1 Summary of Lindhard's derivation

For Lindhard's derivation, we can start with two variations on the Maxwell's equations:

$$\left(k^2 - \frac{\omega^2}{c^2}\epsilon^{tr}(\mathbf{k},\omega)\right)\mathbf{A}^{tr}(\mathbf{k},\omega) = \frac{4\pi}{c}\mathbf{j}_f^{tr}(\mathbf{k},\omega), \tag{1}$$

$$\epsilon^{lo}(\mathbf{k},\omega)k^2 V(\mathbf{k},\omega) = 4\pi\rho_f(\mathbf{k},\omega). \tag{2}$$

Here, ρ_f and j_f are the free charge density and current. The longitudinal and transverse dielectric functions, ϵ^{lo} and ϵ^{tr}, contain the same information as the traditional ϵ and μ, and are related by

$$\epsilon(\mathbf{k},\omega) = \epsilon^{lo}(\mathbf{k},\omega) \tag{3}$$

$$k^2\left(1 - \frac{1}{\mu(\mathbf{k},\omega)}\right) = \frac{\omega^2}{c^2}\left(\epsilon^{tr}(\mathbf{k},\omega) - \epsilon^{lo}(\mathbf{k},\omega)\right) \tag{4}$$