Julia script: exercises 0

1 PS.0 (harmonic series)

The harmonic series satisfies

$$\sum_{k=1}^{n} \frac{1}{k} \to \infty. \tag{1}$$

a) Write a function $n \mapsto harmonicSum(n)$ that returns $\sum_{k=1}^{n} \frac{1}{k}$.

b) Plot this function between $10 < n < 10^6$.

2 PS.0 (alternate harmonic series)

The alternate harmonic series satisfies

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k} = \ln(2). \tag{2}$$

a) Write a function $n \mapsto harmonicAlternateSum(n)$ that returns $\sum_{k=1}^{n} \frac{(-1)^{k+1}}{k}$.

b) Check numerically if

$$\frac{\left|\sum_{k=1}^{n+1} \frac{(-1)^{k+1}}{k} - \ln(2)\right|}{\left|\sum_{k=1}^{n} \frac{(-1)^{k+1}}{k} - \ln(2)\right|} \to 1 \tag{3}$$

c) Check numerically if

$$\lim_{n\to\infty} \left|\sum_{k=1}^{n} \frac{(-1)^{k+1}}{k} - \ln(2)\right|^{1/n} = 1 \tag{4}$$

3 PS.0 (p-q-formula)

The p-q-formula of quadratic equations leads to

$$x^2 - 2px + q = 0, \qquad x_{1/2} = p \pm \sqrt{p^2 - q}. \tag{5}$$

a) Write a function $(p, q) \mapsto myPQnaive(p, q)$ that returns the zeros x_1 and x_2.

b) nothing to do here (in a few weeks we will have learned why a) is too naive and how to do better)

4 PS.0 (vector of matrices)

Given $v_1, \dots v_m \in \mathbb{R}^{n\times n}$, consider

$$s = v_1 + \dots + v_m, \qquad p = v_1 \cdots v_m.$$

a) Assume that v is a length m (julia-)vector of $n \times n$ matrices. Write a function $v \mapsto mySumProd(v)$ that returns the sum s and the product p all matrices.

b) Test your function $mySumProd$

5 PS.0 (Fibonacci)

The Fibonacci sequence $(a_k)_{k\in\mathbb{N}}$ satisfies

$$a_{k+1} = a_k + a_{k-1}, \qquad a_1 = 1, \quad a_0 = 0.$$

a) Write a recursive function $n \mapsto myFibo(n)$ that returns the n-th Fibonacci number.

b) Write a function $n \mapsto myFiboVector(n)$ that returns the vector of the first n Fibonacci numbers $(a_0, \dots, a_{n-1})^\top$.