1 Summary of Lindhard's derivation

For Lindhard's derivation, we can start with two variations on the Maxwell's equations:

$$\left(k^2 - \frac{\omega^2}{c^2}\epsilon^{tr}(\mathbf{k},\omega)\right)\mathbf{A}^{tr}(\mathbf{k},\omega) = \frac{4\pi}{c}\mathbf{j}_f^{tr}(\mathbf{k},\omega), \tag{1}$$

$$\epsilon^{lo}(\mathbf{k},\omega)k^2 V(\mathbf{k},\omega) = 4\pi\rho_f(\mathbf{k},\omega). \tag{2}$$

Here, ρ_f and j_f are the free charge density and current. The longitudinal and transverse dielectric functions, ϵ^{lo} and ϵ^{tr}, contain the same information as the traditional ϵ and μ, and are related by

$$\epsilon(\mathbf{k},\omega) = \epsilon^{lo}(\mathbf{k},\omega) \tag{3}$$

$$k^2\left(1 - \frac{1}{\mu(\mathbf{k},\omega)}\right) = \frac{\omega^2}{c^2}\left(\epsilon^{tr}(\mathbf{k},\omega) - \epsilon^{lo}(\mathbf{k},\omega)\right) \tag{4}$$

Lindhard starts by looking at the Boltzmann equation:

$$\frac{\partial f}{\partial t} + \mathbf{F}\cdot\boldsymbol{\nabla}_p f + \mathbf{v}\cdot\boldsymbol{\nabla}_r f = -\frac{f - f_0}{\tau} \tag{5}$$

where $f(r,p,t)$ is the electron distribution function, $\mathbf{F}$ is the external force and τ is the relaxation time. We can define $f(r,p,t) = f_0(r,p) + f_1(r,p,t)$, where f_0 represents an equilibrium, time-independent electron distribution. Then, simplifying gives us:

Include note about the regimes where the Boltzmann equation holds. This could be where errors creep in.

$$-\frac{f - f_0}{\tau} = \frac{\partial f}{\partial t} + \mathbf{F}\cdot\boldsymbol{\nabla}_p f + \mathbf{v}\cdot\boldsymbol{\nabla}_r f \tag{6}$$

$$-\frac{f_1}{\tau} = \frac{\partial f_1}{\partial t} + \mathbf{F}\cdot\boldsymbol{\nabla}_p\left(f_0 + f_1\right) + \mathbf{v}\cdot\boldsymbol{\nabla}_r\left(f_0 + f_1\right) \tag{7}$$

Because f_0 is an equilibrium solution, we know that it must separately satisfy a time-independent Boltzmann equation:

$$0 = \mathbf{F}\cdot\boldsymbol{\nabla}_p f_0 + \mathbf{v}\cdot\boldsymbol{\nabla}_r f_0, \tag{8}$$

which means we can reduce (7) to

$$-\frac{f_1}{\tau} = \frac{\partial f_1}{\partial t} + \mathbf{F}\cdot\boldsymbol{\nabla}_p\left(f_0 + f_1\right) + \mathbf{v}\cdot\boldsymbol{\nabla}_r\left(f_0 + f_1\right) \tag{9}$$

$$-\frac{f_1}{\tau} = \tag{10}$$