Modeling Probability Knowledge and Choice in Decisions from Experience.

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Abstract

In most everyday decisions, we learn about the outcomes of alternative courses of action through experience: a sampling process. Current models of these decisions from experience emphasize the importance of predicting choice at the expense of explaining how the sample outcomes are used to form a representation of the distribution of outcomes. Moreover, most current models fail to generalize even across quite similar forms of experience-based choice. We develop a new and simple model, the Exemplar Confusion model, which overcomes these limitations. In a novel experiment the model was used to predict participants' choices and their knowledge of outcome probabilities, when choosing among multiple-outcome gambles. The model successfully predicted both types of data in sampling and feedback versions of the experiment. In addition we show that the model performs at least as well as other leading choice models when evaluated against benchmark data from the Technion Prediction Tournament (TPT). Our approach advances current understanding by proposing a psychological mechanism for how probability estimates arise rather than using estimates solely as inputs to choice models. Our principal conclusions are that (1) decisions from experience are best captured by models that assume an exemplar-based memory perturbed by noisy recall, and (2) that models are better constrained by trying to simultaneously capture more than one source of participant data (in our case, choice and probability knowledge).

Keywords: Experience-based choice; Sampling; Feedback; Exemplar; Model.

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1. Introduction

An enduring assumption in models of human choice is that behavior can be described as if people multiply some function of the probability of an outcome by that outcome's value, and then maximize. This framework dates back to Bernoulli (1738/1967) and has undergone many modifications, particularly since von Neumann and Morgenstern (1947) provided an axiomatization for rational choice, but the fundamental idea that people maximize expected utility remains in many successful models of choice. Modern examples include Prospect Theory (Kahneman & Tversky, 1979), Regret Theory (Loomes & Sugden, 1982), Cumulative Prospect Theory (Tversky & Kahneman, 1992), and Security-Potential/Aspiration theory (Lopes & Oden, 1999).[1]

Evaluation of these models has relied heavily on "decisions from description", in which choices are made from explicitly stated outcomes and their associated probabilities (Barron & Erev, 2003; Rakow & Newell, 2010). This approach is understandable given that the primary interest of many models is in capturing the systematic cognitive distortions of utilities and outcome probabilities implied by decision-makers' choices (Prelec, 1998). However, the focus on decisions in which outcome and probability information is readily available neglects important aspects of the cognitive processes that must underlie many of the decisions we face in our daily lives – decisions for which probabilities and outcomes are not explicitly provided. In such "decisions from experience" (Hertwig & Erev, 2009; Rakow & Newell, 2010), decision-makers must explore their environment to establish both the range of potential outcomes and the probability with which each occurs. Notably, the models mentioned above fail to capture the process by which knowledge of potential outcomes and their probabilities is represented and estimated by the decision-maker. Extending these models to achieve such capability is the chief goal of this paper.

In the sections that follow, we develop a new model of experience-based choice – the Exemplar Confusion (ExCON) model – which not only accounts for decision-makers' choices, but also their knowledge of the outcome distribution on which their choices are based. Unlike several recent attempts to model experience-based choice, we retain the core feature of successful description-based models, that is, combining outcome and probability information, and extend this approach to explain how a representation of the outcome distribution is constructed from repeated samples over time. The success of the ExCON model in predicting data from our new experiment, and also benchmark data from the Technion Prediction Tournament (Erev et al., 2010), suggests that the processes underlying experience-based decision-making involve both an exemplar-based, noisy memory system, and also decision rules common to those underlying description-based choices, namely, utility maximization.

1.1. Decisions from Experience

A "decision from experience" is one in which the decision-maker learns about potential outcomes and their respective probabilities via a process of sampling outcomes from

[1] An example of a model that does not adopt this framework is the Priority Heuristic (see Brandstätter, Gigerenzer, & Hertwig, 2006), which eschews the notions of weighting and summing. However, the success of the Priority Heuristic as a general model of choice has been challenged (see Birnbaum, 2008; Johnson, Schulte-Mecklenbeck, & Willemsen, 2008).

the environment. This situation has been modeled in the laboratory using two similar paradigms. In the *feedback paradigm* the outcome of each sample is added to a running total that the decision-maker is tasked with maximizing (e.g., Barron & Erev, 2003). The decision-maker is therefore faced with a tension between the objectives of learning more about the options (to "explore") while also trying to maximize the payoff from an unknown number of choices (to "exploit"). In contrast, the *sampling paradigm* separates these goals by allowing the decision-maker to freely sample during an exploration stage and then, at a time of their choosing, proceed to an exploitation stage during which a single choice is made (e.g., Hertwig, Barron, Weber, & Erev, 2004).

In general, the preferences inferred from the feedback and sampling paradigms are highly correlated (Erev et al., 2010, but see Camilleri & Newell, 2011b for conditions under which the two can be distinguished) and seem to imply that decision-makers *under*weight low-probability outcomes (Camilleri & Newell, 2011b; Ungemach, Chater, & Stewart, 2009). These preferences have sparked the interest of many investigators because they are in contrast to the preferences typically inferred from the conventional description paradigm, which imply that decision-makers *over*weight low-probability outcomes (e.g., Barron & Erev, 2003; Hertwig et al., 2004). More recent work has demonstrated that these format-dependent preference reversals – the "description-experience gap" – might be caused in part by the randomness in the samples. For example, the gap can be reduced or eliminated, at least in the sampling paradigm, under tightly controlled laboratory settings such as when samples are equated across formats (Rakow, Demes, & Newell, 2008) or forced to be representative of the underlying distributions (Camilleri & Newell, 2011a).

The observation that preferences are similar across description and experience paradigms when information about the possible outcome distributions are painstakingly equated suggests that the two formats may share common core features (Camilleri & Newell, in press). The process that is arguably common to both description and experience-based choices is the need to combine probability information with outcome values to inform choice. Interestingly, however, recent models of experience-based choice have had difficulty retaining utility maximization as a core feature. As we will demonstrate below, this may be due to a failure to adequately capture the process by which outcome distributions are represented.

1.2. Modeling Decisions from Experience

Several different approaches have been taken to model experience-based choices. These approaches were tested in the Technion Prediction Tournament (TPT) in which the organizers collected a large dataset in the description, sampling, and feedback paradigms (Erev et al., 2010). A noteworthy feature of the competition was that the winner in each paradigm was determined by the model that best predicted average choice in its own paradigm (i.e., there was no incentive to generate models that generalized across different tasks, such as the sampling and feedback paradigms). As a result, model developers did not prioritize capturing how sequentially observed outcomes were integrated into a representation nor how this representation could be used to model knowledge of the outcome probabilities. In fact, some models took probability information as *inputs* to the modeling process. Examples include Bernoullian-inspired models that combined weighted outcome and probability information, such as cumulative prospect theory (CPT; Kahneman & Tversky, 1979; Tversky & Kahneman, 1992). Other models were designed with no capacity to represent probability information at all, such as associative-learning models that rely on feedback and reinforcement of option attractiveness (Sutton & Barto, 1998).

An alternative approach is illustrated by exemplar-, or instance-, type models such as the ACT-R model (Anderson & Lebiere, 1998). Exemplar models assume that decision-makers record a memory trace each time they encounter a stimulus. This memory trace might include information about the stimulus, the corresponding feedback, and the general context in which the stimulus was encountered. Later, the properties of an unfamiliar stimulus can be predicted from the properties of related exemplars stored in memory. We argue that exemplar-based memory mechanisms provide a general approach for representing and learning about the outcome distributions of lotteries as well as the final choice process. Exemplar models are not new in economic decision making – e.g. the k-sampler model (Erev et al., 2010) and the Instance Based Learning (IBL) model (Gonzalez & Dutt, 2011) – and also have a much longer history in explaining many aspects of higher-level cognition, such as categorization (Hintzman, 1986; Medin & Schaffer, 1978; Nosofsky, 1986), response times (Nosofsky & Palmeri, 1997), absolute identification (Kent & Lamberts, 2005), short-term memory (Nosofsky, Little, Donkin, & Fific, 2011), "base rate neglect" (Estes, Campbell, Hastopoulos, & Hurwitz, 1989; Kruschke, 1992; Nosofsky, Kruschke, & McKinley, 1992), and probability judgment (Dougherty, Gettys, & Ogden, 1999; Juslin & Persson, 2002). For these reasons, we believe that the storage of exemplars in memory provides a simple and psychologically plausible way to explain decision-makers' choices and the estimation of probability information about gambles in risky choice problems.

In developing our modeling approach, we note two important limitations of the current choice modeling enterprise, highlighted by the results of the TPT. First, many current models are highly specific and only successful in the task for which they were originally designed (Cassimatis, Bello, & Langley, 2010). For example, the sampling and feedback paradigms were procedurally very similar and produced preferences that were highly correlated ($r = .84$ in the TPT data set, Erev et al., 2010) and yet the successful models in the sampling and feedback competitions were markedly different and no model performed well across both. Second, there currently exists great difficulty in discriminating between models using existing data and procedures. For example, the models submitted to the competition incorporated a wide variety of assumptions, yet many of these models performed nearly equally well. Such a lack of discriminability is due to the limited variability in the problem sets (a binary choice between a safe option and a two-outcome risky option), limited data sources used to constrain the models (solely choice), and a very general prediction goal (prediction of aggregated choice proportions).

A growing awareness of these issues has prompted the beginnings of a shift towards the development of more generalizable models that can be applied to a range of similar tasks (e.g., Anderson & Lebiere, 2003; Gonzalez & Dutt, 2011). In addition, new streams of data are being sought. For example, Gonzalez and Dutt evaluated their IBL model in terms of proportion of choices as well as the proportion of alternations in sampling between lottery options. In line with this movement, we set out to design and rigorously test a successful model of choice that parsimoniously captures behavior across different contexts that share underlying features. To this end, we constructed a general, exemplar-based model and evaluated it with respect to two streams of data: decision-makers' choices and their estimates of outcome probabilities in both sampling and feedback designs.

1.3. Modeling Probability Knowledge

A number of previous studies of experience-based choice have asked decision-makers to estimate outcome probabilities and found that estimates are either well calibrated (Fox & Hadar, 2006) or that rare events are overestimated (Camilleri & Newell, 2009; Gottlieb, Weiss, & Chapman, 2007; Hau, Pleskac, Kiefer, & Hertwig, 2008; Hertwig, Pachur, & Kurzenhauser, 2005; Ungemach et al., 2009). These observations are interesting because they reveal a subtle paradox: decision-makers appear to overestimate the probability of rare events yet make choices as if they underweight rare events. For example, Camilleri and Newell (2011c) found that when presented with the choice between a gamble offering a certain \$3 versus an 80% chance of \$4 and 20% chance of \$0, decision-makers estimated that \$0 occurred 27% of the time, yet the majority still preferred this alternative. A successful model of experience-based choice should account for this overestimation-underweighting paradox, beginning with an account of decision-makers' probability knowledge.

To the best of our knowledge, no model has yet attempted to simultaneously model choices and the outcome distributions associated with the alternative options. Indeed, a major limitation of some previous work is that the elicited probability estimates have served only as inputs to models such as Prospect Theory rather than dependent variables used to constrain the model (e.g., Camilleri & Newell, 2009). Moreover, the outcome distributions used to examine the model have been very simple – just two outcomes per option – which severely limits their potential to constrain any model, even if attempted (Reiger, 2003). Given that participants in the standard experiments are tasked with evaluating probability distributions in order to make choices that maximize their earnings, it is desirable to develop a model which can simultaneously perform both tasks.

1.4. The Experiment

We designed an experiment to address the limitations outlined above, therefore providing a rigorous test of the Exemplar Confusion model (ExCON, outlined in detail below). We presented decision-makers with a number of multi-outcome gambles in both the sampling and feedback paradigms and asked them to choose between the alternatives and also estimate the probability of each outcome. The gambles were constructed from the six lotteries described by Lopes and Oden (1999). Each of these lotteries had the same expected value, and contained five outcomes that differed in the distribution of payoffs around the mean (see Figure 1). Presenting these lotteries in pairs allowed us to collect ten outcome probability estimates per gamble (five for each lottery) in addition to choice behavior. These data were used to constrain the ExCON model in subsequent analyses. Although we did not specifically test a decision-from-description condition, since rare events were asymmetrically present in only a few pairings we did not expect patterns of choice to depart markedly from those originally observed by Lopes and Oden (1999); that is, stronger preferences for the alternatives with fewer chances of obtaining zero. We obliged participants to take a fixed 100 samples in the feedback condition and therefore expected more accurate probability estimates than those provided by participants in the sampling condition where participants were able to choose their own stopping point, resulting, presumably, in fewer samples. Nonetheless, and in line with the TPT data, we expected similar patterns of choice between the sampling and feedback paradigms.

2. Method

2.1. Participants

The participants were 107 undergraduate university students from two Australian universities who participated in exchange for payment contingent on performance. Data from two participants were excluded because of a failure to follow instructions.

2.2. Design

Participants were allocated to either the sampling task or the feedback task. Participants always made choices between two competing lotteries; we denote such a pair of lotteries as a "problem". The primary dependent variables were participants' preferences in each problem, and also their estimates of the probabilities of the outcomes from the lotteries in the problem. In the sampling group, lottery preference was operationalized as the one-shot choice made after learning about the lotteries in the free sampling phase. In the feedback group, lottery preference was operationalized as the deck selected most frequently in the final 50 samples.[2] From the six different lotteries defined by Lopes and Oden (1999) there were 15 possible pairings (ignoring order, and without identical choices), and participants in the sampling group played each of these pairings once, with the order randomized across participants. In order to equate the length of the experiment, participants in the feedback group played a random sample of just six of the fifteen problems. Data were excluded from participants who failed to sample from one of the lotteries during any problem. Some participants did not complete all problems during the one hour experiment and therefore not all problems have equal sample sizes. Specifically, there were 40 participants in the sampling group who experienced a total of 528 problems, corresponding to 32–39 participants experiencing each problem. There were 67 participants in the feedback group who experienced a total of 386 problems, corresponding to 22–32 data points for each problem.

2.3. Materials

Each problem consisted of a choice between two lotteries. We used six lotteries in total, taken from Lopes and Oden (1999). Each lottery was associated with five possible outcomes that ranged from between \$0 and \$348. As shown in Figure 1, the outcome distribution for each lottery was unique but all had expected values close to \$100. Although the lotteries were unlabeled during the experiment, for purpose of discussion we adopt the lottery names used by Lopes and Oden: Riskless, Rectangular, Peaked, Bimodal, Shortshot or Longshot, depending on the specific distribution of outcomes.

2.4. Procedure

The experiment took on average one hour to complete. After signing consent forms, participants were presented with instructions indicating that they would face a choice be-

[2]Similar results were obtained when we used the mode across all 100 samples, as well as just the final sample. We preferred the mode of the last 50 samples because we assumed many trials near the beginning of the task would be for the purpose of exploration rather than an indication of preference. Additionally, we assumed that participants would intermittently take reminder samples from their non-preferred option towards the latter part of each problem as study for the probability estimation task.

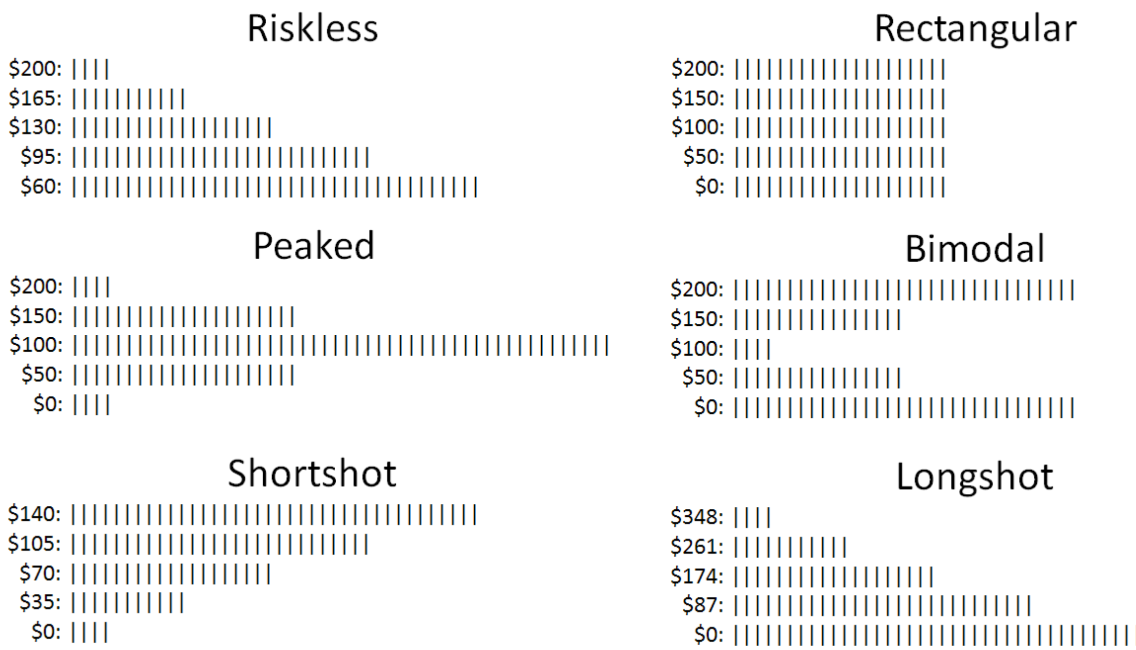

Figure 1. Visual representation of the outcome distribution for the six lotteries, adapted from Lopes and Oden (1999). Note that these labels and figures were not presented to the participants.

tween two decks of cards that were displayed on screen. Participants were instructed to use their choices to earn as much money as possible. We randomized the allocation of lotteries to the left and right decks of cards on screen, and the order of the problems. Both the sampling and feedback versions of the task presented participants with two unlabeled images of decks of cards. Each deck was associated with a distribution of outcomes from the lotteries of Lopes and Oden (1999, the names of the lotteries were not presented). When the participant clicked on a deck, an outcome was selected randomly from the associated lottery, with replacement, and displayed briefly as if the participant had turned over a playing card. We call the act of selecting a deck and observing an outcome a "sample".

The sampling task began with an exploration phase in which each sample had no consequence, so the participant could learn about the lotteries. The participants were free to terminate the exploration phase at any time, after which they made a single and final choice indicating their preference. The outcome of this final choice was added to the participant's running total for the experiment. In the feedback task, each problem granted 100 samples before moving to the next problem (participants were not informed about the number of samples). Each of the 100 samples were consequential: the outcome of the sample was added to the participant's running score, which was constantly displayed on screen.

After making a choice (in the sampling paradigm) or observing 100 samples (in the feedback paradigm), participants were asked to estimate the probability of different types of cards that were in each deck, corresponding to the probability of the different outcomes in the lotteries. As shown in Figure 2, six different outcome values were presented beside adjustable sliders in a random order across problems and participants. Participants were required to move each of these sliders to a point between 0 and 100 indicating the estimated percentage. The default starting estimate was 0. One of the six outcomes was a "foil" that was not an outcome from the deck in question, but was selected from one of the other decks. The foil card was included to identify participants who may not have been paying attention to the sampling process. Participants were only allowed to proceed to the next problem when the sum of the sliders for each deck equaled 100%. Also note that we specifically asked participants to estimate the probability of each of the different cards being observed on some future (hypothetical) sample – this is different from asking for estimates of the sample frequencies of the outcomes they experienced. At the conclusion of the experiment participants were debriefed, thanked, and reimbursed. Reimbursement was calculated by converting every 100 experiment dollars to 1 Australian dollar, where reimbursement was contingent on choices and not probability estimates.

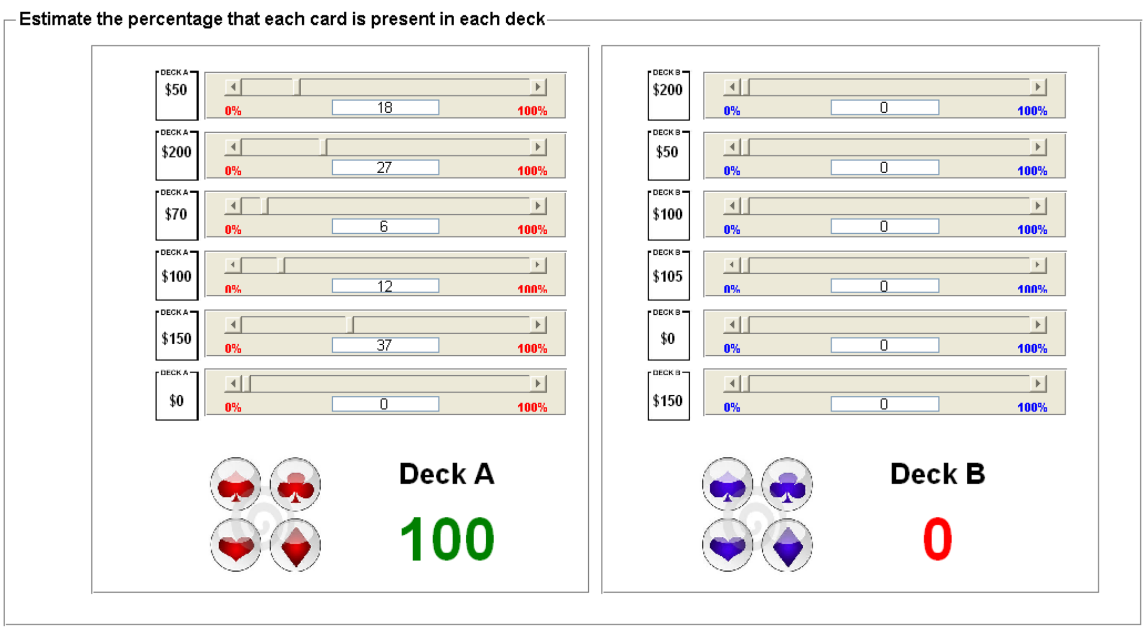

Figure 2. Screenshot of the probability knowledge task. The participant has already used the sliders to make their estimates of the outcome probabilities for Deck A, but not yet for Deck B.

3. Results

3.1. Preferences

To gauge the extent to which the participants' actual samples differed from the population outcome distributions, we calculated the absolute difference between these two distributions averaged across outcomes. Participants' sampled frequencies were closer to the objective probabilities in the feedback group than in the sampling group ($t(912) = 12.02$, $p < .001$). This makes sense given that the median number of samples taken in the sampling group (51) was half the number of samples taken in the feedback group. Even so, the sample size was still considerably higher than previous experiments with similar incentives (e.g., 15 – 19 as reported in Hau, Pleskac, & Hertwig, 2010).

Figure 3 displays the percentage of participants preferring each lottery, averaged over all problems in which the lottery was presented. In general, preferences favored the lotteries that minimized the possibility of obtaining zero (i.e., the riskless, peaked, and shortshot decks), which is consistent with the idea of a negatively accelerated utility function for gains, as assumed by many theories of choice. There was a strong, positive correlation between preferences in the feedback and sampling paradigms (r = .64, p = .01), which is consistent with previous studies where participants were free to choose their own stopping point in the sampling task (Erev et al., 2010). There were only two notable points of departure between preferences observed in the feedback and sampling groups: participants in the feedback group showed a relatively stronger preference for the rectangular lottery ($\chi^2_{(1,N=330)} = 4.89$, $p = .027$) but a relatively weaker preference for the bimodal lottery ($\chi^2_{(1,N=319)} = 7.07$, $p = .008$). Additionally, and despite a number of important differences between our design and the one used by Lopes and Oden (1999), the lottery preferences were qualitatively similar between the studies; the major points of departure were that our participants showed greater overall indifference and also less preference for the riskless lottery. The absence of a clear "description-experience gap" is unsurprising given that this phenomenon is traditionally perceived as being a function of asymmetric rare events, which were absent in the majority of our problems.

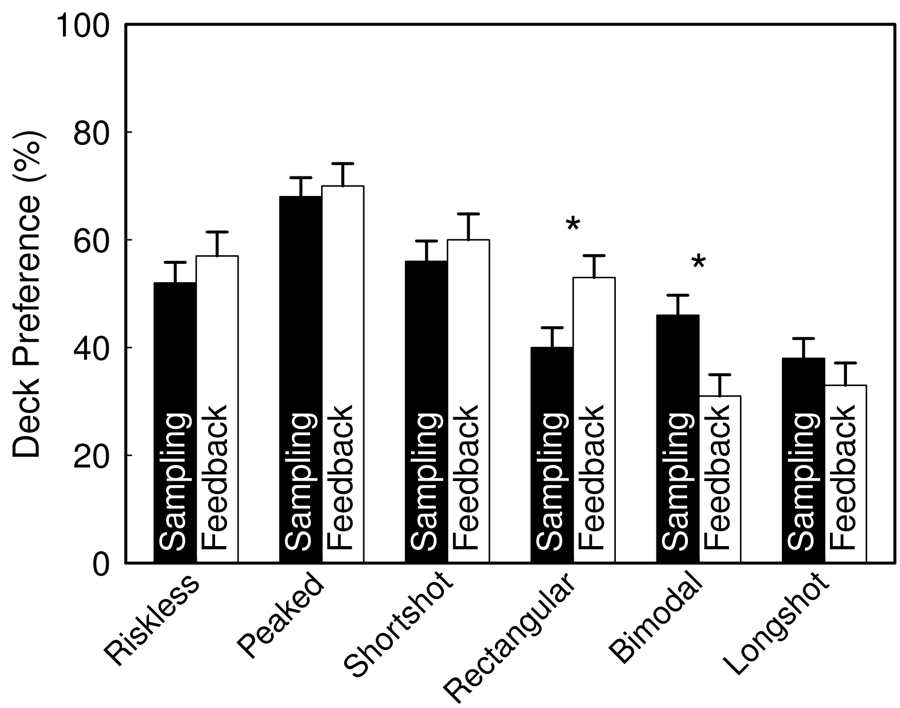

Figure 3. Percentage of participants preferring each lottery, averaged over problems.

Preferences between pairs of lotteries are displayed in Table 1. Each score indicates the average preference for the column-named lottery over the row-named lottery, and asterisks denote preferences in which a lottery was significantly preferred over indifference (i.e., 50-50) by a z-test. For example, in the first row of Table 1 the 79 value is asterisked, which indicates that a significant proportion of participants preferred the peaked lottery over the longshot lottery in the sampling task. Only a few contrasts showed significant differences in preference between the groups which is to be expected given the lotteries all had identical expected values. In general, two decks associated with the largest number of zero outcomes (the longshot and bimodal decks) were least preferred, particularly when paired with the two decks that had the fewest number of zero outcomes (the shortshot and peaked decks).

Table 1: Percentage of participants selecting the column-named lottery over the row-named lottery, separately for both experimental conditions. Abbreviations refer to deck type: RL=riskless, PK=peaked, SS=shortshot, RC=rectangular, BM=bimodal, LS=longshot. * $p < .05$ by (unadjusted) z-test.

	Sampling						Feedback				
	RL	PK	SS	RC	BM		RL	PK	SS	RC	BM
LS	60	79*	75*	46	49	LS	62	72	88*	63	55
BM	55	69	59	39		BM	82*	70	88*	62	
RC	59	64	58			RC	59	79*	56		
SS	47	66				SS	44	70			
PK	39					PK	39				

3.2. Probability Knowledge

The median probability estimate assigned to outcomes is plotted against the actual sampled frequency of the outcome, in the samples observed by participants, in Figure 4 (the whiskers show the 5^{th} and 95^{th} percentiles of these estimates, calculated across participants and problems).[3] The probability distribution estimates demonstrate that participants had quite good knowledge of the probabilities of the lottery outcomes: in both conditions the median estimate assigned to outcomes increased almost always with increasing sample probability. Nevertheless, there was a tendency to overestimate the probability of rare outcomes and underestimate the probability of frequent outcomes, which is shown by the inverted-S shapes in Figure 4. This pattern appeared in both sampling and feedback paradigms, and was also almost unchanged when we instead graphed the probability estimates against the *population* probability of the outcome (i.e., the proportion of times it would appear in the long run, defined by Lopes & Oden, 1999) rather than the sampled frequency of the outcome (i.e., the proportion of times it really did appear, in the samples observed by participants).

We confirmed the statistical reliability of differences between the pattern of estimated

[3]We first confirmed that participants' estimates of the probability of foil outcomes (i.e., outcomes that were not part of the lottery) were accurate. Across participants and problems, foil cards were assigned zero probability 62% of the time. On the remaining 38% of problems in which the foils were assigned some non-zero probability, this estimate was still small – 11% on average – and on 37% of these trials the foil was assigned the smallest of the estimated probabilties. Since the foils were mostly well identified by participants, we do not analyse those data further

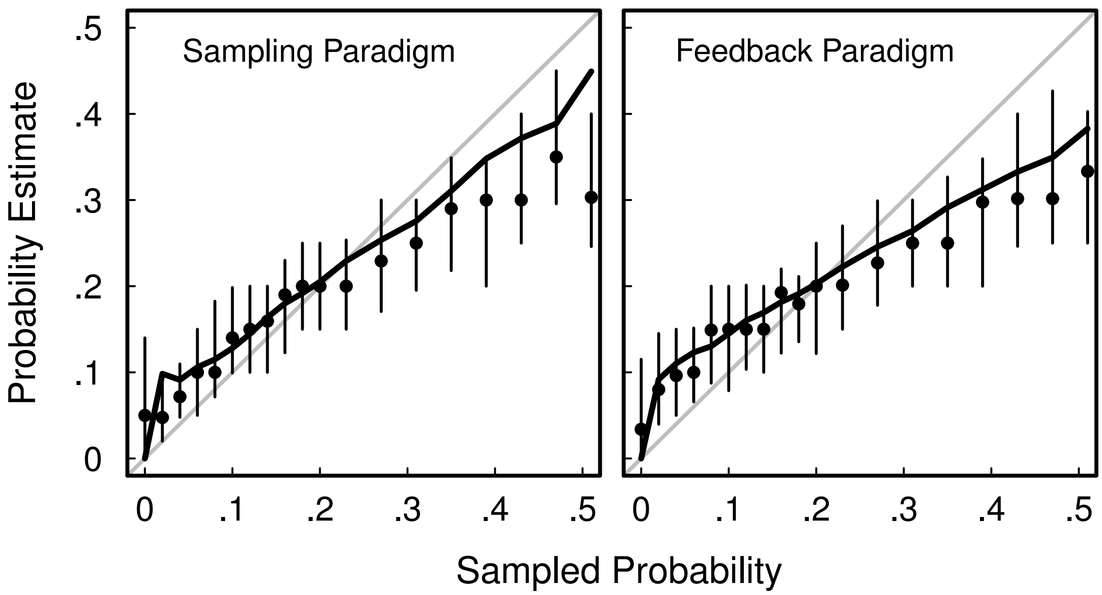

Figure 4. Probability estimates from participants (y-axes) against the sampled probability for the corresponding outcome (x-axis) in the sampling (left panel) and feedback (right panel) conditions. The circles are medians, the whiskers show 5^{th} and 95^{th} percentiles across participants. The gray identity lines indicate the trend that probability estimates would follow if they were perfectly reflective of the sampled probabilities. The black lines show median probability estimates of the ExCON model under the parameter values described in text.

versus sampled probabilities and the $y = x$ line (which would indicate probability estimates that perfectly reflect the sampled probabilities) using the Wald-Wolfowitz (or "runs") test. This analysis examines the sign of successive residuals, on the assumption that – under the null hypothesis of perfect calibration – residuals should be randomly distributed either side of zero. The Wald-Wolfowitz test indicated highly significant non-random scatter around the $y = x$ line in both paradigms, both p's $< .001$, reflecting the run of positive residuals for low sample probabilities and negative residuals for high sample probabilities.

A mundane explanation for the inverse-S shape of the probability estimates is a mixture of participants, some who were perfectly calibrated (no inverted-S shape) and others who were simply not engaged in our task. If a participant was not paying attention to the task, then their probability estimates would be unrelated to the observed sample probabilities, and their graphs would show horizontal lines at $y = .2$ (or $y = .17$, if the foil cards were considered). It is possible that a mixture of these two types of participants could lead to the inverted-S shapes, even if no individual participant displayed such a pattern. We ruled out this explanation by removing many participants to leave us with only the very best (those most likely to be engaged in the task), and the inverted-S shape remained. Specifically, we separately implemented two very strict inclusion criteria, and then re-examined the probability estimates. The first check excluded any participant who sampled fewer than 20 outcomes from either lottery on any problem, which is considerably larger than the median number of samples typically observed across *both* lotteries in experience-based choice tasks (e.g., Hau et al., 2010). This criterion excluded many more people from the sampling task, leaving only 10 of 40 (25%) participants, compared to the feedback task, which left 42 of 65 (65%) participants. Beginning again with all data, our second exclusion criterion removed any participant who estimated a non-zero probability for a foil card from either lottery for *any* problem, which left only 5 of 40 (13%) participants in the sampling condition and 10 of 65 (15%) in the feedback condition. For both of these strict exclusion criteria the inverse-S trend in probability estimates remained (graphs not shown), with the Wald-Wolfowitz test indicating highly significant departures from the $y = x$ line for both the sampling and feedback conditions for the two exclusion criteria, all p's $< .001$.

4. The Exemplar Confusion Model: Simultaneously Accounting for Choices and Probability Knowledge

Exemplar models assume that observers record a memory trace each time they encounter a stimulus, and later use these traces to make inferences about their experiences. In this section we develop an exemplar-based model that can simultaneously account for the choice preferences and probability estimates in our data. We then demonstrate that this new model can account for choice behavior in simpler (more standard) problems just as well as the current best performing models of experience-based choice, using the TPT model competition data from Erev et al. (2010).

4.1. The Exemplar Confusion (ExCON) Model

The exemplar confusion (ExCON) model is an extension of the primed sampler models described by Erev et al. (2010), which we refer to as a k-sampler. The standard k-sampler simply draws k samples from each lottery, stores these as k exemplars per lottery, and then chooses whichever lottery has the greater sample mean. Since only k exemplars per lottery can be maintained the k-sampler instantiates a limited memory capacity. Despite its simplicity, the k-sampler accounts for choice data quite well compared to more complex models, as demonstrated in the TPT (Erev et al., 2010). However, the k-sampler fails to capture biased probability knowledge because it necessarily predicts accurate estimates on average.

The ExCON model makes two important modifications to the k-sampler. First, the ExCON replaces the k-sampler's limit on memory *capacity* (k) with a limit on memory *accuracy*. We instantiate memory imperfection by degrading the information content of exemplars. Specifically, we introduce a sample-by-sample confusion process in which new exemplars can interfere with previously stored exemplars (proactive interference; Keppel & Underwood, 1962). Note that this form of confusion occurs with the passing of events rather than the passing of time alone, an assumption that has precedence in the memory literature (e.g., Lewandowsky, Duncan, & Brown, 2004; Lewandowsky, Geiger, & Oberauer, 2008). The confusion process occurs as follows. For each alternative we assume an empty, limitless memory store. Each time a sample outcome is drawn from a lottery, a memory trace is added to the appropriate store. The only assumption we make about this memory trace is that it stores the outcome value for the observed sample. Each time a new sample is added to a store the confusion process operates within that store, leading to a small chance of "mixing up" the exemplars. Specifically, each exemplar in the store has a fixed probability (α) of having its outcome value confused – that is, substituted with the outcome value from another exemplar already in the store. If an exemplar's outcome value is confused, the new outcome value assigned to that exemplar is chosen uniformly from the *list* of all exemplar labels in the store (rather than uniformly from the *distribution* of all exemplars in the store).[4] The α parameter governing the exemplar confusion process is the sole free parameter of the model to be estimated from data.

The second difference between the ExCON and the k-sampler is the replacement of the simple choice rule of preferring the option with the higher sample mean with a utility maximization rule. Specifically, at the conclusion of sampling the ExCON preference is determined by whichever set of exemplars has the highest average utility. The utility function we implement is one of diminishing marginal utility, specifically, we use the utility function for gains specified by Lopes and Oden (1999): $u(x) = x^{0.551}$. This duplication reflects our belief that the choice rule at the core of both description and experience-based choice is the expected utility theory assumption of multiplying some function of probability with an outcome value, and then maximizing.

4.2. ExCON Implementation

The ExCON model was provided with the same sequences of outcomes experienced by the participants on a problem-by-problem basis. The sequence of outcomes for each problem and participant was shown to the model 100 times (i.e., there were 100 synthetic model participants, Monte-Carlo replicates, for every real participant). After the sampling process had finished for each Monte-Carlo replicate, the model preference was inferred differently for the sampling and feedback paradigms – consistent with the human data. In the sampling paradigm, the model choice was determined by the model's decision rule: the lottery with the highest average utility. In the feedback paradigm, after each sample outcome we used the ExCON's decision rule to determine which lottery it would choose on the following sample, for all 100 samples in each problem, and then inferred the preference of the Monte-Carlo replicate as its modal choice over the last 50 trials.[5]

We simulated model predictions for 20 different values of the α parameter, which spanned a feasible range based on conceptual constraints: $[0, .1]$. The upper end of this interval implies a very high degree of confusion: if each sample leads to a 0.1 chance of confusion, the chance of accurately maintaining a single memory trace (exemplar) following 10 samples is only $(1 - .1)^{10} = .349$, and following 100 samples is $(1 - .1)^{100} < .001$. Therefore, the ExCON instantiates that a *single* exemplar is more and more likely to be confused as new exemplars enter the store. Nonetheless, the *set* of exemplars is more and more likely to approximate the true distribution up until a certain sample size, after which

[4]The ExCON as implemented assumes that exemplars can only be confused with outcomes from the same lottery. An alternative model allows confusion of exemplar labels with outcomes from both lotteries. Note that this alternative version of the model only matters when different outcomes are contained in each lottery, which occurred in our experiment in less than half of the problems (see Figure 1). Nonetheless, we investigated this alternative, and found that all model fits, parameter estimates, and general conclusions were essentially unchanged, so we do not report it below.

[5]As with the human data, performance of the three models did not markedly differ when using the modal preference across all 100 samples, the last 50 samples, or the final sample in each problem.

the set of exemplars become noisier. We discuss this counter-intuitive prediction of the model, and its surprising consistency with data, in the conclusion section (cf. Figure 8).

We assessed the goodness-of-fit of the ExCON model on its ability to predict two outcome measures in data: choice and probability knowledge. Choice prediction accuracy was calculated as the proportion of times the model successfully predicted the choice made by the participant when exposed to the same sequence of samples for each problem. We refer to this method as "trial-by-trial agreement". Probability estimate prediction accuracy was calculated as the sum of the squared deviations between participant and model probability estimates across the sampled probability bins. This measures an average distance between the data and the model when represented in a plot such as Figure 4. Probability estimates were derived from the frequency of each outcome in the ExCON memory stores.[6] Parameters were estimated separately for the sampling and feedback groups. As we previously argued, the two outcome measures provide additional constraint that has been lacking in most previous model fitting exercises.

4.3. ExCON Evaluation

Trial-by-trial agreement and the sum-of-squares error in probability knowledge for the ExCON, across its parameter space, are shown in Figure 5 separately for the sampling and feedback groups. For trial-by-trial choice agreement larger values indicate better performance, but for probability estimates smaller sum-squared error indicates better performance. As α increased, the trial-by-trial agreement with choice decreased (left panel). In addition, choices in the feedback group were better predicted than choices in the sampling group. One reason for this may be because the feedback data were less noisy given the way we implemented the ExCON model (i.e., modal preference over last 50 trials).

As shown in the right panel of Figure 5, the probability estimates of the model were more sensitive to the parameter setting than the predictions for choice preferences. The ExCON's predictions for probability estimates agreed quite well with the data for any value of $\alpha > .02$, for both groups. Note that, when $\alpha = 0$, the model predicts a $y = x$ line for probability estimates because it perfectly recalls all observed outcomes without noise. However, as α increases, the predicted probability estimates become inverse-S shaped and then progressively flatter as the exemplars are contaminated by random noise.

Figure 5 demonstrates the challenge in simultaneously predicting choice preferences and probability knowledge. Trial-by-trial agreement with choice preferences is generally better when there is less noise in the system (i.e., low α) whereas the inverted-S shape present in the probability estimate data is best captured when the system is subject to substantial levels of noise (i.e., higher α). Since participants simultaneously made choices and estimated probabilities in reference to the same lotteries, we argue that a successful model should capture behavior associated with both measures under the same parameter settings. To this end we sought to locate a parameter "sweet spot" that successfully predicted both outcome measures simultaneously. These parameter settings are indicated by the double circles in Figure 5, described in the upper part of Table 2, and used to illustrate the model predictions in Figures 4, 6, and 8.

4.3.1. Choice Behavior.

[6] Note that the ExCON model as described has no way of giving a non-zero rating to a foil card.

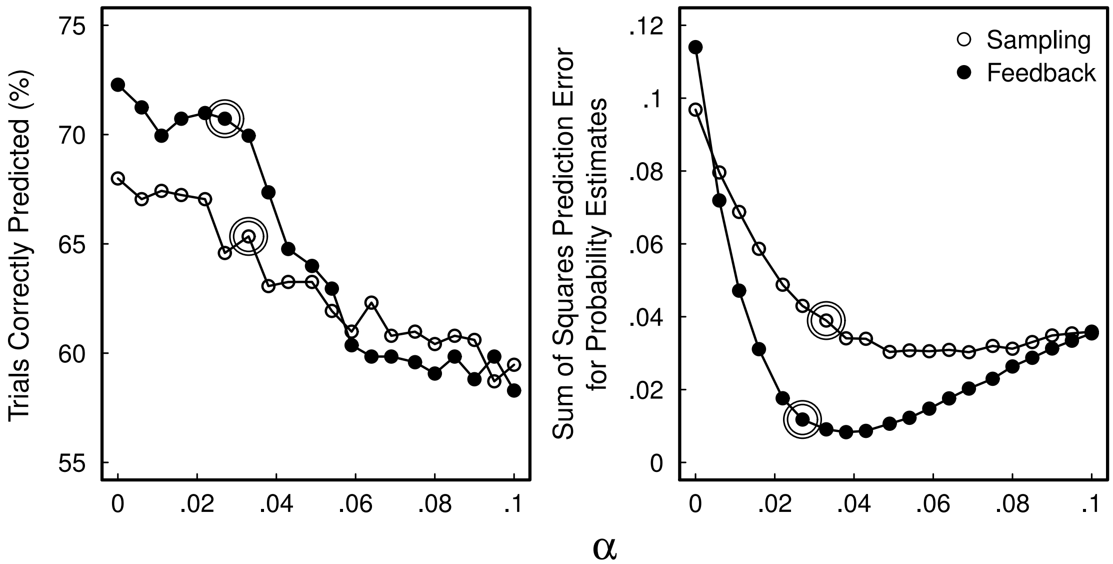

Figure 5. Predictions of the ExCON model as a function of the explored parameter space (α, where α is the probability of exemplar confusion). The left panel displays the percentage of choices correctly predicted on a trial-by-trial basis. The right panel shows the sum-squared prediction error of the ExCON versus the probability estimates in data. Open and filled symbols represent the sampling and feedback paradigms, respectively. In trial-by-trial agreement a larger value indicates better performance whereas a smaller value of sum-squared error indicates better performance. The double circles in each panel represent the parameter values selected to simultaneously maximize trial-by-trial agreement and minimize prediction error in estimating probability distributions, fit separately to data from the sampling and feedback tasks.

Table 2 shows the best fitting parameter estimates and trial-by-trial agreement for the ExCON model. The ExCON predicted approximately 65% trial-by-trial agreement in the sampling paradigm, and a little over 70% in the feedback paradigm. In isolation it is difficult to determine whether these two values indicate good or poor performance, so we provide comparison of the ExCON's performance with two benchmarks.

Table 2: Trial-by-trial agreement and probability estimates for the ExCON model, the natural mean heuristic and participants' probability estimates, for the sampling and feedback conditions. Values in parentheses show the parameter estimates for the ExCON model.

	ExCON	Natural Mean Heuristic	Prob. Estimates
Sampling	65.3% ($\alpha = .033$)	62.9%	62.9%
Feedback	70.7% ($\alpha = .027$)	64.5%	56.7%

The first benchmark was the "natural mean" heuristic, which assumes that a decision-maker prefers the lottery that has the highest observed sample mean (Hertwig & Pleskac, 2008). Note that the ExCON reduces to the natural mean heuristic when $\alpha = 0$ and assuming a linear utility function. As shown in the middle column of Table 2, the natural mean heuristic was outperformed by the ExCON in both the feedback and sampling group data. The second benchmark maximized choice in light of the inferred expected value of each lottery as indicated by participant's probability estimates. As shown in the last column of Table 2, this method was also outperformed by the ExCON in both the feedback and sampling group data. The relative success of the ExCON against these two benchmarks highlights the benefit of implementing a choice rule that maximizes utility rather than value.

We can gain greater insight to the operation of the ExCON model by exploring the model's preference for specific lottery pairs. The average ExCON choice between pairs of lotteries is displayed in Table 3, in the same format as Table 1. In this table, bold face shows those lottery pairings for which ExCON produced the same modal preference as the participants (i.e., both data and model scored above or below 50%). Impressively, on this lottery-by-lottery basis, the ExCON correctly predicts 24 out of 30 lottery preferences observed in data even though the model was not explicitly fit to this table (as would occur in descriptive models, such as CPT). A more detailed comparison of Tables 1 and 3 reveals a strong correspondence between the ExCON predictions and actual data; the only point of departure from this general trend is that the ExCON more strongly prefers the Riskless deck (because it has the highest expected utility under an assumption of diminishing marginal utility).

Table 3: Percentage of ExCON Monte Carlo replicates selecting the column-named lottery over the row-named lottery, separately for the Sampling and Feedback conditions. Abbreviations are as described in Table 1. Lottery pairings for which data and model had the same modal preference are shown in bold face.

	Sampling						Feedback				
	RL	PK	SS	RC	BM		RL	PK	SS	RC	BM
LS	**73**	**62**	48	**39**	**23**	**LS**	**84**	**73**	**61**	46	**52**
BM	**88**	**77**	**59**	**48**		BM	**99**	**76**	**79**	**80**	
RC	**89**	**73**	**55**			RC	**95**	**73**	**56**		
SS	73	**82**				SS	95	**63**			
PK	78					PK	86				

We also attempted to understand how well the ExCON model addressed the overestimation-underweighting paradox. It is clear from Figure 4 that participants overestimated the probability of rare events, however, in the current stimulus set it is more difficult to demonstrate that choice behavior was consistent with underweighting of rare events (compared to decreasing marginal utility). For example, preferences for a model that maximizes value while underweighting rare events (perceived probability = probability^2) and preferences for a model that maximizes utility ($u(x) = x^{0.551}$) agree in 80% of the choice sets. Because of this overlap, it is difficult to conclude that participants underweighted rare events because similar preferences could be produced by appropriate weighting combined with decreasing marginal utility. For this reason we cannot state that the ExCON solves the overestimation-underweighting paradox because it is unclear whether such a paradox exists in our data. The safest conclusion is that the ExCON is consistent with the data and the data are consistent with both underweighting and diminishing marginal utility.

4.3.2. Probability Estimates.

The black lines overlaid on Figure 4 illustrate the ExCON probability estimate predictions, under the parameter settings described in Table 2. The model captures the qualitative patterns in the probability estimates, namely, the overestimation of rare outcomes and underestimation of more common outcomes. The inverse-S shape is reminiscent of the Prospect Theory (PT) probability weighting function. Combining this probability weighting function with a value function corresponding to diminishing marginal utility from a reference point allows PT to account for the "four fold" pattern: risk-*averse* behavior in gains involving moderate probabilities and for small probability losses but risk-*seeking* behavior in losses involving moderate probabilities and for small probability gains (Gonzalez & Wu, 1999; Wu & Gonzalez, 1996, 1998).

A limitation of PT and its variants (including Cumulative Prospect Theory, CPT) is that it is a descriptive theory of choice and therefore does not provide a *process* understanding for why the impact of probability on the attractiveness of a prospect varies as a function of the prospect rarity. The common explanation for the inverse-S shape of the probability weighting function is that decision-makers exhibit diminishing sensitivity to the impossibility and certainty reference points (Fox & See, 2003). Indeed, it is most common in the literature for probability weighting and probability estimates to be inferred from choice or used as model inputs, rather than model outputs used to constrain predictive models. The ExCON model, in contrast, was designed to provide a process understanding for how probability knowledge is gained and represented.

To illustrate the connection between CPT, ExCON and our probability estimate data, we reproduce the probability estimates from the participants and ExCON from Figure 4 in Figure 6, as circles and light gray lines, respectively. In Figure 6 we add two new lines that represent CPT's probability weighting function. We fit CPT's probability weighting function (parameter γ) to the probability estimates in our data by minimizing sum-squared deviations between model and data, shown as solid black lines. These fits of CPT were solely to probability estimates and not choices, as this provides CPT's probability weighting function the best possible chance to predict our probability estimate data (the fit to probability estimates can only get worse by fitting the model to choice data). The sampling and feedback conditions led to very similar parameter estimates for γ. CPT does a reasonable job of capturing the trends in probability estimate data, except that it overestimates low probability events relative to data and the ExCON. The ExCON arguably predicts probability estimates closer to data than CPT for rare events (up until probability $\approx .25$), even though ExCON was fit to both choices and probability estimates (thus providing a harsher test of the ExCON relative to CPT's probability weighting function). In contrast, CPT makes predictions closer to data for common events (probability $> .3$). The dark gray lines shown in Figure 6 show CPT's probability estimate predictions under the γ parameter estimate obtained by Lopes and Oden (1999) in their fit of CPT to their data. It is interesting that even though this parameter estimate comes from different data in a description-based choice paradigm, CPT's probability estimate predictions are not too discrepant from our participants' actual estimates. The most important point here is that ExCON's ability to not only predict choice but also probability estimates consistent with data is a significant advance: rather than just using probability estimates as inputs (like CPT), we provide a psychological process describing how probability knowledge arises.

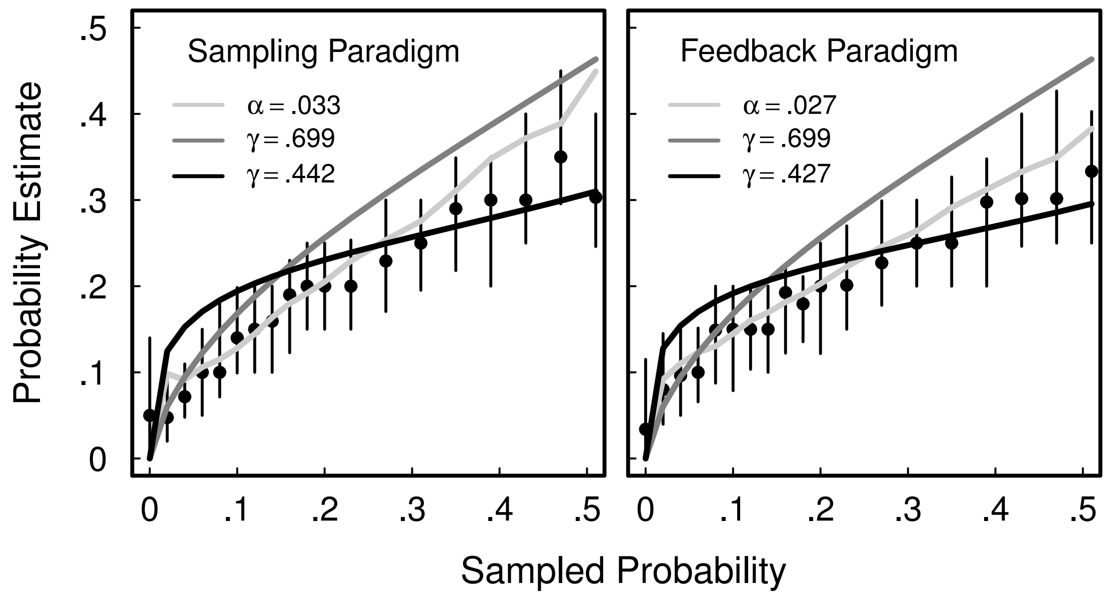

Figure 6. Probability estimates from participants (y-axes) against the sampled probability for the corresponding outcome (x-axis) in the sampling (left panel) and feedback (right panel) conditions. The circles are medians, the whiskers show 5^{th} and 95^{th} percentiles across participants. The light gray lines show show median probability estimates of the ExCON model under the parameter values described in text. The dark gray and black lines show predictions of CPT using the probability weighting function parameter estimate (γ) described in Lopes and Oden (1999), and when fit to our data, respectively.

5. Technion Prediction Tournament

Several features of our experiment were different from typical experience-based choice tasks, such as our use of complex 5-outcome lotteries compared to the standard 2-outcome lotteries, and requesting estimates of probability distributions from participants. Moreover, the way we chose to evaluate the performance of the ExCON differed to the measures used in other recent investigations of experience-based choice. In this section we evaluate the performance of the ExCON model against recent models, on a level playing field, by using the data from the TPT (Erev et al., 2010). Our aims are 1) to highlight that the ExCON performs as well in the TPT as two leading competitor models in accounting for experience-based choice; 2) that important lessons can be learned from examining different methods of evaluating models; and 3) to illustrate the importance of multiple sources of data for constraining model fits. To achieve these aims we first outline the structure of the TPT, followed by a description of the IBL model and how the three models – ExCON, k-sampler and IBL – were fit to the TPT data.

The TPT evaluated models separately using data from three conditions: decisions from description, and decisions from experience either in feedback or sampling conditions. In each of the three conditions there were data from 60 choice problems between pairs of lotteries. In each problem, one lottery had a low probability of a high payoff outcome, otherwise a low payoff outcome, and the other had a certain, medium payoff outcome. Across choices, one third of gambles were gain-framed, one third were loss-framed, and the remaining third contained a mix of gain and loss outcomes. The TPT used one data set for model calibration, and another data set (from an identical problem distribution as the original problems) for the evaluation.

5.1. The Instance Based Learning (IBL) model

The IBL model (Gonzalez & Dutt, 2011; Lejarraga, Dutt, & Gonzalez, 2011) is another exemplar based model that goes further than the k-sampler or ExCON models by also describing the processes involved in choosing samples from the lotteries, and the corresponding tension between exploration and exploitation. This is accomplished by two extra model components: a "swapping" rule, which describes when to switch sampling from one lottery to the other; and a "stopping" rule, which describes when to stop sampling and make a final, consequential choice (in the sampling paradigm).

To more precisely evaluate choice behavior, and retain our trial-by-trial agreement measure of predictive adequacy, we evaluate the performance of the IBL model without the swapping and stopping rules. Instead, we provide the model with the exact sequences of outcomes sampled by TPT participants, as we did for the ExCON model in our experiment. By doing this for each model, we have effectively endowed all models with omniscient swapping and stopping rules, allowing fair comparsion. For rigorous evaluation of the swapping and stopping rules in the IBL model, see Gonzalez and Dutt (2011), Lejarraga et al. (2011).[7]

After sampling the lotteries, the IBL makes a decision by selecting the lottery with the highest "blended value". The blended value is the total of all instances from that lottery stored in memory, weighted by the probability of retrieving each instance from memory. Retrieval probability depends on each outcome's frequency of occurrence and also the recency of those occurrences (for greater detail on IBL model equations see Gonzalez & Dutt, 2011; Lejarraga et al., 2011). Our implementation of the IBL model contains two free parameters in the sampling condition: the decay of instances over time (i.e., the influence of recency in sampling, d), and a noise parameter (σ). There was an additional free parameter in the feedback condition: the probability of inertia (pInertia). pInertia refers to the probability of simply repeating the choice on the previous sample regardless of the obtained outcome. This is not the same as including the swapping rule, because we still provided the model with the same sequence of outcomes as experienced by the participants. Including the inertia process influences the preference about the next lottery to sample following each outcome, and hence influences decisions. Removing the swapping and stopping rules from the IBL leaves all three models to be evaluated solely on their decision mechanisms: highest blended value in the IBL; highest average utility in the ExCON; and the highest sample mean in the k-sampler.

[7] To check that inclusion or exclusion of the stopping and swapping rules did not greatly alter a model's predictions for choice preferences, we re-ran our simulations of the ExCON model with these rules incorporated using the data from our experiment. The results were very similar – but a little noisier – than without the rules, for both choice preferences and probability estimates.

5.2. Entering the Models in the TPT

We simulated the process of entering the k-sampler, IBL, and ExCON models in the TPT, by first estimating their parameters using data from the estimation set, and then evaluating their predictive performance against the competition data set. As before, we evaluated the performance of the models in predicting choice behavior using the fine-grained measure of trial-by-trial agreement between the models' modal choices and participants' individual responses. This analysis is similar in spirit to that used in the TPT, but differs in detail. In the TPT, the performance of each model was reported as the average proportion of agreement (across problems) between the modal preference of the model and of the participants (e.g., Erev et al., 2010), which we refer to as "PAgree". Our measure of trial-by-trial agreement maintains more information than PAgree. For instance, suppose there was a choice problem between lotteries A and B for which 51% of participants preferred lottery A. The PAgree method would treat a model that perfectly agreed with the data (51% preference) as equal to a model that always preferred lottery A (100% preference), but the trial-by-trial measure captures the large difference between these models. Such considerations mean that the trial-by-trial measure of agreement is generally lower than PAgree (compare our Tables 2 and 4 with Erev et al., 2010). For ease of comparison, we also report Erev et al.'s PAgree.

Each model was simulated in as similar manner as possible as when fitting the ExCON to our own data, but there were three differences. Firstly, because the TPT lotteries included both positive and negative outcomes, we used both the positive and negative branches of the utility function estimated by Lopes and Oden (1999) for the ExCON:

$$f(n) = \begin{cases} x^{0.551} & \text{if } x \geq 0 \\ -\left(-x^{0.970}\right) & \text{if } x < 0 \end{cases}. \tag{1}$$

The second change was that we did not calculate predictions for probability estimates from the models, since there were no data for comparison. The third change was that in addition to simulating predictions across a range in each model's parameter space, we used an optimization algorithm to calibrate the parameters.

We measured the goodness-of-fit of each model to the estimation data set based on its ability to predict trial-by-trial agreement. We adjusted the models' parameters to maximize trial-by-trial agreement using particle swarm optimization (PSO: for more detail see Kennedy & Eberhart, 1995). As when fitting data from our experiment, we simulated 100 Monte Carlo replicates for each participant from each model per PSO iteration, where each Monte Carlo replicate was shown the same sequences of outcomes as experienced by the participants in the TPT, on a problem-by-problem basis. When implementing PSO we established the lower and upper limits on the parameters of each model from previous implementations of the model and/or conceptual constraints. For the ExCON model we set lower and upper limits on the α parameter equal to those used when fitting our own data: $[0, .1]$. In the k-sampler we set the limits on the k parameter, rounded to whole numbers, on the interval $[1, 50]$. If the number of observed outcomes from a participant was lower than k, we simply sampled k outcomes from the observed samples with replacement. Finally, the parameters of the IBL were constrained for the decay parameter to the interval $[.01 - 10]$ and σ between $[.01, 2]$, and for the feedback task there was also the pInertia parameter, constrained to the unit interval. These parameter ranges for the IBL model covered the best fitting values of previous implementations of the IBL model to similar experience-based choice data (e.g., Gonzalez & Dutt, 2011; Lejarraga et al., 2011).

Before discussion of the best fitting parameter estimates, we explore the fit of the models against the estimation data set, across the feasible parameter ranges, using the profile plots shown in Figure 7. For each free parameter in each model, we calculated model predictions for 20 different values that spanned the feasible range for that parameter. For the k-sampler we estimated model predictions for 20 integers in the range $k = 1 \ldots 50$. In the ExCON, we examined 20 log-spaced values for α in the interval $[0, .1]$. For the IBL model we simulated selected values of decay, log-spaced between $d = .1 - 10$, as a function of 20 log-spaced values of the noise parameter ($\sigma = .1 - 2$), and a range in pInertia (.05, .5, .95) for the feedback paradigm.

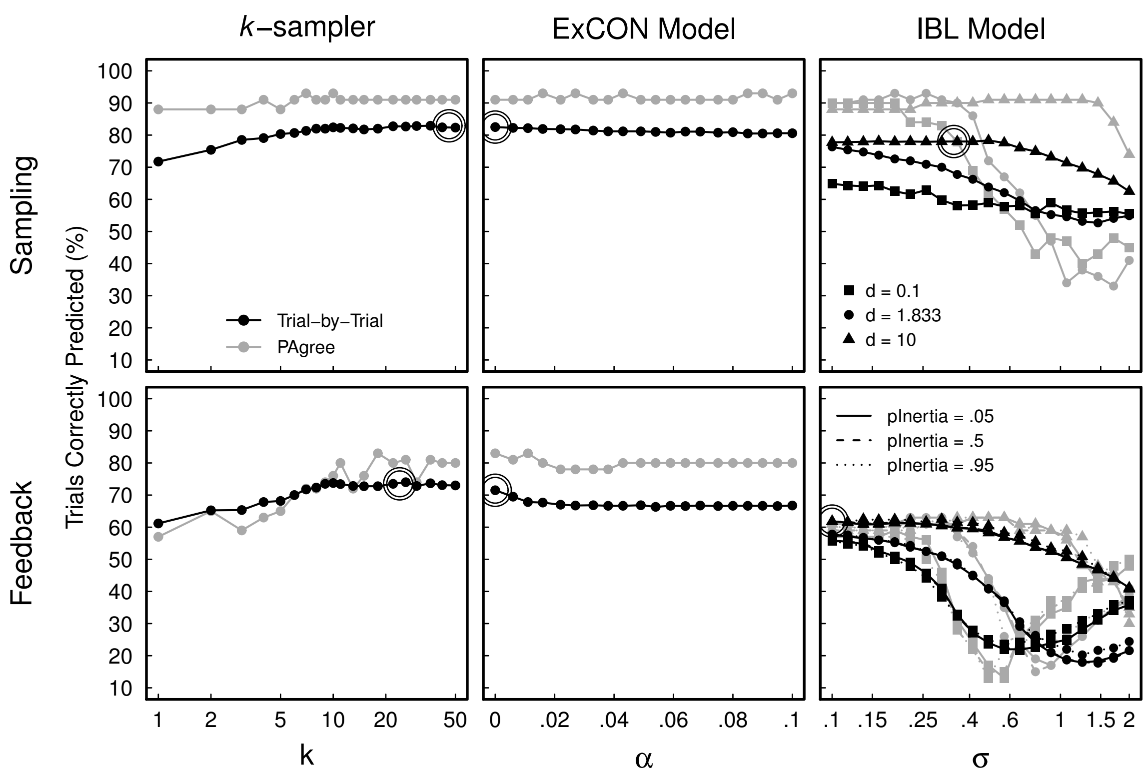

Figure 7. Percentage of choices correctly predicted by the k-sampler, ExCON and IBL models (left, middle, and right columns, respectively) across their respective parameter ranges in the sampling (top row) and feedback (bottom row) paradigms for the TPT estimation set. For the IBL we plot three representative values of decay from the 20 simulated values (as different symbols), and for the feedback condition we plot three representative values of pInertia (as different line types). Trial-by-trial agreement is shown as black lines and PAgree as gray lines. The double circle in each panel represents the parameter estimate that maximized trial-by-trial agreement.

The profile plots demonstrate that our method of calculating trial-by-trial agreement (black lines) is qualitatively consistent with Erev et al.'s (2010) PAgree (gray lines), although the trial-by-trial measure provides a less optimistic view of the models. The k-sampler predicted relatively constant trial-by-trial agreement when $k > 7$ in both sampling and feedback paradigms. Re-assuringly, when $k = 5$ in the sampling paradigm, the k-sampler predicted a PAgree value of 90%, which is the same outcome that Erev et al. observed in their model competition (see their Table 3), suggesting that our implementation of their model and the PAgree method is the same.

Predictions of the ExCON model agreed well with the data for almost all of its parameter range (middle column of Figure 7), with a slight increase in predicted trial-by-trial agreement for very small values of α. The right column of Figure 7 shows that performance of the IBL model was much more sensitive to parameterization. For example, when there was strong decay of instances over time, the noise parameter had relatively little influence on trial-by-trial agreement. This occurs because, if there is large decay, an instance is forgotten very soon after it was observed, and before it can be influenced by recollection noise. In contrast, when decay is low there is a large and detrimental influence of noise on choice prediction accuracy, particularly in the feedback paradigm. The lower right panel of Figure 7 also suggests that the pInertia parameter of the IBL model has very little influence on choice prediction accuracy, for both trial-by-trial agreement and PAgree.[8]

The best fitting parameter estimates, trial-by-trial agreement and PAgree (in parentheses) for each model are shown in Table 4. The performance of each model was impressive across both the estimation and competition data sets, although better in the sampling than feedback paradigm. Overall, the k-sampler performed best, followed closely by the ExCON model and then the IBL. To compare the adequacy of our model fits with those reported for other models in the TPT we use PAgree values, since this was the common metric to both studies. According to PAgree, the models reported here performed about as well as, or better than, those reported in the TPT (Table 3, Erev et al., 2010) – except for the IBL model in the feedback paradigm. The IBL also performed a little more poorly on these data than previously reported by Gonzalez and Dutt (2011). These discrepancies are noteworthy because they highlight the impact of choices about model implementation, in particular whether choices should be aggregated across individuals (see also Hills & Hertwig, in press; Gonzalez & Dutt, in press), but also whether the model inputs should be simulated outcomes from gambles or the actual outcomes experienced by participants. Previous investigations have used the former method (Gonzalez & Dutt, 2011; Lejarraga et al., 2011); we chose the latter in order to stay closer to the data. Strong cases can be made for both methods, but in our implementation we wanted to focus tightly on the decision mechanisms of the models thus making the second method, in which the models are not allowed to choose their own balance between exploration and exploitation, the more suitable one.

The three models reported here performed about as well as the winners and runners-up in both the sampling *and* feedback paradigms, in contrast to the TPT where each model was successful only in the sampling *or* feedback paradigm. The best fitting parameter estimates of the three models tend towards extremes in their parameter ranges, where the

[8]We repeated the PSO model fits of the IBL model to data from the feedback condition excluding the pInertia parameter (Dutt & Gonzalez, 2012). Parameter estimates for d and σ were virtually identical to the fits that included the pInertia parameter, as was the quality of choice prediction.

Table 4: Percentage of participants' preferences correctly predicted on a trial-by-trial basis by the k-sampler, ExCON and IBL models for the estimation and competition data sets of the Technion Prediction Tournament, using the best fitting parameter values from the estimation data set (described in text). Numbers in parentheses represent PAgree values.

		k-sampler	ExCON	IBL
Parameters	Sampling	$k = 46$	$\alpha = 0$	$d = 7.037$, $\sigma = .341$
	Feedback	$k = 24$	$\alpha = 0$	$d = 9.883$, $\sigma = .100$, pInertia= .948
Estimation Data	Sampling	82.9 (91.4)	82.5 (91.4)	78.0 (89.7)
	Feedback	73.5 (77.8)	71.5 (83.3)	61.9 (59.3)
Competition Data	Sampling	84.7 (92.7)	84.2 (92.7)	71.7 (80.0)
	Feedback	74.2 (91.7)	73.8 (83.3)	66.7 (83.3)

models approximate simpler accounts of the choice process, as in the simpler "natural mean" heuristic (i.e., large k, $\alpha = 0$, low σ). This is a concern for model plausibility, particularly in the feedback paradigm where each trial involves many samples. However, as suggested in Figure 7, selecting a parameter setting – for any model – with a much higher noise value confers only a slight loss of performance. For example, in the sampling paradigm if we changed to $k = 7$ for the k-sampler, and $\alpha = .011$ for the ExCON, and for the feedback paradigm to $k = 10$ and $\alpha = .006$, trial-by-trial agreement is almost unchanged. This insensitivity highlights the third aim of this section – to illustrate that because the TPT has only one source of data (choices) the parameters of the models are under-constrained. More precise model evaluation is possible with more data, such as the two streams from our experiment (choice and probability estimates).

6. Conclusions

We have attempted to make three general points regarding experience-based choice. The first is that a model of experience-based choice should account not only for participants' choices in multiple paradigms and datasets but also describe the process by which participants construct and represent the probability distributions upon which those choices are based. To that end we designed a new model – the exemplar confusion model (ExCON) – that provides a process explanation for how people might acquire knowledge of outcome distributions. We assumed that stored memories for each sampled outcome – exemplars – were used to represent the outcome distribution for each alternative option. We also assumed an imperfect, noisy memory system that we implemented by way of a confusion process. The confusion process operated each time a new exemplar was stored, in a similar fashion to proactive memory interference. We showed that the ExCON could successfully predict decision-makers' behavior across two experience-based choice paradigms both in new data that we collected (Table 2) as well as an important existing dataset (the Technion Prediction Tournament, Table 4).

The confusion process of the ExCON model leads to a surprising prediction about the relationship between the accuracy of probability knowledge and the number of samples drawn from a lottery. Any reasonable model, including the ExCON, predicts that increasing sample size leads to more accurate probability knowledge. The surprising thing is that the ExCON only predicts this relationship up until a certain sample size, after which it predicts either increasing or unchanging error in probability knowledge. This prediction arises as a result of the confusion process in the exemplars: as sample size increases there is a greater chance of confusion amongst the set of exemplars. More generally, this prediction relates to the ExCON's notion of a memory system with limited accuracy rather than limited capacity; we can keep filling our memory with exemplars, but only a portion of those exemplars remain "functional" in the sense that they produce accurate inferences.

The quantitative details of the ExCON prediction, such as the point at which more samples no longer improve prediction, depend on the model's parameter (α) and the lottery. Although counter-intuitive, Figure 8 shows that the data from our experiment might be consistent with this model prediction. The large gray symbols represent the median absolute error in participants' estimates of the probability of lottery outcomes, across the 5^{th}, 10^{th}, ..., 95^{th} percentiles of the sample size distribution, as a function of the number of samples observed from the lottery, across all lotteries and participants. The black lines represent ExCON's predictions under the parameter settings described in Table 2. The agreement between model and data is particularly impressive given the model was not fit to this aspect of the data, nor was the experiment designed to test this prediction. Future research should more rigorously test this prediction of the ExCON, by explicitly manipulating sample size.

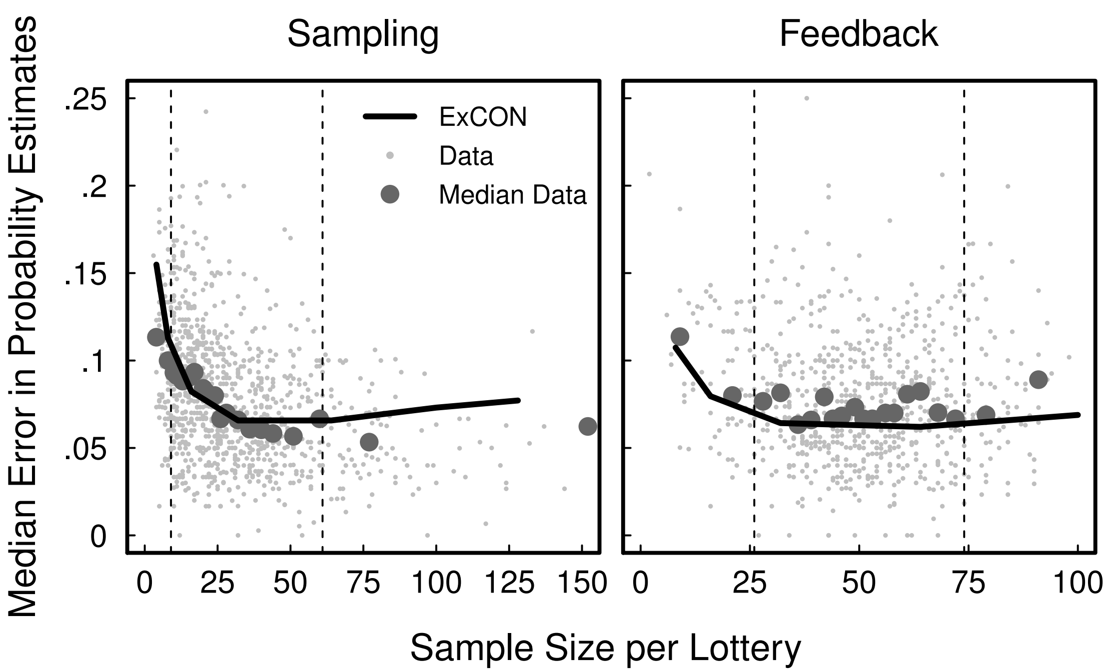

Figure 8. Median absolute error in estimating the probability of lottery outcomes (relative to population probabilities, y-axes) as a function of sample size (x-axes) in the sampling and feedback conditions (left and right panels, respectively). The small gray symbols show data from all problems and participants. The large, dark gray symbols represent median data across the 5^{th}, 10^{th}, ..., 95^{th} percentiles of the sample size distribution, with the dashed vertical lines representing the 10^{th} and 90^{th} percentiles of the sample size distribution, bounding the middle 80% of data. The black lines show predictions of the ExCON model under the parameter settings described in Table 2.

Our second point relates to the method by which models are evaluated. Traditional datasets upon which models are evaluated may not provide sufficient constraint and therefore permit models with very different assumptions to perform equally well. Such under-constraint is apparent from Figure 7, which shows the model fits to data from the TPT. This under-constraint is due to the limited variability in the problem sets (a binary choice between a safe option and a two-outcome risky option), limited data sources used to constrain the models (only choice), and a very general prediction goal (prediction of aggregated choice proportion). To address these limitations we conducted a new experiment that presented decision-makers with more complex problem sets (binary choice between 5-outcome options; see Figure 1), and evaluated model success by simultaneously predicting probability knowledge and trial-by-trial choices. In collecting probability estimates we depart from previous models of risky choice (e.g. Prospect Theory) which infer probability weighting from choices or measure probability estimates to serve as model inputs. One of our novel contributions is to instead use this stream of data to constrain model development.

The third point relates to the similarities between description- and experience-based choice. Recent studies have highlighted the preference differences observed between situations when decision-makers make choices based on explicit knowledge of the outcome distribution and situations when decision-makers make choices after integrating outcome knowledge sequentially encountered over time (Hertwig & Erev, 2009). Although these situations are different, they nonetheless share core features, such as the need to integrate outcome and probability information. For this reason the ExCON is fundamentally grounded in existing research on description-based choice via its incorporation of a utility maximization choice rule using a parameter estimated directly from the description data of Lopes and Oden (1999). The role played by this decision rule was highlighted by its superiority over rules that maximized value rather than utility (Table 2). Demonstrating that ExCON was able to closely model the TPT data using such a choice rule is perhaps surprising in light of the observation made by the TPT competition organizers that the concept of probability did not play an important role in the models submitted to the feedback competition (Erev et al., 2010, p. 35). The superiority of the utility maximizing rule also has implications for current debates about the explanatory power of value-based models over comparison-based models of choice (e.g., Vlaev, Chater, Stewart, & Brown, 2011). The latter class of models eschew any form of intermediate value computation, and thus would not fare as well in accounting for our data.

A complete model of experience-based choice will need to address all components of the decision-making process: how search is conducted, how discovered information is used to construct a representation, and how that representation is used to form a preference. Most models are only concerned with explaining the preference component. Here we have explicitly modeled how discovered information is used to form a representation and how that representation is used to form a preference. A challenge for future research is to develop a comprehensive model that also incorporates the search aspect: how is the exploration/exploitation trade-off reconciled? What determines the order and nature in which options are searched (cf. Hills & Hertwig, 2010) and how are evidence-thresholds determined? A valuable approach might be to combine the admirably simple "swapping" and "stopping" rules of the IBL model (which already address these search questions, though for debate about these mechanisms see Hills & Hertwig, in press; Gonzalez & Dutt, in press) with the probability representation and decision mechanisms of the current ExCON model. This would be an extensive undertaking, and the lessons learned from the current investigation regarding model-constraint, measures of fit and methods of implementation would be important to heed, but the potential benefit of a generalizable comprehensive model of the processes underlying experience-based choice would, we argue, be well worth the effort.

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