# Exercise sheet 6

Numerical Analysis 2022

## 1 PS.6 (conjugated gradient)

Explain the notebook myCG.ipynb and address the following aspects:

a) CG is fast, in particular, faster than SGD.

b) Preconditioning is useful.

c) CG is useful for linear least squares problems, in particular, for sparse matrices.

## 2 PS.6 (iterative schemes)

Consider the equation

$$x + ln(x) = 0.$$

To determine a solution $x_*$, we make use of two iterative schemes.

a) Verify that $x_{n+1} = \mathrm{e}^{-x_n}$ provides locally linear convergence.

b) Verify that $x_{n+1} = \frac{1}{2}(x_n + \mathrm{e}^{-x_n})$ provides locally linear convergence.

## 3 PS.6 (iterative schemes, quadratic convergence)

a) Determine $a \in \mathbb{R}$ such that

$$x_{n+1} = \frac{ax_n + \mathrm{e}^{-x_n}}{a+1}$$

is locally convergent of order 2.

b) We now replace $a$ in part a) by $x_n$. Verify that

$$x_{n+1} = \frac{x_n x_n + \mathrm{e}^{-x_n}}{x_n + 1}$$

is locally convergent of order 2.

## 4 PS.6 (root finding)

Find $x$ such that

$$f(x) = 0$$

## 4.1 bisection method

Define $A_0 := [a_0, b_0] := [a, b]$ and denote $A_n := [a_n, b_n]$, for $n \in \mathbb{N}$, and

$$x_n = \frac{a_n + b_n}{2}.$$

If $f(x_n) = 0$, then simply $\hat{x} = x_n$. We consider

$$A_{n+1} := \begin{cases} [a_n, x_n], & f(a_n)f(x_n) < 0, \\ [x_n, b_n], & \text{otherwise.} \end{cases}$$

a) Implement the bisection method.

## 4.2 secant method

Given $x_0$ and $x_1$, define

$$x_{k+1} := x_k - f(x_k)\frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})} \tag{1}$$

b) Implement the secant method.

## 4.3 Newton's method

Given $x_0$, define

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \tag{2}$$

c) Implement the Newton's method.

## 4.4 Tests for root finding

d) Run the tests and explain the plots.