1 Qubit Relaxation Time

1.1 The quasi-static limit

We can start by looking at

$$\chi_{zz}^{E}(z,z,\omega) = \frac{\hbar}{\epsilon_0} \operatorname{Re} \int_0^\infty dp\, \frac{p^3}{q} e^{2iqz} r_p(p) \tag{1}$$

Here, we have

$$q = \begin{cases} \sqrt{\frac{\omega^2}{c^2} - p^2}, & p^2 \leq \frac{\omega^2}{c^2} \\ i\sqrt{p^2 - \frac{\omega^2}{c^2}}, & p^2 > \frac{\omega^2}{c^2} \end{cases} \tag{2}$$

If we look at the case where $\omega = 6\pi \times 10^8\,\mathrm{s}^{-1}$, the cutoff for real or imaginary q will be when $p = 2\pi\,\mathrm{m}^{-1}$.

If we assume that r_p doesn't decay too quickly, this integral will be dominated by values of p larger than this, which lets us make the substitution that $\frac{\omega}{c} \to 0$, which means this integral will reduce to

$$\chi_{zz}^{E}(z,z,\omega) = \frac{\hbar}{\epsilon_0} \int_0^\infty dp\, p^2 e^{-2pz} \operatorname{Im} r_p(p,\omega) \tag{3}$$

The note that we're effectively taking $c \to \infty$ is important, as we still shouldn't necessarily assume that we can take $\omega \to 0$ in r_p.