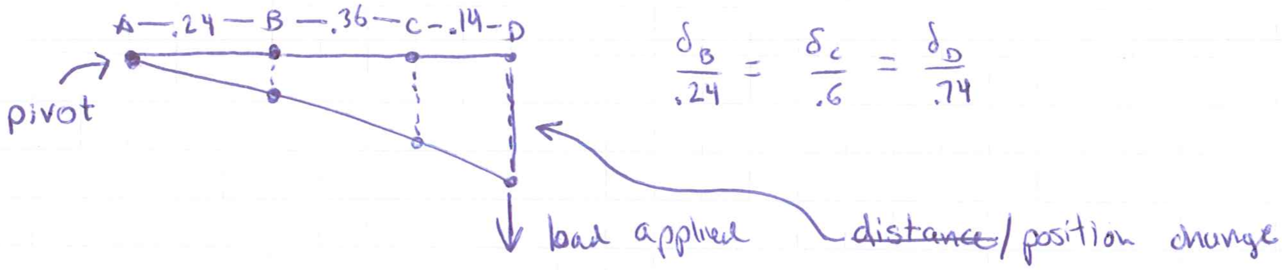




Notes 1-20-2010

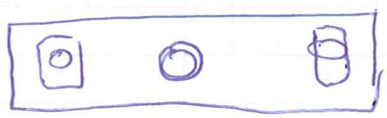
elongation $= \Delta L = \delta$ (delta)

axial strain $= \dfrac{\Delta L}{L_0} = \varepsilon$ (epsilon)



$$\frac{\delta_B}{.24} = \frac{\delta_C}{.6} = \frac{\delta_D}{.74}$$

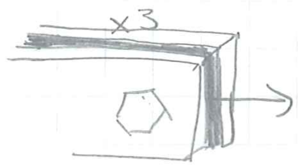
— gaps —



1 2

1| 17, 20, 21, 23, 24 2| 1, 3, 4

17)



16 mm diam 210 Mpa = τ

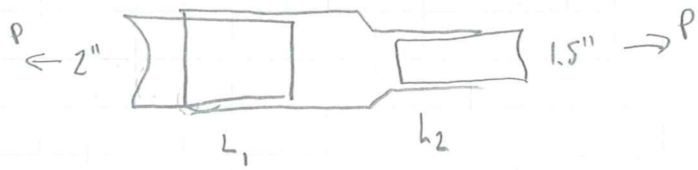
$A = \pi(8\,mm)^2$

$\frac{V}{A} = \frac{V\ kN}{.0002011\ m^2} \leq 210{,}000\ kPA = MN/m^2$

$V = 42.223 \quad \times 6 = \boxed{253.34\ kN}$

20) 400 psi

$SA_1 = L_1 \pi d_1 = L_1(6.283)$



$5000\ lb = 400(6.283\ L_1) = \boxed{1.989''}$

$5000\ lb = 400(4.712\ L_2) = \boxed{2.653''}$

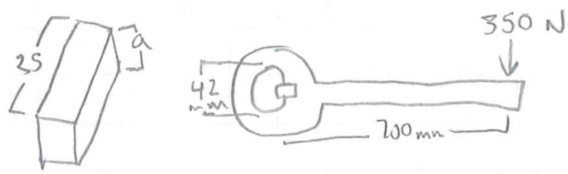
21) Beam $A = 11{,}400\ mm^2$ 110 MPa max

$110\ MPa = \frac{N}{11{,}400\ mm^2}$ $N = 1{,}254\ kN$

$8\ MPa = \frac{N}{A} = \frac{N}{a^2}$ $\boxed{a \geq .396\ m}$

23)

$F_{key}(21\ mm) = 350\ N\,(700\ mm)$



$F_{key} = 11.666\ kN$

$\sigma_{key} = 80\ MPa = \frac{11.667\ kN}{.025\,a\ m}$ $a = .00583\ m$ $\boxed{a = 5.834\ mm}$

24) o.d. = 8.625" .25" wall beam 10.75" F = 80 kips

$a_{Steel} = 58.426 - 51.849$
$= 6.577\ in^2$

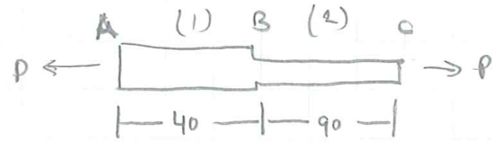
a) $\sigma_b = \frac{80\ kip}{6.577\ in^2} = \boxed{12.164\ Kpsi}$

b) $500\ psi = \frac{80\ kip}{L(10.75'')}$ $\boxed{L = 14.884''}$

c) $900\ psi = \frac{80\ kip}{a^2}$ $\boxed{a = 9.428''}$

2| 1, 3, 4

1) a) $1{,}100 \frac{\mu in}{in} (90 in)$



$= .099 in$

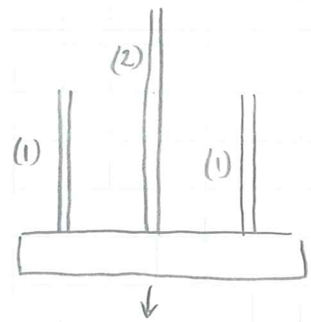
$\delta_1 + \delta_2 = .125 in$ $\quad \delta_1 = .125 - .099 = .026''$

$\varepsilon_1 = \frac{.026''}{40''} = \boxed{650 \mu in/in}$

3) L_1

$L_0 1 = 1250 mm$

$L_0 2 = 2000 mm$



$2800 \mu m/m (1.250 m) = 3.5 mm$

a) $\varepsilon_2 = 3.5 mm / 2000 mm = \boxed{1750 \mu m/m}$

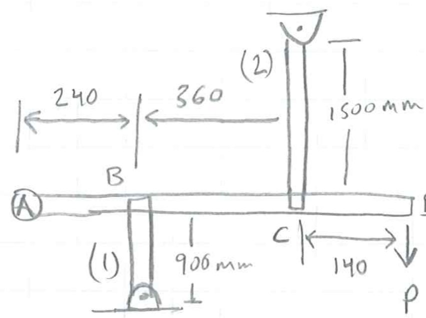
b) $\varepsilon_2 = (3.5 mm + 2 mm) / 2 m$ $\boxed{2750 \mu m/m}$

$\boxed{\mu mm/mm \ ?}$

c) $3.5 mm - 2 mm / 2 m = \boxed{750 \mu m/m}$

4)

$\varepsilon_1 = -1200 \mu m/m$ $\quad \varepsilon/L = \delta = -1080 \mu m$



a) $\delta_2 = -\frac{600}{240} \delta_1 = 2700 \mu m$

$\varepsilon_2 = \frac{\delta_2}{L} = \boxed{1800 \mu m/m}$

b) 1mm gap pin C $\Rightarrow \delta_2 = 2700 \mu m - 1 mm \Rightarrow \varepsilon_2 = \boxed{1133 \mu m/m}$

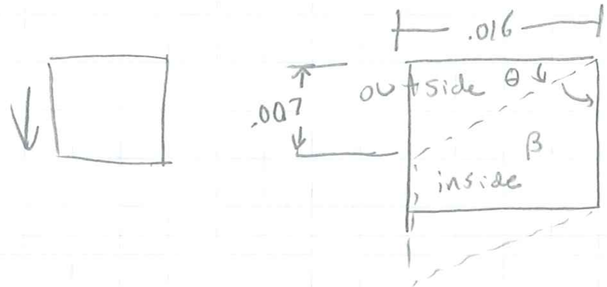
c) 1mm gap pin B $\Rightarrow \delta_2 = 2700 \mu m + \frac{600}{240}(1 mm) = 4700 \mu m$

$\Rightarrow \varepsilon_2 = \frac{4700 \mu m}{1.5 m} = \boxed{3133 \mu m/m}$

Notes 1-21-10

Shear strain

2.9)



$\theta = \tan^{-1} \frac{.007}{.016} = \gamma$

$\beta = 90 - \theta$

$\tau = \frac{V}{A} = \frac{P/2}{.022(.025)}$

γ = change in angle from 90°

2.12) hint

γ = arctan 1 outside angle + arctan 2nd outside angle

$\varepsilon = \frac{\delta}{L}$ $\quad \gamma = \Delta$ angle $\quad \epsilon_t = \alpha \Delta T$ $\quad \delta = \alpha \Delta T L$

2.9, 11, 12, 14, 15, 16, 17

11) find shear strain at Q' $\quad \gamma$ = change in angle = $90 - (\arctan \frac{135}{226} + \arctan \frac{375}{226}$

$= 90 - 89.7758 = .2242° \,(\pi/180) = .003913$ rad

12) $\gamma_{xy} = \arctan \frac{.8}{800} + \arctan \frac{1.7}{1000} = \frac{1738}{\cancel{.07642}}° \times \pi/180 = \frac{\boxed{3033 \mu rad}}{\cancel{.001334 \text{ rad}}}$

14) $.1428° \,(\pi/180) = \boxed{2492 \,\mu \text{ rad}}$

15) $\alpha_A = 22.5 \times 10^{-6}$ $\quad$ span 40m $\quad$ 40°C → -40°C

$\delta = \alpha \Delta T L = \frac{22.5}{°C}(10^{-6})(80°)(40m) = -.072$ [redacted] m $\quad \boxed{-72 mm}$

17 ~~16~~) d = 70 mm D = 105 mm L = 2.5m $\quad \alpha_I = 12.1(10^{-6})/°C$

a) $12.1(10^{-6})/\text{deg.}\,(+70°)(2.5m) = +2.118$ mm L $\quad$ b) (-85°) ⇒ -2.571 mm L

(70) mm = + .05929 mm id $\quad$ - .0720 id

(105) mm = + .08894 mm od $\quad$ - .1080 od

(-85)° (2.5)

(70)

16) L = 225 ft $\quad$ D = 12 ft $\quad \alpha_s = 6.5 \times 10^{-6}/°F$ $\quad$ +250°F $\quad \delta_L = +.3656$ ft (4.388")

$\delta_d = +.0195$ ft (.234")

normal stress σ | shear modulus G
normal strain ε | Elastic modulus E

Hooke's Law $\sigma = E\varepsilon$

$\tau = G\gamma$

Notes 1-27

strain : deformation / original length

$\frac{N}{A} = \sigma = E\varepsilon$

$\tau = G\gamma$

Poisson's ratio $\nu = -\varepsilon_{transverse} / \varepsilon_{axial}$

$F = k \Delta l$ (lb/in)

E . modulus of elasticity

G = shear modulus

$G = \frac{E}{2(1+\nu)}$

factor of safety = actual strength / design strength

$F.S = \frac{F_{max}}{A} \Big/ \frac{F_{actual}}{A} = \frac{F_{max}}{F_{actual}} \Rightarrow F_{max} = F_{design}(F.S)$

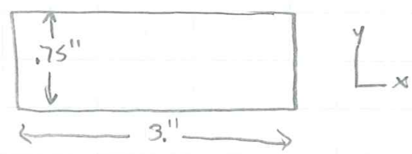
Ch 5 - axial deformation

$\delta = NL_0/EA + \alpha \Delta T L_0$

Homework 3.2 ~~10 12, 4, 8, 17~~ ✓✓ 4, 8, 10, 12, 17

3.4) axial $\varepsilon_x = 2136\,\mu\varepsilon$

transverse $\varepsilon_y = -673\,\mu\varepsilon$



poisson's ratio $\nu = -\frac{\varepsilon_t}{\varepsilon_a} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\cancel{2136\mu\varepsilon/3}}{\cancel{-673\mu\varepsilon/.75}}$

$\nu = .315$

$\sigma =$

P = 50 kips find E mod. elasticity $\sigma = E\varepsilon = \frac{N}{A}$

$\frac{50\,kip}{3(.75)\,in^2} = E\varepsilon$ $E = \frac{22.22\ kpsi}{2136(10^{-6})\ \varepsilon}$

$\boxed{E = 10{,}400{,}000\ psi}$

8) $d_0 = .500$ in $l_0 = 2.00$ in $d_f = .260$ $l_f = 3.08$

% elongation % reduction area

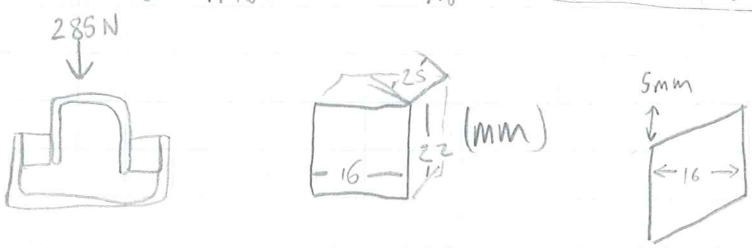
$\frac{3.08 - 2}{2} = \boxed{54\%\ \text{elongation}}$

$A_f = .053093$

$A_0 = .19635$

$\frac{A_f - A_0}{A_0} = \boxed{73\%\ \text{reduction in area}}$

10)



shear modulus $G = \tau/\gamma$, $\tau = \frac{V}{A} = \frac{142.5}{550(10^{-6})}$

γ = change in angle

$= 17.354° = .303$ rad

$\tau = 259091$ Pa

$\boxed{G = 855{,}410\ Pa}$

~~Lab Notes 2-1-10~~

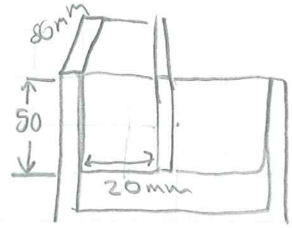
$\boxed{\text{HomeWork 3-12, 17, 18}}$

$A = 50(80) = 4000E^{-6}\ m^2$

$V = 450\ N$

$\tan^{-1}\left(\frac{d\,(mm)}{20\,(mm)}\right)\left(\frac{\pi}{180}\right) = \gamma$

12)



$P = 900N$

$G = 350\ kPa$

$G = \tau/\gamma = \frac{V/A}{\gamma}$

γ = change in angle

$350\,kPa = \frac{112500}{\tan^{-1}(\frac{d}{20})(\frac{\pi}{180})}$

$\tan^{-1}\left(\frac{d}{20}\right) = \frac{112500}{350\,kPa\,(\frac{\pi}{180})}$

$\tan^{-1}\left(\frac{d}{20}\right) = 18.4165$

$\boxed{d = 6.60\ mm}$

17) $d = .495$ in $l_0 = 2.00$ in find:

a) elastic Modulus, $E = \frac{\sigma}{\varepsilon}\ \frac{\text{stress}}{\text{strain}} = \frac{60\,ksi}{.002\ in/in} = \boxed{30{,}000\ ksi}$

b) proportional limit ~~69 ksi~~ $\boxed{60\ ksi}$ end of linearity

c) ultimate strength $\boxed{159\ ksi}$

d) yield strengt, 20% offset $\boxed{80\ ksi}$

e) fracture stress $\boxed{135\ ksi}$

f) final diameter = .350

stress, fracture $= 135\,ksi\left(\frac{(.350)^2\pi}{4} \Big/ \frac{(.495)^2\pi}{4}\right)$

$= 135\,ksi\left[\frac{(.495)^2}{(.395)^2}\right] = \boxed{270\,ksi}$

18) $d_0 = 12.8mm$ $l_0 = 50mm$

a) $= \frac{\sigma}{\varepsilon}\ \frac{200\ MPa}{.002\ mm/mm} = 100{,}000\ MPa$

b) proportional limit = $\boxed{220\ MPa}$

c) ult. strength $\boxed{380\ MPa}$

d) yield st. = $\boxed{280\ MPa}$

e) fracture $\boxed{320\ ksi}$

f) $d_f = 10.5mm$

$\frac{12.8^2}{10.5^2}(320\ Mpa) =$

$\boxed{475.5\ MPa}$

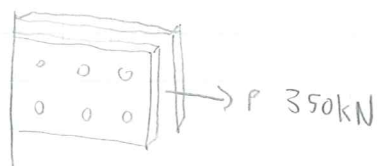
$$\sqrt{4^2 p^2 + 5^2 p^2}$$

$$\sqrt{(4^2 + 5^2)(\cdot p^2)}$$

$$\sqrt{4^2 + 5^2} \; p$$

4) 2, 5, 6, 8, 9, 11

2) shear strength 300 MPa 4.0 s.f. min diameter D

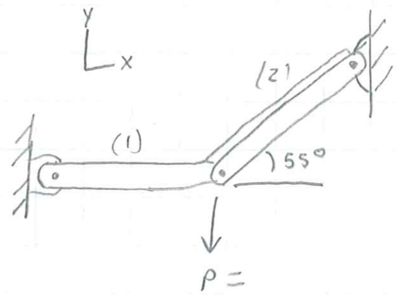


$\frac{V}{A} = 300\,MPa = \frac{V}{d^2 \frac{\pi}{4}}$ $V = \frac{P}{12}(4) = 116.7\ kN$

$d^2 \frac{\pi}{4} = \frac{V}{300}$ $d^2 = \frac{4(116.7)\ kN}{\pi\ 300\ MPa}$ $d = \sqrt{.49529}$

$d = .0223\ m$
$= 22.3\ mm$

5) ① $1.75\,in^2$ 50ksi yield str. steel
② 6061 T6 Al combined X-section $4.5\,in^2$ yield str. 40ksi
s.f = 1.5 for both



(2) $V_y = P$ $V \sin 55 = P$ $V = 1.2208\,P$
$V_x =$
(1) $V = .7002\,P$
$\frac{.7002P(1.5)}{1.75\,in^2} = 50\,ksi \Rightarrow P = \boxed{83.31\ kip}$

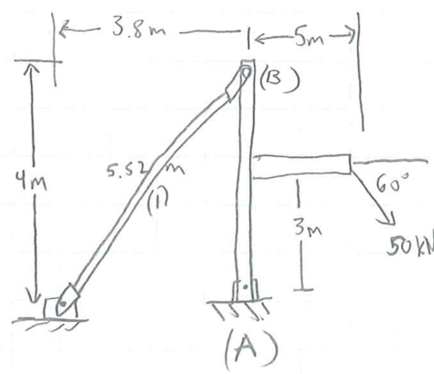
(2) $\frac{1.2208P(1.5)}{4.5\,in^2} = 40\,ksi \Rightarrow P = 98.30\ kip$

factor of safety for (2) is $\frac{98.3}{83.31}(1.5) = \boxed{1.770}$

6) a) $N_{(1)} = ?$

	diameter	Ultimate, or Shear strength
(1)	35mm	250 MPa
Pin B, C	24mm double	330 MPa
Pin A	30mm single	330 MPa

$\circlearrowleft \Sigma M_A$ $-50\sin 60(5) - 50\cos 60(3)$
$+ \frac{3.8}{5.52}(4)N_1 + \frac{4}{5.52}(0)N = 0$
$291.51 = 2.75\,N_1$
$\boxed{N_1 = 105.86\ kN}$

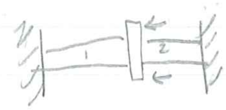


b) shear @ A $A = 707(10^{-6})\,m^2$ $F_y = \frac{4}{5.52}N_1 + 50kN\sin 60 = 120.0\ kN$
$F_y = \frac{3.8}{5.52}N_1 - 50\cos 60 = F_x = 47.88\ kN$ $F_A = 129.2$
$\tau_A = \frac{F}{A} = \frac{129.2\ kN}{707(10^{-6})}$ 182,744 kPa $\boxed{\tau_A = 182.8\ MPa}$
shear @ B $= \frac{105.86}{905(10^{-6})}$ $A = 2(.024^2 \frac{\pi}{4})$ $\tau_B = \boxed{117.0\ MPa}$

c) F.S. for (1) $\frac{F}{A} = \frac{105.8}{.035^2 \frac{\pi}{4}}$ 110 MPa $\Rightarrow \frac{110}{250} = \boxed{F.S = 2.27}$

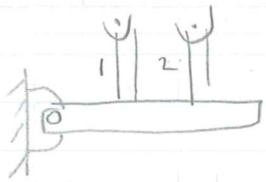
d) Pin A F.S. $\frac{330}{183} = \boxed{1.81}$ F.S. (A) ($\cancel{182.8}$)
b) $\frac{330}{117} = F.S._B = \boxed{2.82}$

compatibility

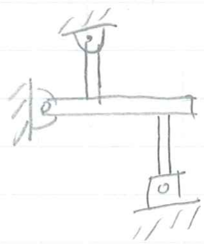


$$\delta_1 + \delta_2 = 0$$

or $\delta_1 + \delta_2 = \text{gap}$



$$\frac{\delta_1}{a} = \frac{\delta_2}{b}$$

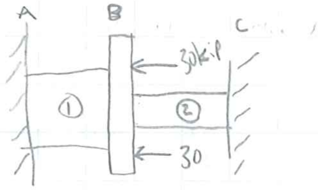


$$\frac{\delta_1}{a} = \frac{-\delta_2}{b}$$

Notes 2-3-10

deflections

$\boxed{\delta = NL/EA}$ formula



1. F.B.D
2. sum forces $\Sigma F_x = 0$

3) compatibility: $\delta_1 + \delta_2 = 0$
4) Force-deform. relationships $\delta_1 = \frac{N_1 L_1}{E_1 A_1}$ $\delta_2 = \frac{N_2 L_2}{E_2 A_2}$

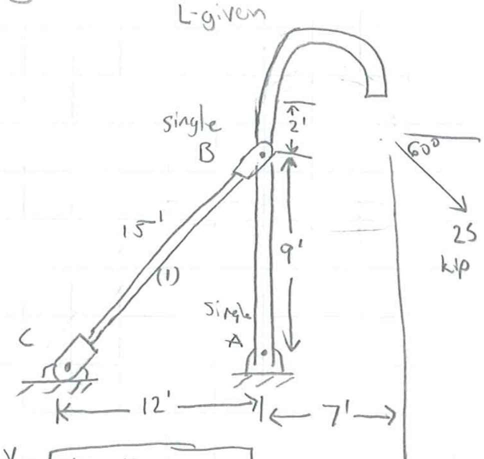
5) combine & solve for N_1 & N_2 $\frac{-N_1 L_1}{E_1 A_1} + \frac{N_2 L_2}{E_2 A_2} = 0$ * compression is (−)

$\frac{-N_1 L_1}{E_1 A_1} + \frac{(60{,}000 - N_1) L_2}{E_2 A_2} = 0$ * E-given, calculate A, L-given

$\delta_1 = -\delta_2 + \text{gap}$

Homework 4) 8, 9, 11

	d	A	strength
A	1.25"	1.23"²	80 ksi
B, C	.75"	.442"²	80 ksi
(1)		1.5"²	36 ksi



8) a) $N_{(1)}$

$\circlearrowleft \Sigma M_A$: $9'(\frac{4}{5})N + 0(\frac{3}{5})N = 25\sin 60(7) + 25(\cos 60)(11)$

$\boxed{N = 40.15 \text{ kip}}$ $\sigma_1 = \frac{40.15}{1.5 in^2} = \boxed{26.8 \text{ ksi}}$ normal stress

b) shear stress A − $A_y = \frac{3}{5}(40.15) + 25\sin 60 = 45.74$ $F_A = 49.8$ $\frac{V}{A} = \boxed{\tau_A = 40.49 \text{ ksi}}$

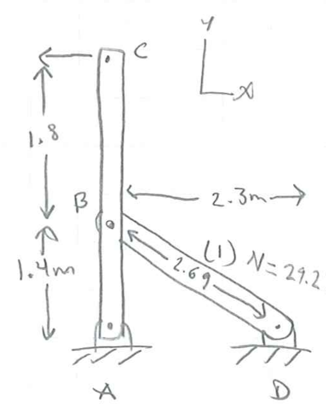
$-A_x = \frac{4}{5}(40.15) - 25\cos 60 = 19.62$

c) F.S. (1) = $\frac{36}{26.8} = \boxed{1.34}$ $\frac{V}{2A} = \boxed{\tau_B = 45.4 \text{ ksi}}$

d) F.S. $\boxed{\begin{matrix} A) = 1.98 \\ B) = 1.76 \end{matrix}}$

9) $\sigma_{max} = \frac{V_{max}}{A \cdot FS}$

F.S. min		d	A (mm²)	Y.S.
1.67	(1)		250 mm²	255 MPa
2.5	B	12 mm	(2) 113	300 MPa



ΣM_A: $P(3.2) = 1.4(\frac{2.3}{2.69})N$

$P = .374N$ $\sigma_1 = \frac{N_{max}}{250 mm^2} = \frac{25.5 MPa}{1.67}$, $N_{max} = 38.17$ kPa

☆ $\boxed{P_{max} = 14.28 \text{ kN}}$

a) $\tau_B = \frac{V_{max}}{226(10^{-6})} = \frac{300 MPa}{2.5}$ $V_{max} = 27.12$ kPa

$\boxed{P_{max} = 10.14 \text{ kN}}$

b) F.S. = 2.5 min diam $\circlearrowleft \Sigma M_B$: $10.14(18) = A_x(1.4)$ $A_x = 13.04$ ΣF_y: $\frac{1.4}{2.69}(29.22) = A_y = 15.21$

$V_A = 20.03$ kN $\frac{V_A}{A} = \frac{\tau_{max}}{F.S.}$ $\frac{20.03 kN}{\frac{\pi}{4}d^2} = \frac{300 MPa}{2.50}$ $\frac{1}{A} = 5991$ $A = .000167 m^2$

$\frac{\pi}{4}d^2 = .000083$

$\boxed{d_{pin} = 10.31 mm}$ $d = .01031$ m

Homework 4/11 F.S. 2.0 $P_{max} = ?$ — Skipped —

5) 1, 4, 8, 9, 15, 16, 17

1) $E = 200\,GPa$ $l = 6m$ Minimum diameter $\sigma_{allow} = 180\,MPa$

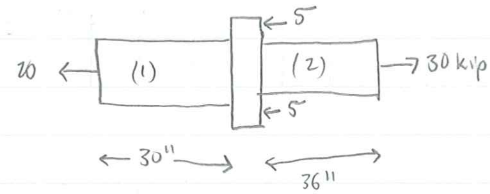
$F = 30kN$ stretch $= \delta = 5mm$

$\sigma = \frac{N}{A} = 180\,MPa = \frac{30\,kN}{\frac{\pi}{4} d^2}$ $d^2 = .212\,m^{-3}$ $\boxed{d = 14.57\,mm}$ X

$\delta = \frac{NL}{EA} \le \delta_{max}$ $\frac{30\,kN\,(6m)}{200\,GPa\,(d^2 \frac{\pi}{4})} \le 5mm$ $\boxed{d = 15.14\,mm}$ ✓

4) (1) $2.2\,in^2$ $E = 2400\,ksi$

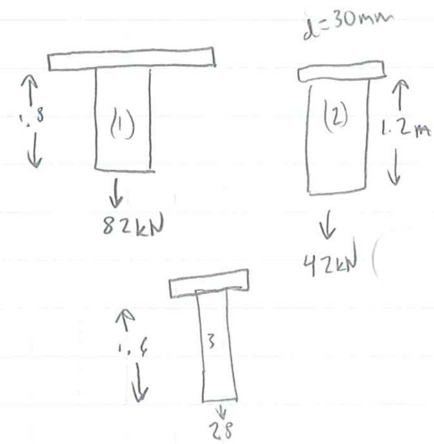
(2) $1.3\,in^2$ $E = 4000\,ksi$



$\delta = \frac{NL}{EA} = \frac{20\,kip\ 30''}{2400\,(2.2)} + \frac{30\,kip\,(36'')}{4000\,(1.3)} =$

$.0947 + .2077 = .3024''$

8) $E = 100\,GPa = 100(10^6)\,kPa$



a) $\frac{42kN\,(1.2m)}{100(10^6)(707)10^{-6}}$ $A_2 = 707\,(10^{-6})\,m^2$ $\boxed{\delta = .713\,mm}$

b) $\delta_{(1)} = \frac{82kN\,(1.8m)}{100(10^6)(707)10^{-6}} =$ $\boxed{\delta = 2.088\,mm}$

$\delta_{(3)} = \frac{28\,(1.6)}{100\,(10^6)(201)(10^{-6})}$ $\boxed{\delta_{(3)}\ 2.229\,mm}$

$\boxed{\delta = 5.03}$

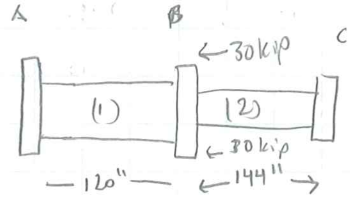
c) σ_1 $82/(707)(10^{-6})$

σ_2

σ_3 $28/(201)(10^{-6})$ $\boxed{\sigma_3 = 139\,kN/m}$

9)

5- 27, 29, 30, 36, 38, 40, 43,



27) $E = 30{,}000 \text{ ksi} \quad A_1 = 5.60 \text{ in}^2$

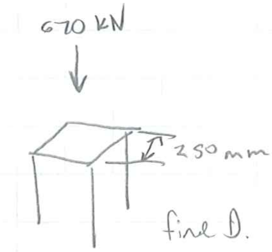
$E = 10{,}000 \text{ ksi} \quad A_2 = 4.40 \text{ in}^2$

$$\frac{N_1 L_1}{E_1 A_1} = -\frac{N_2 L_2}{E_2 A_2} \qquad \frac{N\,(120'')}{30{,}000k\,(5.6)} = \frac{(60-N)(144)}{10{,}000\,(4.40)}$$

$$.000714\,N \qquad + .00327N = .1964$$

a) $.003984N = .1964 \quad \boxed{N = 49.30 \text{ kN}}$

b) $\delta \; \frac{49.3 \text{ kN}\,(120'')}{30{,}000\,(5.6)} = \boxed{.0352 \text{ in}}$



find D.

20% vertically steel

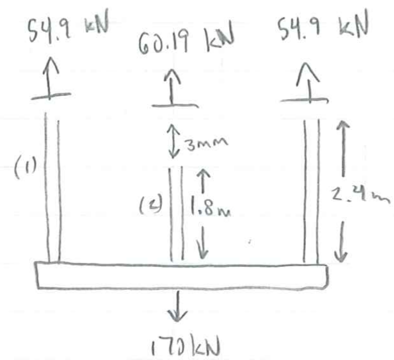
29) $E = 29 \text{ GPa} \qquad E = 200 \text{ GPa}$

$$\frac{(.8)\,670 \text{ kN}\,(1.5\text{m})}{29(10^6)\text{kPa}\,(.0625 - \pi d^2)} = \frac{.2\,(670)\,(1.5)}{200\,(10^6)\,(\pi d^2)}$$

$$\frac{.0276}{.0625 - \pi d^2} = \frac{1.001\,(.0625) - (\pi d^2)}{\pi d^2}$$

$$.0276 = \frac{6.25(10^{-5})}{\pi d^2} - 1$$

? $\qquad 1.0276 = \frac{6.25\,(10^{-5})}{\pi d^2}$



30) $\frac{NL}{EA} = \delta$

			A	
(1)	100 GPa	20mm	$.314(10^{-3})$	
(2)	200 GPa	24mm	$.452(10^{-3})$	

$$\frac{N_1 L_1}{E_1(2A)} = 3\text{mm} + \frac{N_2 L_2}{E_1\, 2A_1}$$

$$\frac{N_1\,(2.4\text{m})}{100(10^6)\,.628(10^{-3})} = .003 + \frac{(170 - N_1)(1.8)}{200(10^6)\,.452(10^{-3})}$$

$$N_1\,(38.2E^{-6}) = .003 + 3.38\,(10^{-3}) - N_1\,(.0199)10^{-3}$$

$$N\,(5.81\,E^{-5}) = 6.38\,(10^{-3})$$

$N = 109.81 \text{ kN}$

$$\sigma_1 = \frac{54.9}{.314(10^{-3})} = \boxed{175 \text{ MPa}}$$

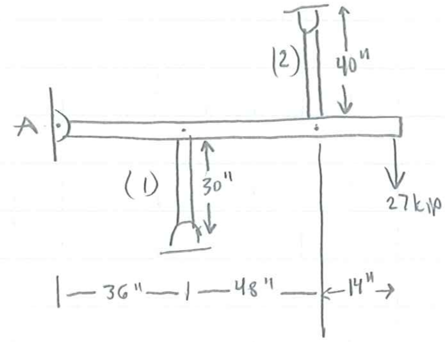
$$\sigma_2 = \boxed{133 \text{ MPa}}$$

$$175\text{MPa}\;\frac{2.4\text{m}}{100\text{ GPa}} = \boxed{4.2\text{mm}}$$

S-43)   (1) 15,000 ksi $1.25 in^2$

(2) 10,000 ksi $2.0 in^2$

a) σ_1 & σ_2 ?



$\delta_1 = -\frac{3}{7}\delta_2$

$\frac{N_1\,30''}{15{,}000\text{ ksi}(1.25)} = -\frac{N_2(40'')}{10{,}000(2.0)}\frac{3}{7}$

$.0016 N_1 = .000857 N_2$

$\boxed{N_1 = .536 N_2}$

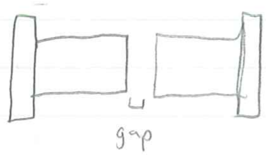
$\circlearrowleft \Sigma M_A : \quad 36 N_1 + 84 N_2 - 98(27\text{kip})$

$2646 = 36N_1 + 84N_2$

$N_1 = 73.5 - 2.33N_2$

$.536 N_2 + 2.33N_2 = 73.5$

$\boxed{N_1 = 13.75 \text{ kip}}$ $\boxed{N_2 = 25.68 \text{ kip}}$



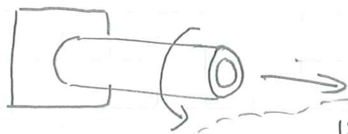
$\frac{-N_1 L_1}{E_1 A_1} + \alpha_1 \Delta T L_1 + \frac{-N_2 L_2}{E_2 A_2} + \alpha_2 \Delta T L_2$

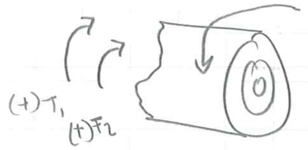
$\nu = \frac{\varepsilon_{transverse}}{\varepsilon_{axial}}$

$T = T_1 + T_2$

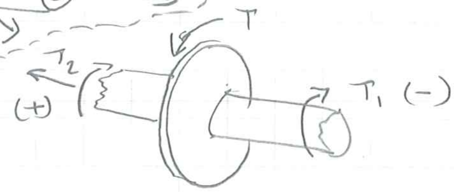
$\phi_1 = \phi_2$

Internal torque is positive when vector points away from cut section





$T_1 + T_2 - T = 0$

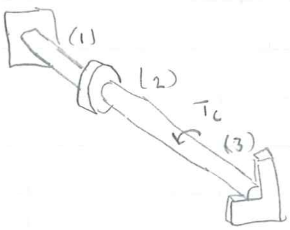


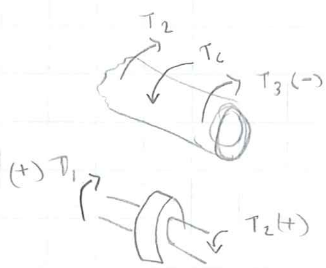
$T_1 + T_2 = T$

$\phi_1 + \phi_2 = 0$

$$-\frac{T_1 L_1}{G_1 J_1} + \frac{T_2 L_2}{G_2 J_2} = 0$$

6.81)





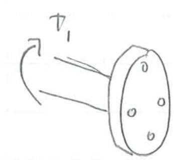
$T_2 + T_3 = T_c$

$T_1 = T_2$

$T_c =$ ____

$\phi_1 + \phi_2 + \phi_3 = 0$ $\quad \frac{T_1 L_1}{G_1 J_1} + \frac{T_2 L_2}{G_2 J_2} + \frac{-T_3 L_3}{G_3 J_3} = 0$

$\tau = \frac{V}{A} \Rightarrow V =$ ____



$\Sigma M_A = T_1 - 4V(.6) = 0$

6.90) $\quad r_2 T_1 = r_1 T_2 \qquad r_1 \omega_1 = r_2 \omega_2$

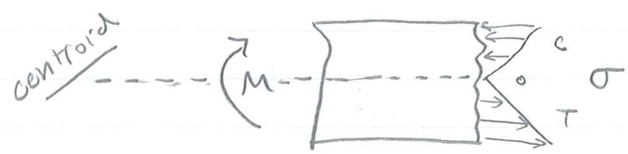
these eq's don't work here. this problem type won't be on Quiz

2-26

$$\sigma = \frac{-My}{I} = \frac{M}{S}$$

$$\bar{y} = \frac{\Sigma \bar{y}_i A_i}{\Sigma A_i}$$

$$I = \Sigma(\underset{\uparrow \atop \frac{bh^3}{12}}{I_i} + d_i^2 A_i)$$



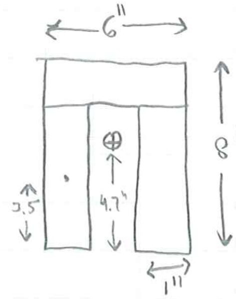
2-26

Torsion & equilibrium of beams

6] 5, 10, 16, 18, 22, 27, 28, 33
6] 34, 35, 38, 42, 48, 50, 58, 59
63, 68, 69, 74, 78, 81, 90
7] 24, 27, 30, 32, 34, 36, 46

did not do these

8] 6, 8, 10, 18, 20, 22, 29 32



6)

a) centroid $\bar{y} = \cancel{3.475}\ 4.7$

$\bar{x} = 3.00$

moment of Inertia about z axis

$$I = \left[\frac{1(7)^3}{12} + (1.2)^2(1)(7)\right]2 + \frac{6(1)^3}{12} + (7.5-4.7)^2(6)(1)$$

$= 124.867$

$$\sigma_H = \frac{-(-8500)(12)\ \text{in-lb}\ (2\text{in})}{124.9\ \text{in}^4} = 1633.31$$

b)

$$\sigma_{max} = \frac{-(-8500)(12)\left(\begin{smallmatrix}+3.3 \\ \text{or} \\ -4.7\end{smallmatrix}\right)}{124.9}$$ top (+) bottom (c)

$$S = \frac{I}{y} = \overset{37.85}{\frac{124.9}{3.3}} \text{ or } \overset{26.57}{\frac{124.9}{4.7}}$$

8)

8-18) $\bar{y} = 1.15625$

do something like this

8-20) $\sigma_{max} \quad \dfrac{-\begin{pmatrix} M_1 \\ M_2 \end{pmatrix}\begin{pmatrix} +3.3 \\ -4.7 \end{pmatrix}}{I}$ top / bottom find maximum negative moment & max pos. moment

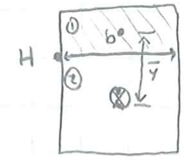
8-29) look up HSS 12 x 8 x 1/2 p685

8-32) $\sigma = \dfrac{-(-)(136\ N \cdot m)\left(\pm \frac{d}{2}\right)}{\frac{\pi}{64} d^{\cancel{4}\,3}}$ $d \geq$ ______

signs $\quad \sigma = \dfrac{M y_{*}}{I} \leftarrow$ dist from centroid to where you need stress

magnitude $\quad \tau_{beam} = \dfrac{VQ}{Ib}$ for "shaded" area

$Q = \bar{y} A$ dist from centroid to middle of shaded area



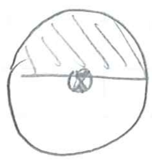
shade away from centroid

-use this with p 9.12

$$\tau_H = \frac{3000\left(4.5 + \frac{3}{2}\right)\cancel{(6)}(3)}{\frac{6(15)^3}{12}\cancel{(6)}} = 32$$

$\tau_K = 12.44$

$$\tau_{max} = \frac{300\left(\frac{7.5}{2}\right)(7.5)(6)}{\frac{6(15)^3}{12}(6)} = 50$$



$$\tau_{max} = \frac{V\left(\frac{4r}{3\pi}\right)\left(\frac{\pi r^2}{2}\right)}{\left(\frac{\pi r^4}{4}\right)(2r)} = \frac{4V}{3A}$$

 $= \dfrac{3V}{2A}$

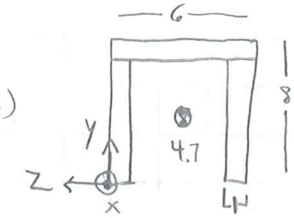
$\dfrac{\pi d^4}{64}$

 $= \dfrac{4V}{3A}\left(\dfrac{r_2^2 + r_2 r_1 + r_1^2}{r_2^2 + r_1^2}\right)$

$$\sigma_H = \frac{My}{I}$$

Homework 8. 6, 8, 10, 18, 20, 22, 29, 32

6) a) $\bar{x} = 3''$ $\bar{y} = \dfrac{16(4) + 4(7.5)}{20} = 4.7$ $\boxed{(3, 4.7)}$



$$I_z = \left[\frac{1(7)^3}{12} + (4.7-3.5)^2(1)(7)\right]2 + \frac{6(1)^3}{12} + (7.5-4.7)^2(6)$$

$= ~~(41.083)2 + 47.54~~$

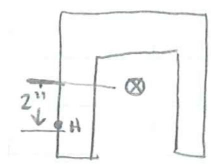
$= \boxed{124.9 \text{ in}^4}$

$$I = \frac{bh^3}{12} + d^2A$$

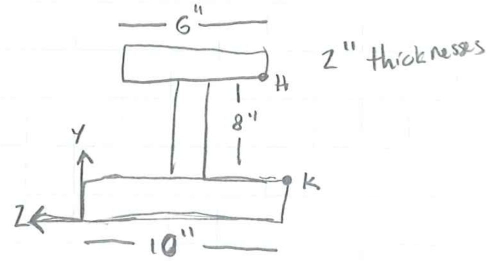
controlling section modulus about z axis

$\dfrac{I}{3.3}$ or $\dfrac{I}{4.7}$, 37.85 or $\boxed{26.57 \text{ in}^3}$

b) σ_H = bending stress at H $= \dfrac{(8.5 \text{ kip-ft})(12 \text{ in/ft})(2'')}{124.9} = \boxed{1.63(10^3) \text{ psi}}$ compression



c) $\sigma_{max} = \dfrac{4.7}{2}\sigma_H = \boxed{3.838(10^3) \text{ psi}}$ compression



8) Centroid $= (5, 5.167)$ $\dfrac{12(11) + 16(6) + 20(1)}{48} =$

$$I_z = \frac{10(2)^3}{12} + 20(4.167)^2 + \frac{2(8)^3}{12} + 16(.833)^2 + \frac{6(2)^3}{12} + 12(5.833)^2$$

353.94 + 96.44 + 412.287

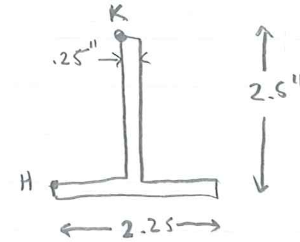
$I_z = \boxed{862.667}$

$S = \dfrac{I}{y} = \dfrac{862.7}{(12-5.167)} = \boxed{126.25 \text{ in}^3}$

$\sigma_H = \dfrac{10.5^{\text{kip-ft}}(12'')(10-5.167)''}{862.7 \text{ in}^4}$ $\boxed{705.9 \text{ psi}}$

$\sigma_K = \dfrac{10.5(12)(5.167-2)}{862.7} = \boxed{462.6 \text{ psi}}$

$\sigma_{max} = \dfrac{10.5(12)(6.83)}{862.7} = \boxed{998.0 \text{ psi}}$



10) $\sigma_K = 2600$ psi T $\bar{y} = .75''$ $\sigma_K = \dfrac{My}{I} =$

$$I_z = \frac{2.5^3(.25)}{12} + 2.5(.25)(.75)^2 + \frac{2(.25)^3}{12} + 2(.25)(.625)^2$$

.6771 + .198

$I_z = ~~.875~~$ .6797 $\quad 2600 \text{ psi} = \dfrac{M(1.75)}{~~.875~~ \; .6797}$ $\boxed{~~M = 1300 \text{ in-lb}~~}$

$\boxed{M = -1010 \text{ in lb}}$

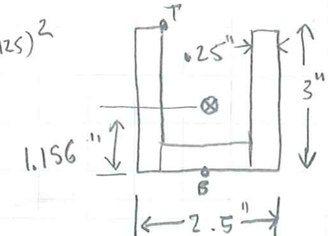
$\sigma_H = \dfrac{-1010(.75)}{.6797} = \boxed{1114.46}$ psi (C)

$$\sigma = \frac{My}{I}$$

$$I = \frac{b^3 h}{12} + d^2 A$$

18, 20, 22, 29, 32

18) $\bar{Y} = \dfrac{3(.25)(1.5) + (2)(.25)(.125)}{5(.25)} = 1.156$



$I_z = \left[\dfrac{(.25)(3)^3}{12} + (.25)(3)(1.5-1.156)^2\right](2) + \dfrac{2(.25)^3}{12} + 2(.25)(1.156-.125)^2$

$= 1.836 = \boxed{1.84}$

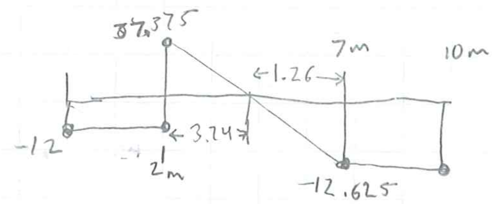
limiting

Max tension @ bottom $= \dfrac{My}{I} = \dfrac{600(12)1.156}{1.836}$ $\boxed{4533\text{ psi }(t)}$

Max $\sigma_{(c)}$ = top $= \dfrac{600(12)(1.844)}{1.836}$ $\boxed{7231\text{ psi }(c)}$

20) WT 305 x 41

$I = 48.7(10^6)\text{ mm}^4$



$\dfrac{1.26(10)}{2} + 12625(2)$

$\dfrac{My}{I}$ tension = bottom $\dfrac{31.55(211.1)\text{mm}}{48.7(10^6)\text{ mm}^4}$ N·mm(10^3)

M = ~~31.55 N·m~~

$M_{5.74m} = 45.89$ kN·m

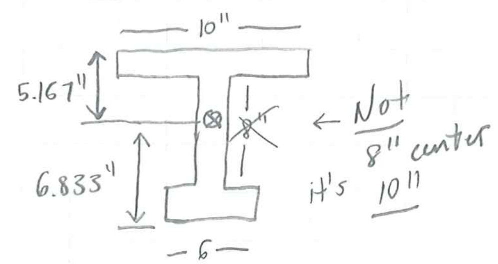
$x = 2m$ $\dfrac{24000(.211)}{48.7(10^6)} = 104\text{ MPa }(t)$

$\dfrac{''(.088)}{''} = 43.4\text{ MPa }(c)$

$x = 5.74$ $\dfrac{45890(.211)}{48.7(10^6)} = 199\text{ MPa }(c)$

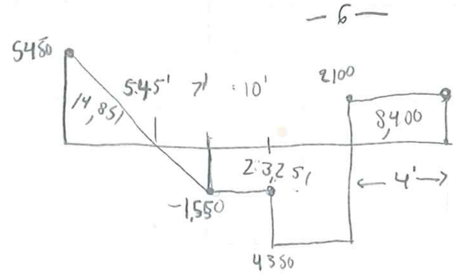
$\dfrac{''(.088)}{''} = 82.9\text{ MPa }(t)$

22) Centroid = (5, 6.833)



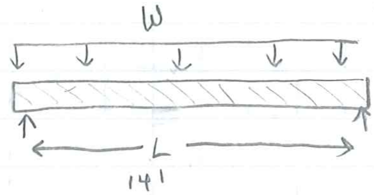
$\sigma = \dfrac{My}{I} =$

$x = 5.45'$ $\dfrac{14{,}851\begin{pmatrix}6.833\\5.167\end{pmatrix}}{862.7} = \begin{matrix}117.627\ (c)\\88.947\ (t)\end{matrix}$

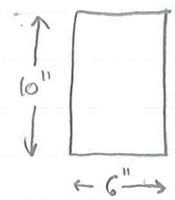


$x = 14''$ $M = -8{,}400$ $\dfrac{-8400\begin{pmatrix}6.833\\5.167\end{pmatrix}}{862.7}$ $66.53\ (t)$

Homework



8.34) $\sigma_{max} = 900 psi$ $\quad W_{max} = ?$

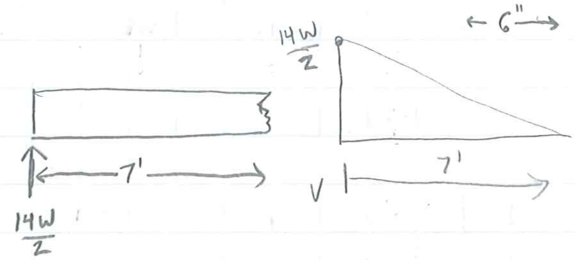


$$I = \frac{bh^3}{12} + \cancel{d^2 A}^{\,0} = 500 \text{ in}^4$$

$$\sigma = \frac{My}{I} \qquad 900 = \frac{M(5)}{500} \qquad M_{max} = 90{,}000 \text{ in·lb} = 7500 \text{ ft·lb}$$

$$\frac{w(7')(7')}{2} = M = 7500 \text{ ft·lb}$$

$W = \cancel{110.204}$ lb/ft. $\boxed{306.12 \text{ lb/ft}}$ ✓



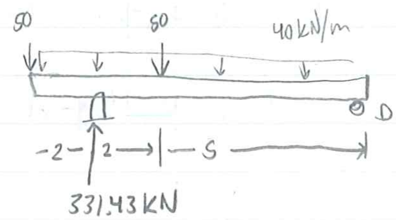
8. 38) $\sigma_{max} = 165 MPa = \frac{My}{I} \Rightarrow \frac{y}{I} \geq .801$

$\boxed{W 360 : x72}$ $\qquad \frac{d}{I} < 1.602$

$\sigma < 165 MPa \Rightarrow \frac{My}{I} < 165, \Rightarrow \frac{y}{I} < .801$

$\boxed{W 410 \times 75}$

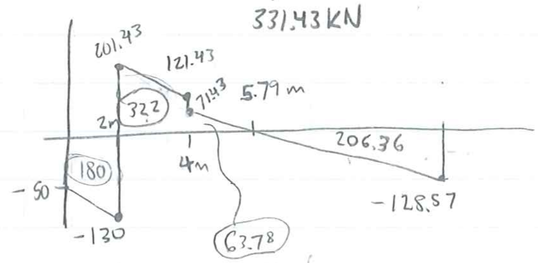
$\boxed{W 460 \times 74}$ $\quad \boxed{W 530 \times 66}$

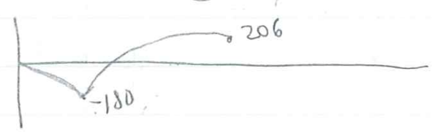


$$\sigma = \frac{M}{S}$$ * you can look up S in the table

$$\sigma = \frac{206 kN}{S} \leq 165(10^6)$$

$$S_x \geq 1.248(10^{-3})$$





$M_{max} = 206 kN·m$

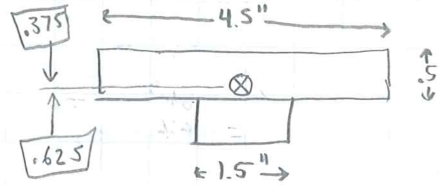
47, 51, 52, 57, 60

8.45) Al: $E = 10{,}000$ ksi $M = 300$ lb·ft $\hat{k}$ a) find max bending stresses Al & Steel

Steel: $E = 30{,}000$ ksi b) stress at joint

$$n = \frac{E_{steel}}{E_{Al}} = \boxed{3}$$

equivalent to all aluminum



$$\bar{Y} = \frac{4.5(.5)(.75) + 1.5(.5)(.25)}{6(.5)}$$

$\bar{Y} = .625$

$$I_z = \frac{4.5(.5)^3}{12} + (.75-.625)^2(.5)(4.5) + \frac{1.5(.5)^3}{12} + (.625-.25)^2(1.5)(.5)$$

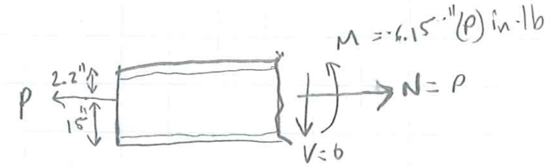
$I_z = \cancel{.1094}$ .203125

$$\sigma_{st} = \frac{300(12)(.375)}{.2031}(3) = \boxed{19.94 \text{ ksi}} \text{ compression}$$

$$\sigma_{Al} = \frac{300(12)(.625)}{.2031}(1) = \boxed{11.078 \text{ ksi}} \text{ tension}$$

$$\boxed{\sigma_{max} = \frac{M y_{max}}{I}(n)}$$

8.57) W18x35 $A = 10.3\text{in}^2$ $d = 17.7$in $I = 510\text{ in}^4$



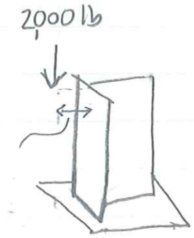
top has most load $$\sigma_{top} = \frac{N}{A} - \frac{(-6.15P)(8.85)}{(510)}$$

$$15{,}000 \text{ psi} = \frac{P}{10.3\text{in}^2} + \frac{(6.15)(8.85)P}{510}$$

$$\boxed{P \leq 73{,}600 \text{ lb}}$$

$$\boxed{\sigma = \frac{N}{A} + \frac{My}{I}}$$

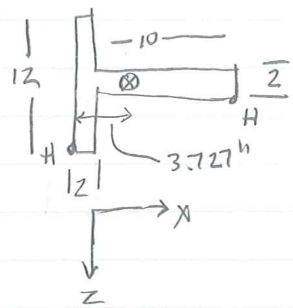
8.60)



$\bar{X} = 3.727''$

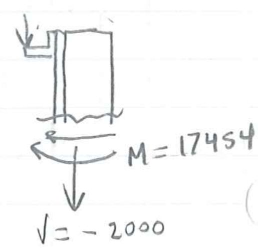
$$I_z = \frac{12(2)^3}{12} + 24(3.727-1)^2 + \frac{2(10)^3}{12} + (7-3.727)^2(20)$$
$$= \boxed{567.4\ in^4}$$

$M = 17454$ ft.lb $\quad A = 44\ in^2$

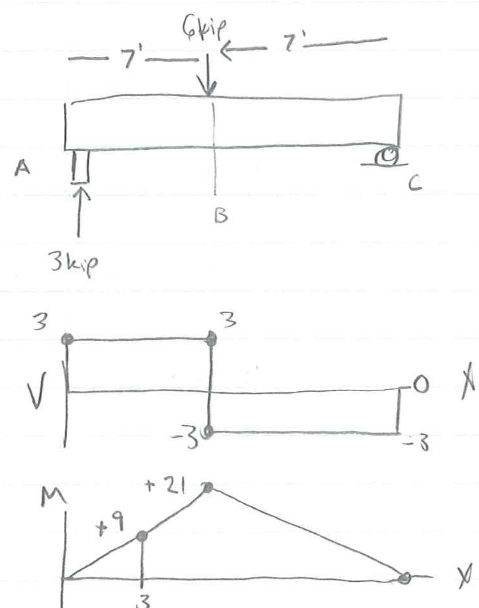


$$\sigma_H = \frac{17454(3.727)}{567.4} + \frac{2000}{44} \quad \boxed{160.1\ psi\ (C)}$$

$$\sigma_K = \frac{17454(-8.273)}{567.4} + \frac{2000}{44} \quad \boxed{209.0\ psi\ (t)}$$



9.12) Shear stress in beam @ H



$$\tau_H = \frac{VQ}{Ib} \quad Q = \Sigma \bar{y} A_i = (3'')(6'')(7.5''-1.5'')$$
$Q = 108''^3$ $\quad V = 3,000$ lb $\quad I = 1688''^4$

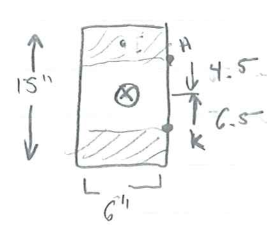
$$\boxed{\tau_H = \boxed{32\ psi}}$$

$$\tau_K = \frac{3000(42)}{1688(6)} \quad Q = (6)(1)(7)$$

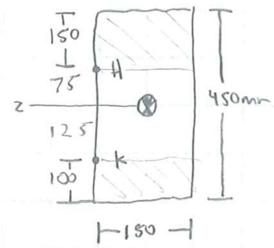
$$\boxed{\tau_K = 12.44\ psi}$$

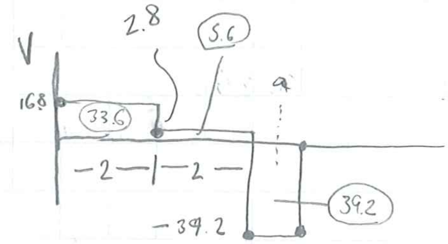
$$\tau_{max} = \frac{3000(6)(7.5)(3.75)}{1688(6)} \quad \boxed{49.99\ psi}$$

$$\sigma_{max} = \frac{My}{I} \quad \frac{21{,}000(7.5)(12''/ft)}{1688} = \boxed{1120\ psi}$$



9.14) $I = 1.139(10^{-3})\ m^4$





$$\sigma_H = \frac{-39.2\,\overbrace{(.15)}^{\bar{y}}\,\overbrace{(.15)(.15)}^{A}}{1.139(10^{-3})(.15)}$$

τ_H

$\boxed{\cancel{\sigma_H} = 774.4\ kPa}$

$$\sigma_k = \frac{-39.2(.175)(.15)(.1)}{(1.139E^{-3})(.15)}$$

τ_k $\boxed{\cancel{\sigma_k} = 602.3\ kPa}$

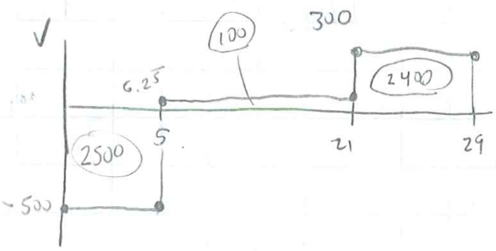
$$\sigma_{max} = \frac{39.2(.1125)(.225)(.15)}{1.139E^{-3}(.15)} = \boxed{\cancel{\sigma_{max}} = 871.2\ kPa}$$

τ_{max}

$M_a = +19.6\ kN\cdot m$

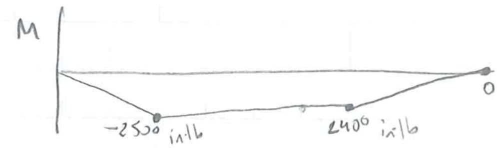
$V_A = -39.2\ kN$

$$\sigma_{max} = \frac{My}{I} = \frac{(39.2)(.225)}{1.139E^{-3}} =$$



9.20) Max horizontal shear stress τ

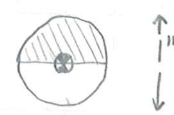
$I = .0491\ in^4$ $\quad \frac{VQ}{IB} = \frac{-500(.1667)}{(.0491)(1)}$



$Q = \bar{y}A = (\quad).7854 = .16\bar{6}$

Q is not correct. should be $2(.16\bar{6}) = .0833$

$$\tau_{max} = \frac{VQ}{IB} = \frac{500(.08\bar{3})}{(.0491)(1)} = \boxed{849.0}\ psi$$



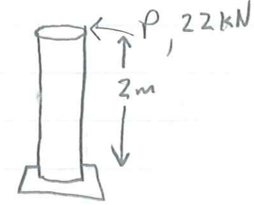
Review #20

$$Q = \Sigma\bar{y}A = \frac{1}{2}\pi R^2\left(\frac{4R}{3\pi}\right) = \frac{2}{3}R^3 \text{ or } \frac{2d^3}{8(3)} = \frac{d^3}{12}$$

$$\text{Max tension bending stress} = \frac{M_{max}\,y_{max}}{I} = \frac{2500(.5)}{.0491} = \boxed{25{,}460\ psi}$$

9.26)

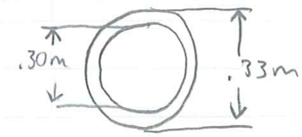
Max vertical shear stress $= \tau_{max} = \frac{VQ}{IB}$



(22 kN)(

$I = \Sigma(I_i + d_i^2 A_i) = \frac{\pi(.33)^4}{4} - \frac{\pi(.30)^4}{4}$ $\boxed{.000185 \ m^4}$

$Q = \Sigma \bar{y}_i A_i = \frac{4R}{3\pi}\left(\frac{1}{2}\pi R^2\right) - \frac{4R}{3\pi}\left(\frac{\pi R^2}{2}\right)$ $\boxed{.000745 \ m^3}$



$\tau = \frac{22000 \ Q}{I \ (.03)} = 2.953 \ E^6 \ Pa$

Max tension bending stress

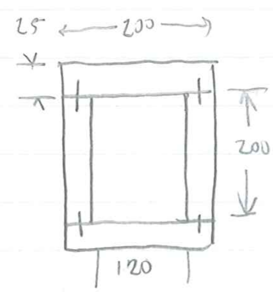
decimal point is off

$= \frac{My}{I} = \frac{44(10^3)(.165)}{1.85 \ E^{-4}} = .3924$ 39.3 MPa

9|40, 42, 44, 47 10|3, 6

40) $I_z = \frac{1}{12}(200)(250)^3 - \frac{1}{12}(120)(200)^3 = 1.804 \ E8 \ mm^4$

1st moment of area $Q = (200)(25)(112.5) = 562{,}500 \ mm^3$



$qs \leq n_f V_f \qquad \frac{VQ}{I_z} \leq n_f V_f$

$q \leq \frac{2 \text{ nails } (500 \ N/\text{nail})}{125 \ mm} = 8 \ N/mm$

$V \leq \frac{8 \ N/mm \ 1.804 E8 \ mm^4}{562500 \ mm^2}$

$V \leq 2566 \ N = \boxed{2.566 \ kN}$

$S = 125 \ mm$
$V_f = 500 \ N$

Homework

$$F_H = \frac{VQ}{I_z} \Delta x$$

F_H : horizontal force
V : shear force
Q : shear area

$$\frac{F_H}{\Delta H} = q = \frac{VQ}{I_z}$$

q = shear flow

$$F_H = qs$$

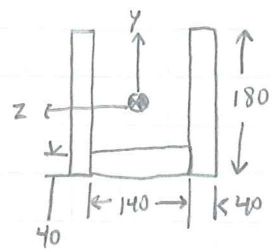
S: spacing interval

Fastener Force spacing relationship

$qs \le n_f V_f$ or $qs \le n_f \tau_f A_f$

HW 10-3

47) $M_z = -4.5 \text{ kN·m}$ $S = 150 \text{ mm}$
$V_y = -2.25 \text{ kN}$
max horizontal shear stress



$\bar{y} = 70.4 \text{ mm}$

$$I_z = 2\left[\frac{(40)(180)^3}{12}\right] + \left[(19.6)^2(180)(40)\right]2 + \frac{140(40)^3}{12} + (140)(40)(50.4)^2$$

$= 5.938 \cdot 10^7 \text{ mm}^4$

$Q = (109.6)(40)(2)(54.8) = 480486 \text{ mm}^3$

a) $\tau_{max} = \frac{VQ}{It} = \frac{2.25(10)^3 \; 480,486}{5.938(10^7)(2)(40\text{mm})}$

$\tau_{max} = 228 \text{ kPa}$

b) shear force in each screw: $qs \le n_f V_f$
(bottom board)
$Q = 140(40)(70.4 - 20) = 282,240 \text{ mm}^3$

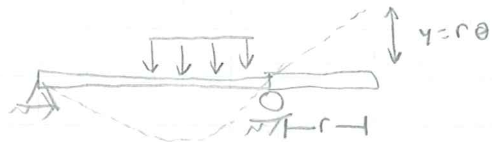
$V = \frac{qs}{n}$ $q = \frac{VQ}{I} = 10.69 \text{ N/mm}$

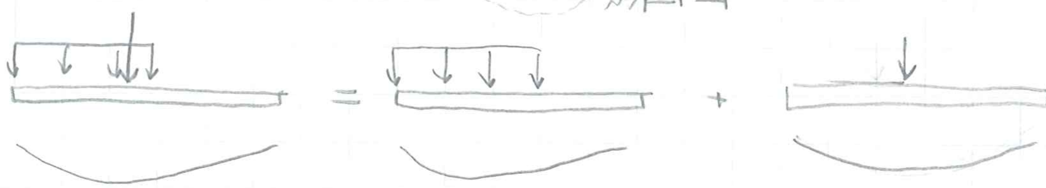
$\boxed{V = 801.75 \text{ N per screw}}$

BC = boundary conditions
Cont: continuity

10.31 find moment

ignore force supported by ground

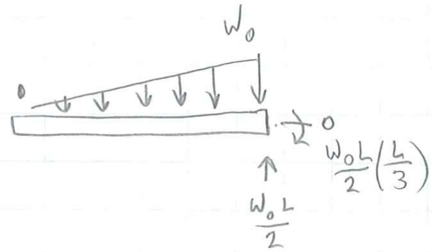




pay attention to where deflection tables are valid. (according to discontinuities)

10.3)

1) FBD



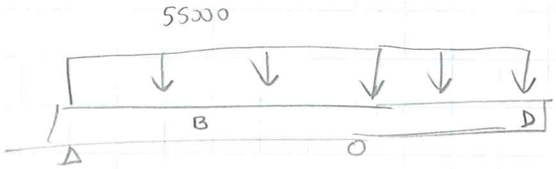
3) BC's $V_0 = \frac{W_oL}{2}$ $\quad y(0)=0$

$M_0 = -\frac{W_oL^2}{6}$ $\quad \theta(0)=0$

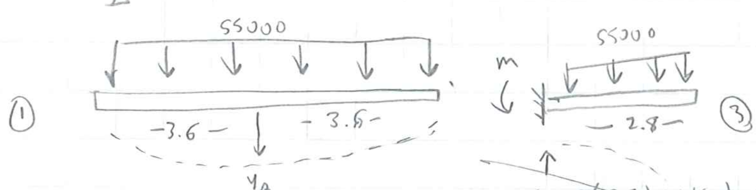
4) integrate $w(x)$

$$w(x) = w(x_o) - \frac{W_o}{L}x$$

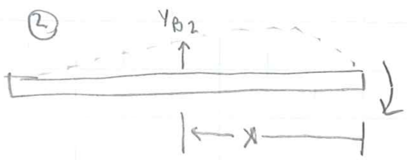
Notes 3-19



$+\circlearrowleft \sum M_{wall} = M - 1.4(55{,}000)(2.8) = 0$

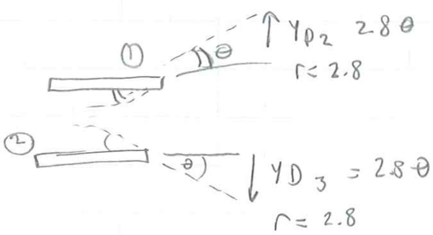


$55000(2.8)$: wont affect bending.

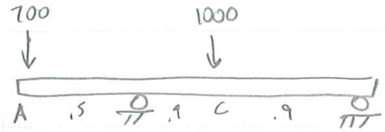


① $V_{max} = \frac{5WL^4}{384EI}$

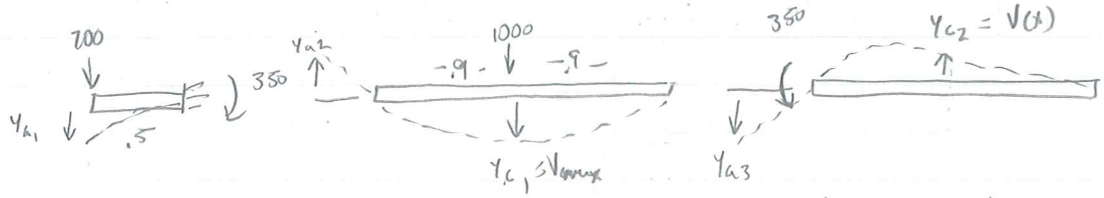
② $V = -\frac{Mx}{6LEI}(2l^2 - 3lx + x^2)$



Notes 3-19

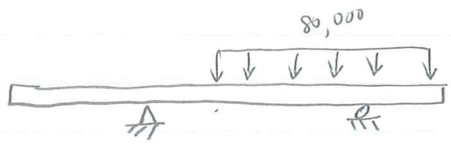


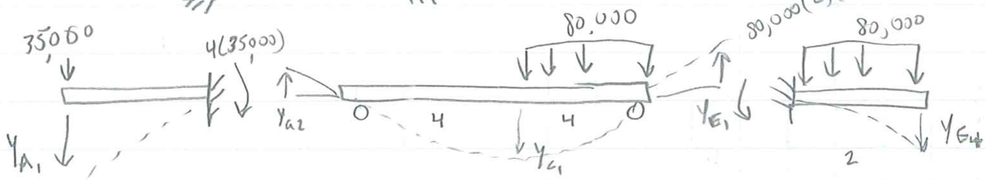
44)

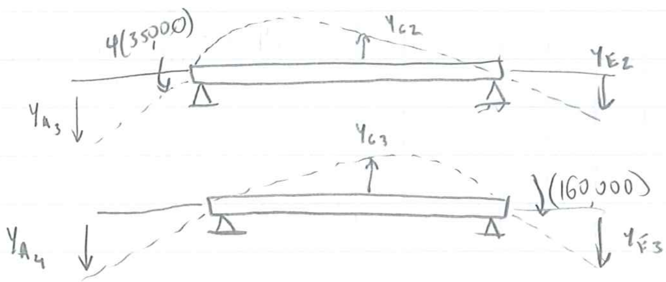


$Y_A = 4.139$ mm

48)



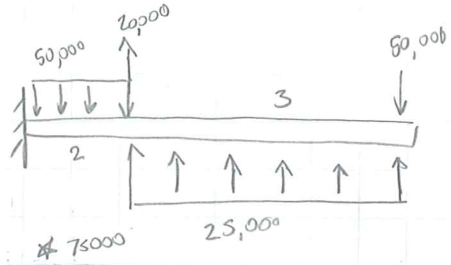


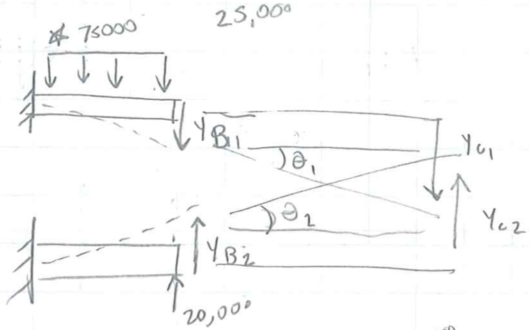


$Y_A = Y_{A_1} + r_\theta + r_\theta + r_\theta$

$Y_C = Y_{C_{1,2,3}}$ $(-, +, +)$

52)

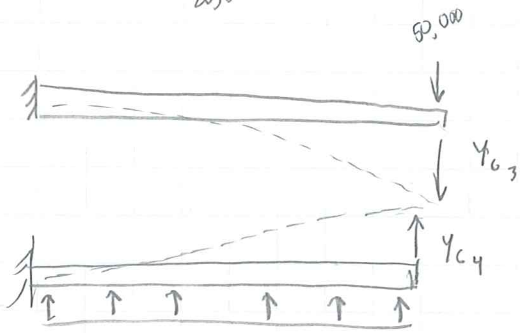




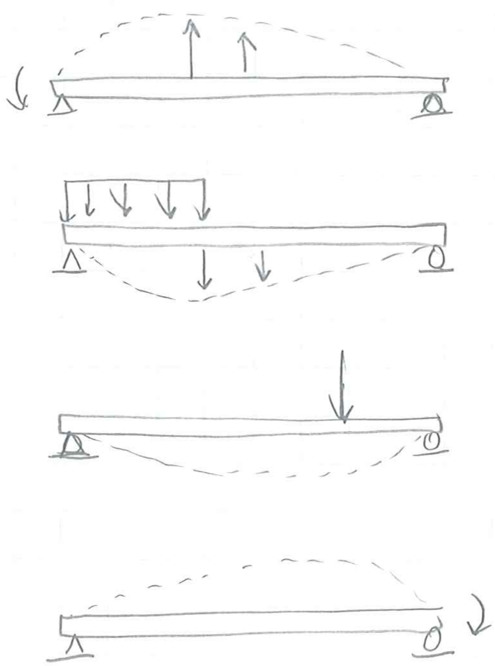
$$y_{C_1} = y_{B_1} + r\theta$$

$y_B = 2.595 \downarrow$

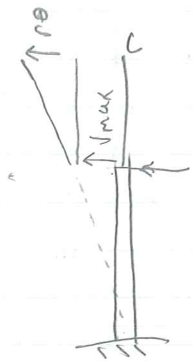
$y_C = 21.41 \uparrow$

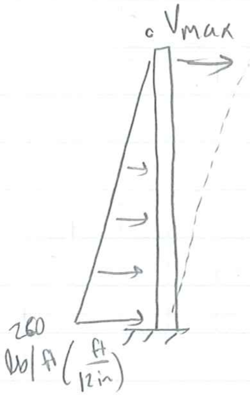


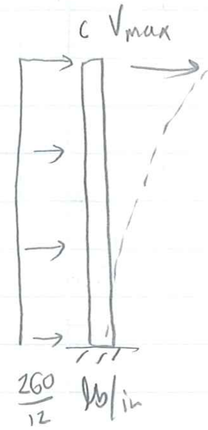
56)



58

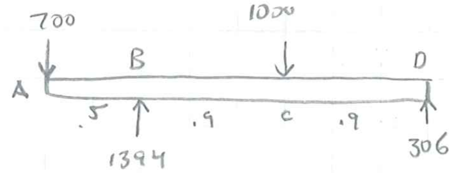






Homework 3-21

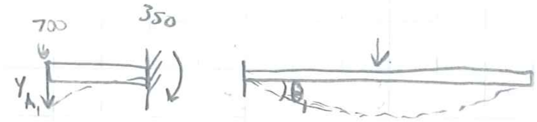
(10-44)

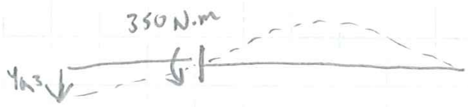


$E = 200\ GPa$

$I = \frac{\pi}{64}(30mm)^4 = 39{,}761\ mm^4$

$EI = 7952.2$





$Y_{A_1} = \frac{700(.5)^3}{3EI} = -3.668\ mm$

$\theta_1 = -\frac{PL^2}{16EI} = \frac{1000(1.8)^2}{16\ EI} = .02546\ rad$

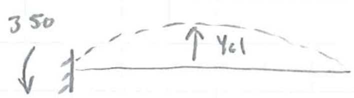
$Y_{A_2} = .00197(.5) = +12.73mm$

$\theta_2 = \frac{ML}{3EI} = \frac{350(1.8)}{3EI}$ $.026408\ rad$

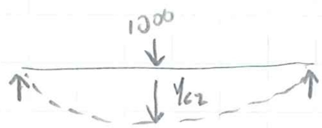
Y_{A_3} $-13.204mm$

$$\boxed{Y_A = Y_{A_1} + Y_{A_2} + Y_{A_3} = 4.1419mm}$$

Y_c:



$Y_{c_1} = \frac{ML^2}{9\sqrt{3}EI}$ $\frac{350(1.8)^2}{9\sqrt{3}\ 7952}$ $8.913\ mm\ \uparrow$



$Y_{c_2} = \frac{PL^3}{48EI} = \frac{1000(1.8)^3}{48(7952} = 15.279\ mm\ \downarrow$

$$\boxed{Y_c = 6.366\ mm\ \downarrow}$$

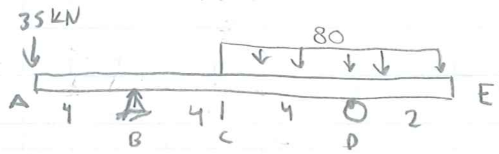
$Y_{c_1} = -\frac{Mx}{6LEI}(2L^2 - 3Lx + x^2) =$

$x = .900$

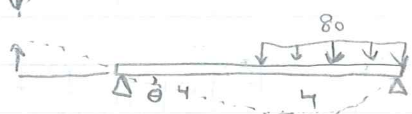
a) deflection at A

10.48) $EI = 7.02(10^7)$

$\circlearrowleft \Sigma M = 0 = 35(4) - 80(6)(7) + F_D(8)$ $F_D = 402.5$



Y_{A_1}  $Y_{A_1} = \frac{3L^3}{3EI} = 10.636\text{mm} \downarrow$

Y_{A_2} 

$\theta = \frac{wa^2}{24EIL}(2L^2 - a^2) = .01064 \text{ rad}$ $Y_{A_2} = 42.545\text{mm}$

Y_{A_3} 

$\theta = \frac{ML}{3EI} = .005318 \text{ rad}$ $Y_{A_3} = 21.273\text{mm} \downarrow$

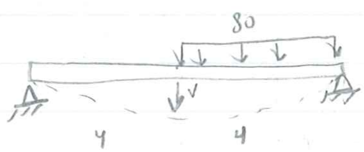
Y_{A_4} 

$\theta = \frac{ML}{6EI} = .003039 \text{ rad}$ $Y_{A_4} = 12.156 \text{ mm} \downarrow$

$\boxed{Y_A = 1.52 \downarrow \text{mm}}$

b) @ C.

$v_1 = \frac{Mx}{6LEI}(2L^2 - 3Lx + x^2) = 7.977 \text{ mm} \uparrow$



$V = \frac{Wa^3}{24LEI}(4L^2 - 7aL + 3a^2) = 30.389 \text{ mm} \downarrow$

$V = \frac{Mx}{6LEI}(2L^2 - 3Lx + x^2) = 9.117\text{mm} \uparrow$

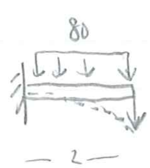
$\boxed{V_c = \downarrow 13.295 \text{ mm}}$

c) @ E

$\theta = \frac{ML}{6EI} = $ ~~~ $\Rightarrow$ 5.318mm $\downarrow$

$\theta = \frac{wa^2}{24LEI}(2L - a)^2 = $ ~~~ $\Rightarrow$ 27.35 mm $\uparrow$

$\theta = \frac{ML}{3EI}$ $\Rightarrow$ 12.156 mm $\downarrow$



$V = \frac{WL^4}{8EI} = 2.279 \text{ mm} \downarrow$

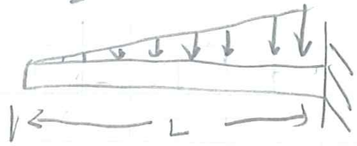
$\boxed{7.597 \text{ mm} \uparrow}$

dist Force

10.3) $\cancel{\Sigma M = 0 = M + (L-x)\,\frac{W_0\left(\frac{x}{L}\right) + W_0}{2}}$

$\cancel{w = W_0\left(\frac{x}{L}\right) \quad M = \frac{W_0\left(\frac{x}{L}\right)(L)}{2} + \frac{W_0 L}{2} - \frac{W_0\left(\frac{x}{L}\right)x + W_0 x}{2}}$

$w = W_0\left(\frac{x}{L}\right)$



$M \quad \left(\frac{x}{3}\right) W_0\left(\frac{x}{L}\right)\left(\frac{1}{2}\right)(x) = \frac{W_0 x^3}{6L}$

$EI\frac{d^2v}{dx^2} = -\frac{W_0 x^3}{6L}$

$EI\frac{dv}{dx} = -\frac{W_0 x^4}{24L} + C_1$

$EIv = -\frac{W_0 x^5}{120L} + C_1 x + C_2$

B.C.: $x = L,\ v = 0$

$x = L,\ \frac{dv}{dx} = 0$

$-\frac{W_0 L^4}{24L} + C_1 = 0 \qquad \boxed{C_1 = +\frac{W_0 L^3}{24}}$

$-\frac{W_0 L^4}{120} + \frac{W_0 L^3}{24}(L) + C_2 = 0 \qquad C_2 = W_0\left(\frac{L^4}{120} + \frac{L^3}{24}\right)$

$C_2 = W_0 L^4\left(\frac{1}{120} - \frac{1}{24}\right)$

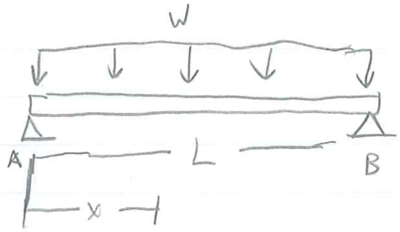
$\boxed{C_2 = -\frac{W_0 L^4}{30}}$

$EIv = -\frac{W_0 x^5}{120L} - \frac{W_0 L^3}{24}x - \frac{W_0 L^4}{30} = \frac{-W_0}{120L}\left[x^5 + 5L^4 x + 4L^5\right]$

$EIv(x=0) = -\frac{W_0}{120L}\left(4L^5\right) = -\frac{W_0 L^4}{30} \qquad \boxed{v = -\frac{W_0 L^4}{30EI}}$ at free end

$\frac{dv}{dx}(x=0) = \frac{W_0 L^3}{24EI}$

10.6)



$\circlearrowright \Sigma M = 0$

$0 = -F_A(x) + F_W\left(\frac{x}{2}\right)$

$= -\frac{WL}{2}(x) + \frac{Wx}{2}(x) + M$ $\qquad M = \frac{W}{2}\left(-x^2 + Lx\right)$

$EI\frac{d^2v}{dx^2} = -\frac{Wx^2}{2} + \frac{WLx}{2}$ $\qquad$ BC| $\frac{dv}{dx} = 0$ @ $x = L/2$

a) $EI\frac{dv}{dx} = -\frac{Wx^3}{6} + \frac{WLx^2}{4} + C_1$ $\qquad x = 0,\ v = 0$

$x = L,\ v = 0$

b) $EIv = -\frac{Wx^4}{24} + \frac{WLx^3}{12} + C_1x + C_2$

$\boxed{C_2 = 0}$ $\qquad 0 = -\frac{W\left(\frac{L}{2}\right)^3}{6} + \frac{WL\left(\frac{L}{2}\right)^2}{4} + C_1$

$C_1 = +\frac{WL^3}{48} - \frac{WL^3}{16}$ $\qquad \boxed{C_1 = -\frac{WL^3}{24}}$

$EIv = -\frac{Wx^4}{24} + \frac{WLx^3}{12} + (-)\frac{WL^3}{24}x$

$v = \frac{Wx}{24EI}\left(-x^3 + 2Lx^2 - L^3\right)$

Max deflection: $x = \frac{L}{2}$ $\qquad v = \frac{WL}{48EI}\left(-\frac{L^3}{8} + \frac{2L^3}{4} - L^3\right)$

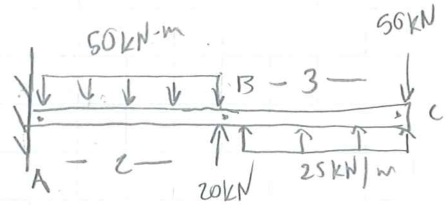
$\boxed{v_{max} = \frac{WL^4}{EI}\left(\frac{-5}{384}\right)}$

Slope at A| $EI\frac{dv}{dx} = C_1 = -\frac{WL^3}{24}$

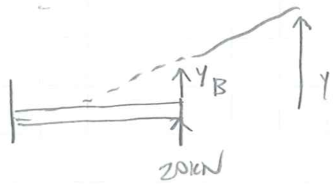
$\boxed{\frac{dv}{dx} = -\frac{WL^3}{24EI}}$

Quiz Prep Express:

10-52 $E = 200\,GPa$ $I = 95E6\ mm^4$ $IE = 19(10^6)$



v_B caused by 20 kN load



$$v_B = \frac{PL^3}{3EI} = \frac{20\,000(2)^3}{3(19E6)}$$

$$= \boxed{2.807\ mm}\ \downarrow$$

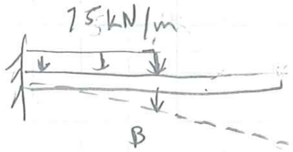
v_C caused by 20 kN pt load $\frac{PL^3}{3EI} + 3m\left(\frac{PL^2}{2EI}\right) = 2.807\,mm + 6.316\,mm = \boxed{9.123\,mm}$ ✓

v_B caused by 50 kN. $= \frac{Px^2}{6EI}(3L - x) = \frac{50,000(4)}{6(19E6)}(15-2) = \boxed{22.807\ mm}$

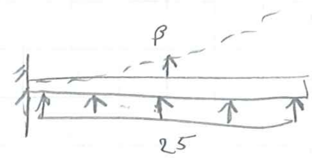
$L = 5$ $x = 2$

v_C caused by 50 kN $= \frac{PL^3}{3EI} = \frac{50000(5)^3}{3(19E6)} = .1096\ m$

v_B caused by dist. loads



+



$$= \frac{wx^2}{24EI}(6L^2 - 4Lx + x^2)\uparrow + \frac{wL^4}{8EI}\downarrow$$

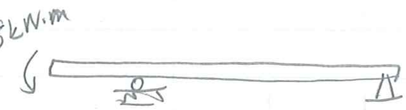
25 mm $\uparrow$ 7.895 mm $\downarrow$ $\boxed{17.1\ mm \uparrow}$

Quiz Prep

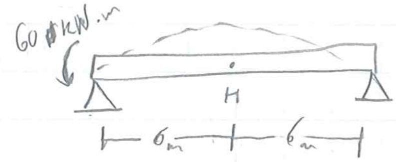
10.31). $EI = 6(10^{10})\ kN\ mm^2 =$

$= 6(10^4)\ kN\ m$

$= 6(10^7)\ N \cdot m$



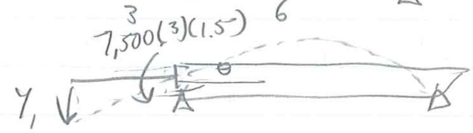
a) $\frac{-Mx}{6LEI}(2L^2 - 3Lx + x^2)$



$= \frac{60{,}000\,(6)}{6(12)\,6(10^{10})}\left[2(12)^2 - 3(12)(6) + 6^2\right]$

~~15.1~~ $\boxed{9mm \uparrow}$

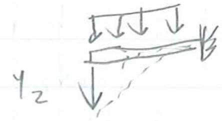
b) $\theta = \frac{ML}{3EI} = \frac{7{,}500(3)(1.5)(6)}{3\,(6E7)}$



$\Rightarrow Y_1 = 3.375\ mm$

$Y_2 = 1.266\ mm \downarrow$

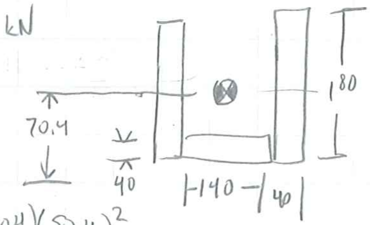
$\boxed{Y_A = 1.641}\ mm \downarrow$



$\frac{WL^4}{8EI} = \frac{7500(3)^4}{8\,(6E7)}$

9.47)

$S = 150\,mm \quad M_z\,max = -4.50\,kN \cdot m \quad V_{y\,max} = -2.25\,kN$



a) max horizontal shear stress

$\bar{Y} = 70.4mm$

$$I = 2\left[\frac{(.04)(.18)^3}{12} + (.04)(.18)(.0196)^2\right] + \frac{(.140)(.040)^3}{12} + (.140)(.04)(50.4)^2$$

$\boxed{5.938\ m^4\ E^{-5}}$

$Q_{max} = \cancel{\bar{y}A = 2[35.2mm(70.4)(40)] + (50.4mm)(.14)(.04)}$

$$= \left[\overbrace{(109.6)(40)}^{A}\Big(\overbrace{\frac{109.6}{2}}^{\bar{y}}\Big)\right]2 = 480486\ mm^4 = .4805\,(10^{-3})\,m^4$$

$4.805\,(10^{-4})m^4$

$$\tau_{max} = \frac{VQ}{I_z b} = \frac{-2.25\ kN(4.805\,E^{-4})}{5.938\ E{-5}\,(.8)} = 2.276\,(10^4)\ Pa$$

$22.76\ kPa$

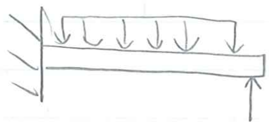
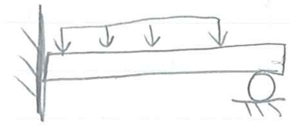
shear force in each screw $= \frac{nV_f}{s} = \frac{VQ}{I}$ $\quad Q_{screws} = (.14)(.04)(.050.4) \quad 2.822\,E^{-4}$

$$V_{fastener} = \frac{V_m Q\, s}{I\, n} = \frac{2250\,(2.822E^{-4})(.15)}{5.938\ E{-5}\ (2)} = 801.98\ N$$

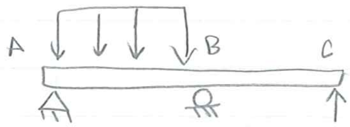
bending stress max $= \sigma_{max}\ \frac{My}{I} = \frac{4500(N \cdot m)(.1096)}{5.938\ E{-5}}$ $\quad 8.306\ MPa$

Notes 3-23

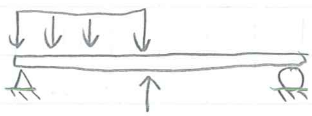
make this   out of this

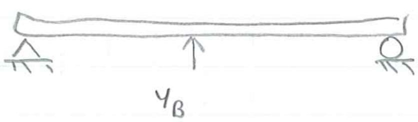
Say $Y_B = 0 \Rightarrow B = \underline{?}$



$Y_C = 0$
$\Rightarrow C = \_\_\_\_$

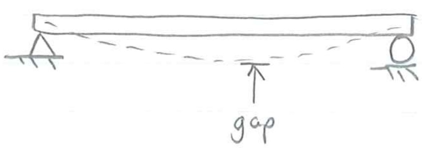


$Y_B = 0$
$\Rightarrow B = \_\_\_\_$



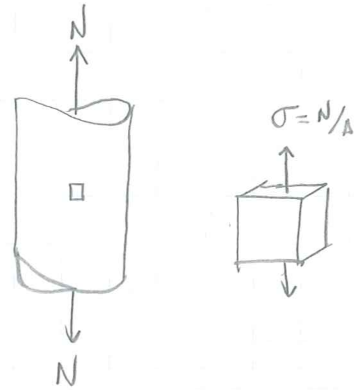
$Y_B = \frac{NL}{EA}$

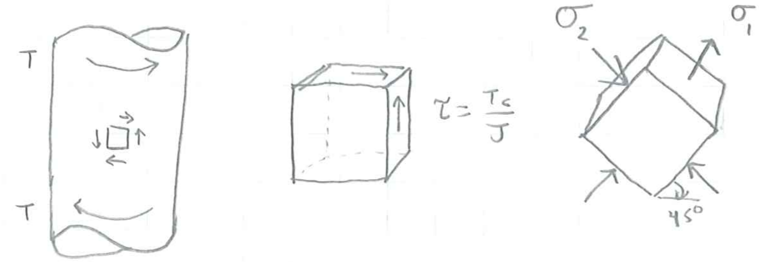
$\delta = \frac{NL}{EA}$



$Y = gap$

Notes 3-23

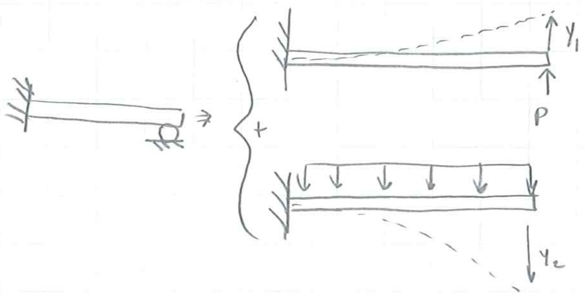




ductile metals: $\tau = \frac{Tc}{J}$ is determining factor
brittle metal: σ_2 & σ_1

Mohr's circle

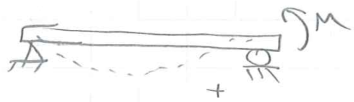
Notes 3-26

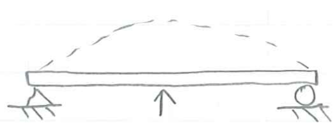


$$y = 0 = y_1 + y_2 = \frac{PL^3}{3EI} - \frac{wL^4}{8EI}$$

$$P = \frac{3wL}{8} = \frac{3}{8}\left(\frac{4\text{ kip}}{\text{ft}}\right)\left(\frac{1000\text{ lb}}{\text{kip}}\right)\left(\frac{\text{ft}}{12\text{ in}}\right)(13 \cdot 12)$$

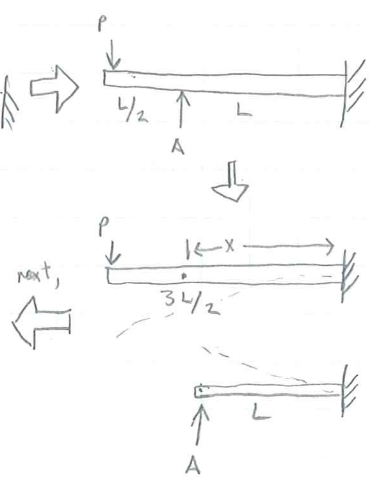
11.19)





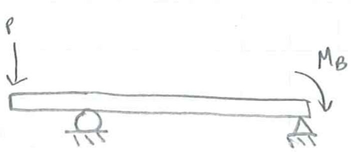
$$\Theta_B = 0 = \frac{-PL^2}{16EI} + \frac{ML}{3EI} \qquad M = \frac{3}{16}PL$$

11.24)



$$y_A = 0 = -\frac{PL^2}{6EI}\left[3\left(\frac{3L}{2}\right) - L\right] + \frac{AL^3}{3EI}$$

$$\Rightarrow A = \frac{P}{2L}\left[\frac{7L}{2}\right] = \frac{7P}{4}$$



$$* = \frac{\frac{PL}{2}(L)}{6EI} - \frac{M_B L}{3EI}$$

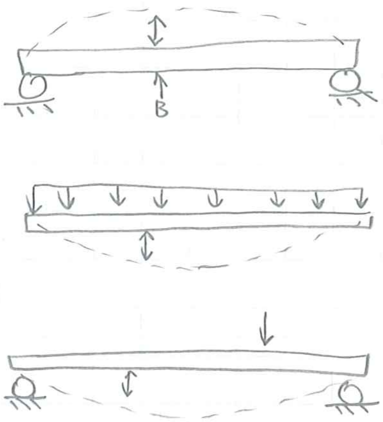
$$\boxed{M_B = -\frac{PL}{4}}$$

approach #1 $\Theta_B = 0 = *$

OR $\sum M_B \circlearrowleft = \frac{3L}{2}P - \frac{7PL}{4} - M_B = 0$

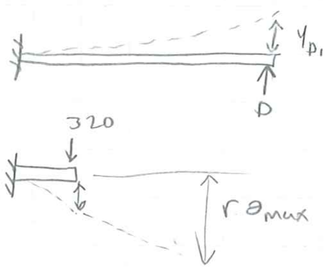
$$\boxed{M_B = -\frac{PL}{4}}$$

11.36)



$y_B = \underline{0} = +B - \_\_\_ - \_\_\_ \Rightarrow B = \_\_\_$

11.37)

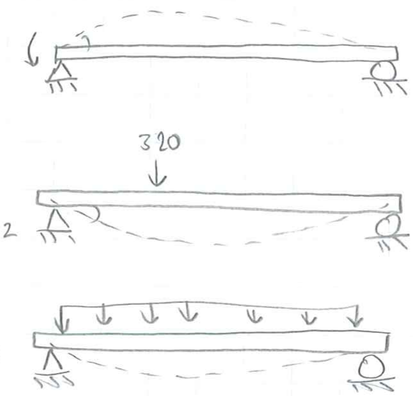


The easy way

θ_1

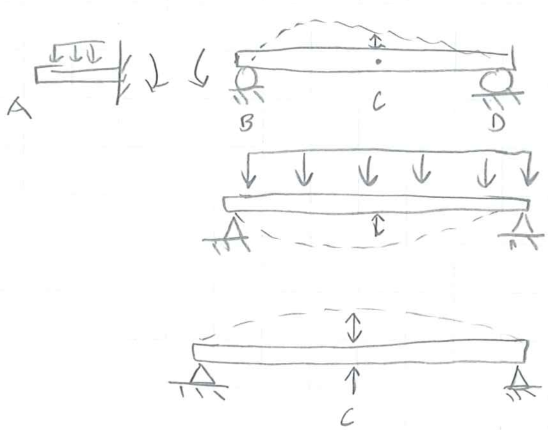
θ_2

θ_3



$\Rightarrow M_A = \_\_\_$

11.40)



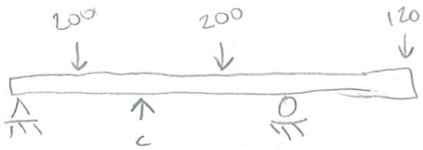
$Y_c = 0 = +\_\_\_ - \_\_\_ + \underline{C}$

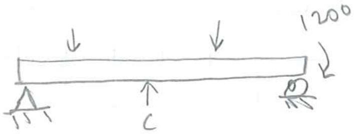
$C = \_\_\_$

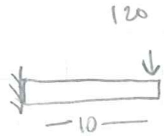
find deflection at A:

use $\underline{\text{cantilever}}$ + $\underline{3\ r\theta\text{'s}}$

11-44)





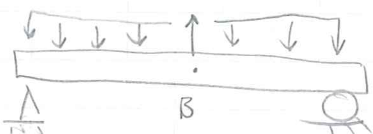


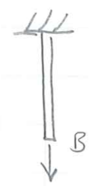
$$Y_c = 0 = \frac{C(60)^3}{48EI} - \left[\frac{200(15)(30)}{6(60)EI}\left(60^2 - 15^2 - 30^2\right)\right](2) + \frac{1200(30)}{6(60)EI}\left(2(60)^2 - 3(60)(30) + 30^2\right)$$

for both loads

$\Rightarrow C = 215$ lbs $E = 232.5$ $A = 72.5$

11-49)



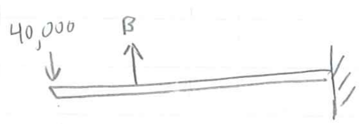


$$\delta_B = \frac{B(16)(12)}{E_s A_s}$$

$$\underset{\text{Beam}}{Y_B} = \underbrace{\frac{-B(16)(12)}{E_s A_s}}_{\text{rod}} = \underbrace{\frac{B[(14)(12)]^3}{48 E_w I_w} - \frac{5\left(\frac{900}{12}\right)(14\times 12)^4}{384 E_w I_w}}_{\text{beam}}$$

$\Rightarrow B =$ ____

11-50)



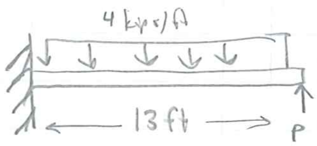
$$Y_B = \frac{-BL_r}{E_r A_r} = \frac{-40{,}000\ldots}{} + \frac{B\ldots}{}$$

11.52) $Y_B = \underline{-.015\,m}$

11 - 19, 21, 24, 36, 37, 40, 42 — Practice

19) $EI = 5\times10^6$ kip-in^2

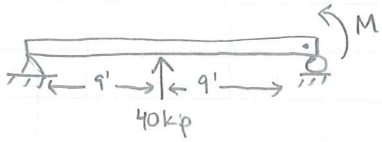
a)



$V_B = 0 = -\frac{wL^4}{8EI} + \frac{PL^3}{3EI} \Rightarrow \frac{wL^{\cancel{4}}}{8\cancel{EI}} = \frac{P\cancel{L^3}}{3\cancel{EI}}$

$P = \frac{wL}{8}(3) = \frac{4(13\cdot\cancel{12})(3)}{8} = \boxed{19.5 \text{ kips}}$

b) $\theta_c = 0$

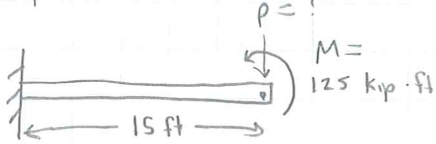


$-\frac{M\cancel{L}}{3\cancel{EI}} = -\frac{PL^{\cancel{2}}}{16\cancel{EI}}$

$M = \frac{40\text{kip}(18')(3)}{16} = \boxed{135 \text{ kip}\cdot\text{ft}}$

21) $EI = 8\times10^6$ kip-in^2

a) P = ?

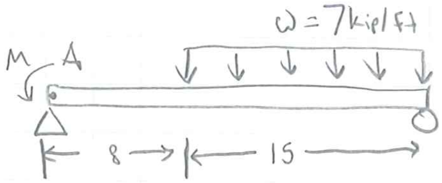


$V_B = 0$

$\boxed{P = 12.5 \text{ kip}}$

$-\frac{PL^{\cancel{3}}}{3\cancel{EI}} = -\frac{M\cancel{L^2}}{2\cancel{EI}}, \quad P = \frac{M}{2}\left(\frac{3}{L}\right) = \frac{125}{2}\left(\frac{3}{15}\right)$

b)

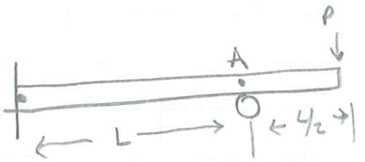


$\theta_A = 0 \quad a = 15 \quad L = 23 \quad w =$

$\frac{wa^2}{24L\cancel{EI}}(2L^2 - a^2) = \frac{ML}{3\cancel{EI}}$

$M\left(\frac{23}{3}\right) = 2376.8 \qquad \boxed{M = 310 \text{ kip}\cdot\text{ft}}$

24) $F_{A,B} = ?$



$V_A = 0 \quad L = 1.5l \quad x = l$

$\frac{P\cancel{l^2}}{6\cancel{EI}}(3(1.5l) - l) = \frac{F_A l^{\cancel{3}}}{3\cancel{EI}}$

$\frac{P}{6}(3.5\cancel{l}) = \frac{F_A \cancel{l}}{3} \qquad F_A = \frac{P}{6}3.5(3) = \boxed{1.75P\uparrow}$

$\boxed{F_B = .75P\downarrow}$

$+\circlearrowleft \Sigma M$

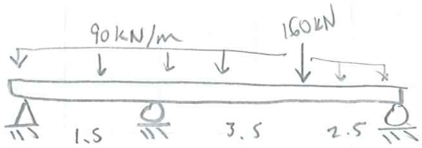
$M_B = M_A - M_P$

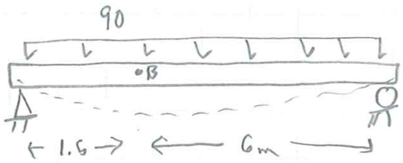
$M_B = 1.75pl - p\left(\frac{3l}{2}\right)$

$\boxed{M_B = .25pl \circlearrowleft}$

11) 36, 37, 40, 42

36) $EI = 2.24 \; 10^8$ GPA (m^4) $R_{A, B, D}$ W610 × 140

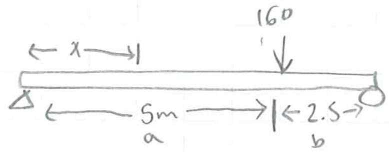




V_B $x = 1.5$ $L = 7.5$

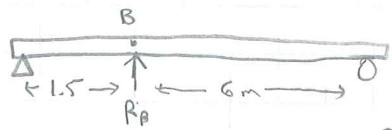
$$V_B = \frac{wx}{24EI}(L^3 - 2Lx^2 + x^3) = \frac{90(1.5)}{24EI}(7.5^3 - 2(7.5)(1.5)^2 + (1.5)^3)$$

$$= \frac{2202}{EI}$$



$a = 5$ $b = 2.5$ $L = 7.5$ $x = 1.5$

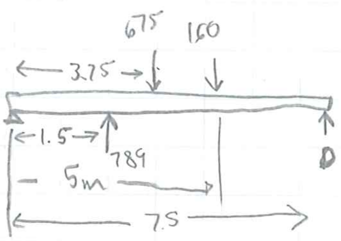
$$V_B = \frac{Pbx}{6LEI}(L^2 - b^2 - x^2) = \frac{160(2.5)(1.5)}{6(7.5)EI}(7.5^2 - 2.5^2 - 1.5^2) \quad V_B = \frac{637}{EI}$$



$a = 1.5$ $b = 6$ $L = 7.5$

$$\frac{R_B\, ba}{6LEI}(L^2 a^2 - b^2) = \frac{R_B\, 6(1.5)}{6(7.5)EI}(7.5^2 - 1.5^2 - 6^2)$$

$$\frac{3.6\, R_B}{\cancel{EI}} = \frac{2202}{\cancel{EI}} + \frac{637}{\cancel{EI}} \qquad R_B = 788.6 = \boxed{789 \text{ kN}}$$



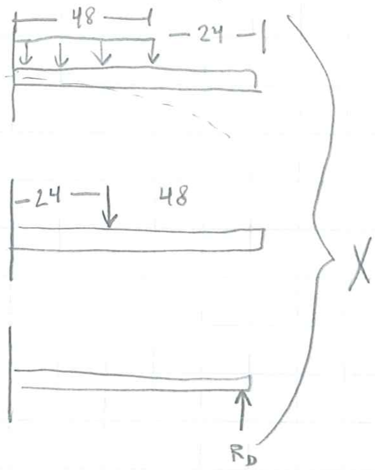
$\boxed{R_D = 286.4 \text{ kN}}$

$\boxed{R_A = 240 \text{ kN}}$

b) find maximum stress $\sigma_{beam} = \frac{My}{I}$ ...

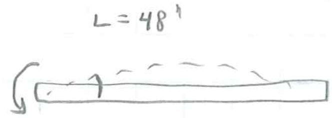
11-37)

$R_{A,D} = ?$ $EI = 12.8(10^6)\,lb\cdot in^2$



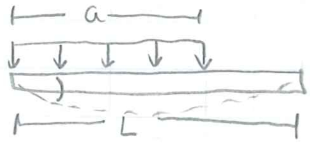
$L = 48'$

$\theta = 6$



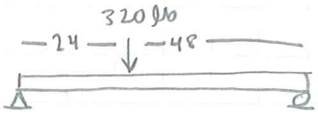
$\theta = \dfrac{ML}{3EI} = \dfrac{M}{EI}\left(\dfrac{72}{3}\right)$

$a = 48''$ $L = 72''$ $w = 20\,lb/in$



$\theta = \dfrac{wa^2}{24LEI}(2L-a)^2 = \dfrac{20(48)^2}{24(72)EI}(2(72)-48)$

$a = 24''$ $b = 48$



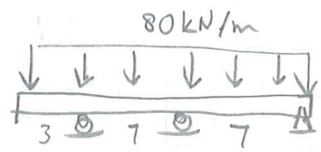
$\theta = \dfrac{Pb(L^2-b^2)}{6LEI} = \dfrac{320(48)}{6(72)EI}(72^2-48^2)$

$\dfrac{M}{\cancel{EI}}(24) = \dfrac{245760}{\cancel{EI}} + \dfrac{102400}{\cancel{EI}}$ $\boxed{M = 14507\,lb\cdot in}$

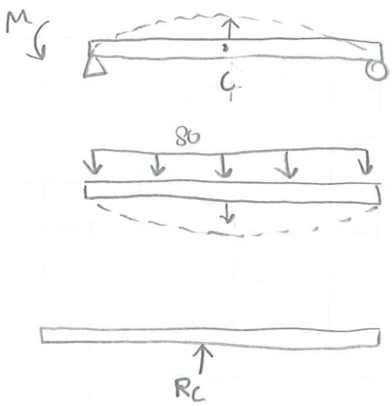
$14507 = 20(48)(24) + 320(24) + R_D(72)$ $\boxed{R_D = 225\,lb\uparrow}$

$20(48) + 320 - 225 = \boxed{R_A = 1055\,lb\uparrow}$

11-40) W610 x 82 R_C, $V_C = 0$



a)



$M = 3(80)(1.5) = 360\,kN\cdot m$ $x = 7'$ $L = 14'$

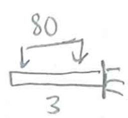
$V_C = \dfrac{Mx}{6LEI}(2L^2 - 3Lx + x^2) = \dfrac{360(7)}{6(14)EI}(2\cdot 14^2 - 3\cdot 14\cdot 7 + 7^2)$

$= 4410/EI \uparrow$

$V_C = \dfrac{5wL^4}{384EI} = \dfrac{5(80)(14)^4}{384EI} = \dfrac{40017}{EI}\downarrow$

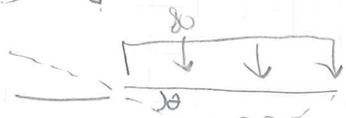
$V_C = \dfrac{PL^3}{48EI} = \dfrac{R_C(14)^3}{48} = 40{,}017 - 4410$ $\boxed{R_C = 623\,kN}$

b)

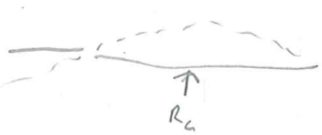


$\dfrac{wL^4}{8EI} = \dfrac{80(3)^4}{8EI}$ (1) $\downarrow$ 810

$\theta R = 3\left(\dfrac{ML}{3EI}\right) = \dfrac{3\ 360(14)}{3EI}$ (2) $\downarrow$



$\theta R = 3\left(\dfrac{wL^3}{24EI}\right) = 3\left(\dfrac{80(14)^3}{24\ EI}\right)$ (3) $\uparrow$

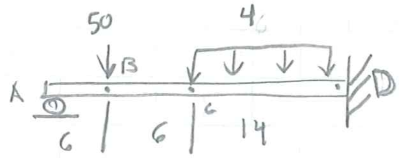


$\theta R = 3\left(\dfrac{PL^2}{16EI}\right) = 3\left(\dfrac{623(14)^2}{16\ EI}\right)$ (4) $\downarrow$

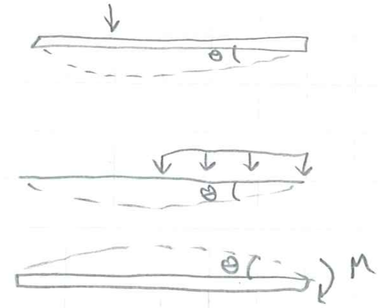
$-\dfrac{810}{EI} - \dfrac{5040}{EI} + \dfrac{27440}{EI} - \dfrac{22895}{EI} \approx \delta_A = \dfrac{-1305}{EI} = \boxed{11.61\,mm}$

$EI = 200(10^9)\,Pa \cdot 562(10^{-6})\,m^4 \Rightarrow Pa\cdot m^4$

11-42



$R_{A,D} = ?$

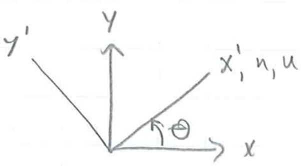


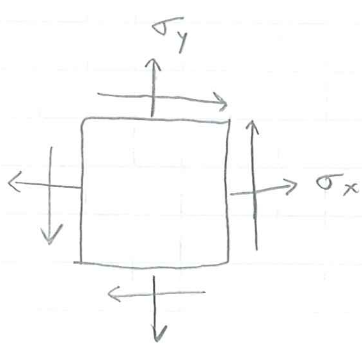
Notes

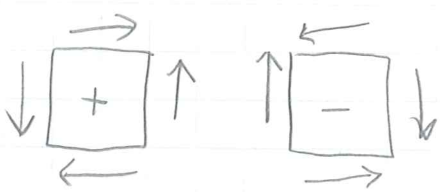
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \;'' \; - \; '' \; - \; ''$$

$$\tau_{x'y'} = \;'' \; - \; '' \sin 2\theta + \; '' \cos 2\theta$$







12.9) $\theta = -40°$

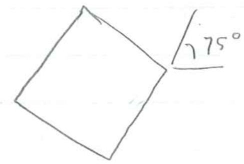
$\sigma_x = 3208$

$\sigma_y = 2792$

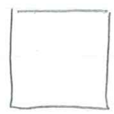
$\tau_{xy} = +1182$

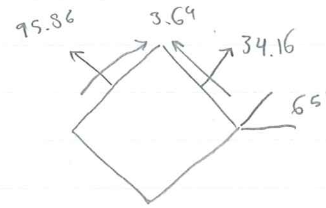
12.13)

$\theta = 75$

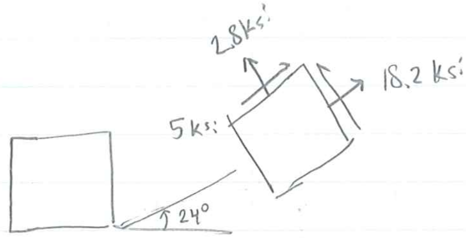


12- 14)





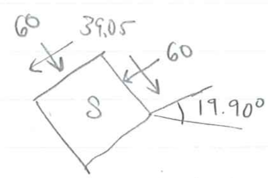
12- 24)



$\theta = -24°$ $\sigma_x = 11.94$ $\sigma_y = 9.06$ $\tau_{xy} = 9.07$

Principals & max shear

12- 29)



$\sigma_x = -60$
$\sigma_y = -60$
$\theta_s = 19.90°$
$\tau_{xy} = -39.05$
Mpa

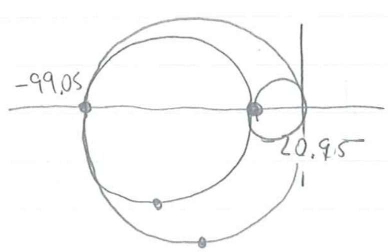
max

principles Mpa
$\tau_{xy} = 0$
$\theta_p = 25.10$
$\sigma_x = -20.95$
$\sigma_y = -99.05$

$$\boxed{\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)}$$

$$\boxed{\theta_s = \theta_p \pm 45°}$$

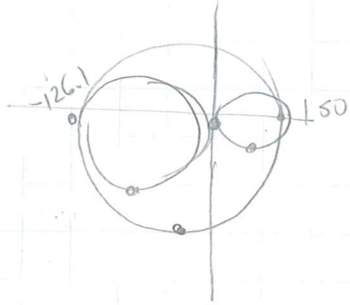
absolute maximum shear stress



abs. max shear stress = radius of big circle
in plane max shear stress = radius of medium circle

12-30

31)



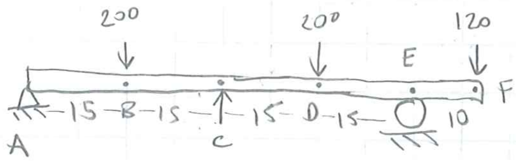
12-46) wont be on quiz → uses shortcut formulas

-69) not on quiz

4-7-10

Homework 11-44 ✓, 46 ✓, 49

44) $R_{A,C,E}$



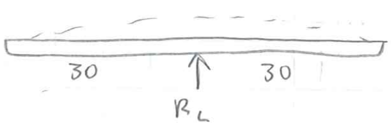
$V_c = \frac{Mx}{6LEI}(2L^2 - 3Lx + x^2)$

$\frac{270,000}{EI} \leftarrow = \frac{1200(30)}{6(60)EI}(2(60)^2 - 3(60)(30) + 30^2)$

(2) 

$b = 15 \quad x = 30 \quad L = 60$

$V = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2) = -\frac{200 \cdot 15 \cdot 30}{6(60)EI}(60^2 - 15^2 - 30^2)(2) = \frac{1,237,500}{EI}$



$V_c = \frac{PL^3}{48EI} = \frac{P(60)^3}{48EI} = \frac{P\,4500}{EI}$

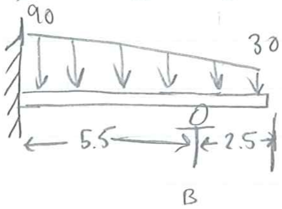
$R_c(4500) = 1,237,500 - 270,000 \qquad \boxed{R_c = 215\,lb}$

$\Sigma M_A = 200(1) - C(2) + 200(3) - E(4) + 120(4.66) = 0 \qquad \boxed{R_E = 232.3\,lb}$

$R_A = 520 - (215 + 232.5) \qquad \boxed{R_A = 72.5}$

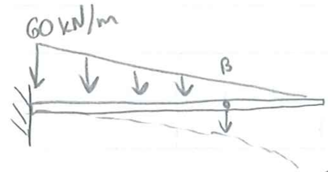
b) max stress?

11-46) R_a & R_B = ?



$x = 5.5 \quad L = 8$

(1)

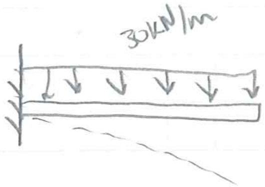


$$V_B = \frac{W_o x^2}{120LEI}\left(10L^3 - 10L^2x + 5Lx^2 - x^3\right)$$

$$= \frac{60(5.5)^2}{120(8)EI}\left(10(8)^3 - 10\cdot 8^2(5.5) + 5(8)(5.5)^2 - (5.5)^3\right)$$

$$= \frac{4998.1}{EI}$$

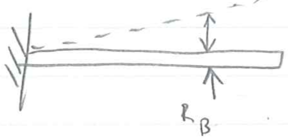
(2)



$$V_B = \frac{wx^2}{24EI}\left(6L^2 - 4Lx + x^2\right)$$

$$= \frac{30\cdot 5.5^2}{24EI}\left[6(64) - 4\cdot 8\cdot 5.5 + (5.5)^2\right] = \frac{9008.8}{EI}$$

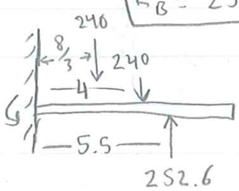
(3)



$$V_B = \frac{R_B L^3}{3EI} = \frac{R_B(5.5)^3}{3EI} \Rightarrow 55.46\,R_B = 4998 + 9009$$

$$\boxed{R_B = 252.6 \text{ kN}}$$

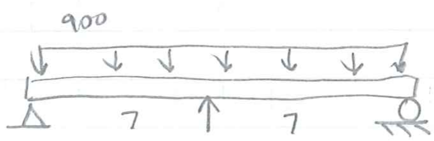
ΣM



$$\boxed{M = 210.7 \text{ kN}\cdot\text{m}} \qquad \boxed{R_a = 227.4 \text{ kN}}$$

11-49

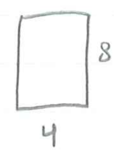
$EI = 3.0726E6$



$$V_B = \frac{5wL^4}{384EI} = \frac{5(900)(14.12)^4}{384EI} = \frac{450188}{EI}$$

$$\text{rod } V_B = \frac{PL^3}{48EI} = \frac{P\,57.17}{EI} \qquad A_{rod} = .19635$$

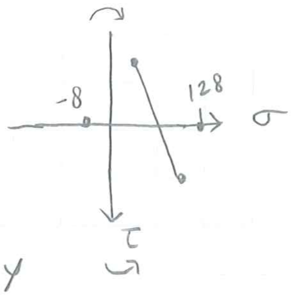
$$P\frac{57.7}{EI} - \frac{450188}{EI} = \delta = \frac{NL}{EA} = \frac{P\,14}{EA}$$



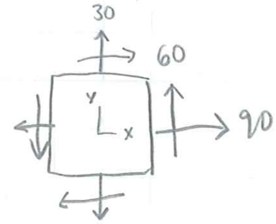
$$I = \frac{1}{12}(4)(8)^3 = 170.7$$

$F_1 = 7150$ lb

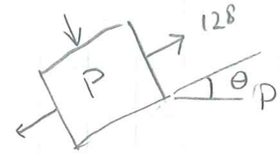
12.48



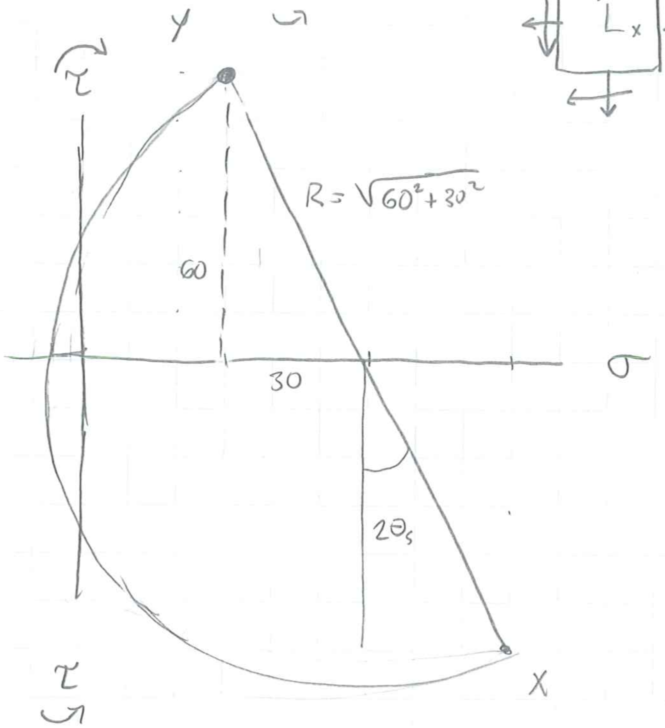
$\sigma_x = 90$
$\sigma_y = 30$
$\tau_{xy} = 60$



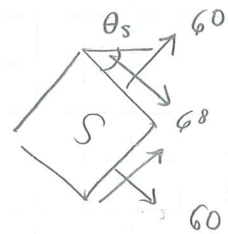
$\sigma_{max} \approx 128 = C + R$
$\sigma_{min} \approx -8 = C - R$



1 grid square = 10ksi



$\tau_{max,\ in\ plane} = R \approx 68$

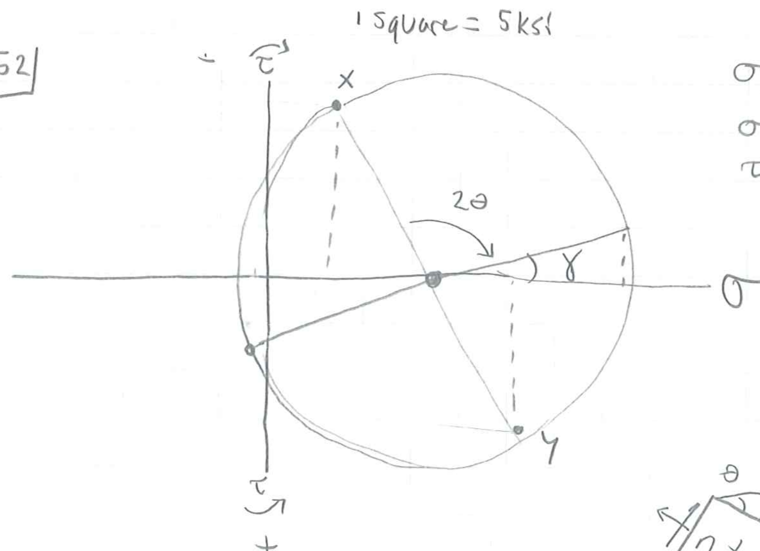


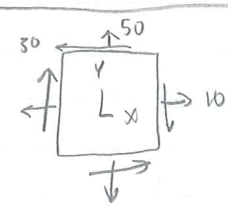
Next Quiz a few detailed Ch. 11 problems
several little Ch 12. problems

Mohr's Circle
X: (σ_x, τ_{xy})
Y: $(\sigma_y, -\tau_{xy})$
$(30, -60)$

12.52

1 square = 5ksi

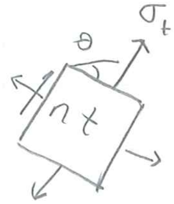




$\sigma_x = 10$
$\sigma_y = 50$
$\tau_{xy} = -30$

$R = \sqrt{20^2 + 30^2}$

$\theta = \frac{1}{2}(180 - \tan^{-1}(\ ) - \tan^{-1}(\ ))$
$= 53.87$



$\sigma_t = C - R\cos\gamma$

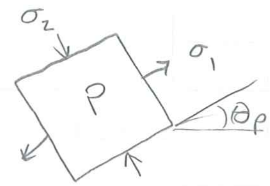
$\sigma_n = 30 + R\cos\gamma$

$\boxed{12.57}$

$\sigma_x = 6$
$\sigma_y = 18$
$\tau_{xy} = 30$
$X : (6, 30)$
$Y : (18, -30)$

$\sigma_{max} = 12 + R = \sigma_1$
$\sigma_{min} = 12 - R = \sigma_2$
$\theta_p = \frac{1}{2}\left(180 - \tan^{-1}\left(\frac{30}{6}\right)\right)$

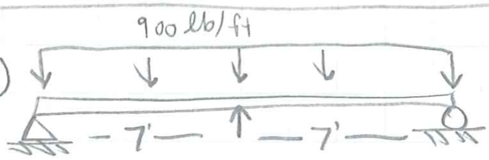
$R = \sqrt{6^2 + 30^2}$



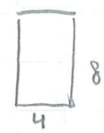
$\tau_{max,\ in\ plane} = R$

$\sigma_{avg} = 12$

11-49)

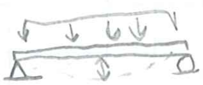


$\frac{5wL^4}{384EI}$



Practice

$E = 1800\,ksi \quad I = \frac{1}{12}4(8)^3 = 170.7\,in^4 \quad EI = 307200\ ksi \cdot in^4 = 3.072\,E8\ lb \cdot in^2$



$$\frac{5wL^4}{384EI} = \frac{5\,(900\,lb/ft)\left(\frac{1\,ft}{12\,in}\right)(14\,ft)^4\left(\frac{12\,in}{1\,ft}\right)^4}{(384)\,3.072\,E8\ lb \cdot in^2} = 2.532 \text{ inches}$$

$$= \frac{PL^3}{48EI} = \frac{P\,(14\,ft)^3(12\,in/ft)^3}{48(3.072\,E8)\ lb \cdot in^2} = P(.322\,E^{-3}\,in)/lb \quad \checkmark$$

$$\frac{NL}{EA} = \frac{P(16\,ft)(12\,in/ft)}{30E6\ lb/in^2\,(.19635)\,in^2} = P\,(3.259\ E{-5}\ in)/lb \quad \checkmark$$

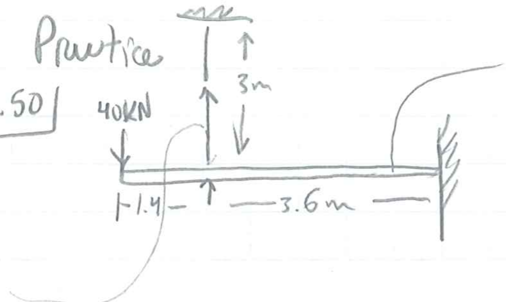
(1.5) in

$2.532\,in - P(.322\,E{-3})\,in = P(3.259\,E{-5})\,in$

$\boxed{P = 7140.64\ lb}$ ✓

Max beam deflection = $\boxed{.2326\ in}$ ✓

Practice



$\underline{11.50}$

$E = 200$ GPa $\quad$ GN/m^2 $\quad I = 201(10^6)\,mm^4 = 201(10^{-6})\,m^4$ $\quad EI = 4.02\,E7\ N\cdot m^2 = 4.02\,E4\ kN\cdot m^2$

$E = 70$ GPA
$= 70\,E6$ kPA
$= 70\,E6\ kN/m^2$

$$v_1 = \frac{Px^2}{6EI}(3L - x) = \frac{40kN(3.6m)^2}{6(4.02E4)kN\cdot m^2}(3.5 - 3.6)m = .0245\ m \downarrow$$

$$v_2 = \frac{PL^3}{3EI} = \frac{P(3.6m)^3}{3(4.02E4\,kN\cdot m^2)} = P\,.387E\text{-}3\ m \uparrow$$

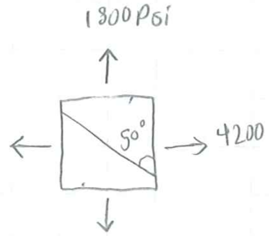
$$\frac{NL}{EA} = \frac{P(3m)}{70E6\ kN/m^2 (3.142E^{-4})m^2} = P\ 1.364\,E\text{-}4\ m$$

P in kN: $\quad .0245 - P\,.387E\text{-}3 = P\,1.364E\text{-}4 \quad \boxed{P = 46.81\ kN}$

deflection @ B: $= .0245m - P(.387\,E\text{-}3) = \boxed{6.385\ mm} \downarrow$

12- 9, 13, 14, 19

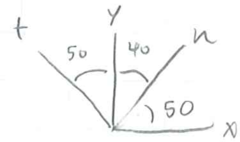
12.9)



$\sigma_x = 4200$

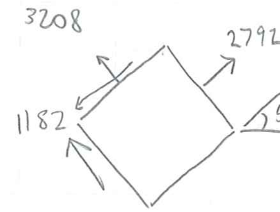
$\sigma_y = 1800$

$\tau_{xy} = 0$



$\theta = 50$

$\sigma_x =$



$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta = \boxed{2792 \text{ psi}}$

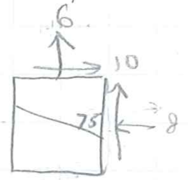
$4200 \cos^2 50 + 1800 \sin^2\theta + 0$

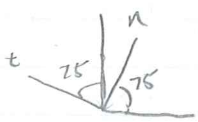
$\tau_{nt} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$

$-(4200 - 1800)\sin 50 \cos 50 + 0 = \boxed{-1182 \text{ psi}}$

$\sigma_t = \sigma_y \cos^2\theta + \sigma_x \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta = 3208$

12.13)





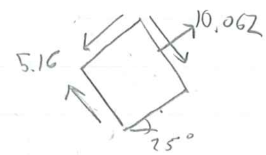
$\sigma_x = -8$

$\sigma_y = 6$

$\tau_{xy} = 10$

$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$

$= 10.062 \text{ ksi } t$

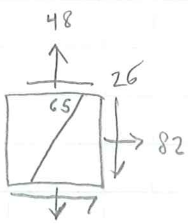


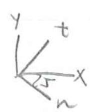
$\tau_{xy} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$

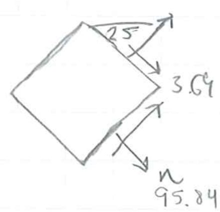
$= -(-14)\sin 75 \cos 75 + 10(\cos^2 75 - \sin^2 75)$

$= -5.16$

12.14)







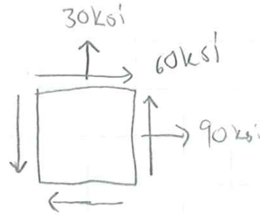
$\sigma_n = 82\cos^2(-25) + 48\sin^2(-25) + 2(-26)\sin(-25)\cos(-$

$= \boxed{95.84}$

$\tau_{nt} = -(82-48)\sin(65)\cos(65) + -26(\cos^2 65 - \sin^2 65)$

$= \boxed{3.69}$

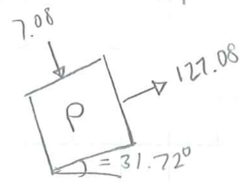
Principals., Max in plane



12-48) $\sigma_x = 90$ ksi

$\sigma_y = 30$ ksi 6 ± 6.708 $\sigma_{avg} = C = 60$ ksi

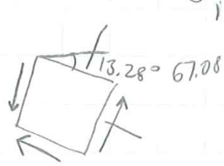
$\tau_{xy} = \cancel{+20}\ 60$ 12.708, $-.708$ $\tau_{max} = 67.08$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)} = \frac{2(60)}{(90-30)}$$

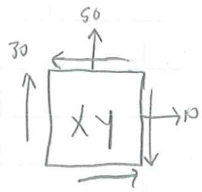
$$\theta_p = 31.72°$$

$$\sigma_{1,2} = \frac{90+30}{2} \pm \text{radius} = 60 \pm 67.08$$



$\sigma = \sigma_{avg} = C = 60$ ksi

nt = Principals

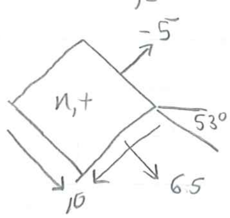


12-52) $\sigma_x = 2(5) = 10$ ksi $\tau_{xy} = 6(5) = -30$ ksi

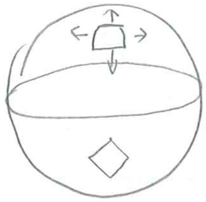
$\sigma_y = 50$ ksi

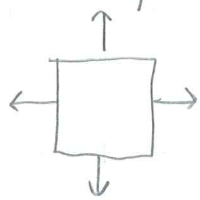
$$\theta_p = .5\left[\tan^{-1}\left(\overset{7/2}{\frac{12}{7}}\right) + \tan^{-1}\left(\overset{4/6}{\frac{6}{4}}\right)\right] = \cancel{24.82°}$$ Guide: $-53.87°$

$\sigma_{1,2} = 30 \pm \text{radius} = 30 \pm 7.28 = 37.28$ or



Notes 4-14-10



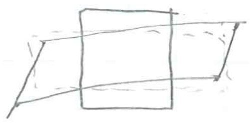


$\sigma_y = \frac{pr}{2t}$

$\sigma_x = \frac{pr}{2t}$

$\tau_{xy} = 0$

Pressure Vessel



$+\epsilon_x$

$-\epsilon_y$

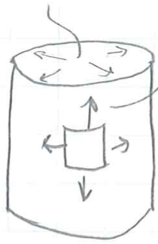
$+\gamma_{xy}$ change in angle

p = gage pressure, relative to 1atm

outside pressure = 0

inside pressure

$\sigma_z = -p$

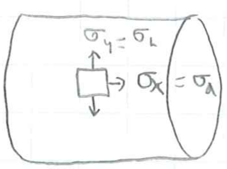


$\frac{pr}{2t}$

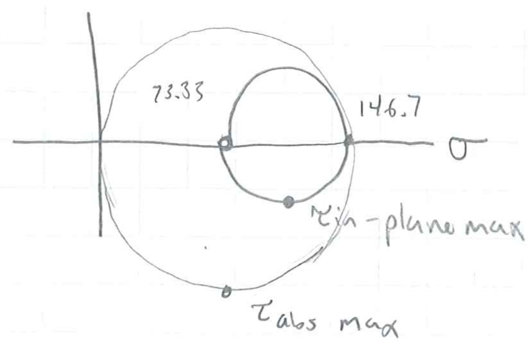
$\sigma_x = \frac{pr}{t}$

σ_{radial}

14.6



$\theta_p = 45°$



$\sigma_{sphere} = \frac{pr}{2t}$

$\sigma_{cyl,\ axial} = \frac{pr}{2t}$

$\sigma_{cyl,\ hoop} = \frac{pr}{t}$

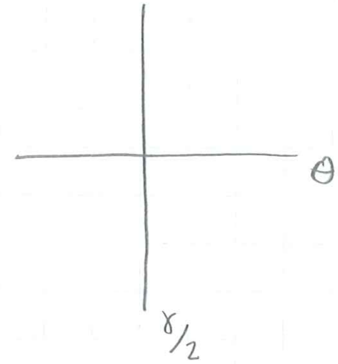
$\sigma_{radial,\ outside} = 0$

$\sigma_{radial,\ inside} = -p$

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \quad (1)$$

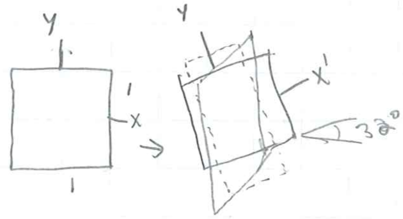
$$\epsilon_y = \quad '' \quad - \quad '' \quad - \quad '' \quad (2)$$

$$\frac{\gamma_{x'y'}}{2} = \quad '' \quad - \quad '' \quad \sin 2\theta + \quad '' \quad (3)$$

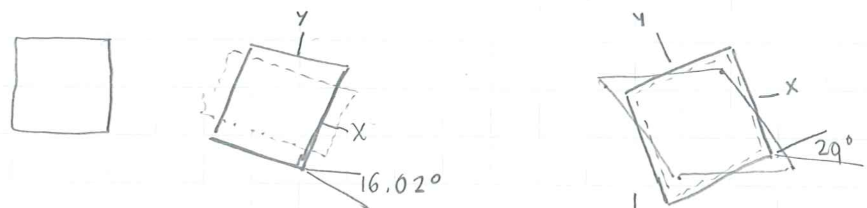


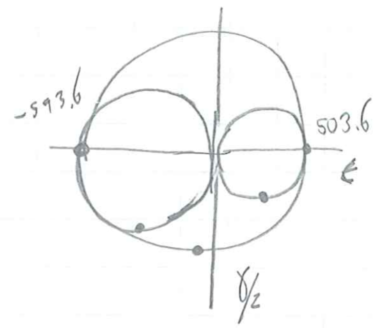
$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}\right)$$

$$\theta_s = \theta_p \pm 45°$$

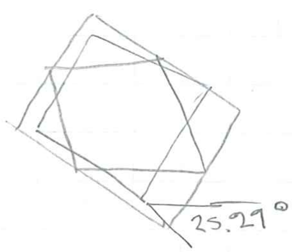


13.11)

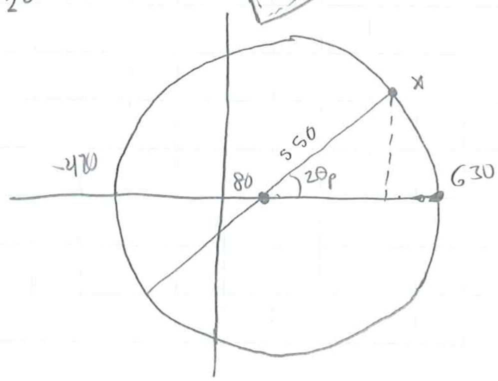




13.18)



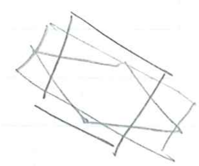
13-23)



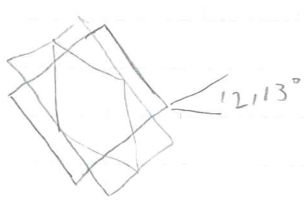
$C = \frac{630 - 470}{2} = 80$

Un-rotate 20.16° x 2

$\epsilon_1 = 630\mu \qquad \epsilon_2$



13.27)



13.39)

550
ϵ_a with eq. above (1), using θ_a

730
ϵ_b = eq. (1) using θ_b (positive 45°)(2)

$$\underset{-375}{\cancel{\epsilon_c}} = = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos \overset{+180}{\cancel{2\theta_c}} + \frac{\gamma_{xy}}{2}\sin \overset{180°}{\cancel{2\theta_c}}$$

$$= \frac{\epsilon_x}{2} + \frac{\epsilon_y}{2} + \frac{\epsilon_y}{2} - \frac{\epsilon_y}{2} = \epsilon_x$$

$$\sigma_x = \frac{E(\epsilon_x - \nu \epsilon_y)}{1-\nu^2} \qquad \epsilon_x = \frac{\sigma_x - \nu \sigma_y}{E} = \frac{\Delta AB}{b}$$

$$\sigma_y = \frac{E(\epsilon_y - \nu \epsilon_x)}{1-\nu^2} \qquad \epsilon_z = \frac{-\nu(\sigma_x + \sigma_y)}{E} = \frac{\Delta t}{t}$$

$$\tau_{xy} = G\gamma_{xy} = \frac{E\gamma_{xy}}{2(1+\nu)}$$

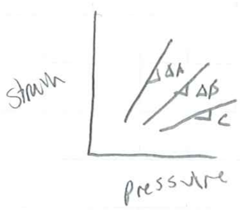
1) Strain gages $\rightarrow \epsilon_x, \epsilon_y, \gamma_{xy}$
2) Hookes law $\rightarrow \sigma_x\ \sigma_y, \tau_{xy}$
3) θ_p $\quad \sigma_1, \sigma_2, 0$
4) θ_s $\quad \sigma_{avg}\ \sigma_{avg}, \tau_{in\text{-}plane\ max}$
5) abs max

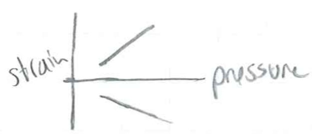
Lab #11

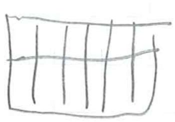
Team Worksheet

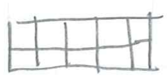
P1 — thin walled

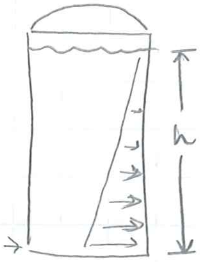
Page 2 — thick walled



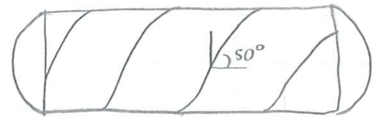


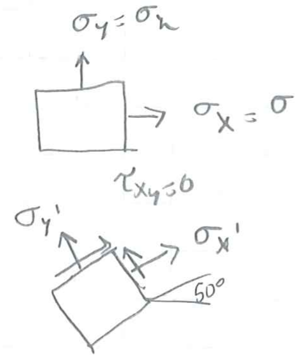




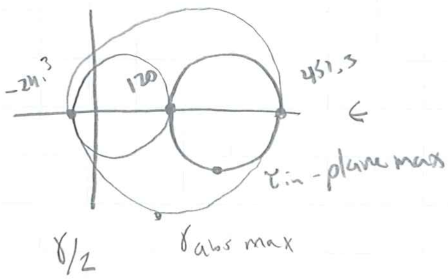


$p = \gamma h$
$= \rho g h = 10.4\,psi$

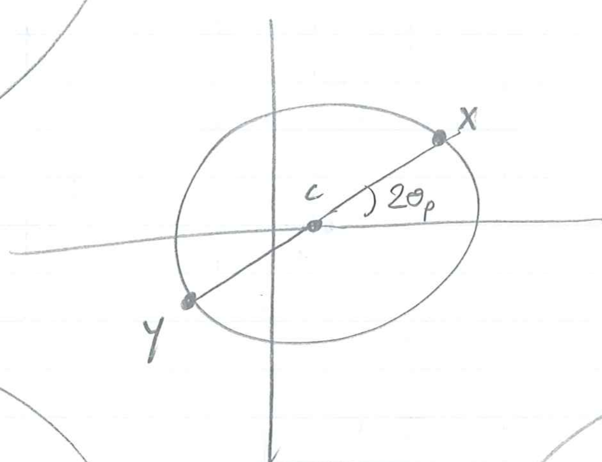




14.14)



$$\epsilon_z = -\frac{\sigma_x + \sigma_y}{E} = \frac{\Delta t}{t}$$



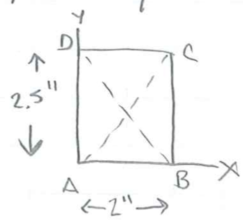
point x below center $\Rightarrow$ principal axis (1) is θ_p CCW

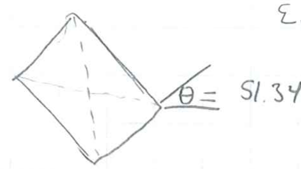
X above C, P_1 is θ_p CW

13.2, 9, 11, 18, 23, 27, 31, 39

2) $\varepsilon_x = -475\,\mu\varepsilon \quad \varepsilon_y = 750\,\mu \quad \gamma_{xy} = -1320\,\mu rad$

a) $\varepsilon_{AC} = ?$





$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$

$-475\cos^2 51.34 + 750\sin^2 51.34 + 2(-1320)\sin 51.34\cos 51.34$

$\boxed{\varepsilon_{AC} = -372\,\mu\varepsilon}$ b) strain $\varepsilon_{BD} = ?$ same, but -51.34

$\boxed{\varepsilon_{BD} = 916\,\mu\varepsilon}$

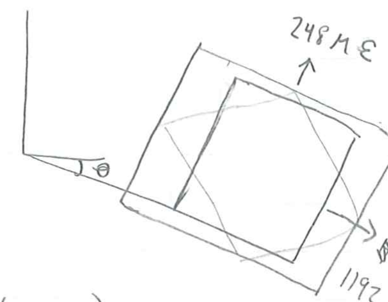
18) plane strain $\varepsilon_x = 1020\,\mu\varepsilon \quad \varepsilon_y = 420\,\mu\varepsilon \quad \gamma_{xy} = -730\,\mu rad$

$\varepsilon_{principal}$, $\varepsilon_{max(inplane)}$ abs. max shear strain

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 720 \pm 472 = \boxed{\varepsilon_2 = 248\,\mu\varepsilon \quad \varepsilon_1 = 1192\,\mu\varepsilon}$$

$\gamma_{max} = 2(472) = \boxed{944\,\mu rad}$

$\tan 2\theta_p = \dfrac{\gamma_{xy}}{(\varepsilon_x - \varepsilon_y)}$ $\boxed{\theta_p = -25.29}$ $\therefore$ θ_p is clockwise



$\theta_s = 19.71°$ $\quad \varepsilon_x = \varepsilon_y = 720$

$\gamma_{xy} = -944\,\mu rad$

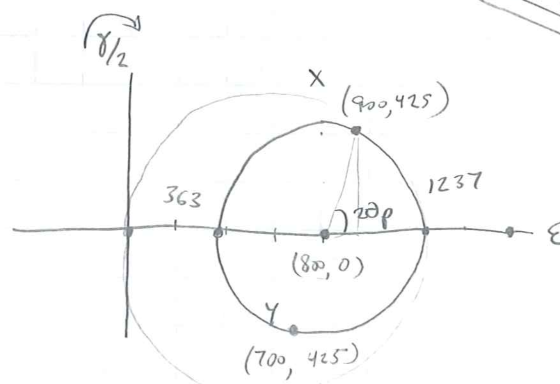
31) plane strain

ε_x	ε_y	γ_{xy}
$900\,\mu\varepsilon$	$700\,\mu\varepsilon$	$-850\,\mu rad$

$r = 436.6 = \sqrt{\left(\frac{900-700}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$

$c = 800\,\mu\varepsilon$ $\quad 425 = \dfrac{\gamma_{xy}}{2}$

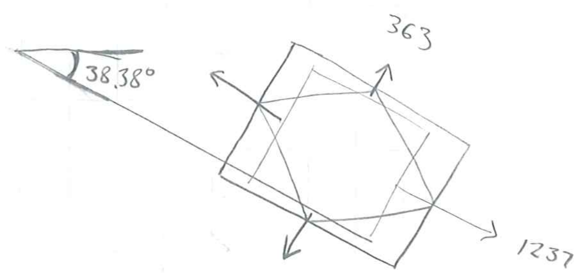
Max in-plane $= \boxed{1237\,\mu\varepsilon}$



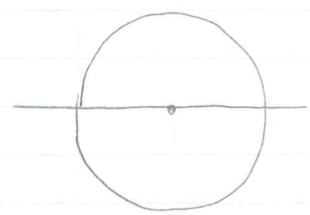
$\theta = \tan^{-1}\left(\frac{425}{100}\right) = 38.38°$

$\varepsilon_p = 800 \pm (436.6)$

$\varepsilon_1 = 1237 \quad \varepsilon_2 = 363\,\mu\varepsilon$

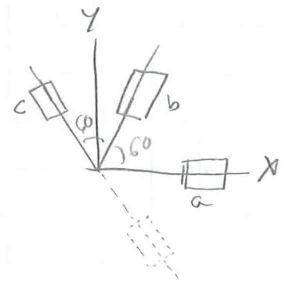


- $\varepsilon_x, \varepsilon_y, \gamma_{xy}$
- principal strains max inplane shear strain
- sketch



When strain gauges are given,
θ_p is reversed

13.42, 45, 49, 52, 57 70, 76



13.42)

ε_a	ε_b	ε_c	ν
-1320	-840	-215	$.33$

$\varepsilon_x, \varepsilon_y, \gamma_{xy} = ?$

$\boxed{\varepsilon_x = -1320\ \mu\varepsilon}$

$$-840 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$-840 = \frac{-1320 + \varepsilon_y}{2} + \frac{-1320 - \varepsilon_y}{2}\underbrace{\cos 120}_{(-.5)} + \frac{\gamma_{xy}}{2}\underbrace{\sin 120}_{.866}$$

$$-215 = \frac{-1320 + \varepsilon_y}{2} + \frac{-1320 - \varepsilon_y}{2}(-.5) + \frac{\gamma_{xy}}{2}(-.866)$$

$\boxed{\varepsilon_x = -1320\mu\varepsilon}$

$$-1055 = -1320 + \varepsilon_y \ (-)\left(\frac{-1320 - \varepsilon_y}{2}\right)$$

$$-1055 = -1320 + \varepsilon_y + 660 + \frac{\varepsilon_y}{2}$$

$$-395 \ \cancel{-915} = \frac{3}{2}\varepsilon_y$$

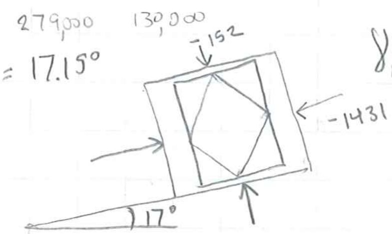
$\boxed{\varepsilon_y = -263.3\ \mu\varepsilon}$ $\boxed{\gamma_{xy} = -721.7\ \mu}$

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = -792 \pm 640$$

$\boxed{\varepsilon_1 = -152\ \mu\varepsilon}$ $\boxed{\varepsilon_2 = -1431\ \mu\varepsilon}$

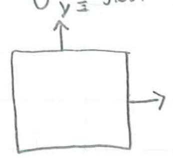
279,000 130,000

$$\tan(2\theta_p) = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \qquad \theta_p = 17.15°$$



$\gamma_{max, in\ plane} = 2(640) = \boxed{1280\ \mu\varepsilon}$

52)



$\varepsilon_x = +950\ \mu\varepsilon \quad \varepsilon_y = 335\ \mu\varepsilon \qquad \nu = ? \quad E = ?$

$$950 = \frac{(8ksi - \nu\, 5ksi)}{E} \qquad 335 = \frac{5 - \nu(8)}{E}$$

$$950E = 8 - 5\nu \qquad 335E = 5 - 8\nu \qquad \nu = .3494 \qquad E = 6.582\ \cancel{(10^{-3})}\ (10^6)$$

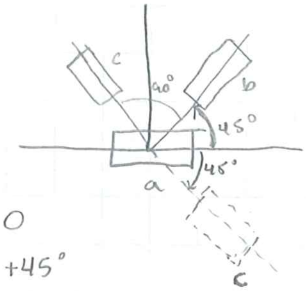
~~70~~)

$$\varepsilon_u = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_u = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

13-76) $\varepsilon_x, \varepsilon_y, \gamma_{xy}$? $\varepsilon_p, \varepsilon_{max}$ in plane

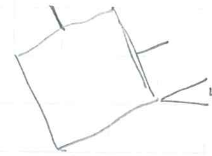
ε_a	ε_b	ε_c	E	ν
$-400\mu\varepsilon$	-240	-1280	212 GPa	.3



$\theta_a = 0$

$\theta_b = +45°$

$\theta_c = -45°$



$\varepsilon_u = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\theta + \gamma_{xy}/2\left(\sin 2\theta\right)$

$-400 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cancel{\cos 0}^{1} + 0$

$-240 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cancel{\cos(-90)}^{0} + \frac{\gamma_{xy}}{2}\cancel{\sin(-90)}^{-1}$

$-1280 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cancel{\cos(90)}^{0} + \frac{\gamma_{xy}}{2}\cancel{\sin 90}^{1}$

$-1920 = \frac{3}{2}(\varepsilon_x + \varepsilon_y) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)$

$-400\mu\varepsilon = \varepsilon_x \cos^2\theta + \varepsilon_y \cancel{\sin^2\theta}^{0} + \gamma_{xy}\cancel{\sin\theta}^{0}\cos\theta$

$\boxed{\varepsilon_x = -400\mu\varepsilon}$

$-240\mu\varepsilon = -400\cos^2 45 + \varepsilon_y \sin^2 45 + \gamma_{xy}\sin 45\cos 45$

$-40 = \frac{\varepsilon_y}{2} + \gamma_{xy}/2$

$-1280 = \frac{-400}{2} + \frac{\varepsilon_y}{2} - \frac{\gamma_{xy}}{2}$

$-1080 = \frac{\varepsilon_y}{2} - \frac{\gamma_{xy}}{2}$

$\boxed{\varepsilon_y = -1120}$ $\boxed{\gamma_{xy} = 1040}$

$\varepsilon_p = -760 \pm \sqrt{360^2 + 520^2}$

$\boxed{\varepsilon_1 = -128}$

$\boxed{\varepsilon_2 = -1392}$

$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{(1040)}{-400+1120}\right)$

$\tan 2\theta_p = \left(\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}\right)$ $\boxed{\theta_p = 27.65° \text{ ccw}}$ $\gamma_{max} = 2\left[\sqrt{\left(\frac{x-y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}\right] = \boxed{1265}$

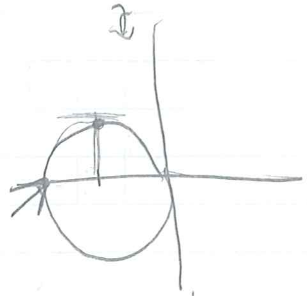
$\boxed{\theta_s = -17.35°}$

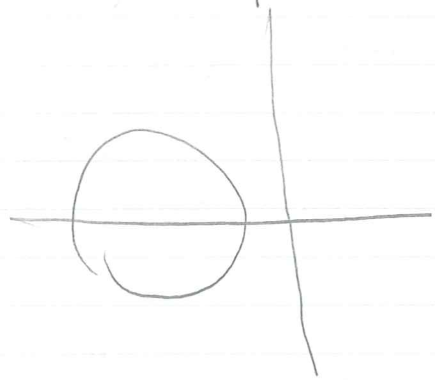
$\sigma_1, \sigma_2, \tau_{max} = ?$

$\sigma_x = \frac{E(\varepsilon_x + \nu\varepsilon_y)}{1-\nu^2} = \frac{212(10^9)(-128 + .3(-1392))}{1-(.09)} = -1.27(10^8) = 127.$

$\sigma_y = \ldots$ $\underline{333\ E6}$

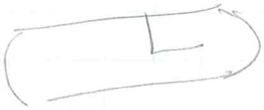
$\tau_{max} = \tau_{xy} = \frac{E\gamma_{xy}}{2(1+\nu)} = 103$ $\boxed{\tau_{max} = 103}$





14-6) OD = .2 m $\quad t = .012\,m \quad p = 20\,MPa \quad r = $ .1 -

$\sigma_{hoop} = \frac{pr}{t} = \frac{20\,E6\left(\frac{.2}{2} - .012\right)}{.012\,m}$ $\quad 1.467\ 10^8\ Pa = 146.7\ MPa$



$\sigma_{longditude} = 73.3\ MPa \qquad \sigma_x, \sigma_y\ \tau_{xy}$

$\sigma_x = 73.3\ MPa = P_2$
$\sigma_y = 146.7\ MPa = P_1$
$\tau_{xy} = 0$

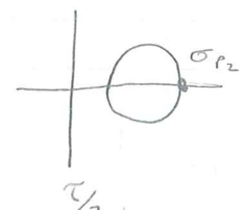
Max shear stress $\tau_{xy} = \frac{146.7 - 73.3}{2} = 36.67$

Max shear in plane

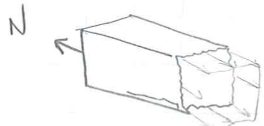
abs. max shear stress $\Rightarrow$ $\tau_{abs,\ outside} = \tau_{abs,\ inside}$

$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{73.3}{2}$

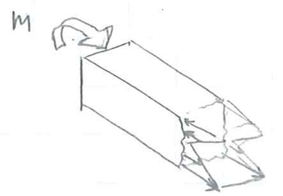
σ_{P_1} & σ_{P_2} are positive, $\quad \tau_{abs\ max} = \frac{\sigma_{P_1}}{2}$



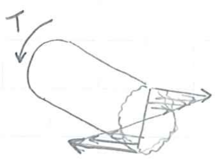
Notes 4-30



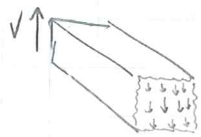
$\sigma = \frac{N}{A}$



$\sigma = \frac{My}{I}$?

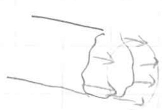


$\tau = \frac{Tc}{J}$

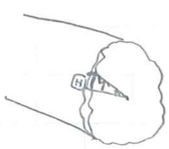


$\tau = \frac{VQ}{Ib}$

15.4)

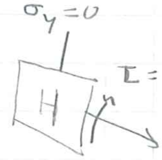


$\sigma_x = \frac{P}{A}$



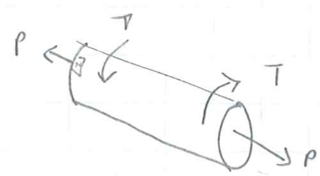
$\tau = \frac{Tc}{J}$

$\sigma_y = 0$



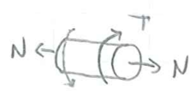
$\tau = \frac{Tc}{J} = \tau_{xy}$

$\sigma_x = \frac{P}{A}$



c goes wherever stress element is. (usually outside surface of shaft)

15.6)



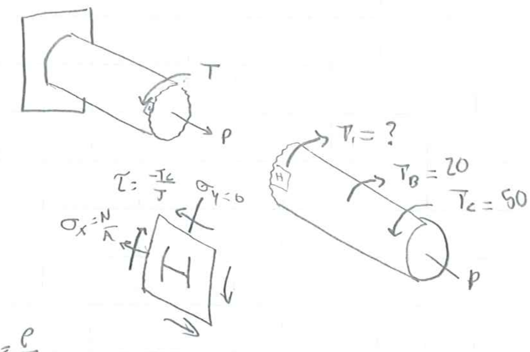
$P = T\omega$

$T = \frac{100{,}000\ W \left(\frac{J/s}{W}\right)\left(\frac{Nm}{J}\right)}{1600\ \frac{rev}{min}\left(\frac{2\pi\,rad}{rev}\right)\left(\frac{min}{60\,s}\right)}$

15.10

$\overset{+}{\curvearrowleft} \Sigma M_x = T + T_B - T_c = 0$

$\Sigma M_{x\text{-axis}} = T_B - T_c \qquad T = 30$



15.16 $\sigma_x =$

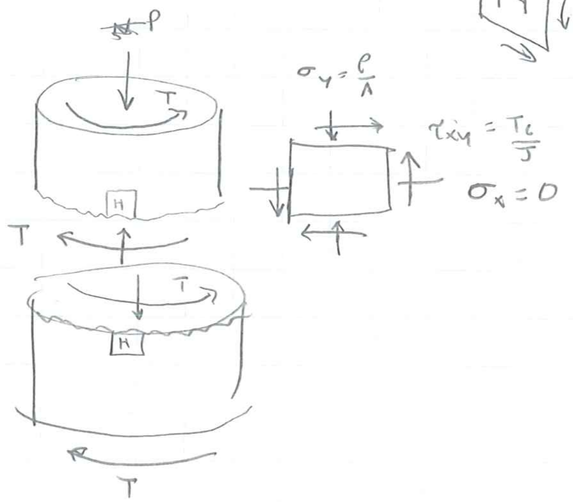
a

b 0 ✓

c

d

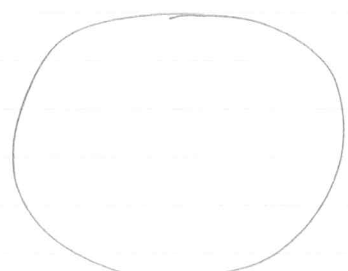
e



15.18 N Will not be on Quiz

10:45 program yar calc for shear strains

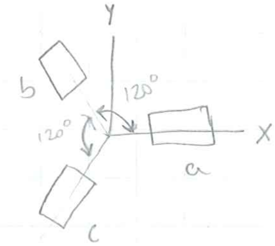
$\frac{Pr}{2t}$: r= inner radius



F505 Problem #1

$\varepsilon_a = -555\,\mu\varepsilon$ a) principal strains?
$\varepsilon_b = 925\,\mu\varepsilon$ b) max shear strain
$\varepsilon_c = 740\,\mu\varepsilon$ c) θ to max tensile strain

$\theta_a = 0$
$\theta_b = -30^\circ$
$\theta_c = 30^\circ$



ε_p)

$$-555 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(0) + \cancel{\tau_{xy}\sin 0}^{\,0}$$

$$925 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(-60) + \cancel{\tau_{xy}\sin(-60)}$$

$$740 = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(60) + \cancel{\tau_{xy}\sin(60)} \Big\} 0$$

$$(\varepsilon_x + \varepsilon_y) + \frac{(\varepsilon_x - \varepsilon_y)(1)}{2}$$

$$925+740 = (\varepsilon_x) + \varepsilon_y + \frac{\varepsilon_x}{2} - \frac{\varepsilon_y}{2} \qquad 1.5\,\varepsilon_x + .5\,\varepsilon_y = 1665$$

$$\varepsilon_u = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta \quad \gamma_{xy}\cos\theta\sin\theta$$

$$925 = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \cancel{\gamma_{xy}\sin\theta\cos\theta}$$

$$740 = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \cancel{\gamma_{xy}\sin\theta\cos\theta}$$

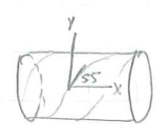
$$1665 = \frac{3}{2}\varepsilon_x + \frac{1}{2}\varepsilon_y + \qquad \varepsilon_y = 1665 \qquad 1387.5 \pm \gamma_{xy}$$

$\varepsilon_1 = 1301$ $\quad \varepsilon_2 = -561$ $\quad \varepsilon_y = 4995$

S507 Prob 6)

$$\sigma_x = \frac{Pr}{2t} = \frac{1.25^{E^3}(.3875\,m)}{2(.0025\,m)}$$

OD 780 mm
wall 2.5 mm
P = 1.25 MPa

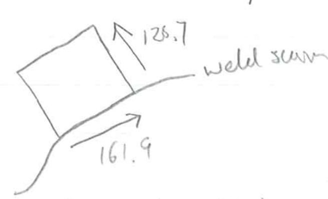


OK ✓

$\sigma_x = 96.875$ MPa
$\sigma_y = 193.75$ MPa

$$\varepsilon_n = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) - \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos(2\theta) - \cancel{\frac{\gamma_{xy}}{2}}^{\,0}\sin(2\theta) \qquad \theta = 55, \qquad \gamma_{xy} =$$

$$\sigma_1 = 145.3 - (\cancel{-48.44})^{16.567} = \boxed{128.7}\ \text{MPa}$$



$$\sigma_2 = \qquad + \qquad = \cancel{\boxed{161.9}\ \text{MPa}}$$

~~τ_{xy} =~~ τ shear stress parallel to weld seams $= \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin(2\theta)$

$$\boxed{\tau = 45.52\ \text{MPa}}$$

Exams/Quiz

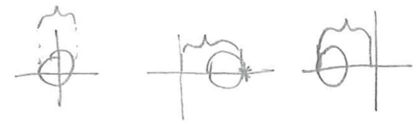
— Mec. Mat Quiz 7 prep —

$$\varepsilon_u = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

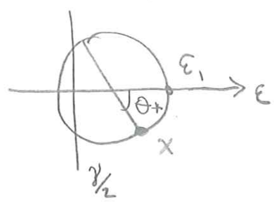
$$\gamma_{\text{max in plane}} = 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 2R$$

$\gamma_{\text{abs max}}$ = $|\text{largest } \varepsilon_p|$ or $|\varepsilon_1 - \varepsilon_2|$ if opposite signs



= radius of large circle (x 2 to go from ε to γ)

moore's circle θ positive, x is on bottom, ccw rotation from x to principal



$P = g\rho h$

$\tau_{xy} = 0$ for pressure vessel

τ_{xy} corresponds to $\frac{\gamma_{xy}}{2}$

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\cos\theta \sin\theta_a$$

$$\cancel{\varepsilon_x \cos^2\theta - \varepsilon_y \sin^2\theta} - \gamma_{xy}\cos\theta\sin\theta$$

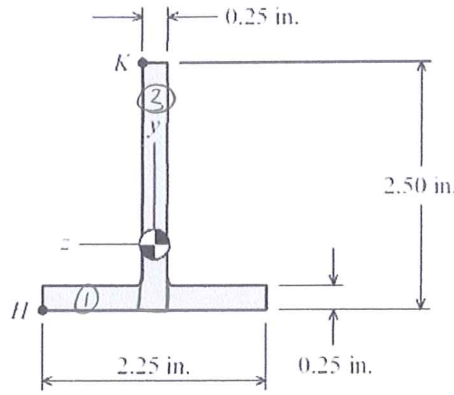
Quiz 4 practice

 Name: ____________________

1. A cross section of a beam is shown. For any moment M about the z-axis, which point will have the greater bending stress magnitude (either tension or compression)?

   __X__ Point K

   _____ Point H



2. If the beam in Problem 1 is subjected to a negative moment about the z-axis, which bending stress will have the greater magnitude?

   _____ Compression bending stress

   __X__ Tension bending stress

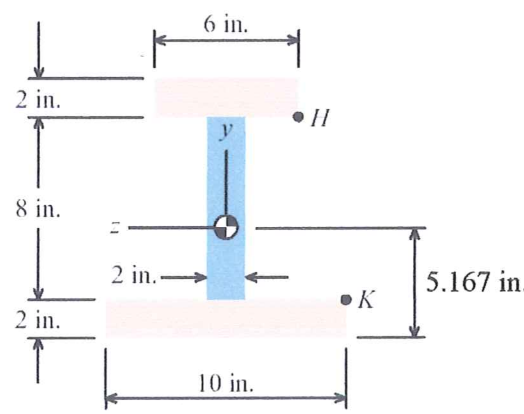
3. Determine the distance from the bottom of the cross section in Problem 1 to the centroid.

   y from bottom = __.75__ in.

$$\frac{\overset{A_1}{2(.25)}\overset{\bar{y}}{(.125)} + \overset{A_2}{(2.5)(.25)}\overset{\bar{y}}{(1.25)}}{(4.5)(.25)}$$

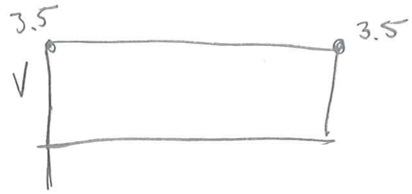
4. Determine the moment of inertia about the z-axis for the cross section shown. Note that the centroid is 5.167 in. from the bottom.

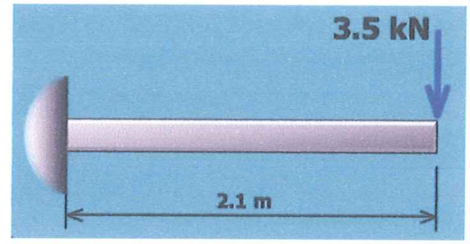
   I_z = ________ in.^{4}



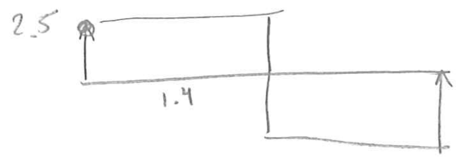
5. Determine the maximum bending moment in the following beams.

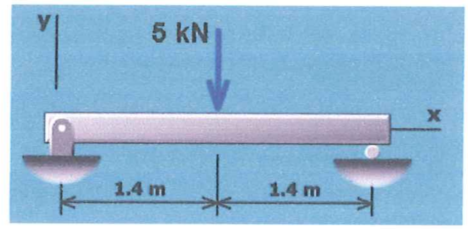
M_{max} = 7.35 kN-m



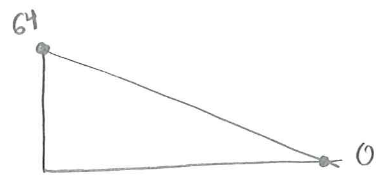


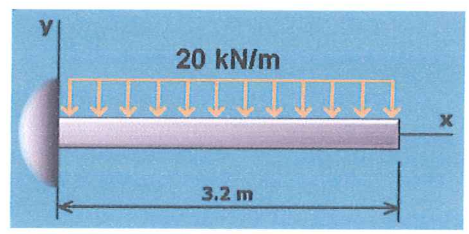
M_{max} = 3.5 kN-m



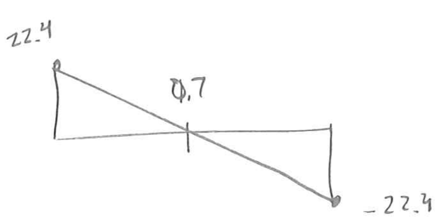


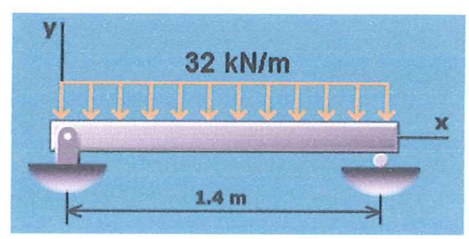
M_{max} = 102.4 kN-m



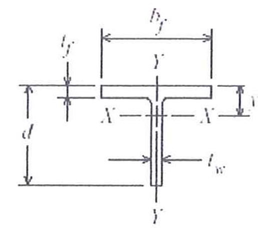


M_{max} = 7.84 kN-m





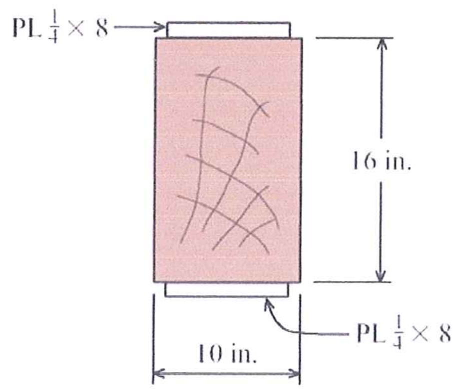
9. Circle the most economical WT beam in the following table if a section modulus $S_x \geq 8$ in.^{3} is required.



Shapes Cut from Wide-Flange Sections or WT Shapes

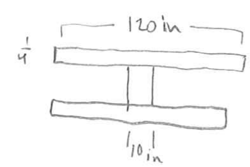
Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	Centroid $\bar{y}$	I_x	S_x	r_x	I_y	S_y	r_y
	in.^{2}	in.	in.	in.	in.	in.	in.^{4}	in.^{3}	in.	in.^{4}	in.^{3}	in.
WT12×47	13.8	12.2	0.515	9.07	0.875	2.99	186	20.3	3.67	54.5	12.0	1.98
WT12×38	11.2	12.0	0.440	8.99	0.680	3.00	151	16.9	3.68	41.3	9.18	1.92
WT12×34	10.0	11.9	0.415	8.97	0.585	3.06	137	15.6	3.70	35.2	7.85	1.87
WT12×27.5	8.10	11.8	0.395	7.01	0.505	3.50	117	14.1	3.80	14.5	4.15	1.34
WT10.5×34	10.0	10.6	0.430	8.27	0.685	2.59	103	12.9	3.20	32.4	7.83	1.80
WT10.5×31	9.13	10.5	0.400	8.24	0.615	2.58	93.8	11.9	3.21	28.7	6.97	1.77
WT10.5×25	7.36	10.4	0.380	6.53	0.535	2.93	80.3	10.7	3.30	12.5	3.82	1.30
WT10.5×22	6.49	10.3	0.350	6.50	0.450	2.98	71.1	9.68	3.31	10.3	3.18	1.26
WT9×27.5	8.10	9.06	0.390	7.53	0.630	2.16	59.5	8.63	2.71	22.5	5.97	1.67
WT9×25	7.33	9.00	0.355	7.50	0.570	2.12	53.5	7.79	2.70	20.0	5.35	1.65
WT9×20	5.88	8.95	0.315	6.02	0.525	2.29	44.8	6.73	2.76	9.55	3.17	1.27
WT9×17.5	5.15	8.85	0.300	6.00	0.425	2.39	40.1	6.21	2.79	7.67	2.56	1.22
WT8×28.5	8.39	8.22	0.430	7.12	0.715	1.94	48.7	7.77	2.41	21.6	6.06	1.60
WT8×25	7.37	8.13	0.380	7.07	0.630	1.89	42.3	6.78	2.40	18.6	5.26	1.59
WT8×20	5.89	8.01	0.305	7.00	0.505	1.81	33.1	5.35	2.37	14.4	4.12	1.56
WT8×15.5	4.56	7.94	0.275	5.53	0.440	2.02	27.5	4.64	2.45	6.2	2.24	1.17

10. Two ¼ in.×8 in. steel [E = 30,000 ksi] plates are securely attached to a pine [E = 2,000 ksi] timber to form a composite beam. Determine the maximum bending stress magnitude in the steel if a moment of 100 kip-ft is applied about the horizontal axis of the beam.



$$n = \frac{E_{st}}{E_t} = 15$$

$\sigma_{\text{max steel}}$ = 20.135 ksi

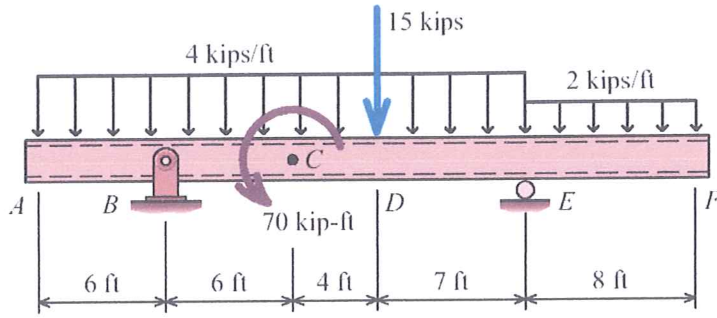


$$I = 2\left[\frac{(.25)^3(120)}{12} + (8.125)^2(.25)(120)\right] + \frac{(10)(16)^3}{12}$$

$$I = 7375 \text{ in}^4$$

$$\frac{My}{I}(n) = \frac{100000(12)(8.25)(15)}{7375}$$

8. A HSS10×4×1/2 standard steel shape is used to support the loads shown on the beam. The shape is oriented so that bending occurs about the strong axis. Determine the magnitude of the maximum bending stress in the beam. Note that the shear-force and bending-moment diagrams have been provided in kips and kip-ft.

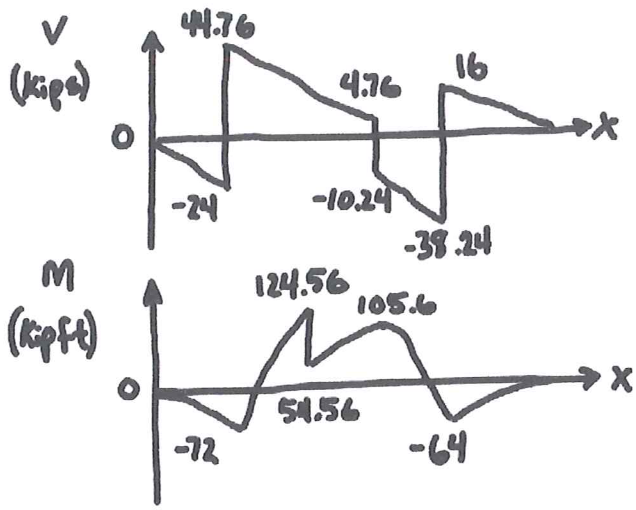


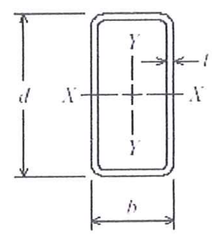
$\sigma_{max} =$ 57.93 ksi

Max bending stress

$$= \frac{My}{I} = \frac{124.56 \text{ kip}\cdot\text{ft}\,(12)(5")}{129 \text{ in}^4}$$

$\sigma_{max} = 57.93$ ~~psi~~ ksi

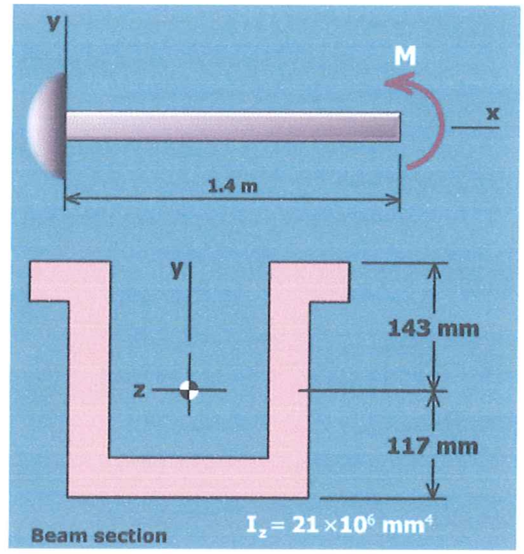




Hollow Structural Sections or HSS Shapes

Designation	Depth d	Width b	Wall thickness (nom.) t	Weight per foot	Area A	I_x	S_x	r_x	I_y	S_y	r_y
	in.	in.	in.	lb/ft	in.^{2}	in.^{4}	in.^{3}	in.	in.^{4}	in.^{3}	in.
HSS12×8×1/2	12	8	0.5	62.3	17.2	333	55.6	4.41	178	44.4	3.21
×8×3/8	12	8	0.375	47.8	13.2	262	43.7	4.47	140	35.1	3.27
×6×1/2	12	6	0.5	55.5	15.3	271	45.2	4.21	91.1	30.4	2.44
×6×3/8	12	6	0.375	42.7	11.8	215	35.9	4.28	72.9	24.3	2.49
HSS10×6×1/2	10	6	0.5	48.7	13.5	171	34.3	3.57	76.8	25.6	2.39
×6×3/8	10	6	0.375	37.6	10.4	137	27.4	3.63	61.8	20.6	2.44
×4×1/2	10	4	0.5	41.9	11.6	129	25.8	3.34	29.5	14.7	1.59
×4×3/8	10	4	0.375	32.5	8.97	104	20.8	3.41	24.3	12.1	1.64
HSS8×4×1/2	8	4	0.5	35.1	9.74	71.8	17.9	2.71	23.6	11.8	1.56
×4×3/8	8	4	0.375	27.4	7.58	58.7	14.7	2.78	19.6	9.80	1.61
×4×1/4	8	4	0.25	19.0	5.24	42.5	10.6	2.85	14.4	7.21	1.66
×4×1/8	8	4	0.125	9.85	2.70	22.9	5.73	2.92	7.90	3.95	1.71
HSS6×4×3/8	6	4	0.375	22.3	6.18	28.3	9.43	2.14	14.9	7.47	1.55
×4×1/4	6	4	0.25	15.6	4.30	20.9	6.96	2.20	11.1	5.56	1.61
×4×1/8	6	4	0.125	8.15	2.23	11.4	3.81	2.26	6.15	3.08	1.66
×3×3/8	6	3	0.375	19.7	5.48	22.7	7.57	2.04	7.48	4.99	1.17
×3×1/4	6	3	0.25	13.9	3.84	17.0	5.66	2.10	5.70	3.80	1.22
×3×1/8	6	3	0.125	7.30	2.00	9.43	3.14	2.17	3.23	2.15	1.27

6. For the beam shown, the allowable compression bending stress is 60 MPa, and the allowable tension bending stress is 70 MPa. Determine the maximum value of *M* that can be applied as shown to the beam.



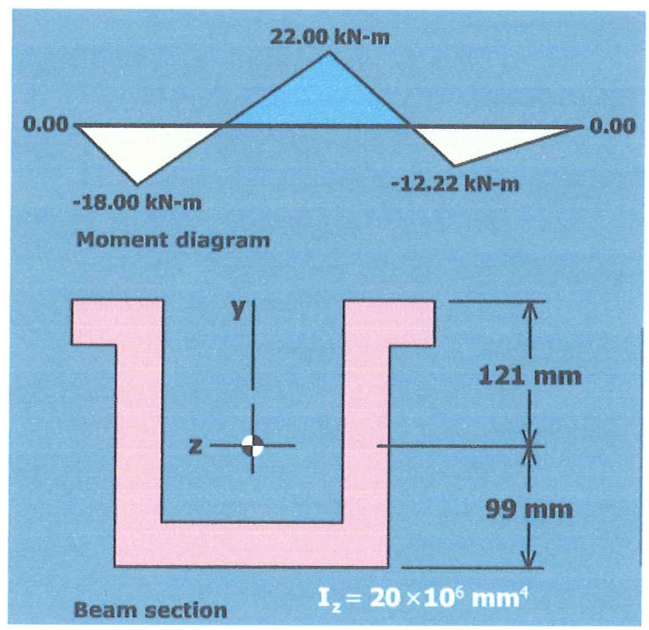
M = 8.811 kN-m

$$\sigma_t = \frac{My}{I} > 70\text{MPa}$$

$$\frac{M\,.117}{21(10^{-6})} > 70 \qquad M < 12.564$$

$$\sigma_c = \frac{M\,.143}{21(10^{-6})} > 60 \qquad M < 8.811$$

7. For the moment diagram and cross section shown, compute the maximum tension and compression bending stresses produced at any location along the beam span.



σ max tension = 108.9 MPa

σ max compression = 133.1 MPa

$$\frac{\begin{pmatrix} 22,000 \\ -18,000 \end{pmatrix}\begin{pmatrix} .121 \\ -.099 \end{pmatrix}}{20(10^{-6})\ m^4}$$

$$22 \begin{cases} 133.1 \\ -109.0 \end{cases}$$

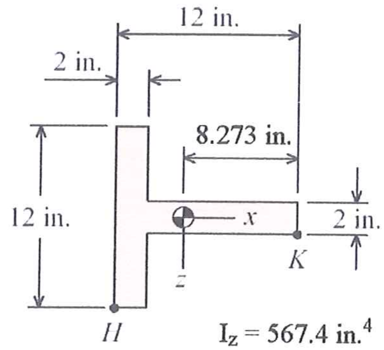
$$-18 \begin{cases} -108.9 \\ 89.1 \end{cases}$$

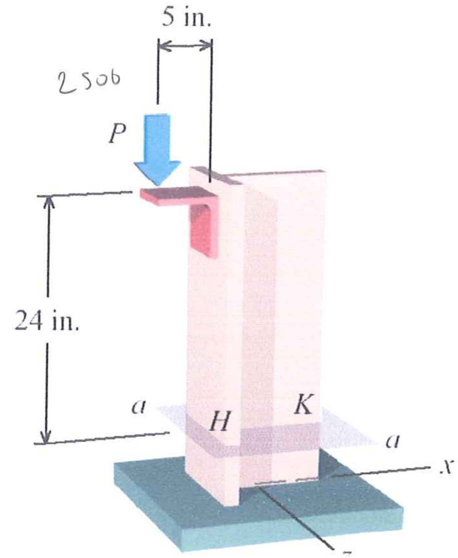
11. The tee shape is used as a short post to support a load of $P = 2{,}500$ lb. The load P is applied at a distance of 5 in. from the surface of the flange. Determine the normal force and bending moment located at section *a-a*. Also determine the magnitude of the bending stress at point *K*. Note that the centroid location and moment of inertia are provided.

$N =$ 2500 lb

$M_z =$ 21,800 lb-in.

$\sigma_K =$ 261.04 psi (t)

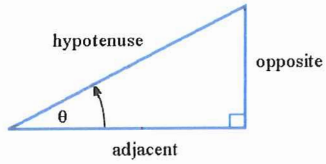




$$\sigma_K = \frac{N}{A} + \frac{My}{I}$$

$$\frac{2500 \text{ lb}}{44 \text{ in}^2} + \frac{-21{,}800\,(8.273)}{567.4}$$

TRIGONOMETRY

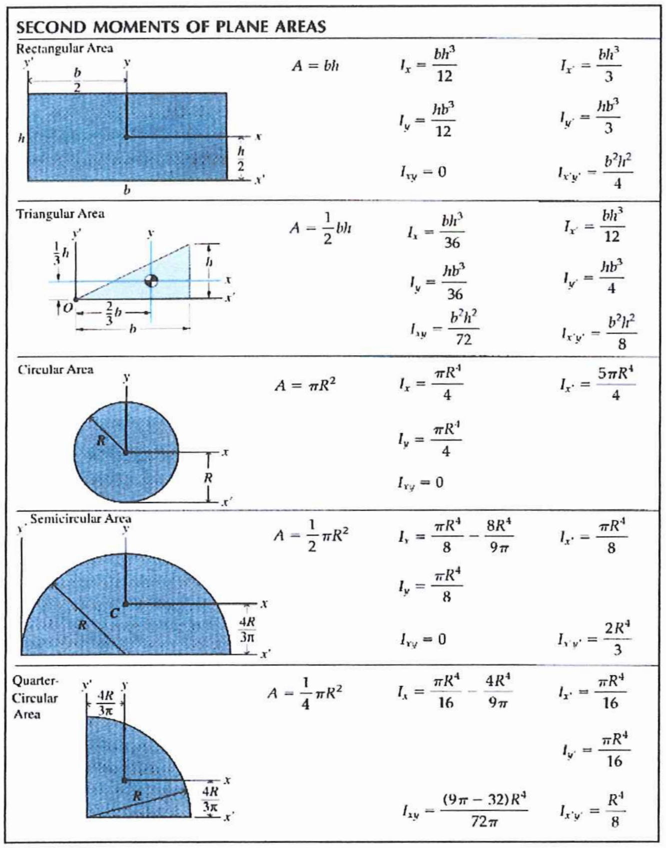


$\sin\theta = \text{opp} / \text{hyp}$
$\cos\theta = \text{adj} / \text{hyp}$
$\tan\theta = \text{opp} / \text{adj}$

1 hp = 550 ft.lb/s
power through gears stays same (Torque x rpm)

STATICS

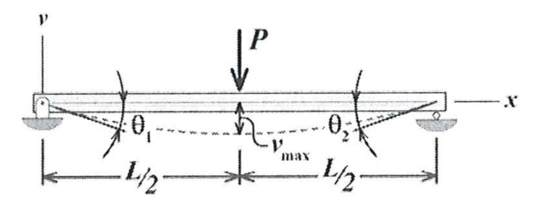
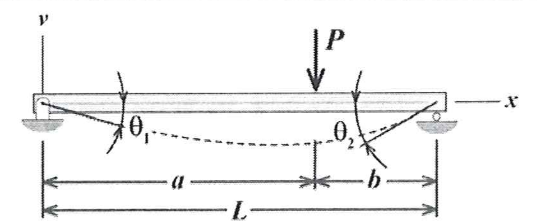
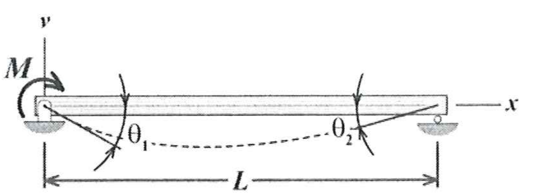
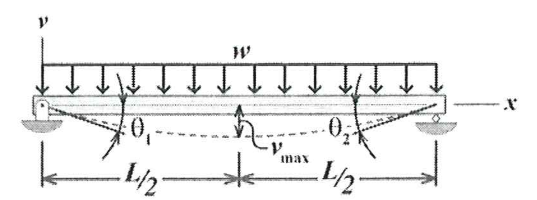
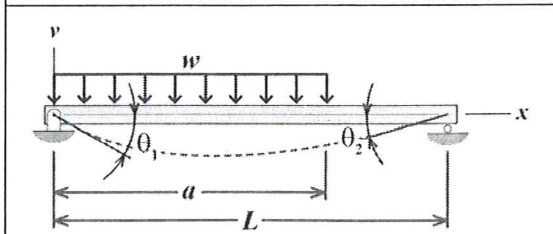
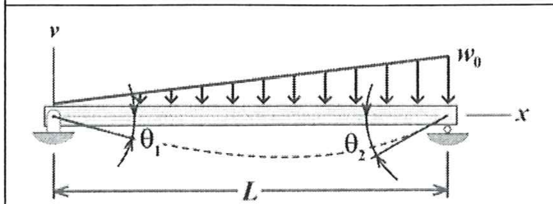
Symbol	Meaning	Equation	Units
$\bar{x}, \bar{y}, \bar{z}$	centroid position	$\bar{y} = \Sigma y_i A_i / \Sigma A_i$	in, m
I	moment of inertia	$I = \Sigma(I_i + d_i^2 A_i)$	in^4, m^4
J	polar moment of inertia	$J_{\text{solid circular shaft}} = \pi d^4/32$ $J_{\text{hollow circular shaft}} = \pi(d_o^4 - d_i^4)/32$	in^4, m^4
N	normal force		lb, N
V	shear force	$V = \int -w(x)\,dx$	lb, N
M	bending moment	$M = \int V(x)\,dx$	in-lb, Nm
equilibrium		$\Sigma \mathbf{F} = \mathbf{0}$ $\Sigma \mathbf{M}_{(\text{any point})} = \mathbf{0}$	lb, N in-lb, Nm



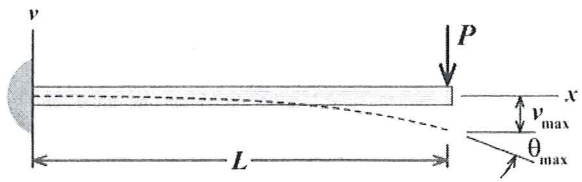
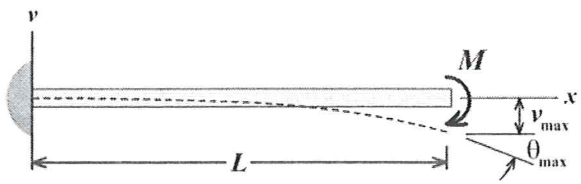
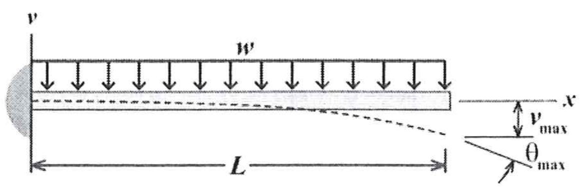
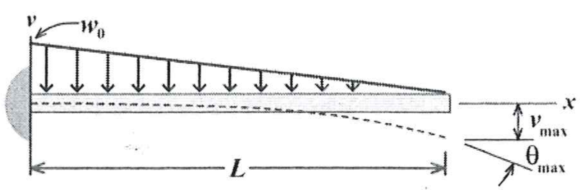
MECHANICS OF MATERIALS

Topic	Symbol	Meaning	Equation	Units
axial	σ, sigma	normal stress 	$\sigma_{\text{axial}} = N/A$ $\tau_{\text{cutting}} = V/A$ $\sigma_{\text{bearing}} = F_b/A_b$	psi, Pa
	ε, epsilon	normal strain	$\varepsilon_{\text{axial}} = \Delta L/L_o = \delta/L_o$ $\varepsilon_{\text{transverse}} = \Delta d/d$	in/in, m/m
	γ, gamma	shear strain	γ = change in angle,	rad
	E	Young's modulus, modulus of elasticity	$\sigma = E\varepsilon$ (one-dimensional only)	psi, Pa
	G	shear modulus, modulus of rigidity	$G = \tau/\gamma = E / 2(1+\nu)$	psi, Pa
	ν, nu	Poisson's ratio	$\nu = -\varepsilon'/\varepsilon$	
	δ, delta	deformation, elongation, deflection	$\delta = NL_o/EA + \alpha\Delta T L_o$	in, m
	α, alpha	coefficient of thermal expansion (CTE)		in/inF, m/mC
	F.S.	factor of safety	F.S. = actual strength / design strength	

SIMPLY SUPPORTED BEAMS

Beam	Slope	Deflection	Elastic Curve
	$\theta_1 = -\theta_2 = -\dfrac{PL^2}{16EI}$	$v_{\max} = -\dfrac{PL^3}{48EI}$	$v = -\dfrac{Px}{48EI}(3L^2 - 4x^2)$ $for\ 0 \le x \le L/2$
	$\theta_1 = -\dfrac{Pb(L^2 - b^2)}{6LEI}$ $\theta_2 = +\dfrac{Pa(L^2 - a^2)}{6LEI}$	$v\big\|_{x=a} = -\dfrac{Pba}{6LEI}(L^2 - b^2 - a^2)$	$v = -\dfrac{Pbx}{6LEI}(L^2 - b^2 - x^2)$ $for\ 0 \le x \le a$
	$\theta_1 = -\dfrac{ML}{3EI}$ $\theta_2 = +\dfrac{ML}{6EI}$	$v_{\max} = -\dfrac{ML^2}{9\sqrt{3}EI}$ $@\ x = L\left(1 - \dfrac{\sqrt{3}}{3}\right)$	$v = -\dfrac{Mx}{6LEI}(2L^2 - 3Lx + x^2)$
	$\theta_1 = -\theta_2 = -\dfrac{wL^3}{24EI}$	$v_{\max} = -\dfrac{5wL^4}{384EI}$	$v = -\dfrac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$
	$\theta_1 = -\dfrac{wa^2}{24LEI}(2L - a)^2$ $\theta_2 = +\dfrac{wa^2}{24LEI}(2L^2 - a^2)$	$v\big\|_{x=a} = -\dfrac{wa^3}{24LEI}(3a^2 - 7aL + 4L^2)$	$v = -\dfrac{wx}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 2a^2x^2 - 4aLx^2 + Lx^3)$ $for\ 0 \le x \le a$ $v = -\dfrac{wa^2}{24LEI}(-a^2L + 4L^2x + a^2x - 6Lx^2 + 2x^3)$ $for\ a \le x \le L$
	$\theta_1 = -\dfrac{7w_0L^3}{360EI}$ $\theta_2 = +\dfrac{w_0L^3}{45EI}$	$v_{\max} = -0.00652\dfrac{w_0L^4}{EI}$ $@\ x = 0.5193L$	$v = -\dfrac{w_0x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$

CANTILEVER BEAMS

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = -\dfrac{PL^2}{2EI}$	$v_{max} = -\dfrac{PL^3}{3EI}$	$v = -\dfrac{Px^2}{6EI}(3L - x)$
	$\theta_{max} = -\dfrac{ML}{EI}$	$v_{max} = -\dfrac{ML^2}{2EI}$	$v = -\dfrac{Mx^2}{2EI}$
	$\theta_{max} = -\dfrac{wL^3}{6EI}$	$v_{max} = -\dfrac{wL^4}{8EI}$	$v = -\dfrac{wx^2}{24EI}(6L^2 - 4Lx + x^2)$
	$\theta_{max} = -\dfrac{w_0 L^3}{24EI}$	$v_{max} = -\dfrac{w_0 L^4}{30EI}$	$v = -\dfrac{w_0 x^2}{120LEI}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$