1 1-D Ising Model Long-Range interactions

For this problem, we want to evaluate the Peierls argument for long-range interactions modelled by the Hamiltonian:

$$\mathcal{H} = -J \sum_{i \neq j}^{N} \frac{s_i s_j}{|i-j|^a} \tag{1}$$

In addition, we're given that $s \in \{-1, 1\}$, and $a > 0$. We want to show that the Peierls argument fails for $a < 2$, meaning that with a in that range, there is indeed a phase transition.

Like with the original argument, we can find the free energy A for $m = 0$ and $m = 1$. We note that the entropies are unchanged:

$$S_{m=0} < k_{\mathrm{B}} \log \frac{N}{2} \tag{2}$$

$$S_{m=1} = 0 \tag{3}$$

We need to find the energies. For $m = 1$:

$$U = \sum_{i \neq j}^{N} \frac{s_i s_j}{|i-j|^a} \tag{4}$$

$$= \sum_{i \neq j}^{N} \frac{1}{|i-j|^a} \tag{5}$$

$$\tag{6}$$