

## 1 General discussion of dielectric function

We can start by looking at some basic relationships between  $\mathbf{D}$ ,  $\mathbf{E}$ . In SI units, and writing  $\epsilon$  instead of  $\epsilon_r$  to avoid unnecessary subscripts,

$$\mathbf{D}_\alpha(r) = \int d^d r' \epsilon_{\alpha\beta}(r, r') \epsilon_0 \mathbf{E}_\beta(r'). \quad (1)$$

We can start by making the large assumption that the system is isotropic, and thus, the position dependence for  $\epsilon$  must be of the form  $\epsilon_{\alpha\beta}(r - r')$ . This justifies the following:

$$\mathbf{D}_\alpha(r) = \int d^d r' \epsilon_{\alpha\beta}(r - r') \epsilon_0 \mathbf{E}_\beta(r') \quad (2)$$

$$= \int d^d r' \epsilon_{\alpha\beta}(r - r') \epsilon_0 \mathbf{E}_\beta(r') e^{iqr'} e^{-iqr'} \quad (3)$$

$$= \int d^d r' \epsilon_{\alpha\beta}(r - r') \epsilon_0 \mathbf{E}_\beta(r') e^{iqr'} e^{-iqr'} \quad (4)$$

$$= \int d^d r' \epsilon_{\alpha\beta}(r - r') e^{iqr'} \epsilon_0 \mathbf{E}_\beta(r') e^{-iqr'} \quad (5)$$

$$\mathbf{D}_\alpha(r) e^{-iqr} = \int d^d r' \epsilon_{\alpha\beta}(r - r') e^{iqr'} e^{-iqr} \epsilon_0 \mathbf{E}_\beta(r') e^{-iqr'} \quad (6)$$

$$\mathbf{D}_\alpha(r) e^{-iqr} = \int d^d r' \epsilon_{\alpha\beta}(r - r') e^{-iq(r-r')} \epsilon_0 \mathbf{E}_\beta(r') e^{-iqr'} \quad (7)$$

If we integrate this over  $r$ , and recognise our Fourier transforms, this becomes

$$D_\alpha(q) = \epsilon_{\alpha\beta}(q) E_\beta(q) \quad (8)$$

This is a fair result for the assumption of isotropy. We have an implicit sum over  $\beta$  here to look at.

Skipping over some details to fill in later, we end up with

$$\frac{1}{\epsilon_r(q, \omega)} = 1 + \frac{4\pi e^2}{q^2} \Pi(q, \omega), \quad (9)$$

where  $\Pi(q, \omega)$  is a response function satisfying

$$n_{ind}(q, \omega) = \Pi(q, \omega) V_f(q, \omega) \quad (10)$$

Here  $n_{ind}$  is the number density of induced electrons, and  $V_f$  is the voltage created by any free electrons in the metal (which isn't quite the same as an external voltage, but I think you might be able to ignore that difference).