

1 Nam form of conductivity

Starting with the form in the Nam paper, we have for the conductivity σ :

$$\sigma(q, \omega) = -i \frac{3}{4} \frac{ne^2}{m} \frac{1}{\omega} \left[\int_{\Delta-\omega}^{\Delta} d\omega' \tanh\left(\frac{\omega + \omega'}{2T}\right) I_1 + \int_{\Delta}^{\infty} d\omega' \left(\tanh\left(\frac{\omega + \omega'}{2T}\right) I_1 - \tanh\left(\frac{\omega'}{2T}\right) I_2 \right) \right] \quad (1)$$

with

$$I_1 = F(q, \text{Re}[\sqrt{(\omega + \omega')^2 - \Delta^2} - \sqrt{\omega'^2 - \Delta^2}]) (g + 1) \\ + F(q, \text{Re}[-\sqrt{(\omega + \omega')^2 - \Delta^2} - \sqrt{\omega'^2 - \Delta^2}]) (g - 1) \quad (2)$$

$$I_2 = F(q, \text{Re}[\sqrt{(\omega + \omega')^2 - \Delta^2} - \sqrt{\omega'^2 - \Delta^2}]) (g + 1) \\ + F(q, \text{Re}[\sqrt{(\omega + \omega')^2 - \Delta^2} + \sqrt{\omega'^2 - \Delta^2}]) (g - 1) \quad (3)$$

$$F(q, E) = \frac{1}{qv_0} \left[2S(E) + (1 - S(E)^2) \ln\left(\frac{S(E) + 1}{S(E) - 1}\right) \right] \quad (4)$$

$$S(E) = \frac{1}{qv_0} \left(E - i \left(\text{Im}[\sqrt{(\omega + \omega')^2 - \Delta^2} + \sqrt{\omega'^2 - \Delta^2}] + \frac{2}{\tau} \right) \right) \quad (5)$$

$$g = \frac{\omega' (\omega + \omega') + \Delta^2}{\sqrt{\omega'^2 - \Delta^2} \sqrt{(\omega + \omega')^2 - \Delta^2}} \quad (6)$$

1.1 Removing units

To remove units, we'll want to represent all the various quantities in terms of Δ :

$$\xi = \frac{\omega}{\Delta} \quad (7)$$

$$\xi' = \frac{\omega'}{\Delta} \quad (8)$$

$$\nu = \frac{1}{\tau \Delta} \quad (9)$$

$$\kappa = \frac{qv_0}{\Delta} \quad (10)$$

$$t = \frac{T}{\Delta} \quad (11)$$

$$\sigma_0 = \frac{ne^2}{m\Delta} \quad (12)$$

This gives us

$$\sigma(\kappa, \xi) = -i \frac{3\sigma_0}{4} \frac{1}{\xi} \left[\int_{1-\xi}^1 d\xi \tanh\left(\frac{\xi + \xi'}{2t}\right) I_1 + \int_1^\infty d\xi' \left(\tanh\left(\frac{\xi + \xi'}{2t}\right) I_1 - \tanh\left(\frac{\xi'}{2t}\right) I_2 \right) \right] \quad (13)$$

with

$$I_1 = F(\kappa, \text{Re}[\sqrt{(\xi + \xi')^2 - 1} - \sqrt{\xi'^2 - 1}](g + 1) + F(\kappa, \text{Re}[-\sqrt{(\xi + \xi')^2 - 1} - \sqrt{\xi'^2 - 1}](g - 1) \quad (14)$$

$$I_2 = F(\kappa, \text{Re}[\sqrt{(\xi + \xi')^2 - 1} - \sqrt{\xi'^2 - 1}](g + 1) + F(\kappa, \text{Re}[\sqrt{(\xi + \xi')^2 - 1} + \sqrt{\xi'^2 - 1}](g - 1) \quad (15)$$

$$F(\kappa, E) = \frac{1}{\kappa} \left[2S(E) + (1 - S(E)^2) \ln\left(\frac{S(E) + 1}{S(E) - 1}\right) \right] \quad (16)$$

$$S(\kappa, E) = \frac{1}{\kappa} \left(E - i \left(\text{Im}[\sqrt{(\xi + \xi')^2 - 1} + \sqrt{\xi'^2 - 1}] + 2\nu \right) \right) \quad (17)$$

$$g = \frac{\xi'(\xi + \xi') + 1}{\sqrt{\xi'^2 - 1}\sqrt{(\xi + \xi')^2 - 1}} \quad (18)$$

For future reference, F carried units, which when included out a Δ in the integrals.

1.2 Comparing to normal conductivity

We can also compare this to the normal conductivity: $\sigma_N = \frac{ne^2\tau}{m}$:

$$\sigma_N = \frac{ne^2\tau}{m} \quad (19)$$

$$= \frac{ne^2}{m\Delta} \tau \Delta \quad (20)$$

$$= \sigma_0 \frac{1}{\nu} \quad (21)$$

This, we can find the ratio $\Sigma = \frac{\sigma}{\sigma_N} = \frac{\sigma\nu}{\sigma_0}$,

$$\Sigma(\kappa, \xi) = -i \frac{3}{4} \frac{\nu}{\xi} \left[\int_{1-\xi}^1 d\xi \tanh\left(\frac{\xi + \xi'}{2t}\right) I_1 + \int_1^\infty d\xi' \left(\tanh\left(\frac{\xi + \xi'}{2t}\right) I_1 + \tanh\left(\frac{\xi'}{2t}\right) I_2 \right) \right] \quad (22)$$