

1 Qubit Relaxation Time

1.1 The quasi-static limit

We can start by looking at

$$\chi_{zz}^E(z, z, \omega) = \frac{\hbar}{\epsilon_0} \operatorname{Re} \int_0^\infty dp \frac{p^3}{q} e^{2iqz} r_p(p) \quad (1)$$

Here, we have

$$q = \begin{cases} \sqrt{\frac{\omega^2}{c^2} - p^2}, & p^2 \leq \frac{\omega^2}{c^2} \\ i\sqrt{p^2 - \frac{\omega^2}{c^2}}, & p^2 > \frac{\omega^2}{c^2} \end{cases} \quad (2)$$

If we look at the case where $\omega = 6\pi \times 10^8 \text{ s}^{-1}$, the cutoff for real or imaginary q will be when $p = 2\pi \text{ m}^{-1}$.

If we assume that $\operatorname{Im} r_p$ doesn't decay too quickly, this integral will be dominated by values of p larger than this, which lets us make the substitution that $\frac{\omega}{c} \rightarrow 0$, which means this integral will reduce to

$$\chi_{zz}^E(z, z, \omega) = \frac{\hbar}{\epsilon_0} \int_0^\infty dp p^2 e^{-2pz} \operatorname{Im} r_p(p, \omega) \quad (3)$$

The note that we're effectively taking $c \rightarrow \infty$ is important, as we still shouldn't necessarily assume that we can take $\omega \rightarrow 0$ in r_p .

1.2 Non-local reflection coefficient

We can look specifically at what r_p will be in the non-local case:

$$r_p(p, \omega) = \frac{1 - \frac{2p}{\pi} \int_0^\infty d\kappa \frac{1}{k^2 \epsilon_l(k\omega)}}{1 + \frac{2p}{\pi} \int_0^\infty d\kappa \frac{1}{k^2 \epsilon_l(k\omega)}} \quad (4)$$

where $k^2 = p^2 + \kappa^2$ and

$$\epsilon_l = 1 + \frac{3\omega_p^2}{k^2 v_F^2} \frac{(\omega + i\nu) f_l((\omega + i\nu)/kv_f)}{\omega + i\nu f_l((\omega + i\nu)/kv_f)}. \quad (5)$$

All of the interesting behaviour here comes from the integral, which we might name $I = \int_0^\infty d\kappa \frac{1}{k^2 \epsilon_l(k, m\omega)}$. Knowing that we will eventually need to find $\operatorname{Im} r_p$, we might find utility in also writing $I = I_1 + iI_2$ and noting that

$$\operatorname{Im} r_p = \operatorname{Im} \frac{1 - I}{1 + I} \quad (6)$$

$$= \text{Im} \frac{1 - I_1 - iI_2}{1 + I_1 + iI_2} \quad (7)$$

$$= \text{Im} \frac{1 - I_1 - iI_2}{1 + I_1 + iI_2} \frac{I + I_1 - iI_2}{I + I_1 - iI_2} \quad (8)$$

$$= \text{Im} \frac{(1 - I_1)(1 + I_1) - I_2^2 - iI_2(1 - I_1 + 1 + I_1)}{(1 + I_1)^2 + I_2^2} \quad (9)$$

$$= \text{Im} \frac{(1 - I_1)(1 + I_1) - I_2^2 - 2iI_2}{(1 + I_1)^2 + I_2^2} \quad (10)$$

$$= \frac{-2I_2}{(1 + I_1)^2 + I_2^2}. \quad (11)$$

This gives us some idea of how $\text{Im} r_p$ should behave, at least once we can write out the integral I .

$$I = \int_0^\infty d\kappa \frac{1}{k^2 \epsilon_l(k, \omega)} \quad (12)$$