

1 Summary of Lindhard's derivation

For Lindhard's derivation, we can start with two variations on the Maxwell's equations:

$$\left(k^2 - \frac{\omega^2}{c^2} \epsilon^{tr}(\mathbf{k}, \omega)\right) \mathbf{A}^{tr}(\mathbf{k}, \omega) = \frac{4\pi}{c} \mathbf{j}_f^{tr}(\mathbf{k}, \omega), \quad (1)$$

$$\epsilon^{lo}(\mathbf{k}, \omega) k^2 V(\mathbf{k}, \omega) = 4\pi \rho_f(\mathbf{k}, \omega). \quad (2)$$

Here, ρ_f and j_f are the free charge density and current. The longitudinal and transverse dielectric functions, ϵ^{lo} and ϵ^{tr} , contain the same information as the traditional ϵ and μ , and are related by

$$\epsilon(\mathbf{k}, \omega) = \epsilon^{lo}(\mathbf{k}, \omega) \quad (3)$$

$$k^2 \left(1 - \frac{1}{\mu(\mathbf{k}, \omega)}\right) = \frac{\omega^2}{c^2} \left(\epsilon^{tr}(\mathbf{k}, \omega) - \epsilon^{lo}(\mathbf{k}, \omega)\right) \quad (4)$$

Lindhard starts by looking at the Boltzmann equation:

$$\frac{\partial f}{\partial t} + \mathbf{F} \cdot \nabla_p f + \mathbf{v} \cdot \nabla_r f = -\frac{f - f_0}{\tau} \quad (5)$$

where $f(r, p, t)$ is the electron distribution function, \mathbf{F} is the external force and τ is the relaxation time. We can define $f(r, p, t) = f_0(r, p) + f_1(r, p, t)$, where f_0 represents an equilibrium, time-independent electron distribution. Then, simplifying gives us:

$$-\frac{f - f_0}{\tau} = \frac{\partial f}{\partial t} + \mathbf{F} \cdot \nabla_p f + \mathbf{v} \cdot \nabla_r f \quad (6)$$

$$-\frac{f_1}{\tau} = \frac{\partial f_1}{\partial t} + \mathbf{F} \cdot \nabla_p (f_0 + f_1) + \mathbf{v} \cdot \nabla_r (f_0 + f_1) \quad (7)$$

Include note about the regimes where the Boltzmann equation holds. This could be where errors creep in.

Because f_0 is an equilibrium solution, we know that it must separately satisfy a time-independent Boltzmann equation:

$$0 = \mathbf{F} \cdot \nabla_p f_0 + \mathbf{v} \cdot \nabla_r f_0, \quad (8)$$

which means we can reduce (7) to

$$-\frac{f_1}{\tau} = \frac{\partial f_1}{\partial t} + \mathbf{F} \cdot \nabla_p (f_0 + f_1) + \mathbf{v} \cdot \nabla_r (f_0 + f_1) \quad (9)$$

$$-\frac{f_1}{\tau} = \quad (10)$$