

Endogenous Technological Change Along the Demographic Transition^{*}

Markus Pettersson[†]

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Abstract

Does population ageing hurt output per capita in the advanced economies? Standard calibrations of life-cycle models with exogenous growth that consider two fundamental and opposing forces, capital deepening versus a declining employment rate, predict yes. Using a quantitative overlapping generations model with realistic demographics and R&D-driven endogenous growth, this paper challenges the standard prediction through a third possibility: that current demographic trends boost R&D investment and thus generate technological change. Calibrated to the United States, the model indicates that the demographic transition between 1950 and 2100 increases output per capita by 0.41 percent per year until 2000 and by 0.18 percent per year overall, thereby explaining 10 to 20 percent of observed US growth. The key mechanism is the endogeneity of technological change, whose growth contribution triples that of capital deepening, and removing this channel eliminates the positive impact.

Keywords: demographic transition, endogenous growth, OLG model.

JEL Codes: E17, E25, J11, O30, O40.

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[†] Department of Economics, Stockholm University. Email: markus.pettersson@su.se.

1 Introduction

In 1950 there were 14 people aged 65 and above for every 100 people aged 20 to 64 in the United States. This number doubled by 2020 and is projected to quadruple by the end of the century. [Figure 1](#) shows that similar trends apply worldwide. Also illustrated is the notion that economic growth is slowing down across high-income countries, and there is a widespread view that the ongoing population ageing is an important driver of this decline. How do current demographic trends actually affect output per capita? And how will they impact future growth? These questions lie at the heart of any policy discussion surrounding these shifts in the population structure.

It is widely acknowledged that population ageing influences output per capita through two major channels. First, ageing reduces the number of workers relative to the rest of the population, thereby decreasing output per capita. Second, rising life expectancy increases savings in anticipation of longer retirement, which raises investment and capital accumulation. The subsequent capital deepening increases output per capita. Life-cycle models calibrated to the advanced economies that compare these channels typically predict the former to dominate. Krueger and Ludwig (2007) and Ludwig, Schelkle and Vogel (2012) for instance find that demographic change during the twenty-first century generates cumulative declines in output per capita on the order of 10 percent.

This paper emphasises a third key channel that reverses both the sign and the magnitude of this prediction: the increase in savings induced by demographic change may also be used to finance research and development (R&D), which improves per-capita output through innovations that generate technological progress. The need to distinguish R&D from capital investment is essential in these analyses due to the nonrival nature of technology. An innovation can be used simultaneously by any number of workers at no additional cost and therefore raises everyone's productivity. An extra unit of capital, on the other hand, improves productivity only insofar as it increases capital per effective worker. Technological change therefore provides a more potent mechanism than capital deepening.

To formalise this argument and to explore its importance, I develop a quantitative general equilibrium model that combines two standard macroeconomic frameworks. On the supply side, the model features endogenous growth à la Romer (1990), with monopolistically competitive intermediate producers and an R&D sector whose innovations improve productivity by expanding the variety of intermediate goods. The household side follows in the Auerbach and Kotlikoff (1987) tradition with a large number of overlapping generations and a realistic population structure. These life-cycle households consume, save, and make labour supply decisions on both hours worked and the timing of retirement subject to changes in household size, mortality risk, income risk, borrowing constraints, and progressive income taxation. Savings are either invested in physical capital or provide funding for R&D in exchange for ownership stakes in the new firms it generates, the latter creating the link between household behaviour and technological progress.

The model exhibits semi-endogenous growth as in Jones (1995), Kortum (1997), and Segerstrom (1998), but the rich life-cycle setup goes substantially beyond the representative agent and perpetual-youth environments typically found in these growth models.¹ This contribution is crucial for the question at hand. In semi-endogenous growth theory, long-run economic growth is proportional to population growth. A long-run interpretation of the decline in rich-world population growth observed since the mid-twentieth century therefore suggests that current demographic trends negatively impact economic

¹ Semi-endogenous growth analyses of demographic change with a representative household or perpetual-youth framework include Prettner (2013), Prettner and Trimborn (2017), Peters and Walsh (2021), and Jones (2022a).

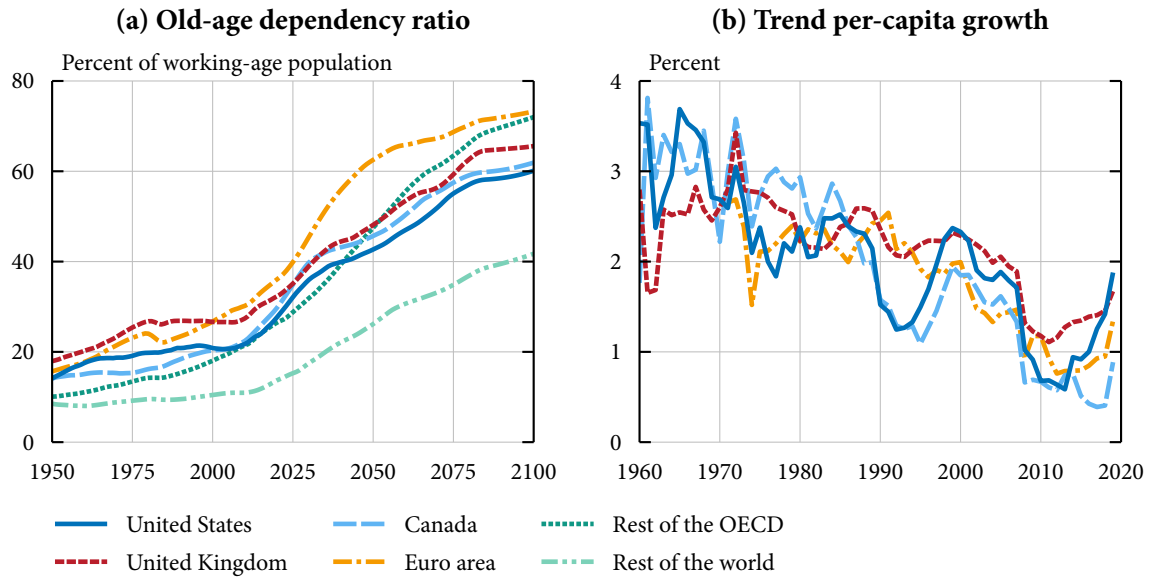


Figure 1. Ageing and growth across major economies.

Source: Holston, Laubach and Williams (2017) and the United Nations (2022).

growth. Yet, in advanced economies like the United States, the United Kingdom, Canada, and Western Europe, the demographic shift since World War II primarily reflects two temporary forces: an initial surge in fertility (the baby boom) and steadily rising survival rates beyond the age of 50.² Neither affects long-run population growth when people have finite lifespans. Such shifts in the population structure generate temporary growth that permanently affects the *level* of income, but these effects cannot be properly examined through the lens of steady-state growth in a representative agent or perpetual-youth model.

After analytically deriving the mechanisms in a simplified three-generation setting, I outline and calibrate the full model and quantify these effects and their impact on output per capita along the equilibrium path induced by the demographic transition between 1950 and 2100. In doing so, I make sure to keep the long-run trend fixed so that the demographic transition and the corresponding growth effects are treated as exclusively transitory. This quantitative analysis focuses on the United States, a large economy at the technological frontier, since the endogenous growth framework arguably describes US technological change better than it does for smaller economies in which most technology is imported. As shown in Figure 1a, the United States is also a useful demographic benchmark since its ageing process lies roughly in between the fastest ageing rich countries and the younger non-OECD countries.

Contrary to what a standard long-run analysis suggests, I find that the demographic transition is a net positive. On average, output per capita increases by 0.41 percent per year between 1950 and 2000 and by 0.18 percent per year overall. The former corresponds to roughly 20 percent of observed US growth over the same period, thereby making demographics as important to US post-war growth as rising educational attainment according to estimates by Fernald and Jones (2014). Counterfactual simulations also reveal that the baby boom and rising life expectancy after the age of 50 explain around a quarter each of these results. Changes in the age structure due to the nonstationary population already present in 1950 and general increases in population size explain the remainder. As far as the impact on output per capita is

² Figures E.1 and E.2 in Online Appendix E document these two facts. Besides the baby boom, fertility remains stable from the mid-1930s onward. Likewise, mortality during the first 50 years of life is largely unchanged after 1950.

concerned, these findings leave little reason to worry about current demographic change in general and about population ageing in particular.

The positive effect dissipates after the turn of the millennium when the baby boom retires, but the growth impact does not turn negative on average during the twenty-first century. Yet, the shift when this happens is sizeable: the growth rate declines by 0.60 percentage points between 1995 and 2030. Demographics can therefore explain a large chunk of the decline in observed growth rates. It is important to stress though that this decline is *not* a result of demographic change being inherently detrimental to economic output, but rather because of a reversion back from the above-average growth that it induced in the first place. This interpretation contrasts with the short-run secular stagnation view of Eggertsson, Lancastre and Summers (2019), Eggertsson, Mehrotra and Robbins (2019), and Jones (2023), which sees current population ageing as a significant drag on output per capita.

At the core of these results lies technological change, which contributes three to four times more to output growth than capital deepening. This difference reflects the extra bang for the buck obtained from technology being nonrival. Together these two channels more than offset the negative impact of a growing share of retired households. Capital deepening alone cannot accomplish this: treating technology as exogenous (which is readily achieved as a special case of the benchmark model) eliminates the positive effect and leads to an 11.1 percent cumulative decline in output per capita between 1950 and 2100. This decline is quantitatively similar to Krueger and Ludwig (2007) and contrasts with a 31.5 percent increase in the baseline scenario, thus underlining that whether the demographic transition raises or lowers output per capita depends crucially on whether or not we account for its impact on technological progress.³

The household heterogeneity in the model separates this paper not only from the growth literature but also from other quantitative macroeconomic papers incorporating R&D-based growth. This line of research typically examines business cycle dynamics under a representative household assumption.⁴ Two closely related exceptions are Aksoy *et al.* (2019) and Basso and Jimeno (2021), who modify the Comin and Gertler (2006) framework to study the growth effects of demographic change in the early twenty-first century. Both employ Gertler's (1999) perpetual-youth model, which effectively boils down to a stylised two-generation consumption-savings model of workers and retirees, and simulate transitions from high-growth to low-growth steady states. In contrast, this paper incorporates full-fledged life-cycle behaviour and realistic population dynamics, pinpoints the key mechanisms analytically, investigates longer transition paths, and considers transitory demographic changes around a fixed long-run trend. As highlighted earlier, the last point matters for interpretation: for comparable decades, both papers find quantitatively similar growth declines to here and consequently conclude that population ageing lowers growth, the opposite conclusion of this paper.

The focus on economic growth also distinguishes the analysis from the abundance of research that employs similar quantitative life-cycle models to examine the macroeconomic impacts of demographic change, which primarily concentrates on topics such as fiscal policy, international capital flows, wealth accumulation, or asset returns.⁵ Studies like Krueger and Ludwig (2007) and Cooley and Henriksen (2018) that nevertheless touch upon the issue of growth typically restrict attention to the comparison

³ The importance of technology also echoes the empirical analyses of Cutler *et al.* (1990) and Acemoglu and Restrepo (2017, 2022), who argue that ageing leads to another type of technical change: automation.

⁴ Examples include Comin and Gertler (2006), Nuño (2011), Benigno and Fornaro (2018), Anzoategui *et al.* (2019), Bianchi, Kung and Morales (2019), and Okada (2023).

⁵ Besides papers mentioned elsewhere, a non-exhaustive list includes De Nardi, Imrohoroglu and Sargent (1999), Storesletten (2000), Fehr, Jokisch and Kotlikoff (2004), Börsch-Supan, Ludwig and Winter (2006), Domeij and Flodén (2006b), Attanasio,

between capital deepening and the declining fraction of people working. While some papers incorporate additional adjustment margins such as human capital accumulation (Ludwig, Schelkle and Vogel, 2012; Vandenbroucke, 2021) or automation (Heer and Irmen, 2014; Benzell *et al.*, 2021), no research to my knowledge conducts these quantitative analyses with endogenous R&D-driven technological change. This paper provides an initial step in this direction.

2 Identifying the Mechanisms: Results From a Simple Model

As stressed in the introduction, the observed and projected demographic transition in the rich world is characterised by two salient features after 1950: the temporary surge in fertility between the late 1930s and early 1970s that caused the baby boom, and an ageing of the population from rising survival rates for people aged 50 and above.⁶ If people have finite lifespans, neither affects the long-run population growth rate. To develop intuition for how these changes interact with the mechanisms outlined in this paper, it is useful to first analyse a simplified version of the quantitative model in the subsequent sections.

To that end, consider the Solow-esque world given in Table 1, in which aggregate household behaviour is captured by an exogenous savings rate sr and a labour supply L_t which grows by the rate of the population n . Households allocate savings either to next-period capital K_{t+1} or to R&D investment Q_t . Together with the current stock of technologies Z_t , the latter generates new technologies through an R&D production function as in Jones (1995). Aggregate output Y_t is pinned down by a Cobb-Douglas production function with capital, technology, and labour as inputs. It is further assumed that capital and technologies fully depreciate after one period and that the allocation of savings follows an exogenous rule in which a fraction $\bar{\rho}$ is invested in capital each period. With full depreciation of capital and technology, this fixed allocation of savings is also the market equilibrium if the model is expanded along the lines of Romer (1990) and Jones (1995) to include the returns on capital and R&D investments, but imposing it directly simplifies the exposition.

We are interested in how the demographic structure affects output per capita, $y_t \equiv Y_t/N_t$, where N_t is the size of the population. It turns out convenient then to consider a decomposition using the Cobb-Douglas production function as follows:

$$y_t = Z_t \left(\frac{K_t}{Y_t} \right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}. \quad (1)$$

Consequently, output per capita is proportional to the stock of technology (or TFP), a capital intensity (captured by the capital-output ratio), and an employment rate. Determining the impact of demographics on Equation (1) requires an understanding of how demographics affect each of these decomposing factors, which is what we turn to next.

2.1 Components of Output Per Capita in Steady State

The components of Equation (1) can be analytically solved for in a steady state (that is, an equilibrium in which all variables grow at constant, possibly zero, rates). First, note that Table 1 outlines a semi-endogenous growth model. To see this, divide (S2) by Z_t to obtain a gross growth rate of TFP equal to $1 + g_{Zt} \equiv \frac{Z_{t+1}}{Z_t} = Q_t^\lambda Z_t^{\phi-1}$. Since R&D investment must grow by the rate of aggregate output in steady state,

Kitao and Violante (2007), Kotlikoff, Smetters and Walliser (2007), İmrohoroğlu and Kitao (2012), Kitao (2014), Carvalho, Ferrero and Nechio (2016), Auclert *et al.* (2021), and Gagnon, Johannsen and López-Salido (2021).

⁶ See Figures E.1 and E.2 in Online Appendix E.

Table 1. Summary of the simple model.

OUTPUT	$Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha},$	$0 < \alpha < 1$	(S1)
R&D OUTPUT	$Z_{t+1} = Q_t^\lambda Z_t^\phi,$	$0 < \lambda \leq 1, \phi < 1$	(S2)
ASSET MARKET	$sr Y_t = K_{t+1} + Q_t$		(S3)
ALLOCATION	$Q_t = (1 - \bar{\rho}) sr Y_t,$	$0 < \bar{\rho} < 1$	(S4)
POPULATION GROWTH	$L_{t+1} = (1 + n) L_t$		(S5)

the right-hand side implies that TFP growth in steady state necessarily satisfies

$$1 + g_Z = (1 + n)^\gamma, \quad \text{where} \quad \gamma \equiv \frac{\lambda}{1 - \phi - \lambda}, \quad (2)$$

and this is a function of exogenous parameters, hence the term “semi”. Equation (2) together with (S3) and (S4) allow us to write the capital-output ratio as

$$\frac{K_t}{Y_t} = \frac{\bar{\rho} sr}{(1 + n)^{\frac{\gamma(1-\phi)}{\lambda}}}, \quad (3)$$

which shows that the capital intensity is the share of output allocated to capital investment divided by the aggregate growth rate, just like the Solow model. Equations (2) and (3) can then be combined with Table 1 to solve for the stock of TFP as

$$Z_t = \left[\frac{1 - \bar{\rho}}{\bar{\rho}} \left(\frac{\bar{\rho} sr}{(1 + n)^{\frac{\gamma(1-\phi)}{\lambda}}} \right)^{\frac{1}{1-\alpha}} \right]^\gamma L_t^\gamma. \quad (4)$$

Equation (4) writes TFP as the product of two factors. The first one is an R&D intensity, which is determined by the savings rate and thus incorporates household behaviour and composition. The second one, L_t^γ , is the scale effect on output per capita present in all semi-endogenous growth models, as discussed by Jones (2005), and reflects the fact that technology is nonrival: when the R&D process generates a new discovery, it can be used simultaneously by everyone at no additional cost. A larger population raises aggregate investment, even without changes in household behaviour, and this raises the stock of TFP. But the nonrivalry of technology means that output per capita depends on the overall stock of TFP rather than TFP per capita. Consequently, output per capita increases simply by virtue of a larger population.

Now add households to endogenise the employment rate and the savings rate. Specifically, consider a three-generation life-cycle model consisting of young, middle-aged, and old households (indexed by $j = 1, 2, 3$, with cohort size N_{jt}), in which young and middle-aged households supply labour inelastically and old households are retired. If the number of young households grows by the rate $1 + n$ in each period and a j -year-old survives till age $j + 1$ with probability s_j , then the steady-state employment rate is given by

$$\frac{L_t}{N_t} = \frac{N_{1t} + N_{2t}}{N_{1t} + N_{2t} + N_{3t}} = \frac{1 + \frac{s_1}{1+n}}{1 + \frac{s_1}{1+n} \left(1 + \frac{s_2}{1+n} \right)}. \quad (5)$$

To keep things simple, suppose households have logarithmic preferences over consumption c , that young households are hand-to-mouth, and that annuity markets are present. Households then maximise expected lifetime utility $\log(c_1) + s_1\beta \log(c_2) + s_1s_2\beta^2 \log(c_3)$ subject to the budget constraints $c_1 = w$, $c_2 + a = w$, and $c_3 = \frac{1+r}{s_2} a$, where β is a subjective discount factor, a denotes savings, w is the wage rate, and r is the interest rate. The solution to the household problem is characterised by a savings policy $a_{t+1} = \frac{s_2\beta}{1+s_2\beta} w_t$. The aggregate savings rate in steady state can then be written

$$sr = \frac{a_{t+1}N_{2t}}{Y_t} = \frac{w_t L_t}{Y_t} \frac{s_3\beta}{1+s_3\beta} \frac{\frac{s_2}{1+n}}{1 + \frac{s_2}{1+n}}, \quad (6)$$

which is a product of the labour share of aggregate output, the share of labour income allocated to savings, and the share of saving, middle-aged workers in the labour force. Note that the labour share of output is constant given the Cobb-Douglas production function and the absence of automation technologies.

2.2 Population Ageing via Increases in Survival Rates

Consider now an ageing of the population through an increase in either of the survival rates s_1 and s_2 . This naturally pushes down the employment rate through a mechanical increase in the old-age dependency ratio, as seen in Equation (5), which reduces output per capita by (1). Meanwhile, Equations (3), (4) and (6) highlight a counteracting force that improves output per capita: higher survival rates raises the savings rate, which in turn leads to larger capital and R&D intensities, the latter of which improves TFP. Additionally, an increase in the middle-age survival rate s_1 also raises the size of the labour force relative to its previous trend, thereby increasing TFP further through the scale effect present in (4).

The intuition for the effects on the capital and R&D intensities is straightforward. A higher likelihood of middle-age survival raises the share of households that save while a higher likelihood of old-age survival increases the life-cycle savings motive, so households save more intensively. These extra savings are allocated proportionately to capital and R&D investment according to (S4). The capital stock, however, increases disproportionately more than TFP because the production of capital is linear in investments whereas the R&D process features decreasing returns to scale: a necessary parameter restriction for a steady state to exist is $\gamma > 0$, which holds for feasible values of λ and ϕ if and only if $\lambda + \phi < 1$. Therefore, both K_t/Y_t and Z_t rise.

2.3 A Baby Boom Through a Temporary Increase in Fertility Rates

Next consider a temporary increase in the population growth rate $1+n$, similar to a baby boom. Clearly, this leaves the age structure unchanged in the long run. Thus, neither the employment rate nor the savings rate (and subsequently the capital and R&D intensities) is affected, as shown by Equations (5) and (6). The only long-run impact is through the scale effect in Equation (4): a temporary fertility boom permanently raises the population size, which leads to higher TFP and subsequently higher output per capita. This is exactly what a standard semi-endogenous growth model with a representative household predicts.

Equations (3) to (6) nevertheless provide some guidance about the transition dynamics for this scenario beyond the scale effect. The savings rate is low when the share of middle-aged households is low, and the employment rate is high when the share of old households is low. The initial boost in the share of young households therefore generates positive growth via the employment rate and reduced growth via the capital and R&D intensities. As this generation reaches middle age and their children enter the labour

Table 2. Comparative statics of the simple model.

Change	Duration	Output per capita	Technology		Capital intensity	Employment rate
			R&D intensity	Scale		
Middle-age mortality: $s_1 \uparrow$	Permanent	Ambiguous	+	+	+	–
Old-age mortality: $s_2 \uparrow$	Permanent	Ambiguous	+		+	–
Fertility: $1 + n \uparrow$	Temporary	+		+		

Notes. Increases denoted by plus signs, decreases denoted by minus signs.

force, the positive impact on the employment rate remains at the same time as the savings rate recovers, thus reverting the capital and R&D intensities back towards their starting points. When this generation finally reaches retirement, the effect on the employment rate also reverts back. These predictions will be useful to understand the quantitative results.

2.4 Taking Stock

The steady-state predictions are summarised in Table 2. Overall, the demographic transition deteriorates the employment rate through the old-age dependency ratio, improves the capital intensity via the savings rate, and generates technical change through both the savings rate and the scale effect associated with the nonrivalry of technology. Yet, the relative strengths of these channels and the net effect on transitional growth are ambiguous. This motivates the need for a quantitative treatment.

3 A Quantitative OLG Model With Endogenous Growth

The mechanisms above are quantified in a closed economy environment populated by overlapping generations of households, production firms, R&D firms, and a government. Time is discrete, with a period set to one year, and exogenous changes in the demographic structure constitute the driving force of the model.⁷ Following the discussion in Section 2, particular care is taken to include a rich set of factors that affect household behaviour over the life cycle, including changes in household size, mortality risk, idiosyncratic income risk, borrowing constraints, progressive income taxation, and intensive (hours worked) and extensive (age of retirement) margins of labour supply. The rest of the framework remains close to the growth literature: the production sector consists of a perfectly competitive final-good firm and monopolistic intermediate-good firms, and technology grows endogenously through the entry of new intermediate firms created from innovations in the R&D sector.⁸

3.1 Demographics and Household Composition

In a given period t , the economy consists of $J + 1$ overlapping generations of sizes N_{0t}, \dots, N_{Jt} , with total population size $N_t = \sum_{j=0}^J N_{jt}$. From a given initial population distribution, the demographic structure in

⁷ For simplicity, I abstract from the possibility that technological change reversely affects demographic variables (birth control, treatment of diseases, and so on). This follows Lee (2016, p. 111), who states “My personal view is that trajectories of fertility, mortality, and health should be taken as exogenous. While theories are available to relate these to individual choices, they have little predictive power and their use might obscure the workings of some better understood mechanisms.”

⁸ The mechanisms emphasised in this paper are present in any standard endogenous growth model, so choosing between the expanding-variety model of Romer (1990) or a quality-ladder model à la Aghion and Howitt (1992) and Grossman and Helpman (1991) is inconsequential here.

subsequent periods is pinned down recursively by age- and time-specific fertility rates f_{jt} , survival rates s_{jt} , and net migration rates m_{jt} according to

$$N_{0,t+1} = \sum_{j=0}^J f_{jt} N_{jt} \quad \text{and} \quad N_{j+1,t+1} = (s_{jt} + m_{jt}) N_{jt}, \quad (7)$$

where $s_{jt} = m_{jt} = 0$ holds for all t such that individuals die with certainty after age J . Migrants bring their accumulated wealth with them when they move and are economically identical to non-migrants. This assumption eliminates the need to separate between natives and migrants in the economic model.

Individuals are children for the first ι years of their lives, after which they form a household and become economically active. A household consists of an adult head and their dependants, including both children and grandchildren of underaged parents. For an adult of age j , the number of dependants of age $i < \iota$ is given by

$$N_{ijt}^d = f_{j-1-i,t-1-i} \frac{N_{j-1-i,t-1-i}}{N_{0,t-i}} \frac{N_{it}}{N_{jt}} + \sum_{k=i+1}^{\iota-1} N_{kjt}^d N_{ikt}^d. \quad (8)$$

The first term is the number of children, which is calculated by multiplying the fraction of the i -year-old cohort born to parents who are now j years old by the current population of i -year-olds, and subsequently dividing that evenly across the current j -year-old population. The second term sums up the number of grandchildren, where the number of i -year-old dependants of an underaged parent is given analogously to the first term on the right-hand side of (8).

3.2 Household Endowments and Preferences

Individuals are endowed with one unit of time and enter adulthood as members of the labour force with no asset holdings. Working households provide an effective labour supply of $\ell_j = \varepsilon_j \eta h_j$ efficiency units, where h_j is hours worked, ε_j is a deterministic, age-dependent productivity level, and η is an idiosyncratic productivity component that evolves stochastically following a time-invariant Markov chain. Idiosyncratic productivity in the initial adult age ι is distributed according to the unique invariant distribution of the associated Markov chain, and workers can partially self-insure against idiosyncratic shocks through asset holdings a subject to a borrowing constraint. Additionally, there are no annuity markets to insure against mortality risk. Instead, the government seizes the assets of households who die prematurely and redistributes them as lump-sum transfers tr to surviving households. Leaving time subscripts implicit in this subsection, the budget constraint facing a household head at age j is thus written as

$$a_{j+1} + (1 + \tau^c) c_j = (1 + r(1 - \tau^k)) a_j + (1 - \tau^w(w \ell_j) - \tau^b) w \ell_j + tr + b(R_j), \quad (9)$$

where c_j is total household consumption, r is the rate of return on savings, w is the wage rate, and τ^c , τ^k , τ^w , τ^b denote tax rates where, in particular, τ^w is allowed to vary with labour income. The final term, $b(R_j)$, is a pension benefit to be specified later that depends on whether a person is retired and, if retired, the age of retirement, both of which are captured by the retirement status R_j .

A household head of age j values their own consumption c_j^a , their leisure time $1 - h_j$, and the consumption c_{ij}^d per i -year-old dependant according to the static utility function

$$u(c_j^a, \{c_{ij}^d\}_{i=0}^{\iota-1}, h_j) = \frac{(c_j^a)^{1-\sigma} - 1}{1-\sigma} + \sum_{i=0}^{\iota-1} \omega_i N_{ij}^d \frac{(c_{ij}^d)^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h_j^{1+1/\theta}}{1+1/\theta}, \quad (10)$$

where $1/\sigma$ is the intertemporal elasticity of substitution for consumption, ω_i is the utility weight of i -year-old dependants, θ is the Frisch elasticity of labour supply, and ψ determines the disutility of working. Households benefit from economies of scale in the sense that total household consumption satisfies the constant elasticity of transformation function

$$c_j = \left[(c_j^a)^\zeta + \sum_{i=0}^{\iota-1} N_{ij}^d (c_{ij}^d)^\zeta \right]^{\frac{1}{\zeta}}, \quad \zeta \geq 1, \quad (11)$$

in which the parameter ζ captures the degree of economies of scale. If $\zeta = 1$, there are no scale benefits and total household consumption is just the sum of the individual consumption levels. Equations (10) and (11) admit an equivalent utility function expressed in terms of total household consumption and hours worked. Taking household consumption as given and maximising (10) with respect to consumption subject to (11), we obtain $c_{ij}^d = \omega_i^{\frac{1}{\zeta-(1-\sigma)}} c_j^a$. This optimality condition combined with Equations (10) and (11) generates a utility function of the form

$$u(c_j, h_j) = \Omega_j \frac{c_j^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h_j^{1+1/\theta}}{1+1/\theta}, \quad (12)$$

where

$$\Omega_j \equiv \left[1 + \sum_{i=0}^{\iota-1} \omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}} N_{ij}^d \right]^{\frac{\zeta-(1-\sigma)}{\zeta}} \quad (13)$$

is an age- and time-specific household-size taste shifter. Specifically, substituting $c_{ij}^d = \omega_i^{\frac{1}{\zeta-(1-\sigma)}} c_j^a$ into Equation (11) reveals that $c_j = \Omega_j^{\frac{1}{\zeta-(1-\sigma)}} c_j^a$, so $\Omega_j^{\frac{1}{\zeta-(1-\sigma)}}$ measures the number of adult equivalents in a household and $\omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}}$ is the equivalence weight of an i -year-old dependant. Equation (12) belongs to the class of balanced-growth preferences characterised by Boppart and Krusell (2020) that allows for falling hours worked in the long run; the income effect of higher wages on leisure dominates the substitution effect if $\sigma > 1$, leading to a decline in hours worked whenever wage growth is positive. These preferences are summed up over time into an expected lifetime utility function of the form

$$\mathbb{E} \left[\sum_{j=\iota}^J \beta^{j-\iota} \left(\prod_{k=\iota}^{j-1} s_k \right) u(c_j, h_j) \right], \quad (14)$$

where β is a subjective discount factor and the expectations are taken over idiosyncratic labour productivity.

Households maximise expected lifetime utility (14) by choosing consumption, hours worked, and their age of retirement. At the start of each period, households observe their state $x_j' = (a_j, \eta, R_{j-1})$ consisting of current-period wealth and idiosyncratic productivity and the previous-period retirement status, and

subsequently make their retirement decisions. Consumption and hours worked are then chosen in a second stage conditional on the post-retirement decision state $x_j = (a_j, \eta, R_j)$. Retirement is an absorbing state, so the intensive- and extensive-margin labour supply choices are then trivial and the household problem reduces to a standard consumption-savings choice.

Formally, denote the retirement choice by a discrete variable d equal to 1 if choosing to remain in the labour force and 0 otherwise and let the retirement status evolve according to $R_j = R_{j-1} + d_j$, starting with $R_{-1} = \iota$. An optimal retirement policy is then a function $d_j(x'_j)$ that solves the first-stage problem

$$V_j(x'_j) = \max_{d_j \in D(R_{j-1})} \{v_j(x_j)\} \quad (15)$$

subject to the motion of the retirement status and to

$$D(R_{j-1}) = \begin{cases} \{0, 1\} & \text{if } R_{j-1} = j, \\ \{0\} & \text{if } R_{j-1} < j, \end{cases}$$

where $V_j(x'_j)$ and $v_j(x_j)$ are the pre- and post-retirement decision value functions at age j . Optimal policies for consumption, savings, and hours worked are functions $c_j(x_j)$, $a_j(x_j)$, and $h_j(x_j)$ that solve the second-stage problem

$$v_j(x_j) = \max_{c_j, h_j} \left\{ u(c_j, h_j) + \beta s_j \mathbb{E}[V_{j+1}(x'_{j+1}) \mid \eta] \right\} \quad (16)$$

subject to the budget constraint (9), the time constraints $h_j \in [0, 1]$ if working and $h_j = 0$ if retired, and the borrowing constraint $a_{j+1} \geq 0$.

3.3 Production

The competitive final-good sector hires labour L_t at the wage w_t and buys a continuum of intermediate capital inputs $\{k_{it}\}$ indexed by i at prices $\{p_{it}\}$ to produce output Y_t . Production follows the Cobb-Douglas function

$$Y_t = L_t^{1-\alpha} \left(\int_0^{z_t} k_{it}^\rho di \right)^{\frac{\alpha}{\rho}}, \quad 0 < \alpha, \rho < 1. \quad (17)$$

where z_t is a measure of the intermediate varieties available at time t . Intermediate-good firms use a linear production function that converts capital into intermediate inputs one for one. Capital is rented from households at the rate $r_t + \delta_k$, where δ_k is the capital depreciation rate. Each firm in the intermediate sector has a patent for their own variety and acts as a monopolist. Conditional on having a patent, a firm j consequently maximises operating profits $\pi_{jt} = (p_{jt} - r_t - \delta_k)k_{jt}$ subject to the final-good producer's demand for j . By profit maximisation in the final-good sector, this demand constraint is given by

$$p_{jt} = \alpha \frac{Y_t}{k_{jt}} \frac{k_{jt}^\rho}{\int_0^{z_t} k_{it}^\rho di}. \quad (18)$$

Symmetry across firms implies that all firms charge the same mark-up over marginal cost, $p_{it} = p_t = \frac{1}{\rho} (r_t + \delta_k)$, and therefore also sell the same quantity k_t and earn the same profits $\pi_t = (1 - \rho)p_t k_t$.

3.4 The R&D Sector

The R&D sector exhibits free entry and invests an aggregate amount Q_t of final output on innovation to develop new designs for intermediate goods. The patents for new designs are sold to prospective intermediate-good firms at the end of a period t for a one-off price $P_{z,t+1}$, which then convert the designs into usable input in period $t + 1$. These patent purchases are financed by households in exchange for equity ownership in the intermediate-good firms. Like the simple model in [Section 2](#), innovation is characterised by an overall production function as in Jones (1995):

$$F(Q_t) = \bar{v}_t Q_t = \nu Q_t^\lambda z_t^\phi, \quad 0 < \lambda \leq 1, \phi < 1, \quad (19)$$

where the productivity term $\bar{v}_t \equiv \nu Q_t^{\lambda-1} z_t^\phi$ incorporates duplication externalities (via λ) and knowledge spillovers (via ϕ) that impact the aggregate development but are external to individual R&D firms. An individual R&D firm therefore chooses expenditures q_t to maximise profits $(P_{z,t+1} \bar{v}_t - 1)q_t$, which together with free entry yields the aggregate zero-profit condition

$$Q_t = P_{z,t+1} \nu Q_t^\lambda z_t^\phi. \quad (20)$$

As in Comin and Gertler (2006) and related papers, an intermediate-good firm is not infinitely lived; in each period a fraction δ_z of firms become obsolete.⁹ The aggregate law of motion of new intermediates is therefore $z_{t+1} = (1 - \delta_z)z_t + F(Q_t)$, thus implying the gross growth rate of intermediate varieties

$$1 + g_{zt} = 1 - \delta_z + \nu Q_t^\lambda z_t^{\phi-1}. \quad (21)$$

Lastly, a prospective intermediate-good firm enters the market only if it is profitable to do so. That is, the firm enters if the sum of expected discounted flow profits π exceeds the fixed cost P_z of purchasing a patent. Free entry into the intermediate-good sector drives the profitability of entry to zero. This is equivalent to saying that the following no-arbitrage condition holds in equilibrium:

$$r_t = \frac{\pi_t + \Delta P_{zt} - \delta_z P_{z,t+1}}{P_{zt}}, \quad (22)$$

where $\Delta P_{zt} \equiv P_{z,t+1} - P_{zt}$ is the change in the patent price in period t .

3.5 The Public Sector

The public sector engages in three activities: (i) it redistributes assets from deceased individuals, (ii) collects taxes on consumption, capital gains and wages via the tax rates τ_t^c , τ_t^k and τ_t^w to finance public consumption G_t , and (iii) maintains a pay-as-you-go social security system. The pension system is financed by the contribution rate τ_t^b on labour earnings. The budget constraint of each public-sector activity is independent of the other two and always balances. In the baseline model, budget balance is ensured through endogenous adjustments in the per-household bequest transfer tr , public consumption G_t , and the social security contribution rate τ_t^b . An ageing population with this setup generates higher contribution rates to maintain pension benefits. Compared to the alternative (constant tax rates and cutting benefits), this is the most growth pessimistic arrangement, since it negatively impacts both the incentive to save for

⁹ This feature is primarily technical to control the level of R&D investment in steady state. We can also set $\delta_z < 0$ to generate exogenous growth, as in the standard neoclassical model, in addition to that created through R&D.

retirement and for late retirement (due to generous pensions) as well as the ability to save (through higher taxation).

Pension benefits are independent of earnings history but depend on the age of retirement.¹⁰ Specifically, a household with retirement status R receives a base-level benefit that is scaled by a factor $ps(R)$ relative to some normal retirement age R^{norm} to capture early retirement penalties and delayed retirement credits. The base-level benefit is a fraction μ of average gross labour income $w_t \bar{\ell}_t$, where $\bar{\ell}_t$ is the average number of efficiency units per worker. An age- j household therefore receives a pension transfer

$$b_t(R) = \begin{cases} ps(R)\mu w_t \bar{\ell}_t & \text{if } j \geq \max\{R^{min}, R\}, \\ 0 & \text{if } j < \max\{R^{min}, R\}, \end{cases} \quad (23)$$

where no pension is paid out to working households or to retirees younger than some threshold age R^{min} .

3.6 Equilibrium

In any period t , the state of the economy is pinned down by the aggregate capital stock $K_t \equiv \int_0^{z_t} k_{it} di$, the number of intermediate varieties z_t , the patent price P_{zt} , and measures $\{\Phi_{jt}(x)\}_{j=1}^J$ characterising the distribution of households after their retirement decisions are made. Since the state of a household is determined by their asset wealth, idiosyncratic productivity, and retirement status, the state space for the household measures is given by $X = \mathbb{R}_+ \times H \times \{1, \dots, J+1\}$, where H denotes the state space of the Markov chain for idiosyncratic productivity. Given an evolution of demographic variables and initial conditions $K_0, z_0, P_{z,0}, \{\Phi_{j0}(x)\}_{j=1}^J$, we then have:

Definition 1 (General equilibrium). An equilibrium consists of paths for household decision rules $\{d_{jt}(\cdot), c_{jt}(\cdot), a_{jt}(\cdot), h_{jt}(\cdot)\}_{t=0}^\infty$ and measures $\{\Phi_{jt}(x)\}_{t=0}^\infty$ for all ages j , prices and profits $\{r_t, w_t, \pi_t, P_{zt}\}_{t=0}^\infty$, taxes and transfers $\{\tau_t^c, \tau_t^k, \tau_t^b, \tau_t^w(\cdot), tr, \{b_{jt}(\cdot)\}_{j=1}^J\}_{t=0}^\infty$, and aggregate quantities such that for all periods t :

- (i) Household decision rules solve problems (15) and (16).
- (ii) Profit maximisation of final- and intermediate-good firms yields the consolidated production function

$$Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha}, \quad \text{where} \quad Z_t \equiv z_t^{\frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}, \quad (24)$$

with corresponding factor prices and profits

$$w_t = (1-\alpha) \frac{Y_t}{L_t}, \quad r_t = \alpha \rho \frac{Y_t}{K_t} - \delta_k, \quad \pi_t = \alpha(1-\rho) \frac{Y_t}{Z_t}. \quad (25)$$

- (iii) The measure of intermediate-good varieties z_t , the patent price P_{zt} and aggregate R&D investment Q_t satisfy Equations (20) to (22).
- (iv) The public sector budgets balance:

$$G_t = \tau_t^c C_t + \tau_t^k r_t A_t + \sum_{j=1}^J N_{jt} \int_X \tau^w(w_t \ell_{jt}(x)) w_t \ell_{jt}(x) d\Phi_{jt}, \quad (26)$$

¹⁰ Earnings-dependent pension would introduce another continuous state variable in the household problem and I abstract from this feature to avoid the additional computational complexity that it entails, as is common in the OLG literature.

$$\tau_t^b w_t L_t = \sum_{j=t}^J N_{jt} \int_X b_t(R) d\Phi_{jt}, \quad (27)$$

$$tr_{t+1} = \frac{1 + r_{t+1}(1 - \tau_{t+1}^k)}{\sum_{j=t}^J N_{j,t+1}} \sum_{j=t}^J (1 - s_{jt}) N_{jt} \int_X a_{jt}(x) d\Phi_{jt}, \quad (28)$$

where $C_t = \sum_{j=t}^J N_{jt} \int_X c_{jt}(x) d\Phi_{jt}$ is aggregate consumption and $A_{t+1} = \sum_{j=t}^J (1 + m_{jt}) N_{jt} \int_X a_{jt}(x) d\Phi_{jt}$ is aggregate wealth.

(v) The markets for labour, capital, and goods clear:

$$L_t = \sum_{j=t}^J N_{jt} \int_X \ell_{jt}(x) d\Phi_{jt}, \quad (29)$$

$$A_t = K_t + P_{zt} z_t, \quad (30)$$

$$Y_t + A_{t+1}^M = C_t + G_t + [K_{t+1} - (1 - \delta_k)K_t] + Q_t, \quad (31)$$

where $A_{t+1}^M = \sum_{j=t}^J m_{jt} N_{jt} \int_X a_{jt}(x) d\Phi_{jt}$ is the net wealth brought by migrants.

(vi) For a given Markov kernel Π and for all Borel sets $S = \mathcal{A} \times \mathcal{H} \times \mathcal{R}$ on X (and dropping time subscripts), the household distributions evolve according to

$$\Phi_{j+1}(S) = \int_{X_W} \left[\int_{\mathcal{H}} d_{j+1}(x') \Pi(\eta, d\eta') \right] d\Phi_j + \int_{X_R} \left[\int_{\mathcal{H}} (1 - d_{j+1}(x')) \Pi(\eta, d\eta') \right] d\Phi_j, \quad (32)$$

where $X_W = \{x \in X : a_j(x) \in \mathcal{A}, R + 1 \in \mathcal{R}\}$ and $X_R = \{x \in X : a_j(x) \in \mathcal{A}, R \in \mathcal{R}\}$, and where $x' = (a_j(x), \eta', R)$, $\eta' \in \mathcal{H}$, are the relevant pre-retirement decision states at age $j + 1$. Letting Γ denote the unique invariant distribution of Π , the initial household distribution is equal to

$$\Phi_i(\{0\} \times \mathcal{H} \times \{R\}) = \begin{cases} \int_{\mathcal{H}} d_i(0, \eta, \iota) d\Gamma & \text{if } R = \iota + 1, \\ \int_{\mathcal{H}} (1 - d_i(0, \eta, \iota)) d\Gamma & \text{if } R = \iota, \end{cases} \quad (33)$$

and zero for all other asset levels and retirement statuses. \triangleleft

As in Romer (1990), the model features endogenous growth in total factor productivity Z_t through changes in the measure of intermediate varieties z_t . As in Jones (1995), this growth is semi-endogenous in that it is determined exogenously by the rate of population growth along a balanced growth path:

Definition 2 (Stationary equilibrium). A stationary equilibrium, or steady state, is an equilibrium in which all variables grow at constant rates (possibly zero) and all growth rates are determined by the population growth rate $1 + n \equiv \frac{N_{t+1}}{N_t}$. In particular, the growth rate of new intermediate-good firms is

$$1 + g_z = (1 + n)^\chi, \quad \text{where} \quad \chi \equiv \frac{\lambda}{1 - \phi - \lambda \frac{1+\theta}{1+\theta\sigma} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}} > 0. \quad (34)$$

The growth rates of TFP, output per capita, and hours per worker are similarly

$$1 + g_Z = (1 + n)^{\gamma_Z}, \quad 1 + g_Y = (1 + n)^{\gamma_Y}, \quad 1 + g_h = (1 + n)^{\gamma_h},$$

where $\gamma_Z \equiv \chi \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}$, $\gamma_Y \equiv \frac{1+\theta}{1+\theta\sigma} \gamma_Z$, and $\gamma_h \equiv \frac{\theta(1-\sigma)}{1+\theta\sigma} \gamma_Z = \gamma_Y - \gamma_Z$. ◀

The growth rates in [Definition 2](#) are derived in Online Appendix A. Note that the TFP growth rate reduces to [Equation \(2\)](#) in the simple model if we consider a steady state with constant hours worked (via log preferences, $\sigma \rightarrow 1$) and a substitution parameter for intermediate goods exactly equal to the capital share parameter ($\rho = \alpha$).

4 Numerical Experiment and Implementation

The quantitative exercise considered is a demographic transition between 1950 and 2100 which mimics the observed and projected population trends during this period. The transition ultimately converges to a steady state consistent with the projected fertility, mortality, and migration rates expected by the end of the twenty-first century. To reduce the impact of the initial steady state on the period of interest, I simulate the model from a stationary equilibrium imposed already in 1900. In 1901, the economy is then shocked by the demographic transition, during which there is perfect foresight of aggregate variables until the new steady state. I also assume the same population growth rate in the initial steady state as in the final steady state. This ensures that the long-run growth trend does not change, allowing any growth beyond this trend to be attributed to transitional factors.

Numerically, I solve the household problem with the discrete-continuous endogenous grid method introduced by Iskhakov *et al.* (2017) and approximate the household distributions using histograms over wealth, idiosyncratic productivity, and retirement status. Iskhakov *et al.* extend the standard endogenous grid method of Carroll (2006) to incorporate discrete choices, which simplifies the computational difficulty associated with the endogenous retirement decision. The overall solution method iterates over paths of interest rates (r_t), pension contribution rates (τ_t^b), average labour supplies ($\bar{\ell}_t$), lump-sum transfers (tr_t), and growth rates of intermediate varieties (g_{zt}) until all equilibrium conditions hold. These paths are updated with Ludwig's (2007) modified quasi-Newton algorithm, which exploits the Jacobian of the equilibrium conditions related to $\{r_t, \tau_t^b, \bar{\ell}_t, tr_t, g_{zt}\}$ in the final steady state to provide an initial guess for the Jacobian in the transition algorithm.

5 Calibration

The model is calibrated to match the US economy and stays as close as possible to standard parameter values used in the literature. The resulting choices, discussed in detail below, are summarised in [Table 3](#).

5.1 Demographics

The United Nations (2022) World Population Prospects provides annual estimates on age-specific survival rates, age-specific fertility rates (births per woman), and aggregate net migration between 1950 and 2100. For earlier years, I collect age-specific survival rates and fertility rates from Bell and Miller (2005) and Heuser (1976), and use aggregate immigration data from the US Department of Homeland Security (2020) as a proxy for aggregate net migration. The age distribution of migration is assumed throughout to match

Table 3. Calibrated parameters of the baseline model.

Parameter	Description	Value	Target/source
<i>Households</i>			
ι	Initial adult age	20	Children between 0–19
J	Maximum age	99	Certain death at 100
β	Discount factor	1.022	Capital/output = 2.8
ψ	Leisure weight	19.718	50 hours/week in 1900
σ	Inverse IES	1.75	Boppart and Krusell (2020)
θ	Frisch elasticity	0.5	Chetty <i>et al.</i> (2011)
ζ	Household consumption scale	1.49	Browning and Ejrnæs (2009)
$\{\omega_i\}_{i=0}^{\iota-1}$	Utility weights on dependants	Fig. 3a	Browning and Ejrnæs (2009)
<i>Individual productivity</i>			
$\{\varepsilon_j\}_{j=\iota}^J$	Deterministic productivity	Fig. 4a	PSID
φ	Persistence shock	0.97	Heathcote <i>et al.</i> (2010)
σ_ε^2	Variance shock	0.02	Heathcote <i>et al.</i> (2010)
<i>Production</i>			
α	Intermediate goods share	0.36	Labour share = 0.64
ρ	Elasticity of substitution	0.71	Profit share = 0.10
δ_k	Capital depreciation rate	0.049	Investment/output = 0.136
<i>R&D</i>			
δ_z	Firm obsolescence rate	0.005	R&D/output = 0.014
ν	R&D productivity	0.009	$z = Z = 1$ in initial period
λ	Duplication externality	0.75	Comin and Gertler (2006)
ϕ	Knowledge spillovers	0.117	$g_Q = 6.73\% \implies g_Z = 1.26\%$
<i>Social security</i>			
μ	Replacement rate	0.413	Clingman <i>et al.</i> (2021)
R^{norm}	Normal retirement age	65	Social Security Administration
R^{min}	Lowest retirement age	62	Social Security Administration
$ps(R_j)$	Early/delayed scaling	0.8–1.15	Social Security Administration
<i>Taxes</i>			
τ^c	Consumption tax rate	0.080	BEA national accounts
τ^k	Capital gains tax rate	0.368	BEA national accounts
$\tau^w(w\bar{\ell})$	Income tax rate at mean income	0.115	BEA national accounts
κ_0	Asymptotic income tax rate	0.631	OECD tax database
κ_1	Income tax progressivity	0.574	OECD tax database
κ_2	Income tax scale parameter	0.505	OECD tax database
κ_3	Income tax rate at zero income	−0.207	OECD tax database

the US Census Bureau’s (2018) population projection. Fertility rates by age are only available from 1917, so for earlier years I adjust the age-specific fertility rates in 1917 by the change in the total fertility rate between these years and 1917 using estimates from the Gapminder Foundation (2017). The fertility rates and migration levels are then converted into births and migrants per person using population estimates from the Census Bureau’s intercensal tables and the UN World Population Prospects. I also smooth all demographic variables with an HP-filter using an annual smoothing parameter of 6.25.

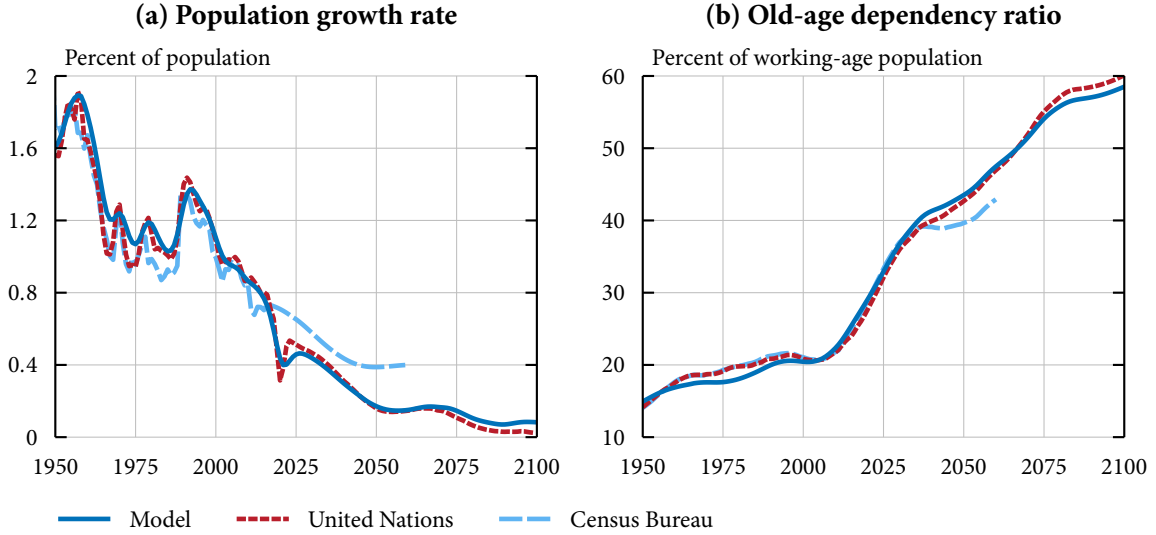


Figure 2. Demographic evolution.

The resulting demographic variables for the year 2100 generate a steady-state population with near-zero growth: -0.02 percent. For simplicity, I marginally increase migration rates by a common term so that population growth is exactly zero in the final steady state. I similarly impose a stable and constant population in the initial steady state which is consistent with the data on survival rates, fertility rates, and population size, again by adjusting migration rates with a common term.¹¹ Figure 2 plots the implied demographic development and shows that the calibration reasonably matches official estimates and projections from the United Nations and the Census Bureau.

5.2 Preferences and Labour Productivity

Households start their economic lives at 20 and die with certainty at 100. The discount factor β and the weight on labour supply ψ are calibrated to match a capital-output ratio of 2.8 and an average labour supply per worker of 0.45 in the initial steady state. The former yields a discount factor in the vicinity of Hurd (1989), who explicitly accounts for mortality risk and estimates a β of 1.011. Given a time endowment of 16 hours per day, the latter implies an average of 50 hours worked per week and worker in 1900, which is consistent with the evidence in Ramey and Francis (2009, Figure 1A).

I set the Frisch elasticity of labour supply θ to 0.5, as recommended by Chetty *et al.* (2011) along the intensive margin. This value is also consistent with Domeij and Flodén (2006a), who explicitly account for biases arising from uninsurable income risk and borrowing constraints. The inverse of the intertemporal elasticity of substitution, σ , is calibrated following Boppart and Krusell (2020). Based on long-run macro evidence, Boppart and Krusell argue that 2 percent productivity growth implies a fall in hours worked by roughly 0.4 percent. Noting from Definition 2 that the steady-state growth rates of hours worked and technology are linked via $1+g_h = (1+g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}}$, this empirical pattern suggests that $\theta(1-\sigma)/(1+\theta\sigma) \approx -0.2$. Given $\theta = 0.5$, this generates a σ of 1.75. Although based on long-run macro evidence, these parameter values are also consistent with micro evidence: Heathcote, Storesletten and Violante (2014) consider an

¹¹ If x is a common migration rate shifter, a steady-state population satisfies $N_{j+1} = \frac{s_j+m_j+x}{1+n} N_j = \left(\prod_{k=0}^j \frac{s_k+m_k+x}{1+n} \right) N_0$. Consistency with fertility rates requires that $(1+n)N_0 = \sum_{j=0}^J f_j N_j$. Combining these yields the condition $1+n = \sum_{j=0}^J \left(\prod_{k=0}^{j-1} \frac{s_k+m_k+x}{1+n} \right) f_j$, which can be solved for x numerically.

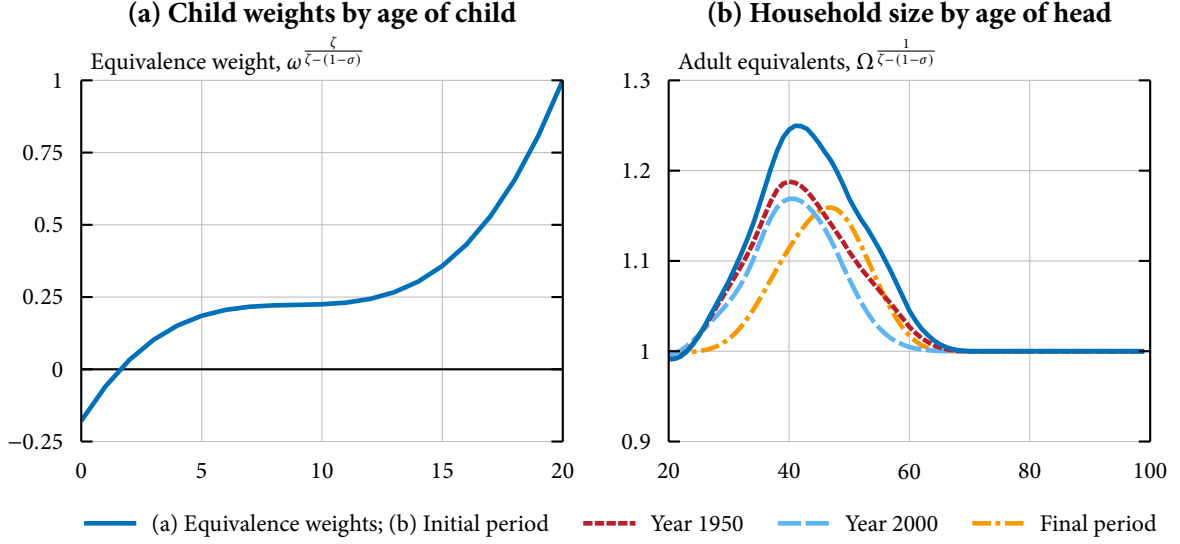


Figure 3. Household size.

incomplete-markets model with similar preferences and estimate $(\sigma, \theta) = (1.71, 0.46)$ using US earnings and consumption survey data.

The equivalence weights $\omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}}$ and the household scale parameter ζ follow Browning and Ejrnæs (2009), who estimate equivalence scales for low- and high-educated households that include age-specific weights for children and economies of scale. Browning and Ejrnæs use UK data, though Fernández-Villaverde and Krueger (2007) demonstrate with US data that a similar age-varying equivalence scale explains the hump in the life-cycle consumption profile better than a traditional scale. The equivalence weights, shown in Figure 3 together with the household size dynamics, are set to the average of Browning and Ejrnæs' estimates for the two education groups. Browning and Ejrnæs' economies-of-scale parameter, call it ζ_{BE} , is linked to the scale parameter here via $\zeta_{BE} \sigma = \frac{\zeta-(1-\sigma)}{\zeta}$. Given $\sigma = 1.75$ and choosing $\zeta_{BE} = 0.86$, again the average of Browning and Ejrnæs' estimates, then yields $\zeta = 1.49$.

I estimate the deterministic age-efficiency profile $\{\varepsilon_j\}_{j=t}^J$ by a fixed-effects regression of real log wages on a quadratic in age using earnings data from the 1968–2019 family files of the Panel Study of Income Dynamics (PSID). This procedure follows the usual steps in the literature and the details are relegated to Online Appendix B. The resulting profile is shown in Figure 4a and features a standard hump shape which peaks between the ages of 40 and 50. The idiosyncratic productivity shocks are assumed to follow the AR(1) process $\log \eta' = \varphi \log \eta + \epsilon$, where $\epsilon \sim N(0, \sigma_\epsilon^2)$, with the persistence parameter φ and error variance σ_ϵ^2 set to 0.97 and 0.02, roughly following the evidence presented by Heathcote, Storesletten and Violante (2010). This process is discretised into a five-state Markov chain using Rouwenhorst's method (Kopecky and Suen, 2010).

5.3 Production and R&D

The labour share of output is set to $1 - \alpha = 0.64$, a standard value, while the markup of intermediate producers is set to $1/\rho = 1.4$ to match an aggregate profit share of 10 percent, also a standard benchmark. To determine the depreciation rates δ_k and δ_z , I proxy R&D investment in the model by the national accounts measure of private fixed investment in intellectual property products (IPP) from the Bureau of Economic Analysis (BEA, NIPA Table 1.1.5). Besides formal R&D, IPP also includes nonrival goods

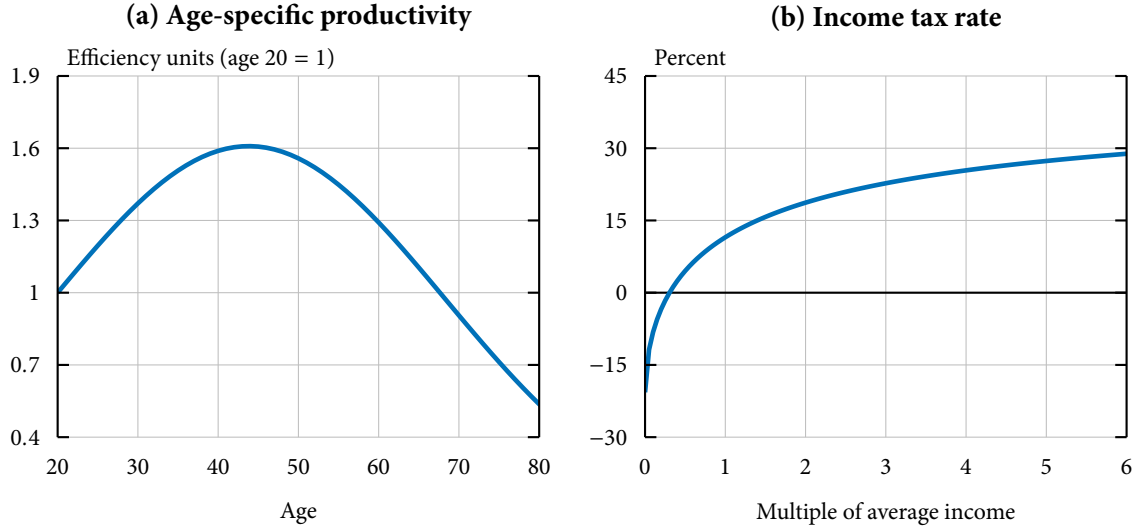


Figure 4. Calibrated productivity and tax profiles.

such as computer software and entertainment, literary, and artistic originals, thus capturing the model's notion of R&D better than formal R&D does. IPP investment is stable at 0.7 percent of GDP before 1950, after which it trends upwards. I set δ_z to get R&D investment within the same ballpark in the initial steady state at 1.1 percent of GDP; an exact match requires such a low δ_z that the transition paths become infeasibly long. Remaining investment in the national accounts (gross private domestic investment minus IPP investment) is roughly constant over time at 13.6 percent of GDP, and δ_k is chosen to match this value in the initial period.

The R&D production function contains three parameters: the productivity level ν , the duplication externality λ , and the knowledge spillover ϕ . The first one is a scale parameter; I set it so that $z = Z = 1$ in the initial period. Bloom *et al.* (2020) highlight that there is no consensus on the correct value for λ , so I just follow the calibrations in Jones and Williams (2000) and Comin and Gertler (2006) and set it to 0.75. The procedure to calibrate ϕ is identical to that of Bloom *et al.* and Jones and Williams. Specifically, Figure 5 shows that US TFP and aggregate IPP investment have grown at roughly constant rates of 1.26 and 6.73 percent per year since World War II. From the intermediate variety growth rate in (21), $1 + g_z = 1 - \delta_z + \nu Q^\lambda z^{\phi-1}$, it follows that the right-hand side of this equation must also have been constant. Log-differencing the last term then yields $\phi = 1 - \lambda \frac{g_Q}{g_z}$, where g_Q is the growth rate of R&D investment. With TFP defined as $Z = z^{\frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}$, the growth rates in Figure 5 together with the parameter values for α , ρ , and λ then gives $\phi = 0.117$.¹² This calibration yields steady-state growth exponents γ_Z and γ_Y equal to 0.23 and 0.18, which is close to the midpoint of the range of estimates in Jones (2002).¹³

5.4 Public Sector

I set the gross pension replacement rate μ to 0.413 based on Clingman, Burkhalter and Chaplain's (2021) estimates for average-income workers who retire at the normal retirement age. The normal retirement age,

¹² With $\phi > 0$, R&D exhibits positive knowledge spillovers: the more we know, the easier it is to discover new ideas. This value contrasts with for example Jones (2002) and Bloom *et al.* (2020), who find negative values of ϕ for the aggregate US economy with similar approaches. However, these papers compute ϕ under the implicit assumption that $\rho = \alpha$. If the same restriction is imposed here, I also find a negative value for ϕ .

¹³ A steady state with $(\gamma_Z, \gamma_Y) = (0.23, 0.18)$ and for example 1.2 percent population growth exhibits TFP and output per capita growth of 0.27 and 0.21 percent per year.

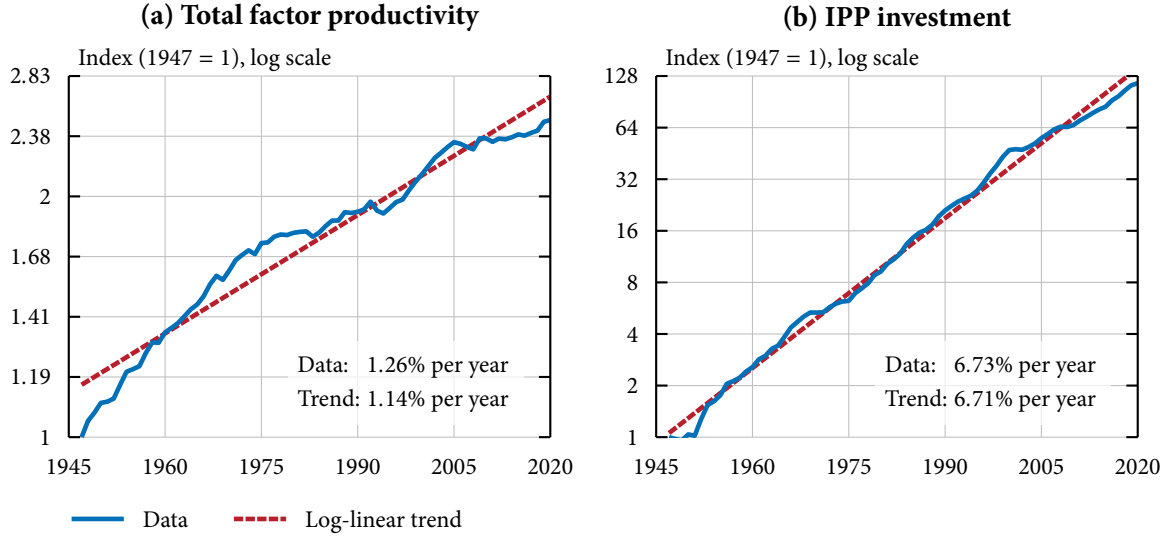


Figure 5. US TFP and aggregate gross IPP investment in the data.

Source: Fernald (2014) and BEA NIPA Table 5.6.3.

R^{norm} , is 65 and the earliest age to collect retirement benefits, R^{min} , is 62. Early and delayed retirement adjustment via $ps(R)$ is similar to that of the US social security system. For every year of early retirement, the base level benefit is reduced by 6 ⅔ percent per year for the first three years and 5 percent per any additional year. For every year of delayed retirement, the base level benefit is scaled up by 3 percent up until a maximum age of 70. After the age of 70, no extra benefit is given for delaying retirement.¹⁴

The individual income tax rate $\tau^w(e)$ at household earnings e is parametrised by Gouveia and Strauss's (1994) functional form:

$$\tau^w(e) = \kappa_0 \left[1 - \left(\kappa_2 \left(\frac{e}{\bar{e}} \right)^{\kappa_1} + 1 \right)^{-\frac{1}{\kappa_1}} \right] + \kappa_3, \quad (35)$$

where κ_0 controls the asymptotic tax rate $\lim_{e \rightarrow \infty} \tau^w(e) = \kappa_0 + \kappa_3$, κ_1 determines the degree of tax progressivity, and κ_2 is a scale parameter. Equation (35) augments Gouveia and Strauss' original function with average labour earnings \bar{e} , which makes the tax function invariant to units of measurement, and the parameter κ_3 , which allows for a non-zero marginal tax rate at zero earnings.

To calibrate the taxes, I first construct tax rates on consumption, capital, and labour income using national accounts data from the BEA by dividing the aggregate revenues of each tax with its corresponding tax base. Taking the average of each tax rate between 1950 and 2020 gives τ^c , τ^k , and $\tau^w(\bar{e})$, the latter being the income tax rate at average earnings. Next, I estimate $\{\kappa_0, \kappa_1, \kappa_2, \kappa_3\}$ using the OECD tax database, which provides income-specific tax rates that incorporate federal, state, and local government taxes plus several deductions and credits. These estimates are available annually since 2000, but only for a few representative income levels. I therefore replicate the OECD's methodology for each year to compute tax rates over a full grid of incomes and fit (35) to these estimates. Finally, for consistency with the aggregate data, I shift the level of the tax curve by adjusting the estimated parameters until the tax rate at average earnings, $\tau^w(\bar{e})$, equals the tax rate computed from the national accounts. Figure 4b plots the resulting income tax function and Online Appendix C outlines all the details of these steps.

¹⁴ These are the exact retirement ages and scaling rules used by the Social Security Administration for cohorts born before 1924. Later cohorts have higher normal retirement ages and more generous delayed retirement credits, though Online Appendix D shows that the quantitative results are insensitive to a more accurate development of R^{norm} and $ps(R)$.

6 Quantitative Results

The lack of population growth in the initial and final periods means that there is no economic growth in the model steady state. The growth impact of the demographic transition is therefore purely transitional. This section illustrates and quantifies this transitional impact and its underlying mechanisms through three quantitative exercises: (i) a growth accounting analysis of per-capita growth in the baseline model, (ii) counterfactual simulations that identify the key demographic factors at work, and (iii) a comparison to an equivalent model without endogenous growth to evaluate the importance of the TFP channel.¹⁵ Throughout, I take “per capita” to mean “per adult equivalent person”, since children only affect the economy through the household size.

The growth accounting exercise utilises that aggregate labour is the sum of efficiency units supplied by households to decompose labour into total employment (E), average hours per worker (\bar{h}), and average productivity per hour worked ($\bar{\epsilon}$): $L = E \bar{h} \bar{\epsilon}$. Combining this decomposition with output per capita as in Equation (1), $y = Z \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} \frac{L}{N}$, and log-differencing between two subsequent periods gives the growth accounting identity

$$g_y = g_Z + \frac{\alpha}{1-\alpha} g_{K/Y} + g_{E/N} + g_h + g_\epsilon, \quad (36)$$

where g_x denotes the net growth rate of a variable x . Equation (36) splits the change in output per capita into changes in technology, the capital intensity, the employment rate, average hours per worker, and average productivity per hour worked, which makes it easy to quantify the transitional mechanisms in operation.¹⁶

6.1 Comparative Statics

Following the discussion in Section 2, consider first comparative statics of the baseline scenario. Figure 6 displays the overall change in the age structure and the cumulative percentage change in output per capita between the initial and final periods, which reveals that the change in the population structure raises output per capita by 55 percent. This finding follows primarily from a substantial increase of 79 percent of TFP relative to its initial level. The capital intensity also increases by 17 percent while the higher share of old-age households in the population reduces the employment rate by 22 percent. The qualitative changes in the underlying channels are therefore precisely as the simple model predicts.

A natural question immediately emerges from Figure 6: why is the magnitude of the change in TFP so much larger than any of the other changes? It is not because an overwhelming majority of household savings are allocated to R&D rather than to capital investments.¹⁷ Rather, this is a result of the scale effect discussed in Section 2, which occurs when the demographic transition increases the population size. A simple decomposition of TFP illustrates this point. Specifically, per Definition 2 we can exploit that TFP is proportional to the size of the labour force E raised to γ_Z in steady state. The corresponding proportionality constant provides a measure of the R&D intensity, just as in Equation (4) of the simple model. The scale effect is pinned down by the overall change in employment, E_t/E_0 where 0 and T denote

¹⁵ Additional figures of variables beyond those presented in this section are available in Online Appendix E.

¹⁶ Generally, for any fixed long-run population growth rate \tilde{n} , we can identify and decompose transitional growth as in Jones (2002) via $g_y - \gamma_y \tilde{n} = (g_Z - \gamma_Z \tilde{n}) + \frac{\alpha}{1-\alpha} g_{K/Y} + g_{E/N} + (g_h - \gamma_h \tilde{n}) + g_\epsilon$.

¹⁷ Capital investment exceeds R&D investment by a factor of 4 during most of the transition and the growth rate of the capital stock relative to TFP is even higher; see Figure E.5 in Online Appendix E.

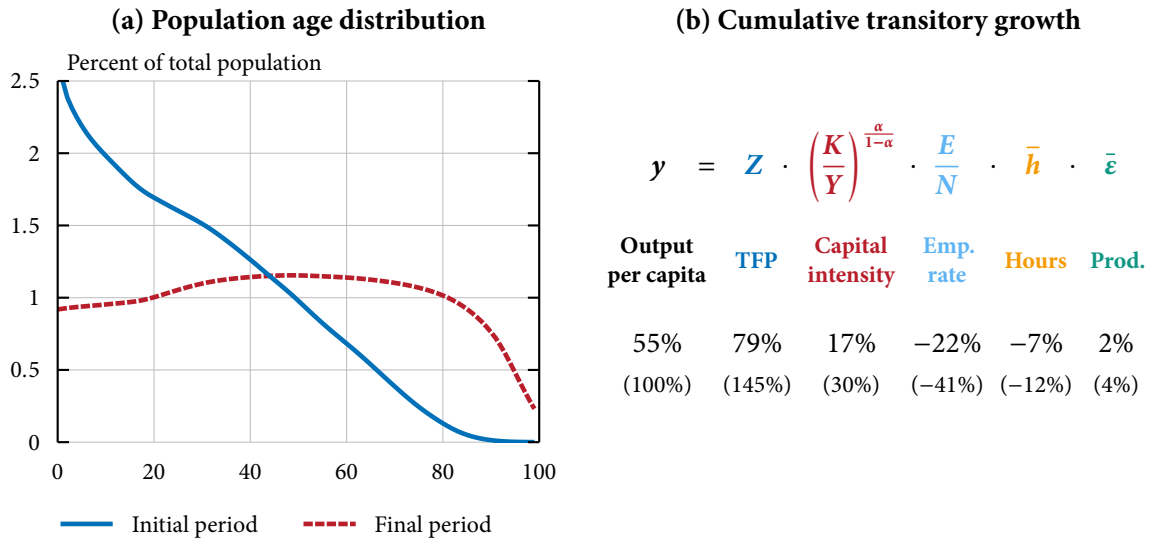


Figure 6. Comparative statics: population structure and output per capita.

Notes. Figure 6b shows the cumulative growth in output per capita and each of its contributing factors along the demographic transition. The numbers in parentheses displays their relative contributions to overall growth.

the initial and final periods, and we then obtain the following decomposition:

$$\text{TFP change} = \frac{Z_T}{Z_0} = \frac{Z_T / E_T^{\gamma_Z}}{Z_0 / E_0^{\gamma_Z}} \cdot \left(\frac{E_T}{E_0}\right)^{\gamma_Z} = \text{Change in R\&D intensity} \times \text{Scale effect},$$

$\uparrow 79\%$

$\uparrow 21\%$

$\uparrow 48\%$

where the percentages show the cumulative change in each factor between the initial and the final steady states. The 79 percent rise in TFP is due to a 21 percent increase in the R&D intensity and a 48 percent transitory increase in scale. Changes in household behaviour and composition thus impact output per capita through the R&D intensity much like they do through the capital intensity. The key difference between TFP and the other factors of production instead lies in the fact that technology is a nonrival good, as seen from the large scale effect.¹⁸

6.2 Growth Dynamics

The comparative statics is informative but says little about the growth dynamics during the transition. Therefore, we now turn to the main period of interest: 1950 to 2100. Figure 7 illustrates the growth rate and its corresponding decomposition (that is, the right-hand side of (36)) during this period. Focusing first on the overall growth rate, three key developments emerge. First, the demographic transition positively affects output per capita throughout the second half of the twentieth century, with growth firmly above zero. Then, this effect fades at the turn of the century, causing a significant drop in the growth rate. Finally, despite this decline, the demographic development does not negatively affect output per capita. Instead, growth remains around zero throughout the twenty-first century.

Looking at the underlying mechanisms, we again find a qualitative development as predicted in Section 2: the demographic transition generates positive TFP growth and capital deepening, but lowers the employ-

¹⁸ A caveat here is that using employment as the relevant scaling variable is somewhat arbitrary. We could just as well use the overall population size, since it grows in parallel with employment in steady state. Switching to the population size (measured in adult equivalents) yields a slightly larger scale effect, 57 percent, although this does not change the basic point of the exercise.

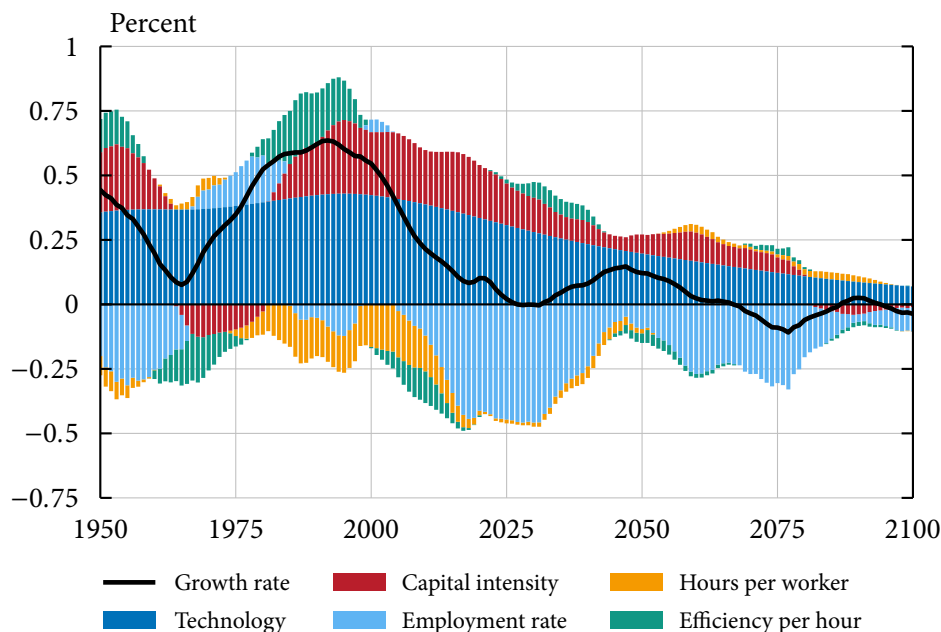


Figure 7. Growth accounting of the baseline model.

ment rate. Moreover, average hours worked declines in periods of positive growth (and vice versa) while average efficiency per hour worked rises in periods where the share of middle-aged workers is higher. The former is due to the parametrisation of household preferences, where the income effect of higher wages on leisure dominates the substitution effect, while the latter follows from the hump shape in workers' life-cycle productivity profile.

[Table 4](#) quantifies these observations by summarising the average annual growth rates of output per capita and its components, first for the entire time period considered and then separately for each century. Overall, the demographic transition boosts output per capita by 0.18 percent per year. The effect is primarily driven by the twentieth-century development, where the average annual growth rate is 0.41 percent. By comparison, the observed long-run growth rate of GDP per person in US data is approximately 2 percent per year. From this viewpoint, the impact obtained here is quantitatively significant; taken at face value it implies that over 20 percent of actual US post-war growth can be attributed to transitory demographic factors. This contribution makes demographics comparable in importance to rising educational attainment, whose contribution US growth over the same period is also around 20 percent according to estimates by Fernald and Jones (2014).

Just as in the comparative statics, what stands out quantitatively from both [Figure 7](#) and [Table 4](#) is the importance of TFP for overall growth. Over the full period, TFP grows by 0.27 percent per year, thus accounting for more than 150 percent of economic growth. Its contribution is almost three times as large as that of capital deepening. The effect is even larger during the latter half of the twentieth century, with TFP growing by 0.39 percent per year, about four times faster than the capital intensity, thus driving the bulk of output growth.

A declining share of workers in the population constitutes the main drag on output growth, depressing the annual growth rate by on average 0.16 percentage points overall and by 0.21 percentage points during the twenty-first century. This happens as increases in the average retirement age do not keep up with increasing life expectancy; [Figure 8](#) shows that, although there is a shift towards later retirement over time,

Table 4. Growth accounting of the baseline model.

	Output per capita	TFP	Capital intensity	Employment rate	Hours per worker	Efficiency per hour
Period	g_y	g_Z	$\frac{\alpha}{1-\alpha} g_{K/Y}$	$g_{E/N}$	g_h	g_ε
1950–2100	0.18 %	0.27 %	0.10 %	−0.16 %	−0.04 %	0.01 %
1950–2000	0.41 %	0.39 %	0.10 %	−0.06 %	−0.07 %	0.04 %
2001–2100	0.07 %	0.22 %	0.10 %	−0.21 %	−0.02 %	−0.01 %
1995	0.60 %	0.43 %	0.29 %	−0.12 %	−0.14 %	0.15 %
2030	0.00 %	0.28 %	0.13 %	−0.46 %	−0.02 %	0.06 %
Difference	−0.60 pp.	−0.15 pp.	−0.15 pp.	−0.34 pp.	0.13 pp.	−0.09 pp.

Notes. The table reports average annual growth rates according to the growth decomposition in [Equation \(36\)](#). Individual growth rates may not sum to totals due to rounding.

the increase in the retirement age is too marginal to affect the share of retirees in the population aged 65 and above.

Due to concerns about recent declines in observed growth rates, it is also interesting to zoom in on the decline in model growth from peak in 1995 to trough in 2030. The decomposition for this period is shown in the lower half of [Table 4](#). The growth rate and most of its components change monotonically between these years, so it suffices to consider snapshots at the beginning and end of this period and the differences between them. Overall, changes in the demographic structure leads to a 0.60 percentage point drop in the growth rate over the last three decades, thus suggesting that demographics explain a significant chunk of the growth decline observed in the data. The decline stems in part from roughly similar declines of 0.1 to 0.2 percentage points in the growth rates of TFP, capital intensity, and average efficiency. The growth rate of hours increases by a similar magnitude, therefore marginally counteracting the overall development. The majority, however, comes from the retirement of the baby boom: growth in the employment rate declines by 0.34 percentage points, accounting for 56 percent of the total decline.

6.3 It Is Not All About Boomers: Identifying the Demographic Effects

Related to the last point above, at first sight it appears that [Figure 7](#) just reflects the baby boom dynamics predicted in [Section 2](#) more broadly. The share of young workers increases when the baby boomers enter adulthood in the 1960s and 1970s, which improves the employment rate and worsens the capital intensity and average productivity. When these cohorts become middle-aged in the 1980s and 1990s, growth through TFP, the capital intensity, and average productivity improves. Finally, they retire in the early decades of the twenty-first century, thereby causing a negative growth impact via the employment rate. Yet, [Section 2](#) also stresses the increase in life expectancy from rising middle- and old-age survival rates. Which of these forces, if any, is the key driver of the results above?

To answer this question, I consider three counterfactual scenarios. First, I simulate the model under the assumption that neither the baby boom nor the rise in survival rates happens. That is, I replace fertility rates between 1935 and 1975 with interpolated values and hold survival rates above the age of 50 fixed from 1950 onward. As shown in [Figure 9](#), these adjustments remove the hump shape in fertility associated

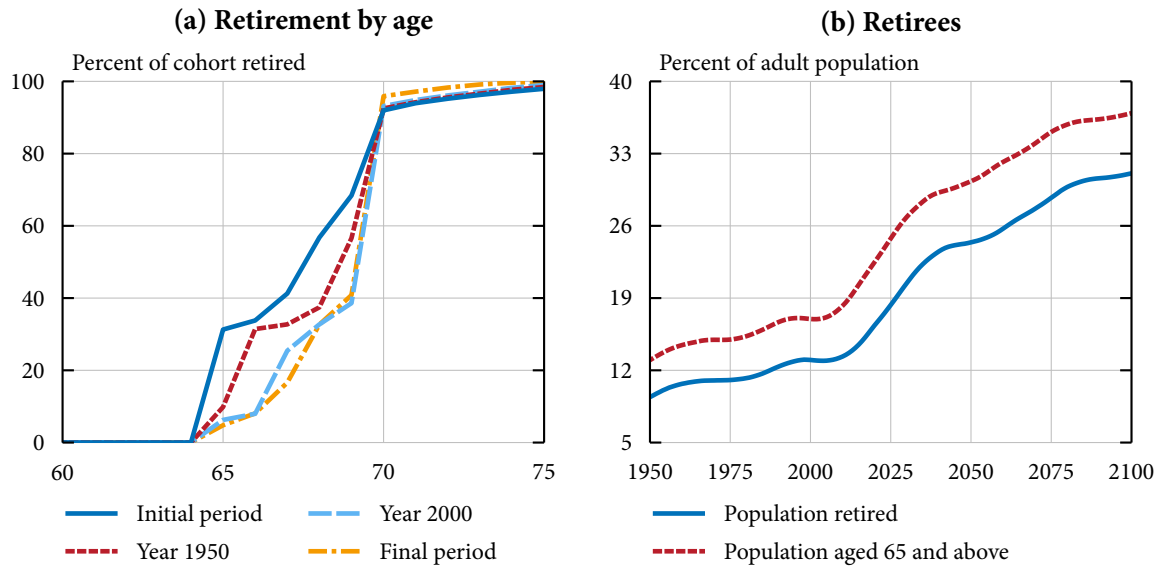


Figure 8. Retirement in the baseline model.

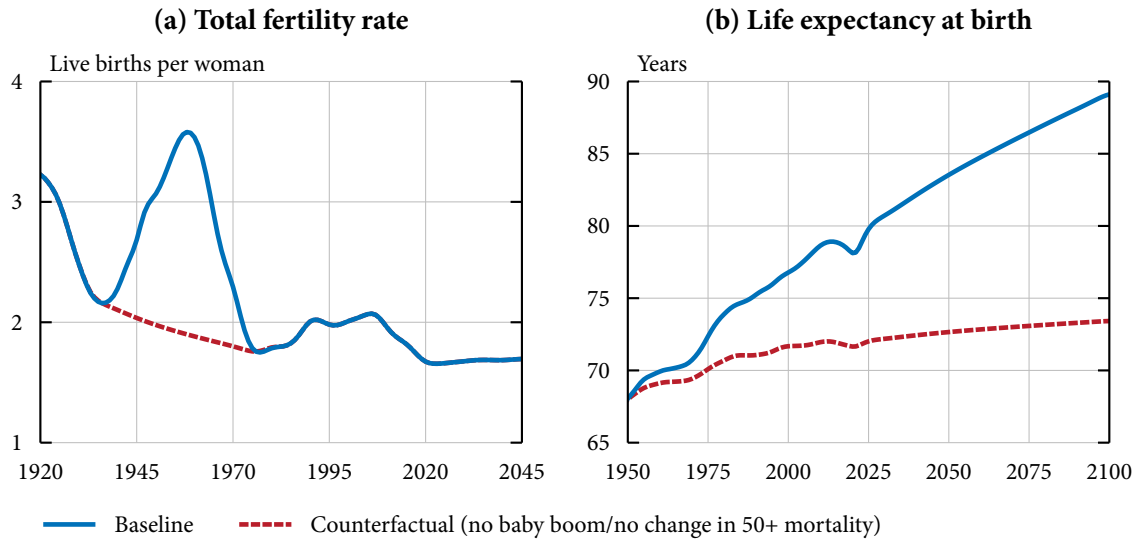


Figure 9. Demographic counterfactual scenarios.

Notes. For both scenarios, the measures in Figure 9 are computed under the assumption that the fraction of women in an age group and the average fraction of the year lived at the age of death remain the same as in official data and projections.

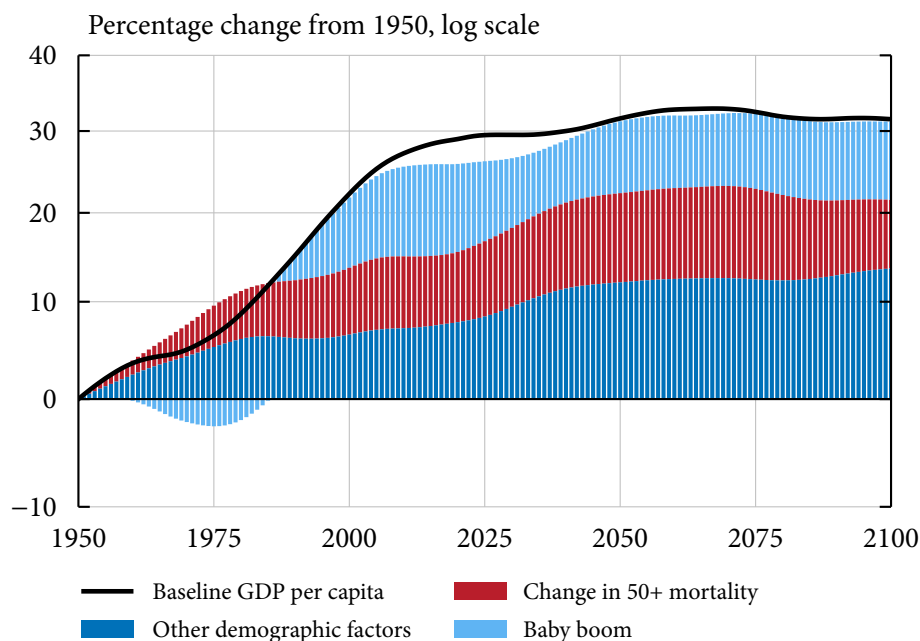


Figure 10. Identifying the impact of the demographic forces.

with the baby boom and almost all life expectancy improvements after 1950. I then switch either fertility or mortality back to the baseline calibration and look at the change relative to the first counterfactual. Without strong interaction effects, these scenarios isolate the impacts of the baby boom, of the change in middle- and old-age mortality, and of all other demographic changes.

Figure 10 plots the resulting decomposition of the cumulative change in output per capita into each demographic factor. While the growth swings in Figure 7 are clearly attributed to the life-cycle phases of the baby boomers here, they are by no means the sole explanation behind the overall development. Between 1950 and 2100, output per capita rises by a total of 31.5 percent, which is accounted for by an 8.0 percent increase due to the baby boom, a 7.0 percent increase due to middle- and old-age mortality changes, and a 13.6 percent increase due to other demographic factors. The baby boom and changing mortality consequently explain around a quarter each of the baseline results, with remaining demographic changes accounting for the rest.

Table 5 displays the demographic contributions to each component of the average growth rate between 1950 and 2100.¹⁹ As with output per capita, the baby boom and changing mortality generate around a quarter each of the total technological progress. Transition dynamics notwithstanding, this TFP contribution (and the subsequent income effect on hours worked) is the baby boom's only impact in the end, thus confirming the prediction from the simple model. This is unsurprising since the counterfactual fertility rates affect the cohorts that become adults after 1955 and die with certainty no later than 2075. Before and after these years, the age structure of the adult population is nearly identical to the baseline. Most changes in the capital intensity and the employment rate are instead driven by life expectancy improvements. Despite its positive impact on output per capita, longer lifespans also increase hours per worker, reflecting households' need to finance longer periods of retirement.

¹⁹ Decomposition plots similar to Figure 7 of the growth impact of each demographic factor are provided in Online Appendix E.

Table 5. Growth decomposition of the demographic forces.

	Output per capita	TFP	Capital intensity	Employment rate	Hours per worker	Efficiency per hour
	g_y	g_Z	$\frac{\alpha}{1-\alpha} g_{K/Y}$	$g_{E/N}$	g_h	g_ϵ
Average growth rate	0.18 %	0.27 %	0.10 %	−0.16 %	−0.04 %	0.01 %
Baby boom	0.05 pp.	0.07 pp.	0.00 pp.	0.00 pp.	−0.02 pp.	0.00 pp.
50+ mortality	0.04 pp.	0.07 pp.	0.06 pp.	−0.12 pp.	0.04 pp.	0.00 pp.
Other changes	0.08 pp.	0.13 pp.	0.04 pp.	−0.04 pp.	−0.06 pp.	0.01 pp.

Notes. The table reports average annual growth rates between 1950 and 2100 according to the growth decomposition in Equation (36) and the percentage point contributions by each demographic factor. Individual growth rates may not sum to totals due to rounding.

What explains the residual factor? First, the population in 1950 is not stationary. Changes in the population structure therefore occur even without further changes in the underlying demographic variables, thus impacting the savings rate, the employment rate, and the average productivity per hour worked. This shift in the age structure is largely complete by the end of the twentieth century. Additionally, the changes in fertility, mortality, and migration rates that nevertheless occur raise the population size, so the scale effect on TFP is also at work here, as seen from the large impact on TFP growth.

6.4 It Is All About Technological Progress: The Exogenous Growth Case

The finding that the demographic transition does not negatively impact per-capita output contrasts with the general notion of population ageing as a major drag on economic activity. For instance, Krueger and Ludwig (2007), the perhaps closest paper to the analysis here, use a similar quantitative model and find a 12.6 percent cumulative drop in US output per capita associated with the demographic transition between 2005 and 2080. However, most previous work, including Krueger and Ludwig, ignore the TFP channel that I incorporate. Another important question is thus to what extent the difference can be explained by this additional mechanism.

As a final exercise, I therefore analyse a version of the model without technological change. Specifically, the benchmark model nests a standard model without endogenous growth as the special case with perfect substitution between intermediate firms ($\rho = 1$) and a zero intermediate-firm exit rate ($\delta_z = 0$). The former eliminates profits, thus forcing the patent price and R&D investment to zero, which in turn reduces the intermediate-firm dynamics to $z_{t+1} = (1 - \delta_z)z_t$. The latter ensures that the measure of intermediate firms remains constant over time. I also recalibrate the preference parameters β and ψ here to maintain the baseline calibration targets for the capital-output ratio and average hours worked.

Figure 11a plots the growth rate under this specification against that of the baseline model. The positive effects now disappear, with growth rates consistently around 0.1 to 0.4 percentage points below the baseline. The cumulative decline in output per capita is nearly identical to Krueger and Ludwig (2007): 12.3 percent between 2005 and 2080 and 11.1 percent over the full period. In comparison, for the same periods the baseline model exhibits positive cumulative growth of 5.2 and 31.5 percent, respectively. The difference is driven almost entirely by the TFP difference, as shown in Figure 11b. The impacts on the capital

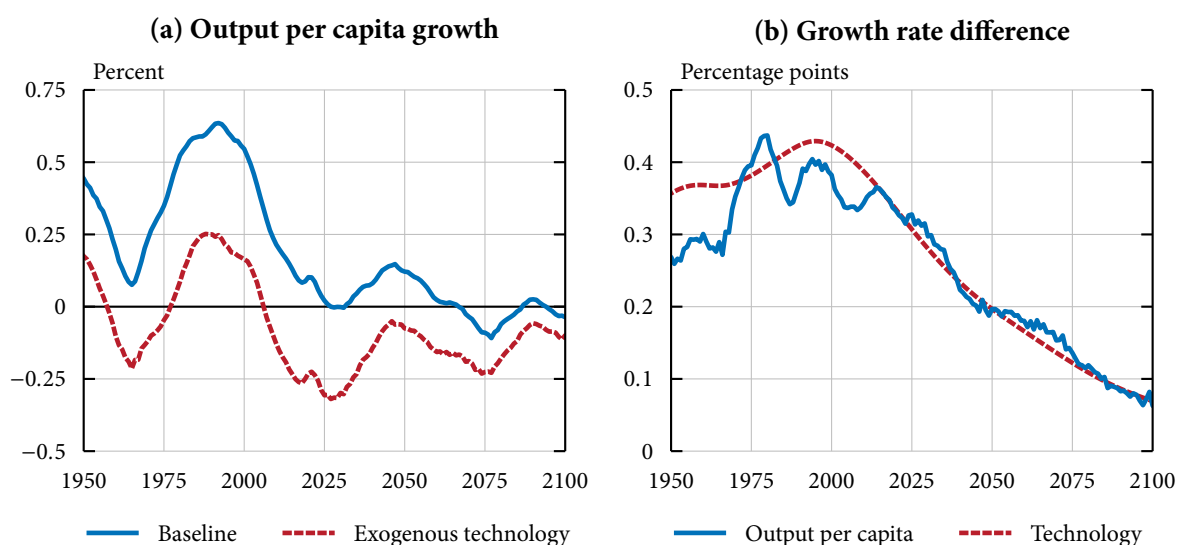


Figure 11. Comparing growth with and without endogenous technological change.

intensity, the employment rate and the average efficiency are virtually unchanged.²⁰ There is also a small counteracting effect in hours worked: lower income leads to a rise in hours, and this raises output. But this effect is too small to offset the lack of technological progress. Thus, whether the demographic transition raises or lowers output per capita turns out to depend crucially on whether or not its impact on technical change is properly accounted for.

6.5 Taking Stock

In sum, neither the comparative statics nor the transition dynamics of the model suggests that the demographic transition, and population ageing in particular, is detrimental to economic growth once we account for its effect on technological progress. If anything, the impact is (temporarily) positive. Although demographics account for a significant decline in growth rates over the last three decades due to a rising share of retirees in the population, it is important to stress that this is *not* because population ageing is particularly bad for economic output. In fact, it is hardly due to population ageing at all. Rather, the temporary rise in births that led to the baby boom generates a long period of above-average growth in the last quarter of the twentieth century when these generations are of prime working age. We are just now experiencing the end of that period as these generations retire. As already stressed in the simple model, the subsequent growth decline would occur even in the absence of improving life expectancy.

7 The Importance of the R&D Production Function

The results above rest on several calibration and parametrisation choices, most of which are standard in the literature or well justified in the data. In Online Appendix D I run a battery of alternative specifications for household preferences, inequality, fiscal policy, and pension rules and show that the baseline findings are generally unaffected. A central element that deserves closer treatment, however, is the R&D process. This section explores the sensitivity of the model with respect to two specific R&D components: the knowledge spillover parameter ϕ and the choice of R&D production function. The former seems particularly important since ϕ governs the degree to which R&D becomes easier or harder over time, and its value is likely

²⁰ See Figure E.6 in Online Appendix E, which fully decomposes the growth rate gap.

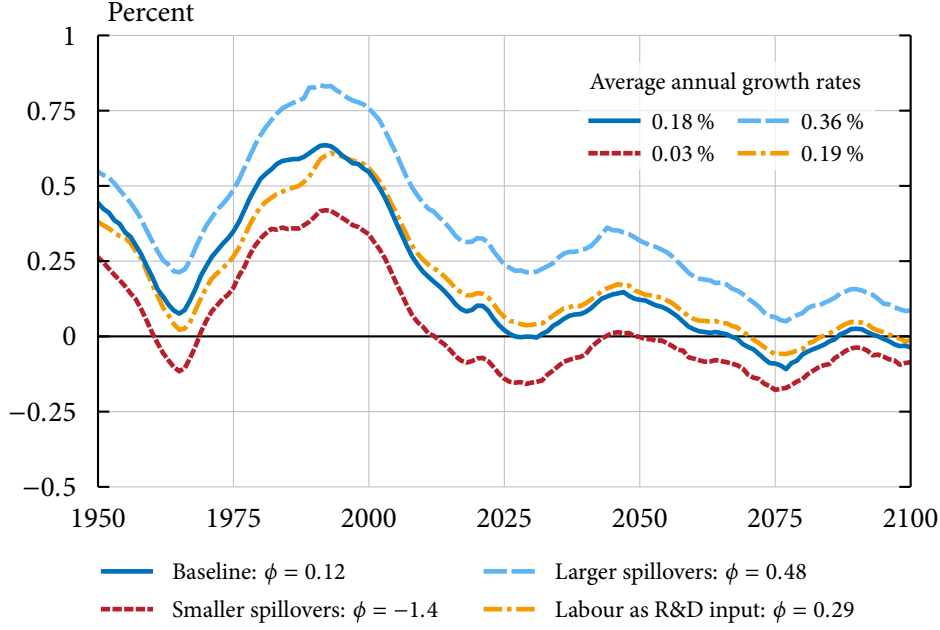


Figure 12. Growth rates with different degrees of knowledge spillovers and R&D inputs.

surrounded by considerable uncertainty given the difficulty to accurately measure productivity growth and R&D investment in the data.

To understand the quantitative significance of ϕ , consider the model with smaller or larger knowledge spillovers. To obtain smaller spillovers, I set ϕ to -1.4 based on Bloom *et al.*'s (2020) estimate for the aggregate US economy when $\lambda = 0.75$. The negative value of ϕ , which they obtain under the assumption that the substitution parameter ρ exactly equals the share parameter α , means that research productivity declines with the stock of knowledge. To obtain larger spillovers, I estimate TFP growth from the Penn World Table as the Solow residual between output and a Cobb-Douglas combination of capital and total hours worked. Applying the baseline calibration method then yields $\phi = 0.48$. Coincidentally, these values of ϕ generate steady-state growth exponents γ_y equal to 0.06 and 0.35, nearly identical to the empirically plausible bounds on γ_y established by Jones (2002). These scenarios thus provide lower and upper bounds on the baseline results with respect to ϕ .

To analyse the importance of the R&D production function, consider an alternative in which labour rather than final goods is used as R&D input. This setting is common in the growth literature and specifies the R&D process as

$$z_{t+1} = (1 - \delta_z)z_t + \nu L_{zt}^\lambda z_t^\phi,$$

where L_{zt} is the total labour devoted to R&D.²¹ Here, I set ϕ to 0.29 to obtain the same long-run growth rate as in the baseline. In this case and in the two above, I again recalibrate the preference parameters β and ψ to match the baseline calibration targets for the capital-output ratio and average hours worked.

Figure 12 displays the per-capita growth rates from these alternatives against that of the baseline. Qualitatively, changing the knowledge spillover is straightforward: the larger the knowledge spillover, the

²¹ The labour and goods market conditions also become $L_t + L_{zt} = \sum_j N_{jt} \int_X \ell_{jt}(x) d\Phi_{jt}$ and $Y_t + A_{t+1}^M = C_t + G_t + [K_{t+1} - (1 - \delta_k)K_t]$. As shown in Online Appendix A, long-run growth again follows the population growth rate via $1 + g_z = (1 + n)^\chi$, but now with $\chi \equiv \frac{\lambda}{1 - \phi - \lambda \frac{\theta(1-\sigma)}{1+\theta\sigma} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}$.

larger the growth impact from R&D, since researchers become more productive the more knowledge is created. [Figure 12](#) confirms this prediction. Meanwhile, changing the R&D input from final goods to labour generates a higher demand for labour relative to capital from producers, with subsequent general equilibrium effects on factor prices. Since the R&D sector is a small share of the total economy, these effects are small. None of the major underlying mechanisms of the model changes here, as the cost of R&D (the wage bill of researchers in this case) and the patent purchases by intermediate-good firms are still financed via household savings. The impact relative to the baseline is therefore negligible.

Growth rates are overall positive across the board (albeit quantitatively insignificant with small knowledge spillovers). Relative to the baseline, the average annual growth rate between 1950 and 2100 decreases and increases by about 0.15 percentage points with the different degrees of knowledge spillovers and remains unaffected when labour is the sole R&D input. Only in the most pessimistic calibration do we observe a negative impact on per-capita output during the twenty-first century, but even here the impact is small at about -0.05 percent per year. Therefore, although reasonable variations of the R&D process impact the quantitative findings, they do not change the basic point of this paper: that the demographic transition and the ageing of the population improves output per capita under endogenous technological change.

8 Conclusion

The model in this paper allows the population structure to affect output per capita via three main channels: through the fraction of people who work, through capital accumulation, and through technological progress. This framework stands in stark contrast to standard macroeconomic life-cycle models, in which technological change is exogenous. It also contrasts with most models of endogenous growth, which hide the entire population structure in a representative household. A key point throughout the paper is that this matters for how we think about demographic change and its impact on output per capita.

My main findings suggest that current and projected US demographic change from 1950 onward raises output per capita and that this effect is quantitatively large; at least on par with the growth contribution from US educational attainment over the second half of the twentieth century. I show that this is primarily due to the inclusion of endogenous technological change. Removing this channel completely reverses the positive impact. Overall, these findings challenge a seemingly conventional wisdom that current demographic developments are detrimental to economic activity.

The framework employed here admittedly leaves out several potentially important channels, such as human capital accumulation, automation, or international technology flows. However, papers that consider endogenous responses in human capital or automation typically find that these extra adjustment margins improve the impact of population ageing on output per capita, much like the inclusion of technological progress in this paper. Moreover, new technologies imported from foreign countries would only serve to raise the productivity of research under my baseline calibration. Thus, if anything, I would expect these mechanisms to only strengthen the main takeaways of the paper.

Lastly, the results also raise questions about the conclusions drawn in other literatures that rely on exogenous growth models. For example, the pension system is a key determinant of the savings rate and, since the savings rate is central to technical change here, an immediate question is therefore whether social security reform becomes more or less costly under R&D-driven growth. Other topics include migration policy, different types of fiscal policy reform, and optimal taxation. A recent paper by Jones ([2022b](#)) for instance

suggests that the optimal progressivity of income taxation is significantly altered by the presence of endogenous technological change. The model considered here could serve as a basis to analyse these questions further.

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