## 1 Summary of Lindhard's derivation

For Lindhard's derivation, we can start with two variations on the Maxwell's equations:

$$\left(k^2 - \frac{\omega^2}{c^2} \epsilon^{tr}(\mathbf{k}, \omega)\right) \mathbf{A}^{tr}(\mathbf{k}, \omega) = \frac{4\pi}{c} \mathbf{j}_f^{tr}(\mathbf{k}, \omega), \tag{1}$$

$$\epsilon^{lo}(\mathbf{k},\omega)k^2V(\mathbf{k},\omega) = 4\pi\rho_f(\mathbf{k},\omega).$$
 (2)

Here,  $\rho_f$  and  $j_f$  are the free charge density and current. The longitudinal and transverse dielectric functions,  $\epsilon^{lo}$  and  $\epsilon^{tr}$ , contain the same information as the traditional  $\epsilon$  and  $\mu$ .