

Exercise sheet 8 (revision)

Numerical Analysis 2022

1 PS.8 (conditioning, stability)

Verify that $\|\cdot\|_2 : \mathbb{R}^n \rightarrow \mathbb{R}$ is partially well-conditioned. Prove that it is also well-conditioned.

2 PS.8 (matrix norms)

Consider the 3 matrix norms $\|\cdot\|_1$, $\|\cdot\|_2$, and $\|\cdot\|_\infty$ that induce the condition numbers κ_1 , κ_2 , and κ_∞ . For $A, B \in \mathbb{R}^{n \times n}$ invertible, verify the following claims for $p = 1, 2, \infty$:

- a) $\kappa_p(A) \geq 1$,
- b) $\kappa_p(AB) \leq \kappa_p(A)\kappa_p(B)$.

3 PS.8 (qr-decomposition)

Suppose that the qr-decomposition of $A \in \mathbb{R}^{5 \times 3}$ is given by $A = QR$ with

$$Q = \begin{pmatrix} * & 1 & * & * & * \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & * & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

where “*” means you still need to reconstruct those entries.

Find the solution $\hat{x} \in \mathbb{R}^3$ of the linear least squares problem

$$\arg \min_{x \in \mathbb{R}^3} \|Ax - b\|_2^2, \quad b = (1, 6, 3, 4, 5)^\top.$$

4 PS.8 (modified Newton’s method)

Implement the modified Newton’s method and apply your program to

$$f(x) = (x^2 - 2)^2$$

with initial value $x_0 = 1$. Apply also Newton’s method (solutions to PS.6 are at the moodle website) and compare the errors of both methods in a logarithmic plot.