Prelim. Q1,

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$$G_j + R_X^o \underset{k_{-,j}}{\overset{k_{+,j}}{\rightleftharpoons}} (G_j : R_X)_C$$

$$(G_j : R_X)_C \xrightarrow{k_{I,j}} (G_j : R_X)_O$$

$$(G_j : R_X)_O \xrightarrow{k_{A,j}} R_X + G_j$$

$$(G_j : R_X)_O \xrightarrow{k_{E,j}} m_j + R_X + G_j$$

$G_j \equiv$ Gene Concentration

$R_X^o \equiv$ free RNAP concentration

$(G_j : R_X)_O \equiv$ open complex concentration

$(G_j : R_X)_C \equiv$ Closed Complex concentration

$$(6)\ r_{X,j} \equiv k_{E,j}(G_j : R_X)_O$$

First perform Conservation Balance analysis:

$$(1)\ \frac{d(G_j : R_X)_O}{dt} = k_{I,j}(G_j : R_X)_C - k_{A,j}(G_j : R_X)_O - k_{E,j}(G_j : R_X)_O$$

$$(2)\ \frac{d(G_j : R_X)_C}{dt} = k_{+,j} G_j R_X^o - k_{-,j}(G_j : R_X)_C - k_{I,j}(G_j : R_X)_C$$

Balance RNAP concentration, Assume RNAP concentration is fixed/constant

Total

$$(3)\ R_{X,T} = R_X^o + (G_j : R_X)_C + (G_j : R_X)_O + \sum_{i=1,j}^{N} [(G_i : R_X)_C + (G_i : R_X)_O]$$

Assume steady state $\Rightarrow$ time derivatives $\Rightarrow 0$

Rearranging (1) & (2)

$$(1) \Rightarrow k_{I,j}(G_j:R_x)_C = (G_j:R_x)_O(k_{A,j}+k_{E,j})$$

$$(4)\quad (G_j:R_x)_O = \underbrace{\frac{k_{I,j}}{(k_{A,j}+k_{E,j})}}_{\equiv \tau_{x,j}^{-1}}(G_j:R_x)_C$$

$$(2) \Rightarrow k_{+,j}G_jR_x^0 = (G_j:R_x)_C(k_{-,j}+k_{I,j})$$

$$(*)\quad (G_j:R_x)_C = \underbrace{\frac{k_{+,j}G_j}{(k_{-,j}+k_{I,j})}}_{\equiv K_{x,j}^{-1}}R_x^0$$

Rearranging (3)

$$\underset{(5)}{\Rightarrow} R_x^0 = R_{x,T} - (G_j:R_x)_C - (G_j:R_x)_O - \sum_{i=1,j}^{N}\left[(G_i:R_x)_O + (G_i:R_x)_C\right]$$

Subbing (*) into (4)

$$(G_j:R_x)_O = \tau_{x,j}^{-1}K_{x,j}^{-1}G_jR_x^0$$

(4) genarlizes for i^{th} genes to: + substitude (*) — generalized to/for i^{th} gene

$$(4g)\quad (G_i:R_x)_O = \tau_{x,i}^{-1}K_{x,i}^{-1}G_iR_x^0$$

(*g) generalized (*) for i^{th} gene:

$$(G_i:R_x)_O = K_{x,i}^{-1}G_iR_x^0$$

Subbing (4), (4g), (*), and (*g) into (3)

$$(3n)\ R_{x,T} = R_x^0 + k_{x,j}^{-1}G_jR_x^0 + k_{x,j}^{-1}\tau_{x,j}^{-1}G_jR_x^0 + R_x^0\sum_{i=1,j}^{N}\left[k_{x,i}^{-1}G_i + k_{x,i}^{-1}\tau_{x,i}^{-1}G_i\right]$$

Reavranging (3n.)

$$(3.n.1)\ R_X^{0} = \frac{R_{X,T}}{1 + K_{X,j}^{-1} G_j + K_{X,j}^{-1} \tau_{X,j}^{-1} G_j + \sum_{i=1,\neq j}^{N} [K_{X,i}^{-1} G_i + K_{X,i}^{-1} \tau_{X,i}^{-1} G_i]}$$

Subbing (3.n.1), (4), into (6)

$$r_{X,j} = \frac{k_{E,j} K_{X,j}^{-1} \tau_{X,j}^{-1} G_j R_{X,T}}{1 + K_{X,j}^{-1} G_j + K_{X,j}^{-1} \tau_{X,j}^{-1} G_j + \sum_{i=1,\neq j}^{N} [K_{X,i}^{-1} G_i + K_{X,i}^{-1} \tau_{X,i}^{-1} G_i]}$$

$$= k_{E,j} R_{X,T} \left(\frac{G_j}{K_{X,j} \tau_{X,j} + \tau_{X,j} G_j + G_j + \sum_{i=1,\neq j}^{N} \left[\frac{K_{X,j} \tau_{X,j} G_i}{K_{X,i}} \left(1 + \frac{1}{\tau_{X,i}}\right) \right]} \right)$$

$$= k_{E,j} R_{X,T} \left(\frac{G_j}{K_{X,j} \tau_{X,j} + (1 + \tau_{X,j}) G_j + \underbrace{\sum_{i=1,\neq j}^{N} \left[\frac{K_{X,j} \tau_{X,j}}{K_{X,i} \tau_{X,i}} (1 + \tau_{X,i}) G_i \right]}_{\equiv \varepsilon_j}} \right)$$

$$\Rightarrow \boxed{r_{X,j} = k_{E,j} R_{X,T} \left(\frac{G_j}{K_{X,j} \tau_{X,j} + (1 + \tau_{X,j}) G_j + \varepsilon_j} \right) \quad \boxed{(7)}}$$

a.)

Thus completing the derivation for P.Q.1.a..

1.b.) If $K_{X,i} \to \infty$ (for all i) i.e $K_{X,i} >> \frac{K_{X,j} \tau_{X,j}}{\tau_{X,i}} (1 + \tau_{X,i}) G_i$

Then $\varepsilon_j \to 0$ and eqn. (7) reduces to,

$$r_{X,j} = k_{E,j} R_{X,T} \left(\frac{G_j}{K_{X,j} \tau_{X,j} + (1 + \tau_{X,j}) G_j} \right)$$

which is the same / approximately equivalent to the 1-gene system

Prelim. Q1. Continued.

1.b.) Also if $G_{x,i} \rightarrow 0$ for all i

Then $\varepsilon_j \rightarrow 0$ and eqn. (7) reduces to,

$$r_{x,j} = k_{E,j} R_{x,T} \left(\frac{G_j}{k_{x,j} \tau_{x,j} + (1 + \tau_{x,j}) G_j} \right)$$

which is the same/ approximately equivalent to the 1-gene system

Also linking / considering $k_{x,i} \rightarrow \infty$, in more depth,

This can occur, without effecting $\tau_{x,i}$, by having $k_{-,i} \rightarrow \infty$ for all i, -OR- if $k_{+,i} \rightarrow 0$ for all i; in both cases $k_{x,i} \rightarrow \infty$ for all i, and $\tau_{x,i}$ is uneffect in both cases, Thus $\varepsilon_j \rightarrow 0$ and eqn. (7) reduces to,

$$r_{x,j} = k_{E,j} R_{x,T} \left(\frac{G_j}{k_{x,j} \tau_{x,j} + (1 + \tau_{x,j}) G_j} \right)$$

Which is the same/ approximately equivalent to the 1-gene system