Endogenous Technological Change Along the Demographic Transition*

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Abstract

Does population ageing hurt output per capita in the advanced economies? Standard calibrations of life-cycle models with exogenous growth that consider two fundamental and opposing forces, capital deepening versus a declining employment rate, predict yes. Using a quantitative overlapping generations model with realistic demographics and R&D-driven endogenous growth, this paper challenges the standard prediction through a third possibility: that current demographic trends boost R&D investment and thus generate technological change. Calibrated to the United States, the model indicates that the demographic transition between 1950 and 2100 increases output per capita by 0.41 percent per year until 2000 and by 0.18 percent per year overall, thereby explaining 10 to 20 percent of observed US growth. The key mechanism is the endogeneity of technological change, whose growth contribution triples that of capital deepening, and removing this channel eliminates the positive impact.

Keywords: demographic transition, endogenous growth, OLG model.

JEL Codes: E17, E25, J11, O30, O40.

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1 Introduction

In 1950 there were 14 people aged 65 and above for every 100 people aged 20 to 64 in the United States. This number doubled by 2020 and is projected to quadruple by the end of the century. Figure 1 shows that similar trends apply worldwide. Also illustrated is the notion that economic growth is slowing down across advanced economies, and there is a widespread view that the ongoing population ageing is an important driver of this decline. How do current demographic trends actually affect output per capita? And how will they impact future growth? These questions lie at the heart of any policy discussion related to these shifts in the population structure.

It is well acknowledged that population ageing influences output per capita through two major channels. First, ageing reduces the number of workers relative to the rest of the population, thereby decreasing output per capita. Second, improving life expectancy increases savings in anticipation of longer retirement, which raises investment and capital accumulation. The subsequent capital deepening increases output per capita. Life-cycle models calibrated to the advanced economies that compare these channels typically predict the former to dominate. Krueger and Ludwig (2007) and Ludwig, Schelkle and Vogel (2012) for instance find that demographic change during the twenty-first century generates cumulative declines in output per capita on the order of 10 percent.

This paper emphasizes a third key channel that reverses both the sign and the magnitude of this prediction: the increase in savings induced by demographic change may also be used to finance research and development (R&D), which improves per-capita output through innovations that generate technological progress. The need to distinguish R&D from capital investment is essential in these analyses due to the nonrival nature of technology. An innovation can be used simultaneously by any number of workers at no additional cost and therefore raises everyone's productivity. An extra unit of capital, on the other hand, improves productivity only insofar as it increases capital per effective worker. Technological change therefore provides a more potent output per capita mechanism than capital deepening.

To formalise this argument and to explore its importance, I develop a quantitative general equilibrium model that combines two standard macroeconomic frameworks. On the supply side, the model features endogenous growth à la Romer (1990), with monopolistically competitive intermediate producers and an R&D sector whose innovations improve productivity by expanding the variety of intermediate goods. The household side follows in the Auerbach and Kotlikoff (1987) tradition with a large number of overlapping generations and a realistic population structure. These life-cycle households consume, save, and make labour supply decisions on both hours worked and the timing of retirement subject to changes in household size, mortality risk, income risk, borrowing constraints, and progressive income taxation. Savings are either invested in physical capital or provide funding for R&D in exchange for ownership stakes in the new firms it generates, the latter creating the link between life-cycle behaviour and technological progress.

The model exhibits semi-endogenous growth as in Jones (1995), Kortum (1997), and Segerstrom (1998), but the rich life-cycle setup goes substantially beyond the representative agent and perpetual-youth environments typically found in these growth models.[1] This contribution is crucial for the question at hand. In semi-endogenous growth theory, long-run economic growth is proportional to population growth. A long-run interpretation of the observed decline in population growth rates across the rich world since the mid-twentieth century therefore suggests that current demographic trends negatively impact

[1] Semi-endogenous growth analyses of demographic change with a representative household or perpetual-youth framework include include Prettner (2013), Prettner and Trimborn (2017), Peters and Walsh (2021), and Jones (2022a).

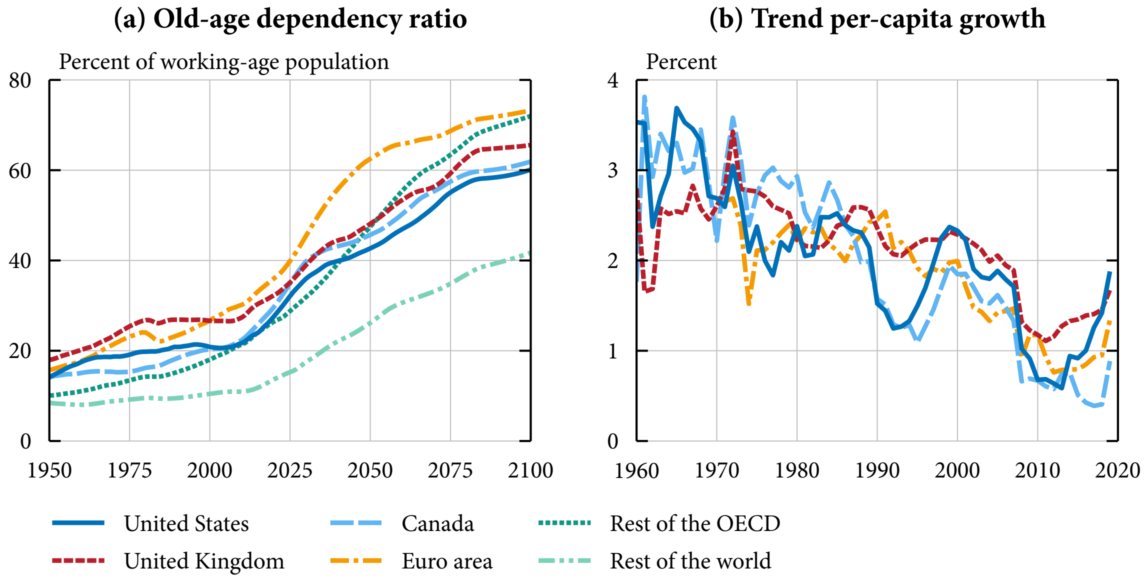


Figure 1. Ageing and growth across major economies.

Source. Holston, Laubach and Williams (2017) and United Nations (2022).

economic growth. Yet, in advanced economies like the United States, the United Kingdom, Canada, and Western Europe, the demographic shift since World War II primarily reflects two temporary forces: an initial surge in fertility (the baby boom) and steadily rising survival rates beyond the age of 50. Neither affects long-run population growth when people have finite lifespans. Such shifts in the population structure generate temporary growth that permanently affects the *level* of income, but these effects cannot be properly examined through the lens of steady-state growth in a representative agent or perpetual-youth model.

After analytically deriving the mechanisms above in a simplified three-generation setup, I calibrate the model and quantify these effects and their net impact on output per capita along the equilibrium path induced by the demographic transition between 1950 and 2100. In doing so, I make sure to keep the long-run trend fixed so that the demographic transition and the corresponding growth effects are treated as exclusively transitory. This quantitative analysis focuses on the United States, a large economy at the technological frontier, since the endogenous growth framework arguably describes US technological change better than it does for smaller economies in which most technology is imported. As shown in Figure 1a, the United States is also a useful demographic benchmark since its ageing process lies roughly in between the fastest ageing rich countries and the younger non-OECD countries.

Contrary to what a standard long-run analysis suggests, I find that the demographic transition is a net positive. Output per capita increases by on average 0.41 percent per year between 1950 and 2000 and by 0.18 percent per year overall. The former corresponds to roughly 20 percent of observed US growth over the same period, thereby making demographics as important to US post-war growth as rising educational attainment according to estimates by Fernald and Jones (2014). Counterfactual simulations also reveal that the baby boom and rising life expectancy after the age of 50 contribute around a quarter each of these results. Changes in the age structure due to the nonstationary population already present in 1950 and general increases in population size explain the remainder. As far as the impact on output per capita is concerned, these findings leave little reason to worry about current demographic change in general and about population ageing in particular.

The positive effect dissipates at the turn of the millennium when the baby boom retires, but the growth impact does not turn negative on average during the twenty-first century. Yet, the shift when this happens is sizeable: the growth rate declines by 0.60 percentage points between 1995 and 2030. Demographics can therefore explain a large chunk of the decline in observed growth rates. It is important to stress though that this decline is *not* a result of demographic change being inherently detrimental to economic output, but rather because of a reversion back from the above-average growth that it induced in the first place. This interpretation contrasts with the short-run secular stagnation view of Eggertsson, Lancastre and Summers (2019), Eggertsson, Mehrotra and Robbins (2019), and Jones (forthcoming), which sees current population ageing as a significant drag on output per capita.

At the core of these results lies technological change, which contributes three to four times more to output growth than capital deepening. This difference reflects the extra bang for the buck obtained from technology being nonrival. Together these two channels more than offset the negative impact of a growing share of retired households. Capital deepening alone cannot accomplish this: treating technology as exogenous (which is readily achieved as a special case of the benchmark model) eliminates the positive effect and leads to an 11.1 percent cumulative decline in output per capita between 1950 and 2100. This decline is quantitatively similar to Krueger and Ludwig (2007) and contrasts with a 31.5 percent increase in the baseline scenario, thus underlining that whether the demographic transition raises or lowers output per capita hinges crucially on whether we account for its impact on technological progress.[2]

The household heterogeneity in the model separates this paper not only from the growth literature but also from other quantitative macroeconomic papers incorporating R&D-based growth. This line of research typically examines business cycle dynamics under a representative household assumption.[3] Two closely related exceptions are Aksoy *et al.* (2019) and Basso and Jimeno (2021), who adapt the Comin and Gertler (2006) framework to study the growth effects of demographic change in the early twenty-first century. Both employ Gertler's (1999) perpetual-youth model, which effectively boils down to a stylised two-generation consumption-savings model of workers and retirees, and simulate transitions from high-growth to low-growth steady states. In contrast, this paper incorporates full-fledged life-cycle behaviour and realistic population dynamics, pinpoints the key mechanisms analytically, investigates longer transition paths, and considers transitory demographic changes around a fixed long-run trend. As highlighted earlier, the last point matters for interpretation: for comparable decades, both papers find quantitatively similar growth declines to here and consequently conclude that population ageing lowers growth, the opposite conclusion of this paper.

The focus on economic growth also distinguishes the analysis from the abundance of research that employs similar quantitative life-cycle models to examine the macroeconomic impacts of demographic change, which primarily concentrates on topics such as fiscal policy, international capital flows, wealth accumulation, or asset returns.[4] Studies like Krueger and Ludwig (2007) or Cooley and Henriksen (2018) that nevertheless touch upon the issue of growth typically restrict attention to the comparison between capital deepening and the declining fraction of people working. While some papers incorporate

[2] The importance of technology echoes the empirical analyses of Cutler *et al.* (1990) and Acemoglu and Restrepo (2017, 2022), who argue that ageing leads to another type of technical change: automation.

[3] Examples include Comin and Gertler (2006), Nuño (2011), Benigno and Fornaro (2018), Anzoategui *et al.* (2019), Bianchi, Kung and Morales (2019), and Okada (2023).

[4] Besides the papers mentioned elsewhere, a non-exhaustive list includes De Nardi, İmrohoroğlu and Sargent (1999), Storesletten (2000), Fehr, Jokisch and Kotlikoff (2004), Börsch-Supan, Ludwig and Winter (2006), Domeij and Flodén (2006b), Attanasio, Kitao and Violante (2007), Kotlikoff, Smetters and Walliser (2007), İmrohoroğlu and Kitao (2012), Kitao (2014), Carvalho, Ferrero and Nechio (2016), Auclert *et al.* (2021), and Gagnon, Johannsen and López-Salido (2021).

additional adjustment margins such as human capital accumulation (Ludwig, Schelkle and Vogel, 2012; Vandenbroucke, 2021) or automation (Heer and Irmen, 2014; Benzell *et al.*, 2021), no research to my knowledge conducts these quantitative analyses with endogenous R&D-driven technological change. This paper provides an initial step in this direction.

2 Identifying the Mechanisms: Results From a Simple Model

The observed and projected demographic transition in the rich world is characterised by two salient features after 1950: the temporary surge in fertility between the late 1930s and early 1970s that caused the baby boom, and an ageing of the population from rising survival rates for people aged 50 and above. If people have finite lifespans, neither affects the steady-state population growth rate. To develop intuition for how these transitory changes interacts with the mechanisms outlined in this paper, it is useful to first analyse a simplified version of the quantitative model in the subsequent sections.

To that end, consider the Solow-esque world given in Table 1, in which aggregate household behaviour is captured by an exogenous savings rate sr and a labour supply L_t which grows by the rate of the population n. Households allocate savings either to next-period capital K_{t+1} or to R&D investment Q_t. Together with the current stock of technologies Z_t, the latter generates new technologies through an R&D production function as in Jones (1995). Aggregate output Y_t is pinned down by a Cobb-Douglas production function with capital, technology, and labour as inputs. It is further assumed that capital and technologies fully depreciate after one period and that the allocation of savings follows an exogenous rule in which a fraction $\bar{\rho}$ is invested in capital each period. With full depreciation of capital and technology, this fixed allocation of savings is also the market equilibrium if the model is expanded along the lines of Romer (1990) and Jones (1995) to include the returns on capital and R&D investments, but imposing this directly simplifies the exposition.

We are interested in how the demographic structure affects output per capita, $y_t \equiv Y_t/N_t$, where N_t is the size of the population. It turns out convenient then to decompose output per capita using the Cobb-Douglas production function as follows:

$$y_t = Z_t \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}. \tag{1}$$

Consequently, output per capita is proportional to the stock of technology (or TFP), a capital intensity (captured by the capital-output ratio), and an employment rate. Determining the impact of demographics on output per capita requires an understanding of how demographics affect each of these decomposing factors, which is what we turn to next.

2.1 Components of Output per Capita in a Steady State

The components of Equation (1) can be solved for analytically in a steady state (that is, an equilibrium in which all variables grow at constant, possibly zero, rates). First, note that Table 1 outlines a semi-endogenous growth model. To see this, divide (S2) by Z_t to obtain a gross growth rate of TFP equal to $1 + g_{Zt} \equiv \frac{Z_{t+1}}{Z_t} = Q_t^{\lambda} Z_t^{\phi-1}$. Since R&D investment must grow by the rate of aggregate output in steady state,

Table 1. Summary of the simple model.

Output	$Y_t = K_t^{\alpha}(Z_t L_t)^{1-\alpha}$,	$0 < \alpha < 1$	(S1)
R&D output	$Z_{t+1} = Q_t^{\lambda} Z_t^{\phi}$,	$0 < \lambda \leq 1,\ \phi < 1$	(S2)
Asset market	$sr\,Y_t = K_{t+1} + Q_t$		(S3)
Allocation	$Q_t = (1-\bar{\rho})\,sr\,Y_t$,	$0 < \bar{\rho} < 1$	(S4)
Population growth	$L_{t+1} = (1+n)\,L_t$		(S5)

the right-hand side implies that TFP growth in steady state necessarily satisfies

$$1 + g_Z = (1+n)^{\gamma}, \qquad \text{where} \qquad \gamma \equiv \frac{\lambda}{1-\phi-\lambda}, \tag{2}$$

and this is a function of exogenous parameters, hence the term "semi". Equation (2) together with (S3) and (S4) allow us to write the capital-output ratio as

$$\frac{K_t}{Y_t} = \frac{\bar{\rho}\,sr}{(1+n)^{\frac{\gamma(1-\phi)}{\lambda}}}, \tag{3}$$

which shows that the capital intensity is the share of output allocated to capital investment divided by the aggregate growth rate, just like the Solow model. Equations (2) and (3) can then be combined with Table 1 to solve for the stock of TFP as

$$Z_t = \left[\frac{1-\bar{\rho}}{\bar{\rho}}\left(\frac{\bar{\rho}\,sr}{(1+n)^{\frac{\gamma(1-\alpha\phi)}{\lambda}}}\right)^{\frac{1}{1-\alpha}}\right]^{\gamma} L_t^{\gamma}. \tag{4}$$

Equation (4) writes TFP as the product of two factors. The first one is an R&D intensity, which is determined by the savings rate and thus incorporates household behaviour and composition. The second one, L_t^{γ}, is the scale effect on output per capita present in all semi-endogenous growth models, as discussed by Jones (2005), and reflects the fact that technology is nonrival: when the R&D process generates a new discovery, it can be used simultaneously by everyone at no additional cost. A larger population raises aggregate investment, even without changes in household behaviour, and this raises the stock of TFP. But the nonrivalry of technology means that output per capita depends on the overall stock of TFP rather than TFP per capita. Consequently, output per capita increases simply by virtue of a larger population.

At this point, let us add households to endogenise the employment rate and the savings rate. Consider a three-generation life-cycle model consisting of young, middle-aged, and old households (indexed by $j = 1, 2, 3$, with cohort size N_{jt}), in which young and middle-aged households supply labour inelastically and old households are retired. If the number of young households grows by the rate $1 + n$ in each period and a j-year-old survives till age $j + 1$ with probability s_j, then the steady-state employment rate is given by

$$\frac{L_t}{N_t} = \frac{N_{1t} + N_{2t}}{N_{1t} + N_{2t} + N_{3t}} = \frac{1 + \frac{s_1}{1+n}}{1 + \frac{s_1}{1+n}\left(1 + \frac{s_2}{1+n}\right)}. \tag{5}$$

To keep things simple, suppose households have logarithmic preferences over consumption c, that young households are hand-to-mouth, and that annuity markets are present. Households then maximise expected lifetime utility $\log(c_1) + s_1\beta \log(c_2) + s_1 s_2 \beta^2 \log(c_3)$ subject to the budget constraints $c_1 = w$, $c_2 + a = w$, and $c_3 = \frac{1+r}{s_2} a$, where β is a subjective discount factor, a denotes savings, w is the wage rate, and r is the interest rate. The solution to the household problem is characterised by a savings policy $a_{t+1} = \frac{s_2\beta}{1+s_2\beta} w_t$. The aggregate savings rate in steady state can then be written

$$sr = \frac{a_{t+1}N_{2t}}{Y_t} = \frac{w_t L_t}{Y_t} \frac{s_3\beta}{1+s_3\beta} \frac{\frac{s_2}{1+n}}{1+\frac{s_2}{1+n}}, \tag{6}$$

which is a product of the labour share of aggregate output, the share of labour income allocated to savings, and the share of saving, middle-aged workers in the labour force. Note that the labour share of output is constant given the Cobb-Douglas production function and the absence of automation technologies.

2.2 Population Ageing via Increases in Survival Rates

Consider now an ageing of the population through an increase in either of the survival rates s_1 and s_2. This naturally pushes down the employment rate through a mechanical increase in the old-age dependency ratio, as seen in Equation (5), which reduces output per capita by (1). Meanwhile, Equations (3), (4) and (6) highlight a counteracting force that improves output per capita: higher survival rates raises the savings rate, which in turn leads to larger capital and R&D intensities, the latter of which improves TFP. Additionally, an increase in the middle-age survival rate s_1 also raises the size of the labour force relative to its previous trend, thereby increasing TFP further through the scale effect present in (4).

The intuition for the effects on the capital and R&D intensities is straightforward. A higher likelihood of middle-age survival raises the share of households that save while a higher likelihood of old-age survival increases the life-cycle savings motive, so households save more intensively. These extra savings are allocated proportionately to capital and R&D investment according to (S4). The capital stock, however, increases disproportionately more than TFP because the production of capital is linear in investments whereas the R&D process for TFP improvements features decreasing returns to scale: a necessary parameter restriction for a steady state to exist is $\gamma > 0$, which holds for feasible values of λ and ϕ if and only if $\lambda + \phi < 1$. Therefore, both K_t/Y_t and Z_t rise.

2.3 A Baby Boom Through a Temporary Increase in Fertility Rates

Next consider a temporary increase in the population growth rate $1 + n$, similar to a baby boom. Clearly, this leaves the age structure unchanged in the long run. Thus, neither the employment rate nor the savings rate (and subsequently the capital and R&D intensities) is affected, as shown by Equations (5) and (6). The only long-run impact is through the scale effect in Equation (4): a temporary fertility boom permanently raises the population size, which leads to higher TFP and subsequently higher output per capita. This is exactly what a standard semi-endogenous growth model with a representative household predicts.

Equations (3) to (6) nevertheless provide some guidance about the transition dynamics for this scenario beyond the scale effect. The savings rate is low when the share of middle-aged households is low, and the employment rate is high when the share of old households is low. The initial boost in the share of young households therefore generates positive growth via the employment rate and reduced growth via the capital and R&D intensities. As this generation reaches middle age and their children enter the labour

Table 2. Comparative statics of the simple model.

Change	Duration	Output per capita	Technology: R&D intensity	Technology: Scale	Capital intensity	Employment rate
Middle-age mortality: $s_1 \uparrow$	Permanent	Ambiguous	+	+	+	−
Old-age mortality: $s_2 \uparrow$	Permanent	Ambiguous	+		+	−
Fertility: $1 + n \uparrow$	Temporary	+		+		

Notes. Increases denoted by plus signs, decreases denoted by minus signs.

force, the positive impact on the employment rate remains at the same time as the savings rate recovers, thus reverting the capital and R&D intensities back towards their starting points. When this generation finally reaches retirement, the effect on the employment rate also reverts back. These predictions will be useful to understand the quantitative results.

2.4 Taking Stock

The steady-state predictions are summarised in Table 2. Overall, the demographic transition deteriorates the employment rate through the old-age dependency ratio, improves the capital intensity via the savings rate, and generates technical change through both the savings rate and the scale effect associated with the nonrivalry of technology. Yet, the relative strengths of these channels and the net effect on transitional growth are ambiguous. This motivates the need for a quantitative treatment.

3 A Quantitative OLG Model with Endogenous Growth

The mechanisms above are quantified in a closed economy environment populated by overlapping generations of households, production firms, R&D firms, and a government. Time is discrete, with a period set to one year, and exogenously imposed changes in the demographic structure constitute the driving force of the model.[5] Following the discussion in Section 2, particular care is taken to include a rich set of factors that affect household behaviour over the life cycle, including changes in household size, mortality risk, idiosyncratic income risk, borrowing constraints, progressive income taxation, and intensive (hours worked) and extensive (age of retirement) margins of labour supply. The rest of the framework remains close to the growth literature: the production sector consists of a perfectly competitive final-good firm and monopolistic intermediate-good firms, and technology grows endogenously through the entry of new intermediate firms created from innovations in the R&D sector.[6]

3.1 Demographics and Household Composition

In a given period t, the economy consists of $J + 1$ overlapping generations of sizes $N_{0t}, \dots, N_{Jt}$, with total population size $N_t = \sum_{j=0}^{J} N_{jt}$. From a given initial population distribution, the demographic structure in

[5] For simplicity, I abstract from the possibility that technological change reversely affects demographic variables (birth control, treatment of diseases, and so on). This is also the recommendation of Lee (2016, p. 111), who states “While theories are available to relate [fertility, mortality, and health] to individual choices, they have little predictive power and their use might obscure the workings of some better understood mechanisms.”

[6] The mechanisms emphasised in this paper are present in any standard endogenous growth model, so choosing between the expanding-variety model of Romer (1990) or a quality-ladder model á la Aghion and Howitt (1992) and Grossman and Helpman (1991) is inconsequential here.

subsequent periods is pinned down recursively by age- and time-specific fertility rates f_{jt}, survival rates s_{jt}, and net migration rates m_{jt} according to

$$N_{0,t+1} = \sum_{j=0}^{J} f_{jt} N_{jt} \qquad \text{and} \qquad N_{j+1,t+1} = \left(s_{jt} + m_{jt}\right) N_{jt}, \tag{7}$$

where $s_{Jt} = m_{Jt} = 0$ holds for all t such that individuals die with certainty after age J. Migrants bring their accumulated wealth with them when they move and are economically identical to non-migrants. This assumption eliminates the need to separate between natives and migrants in the economic model.

Individuals are children for the first ι years of their lives, after which they form a household and become economically active. A household consists of an adult head and their dependants, including both children and grandchildren of underaged parents. For an adult of age j, the number of dependants of age $i < \iota$ is given by

$$N^d_{ijt} \ = \ f_{j-1-i,t-1-i} \frac{N_{j-1-i,t-1-i}}{N_{0,t-i}} \frac{N_{it}}{N_{jt}} + \sum_{k=i+1}^{\iota-1} N^d_{kjt} N^d_{ikt}. \tag{8}$$

The first term is the number of children, which is calculated by multiplying the fraction of the i-year-old cohort born to parents who are now j years old by the current population of i-year-olds, and subsequently dividing that evenly across the current j-year-old population. The second term sums up the number of grandchildren, where the number of i-year-old dependants of an underaged parent is given analogously to the first term on the right-hand side of (8).

3.2 Household Endowments and Preferences

Individuals are endowed with one unit of time and enter adulthood as members of the labour force with no asset holdings. Working households provide an effective labour supply of $\ell_j = \varepsilon_j \eta h_j$ efficiency units, where h_j is hours worked, ε_j is a deterministic, age-dependent productivity level, and η is an idiosyncratic productivity component that evolves stochastically following a time-invariant Markov chain. Idiosyncratic productivity in the initial adult age ι is distributed according to the unique invariant distribution of the associated Markov chain, and workers can partially self-insure against idiosyncratic shocks through asset holdings a subject to a borrowing constraint. Additionally, there are no annuity markets to insure against mortality risk. Instead, the government seizes the assets of households who die prematurely and redistributes them as lump-sum transfers tr to surviving households. Leaving time subscripts implicit in this subsection, the budget constraint facing a household head at age j is thus written as

$$a_{j+1} + (1+\tau^c)c_j \ = \ \left(1 + r(1-\tau^k)\right)a_j + \left(1 - \tau^w(w\ell_j) - \tau^b\right)w\ell_j + tr + b(R_j), \tag{9}$$

where c_j is total household consumption, r is the rate of return on savings, w is the wage rate, and τ^c, τ^k, τ^w, τ^b denote tax rates where, in particular, τ^w is allowed to vary with labour income. The final term, $b(R_j)$, is a pension benefit to be specified later that depends on whether or not a person is retired and, in the former case, the age of retirement, both of which are captured by the retirement status R_j.

A household head of age j values their own consumption c_j^a, their leisure time $1 - h_j$, and the consumption c_{ij}^d per i-year-old dependant according to the static utility function

$$u\left(c_j^a, \{c_{ij}^d\}_{i=0}^{\iota-1}, h_j\right) = \frac{\left(c_j^a\right)^{1-\sigma} - 1}{1-\sigma} + \sum_{i=0}^{\iota-1} \omega_i N_{ij}^d \frac{\left(c_{ij}^d\right)^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h_j^{1+1/\theta}}{1+1/\theta}, \tag{10}$$

where $1/\sigma$ is the intertemporal elasticity of substitution for consumption, ω_i is the utility weight of i-year-old dependants, θ is the Frisch elasticity of labour supply, and ψ determines the disutility of working. Moreover, households benefit from economies of scale: total household consumption satisfies the constant elasticity of transformation function

$$c_j = \left[\left(c_j^a\right)^\zeta + \sum_{i=0}^{\iota-1} N_{ij}^d \left(c_{ij}^d\right)^\zeta\right]^{\frac{1}{\zeta}}, \qquad \zeta \geq 1, \tag{11}$$

in which the parameter ζ captures the degree of economies of scale. If $\zeta = 1$, there are no scale benefits and total household consumption is just the sum of the individual consumption levels. Equations (10) and (11) admit an equivalent utility function expressed in terms of total household consumption and hours worked. Taking household consumption as given and maximising (10) with respect to consumption under the constraint imposed by (11), we obtain $c_{ij}^d = \omega_i^{\frac{1}{\zeta-(1-\sigma)}} c_j^a$. This optimality condition combined with Equations (10) and (11) generates a utility function of the form

$$u(c_j, h_j) = \Omega_j \frac{c_j^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h_j^{1+1/\theta}}{1+1/\theta}, \tag{12}$$

where

$$\Omega_j \equiv \left[1 + \sum_{i=0}^{\iota-1} \omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}} N_{ij}^d\right]^{\frac{\zeta-(1-\sigma)}{\zeta}} \tag{13}$$

is an age- and time-specific household-size taste shifter. Specifically, substituting $c_{ij}^d = \omega_i^{\frac{1}{\zeta-(1-\sigma)}} c_j^a$ into Equation (11) reveals that $c_j = \Omega_j^{\frac{1}{\zeta-(1-\sigma)}} c_j^a$, so $\Omega_j^{\frac{1}{\zeta-(1-\sigma)}}$ measures the number of adult equivalents in a household and $\omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}}$ is the equivalence weight of an i-year-old dependant. Equation (12) belongs to the class of balanced-growth preferences characterised by Boppart and Krusell (2020) that allows for falling hours worked in the long run; the income effect of higher wages on leisure dominates the substitution effect if $\sigma > 1$, leading to a decline in hours worked whenever wage growth is positive. These preferences are summed up over time into an expected lifetime utility function of the form

$$\mathbb{E}\left[\sum_{j=\iota}^{J} \beta^{j-\iota} \left(\prod_{k=\iota}^{j-1} s_k\right) u(c_j, h_j)\right], \tag{14}$$

where β is a subjective discount factor and the expectations are taken over idiosyncratic labour productivity.

Households maximise expected lifetime utility (14) by choosing consumption, hours worked, and their age of retirement. At the start of each period, households observe their state $x_j' = (a_j, \eta, R_{j-1})$ consisting of current-period wealth and idiosyncratic productivity and the previous-period retirement status, and subsequently make their retirement decisions. Consumption and hours worked are then chosen in a second stage under the post-retirement decision state $x_j = (a_j, \eta, R_j)$. Retirement is an absorbing state, so the intensive- and extensive-margin labour supply choices are then trivial and the household problem reduces to a standard consumption-savings choice.

Formally, denote the retirement choice by a discrete variable d equal to 1 if choosing to remain in the labour force and 0 otherwise and let the retirement status evolve according to $R_j = R_{j-1} + d_j$, starting with $R_{\iota-1} = \iota$. An optimal retirement policy is then a function $d_j(x_j')$ that solves the first-stage problem

$$V_j(x_j') \;=\; \max_{d_j \in D(R_{j-1})} \left\{ v_j(x_j) \right\} \tag{15}$$

subject to the motion of the retirement status and subject to

$$D(R_{j-1}) \;=\; \begin{cases} \{0, 1\} & \text{if } R_{j-1} = j, \\ \{0\} & \text{if } R_{j-1} < j, \end{cases}$$

where $V_j(x_j')$ and $v_j(x_j)$ are the pre- and post-retirement decision value functions at age j. Optimal policies for consumption, savings and hours worked are functions $c_j(x_j)$, $a_j(x_j)$ and $h_j(x_j)$ that solve the second-stage problem

$$v_j(x_j) \;=\; \max_{c_j,\, h_j} \left\{ u(c_j, h_j) + \beta s_j \, \mathbb{E}\left[V_{j+1}(x_{j+1}') \,\middle|\, \eta \right] \right\} \tag{16}$$

subject to the budget constraint (9), the time constraints $h_j \in [0, 1]$ if working and $h_j = 0$ if retired, and the borrowing constraint $a_{j+1} \geq 0$.

3.3 Production

The competitive final-good sector hires labour L_t at the wage w_t and buys a continuum of intermediate capital inputs k_{it} indexed by $i \in [0, z_t]$ at prices p_{it} to produce output Y_t according to the Cobb-Douglas function

$$Y_t \;=\; L_t^{1-\alpha} \left(\int_0^{z_t} k_{it}^{\rho} \, di \right)^{\frac{\alpha}{\rho}}, \qquad 0 < \alpha, \rho < 1. \tag{17}$$

where z_t is a measure of the intermediate varieties available at time t. Intermediate-good firms use a linear production function that converts capital into intermediate inputs one for one. Capital is rented from households at the rate $r_t + \delta_k$, where δ_k is the capital depreciation rate. Each firm in the intermediate sector has a patent for their own variety and acts as a monopolist. Conditional on having a patent, a firm j consequently maximises operating profits $\pi_{jt} = (p_{jt} - r_t - \delta_k) k_{jt}$ subject to the final-good producer's demand for j. By profit maximisation in the final-good sector, this demand constraint is given by

$$p_{jt} \;=\; \alpha \, \frac{Y_t}{k_{jt}} \, \frac{k_{jt}^{\rho}}{\displaystyle\int_0^{z_t} k_{it}^{\rho} \, di}. \tag{18}$$

Symmetry across firms implies that all firms charge the same mark-up over marginal cost, $p_{it} = p_t = \frac{1}{\rho}(r_t + \delta_k)$, and therefore also sell the same quantity k_t and earn the same profits $\pi_t = (1-\rho)p_t k_t$.

3.4 The R&D Sector

The R&D sector exhibits free entry and invests an aggregate amount Q_t of final output on innovation to develop new designs for intermediate goods. The patents for new designs are sold to prospective intermediate-good firms at the end of a period t for a one-off price $P_{z,t+1}$, which then convert the designs into usable input in period $t+1$. These patent purchases are financed by households in exchange for equity ownership in the intermediate-good firms. Like the simple model in Section 2, innovation is characterised by an overall production function as in Jones (1995):

$$F(Q_t) = \bar{v}_t Q_t = \nu Q_t^{\lambda} z_t^{\phi}, \qquad 0 < \lambda \leq 1, \ \phi < 1, \tag{19}$$

where the productivity term $\bar{v}_t \equiv \nu Q_t^{\lambda-1} z_t^{\phi}$ incorporates duplication externalities (via λ) and knowledge spillovers (via ϕ) that impact the aggregate development but are external to individual R&D firms. An individual R&D firm therefore chooses expenditures q_t to maximise profits $(P_{z,t+1}\bar{v}_t - 1)q_t$, which together with free entry yields the aggregate zero-profit condition

$$Q_t = P_{z,t+1} \nu Q_t^{\lambda} z_t^{\phi}. \tag{20}$$

As in Comin and Gertler (2006) and several other papers, an intermediate-good firm is not infinitely lived; in each period a fraction δ_z of firms become obsolete.[7] The aggregate law of motion of new intermediates is therefore $z_{t+1} = (1-\delta_z)z_t + F(Q_t)$, thus implying the gross growth rate of intermediate varieties

$$1 + g_{zt} = 1 - \delta_z + \nu Q_t^{\lambda} z_t^{\phi-1}. \tag{21}$$

Lastly, a prospective intermediate-good firm enters the market only if it is profitable to do so. That is, the firm enters if the sum of expected discounted flow profits π exceeds the fixed cost P_z of purchasing a patent. Free entry into the intermediate-good sector drives the profitability of entry to zero. This is equivalent to saying that the following no-arbitrage condition holds in equilibrium:

$$r_t = \frac{\pi_t + \Delta P_{zt} - \delta_z P_{z,t+1}}{P_{zt}}, \tag{22}$$

where $\Delta P_{zt} \equiv P_{z,t+1} - P_{zt}$ is the change in the patent price in period t.

3.5 The Public Sector

The public sector engages in three activities: (i) it redistributes assets from deceased individuals, (ii) collects taxes on consumption, capital gains and wages via the tax rates τ_t^c, τ_t^k and τ_t^w to finance public consumption G_t, and (iii) maintains a pay-as-you-go social security system. The pension system is financed by the contribution rate τ_t^b on labour earnings. The budget constraint of each public-sector activity is independent of the other two and always balances. In the baseline model, budget balance is ensured

[7] This feature is primarily technical to control the level of R&D investment in steady state. We can also set $\delta_z < 0$ to generate exogenous growth, as in the standard neoclassical model, in addition to that created through R&D.

through endogenous adjustments in the per-capita bequest transfer tr, public consumption G_t, and the social security contribution rate τ_t^b.

Pension benefits are independent of earnings history but depend on the age of retirement.[8] Specifically, a household with retirement status R receives a base-level benefit that is scaled by a factor $ps(R)$ relative to some normal retirement age R^{norm} to capture early retirement penalties and delayed retirement credits. The base-level benefit is a fraction μ of average gross labour income $w_t\bar{\ell}_t$, where $\bar{\ell}_t$ is the average number of efficiency units per worker. An age-j household therefore receives a pension transfer

$$b_t(R) = \begin{cases} ps(R)\mu\, w_t\bar{\ell}_t & \text{if } j \geq \max\{R^{min}, R\}, \\ 0 & \text{if } j < \max\{R^{min}, R\}, \end{cases} \tag{23}$$

where no pension is paid out to working households or to retirees younger than some age R^{min}.

3.6 Equilibrium

In any period t, the state of the economy is pinned down by the aggregate capital stock $K_t \equiv \int_0^{z_t} k_{it}\, di$, the number of intermediate varieties z_t, the patent price P_{zt}, and measures $\Phi_{jt}(x)_{j=\iota}^J$ characterising the distribution of households after their retirement decisions are made. Since the state of a household is determined by their asset wealth, idiosyncratic productivity, and retirement status, the state space for the household measures is given by $X = \mathbb{R}_+ \times H \times \{\iota, \ldots, J+1\}$, where H denotes the state space of the Markov chain for idiosyncratic productivity. Given an evolution of demographic variables and initial conditions K_0, z_0, $P_{z,0}$, $\{\Phi_{j0}\}_{j=\iota}^J$, we then have:

Definition 1 (General equilibrium)**.** An equilibrium consists of paths for household decision rules $\{d_{jt}(\cdot)$, $c_{jt}(\cdot)$, $a_{jt}(\cdot)$, $h_{jt}(\cdot)\}_{t=0}^\infty$ and household measures $\{\Phi_{jt}\}_{t=0}^\infty$ for all ages j, prices and profits $\{r_t, w_t, \pi_t, P_{zt}\}_{t=0}^\infty$, taxes and transfers $\{\tau_t^c, \tau_t^k, \tau_t^b, \tau_t^w(\cdot), tr, \{b_{jt}(\cdot)\}_{j=\iota}^J\}_{t=0}^\infty$, and aggregate quantities such that for all periods t:

(i) Household decision rules solve problems (15) and (16).

(ii) Profit maximisation of final- and intermediate-good firms yields the consolidated production function

$$Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha}, \qquad \text{where} \qquad Z_t \equiv z_t^{\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}, \tag{24}$$

with corresponding factor prices and profits

$$w_t = (1-\alpha)\frac{Y_t}{L_t}, \qquad r_t = \alpha\rho\frac{Y_t}{K_t} - \delta_k, \qquad \pi_t = \alpha(1-\rho)\frac{Y_t}{z_t}. \tag{25}$$

(iii) The measure of intermediate-good varieties z_t, the patent price P_{zt} and aggregate R&D investment Q_t satisfy Equations (20) to (22).

(iv) The public sector budgets balance:

$$G_t = \tau_t^c C_t + \tau_t^k r_t A_t + \sum_{j=\iota}^J N_{jt} \int_X \tau^w\big(w_t \ell_{jt}(x)\big) w_t \ell_{jt}(x)\, \mathrm{d}\Phi_{jt}, \tag{26}$$

[8] Earnings-dependent pension would introduce another continuous state variable in the household problem and I abstract from this feature to avoid the additional computational complexity that it entails, as is common in the OLG literature.

$$\tau_t^b w_t L_t = \sum_{j=\iota}^{J} N_{jt} \int_X b_t(R)\, \mathrm{d}\Phi_{jt}, \tag{27}$$

$$tr_{t+1} = \frac{1 + r_{t+1}(1 - \tau_{t+1}^k)}{\sum_{j=\iota}^{J} N_{j,t+1}} \sum_{j=\iota}^{J} (1 - s_{jt}) N_{jt} \int_X a_{jt}(x)\, \mathrm{d}\Phi_{jt}, \tag{28}$$

where $C_t = \sum_{j=\iota}^{J} N_{jt} \int_X c_{jt}(x)\, \mathrm{d}\Phi_{jt}$ is aggregate consumption and $A_{t+1} = \sum_{j=\iota}^{J} (1+m_{jt}) N_{jt} \int_X a_{jt}(x)\, \mathrm{d}\Phi_{jt}$ is aggregate wealth.

(v) The markets for labour, capital, and goods clear:

$$L_t = \sum_{j=\iota}^{J} N_{jt} \int_X \ell_{jt}(x)\, \mathrm{d}\Phi_{jt}, \tag{29}$$

$$A_t = K_t + P_{zt} z_t, \tag{30}$$

$$Y_t + A_{t+1}^M = C_t + G_t + \left[K_{t+1} - (1 - \delta_k) K_t\right] + Q_t, \tag{31}$$

where $A_{t+1}^M = \sum_{j=\iota}^{J} m_{jt} N_{jt} \int_X a_{jt}(x) \mathrm{d}\Phi_{jt}$ is the net wealth brought by migrants.

(vi) For a given Markov kernel Π and for all Borel sets $S = \mathcal{A} \times \mathcal{H} \times \mathcal{R}$ on X (and dropping time subscripts), the household distributions evolve according to

$$\Phi_{j+1}(S) = \int_{X_W} \left[\int_{\mathcal{H}} d_{j+1}(x')\, \Pi(\eta, \mathrm{d}\eta')\right] \mathrm{d}\Phi_j + \int_{X_R} \left[\int_{\mathcal{H}} \left(1 - d_{j+1}(x')\right) \Pi(\eta, \mathrm{d}\eta')\right] \mathrm{d}\Phi_j, \tag{32}$$

where $X_W = \{x \in X : a_j(x) \in \mathcal{A},\ R + 1 \in \mathcal{R}\}$ and $X_R = \{x \in X : a_j(x) \in \mathcal{A},\ R \in \mathcal{R}\}$, and where $x' = \left(a_j(x), \eta', R\right)$, $\eta' \in \mathcal{H}$, are the relevant pre-retirement decision states at age $j + 1$. Letting Γ denote the unique invariant distribution of Π, the initial household distribution is equal to

$$\Phi_\iota\left(\{0\} \times \mathcal{H} \times \{R\}\right) = \begin{cases} \int_{\mathcal{H}} d_\iota(0, \eta, \iota)\, \mathrm{d}\Gamma & \text{if } R = \iota + 1, \\ \int_{\mathcal{H}} \left(1 - d_\iota(0, \eta, \iota)\right) \mathrm{d}\Gamma & \text{if } R = \iota, \end{cases} \tag{33}$$

and zero for all other asset levels and retirement statuses. ◃

As in Romer (1990), the model features endogenous growth in total factor productivity Z_t through changes in the measure of intermediate varieties z_t. As in Jones (1995), this growth is semi-endogenous in that it is determined exogenously by the rate of population growth along a balanced growth path:

Definition 2 (Stationary equilibrium)**.** A stationary equilibrium, or steady state, is an equilibrium in which all variables grow at constant rates (possibly zero) and all growth rates are determined by the population growth rate $1 + n \equiv \frac{N_{t+1}}{N_t}$. In particular, the growth rate of new intermediate-good firms is

$$1 + g_z = (1 + n)^\chi, \qquad \text{where} \qquad \chi \equiv \frac{\lambda}{1 - \phi - \lambda \frac{1+\theta}{1+\theta\sigma} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}} > 0. \tag{34}$$

The growth rates of TFP, output per capita, and hours per worker are similarly

$$1 + g_Z = (1+n)^{\gamma_Z}, \qquad 1 + g_y = (1+n)^{\gamma_y}, \qquad 1 + g_h = (1+n)^{\gamma_h},$$

where $\gamma_Z \equiv \chi \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}$, $\gamma_y \equiv \frac{1+\theta}{1+\theta\sigma} \gamma_Z$, and $\gamma_h \equiv \frac{\theta(1-\sigma)}{1+\theta\sigma} \gamma_Z = \gamma_y - \gamma_Z$. ◃

The growth rates in Definition 2 are derived in Appendix B. Note that the TFP growth rate reduces to Equation (2) in the simple model if we consider a steady state with constant hours worked (via log preferences, $\sigma \longrightarrow 1$) and a substitution parameter for intermediate goods exactly equal to the capital share parameter ($\rho = \alpha$).

4 Numerical Experiment and Implementation

The quantitative exercise considered is a demographic transition between 1950 and 2100 which mimics the observed and projected population trends during this period. The transition ultimately converges to a steady state consistent with the projected fertility, mortality, and migration rates expected by the end of the twenty-first century. To reduce the impact of the initial steady state on the period of interest, I simulate the model from a stationary equilibrium imposed already in 1900. In 1901, the economy is then shocked by the demographic transition, during which there is perfect foresight of aggregate variables until the new steady state. I also assume the same population growth rate in the initial steady state as in the final steady state. This ensures that the long-run growth trend does not change, allowing any growth beyond this trend to be attributed to transitional factors.

Numerically, I solve the household problem with the discrete-continuous endogenous grid method introduced by Iskhakov *et al.* (2017) and approximate the household distributions using histograms over wealth, idiosyncratic productivity, and retirement status. Iskhakov *et al.* extend the standard endogenous grid method of Carroll (2006) to incorporate discrete choices, which simplifies the computational difficulty associated with the endogenous retirement decision. The overall solution method iterates over paths of interest rates (r_t), pension contribution rates (τ_t^b), average labour supplies ($\bar{\ell}_t$), lump-sum transfers (tr_t), and growth rates of intermediate varieties (g_{zt}) until all equilibrium conditions hold. These paths are updated with Ludwig's (2007) modified quasi-Newton algorithm, which exploits the Jacobian of the equilibrium conditions related to $\{r_t, \tau_t^b, \bar{\ell}_t, tr_t, g_{zt}\}$ in the final steady state to provide an initial guess for the Jacobian in the transition algorithm.

5 Calibration

The model is calibrated to match the US economy and stays as close as possible to standard parameter values used in the literature. The resulting choices, discussed in detail below, are summarised in Table 3.

5.1 Demographics

The United Nations (2022) World Population Prospects provides yearly estimates for age-specific survival rates, age-specific fertility rates (births per woman), and aggregate net migration between 1950 and 2100. For earlier years, I collect age-specific survival rates and fertility rates from Bell and Miller (2005) and Heuser (1976), and use aggregate immigration data from the US Department of Homeland Security (2020) as a proxy for aggregate net migration. The age distribution of migration is assumed throughout to match

Table 3. Calibrated parameters of the baseline model.

Parameter	Description	Value	Target/source
Households			
ι	Initial adult age	20	Children between 0–19
J	Maximum age	99	Certain death at 100
β	Discount factor	1.022	Capital/output = 2.8
ψ	Leisure weight	19.718	50 hours/week in 1900
σ	Inverse IES	1.75	Boppart and Krusell (2020)
θ	Frisch elasticity	0.5	Chetty *et al.* (2011)
ζ	Household consumption scale	1.49	Browning and Ejrnæs (2009)
$\{\omega_i\}_{i=0}^{\iota-1}$	Utility weights on dependants	Fig. 3b	Browning and Ejrnæs (2009)
Individual productivity			
$\{\varepsilon_j\}_{j=\iota}^{J}$	Deterministic productivity	Fig. 4a	PSID
φ	Persistence shock	0.97	Heathcote *et al.* (2010)
σ_ϵ^2	Variance shock	0.02	Heathcote *et al.* (2010)
Production			
α	Intermediate goods share	0.36	Labour share = 0.64
ρ	Elasticity of substitution	0.71	Profit share = 0.10
δ_k	Capital depreciation rate	0.049	Investment/output = 0.136
R&D			
δ_z	Firm obsolescence rate	0.005	R&D/output = 0.014
ν	R&D productivity	0.009	$z = Z = 1$ in initial period
λ	Duplication externality	0.75	Comin and Gertler (2006)
ϕ	Knowledge spillovers	0.117	$g_Q = 6.73\,\% \implies g_Z = 1.26\,\%$
Social security			
μ	Replacement rate	0.413	Clingman *et al.* (2021)
R^{norm}	Normal retirement age	65	Social Security Administration
R^{min}	Lowest retirement age	62	Social Security Administration
$ps(R_j)$	Early/delayed scaling	0.8–1.15	Social Security Administration
Taxes			
τ^c	Consumption tax rate	0.080	BEA national accounts
τ^k	Capital gains tax rate	0.368	BEA national accounts
$\tau^w(w\bar{\ell})$	Income tax rate at mean income	0.115	BEA national accounts
κ_0	Asymptotic income tax rate	0.631	OECD tax database
κ_1	Income tax progressivity	0.574	OECD tax database
κ_2	Income tax scale parameter	0.505	OECD tax database
κ_3	Income tax rate at zero income	−0.207	OECD tax database

the US Census Bureau's (2018) population projection. Fertility rates by age are only available from 1917, so for earlier years I adjust the age-specific fertility rates in 1917 by the change in the total fertility rate between these years and 1917 using estimates from the Gapminder Foundation (2017). The fertility rates and migration levels are then converted into births and migrants per person using population estimates from the Census Bureau's intercensal tables and the UN World Population Prospects. Lastly, I smooth all demographic variables by an HP-filter with an annual smoothing parameter of 6.25.

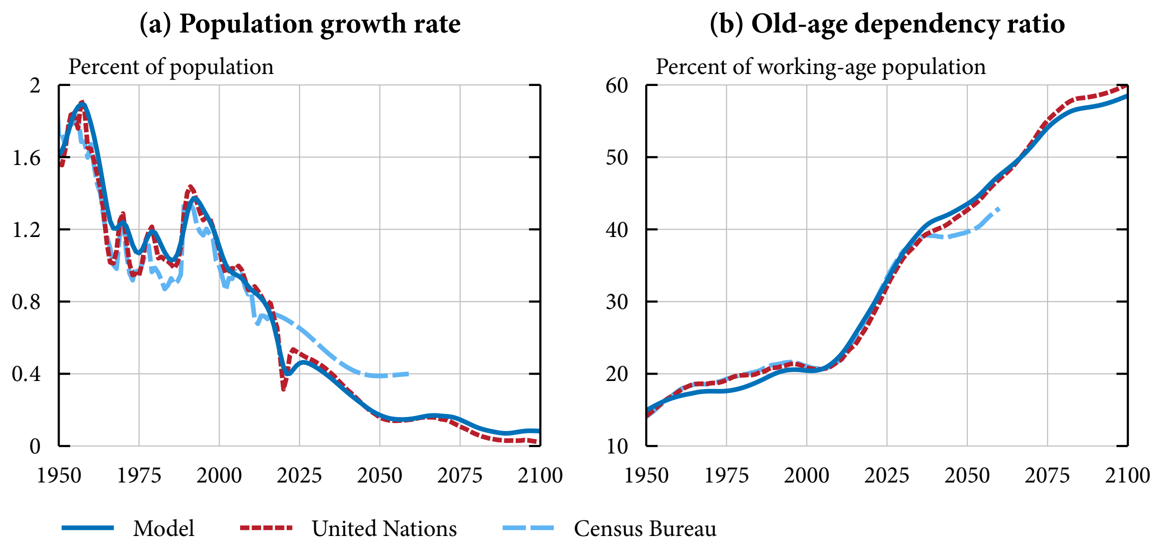


Figure 2. Demographic evolution.

The resulting demographic variables for the year 2100 generate a steady-state population with near-zero growth: -0.02 percent. For simplicity, I marginally increase migration rates by a common term so that population growth is exactly zero in the final steady state. I similarly impose a stable and constant population in the initial steady state that is consistent with the data on survival rates, fertility rates, and population size, again by adjusting migration rates with a common term.[9] Overall, Figure 2 plots the implied demographic development, showing that the calibration reasonably matches official estimates and projections from the United Nations and the Census Bureau.

5.2 Preferences and Labour Productivity

Households start their economic lives at 20 and die with certainty at 100. The discount factor β and the weight on labour supply ψ are calibrated to match a capital-output ratio of 2.8 and an average labour supply per worker of 0.45 in the initial steady state. The former yields a discount factor in the vicinity of Hurd (1989), who explicitly accounts for mortality risk and estimates a β of 1.011. Given a time endowment of 16 hours per day, the latter implies an average of 50 hours worked per week and worker in 1900, which is consistent with the evidence in Ramey and Francis (2009, Figure 1A).

I set the Frisch elasticity of labour supply θ to 0.5, as recommended by Chetty *et al.* (2011) along the intensive margin. This value is also consistent with Domeij and Flodén (2006a), who explicitly account for biases arising from uninsurable income risk and borrowing constraints. The inverse of the intertemporal elasticity of substitution, σ, is calibrated following Boppart and Krusell (2020). Based on long-run macro evidence, Boppart and Krusell argue that 2 percent productivity growth implies a fall in hours worked by roughly 0.4 percent. Noting from Definition 2 that the steady-state growth rates of hours worked and technology are linked via $1+g_h = (1+g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}}$, this empirical pattern suggests that $\theta(1-\sigma)/(1+\theta\sigma) \approx -0.2$. Given $\theta = 0.5$, this generates a σ of 1.75. Although based on long-run macro evidence, these parameter values are also consistent with micro evidence: Heathcote, Storesletten and Violante (2014) consider an

[9] If x is a common migration rate shifter, a steady-state population satisfies $N_{j+1} = \frac{s_j+m_j+x}{1+n} N_j = \left(\prod_{k=0}^{j} \frac{s_k+m_k+x}{1+n}\right) N_0$. Consistency with fertility rates requires that $(1+n)N_0 = \sum_{j=0}^{J} f_j N_j$. Combining these yields the condition $1+n = \sum_{j=0}^{J} \left(\prod_{k=0}^{j-1} \frac{s_k+m_k+x}{1+n}\right) f_j$, which can be solved for x numerically.

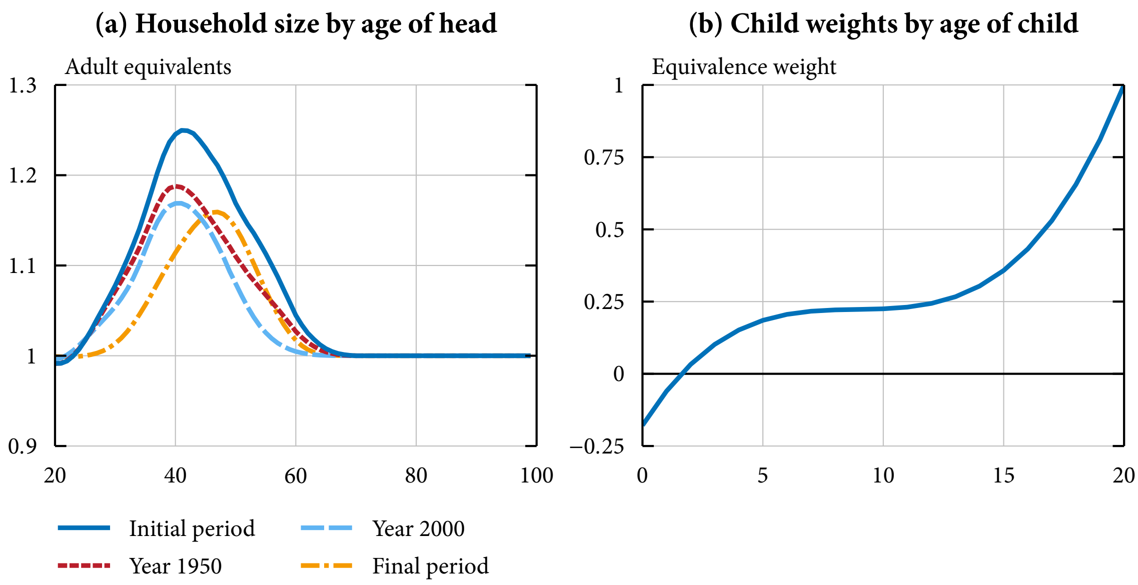


Figure 3. Household size.

Notes. The number $\Omega_j^{\frac{1}{\zeta-(1-\sigma)}}$ of adult equivalents in the left panel and equivalence weights $\omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}}$ in the right panel.

incomplete-markets model with similar preferences and estimate $(\sigma, \theta) = (1.71, 0.46)$ using US earnings and consumption survey data.

The equivalence weights $\omega_i^{\frac{\zeta}{\zeta-(1-\sigma)}}$ and the household scale parameter ζ follow Browning and Ejrnæs (2009), who estimate equivalence scales for low- and high-educated households that include age-specific weights for children and economies of scale. Browning and Ejrnæs use UK data, though Fernández-Villaverde and Krueger (2007) demonstrate with US data that a similar age-varying equivalence scale explains the hump in the life-cycle consumption profile better than a traditional scale. The equivalence weights, shown in Figure 3 together with the household size dynamics, are set to the average of Browning and Ejrnæs' estimates for the two education groups. Browning and Ejrnæs' economies-of-scale parameter, call it ζ_{BE}, is linked to the scale parameter here via $\zeta_{BE}\,\sigma = \frac{\zeta-(1-\sigma)}{\zeta}$. Given $\sigma = 1.75$ and choosing $\zeta_{BE} = 0.86$, again the average of Browning and Ejrnæs' estimates, then yields $\zeta = 1.49$.

I estimate the deterministic age-efficiency profile $\{\varepsilon_j\}_{j=\iota}^{J}$ by a fixed-effects regression of real log wages on a quadratic in age using earnings data from the 1968–2019 family files of the Panel Study of Income Dynamics (PSID). This procedure follows the usual steps in the literature and the details are relegated to Appendix D. The resulting profile is shown in Figure 4a and features a standard hump shape which peaks between the ages of 40 and 50. The idiosyncratic productivity shocks are assumed to follow the AR(1) process $\log \eta' = \varphi \log \eta + \epsilon$, where $\epsilon \sim N(0, \sigma_\epsilon^2)$, with the persistence parameter φ and error variance σ_ϵ^2 set to 0.97 and 0.02, roughly following the evidence presented by Heathcote, Storesletten and Violante (2010). This process is discretised into a five-state Markov chain using Rouwenhorst's method (Kopecky and Suen, 2010).

5.3 Production and R&D

The labour share of output is set to $1 - \alpha = 0.64$, a standard value, while the markup of intermediate producers is set to $1/\rho = 1.4$ to match an aggregate profit share of 10 percent, also a standard benchmark.

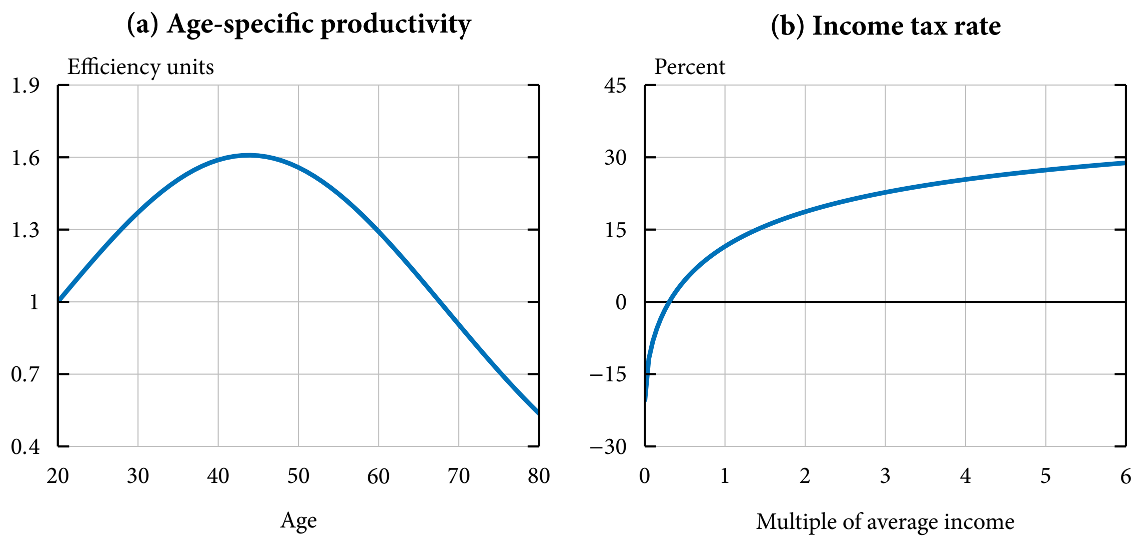


Figure 4. Calibrated efficiency and tax profiles.

To determine the depreciation rates δ_k and δ_z, I proxy R&D investment in the model by the national accounts measure of private fixed investment in intellectual property products (IPP) from the Bureau of Economic Analysis (BEA, NIPA Table 1.1.5). Besides formal R&D, IPP also includes nonrival goods such as computer software and entertainment, literary, and artistic originals, thus capturing the model's notion of R&D better than formal R&D does. IPP investment is stable at 0.7 percent of GDP before 1950, after which it trends upwards. I set δ_z to get R&D investment within the same ballpark in the initial steady state at 1.1 percent of GDP, since an exact match requires such a low δ_z that the transition paths become infeasibly long. Remaining investment in the national accounts (gross private domestic investment minus IPP investment) is roughly constant over time at 13.6 percent of GDP, and δ_k is chosen to match this value in the initial period.

The R&D production function contains three parameters: the productivity level ν, the duplication externality λ, and the knowledge spillover ϕ. The first one is a scale parameter; I set it so that $z = Z = 1$ in the initial period. Bloom *et al.* (2020) highlight that there is no consensus on the correct value for λ, so I just follow the calibrations in Jones and Williams (2000) and Comin and Gertler (2006) and set it to 0.75. The procedure to calibrate ϕ is identical to that of Bloom *et al.* and Jones and Williams. Specifically, Figure 5 shows that US TFP and IPP investment have grown at roughly constant rates of 1.26 and 6.73 percent per year since World War II. From the intermediate variety growth rate in (21), $1 + g_z = 1 - \delta_z + \nu Q^{\lambda} z^{\phi - 1}$, it follows that the right-hand side of this equation must also have been constant. Log-differencing the last term then yields $\phi = 1 - \lambda \frac{g_Q}{g_z}$, where g_Q is the growth rate of R&D investment. With TFP defined as $Z = z^{\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}$, the growth rates in Figure 5 together with the parameter values for α, ρ, and λ then gives $\phi = 0.117$.[10] This calibration yields that the steady-state growth exponents γ_Z and γ_y equal 0.23 and 0.18, which is close to the midpoint of the range of estimates in Jones (2002a).[11]

[10] With $\phi > 0$, R&D exhibits positive knowledge spillovers: the more we know, the easier it is to discover new ideas. This value contrasts with for example Jones (2002a) and Bloom *et al.* (2020), who find negative values of ϕ for the aggregate US economy with similar approaches. However, these papers compute ϕ under the implicit assumption that $\rho = \alpha$. If the same restriction is imposed here, I also find a negative value for ϕ.

[11] A steady state with $(\gamma_Z, \gamma_y) = (0.23, 0.18)$ and for instance 1.2 percent population growth exhibits TFP and output per capita growth of 0.27 and 0.22 percent per year.

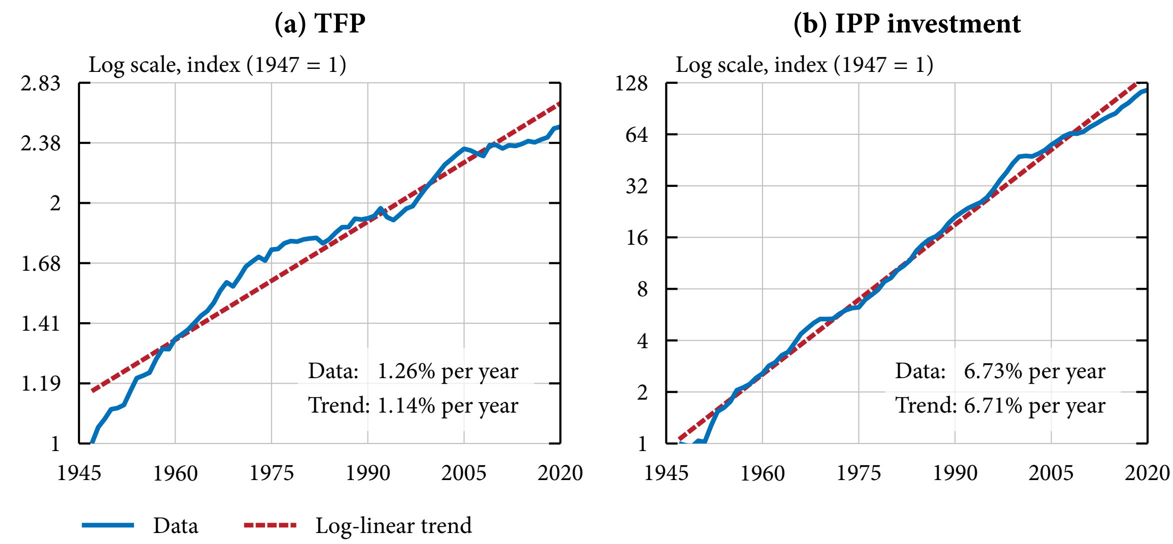


Figure 5. TFP and gross IPP investment in the data.

Source. Fernald (2014) and BEA NIPA Table 5.6.3.

5.4 Public sector

I set the gross pension replacement rate μ to 0.413 based on Clingman, Burkhalter and Chaplain's (2021) estimates for average-income workers who retire at the normal retirement age. The normal retirement age, R^{norm}, is 65 and the earliest age to collect retirement benefits, R^{min}, is 62. Early and delayed retirement adjustment via $ps(R)$ is similar to that of the US social security system. For every year of early retirement, the base level benefit is reduced by 6 ⅔ percent per year for the first three years and 5 percent per any additional year. For every year of delayed retirement, the base level benefit is scaled up by 3 percent up until a maximum age of 70. After the age of 70, no extra benefit is given for delaying retirement.[12]

The individual income tax rate $\tau^w(e)$ at household earnings e is parametrised by Gouveia and Strauss's (1994) functional form:

$$\tau^w(e) = \kappa_0 \left[1 - \left(\kappa_2 \left(\frac{e}{\bar{e}} \right)^{\kappa_1} + 1 \right)^{-\frac{1}{\kappa_1}} \right] + \kappa_3, \qquad \kappa_1 > 0, \tag{35}$$

where κ_0 controls the asymptotic tax rate $\lim_{e\to\infty} \tau^w(e) = \kappa_0 + \kappa_3$, κ_1 determines the degree of tax progressivity, and κ_2 is a scale parameter. Equation (35) augments Gouveia and Strauss' original function with average labour earnings $\bar{e}$, which makes the tax function invariant to units of measurement, and the parameter κ_3, which allows for a non-zero marginal tax rate at zero earnings.

To calibrate the taxes, I first construct tax rates on consumption, capital, and labour income using national accounts data from the BEA by dividing the aggregate revenues of each tax with its corresponding tax base. Taking the average of each tax rate between 1950 and 2020 gives τ^c, τ^k, and $\tau^w(\bar{e})$, the latter being the income tax rate at average earnings. Next, I estimate $\{\kappa_0, \kappa_1, \kappa_2, \kappa_3\}$ using the OECD tax database, which provides income-specific tax rates that incorporate federal, state, and local government taxes plus several deductions and credits. These estimates are available annually since 2000, but only for a few representative

[12] These are the exact retirement ages and scaling rules used by the Social Security Administration for cohorts born before 1924. Later cohorts have higher normal retirement ages and more generous delayed retirement credits, though Appendix C shows that the quantitative results are insensitive to a more accurate development of R^{norm} and $ps(R)$.

income levels. I therefore replicate the OECD's methodology for each year to compute tax rates over a full grid of incomes and fit (35) to these estimates. Finally, for consistency with the aggregate data, I shift the level of the tax curve by adjusting the estimated parameters until the tax rate at average earnings, $\tau^w(\bar{e})$, equals the tax rate computed from the national accounts. Figure 4b plots the resulting income tax function and Appendix E outlines all the details of these steps.

6 Quantitative Results

The lack of population growth in the initial and final periods means that there is no economic growth in the model steady state. The growth impact of the demographic transition is therefore purely transitional. This section illustrates and quantifies this transitional impact and its underlying mechanisms through three quantitative exercises: (i) a growth accounting analysis of per-capita growth in the baseline model, (ii) counterfactual simulations that identify the key demographic factors at work, and (iii) a comparison to an equivalent model without endogenous growth to evaluate the importance of the TFP channel.[13] Throughout, I take "per capita" to mean "per adult equivalent person", since children only affect the economy through the household size.

The growth accounting exercise first utilises that aggregate labour is the sum of efficiency units supplied by households to decompose the former into total employment (E), average hours per worker ($\bar{h}$), and average productivity per hour worked ($\bar{\varepsilon}$): $L = E\,\bar{h}\,\bar{\varepsilon}$. Combining this decomposition with output per capita as in Equation (1), $y = Z\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}}\frac{L}{N}$, and log-differencing between two subsequent periods then gives the growth accounting identity

$$g_y = g_Z + \frac{\alpha}{1-\alpha} g_{K/Y} + g_{E/N} + g_h + g_\varepsilon, \tag{36}$$

where g_x denotes the net growth rate of a variable x. Equation (36) splits the change in output per capita into changes in technology, the capital intensity, the employment rate, average hours per worker, and average productivity per hour worked, which makes it easy to quantify the transitional mechanisms in operation.[14]

6.1 Comparative Statics

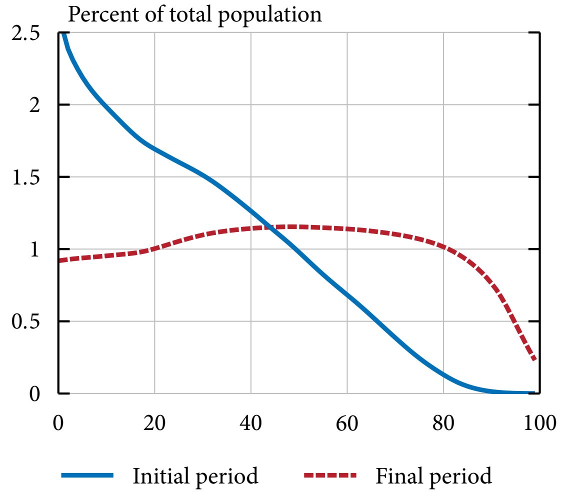
Following the discussion in Section 2, consider first comparative statics of the baseline scenario. Figure 6 displays the overall change in the age structure and the cumulative percentage change in output per capita between the initial and final periods, which reveals that the change in the population structure raises output per capita by 55 percent. This finding follows primarily from a substantial increase of 79 percent of TFP relative to its initial level. The capital intensity also increases by 17 percent while the higher share of old-age households in the population reduces the employment rate by 22 percent. The qualitative changes in the underlying channels are therefore precisely as the simple model predicts.

A natural question immediately emerges from Figure 6: why is the magnitude of the change in TFP so much larger than any of the other changes? It is not because an overwhelming majority of household

[13] Additional figures of variables beyond those presented in this section are available in Appendix A.

[14] Generally, for any fixed long-run population growth rate $\tilde{n}$, we can identify and decompose transitional growth as in Jones (2002a) via $g_y - \gamma_y\tilde{n} = (g_Z - \gamma_Z\tilde{n}) + \frac{\alpha}{1-\alpha}g_{K/Y} + g_{E/N} + (g_h - \gamma_h\tilde{n}) + g_\varepsilon$.

(a) Population age distribution



(b) Cumulative transitory growth

$$y \;=\; Z \cdot \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} \cdot \frac{E}{N} \cdot \bar{h} \cdot \bar{\varepsilon}$$

Output per capita	TFP	Capital intensity	Emp. rate	Hours	Prod.
55%	79%	17%	−22%	−7%	2%
(100%)	(145%)	(30%)	(−41%)	(−12%)	(4%)

Figure 6. Comparative statics: population structure and output per capita.

Notes. Figure 6b shows the cumulative growth in output per capita and each of its contributing factors along the demographic transition. The numbers in parentheses displays their relative contributions to overall growth.

savings are allocated to R&D rather than to capital investments.[15] Rather, this is a result of the scale effect discussed in Section 2, which occurs when the demographic transition increases the population size. A simple decomposition of TFP illustrates this point. Specifically, per Definition 2 we can exploit that TFP is proportional to the size of the labour force E raised to γ_Z in steady state. The corresponding proportionality constant provides a measure of the R&D intensity, just as in Equation (4) of the simple model. The scale effect is pinned down by the overall change in employment, E_t/E_0 where 0 and T denote the initial and final periods, and we then obtain the following decomposition:

$$\underset{\uparrow 79\,\%}{\text{TFP change}} \;=\; \frac{Z_T}{Z_0} \;=\; \frac{Z_T \,/\, E_T^{\gamma_Z}}{Z_0 \,/\, E_0^{\gamma_Z}} \cdot \left(\frac{E_T}{E_0}\right)^{\gamma_Z} \;=\; \underset{\uparrow 21\,\%}{\begin{array}{c}\text{Change in}\\ \text{R\&D intensity}\end{array}} \times \underset{\uparrow 48\,\%}{\text{Scale effect}}\,,$$

where the percentages show the cumulative change in each factor between the initial and the final steady states. The 79 percent rise in TFP is due to a 21 percent increase in the R&D intensity and a 48 percent transitory increase in scale. Changes in household behaviour and composition thus impact output per capita through the R&D intensity much like they do through the capital intensity. The key difference between TFP and the other factors of production instead lies in the fact that technology is a nonrival good, as seen from the large scale effect.[16]

6.2 Growth Dynamics

The comparative statics is informative but says little about the growth dynamics during the transition. Therefore, we now turn to the main period of interest: 1950 to 2100. Figure 7 illustrates the growth rate and its corresponding decomposition (that is, the right-hand side of (36)) during this period. Focusing first

[15] Capital investment exceeds R&D investment by a factor of 4 during most of the transition and the growth rate of the capital stock relative to TFP is even higher (see Figure A.5 in Appendix A).

[16] A caveat here is that using employment as the relevant scaling variable is somewhat arbitrary. We could just as well use the overall population size, since it grows in parallel with employment in steady state. Switching to the population size (measured in adult equivalents) yields a slightly larger scale effect, 57 percent, although this does not change the basic point of the exercise.

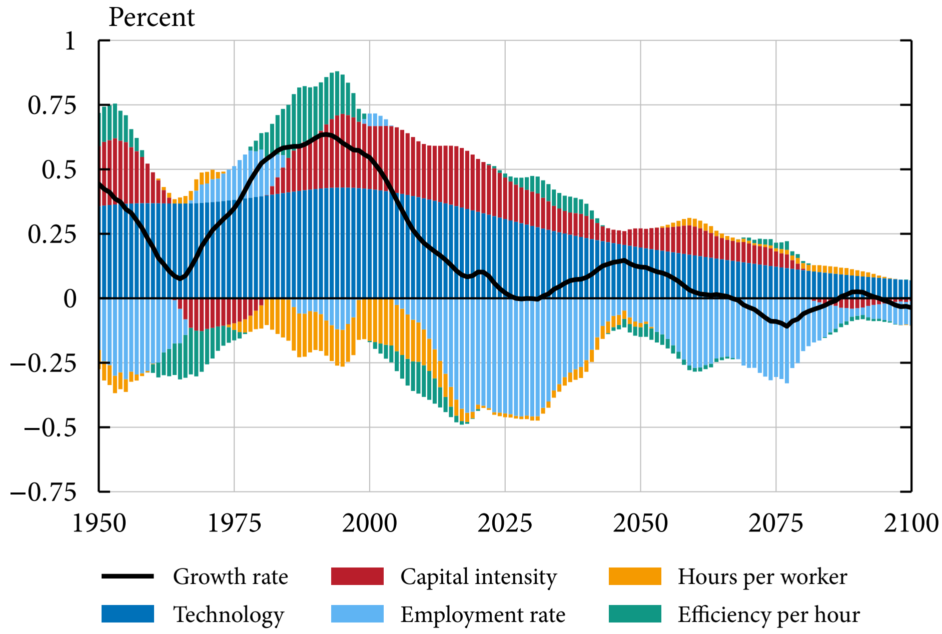


Figure 7. Growth accounting of the baseline model.

on the overall growth rate, three key developments emerge. First, the demographic transition positively affects output per capita throughout the second half of the twentieth century, with growth firmly above zero. Then, this effect fades at the turn of the century, causing a significant drop in the growth rate. Finally, despite this decline, the demographic development does not negatively affect output per capita. Instead, growth remains around zero throughout the twenty-first century.

Looking at the underlying mechanisms, we again find a qualitative development as predicted in Section 2: the demographic transition generates positive TFP growth and capital deepening, but lowers the employment rate. Moreover, average hours worked declines in periods of positive growth (and vice versa) while average efficiency per hour worked rises in periods where the share of middle-aged workers is higher. The former is due to the parametrisation of household preferences, where the income effect of higher wages on leisure dominates the substitution effect, while the latter follows from the hump shape in workers' life-cycle productivity profile.

Table 4 quantifies these observations by summarising the average annual growth rates of output per capita and its decomposition, first for the entire time period considered and then separately for each century. Overall, the demographic transition boosts output per capita by 0.18 percent per year. The effect is primarily driven by the twentieth-century development, where the average annual growth rate is 0.41 percent. By comparison, the observed long-run growth rate of GDP per person in US data is approximately 2 percent per year. From this viewpoint, the impact obtained here is quantitatively significant; taken at face value it implies that over 20 percent of actual US post-war growth can be attributed to transitory demographic factors. This contribution makes demographics comparable in importance to human capital accumulation, whose share of US growth over the same period is also around 20 percent (see for instance Fernald and Jones, 2014).

Just as in the comparative statics, what stands out quantitatively from both Figure 7 and Table 4 is the importance of TFP for overall growth. Over the full period, TFP grows by 0.27 percent per year, thus accounting for more than 150 percent of economic growth. Its contribution is almost three times as large

Table 4. Growth accounting of the baseline model.

	Output per capita	TFP	Capital intensity	Employment rate	Hours per worker	Efficiency per hour
Period	g_y	g_Z	$\frac{\alpha}{1-\alpha} g_{K/Y}$	$g_{E/N}$	g_h	g_ε
1950–2100	0.18 %	0.27 %	0.10 %	−0.16 %	−0.04 %	0.01 %
1950–2000	0.41 %	0.39 %	0.10 %	−0.06 %	−0.07 %	0.04 %
2001–2100	0.07 %	0.22 %	0.10 %	−0.21 %	−0.02 %	−0.01 %
1995	0.60 %	0.43 %	0.29 %	−0.12 %	−0.14 %	0.15 %
2030	0.00 %	0.28 %	0.13 %	−0.46 %	−0.02 %	0.06 %
Difference	−0.60 pp.	−0.15 pp.	−0.15 pp.	−0.34 pp.	0.13 pp.	−0.09 pp.

Notes. The table reports average annual growth rates according to the growth decomposition in Equation (36). Individual growth rates may not sum to totals due to rounding.

as that of capital deepening. The effect is even larger during the latter half of the twentieth century, with TFP growing by 0.39 percent per year, about four times faster than the capital intensity, thus driving the bulk of output growth.

A declining share of workers in the population constitutes the main drag on output growth, depressing the annual growth rate by on average 0.16 percentage points overall and by 0.21 percentage points during the twenty-first century. This happens because increases in the average retirement age do not keep up with increasing life expectancy; Figure 8 shows that, although there is a shift toward later retirement over time, the increase in the retirement age is too marginal to affect the share of retirees in the population aged 65 and above.

Due to concerns about recent declines in observed growth rates, it is also interesting to zoom in on the decline in model growth from peak in 1995 to trough in 2030. The decomposition for this period is shown in the lower half of Table 4. The growth rate and most of its components change monotonically between these years, so it suffices to consider snapshots at the beginning and end of this period and the differences between them. Overall, changes in the demographic structure leads to a 0.60 percentage point drop in the growth rate over the last three decades, thus suggesting that demographics explain a significant chunk of the growth decline observed in the data. The decline stems in part from roughly similar declines of 0.1 to 0.2 percentage points in the growth rates of TFP, capital intensity, and average efficiency. The growth rate of hours increases by a similar magnitude, therefore marginally counteracting the overall development. The majority, however, comes from the retirement of the baby boom: growth in the employment rate declines by 0.34 percentage points, accounting for 56 percent of the total decline.

6.3 It Is *Not* All About Boomers: Identifying the Demographic Effects

Related to the last point above, at first sight it appears that Figure 7 more broadly just reflects the baby boom dynamics predicted in Section 2. The share of young workers increases when the baby boomers enter adulthood in the 1960s and 1970s, which improves the employment rate and worsens the capital intensity and average productivity. When these cohorts become middle-aged in the 1980s and 1990s, growth through TFP, the capital intensity, and average productivity improves. Finally, they retire in the

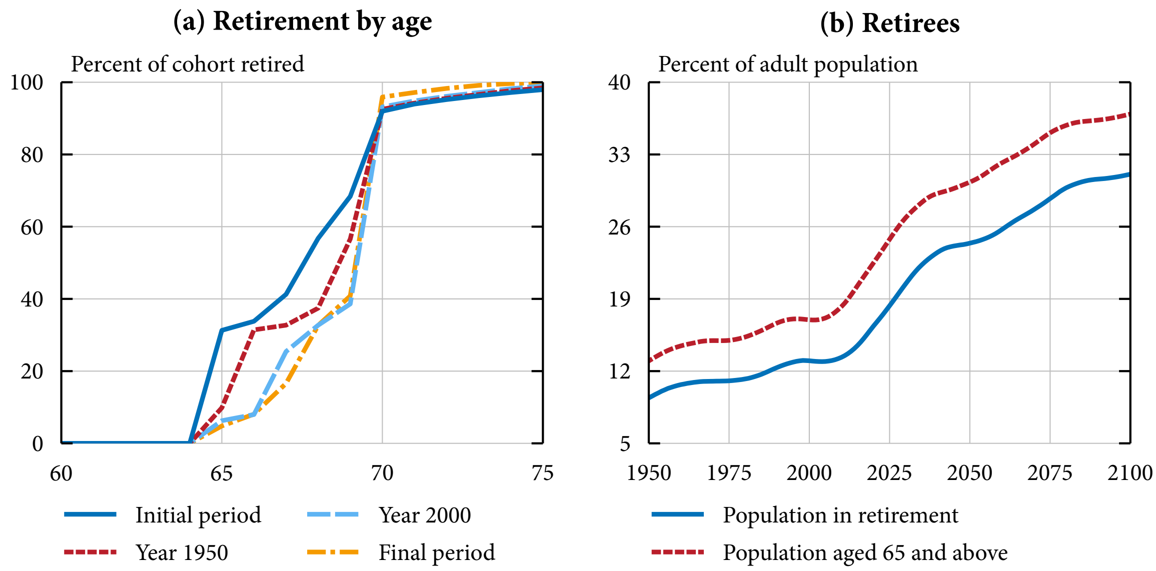


Figure 8. Retirement in the baseline model.

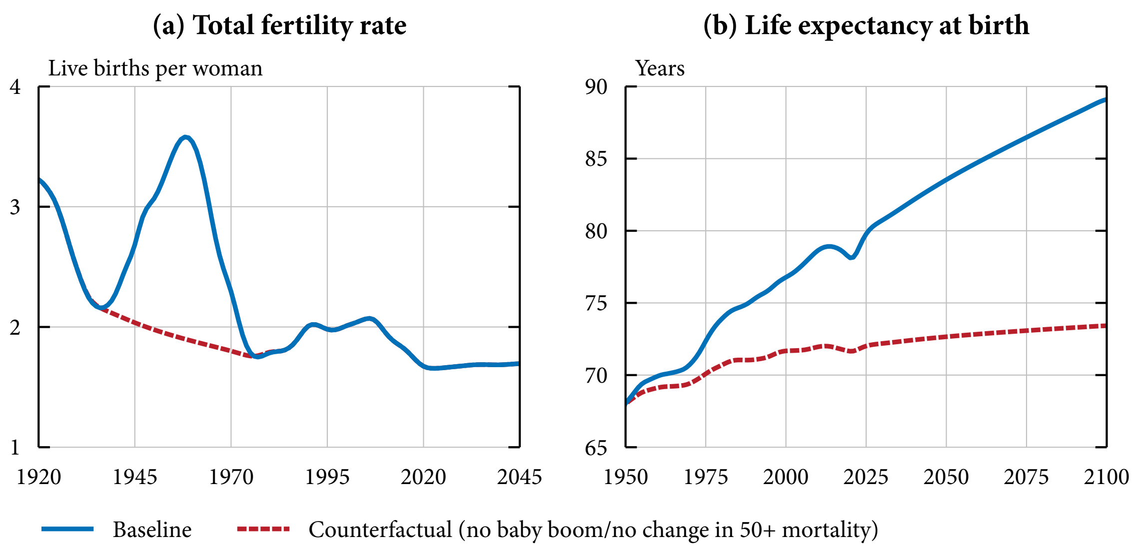


Figure 9. Demographic counterfactual scenarios.

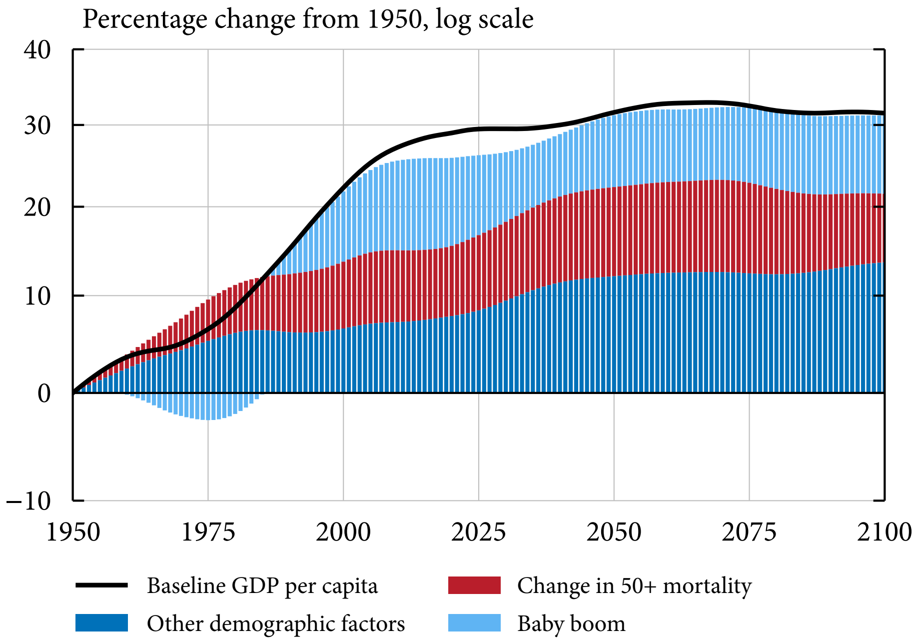


Figure 10. Decomposition of the demographic forces.

early decades of the twenty-first century, thereby causing a negative growth impact via the employment rate. Yet, Section 2 also stresses the increase in life expectancy from rising middle- and old-age survival rates. Which of these forces, if any, is the key driver of the results above?

To answer this question, I consider three counterfactual scenarios. First, I simulate the model under the assumption that neither the baby boom nor the rise in survival rates happens. That is, I replace fertility rates between 1935 and 1975 with interpolated values and hold survival rates above the age of 50 fixed from 1950 onward. As shown in Figure 9, these adjustments remove the hump shape in fertility associated with the baby boom and almost all life expectancy improvements after 1950. I then switch either fertility or mortality back to the baseline calibration and look at the change relative to the first counterfactual. Without strong interaction effects, these scenarios isolate the impacts of the baby boom, of the change in middle- and old-age mortality, and of all other demographic changes.

Figure 10 plots the resulting decomposition of the cumulative change in output per capita into each demographic factor. While the growth swings in Figure 7 are clearly attributed to the life-cycle phases of the baby boomers here, they are by no means the sole explanation behind the overall development. Between 1950 and 2100, output per capita rises by a total of 31.5 percent, which is accounted for by an 8.0 percent increase due to the baby boom, a 7.0 percent increase due to middle- and old-age mortality changes, and a 13.6 percent increase due to other demographic factors. The baby boom and changing mortality consequently explain around a quarter each of the baseline results, with remaining demographic changes accounting for the rest.

Table 5 displays the demographic contributions to each component of the average growth rate between 1950 and 2100. As with output per capita, the baby boom and changing mortality generate around a quarter each of the total technological progress. Its transition dynamics notwithstanding, this TFP contribution (and the subsequent income effect on hours worked) is the baby boom's only impact in the end, thus confirming the prediction from the simple model. This is unsurprising since the counterfactual fertility rates affect the cohorts that become adults after 1955 and die with certainty no later than 2075. Before and

Table 5. Growth decomposition of the demographic forces.

	Output per capita	TFP	Capital intensity	Employment rate	Hours per worker	Efficiency per hour
	g_y	g_Z	$\frac{\alpha}{1-\alpha} g_{K/Y}$	$g_{E/N}$	g_h	g_ε
Average growth rate	0.18 %	0.27 %	0.10 %	−0.16 %	−0.04 %	0.01 %
Baby boom	0.05 pp.	0.07 pp.	0.00 pp.	0.00 pp.	−0.02 pp.	0.00 pp.
50+ mortality	0.04 pp.	0.07 pp.	0.06 pp.	−0.12 pp.	0.04 pp.	0.00 pp.
Other changes	0.08 pp.	0.13 pp.	0.04 pp.	−0.04 pp.	−0.06 pp.	0.01 pp.

Notes. The table reports average annual growth rates between 1950 and 2100 according to the growth decomposition in Equation (36) and the percentage point contributions by each demographic factor. Individual growth rates may not sum to totals due to rounding.

after these years, the age structure of the adult population is nearly identical to the baseline. Most changes in the capital intensity and the employment rate are instead driven by life expectancy improvements. Despite its positive impact on output per capita, longer lifespans also increase hours per worker, reflecting households' need to finance longer periods of retirement.

What explains the residual factor? First, the population in 1950 is not stationary. Changes in the population structure therefore occur even without further changes in the underlying demographic variables, thus impacting the savings rate, the employment rate, and the average productivity per hour worked. This shift in the age structure is largely complete by the end of the twentieth century. Additionally, the changes in fertility, mortality, and migration rates that nevertheless occur raise the population size, so the scale effect on TFP is also at work here, as seen from the large impact on TFP growth.

6.4 It *Is* All About Technological Progress: The Exogenous Growth Case

The finding that the demographic transition does not negatively impact per-capita output contrasts with the general notion of population ageing as a major drag on economic activity. For instance, Krueger and Ludwig (2007), the perhaps closest paper to the analysis here, use a similar quantitative model and find a 12.6 percent cumulative drop in US output per capita associated with the demographic transition between 2005 and 2080. However, most previous work, including Krueger and Ludwig, ignore the TFP channel that I incorporate. Another important question is thus to what extent the difference can be explained by this additional mechanism.

As a final exercise, I therefore analyse a version of the model without technological change. Specifically, the benchmark model nests a standard model without endogenous growth as the special case with perfect substitution between intermediate firms ($\rho = 1$) and a zero intermediate-firm exit rate ($\delta_z = 0$). The former eliminates profits, thus forcing the patent price and R&D investment to zero, which in turn reduces the intermediate-firm dynamics to $z_{t+1} = (1 - \delta_z)z_t$. The latter ensures that the measure of intermediate firms remains constant over time. I also recalibrate the preference parameters β and ψ here to maintain the baseline calibration targets for the capital-output ratio and average hours worked.

Figure 11a plots the growth rate under this specification against that of the baseline model. The positive effects now disappear, with growth rates consistently around 0.1 to 0.3 percentage points below the baseline.

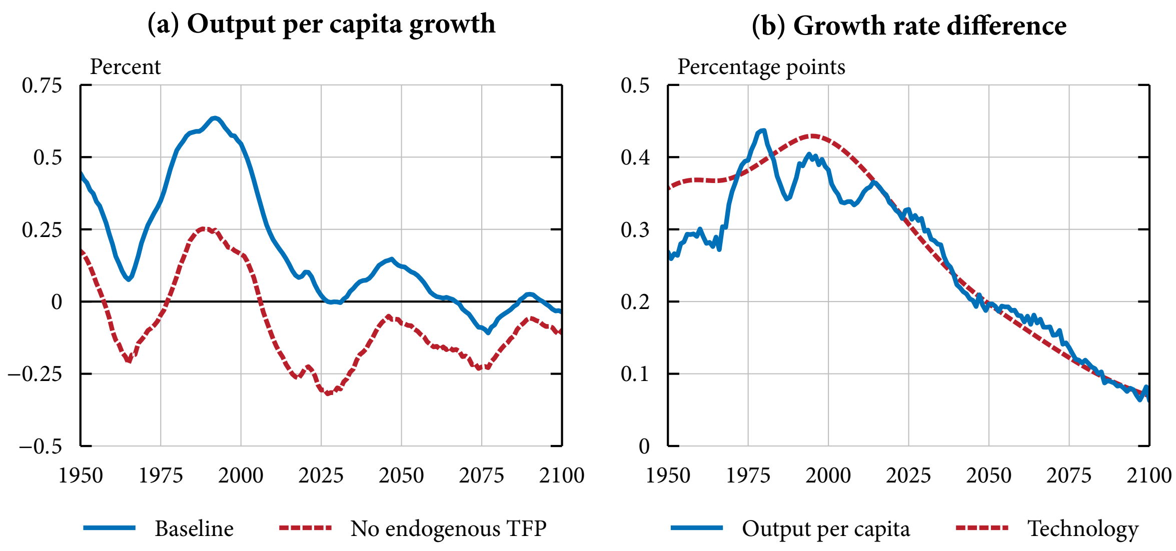


Figure 11. Comparing growth with and without endogenous TFP.

The cumulative decline in output per capita is nearly identical to Krueger and Ludwig (2007): 12.3 percent between 2005 and 2080 and 11.1 percent over the full period. In comparison, for the same periods the baseline model exhibits positive cumulative growth of 5.2 and 31.5 percent, respectively. The difference is driven almost entirely by the TFP difference, as shown in Figure 11b. The impacts on the capital intensity, the employment rate and the average efficiency are virtually unchanged.[17] There is also a small counteracting effect in hours worked: lower income leads to a rise in hours, and this raises output. But this effect is too small to offset the lack of technological progress. Thus, whether the demographic transition raises or lowers output per capita turns out to depend crucially on whether or not its impact on technical change is properly accounted for.

6.5 Taking Stock

In sum, neither the comparative statics nor the transition dynamics of the model suggests that the demographic transition, and population ageing in particular, is detrimental to economic growth once we account for its effect on technological progress. If anything, the impact is (temporarily) positive. Although demographics account for a significant decline in growth rates over the last three decades due to a rising share of retirees in the population, it is important to stress that this is *not* because population ageing is particularly bad for economic output. In fact, it is hardly due to population ageing at all. Rather, the temporary rise in births that led to the baby boom generates a long period of above-average growth in the last quarter of the twentieth century when these generations are of prime working age. We are just now experiencing the end of that period as these generations retire. As already stressed in the simple model, the subsequent growth decline would occur even in the absence of improving life expectancy.

7 The Importance of the R&D Production Function

The results above rest on several calibration and parametrisation choices, most of which are standard in the literature or well justified in the data. In Appendix C I run a battery of alternative specifications for household preferences, inequality, fiscal policy, and pension rules and show that the baseline findings are generally unaffected. A central element that deserves closer treatment, however, is the R&D process. This

[17] See Figure A.6 in Appendix A for a comparison of the full growth decompositions.

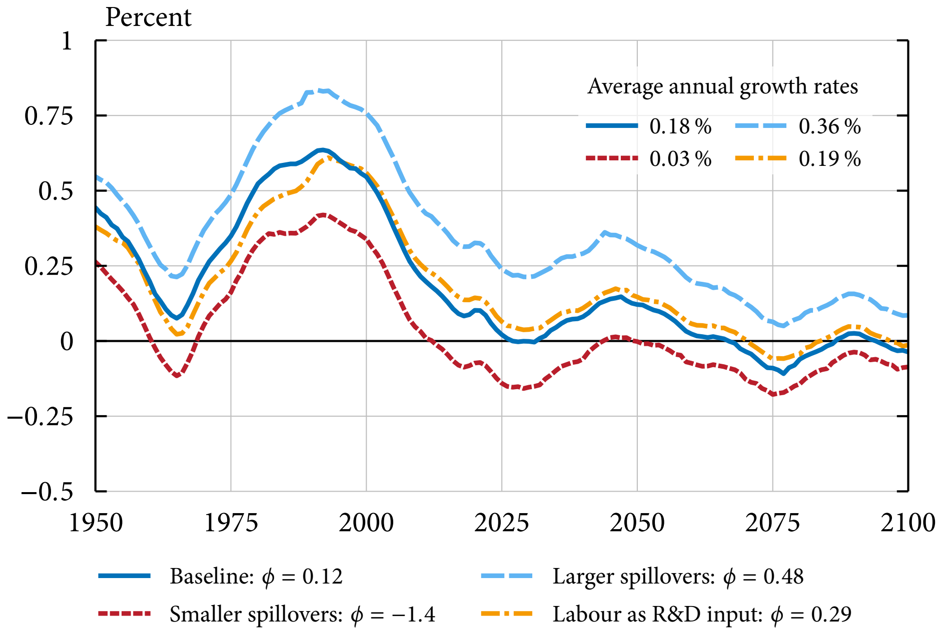


Figure 12. Growth rates with different degrees of knowledge spillovers and R&D inputs.

section explores the sensitivity of the model with respect to two specific R&D components: the knowledge spillover parameter ϕ and the choice of R&D production function. The former seems particularly important since ϕ governs the degree to which R&D becomes easier or harder over time, and its value is likely surrounded by considerable uncertainty given the difficulty to accurately measure productivity growth and R&D investment in the data.

To understand the quantitative significance of ϕ, consider the model with smaller or larger knowledge spillovers. To obtain smaller spillovers, I set ϕ to -1.4 based on Bloom *et al.*'s (2020) estimate for the aggregate US economy when $\lambda = 0.75$. The negative value of ϕ, which they obtain under the assumption that the substitution parameter ρ exactly equals the share parameter α, means that research productivity declines with the stock of knowledge. To obtain larger spillovers, I estimate TFP growth from the Penn World Table as the Solow residual between output and a Cobb-Douglas combination of capital and total hours worked. Applying the baseline calibration method then yields $\phi = 0.48$. Coincidentally, these values of ϕ generate steady-state growth exponents γ_y equal to 0.06 and 0.35, nearly identical to the empirically plausible bounds on γ_y established by Jones (2002a). These scenarios thus provide lower and upper bounds on the baseline results with respect to ϕ.

To analyse the importance of the R&D production function, consider an alternative in which labour rather than final goods is used as R&D input. This setting is common in the growth literature and specifies the R&D process as

$$z_{t+1} = (1 - \delta_z)z_t + \nu L_{zt}^{\lambda} z_t^{\phi},$$

where L_{zt} is the total labour devoted to R&D.[18] Here, I set ϕ to 0.29 to obtain the same long-run growth rate as in the baseline. In this case and in the two above, I again recalibrate the preference parameters β and ψ to match the baseline calibration targets for the capital-output ratio and average hours worked.

[18] The labour and goods market conditions also become $L_t + L_{zt} = \sum_j N_{jt} \int_X \ell_{jt}(x)\, \mathrm{d}\Phi_{jt}$ and $Y_t + A_{t+1}^M = C_t + G_t + \left[K_{t+1} - (1-\delta_k)K_t\right]$. As shown in Appendix B, long-run growth again follows the population growth rate via $1 + g_z = (1+n)^{\chi}$, but now with $\chi \equiv \frac{\lambda}{1-\phi-\lambda\frac{\theta(1-\sigma)}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}$.

Figure 12 displays the per-capita growth rates from these alternatives against that of the baseline. Qualitatively, changing the knowledge spillover is straightforward: the larger the knowledge spillover, the larger the growth impact from R&D, since researchers become more productive the more knowledge is created. Figure 12 confirms this prediction. Meanwhile, changing the R&D input from final goods to labour generates a higher demand for labour relative to capital from producers, with subsequent general equilibrium effects on factor prices. Since the R&D sector is a small share of the total economy, these effects are small. None of the major underlying mechanisms of the model changes here, as the cost of R&D (the wage bill of researchers in this case) and the patent purchases by intermediate-good firms are still financed via household savings. The impact relative to the baseline is therefore negligible.

Growth rates are overall positive across the board (albeit quantitatively insignificant with small knowledge spillovers). Relative to the baseline, the average annual growth rate between 1950 and 2100 decreases and increases by about 0.15 percentage points with the different degrees of knowledge spillovers and remains unaffected when labour is the sole R&D input. Only in the most pessimistic calibration do we observe a negative impact on per-capita output during the twenty-first century, but even here the impact is small at about −0.05 percent per year. Therefore, although reasonable variations of the R&D process impact the quantitative findings, they do not change the basic point of this paper: that the demographic transition and the ageing of the population improves output per capita under endogenous technological change.

8 Conclusion

The model in this paper allows the population structure to affect output per capita via three main channels: through the fraction of people who work, through capital accumulation, and through technological progress. This framework stands in stark contrast to standard macroeconomic life-cycle models, in which technological change is exogenous. It also contrasts to most models of endogenous growth, which hide the entire population structure in a representative household. A key point throughout the paper is that this matters for how we think about demographic change and its impact on output per capita.

My main findings suggest that current and projected US demographic change from 1950 onward raises output per capita and that this effect is quantitatively large; at least on par with the growth contribution from US educational attainment over the second half of the twentieth century. I show that this is primarily due to the inclusion of endogenous technological change. Removing this channel completely reverses the positive impact. Overall, these findings challenge a seemingly conventional wisdom that current demographic developments are detrimental to economic activity.

The framework employed here admittedly leaves out several potentially important channels, such as human capital accumulation, automation, or international technology flows. However, papers that consider endogenous responses in human capital or automation typically find that these extra adjustment margins improve the impact of population ageing on output per capita, much like the inclusion of technological progress in this paper. Moreover, new technologies imported from foreign countries would only serve to raise the productivity of research under my baseline calibration. Thus, if anything, I would expect these mechanisms to only strengthen the main takeaways of the paper.

Lastly, the results also raise questions about the conclusions drawn in other literatures that rely on exogenous growth models. For example, the pension system is a key determinant of the savings rate and, since the savings rate is central to technical change here, an immediate question is therefore whether social security reform becomes more or less costly under R&D-driven growth. Other topics include migration policy, other types of fiscal policy reform, and optimal taxation. A recent paper by Jones (2022b) for instance suggests that the optimal progressivity of income taxation is significantly altered by the presence of endogenous technological change. The model considered here could serve as a basis to analyse these questions further.

References

Aaronson, D., and French, E. (2004). 'The effect of part-time work on wages: Evidence from the social security rules'. *Journal of Labor Economics*, vol. 22(2), pp. 688–720. (Cited on page 43.)

Acemoglu, D., and Restrepo, P. (2017). 'Secular stagnation? The effect of aging on economic growth in the age of automation'. *American Economic Review*, vol. 107(5), pp. 174–179. (Cited on page 3.)

Acemoglu, D., and Restrepo, P. (2022). 'Demographics and automation'. *Review of Economic Studies*, vol. 89(1), pp. 1–44. (Cited on page 3.)

Aghion, P., and Howitt, P. (1992). 'A model of growth through creative destruction'. *Econometrica*, vol. 60(2), pp. 323–351. (Cited on page 7.)

Aksoy, Y., Basso, H. S., Smith, R. P., and Grasl, T. (2019). 'Demographic structure and macroeconomic trends'. *American Economic Journal: Macroeconomics*, vol. 11(1), pp. 193–222. (Cited on page 3.)

Anzoategui, D., Comin, D., Gertler, M., and Martinez, J. (2019). 'Endogenous technology adoption and R&D as sources of business cycle persistence'. *American Economic Journal: Macroeconomics*, vol. 11(3), pp. 67–110. (Cited on page 3.)

Attanasio, O., Kitao, S., and Violante, G. L. (2007). 'Global demographic trends and social security reform'. *Journal of Monetary Economics*, vol. 54(1), pp. 144–198. (Cited on page 3.)

Auclert, A., Malmberg, H., Martenet, F., and Rognlie, M. (2021). *Demographics, wealth, and global imbalances in the twenty-first century* (NBER Working Paper No. 29161). National Bureau of Economic Research. Cambridge, MA. (Cited on page 3.)

Auerbach, A. J., and Kotlikoff, L. J. (1987). *Dynamic fiscal policy*. Cambridge University Press. (Cited on page 1.)

Basso, H. S., and Jimeno, J. F. (2021). 'From secular stagnation to robocalypse? Implications of demographic and technological changes'. *Journal of Monetary Economics*, vol. 117, pp. 833–847. (Cited on page 3.)

Bell, F. C., and Miller, M. L. (2005). *Life tables for the United States Social Security Area 1900–2100* (Actuarial Study No. 120). Social Security Administration, Office of the Chief Actuary. Baltimore, MD. (Cited on page 14.)

Benigno, G., and Fornaro, L. (2018). 'Stagnation traps'. *Review of Economic Studies*, vol. 85(3), pp. 1425–1470. (Cited on page 3.)

Benzell, S. G., Kotlikoff, L. J., LaGarda, G., and Ye, V. Y. (2021). *Simulating endogenous global automation* (NBER Working Paper No. 29220). National Bureau of Economic Research. Cambridge, MA. (Cited on page 4.)

Bianchi, F., Kung, H., and Morales, G. (2019). 'Growth, slowdowns, and recoveries'. *Journal of Monetary Economics*, vol. 101, pp. 47–63. (Cited on page 3.)

Bloom, N., Jones, C. I., van Reenen, J., and Webb, M. (2020). 'Are ideas getting harder to find?' *American Economic Review*, vol. 110(4), pp. 1104–1144. (Cited on pages 18, 28.)

Boppart, T., and Krusell, P. (2020). 'Labor supply in the past, present, and future: A balanced-growth perspective'. *Journal of Political Economy*, vol. 128(1), pp. 118–157. (Cited on pages 9, 15, 16, 39, 43.)

Börsch-Supan, A., Ludwig, A., and Winter, J. (2006). 'Ageing, pension reform and capital flows: A multi-country simulation model'. *Economica*, vol. 73(292), pp. 625–658. (Cited on page 3.)

Browning, M., and Ejrnæs, M. (2009). 'Consumption and children'. *Review of Economics and Statistics*, vol. 91(1), pp. 93–111. (Cited on pages 15, 17.)

Carroll, C. D. (2006). 'The method of endogenous gridpoints for solving dynamic stochastic optimization problems'. *Economics Letters*, vol. 91(3), pp. 312–320. (Cited on page 14.)

Carvalho, C., Ferrero, A., and Nechio, F. (2016). 'Demographics and real interest rates: Inspecting the mechanism'. *European Economic Review*, vol. 88, pp. 208–226. (Cited on page 3.)

Chetty, R., Guren, A., Manoli, D., and Weber, A. (2011). 'Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins'. *American Economic Review*, vol. 101(3), pp. 471–475. (Cited on pages 15, 16.)

Clingman, M., Burkhalter, K., and Chaplain, C. (2021). *Replacement rates for hypothetical retired workers* (Actuarial Note No. 2021.9). Social Security Administration, Office of the Chief Actuary. Baltimore, MD. (Cited on pages 15, 19.)

Comin, D., and Gertler, M. (2006). 'Medium-term business cycles'. *American Economic Review*, vol. 96(3), pp. 523–551. (Cited on pages 3, 11, 15, 18.)

Cooley, T., and Henriksen, E. (2018). 'The demographic deficit'. *Journal of Monetary Economics*, vol. 93, pp. 45–62. (Cited on page 3.)

Cutler, D. M., Poterba, J. M., Sheiner, L. M., and Summers, L. H. (1990). 'An aging society: Opportunity or challenge?' *Brookings Papers on Economic Activity*, vol. 1990(1), pp. 1–73. (Cited on page 3.)

De Nardi, M., İmrohoroğlu, S., and Sargent, T. J. (1999). 'Projected U.S. demographics and social security'. *Review of Economic Dynamics*, vol. 2(3), pp. 575–615. (Cited on page 3.)

Domeij, D., and Flodén, M. (2006a). 'The labor-supply elasticity and borrowing constraints: Why estimates are biased'. *Review of Economic Dynamics*, vol. 9(2), pp. 242–262. (Cited on page 16.)

Domeij, D., and Flodén, M. (2006b). 'Population aging and international capital flows'. *International Economic Review*, vol. 47(3), pp. 1013–1032. (Cited on page 3.)

Eggertsson, G. B., Lancastre, M., and Summers, L. H. (2019). 'Aging, output per capita, and secular stagnation'. *American Economic Review: Insights*, vol. 1(3), pp. 325–342. (Cited on page 3.)

Eggertsson, G. B., Mehrotra, N. R., and Robbins, J. A. (2019). 'A model of secular stagnation: Theory and quantitative evaluation'. *American Economic Journal: Macroeconomics*, vol. 11(1), pp. 1–48. (Cited on page 3.)

Fehr, H., Jokisch, S., and Kotlikoff, L. J. (2004). 'The role of immigration in dealing with the developed world's demographic transition'. *FinanzArchiv / Public Finance Analysis*, vol. 60(3), pp. 296–324. (Cited on page 3.)

Fernald, J. G. (2014). *A quarterly, utilization-adjusted series on total factor productivity* (Federal Reserve Bank of San Francisco Working Paper No. 2012–19). Federal Reserve Bank of San Francisco. San Francisco, CA. (Cited on page 19.)

Fernald, J. G., and Jones, C. I. (2014). 'The future of US economic growth'. *American Economic Review*, vol. 104(5), pp. 44–49. (Cited on pages 2, 22.)

Fernández-Villaverde, J., Guerrón-Quintana, P., Kuester, K., and Rubio-Ramírez, J. (2015). 'Fiscal volatility shocks and economic activity'. *American Economic Review*, vol. 105(11), pp. 3352–3384. (Cited on pages 46, 47.)

Fernández-Villaverde, J., and Krueger, D. (2007). 'Consumption over the life cycle: Facts from Consumer Expenditure Survey data'. *Review of Economics and Statistics*, vol. 89(3), pp. 552–565. (Cited on page 17.)

French, E. (2005). 'The effects of health, wealth, and wages on labour supply and retirement behaviour'. *Review of Economic Studies*, vol. 72(2), pp. 395–427. (Cited on pages 43–45.)

Gagnon, E., Johannsen, B. K., and López-Salido, D. (2021). 'Understanding the new normal: The role of demographics'. *IMF Economic Review*, vol. 69, pp. 357–390. (Cited on page 3.)

Gapminder Foundation. (2017). *Babies per woman (total fertility rate)*. Version 12. Data downloaded on 17 October 2022. (Cited on pages 15, 35.)

Gertler, M. (1999). 'Government debt and social security in a life-cycle economy'. *Carnegie-Rochester Conference Series on Public Policy*, vol. 50, pp. 61–110. (Cited on page 3.)

Gouveia, M., and Strauss, R. P. (1994). 'Effective federal individual income tax functions: An exploratory empirical analysis'. *National Tax Journal*, vol. 47(2), pp. 317–339. (Cited on pages 19, 51.)

Grossman, G. M., and Helpman, E. (1991). 'Quality ladders in the theory of growth'. *Review of Economic Studies*, vol. 58(1), pp. 43–61. (Cited on page 7.)

Guvenen, F., Kuruscu, B., and Ozkan, S. (2014). 'Taxation of human capital and wage inequality: A cross-country analysis'. *Review of Economic Studies*, vol. 81(2), pp. 818–850. (Cited on page 51.)

Heathcote, J., Storesletten, K., and Violante, G. L. (2010). 'The macroeconomic implications of rising wage inequality in the United States'. *Journal of Political Economy*, vol. 118(4), pp. 681–722. (Cited on pages 15, 17, 45.)

Heathcote, J., Storesletten, K., and Violante, G. L. (2014). 'Consumption and labor supply with partial insurance: An analytical framework'. *American Economic Review*, vol. 104(7), pp. 2075–2126. (Cited on page 16.)

Heckman, J., and Robb, R. (1985). 'Using longitudinal data to estimate age, period and cohort effects in earnings equations'. In W. M. Mason and S. E. Fienberg (Eds.), *Cohort analysis in social research* (pp. 137–150). Springer. (Cited on page 44.)

Heer, B., and Irmen, A. (2014). 'Population, pensions, and endogenous economic growth'. *Journal of Economic Dynamics and Control*, vol. 46, pp. 50–72. (Cited on page 4.)

Heuser, R. L. (1976). *Fertility tables for birth cohorts by color; United States, 1917–73* (DHEW Publication No. 76–1152). National Center for Health Statistics, Division of Vital Statistics. Rockville, MD. (Cited on page 14.)

Holston, K., Laubach, T., and Williams, J. C. (2017). 'Measuring the natural rate of interest: International trends and determinants'. *Journal of International Economics*, vol. 108, pp. S59–S75. (Cited on page 2.)

Huggett, M., Ventura, G., and Yaron, A. (2011). 'Sources of lifetime inequality'. *American Economic Review*, vol. 101(7), pp. 2923–2954. (Cited on page 45.)

Hurd, M. D. (1989). 'Mortality risk and bequests'. *Econometrica*, vol. 57(4), pp. 779–813. (Cited on page 16.)

İmrohoroğlu, S., and Kitao, S. (2012). 'Social security reforms: Benefit claiming, labor force participation, and long-run sustainability'. *American Economic Journal: Macroeconomics*, vol. 4(3), pp. 96–127. (Cited on page 3.)

Iskhakov, F., Jørgensen, T. H., Rust, J., and Schjerning, B. (2017). 'The endogenous grid method for discrete-continuous dynamic choice models with (or without) taste shocks'. *Quantitative Economics*, vol. 8(2), pp. 317–365. (Cited on page 14.)

Jones, C. (Forthcoming). 'Aging, secular stagnation and the business cycle'. *Review of Economics and Statistics*. (Cited on page 3.)

Jones, C. I. (1995). 'R & D-based models of economic growth'. *Journal of Political Economy*, vol. 103(4), pp. 759–784. (Cited on pages 1, 4, 11, 13, 39, 40.)

Jones, C. I. (2002a). 'Sources of U.S. economic growth in a world of ideas'. *American Economic Review*, vol. 92(1), pp. 220–239. (Cited on pages 18, 20, 28.)

Jones, C. I. (2005). 'Growth and ideas'. In P. Aghion and S. N. Durlauf (Eds.), *Handbook of Economic Growth* (pp. 1063–1111, Vol. 1B). Elsevier. (Cited on page 5.)

Jones, C. I. (2022a). 'The end of economic growth? Unintended consequences of a declining population'. *American Economic Review*, vol. 112(11), pp. 3489–3527. (Cited on page 1.)

Jones, C. I. (2022b). 'Taxing top incomes in a world of ideas'. *Journal of Political Economy*, vol. 130(9), pp. 2227–2274. (Cited on page 30.)

Jones, C. I., and Williams, J. C. (2000). 'Too much of a good thing? The economics of investment in R&D'. *Journal of Economic Growth*, vol. 5(1), pp. 65–85. (Cited on page 18.)

Jones, J. B. (2002b). 'Has fiscal policy helped stabilize the postwar U.S. economy?' *Journal of Monetary Economics*, vol. 49(4), pp. 709–746. (Cited on pages 46, 47.)

King, R. G., Plosser, C. I., and Rebelo, S. T. (1988). 'Production, growth and business cycles: I. the basic neoclassical model'. *Journal of Monetary Economics*, vol. 21(2–3), pp. 195–232. (Cited on page 43.)

Kitao, S. (2014). 'Sustainable social security: Four options'. *Review of Economic Dynamics*, vol. 17(4), pp. 756–779. (Cited on pages 3, 43.)

Kopecky, K. A., and Suen, R. M. H. (2010). 'Finite state Markov-chain approximations to highly persistent processes'. *Review of Economic Dynamics*, vol. 13(3), pp. 701–714. (Cited on page 17.)

Kortum, S. S. (1997). 'Research, patenting, and technological change'. *Econometrica*, vol. 65(6), pp. 1389–1419. (Cited on page 1.)

Kotlikoff, L. J., Smetters, K., and Walliser, J. (2007). 'Mitigating America's demographic dilemma by pre-funding social security'. *Journal of Monetary Economics*, vol. 54(2), pp. 247–266. (Cited on page 3.)

Krueger, D., and Ludwig, A. (2007). 'On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare'. *Journal of Monetary Economics*, vol. 54(1), pp. 49–87. (Cited on pages 1, 3, 26, 27.)

Lee, R. (2016). 'Macroeconomics, aging, and growth'. In J. Piggott and A. Woodland (Eds.), *Handbook of the Economics of Population Aging* (pp. 59–118, Vol. 1A). Elsevier. (Cited on page 7.)

Ljungqvist, L., and Sargent, T. J. (2014). 'Career length: Effects of curvature of earnings profiles, earnings shocks, taxes, and social security'. *Review of Economic Dynamics*, vol. 17(1), pp. 1–20. (Cited on page 43.)

Ludwig, A. (2007). 'The Gauss–Seidel–quasi-Newton method: A hybrid algorithm for solving dynamic economic models'. *Journal of Economic Dynamics and Control*, vol. 31(5), pp. 1610–1632. (Cited on page 14.)

Ludwig, A., Schelkle, T., and Vogel, E. (2012). 'Demographic change, human capital and welfare'. *Review of Economic Dynamics*, vol. 15(1), pp. 94–107. (Cited on pages 1, 4.)

Mendoza, E. G., Razin, A., and Tesar, L. L. (1994). 'Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption'. *Journal of Monetary Economics*, vol. 34(3), pp. 297–323. (Cited on page 46.)

Nuño, G. (2011). 'Optimal research and development and the cost of business cycles'. *Journal of Economic Growth*, vol. 16(3), pp. 257–283. (Cited on page 3.)

Okada, T. (2023). 'Endogenous technological change and the new keynesian model'. *Review of Economics and Statistics*, vol. 104(6), pp. 1224–1240. (Cited on page 3.)

Peters, M., and Walsh, C. (2021). *Population growth and firm dynamics* (NBER Working Paper No. 29424). National Bureau of Economic Research. Cambridge, MA. (Cited on page 1.)

Prettner, K. (2013). 'Population aging and endogenous economic growth'. *Journal of Population Economics*, vol. 26, pp. 811–834. (Cited on page 1.)

Prettner, K., and Trimborn, T. (2017). 'Demographic change and R&D-based economic growth'. *Economica*, vol. 84(336), pp. 667–681. (Cited on page 1.)

Ramey, V. A., and Francis, N. (2009). 'A century of work and leisure'. *American Economic Journal: Macroeconomics*, vol. 1(2), pp. 189–224. (Cited on page 16.)

Rogerson, R., and Wallenius, J. (2013). 'Nonconvexities, retirement, and the elasticity of labor supply'. *American Economic Review*, vol. 103(4), pp. 1445–1462. (Cited on page 43.)

Romer, P. M. (1990). 'Endogenous technological change'. *Journal of Political Economy*, vol. 98(5), pp. S71–S102. (Cited on pages 1, 4, 7, 13.)

Segerstrom, P. S. (1998). 'Endogenous growth without scale effects'. *American Economic Review*, vol. 88(5), pp. 1290–1310. (Cited on page 1.)

Storesletten, K. (2000). 'Sustaining fiscal policy through immigration'. *Journal of Political Economy*, vol. 108(2), pp. 300–323. (Cited on page 3.)

United Nations. (2022). *World population prospects 2022, online edition*. Data downloaded on 3 March 2023. (Cited on pages 2, 14, 35.)

US Census Bureau. (2018). *2017 national population projections*. Data downloaded on 24 January 2022. (Cited on page 15.)

US Department of Homeland Security. (2020). *Yearbook of immigration statistics 2019*. (Cited on page 14.)

Vandenbroucke, G. (2021). 'The baby boomers and the productivity slowdown'. *European Economic Review*, vol. 132, pp. 1–22. (Cited on page 4.)

Appendix A Additional Figures

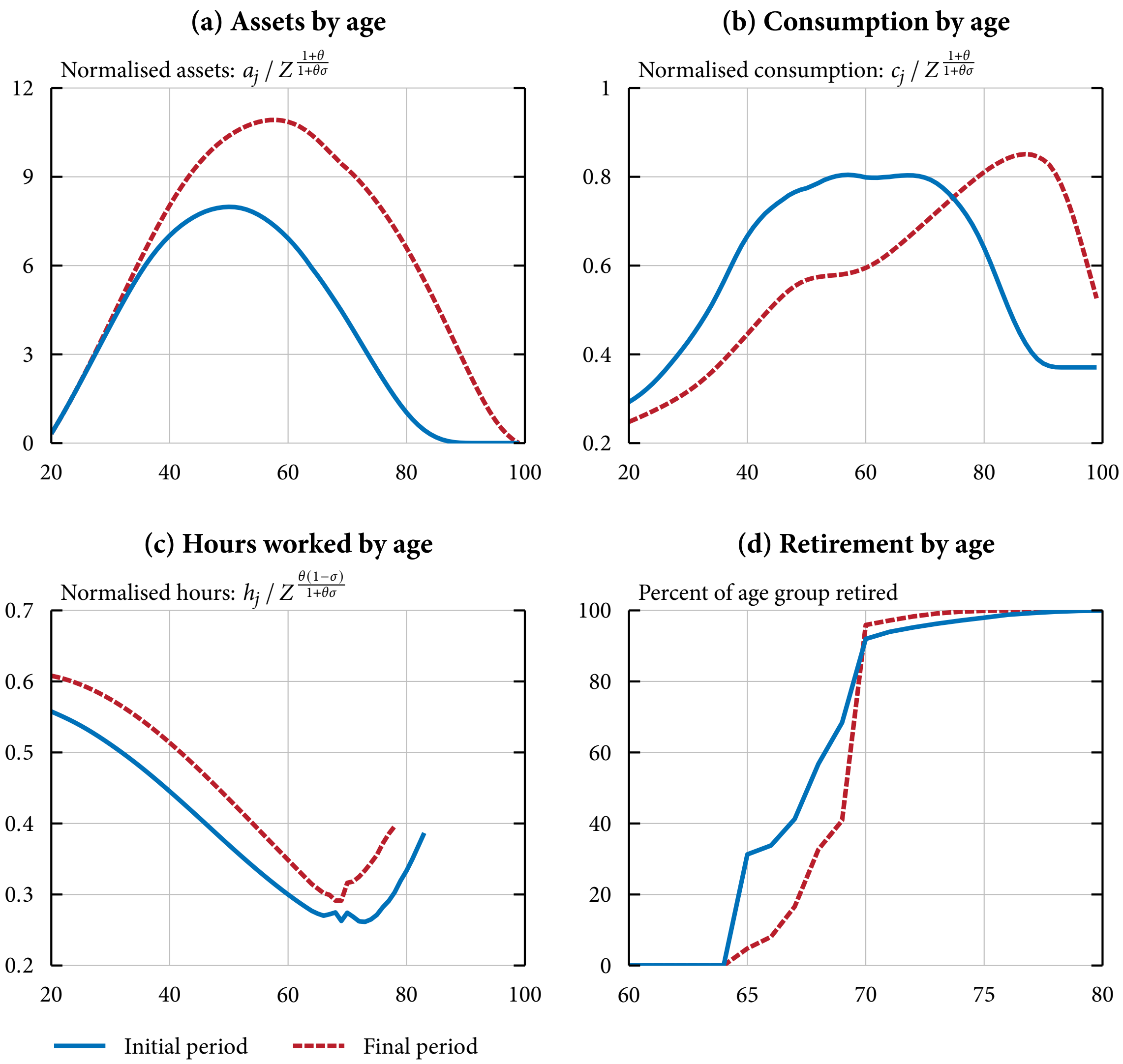


Figure A.1. Average life-cycle profiles in the baseline scenario.

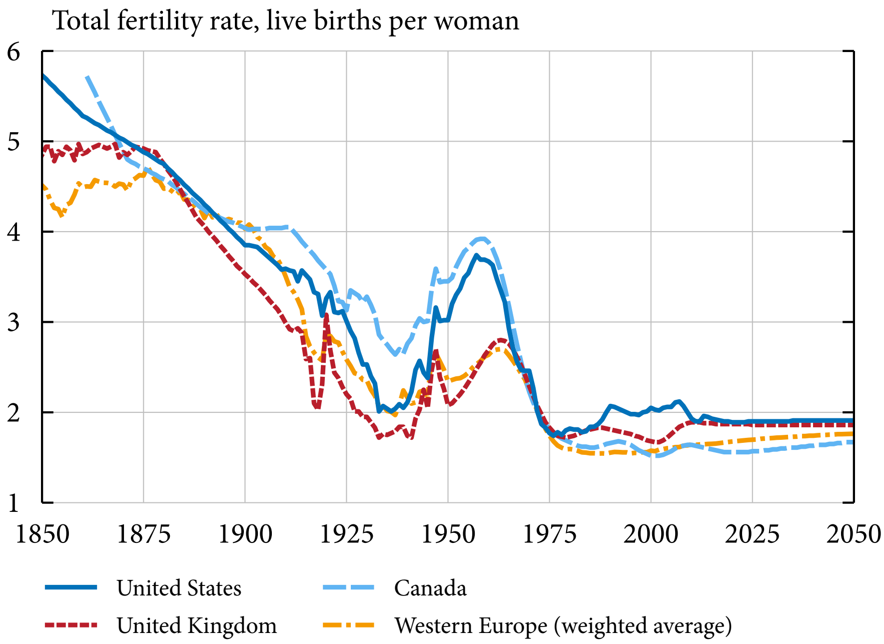


Figure A.2. Fertility across major economies.

Source. Gapminder Foundation (2017).

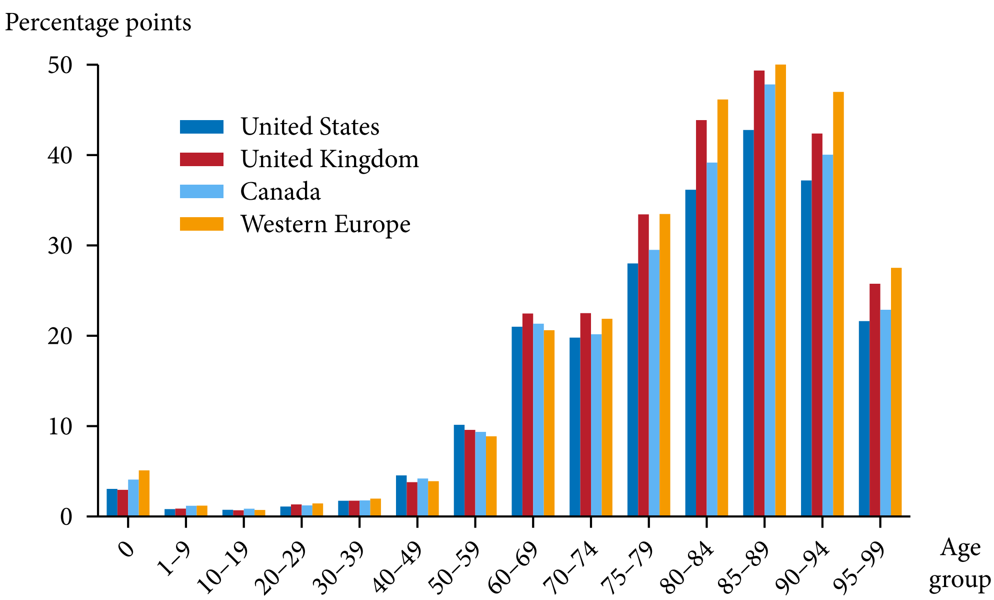


Figure A.3. Increase in survival probabilities by age group, 1950–2100.

Source. United Nations (2022).

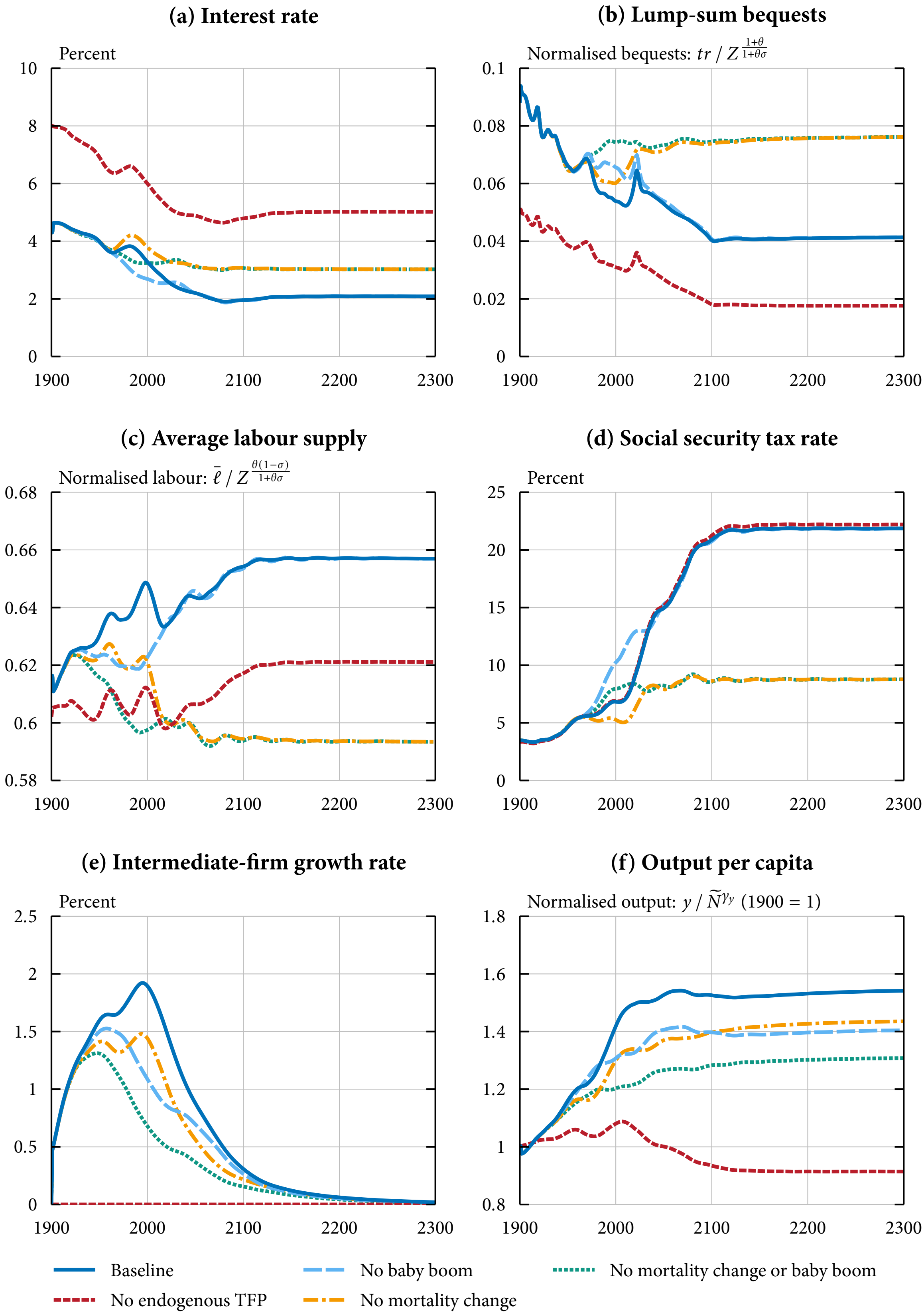


Figure A.4. Transition paths of equilibrium variables and detrended output per capita.

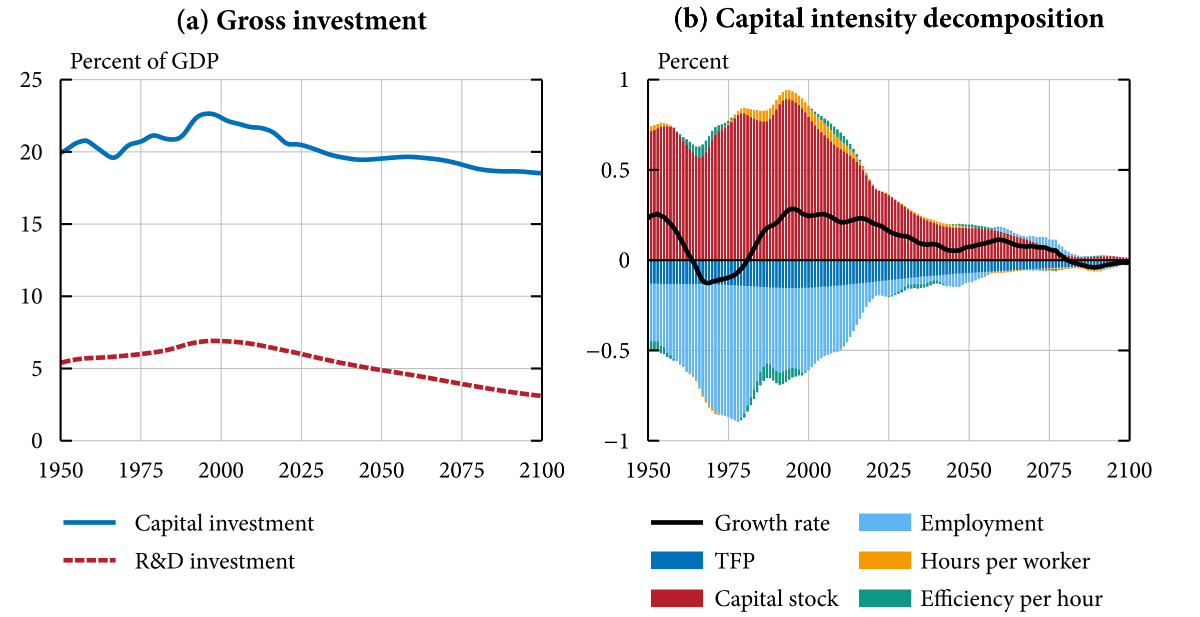


Figure A.5. Investment and capital deepening in the baseline scenario.

Notes. Figure A.5b displays the growth decomposition $\frac{\alpha}{1-\alpha} g_{K/Y} = \alpha\left(g_K - g_Z - g_E - g_{\bar{h}} - g_{\bar{\varepsilon}}\right)$, which is obtained by log differencing the capital intensity $\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{K}{ZL}\right)^{\alpha}$ together with the labour decomposition $L = E\,\bar{h}\,\bar{\varepsilon}$.

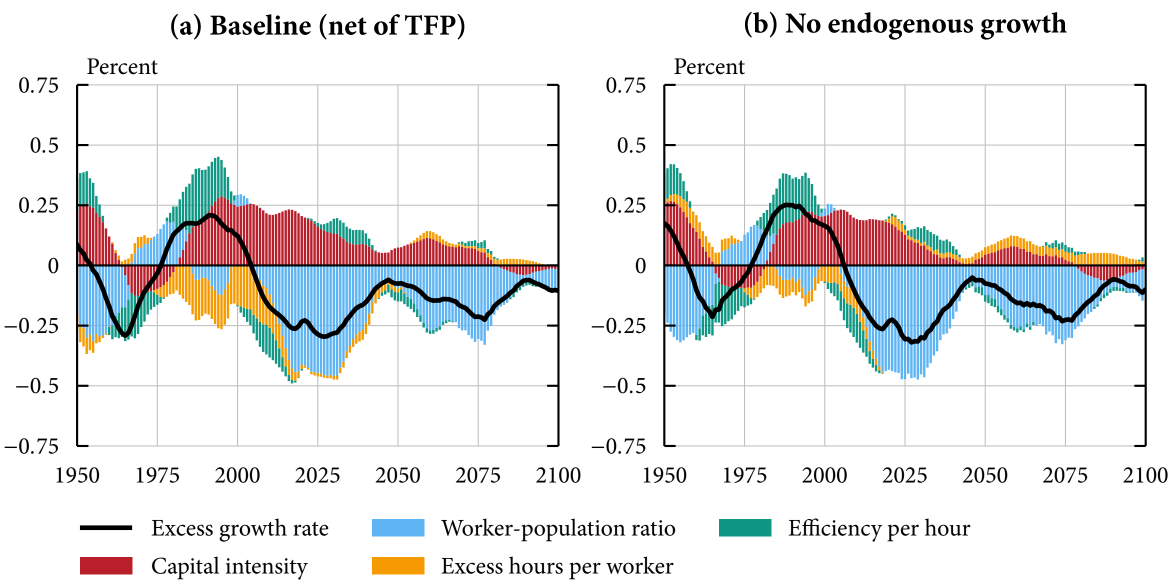


Figure A.6. Growth accounting comparison for the no-endogenous growth scenario.

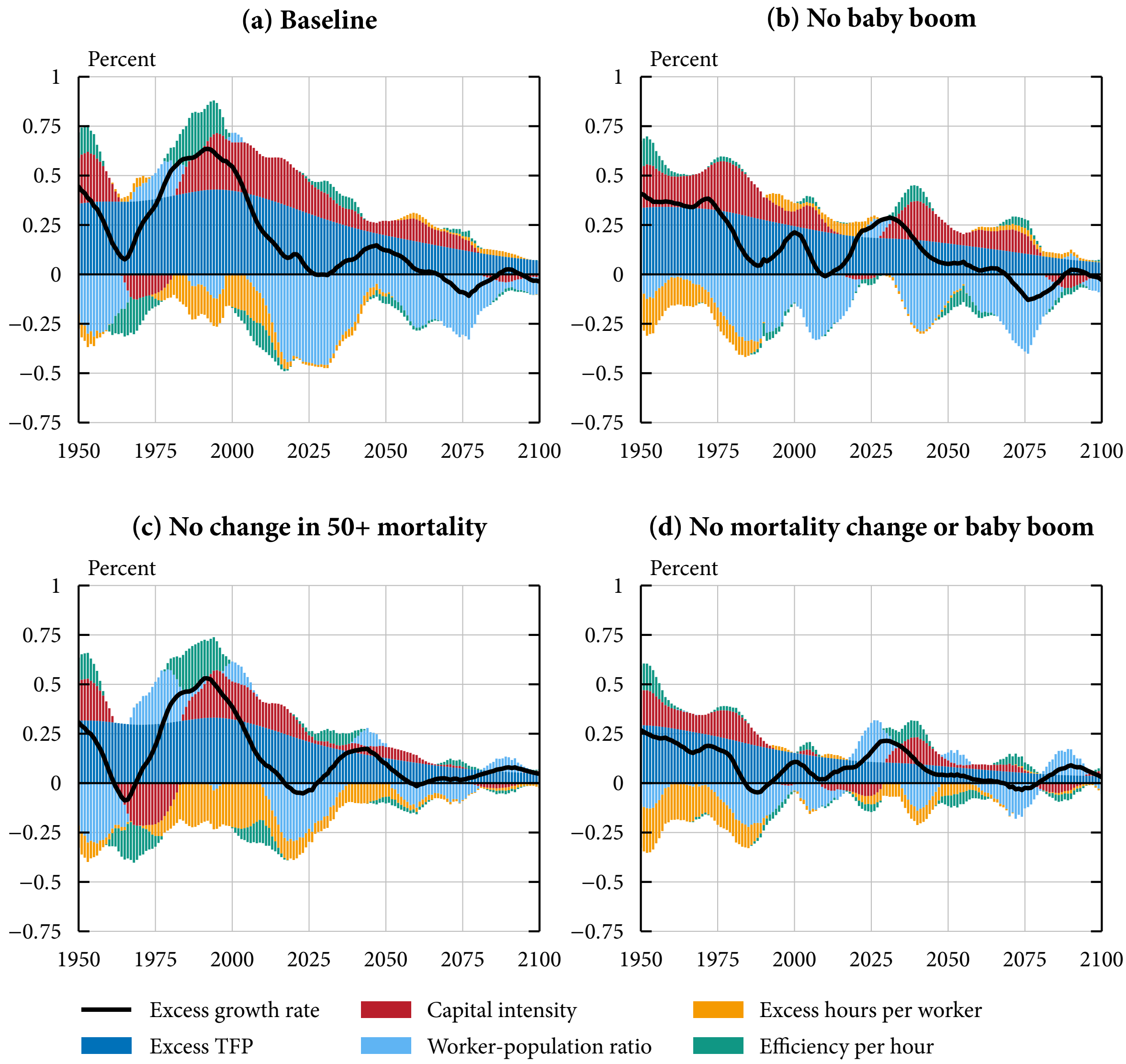


Figure A.7. Growth accounting comparison for the counterfactual scenarios.

Appendix B Deriving the Stationary Growth Rates

The stationary growth rates of the benchmark model and of the version with labour as the only R&D input are straightforwardly derived following Jones (1995) and Boppart and Krusell (2020) as follows.

B.1 TFP, Output per Capita, Hours per Worker

In a stationary equilibrium, interior solutions to the household maximisation problem are characterised by an Euler equation, an intratemporal first-order condition, and a budget constraint of the forms

$$c_j^{-\sigma} = \beta s_j \left(1 + r(1-\tau^k)\right) \frac{\Omega_{j+1}}{\Omega_j} \mathbb{E}\left[c_{j+1}^{-\sigma} \,\middle|\, \eta \right], \tag{B.1}$$

$$\psi h_j^{1/\theta} = \Omega_j \frac{(1 - \tau^{wm}(w\ell_j) - \tau^b) w \varepsilon_j \eta}{c_j^{\sigma}(1+\tau^c)}, \tag{B.2}$$

$$a_{j+1} + (1+\tau^c)c_j = \left(1 + r(1-\tau^k)\right)a_j + \left(1 - \tau^w(w\ell_j) - \tau^b\right)w\ell_j + tr + b_j(R), \tag{B.3}$$

where τ^{wm} in (B.2) denotes the *marginal* income tax rate. Let g_x denote the stationary growth rate of a variable x. Wages are standard neoclassical, so in a stationary equilibrium with a constant interest rate and capital-output ratio, wages grow by the rate of TFP. For the first-order condition (B.2) to hold along a balanced growth path, we then necessarily need $(1+g_h)^{1/\theta} = (1+g_Z)(1+g_c)^{-\sigma}$. Likewise, the budget constraint (B.3) is only consistent with balanced growth if consumption grows by the rate of output per capita and labour income: $1 + g_c = 1 + g_y = (1+g_Z)(1+g_h)$. Meanwhile, TFP is just the measure of intermediate firms raised to a power, $Z = z^{\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}$, so its growth rate is similarly the growth rate of intermediate firms raised to the same power. Combining these conditions yields that the growth rates of TFP, output per capita, and hours per worker must satisfy

$$1 + g_Z = (1+g_z)^{\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}, \qquad 1 + g_y = (1+g_Z)^{\frac{1+\theta}{1+\theta\sigma}}, \qquad 1 + g_h = (1+g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}}, \tag{B.4}$$

and substituting $1+g_z = (1+n)^{\chi}$ for some χ into Equation (B.4) gives the growth rates in Definition 2.

B.2 Intermediate-Firm Growth Rate: Baseline Model

Next, by Equation (21) it holds that $1 + g_z = 1 - \delta_z + \nu Q^{\lambda} z^{\phi-1}$, which is constant if and only if the last term on the right-hand side is constant. The latter holds only if

$$(1+g_z)^{1-\phi} = (1+g_Q)^{\lambda}. \tag{B.5}$$

The growth rate of R&D investment equals the aggregate output growth rate $(1+g_Z)(1+g_L)$ by the goods market condition. Employment grows by the rate of the population $1+n$ whereas labour productivity is constant for a fixed population age structure. The labour force growth rate is therefore given by $1 + g_L = (1+g_h)(1+n)$. Together with Equation (B.4), this allows us to rewrite the growth rate of R&D investment into

$$1 + g_Q = (1+g_Z)(1+g_h)(1+n) = (1+g_z)^{\frac{1+\theta}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\rho}{\rho}}(1+n).$$

Plugging this into (B.5) and rearranging terms yields the growth rate in Definition 2:

$$1 + g_z \;=\; (1+n)^{\chi}, \qquad \text{where} \qquad \chi \;\equiv\; \frac{\lambda}{1 - \phi - \lambda\,\frac{1+\theta}{1+\theta\sigma}\,\frac{\alpha}{1-\alpha}\,\frac{1-\rho}{\rho}}. \tag{B.6}$$

B.3 Intermediate-Firm Growth Rate: R&D With Only Labour

If R&D uses only labour, then the R&D process is given by $1 + g_z \;=\; 1 - \delta_z + \nu L_z^{\lambda} z^{\phi-1}$. In a stationary equilibrium, R&D labour L_z must grow by the rate of total labour supply according to the labour market condition. Again inspecting the right-hand side, we thus get a constant growth rate $1 + g_z$ if and only if

$$(1+g_z)^{1-\phi} \;=\; (1+g_L)^{\lambda}. \tag{B.7}$$

Using Equation (B.4), we can rewrite the labour force growth rate into

$$1 + g_L \;=\; (1+g_h)(1+n) \;=\; (1+g_z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}\,\frac{\alpha}{1-\alpha}\,\frac{1-\rho}{\rho}}(1+n).$$

Plugging this into (B.7) and rearranging terms yields the growth rate

$$1 + g_z \;=\; (1+n)^{\chi}, \qquad \text{where} \qquad \chi \;\equiv\; \frac{\lambda}{1 - \phi - \lambda\,\frac{\theta(1-\sigma)}{1+\theta\sigma}\,\frac{\alpha}{1-\alpha}\,\frac{1-\rho}{\rho}}. \tag{B.8}$$

Note that TFP growth collapses to the benchmark growth rate in Jones (1995) if we consider a steady state with constant hours worked (via log preferences, $\sigma \to 1$) and a substitution parameter for intermediate goods exactly equal to the capital share parameter ($\rho = \alpha$).

Appendix C Additional Robustness Checks

This appendix complements Section 7 with additional robustness checks. Each robustness scenario recalibrates the preference parameters β and ψ to match the same calibration targets as in the baseline if needed. The growth rate of each alternative is plotted against the baseline in Figure C.1 while Figure C.2 shows their growth decompositions. Overall, neither alternative alters any qualitative conclusion and only the case with a different pension system configuration has a quantitatively meaningful impact.

Increased retirement age. The social security system in the benchmark model has a fixed normal retirement age (NRA) and early/delayed pension scaling schedule, contrary to reality. Since the average age of retirement influences all growth mechanisms in the paper, I therefore consider an alternative which more accurately describes the NRA and the delayed retirement credits. Specifically, I increase the NRA to 66 for cohorts born between 1940 and 1956 and to 67 for subsequent cohorts. Moreover, the delayed retirement credit is increased by 0.5 percentage points for every other cohort between 1924 and 1943. That is, the delayed retirement credit is 3 percent for cohorts born before 1925, 3.5 percent for the 1925–1926 cohorts, . . . , 7.5 percent for the 1941–1942 cohorts, and 8 percent for all subsequent cohorts. Overall, these changes nevertheless leave the baseline results unaffected because most households in the baseline already retire between the ages of 65 and 70.

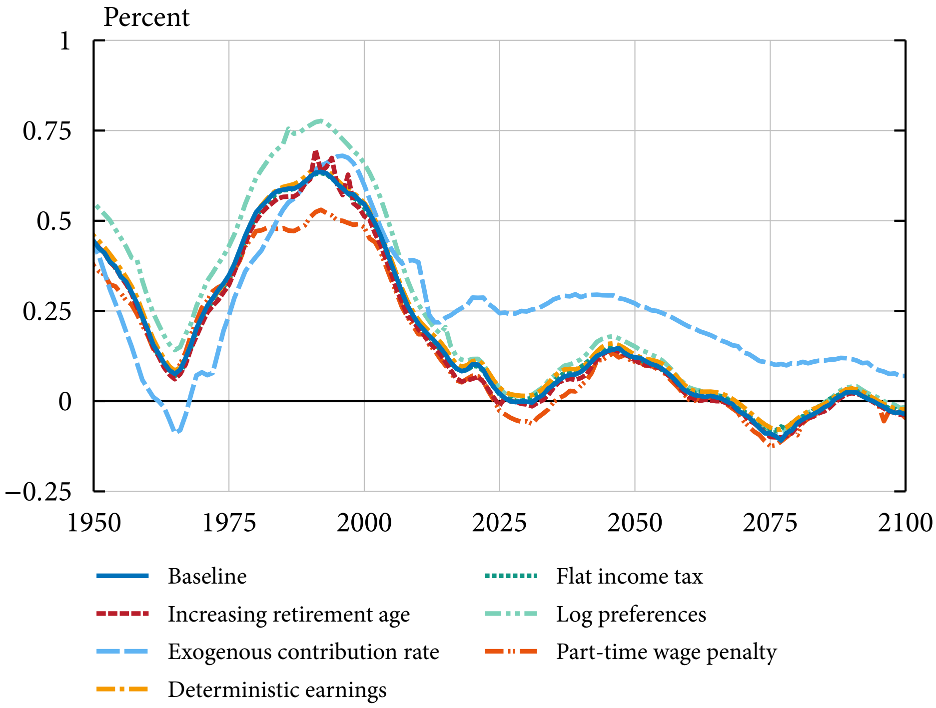


Figure C.1. Robustness checks: growth rates under different model specifications.

Exogenous contribution rate. The benchmark model balances the social security budget by changing the pension contribution rate to maintain a fixed replacement rate. The fiscal pressure of an ageing population makes this the most growth pessimistic setup of the pension system: it reduces the incentive (through higher pension income) and ability (through higher taxation) to save for retirement as well as the incentive for late retirement (through a higher opportunity cost of working once eligible for social security) compared to a case which maintains tax levels by cutting pensions. Here, I consider the more optimistic case with exogenous taxation and endogenous changes to the replacement rate. To this end, I construct a tax rate based on the national accounts measure for social insurance contributions (described in Appendix E) and feed this time series into the model.

Figure E.1 shows that the contribution rate rises throughout the post-war period. In the model, the resulting increase in contributions is more than sufficient to offset the increased pension bill of an ageing population, so benefits grow more generous, peaking at a replacement rate of 0.65 around 1990. After 1990, the contribution rate stabilises, causing the public sector to cut benefits throughout the twenty-first century. The increased generosity during the twentieth century reduces growth relative to baseline by 0.08 percentage points per year as households retire earlier and save less. Conversely, the decline in the replacement rates during the twenty-first century increases growth relative to the baseline by 0.16 percentage points per year.

Deterministic earnings. In the baseline, households face uninsurable idiosyncratic productivity shocks that add a savings motive beyond the standard life-cycle motive. In an alternative, I remove this feature and consider deterministic earnings. Although this reduces inequality and the overall level of household savings, the difference to the baseline turns out to be negligible.

Flat income tax. Rather than considering a progressive income tax, I analyse an alternative in which all households face a constant marginal (and average) income tax rate equal to that obtained from the

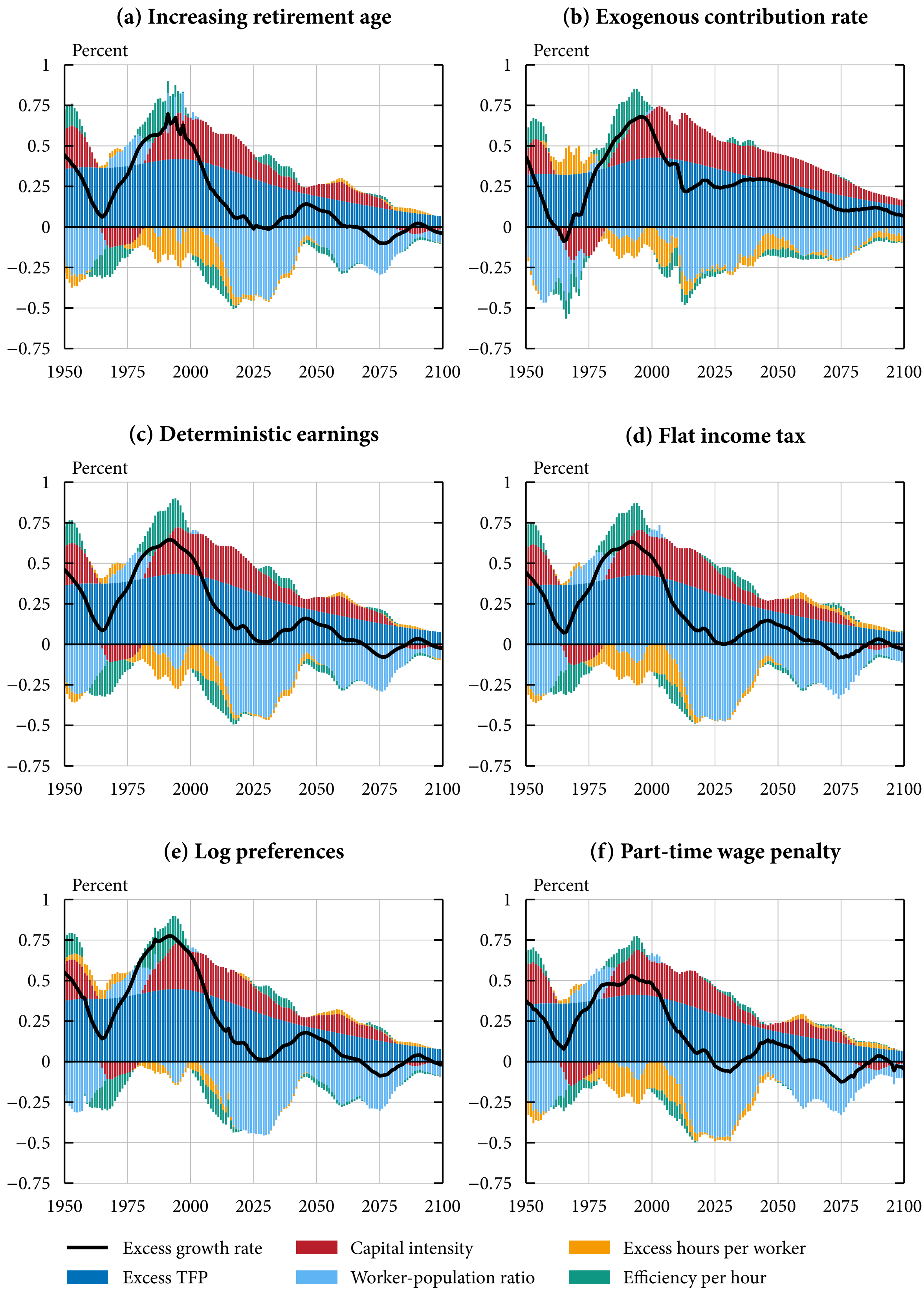


Figure C.2. Robustness checks: growth decomposition.

national accounts: $\tau^w = 0.115$. Contrary to the deterministic earnings scenario, imposing a flat tax increases inequality and the level of household savings. Again, the difference to the baseline is nevertheless negligible.

Log preferences. Another nonstandard feature is that I consider preferences of the Boppart and Krusell (2020) class that generate declining hours worked along a balanced growth path. By contrast, a large part of the macroeconomic literature restricts itself to a subset of this class, defined by King, Plosser and Rebelo (1988), in which hours worked are constant in the long run. Constant long-run hours in my model are obtained as the special case when $\sigma \to 1$, so that flow utility becomes $u(c_j, h_j) = \Omega_j \log(c_j) - \psi \frac{h_j^{1+1/\theta}}{1+1/\theta}$. In this case, the income and substitution effect on leisure exactly offset each other, so hours worked do not fall when growth is positive. Between 1950 and 2000, annual growth with logarithmic preferences is therefore about 0.1 percentage points higher. This difference is explained entirely by the different adjustments in hours worked. Growth during the twenty-first century does not change since, with wage growth around zero, the response in hours is similar in both scenarios.

Part-time wage penalty. Several authors stress the importance of nonconvexities in the budget set to generate endogenous retirement (see for instance Rogerson and Wallenius, 2013, and Ljungqvist and Sargent, 2014). Social security plays this role in the baseline model. Another commonly used nonconvexity is nonlinear wages (see for instance French, 2005, and Kitao, 2014), which is motivated by the empirical observation that part-time work does not pay as high hourly wage as full-time work. Thus, following French (2005), consider household labour earnings given by $w\varepsilon_j\eta h_j^{1+\xi}$, with $\xi \geq 0$. The labour market condition then changes to $L = \sum_j \int_X \varepsilon_j \eta h_j(x)^{1+\xi}\, \mathrm{d}\Phi_j$ and, by similar derivations as in Appendix B, the long-run growth rate of intermediate firms becomes

$$1 + g_z = (1+n)^\chi, \qquad \text{where} \qquad \chi \equiv \frac{\lambda}{1 - \phi - \lambda \frac{1+\theta}{1+\theta\sigma - \xi\theta(1-\sigma)} \frac{\alpha}{1-\alpha} \frac{1-\rho}{\rho}}.$$

The long-run growth rates of output per capita and hours per worker similarly become

$$1 + g_y = (1+g_Z)^{\frac{1+\theta}{1+\theta\sigma-\xi\theta(1-\sigma)}} \qquad \text{and} \qquad 1 + g_h = (1+g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma-\xi\theta(1-\sigma)}}.$$

For $\xi = 0$, wages are a linear function of hours worked and we obtain the benchmark model. Here, I follow French and set the value of ξ to 0.415 based on Aaronson and French's (2004) empirical finding that a 50 percent reduction in hours corresponds to a 25 percent lower hourly wage. I recalibrate the model under the assumption that $\xi = 0.415$ holds, which lowers the intertemporal elasticity of substitution, $\sigma = 1.84$, and flattens the age-efficiency profile.[19]

With these adjustments, the growth rate is similar to the baseline on average but exhibits more stable dynamics during the twentieth century. The latter is due to two changes to the average productivity per hour worked. First, declining hours worked negatively impacts average productivity since the productivity of an individual worker, $\varepsilon_j \eta h_j^\xi$, now includes hours worked. Second, the age-efficiency profile is flatter than in the baseline. Changes in the age composition of the labour force, which are more prominent in the twentieth century, therefore leads to smaller changes in average efficiency.

[19] For the baseline, I estimate the age-efficiency profile $\{\varepsilon_j\}_{j=\iota}^J$ from a PSID wage measure obtained by dividing annual labour income with annual hours. Here, I assume that $\xi = 0.415$ holds and construct PSID wages as (total annual labour income)/(annual hours worked)$^{1.415}$ (see Appendix D for estimation details).

Table D.1. Estimation of deterministic age-efficiency profile.

	Benchmark	Robustness checks					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ϑ_0	-1.3734^{***} (0.2250)	0.9857^{***} (0.0191)	1.1419^{***} (0.0365)	-0.9102^{***} (0.1783)	-1.2920^{***} (0.2043)	-1.0902^{***} (0.1655)	-1.1382^{***} (0.1045)
ϑ_1	0.0734^{***} (0.0020)	0.0954^{***} (0.0010)	0.0835^{***} (0.0018)	0.0813^{***} (0.0018)	0.0709^{***} (0.0018)	0.0610^{***} (0.0015)	0.0606^{***} (0.0010)
ϑ_2	-0.0008^{***} (0.0000)	-0.0010^{***} (0.0000)	-0.0008^{***} (0.0000)	-0.0008^{***} (0.0000)	-0.0008^{***} (0.0000)	-0.0007^{***} (0.0000)	-0.0007^{***} (0.0000)
Individual FE	✓		✓	✓	✓	✓	✓
Time controls[a]	✓			✓	✓	✓	✓
Aggregate wage from	BEA			BLS	BEA	BEA	BEA
Female heads					✓	✓	✓
Spouses/partners						✓	✓
Additional controls[b]							✓
Observations	90,832	90,832	90,832	90,832	110,169	165,034	161,012
Adjusted R^2	0.160	0.121	0.156	0.162	0.158	0.152	0.153

Notes. Dependent variable: log real hourly wage. Wages defined as labour earnings/hours. Regressors of interest: quadratic age polynomial with coefficients ϑ_0, ϑ_1, ϑ_2. Robust standard errors in parentheses. *, **, and *** denote statistical significance at the 10 percent, 5 percent, and 1 percent levels, respectively.
[a] Includes controls for the aggregate real wage level (using BEA or BLS data) and business cycle fluctuations (as proxied by the deviation of the unemployment rate from its long-run mean).
[b] Includes controls for education level (eight groups), family size, marital status (married, single, widowed, divorced, separated), and fixed effects for place of residence (50 states and D.C. plus abroad).

Appendix D Estimating the Life-Cycle Earnings Profile

I parametrise the age-efficiency profile $\{\varepsilon_j\}_{j=\iota}^{J}$ as the exponential of a quadratic age polynomial: $\varepsilon_j = \exp\{\vartheta_0 + \vartheta_1 j + \vartheta_2 j^2\}$. In the model, the hourly wage of an individual i of age j at time t is given by $w_{ijt} = w_t \varepsilon_j \eta_{ijt}$, where w_t is a common wage trend and η_{ijt} captures any idiosyncratic differences. This motivates the fixed effects regression

$$\ln w_{ijt} = \varrho_t + \varrho_i + \vartheta_0 + \vartheta_1 j + \vartheta_2 j^2 + u_{ijt}, \tag{D.1}$$

where ϱ_t is a time fixed effect, ϱ_i is an individual fixed effect, and u_{ijt} is an error term. Equation (D.1) implicitly captures cohort effects through the individual fixed effects and it is well known that collinearity between age, time, and cohorts prohibits simultaneous identification of these effects. As a partial remedy, I use the approach advocated by Heckman and Robb (1985) and replace the time fixed effect by two macroeconomic variables which plausibly proxy for the underlying unobserved time variables in the context of an earnings regression: log of the aggregate real wage level and the percentage point deviation of the unemployment rate from its long-run mean. The former corresponds to $\ln w_t$ and controls for secular wage growth and the latter (which is also used by French, 2005) controls for fluctuations in the business cycle.

I estimate Equation (D.1) with micro data on earnings from the nationally representative SRC sample of the Panel Study of Income Dynamics (PSID) for survey years 1968 to 2019 (which correspond to calendar years 1967 to 2018). Individual wages are imputed as total annual labour income divided by annual hours

Table D.2. Estimation of deterministic age-efficiency profile with part-time wage penalty.

	Benchmark	Robustness checks					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ϑ_0	-4.1083^{***} (0.2291)	-1.8276^{***} (0.0200)	-1.6727^{***} (0.0366)	-3.6135^{***} (0.1809)	-4.0415^{***} (0.2084)	-3.9382^{***} (0.1669)	-3.9195^{***} (0.1121)
ϑ_1	0.0540^{***} (0.0020)	0.0757^{***} (0.0010)	0.0634^{***} (0.0018)	0.0616^{***} (0.0018)	0.0515^{***} (0.0018)	0.0438^{***} (0.0014)	0.0400^{***} (0.0010)
ϑ_2	-0.0006^{***} (0.0000)	-0.0008^{***} (0.0000)	-0.0006^{***} (0.0000)	-0.0006^{***} (0.0000)	-0.0006^{***} (0.0000)	-0.0005^{***} (0.0000)	-0.0004^{***} (0.0000)
Individual FE	✓		✓	✓	✓	✓	✓
Time controls[a]	✓			✓	✓	✓	✓
Aggregate wage from	BEA			BLS	BEA	BEA	BEA
Female heads					✓	✓	✓
Spouses/partners						✓	✓
Additional controls[b]							✓
Observations	90,832	90,832	90,832	90,832	110,169	165,034	161,012
Adjusted R^2	0.123	0.094	0.120	0.125	0.120	0.110	0.113

Notes. Dependent variable: log real hourly wage. Wages defined as labour earnings/hours$^{1.415}$. Regressors of interest: quadratic age polynomial with coefficients ϑ_0, ϑ_1, ϑ_2. Robust standard errors in parentheses. *, **, and *** denote statistical significance at the 10 percent, 5 percent, and 1 percent levels, respectively.
[a] Includes controls for the aggregate real wage level (using BEA or BLS data) and business cycle fluctuations (as proxied by the deviation of the unemployment rate from its long-run mean).
[b] Includes controls for education level (eight groups), family size, marital status (married, single, widowed, divorced, separated), and fixed effects for place of residence (50 states and D.C. plus abroad).

worked. The aggregate wage used to proxy the time fixed effect is obtained from the national accounts by dividing total private industry wages (BEA NIPA Table 2.3) by total private industry hours worked (BEA NIPA Table 6.9). The unemployment rate is taken from the Bureau of Labor Statistics (BLS, series ID LNS14000000). All nominal variables are deflated into 2012 dollars using the PCE price index (BEA NIPA Table 2.3.4).

For the benchmark estimation, I impose standard sample restrictions (see for instance French, 2005, Heathcote, Storesletten and Violante, 2010, and Huggett, Ventura and Yaron, 2011): I select male household heads with no inconsistencies in reported age, who work between 300 and 5,840 hours a year (30 percent of part time and twice full time, respectively), and whose hourly wage exceeds \$3 per hour and does not exceed \$100 per hour in 2012 dollars. I consider individuals between the ages of 18 and 75. This goes against the standard practice of excluding ages at the beginning and end of the working life to avoid sample selection issues relating to work-life entry and exit. This choice is motivated by the need for an efficiency profile for all ages above 20, given that retirement in the model is endogenous. The alternative, estimating the age profile on individuals between, say, the ages of 25 and 60, instead requires extrapolation of the age profile to younger and older ages, and it is not clear that this approach is preferable. An upper bound at 75 is nevertheless imposed to ensure there are at least 100 observations in each age group. Extrapolation beyond 75 is inconsequential, since between 95 and 99 percent of model households retire before 75. The final sample consists of 90,832 person-year observations.

Table D.1 shows the estimation results along with several robustness checks. Column (1) corresponds to the age profile in Figure 4a. Column (2) shows standard OLS estimates and column (3) includes only

Table E.1. Tax data variables.

Variable	Explanation	Source	
C	Personal consumption expenditures	BEA NIPA Table 1.1.5	line 2
EC	Compensation of employees	BEA NIPA Table 1.12	line 2
W	Wages and salaries	BEA NIPA Table 1.12	line 3
PRI	Proprietors' income[a]	BEA NIPA Table 1.12	line 9
RI	Rental income of persons[a]	BEA NIPA Table 1.12	line 12
CP	Corporate profits[a]	BEA NIPA Table 1.12	line 13
NI	Net interest and miscellaneous payments	BEA NIPA Table 1.12	line 18
PCT	Personal current taxes	BEA NIPA Table 3.1	line 3
TPI	Taxes on production and imports	BEA NIPA Table 3.1	line 4
CT	Taxes on corporate income	BEA NIPA Table 3.1	line 5
CSI	Contributions for government social insurance	BEA NIPA Table 3.1	line 7
PRT	Property taxes	BEA NIPA Table 3.3	line 9

[a] With inventory valuation adjustment and capital consumption adjustment.

individual fixed effects. In both cases, secular wage growth is interpreted as life-cycle earnings differences, and this generates steeper profiles; productivity at peak age is 115 to 130 percent larger than the initial age, compared to 70 percent for the main estimation. This underlines the importance of controlling for time effects. Column (4) changes the aggregate wage measure to average hourly earnings of production and nonsupervisory employees (BLS, series ID CES0500000008). This wage exhibits lower growth in recent decades than the imputed wage from the BEA, and the estimation results are therefore similar to columns (2) and (3).[20] Columns (5) to (7) expand the sample to include spouses, partners, and female household heads and add additional individual-level controls that may change over time. These additions lower the point estimates somewhat, although these samples raise additional concerns for sample selection and also provide worse fits to the data as measured by the adjusted R^2.

Lastly, for the model scenario in which wages are an increasing function of hours worked in Appendix C, I run identical estimations to above with the only exception that individual wages are constructed as (total annual labour income)/(annual hours worked)$^{1.415}$ in the PSID data. Table D.2 shows the results from these regressions.

Appendix E Constructing Tax Rates

This section explains the calibration of the model taxes. First, I describe the construction of the aggregate tax rates from the national accounts, then the estimation a progressive income tax function, and finally the calibration of the income tax function used in the model (which builds on the former two).

E.1 Aggregate Tax Rates

The methodology to construct the aggregate tax rates on consumption, capital, and labour income is taken off-the-shelf from Fernández-Villaverde *et al.* (2015), which in turn builds on Jones (2002b) and Mendoza, Razin and Tesar (1994). In short, each tax rate is given by aggregating all relevant tax revenues at the general government level and then dividing by the corresponding tax base. All data for this exercise are taken from the BEA NIPA tables. Table E.1 summarises the variables that I use.

[20] I use the BEA wage measure as the benchmark since the BLS wage is more limited in scope.

The average consumption tax rate τ^c is given by

$$\tau^c = \frac{TPI - PRT}{C - (TPI - PRT)}. \tag{E.1}$$

The numerator of (E.1) is the revenue from consumption taxation. I subtract property taxes from total taxes on production because homeowners in the national accounts are treated as businesses that rent their properties to themselves. Property taxes are therefore incorporated as taxes on capital instead. The consumption tax base in the denominator is total personal consumption expenditures net of consumption taxes paid (that is, the pre-tax value of consumption).

The NIPA tables do not provide a breakdown of personal current taxes into labour and capital income. To make this split, I construct an average personal income tax rate τ^p as an intermediate step via

$$\tau^p = \frac{PCT}{W + PRI/2 + CI}, \qquad \text{where} \qquad CI \equiv PRI/2 + RI + CP + NI.$$

The numerator is the sum of personal current taxes at the federal, state and local levels. The tax base is the sum of wages, proprietors' income, and capital income (CI). Here, proprietors' income is divided evenly between labour and capital income following Jones (2002b), who emphasises that any split of proprietor's income into labour and capital income is arbitrary and therefore chooses the fifty-fifty split as a middle ground.

I then estimate the total revenue from personal taxes on income and capital as $\tau^p(W + PRI/2)$ and $\tau^p CI$, respectively. The average labour income and capital tax rates are subsequently given by

$$\tau^w = \tau^p \frac{W + PRI/2}{EC + PRI/2} \qquad \text{and} \qquad \tau^k = \frac{\tau^p CI + CT + PRT}{CI + PRT}. \tag{E.2}$$

One robustness check in Appendix C also considers an exogenous social security contribution rate τ^b. For this scenario, I construct the social security tax rate as

$$\tau^b = \frac{CSI}{EC + PRI/2}. \tag{E.3}$$

The sum of τ^w and τ^b gives the measure of the average labour income tax rate used by Fernández-Villaverde *et al.* (2015). Figure E.1 plots the estimated tax rates, which highlights that imposing constant tax rates in the model for consumption, labour income and capital is a reasonable assumption.

E.2 Estimating the income tax rate function

To estimate the income tax function, I compute average tax rates at hypothetical levels of income and fit Equation (35) to these synthetic data. This process follows the methodology of the OECD tax database for the United States, which creates effective tax rates by applying applicable tax rules and regulations for given years and earnings levels and then dividing the resulting net tax liabilities by gross earnings. These calculations include taxation at all levels of government for a household assumed to live in Detroit, Michigan.

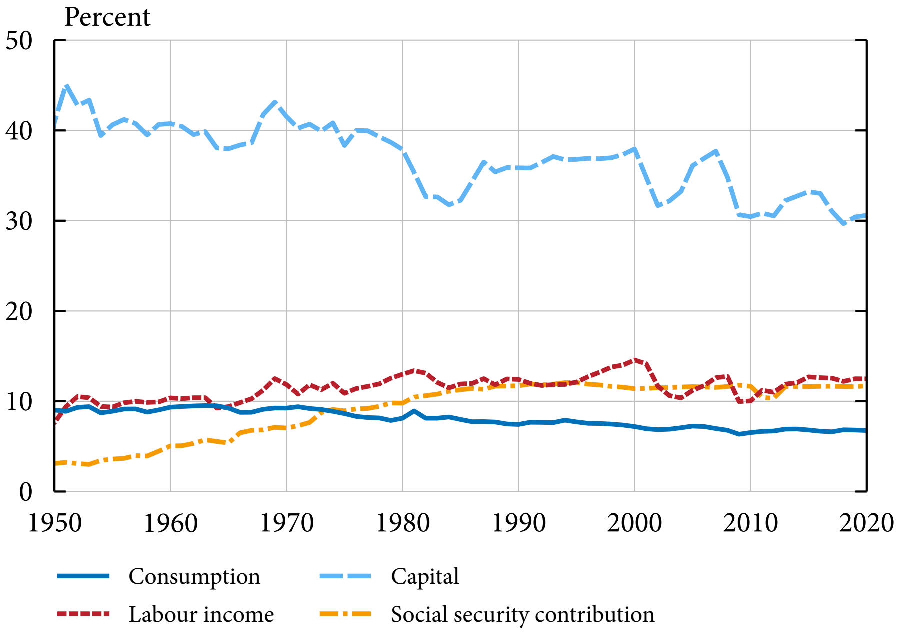


Figure E.1. Aggregate tax rate estimates from the national accounts.

For simplicity, I only consider single households without children, in line with the primary estimates published by the OECD.[21] This choice is of secondary importance since I eventually scale the estimated tax function to match the national accounts. I also abstract from social security contributions since those are modelled separately in my framework. The subsections below outline the formulas, parameters, and parameter values for this particular case.[22]

E.2.1 Taxable Earnings

Taxable earnings at government level $\mathtt{x} \in \{\mathtt{fed}, \mathtt{state}, \mathtt{local}\}$ is given by gross income GI minus a tax allowance $\mathrm{TAXALLOW}_{\mathrm{x}}$, provided that this is positive:

$$e^{\mathrm{x}}(\mathrm{GI}) = \max\{\mathrm{GI} - \mathrm{TAXALLOW}_{\mathrm{x}},\ 0\}.$$

At the federal level, the allowance consists of a standard deduction $\mathrm{STDALLOW}$ and a personal exemption $\mathrm{EXEMPT}_{\mathrm{fed}}$. The personal exemption is reduced at a taper rate $\varphi^{T}_{\mathrm{ex}}$ for every USD 2,500 that gross income exceeds the threshold $\mathrm{THOLD}_{\mathrm{ex}}$. At the state and local levels, the allowances are fixed personal exemptions $\mathrm{EXEMPT}_{\mathrm{state}}$ and $\mathrm{EXEMPT}_{\mathrm{local}}$, respectively. Thus,

$$\begin{aligned}
\mathrm{TAXALLOW}_{\mathrm{fed}} &= \mathrm{STDALLOW} + \mathrm{EXEMPT}_{\mathrm{fed}}\left(1 - \varphi^{T}_{\mathrm{ex}}\left\lceil \frac{\max\{\mathrm{GI} - \mathrm{THOLD}_{\mathrm{ex}},\ 0\}}{2500}\right\rceil\right), \\
\mathrm{TAXALLOW}_{\mathrm{state}} &= \mathrm{EXEMPT}_{\mathrm{state}}, \\
\mathrm{TAXALLOW}_{\mathrm{local}} &= \mathrm{EXEMPT}_{\mathrm{local}},
\end{aligned}$$

[21] See Table I.5, available for download at OECD.Stat.

[22] The implementation code (available upon request) also incorporates different household compositions (with respect to children and marital status) and social security contributions. Supplementary documentation for these cases is given in the OECD publication *Taxing Wages* available at the OECD iLibrary.

where $\lceil \cdot \rceil$ is the ceiling function: $\lceil x \rceil = \min\{n \in \mathbb{Z} : n \geq x\}$.

E.2.2 Taxes

Taxable earnings at the state and local levels are subject to flat tax rates τ^{state} and τ^{local}. The federal income tax is progressive, with higher marginal tax rates at higher levels of income. Consider N federal tax brackets with marginal tax rates $\tau_1^{\text{fed}}, \ldots, \tau_N^{\text{fed}}$ starting at earnings thresholds $\bar{e}_1, \ldots, \bar{e}_N$, where $\bar{e}_1 = 0$. Given a largest applicable federal tax bracket $I = \max\{i : e^{\text{fed}}(\text{GI}) > \bar{e}_i\}$, the tax liability at each level of government is then given by functions $T^{\text{x}}(\text{GI})$ of gross income as follows:

$$T^{\text{fed}}(\text{GI}) = \sum_{i=1}^{I-1} \tau_i^{\text{fed}} \left(\bar{e}_{i+1} - \bar{e}_i\right) + \tau_I^{\text{fed}} \left(e^{\text{fed}}(\text{GI}) - \bar{e}_I\right),$$

$$T^{\text{state}}(\text{GI}) = \tau^{\text{state}} e^{\text{state}}(\text{GI}),$$

$$T^{\text{local}}(\text{GI}) = \tau^{\text{local}} e^{\text{local}}(\text{GI}).$$

E.2.3 Tax Credits

The OECD considers two types of federal tax credits for households without children: the Earned Income Tax Credit (EIC) and the Making Work Pay tax credit (MWP). The EIC and the MWP provide refundable tax credits equal to some fractions φ_{eic} and φ_{mwp} of gross income up to some maximum amounts $\varphi_{\text{eic}} \overline{\text{eic}}$ and $\overline{\text{mwp}}$. The tax credits are phased out at taper rates φ_{eic}^T and φ_{mwp}^T once gross income exceeds thresholds $\text{THOLD}_{\text{eic}}$ and $\text{THOLD}_{\text{mwp}}$. The total tax credit amounts from these programs are thus given by

$$\text{eic}(\text{GI}) = \max\left\{\varphi_{\text{eic}} \min\left\{\text{GI}, \overline{\text{eic}}\right\} - \varphi_{\text{eic}}^T \max\left\{\text{GI} - \text{THOLD}_{\text{eic}}, 0\right\}, 0\right\}$$

and

$$\text{mwp}(\text{GI}) = \max\left\{\min\left\{\varphi_{\text{mwp}} \text{GI}, \overline{\text{mwp}}\right\} - \varphi_{\text{mwp}}^T \max\left\{\text{GI} - \text{THOLD}_{\text{mwp}}, 0\right\}, 0\right\}.$$

Total federal tax credits is the sum of EIC and MWP. At the state level, the OECD includes the Michigan Earned Income Tax Credit, which is an additional refundable credit equal to a fraction φ_{meic} of the federal EIC amount. The local level incorporates the Michigan City Income Tax Credit (CTC) which is a nonrefundable credit equal to some fraction of the total local tax liability $T^{\text{local}}(\text{GI})$ up to some maximum amount $\overline{\text{ctc}}$. Below this upper bound, the CTC credit rates decline with income. Consider N credit rate brackets with marginal credit rates $\varphi_{1,\text{ctc}}, \ldots, \varphi_{N,\text{ctc}}$ starting at tax liability thresholds $\overline{T}_1, \ldots, \overline{T}_N$, where $\overline{T}_1 = 0$. Given a largest applicable tax credit bracket $I = \max\{i : T^{\text{local}}(\text{GI}) > \overline{T}_i\}$, the total tax credit at each level of government is then given by functions $C^{\text{x}}(\text{GI})$ of gross income as follows:

$$C^{\text{fed}}(\text{GI}) = \text{eic}(\text{GI}) + \text{mwp}(\text{GI}),$$

$$C^{\text{state}}(\text{GI}) = \varphi_{\text{meic}} \, \text{eic}(\text{GI}),$$

$$C^{\text{local}}(\text{GI}) = \min\left\{\sum_{i=1}^{I-1} \varphi_{i,\text{ctc}}\left(\overline{T}_{i+1} - \overline{T}_i\right) + \varphi_{I,\text{ctc}}\left(T^{\text{local}}(\text{GI}) - \overline{T}_I\right), \overline{\text{ctc}}\right\}.$$

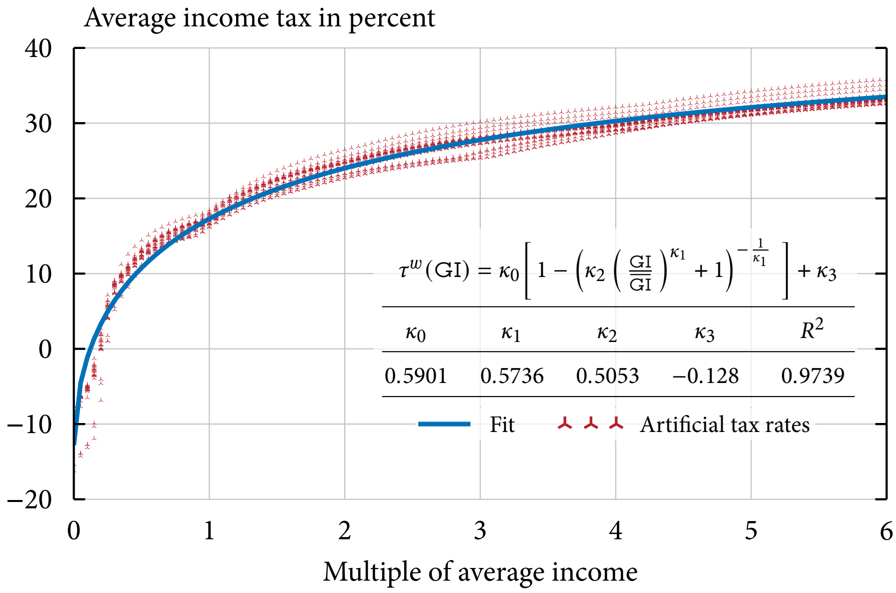


Figure E.2. Estimation of the income tax function.

E.2.4 Effective Income Tax Rate

The effective income tax rate $\tau^w(\mathrm{GI})$ at gross income GI is the total tax liability net of tax credits measured as a percentage of gross income:

$$\tau^w(\mathrm{GI}) = \frac{1}{\mathrm{GI}} \sum_{x \in X} \Big(T^x(\mathrm{GI}) - C^x(\mathrm{GI}) \Big),$$

where $X = \{\texttt{fed}, \texttt{state}, \texttt{local}\}$. In the practical implementation of these tax calculations, we consider an average gross income level $\overline{\mathrm{GI}}$ and then express all other gross incomes as a percentage of that average.

E.2.5 Estimation

Using the methodology above, I create effective income tax rates on a grid of gross incomes for each year between 2000 and 2022. The grid is linearly spaced with 401 points, ranging from 0 to a multiple 20 of average gross income. The parameter values for this exercise are collected from the OECD and are listed in Table E.2. I then fit the income tax function $\tau^w(\mathrm{GI}) = \kappa_0\left[1-\left(\kappa_2\left(\frac{\mathrm{GI}}{\overline{\mathrm{GI}}}\right)^{\kappa_1}+1\right)^{-\frac{1}{\kappa_1}}\right]+\kappa_3$ by a nonlinear OLS to these tax rates.

Figure E.2 shows the constructed tax rates for the lower end of the income grid together with the corresponding fit and its estimated coefficients. Even though the period considered saw two major tax reforms (the Economic Growth and Tax Reconciliation Relief Act of 2001 and the Tax Cuts and Jobs Act of 2017) and underwent three economic downturns (the early 2000s recession, the Great Recession, and the COVID-19 recession), effective income tax rates remain largely stable over time. Therefore, the estimated tax function provides a close fit of the constructed tax rates, as seen by the high R^2 of 0.97.

E.3 Changing the Tax Rate Level While Maintaining Progressivity

Once the income tax function is estimated, I adjust its level so that the tax rate at average earnings matches the tax rate from the national accounts. To this end, I follow Guvenen, Kuruscu and Ozkan (2014) to ensure that the degree of progressivity remains the same before and after. Thus, let $\widetilde{\tau}(e)$ be some average tax rate function of the Gouveia and Strauss (1994) form:

$$\widetilde{\tau}(e) \;=\; \widetilde{\kappa}_0 \left[1 - \left(\widetilde{\kappa}_2 \left(\frac{e}{\bar{e}} \right)^{\widetilde{\kappa}_1} + 1 \right)^{-\frac{1}{\widetilde{\kappa}_1}} \right] + \widetilde{\kappa}_3. \tag{E.4}$$

Denote its corresponding marginal tax rate by $\widetilde{\tau}^m(e) = \frac{\partial}{\partial e}\big(\widetilde{\tau}(e)e\big)$. To change the level of this tax function into a similarly parametrised function $\tau(e)$ with parameters $\kappa_0, \ldots, \kappa_3$ without changing the degree of progressivity, we need the ratio of net take-home shares at any two earnings levels e and e' to be equal between the two tax systems:

$$\frac{1 - \tau^m(e')}{1 - \tau^m(e)} \;=\; \frac{1 - \widetilde{\tau}^m(e')}{1 - \widetilde{\tau}^m(e)}.$$

This expression can be rearranged to obtain

$$\tau^m(e) \;=\; 1 - \bar{k}\big(1 - \widetilde{\tau}^m(e)\big), \qquad \text{where} \qquad \bar{k} \;\equiv\; \frac{1 - \tau^m(e')}{1 - \widetilde{\tau}^m(e')} \tag{E.5}$$

is a level ratio between the two tax systems that we are free to choose. Since $\tau(e)e = \int_0^e \tau^m(x)\,dx$, we can integrate Equation (E.5) to obtain an average tax rate of a similar form:

$$\tau(e) \;=\; 1 - \bar{k}\big(1 - \widetilde{\tau}(e)\big). \tag{E.6}$$

Substituting Equation (E.4) into (E.6) and rearranging terms, we finally get

$$\tau(e) \;=\; \kappa_0 \left[1 - \left(\kappa_2 \left(\frac{e}{\bar{e}} \right)^{\kappa_1} + 1 \right)^{-\frac{1}{\kappa 1}} \right] + \kappa_3,$$

where $\kappa_0 \;\equiv\; \bar{k} \cdot \widetilde{\kappa}_0$, $\kappa_1 \;\equiv\; \widetilde{\kappa}_1$, $\kappa_2 \;\equiv\; \widetilde{\kappa}_2$ and $\kappa_3 \;\equiv\; 1 - \bar{k}(1 - \widetilde{\kappa}_3)$. The calibrated $\kappa_0, \ldots, \kappa_3$ in Table 3 use the estimates in Figure E.2 as $\widetilde{\kappa}_0, \ldots, \widetilde{\kappa}_3$ and set the scale parameter $\bar{k}$ such that the tax rate at average income $\bar{e}$ matches the income tax rate τ^{NA} from the national accounts. Specifically, the latter requires that $\tau(\bar{e}) = \tau^{NA} = 1 - \bar{k}\big(1 - \widetilde{\tau}(\bar{e})\big)$, which can be rearranged to give the scale parameter as

$$\bar{k} \;=\; \frac{1 - \tau^{NA}}{1 - \widetilde{\tau}(\bar{e})}, \qquad \text{where} \qquad \widetilde{\tau}(\bar{e}) \;=\; \widetilde{\kappa}_0 \left[1 - \left(\widetilde{\kappa}_2 + 1 \right)^{-\frac{1}{\widetilde{\kappa}_1}} \right] + \widetilde{\kappa}_3$$

is a function of estimated parameters only.

Table E.2. Income tax parameters.

	Average income	Standard deduction	Federal personal exemption			Earned Income Tax Credit				Making Work Pay Tax Credit			
			Amount	Threshold	Taper	Rate	Threshold	Taper	Max	Rate	Threshold	Taper	Max
Year	$\overline{\mathrm{GI}}$	$\mathtt{STDALLOW}$	$\mathtt{EXEMPT}_{\mathrm{fed}}$	$\mathtt{THOLD}_{\mathrm{ex}}$	φ^T_{ex}	φ_{eic}	$\mathtt{THOLD}_{\mathrm{eic}}$	φ^T_{eic}	$\overline{\mathtt{eic}}$	φ_{mwp}	$\mathtt{THOLD}_{\mathrm{mwp}}$	φ^T_{mwp}	$\overline{\mathtt{mwp}}$
2000	$33,129	$4,400	$2,800	$128,950	2 %	7.65 %	$5,800	7.65 %	$4,600				
2001	$33,998	$4,550	$2,900	$132,950	2 %	7.65 %	$5,950	7.65 %	$4,750				
2002	$35,026	$4,700	$3,000	$137,300	2 %	7.65 %	$6,100	7.65 %	$4,900				
2003	$36,084	$4,750	$3,050	$139,500	2 %	7.65 %	$6,240	7.65 %	$4,990				
2004	$36,739	$4,850	$3,100	$142,700	2 %	7.65 %	$6,390	7.65 %	$5,100				
2005	$37,637	$5,000	$3,200	$145,950	2 %	7.65 %	$6,530	7.65 %	$5,220				
2006	$39,377	$5,150	$3,300	$150,500	1.33 %	7.65 %	$6,740	7.65 %	$5,380				
2007	$42,064	$5,350	$3,400	$156,400	1.33 %	7.65 %	$7,000	7.65 %	$5,590				
2008	$43,196	$5,450	$3,500	$159,950	0.67 %	7.65 %	$7,160	7.65 %	$5,720				
2009	$44,295	$5,700	$3,650	$166,800	0.67 %	7.65 %	$7,470	7.65 %	$5,970	6.2 %	$75,000	2 %	$400
2010	$45,665	$5,700	$3,650			7.65 %	$7,480	7.65 %	$5,980	6.2 %	$75,000	2 %	$400
2011	$46,895	$5,800	$3,700			7.65 %	$7,590	7.65 %	$6,070				
2012	$47,746	$5,950	$3,800			7.65 %	$7,770	7.65 %	$6,210				
2013	$48,774	$6,100	$3,900	$250,000	2 %	7.65 %	$7,970	7.65 %	$6,370				
2014	$50,099	$6,200	$3,950	$254,200	2 %	7.65 %	$8,110	7.65 %	$6,480				
2015	$50,963	$6,300	$4,000	$258,250	2 %	7.65 %	$8,240	7.65 %	$6,580				
2016	$51,945	$6,300	$4,050	$259,400	2 %	7.65 %	$8,270	7.65 %	$6,610				
2017	$53,376	$6,350	$4,050	$261,500	2 %	7.65 %	$8,340	7.65 %	$6,670				
2018	$55,058	$12,000				7.65 %	$8,490	7.65 %	$6,780				
2019	$56,577	$12,200				7.65 %	$8,650	7.65 %	$6,920				
2020	$59,517	$12,400				7.65 %	$8,790	7.65 %	$7,030				
2021	$62,172	$12,550				15.3 %	$11,610	15.3 %	$9,820				
2022	$64,889	$12,950				7.65 %	$9,160	7.65 %	$7,320				

Table E.2. Income tax parameters. (Cont.)

	Federal marginal tax rates							Federal income tax brackets						
Year	τ_1^{fed}	τ_2^{fed}	τ_3^{fed}	τ_4^{fed}	τ_5^{fed}	τ_6^{fed}	τ_7^{fed}	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$	$\bar{e}_4$	$\bar{e}_5$	$\bar{e}_6$	$\bar{e}_7$
2000	15 %	28 %	31 %	36 %	39.6 %			$0	$26,250	$63,550	$132,600	$288,350		
2001	10 %	15 %	27.5 %	30.5 %	35.5 %	39.1 %		$0	$6,000	$27,050	$65,550	$136,750	$297,370	
2002	10 %	15 %	27 %	30 %	35 %	38.6 %		$0	$6,000	$27,950	$67,700	$141,250	$307,050	
2003	10 %	15 %	25 %	28 %	33 %	35 %		$0	$7,000	$28,400	$68,800	$143,500	$311,950	
2004	10 %	15 %	25 %	28 %	33 %	35 %		$0	$7,150	$29,050	$70,350	$146,750	$319,100	
2005	10 %	15 %	25 %	28 %	33 %	35 %		$0	$7,300	$29,700	$71,950	$150,150	$326,450	
2006	10 %	15 %	25 %	28 %	33 %	35 %		$0	$7,550	$30,650	$74,200	$154,800	$336,550	
2007	10 %	15 %	25 %	28 %	33 %	35 %		$0	$7,825	$31,850	$77,100	$160,850	$349,700	
2008	10 %	15 %	25 %	28 %	33 %	35 %		$0	$8,025	$32,550	$78,850	$164,550	$357,700	
2009	10 %	15 %	25 %	28 %	33 %	35 %		$0	$8,350	$33,950	$82,250	$171,550	$372,950	
2010	10 %	15 %	25 %	28 %	33 %	35 %		$0	$8,375	$34,000	$82,400	$171,850	$373,650	
2011	10 %	15 %	25 %	28 %	33 %	35 %		$0	$8,500	$34,500	$83,600	$174,400	$379,150	
2012	10 %	15 %	25 %	28 %	33 %	35 %		$0	$8,700	$35,350	$85,650	$178,650	$388,350	
2013	10 %	15 %	25 %	28 %	33 %	35 %	39.6 %	$0	$8,925	$36,250	$87,850	$183,250	$398,350	$400,000
2014	10 %	15 %	25 %	28 %	33 %	35 %	39.6 %	$0	$9,075	$36,900	$89,350	$186,350	$405,100	$406,750
2015	10 %	15 %	25 %	28 %	33 %	35 %	39.6 %	$0	$9,225	$37,450	$90,750	$189,300	$411,500	$413,200
2016	10 %	15 %	25 %	28 %	33 %	35 %	39.6 %	$0	$9,275	$37,650	$91,150	$190,150	$413,350	$415,050
2017	10 %	15 %	25 %	28 %	33 %	35 %	39.6 %	$0	$9,325	$37,950	$91,900	$191,650	$416,700	$418,400
2018	10 %	12 %	22 %	24 %	32 %	35 %	37 %	$0	$9,525	$38,700	$82,500	$157,500	$200,000	$500,000
2019	10 %	12 %	22 %	24 %	32 %	35 %	37 %	$0	$9,700	$39,475	$84,200	$160,725	$204,100	$510,300
2020	10 %	12 %	22 %	24 %	32 %	35 %	37 %	$0	$9,875	$40,125	$85,525	$163,300	$207,350	$518,400
2021	10 %	12 %	22 %	24 %	32 %	35 %	37 %	$0	$9,950	$40,525	$86,375	$164,925	$209,425	$523,600
2022	10 %	12 %	22 %	24 %	32 %	35 %	37 %	$0	$10,275	$41,775	$89,075	$170,050	$215,950	$539,900

Table E.2. Income tax parameters. (Cont.)

	Personal exemption		Marginal tax rates		Michigan EIC	Michigan City Income Tax Credit						
	State	Local	State	Local		Rates			Credit brackets			Max
Year	$\mathrm{EXEMPT}_{\mathrm{state}}$	$\mathrm{EXEMPT}_{\mathrm{local}}$	τ^{state}	τ^{local}	φ_{meic}	$\varphi_{1,\mathrm{ctc}}$	$\varphi_{2,\mathrm{ctc}}$	$\varphi_{3,\mathrm{ctc}}$	$\overline{T}_1$	$\overline{T}_2$	$\overline{T}_3$	$\overline{\mathrm{ctc}}$
2000	\$2,900	\$750	4.2 %	2.85 %		20 %	10 %	5 %	\$0	\$100	\$150	\$10,000
2001	\$2,900	\$750	4.2 %	2.75 %		20 %	10 %	5 %	\$0	\$100	\$150	\$10,000
2002	\$3,000	\$750	4.1 %	2.65 %		20 %	10 %	5 %	\$0	\$100	\$150	\$10,000
2003	\$3,100	\$750	4 %	2.5 %		20 %	10 %	5 %	\$0	\$100	\$150	\$10,000
2004	\$3,100	\$750	4 %	2.5 %		20 %	10 %	5 %	\$0	\$100	\$150	\$10,000
2005	\$3,200	\$600	3.9 %	2.5 %		20 %	10 %	5 %	\$0	\$100	\$150	\$10,000
2006	\$3,300	\$600	3.9 %	2.5 %		20 %	10 %	5 %	\$0	\$100	\$150	\$10,000
2007	\$3,300	\$600	3.9 %	2.5 %		20 %	10 %	5 %	\$0	\$100	\$150	\$10,000
2008	\$3,300	\$600	4.35 %	2.5 %	10 %	20 %	10 %	5 %	\$0	\$100	\$150	\$10,000
2009	\$3,500	\$600	4.35 %	2.5 %	20 %	20 %	10 %	5 %	\$0	\$100	\$150	\$10,000
2010	\$3,600	\$600	4.35 %	2.5 %	20 %	20 %	10 %	5 %	\$0	\$100	\$150	\$10,000
2011	\$3,700	\$600	4.35 %	2.5 %	20 %	20 %	10 %	5 %	\$0	\$100	\$150	\$10,000
2012	\$3,763	\$600	4.33 %	2.45 %	6 %							
2013	\$3,950	\$600	4.25 %	2.4 %	6 %							
2014	\$3,950	\$600	4.25 %	2.4 %	6 %							
2015	\$3,950	\$600	4.25 %	2.4 %	6 %							
2016	\$4,000	\$600	4.25 %	2.4 %	6 %							
2017	\$4,000	\$600	4.25 %	2.4 %	6 %							
2018	\$4,000	\$600	4.25 %	2.4 %	6 %							
2019	\$4,050	\$600	4.25 %	2.4 %	6 %							
2020	\$4,750	\$600	4.25 %	2.4 %	6 %							
2021	\$4,900	\$600	4.25 %	2.4 %	6 %							
2022	\$5,000	\$600	4.25 %	2.4 %	6 %							