Endogenous technological change along the demographic transition

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Abstract

I study the effect of demographic change on economic growth under endogenous, R&D-driven technological change. Qualitatively, population ageing generates two opposing forces: increased R&D and capital investments on the one hand, and a decreasing share of workers in the population on the other. I evaluate these channels quantitatively along the demographic transition using a calibrated overlapping generations model with idiosyncratic income risk, mortality risk, intensive and extensive labour supply margins and endogenous technological change. Considering the United States between 1950 and 2100, I find that the demographic transition: (i) increased per-capita output by 0.35 percent per year between 1950 and 2000; (ii) accounts for a 0.65 percentage point decline in growth rates between 1995 and 2025 when the positive growth impact reverts back to trend; and (iii) has no net impact on twenty-first century growth. The main positive driver is endogenous technological change, whose growth contribution more than doubles that of capital deepening between 1950 and 2100. Removing this mechanism eliminates all positive growth effects.

Keywords: demographic transition, endogenous growth, OLG model.

JEL Codes: E17, E25, J11, O30, O40.

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1 Introduction

In 1950 there were 14 people aged 65 and above for every 100 people aged 20 to 64 in the United States. By 2020 this number had doubled and by the end of the twenty-first century it is expected to have quadrupled. Similar trends apply worldwide. Meanwhile, a widespread view holds that economic growth is slowing down. Figure 1 illustrates these observations. To what extent can this demographic shift explain the current growth performance? How will it impact future growth? And how important are the different channels through which demographic change affects growth? Concerns about the connection between the population and economic output have been voiced time and time again ever since Malthus (1798) first raised the issue at the dawn of the economics discipline.[1] Yet, despite over two hundred years of discussion, these questions are still subject to considerable debate.

It is well known that population ageing influences output per capita through two major channels (see for instance Samuelson, 1975). First, ageing reduces the size of the labour force relative to the population, thereby decreasing output per capita. Second, longer lifespans increase savings in anticipation of longer periods of retirement, which raises investment and the accumulation of physical capital. The subsequent capital deepening increases output per capita. In this paper, I emphasise a third key mechanism: savings may also be used to finance research and development (R&D), which improves per-capita output through innovations that generate technological progress. Distinguishing between R&D and capital investment is essential because of the nonrival nature of technology. An innovation can be used simultaneously by any number of workers at no additional cost and therefore raises everyone's productivity. An extra unit of capital on the other hand only improves productivity insofar as it increases capital per effective worker. Technological progress therefore makes for a more potent positive channel than capital deepening.

To make this argument, and to explore its importance for output per capita under current demographic trends, I construct a quantitative general equilibrium model by combining two macroeconomic workhorse models. On the supply side, it features endogenous growth as in the original Romer (1990) model, with monopolistically competitive intermediate producers and an R&D sector whose innovations improve productivity by expanding the variety of intermediate goods. The household side follows in the Auerbach and Kotlikoff (1987) tradition with a large number of overlapping generations and a realistic population structure. These households face income and survival risk and exhibit life-cycle behaviour over labour supply along both the intensive and the extensive margins as well as over savings. Household savings are either invested in physical capital at the risk-free rate or are used to finance R&D to earn subsequent dividends from monopoly profits, the latter creating the link between life-cycle savings and technological progress.

Within this set-up lies one of the main contributions of this paper. The underlying machinery is one of semi-endogenous growth, as in Jones (1995), Kortum (1997) and Segerstrom (1998). A feature of these models is that economic growth is proportional to population growth in the long run; see Jones (2005, 2022b) for overviews. Most semi-endogenous growth papers concerned with demographics therefore limit themselves to permanent changes in the overall population growth rate, either for a representative household or in a stylised perpetual-youth framework

[1] The last century contains numerous examples of influential economists chipping in on the topic, including Keynes (1937), Hansen (1939), Kuznets (1960) and, in the recent public debate, Gordon (2016) and Goodhart and Pradhan (2020).

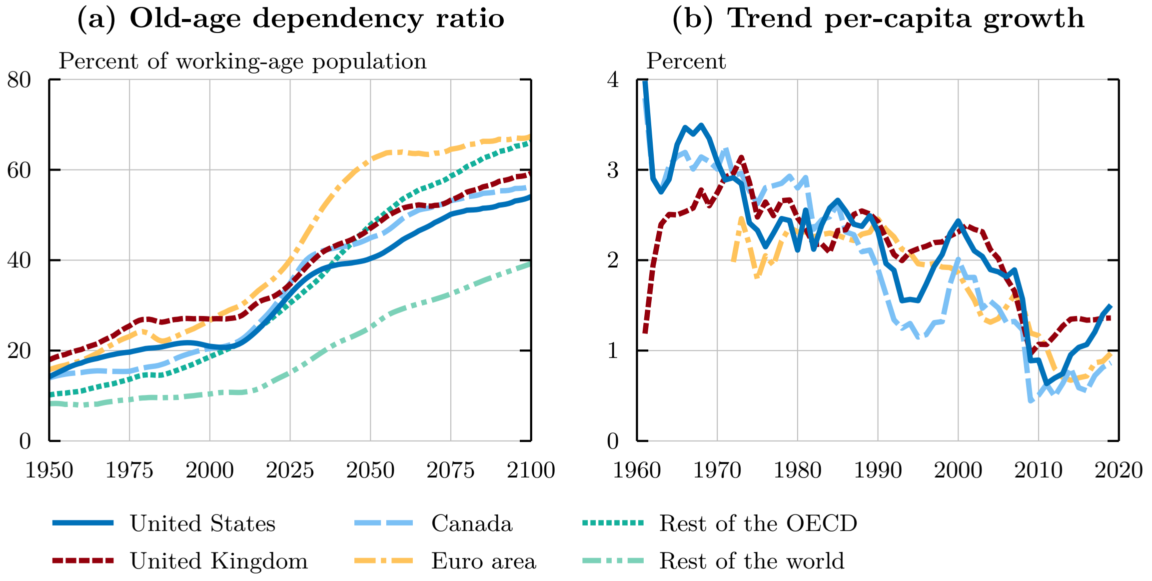


Figure 1. Ageing and growth across major economies.

Source. Holston, Laubach and Williams (2017) and United Nations (2019).

(examples include Prettner, 2013, Prettner and Trimborn, 2017, Peters and Walsh, 2021, and Jones, 2022a). By augmenting the semi-endogenous growth model with fully fledged life-cycle households, I obtain a growth framework that allows for richer analyses of demographic change which ultimately leaves the long-run trend unchanged. Such changes are important, because even temporary growth effects have permanent *level* effects on output per capita.

All three mechanisms emphasised above are present in this model; they are derived analytically in a simplified three-period version in Section 3. To quantify these effects and their net impact on output per capita, I calibrate the model and solve for the equilibrium path induced by the demographic transition between 1950 and 2100. This exercise focuses on the United States, mainly because the United States is a large economy at the technological frontier and serves as the world's innovation engine. It is therefore likely that the endogenous growth framework is a better approximation of US technological progress than of a small open economy in which most technology is imported. Additionally, and as shown in Figure 1a, the United States is a useful demographic benchmark since its ageing process lies roughly in between the fastest ageing rich countries and the younger non-OECD countries.

The quantitative analysis differs from previous work in that I keep the long-run trend fixed and treat the demographic transition and the corresponding growth effects as exclusively transitory. This choice is motivated by the fact that, for the United States, the observed and projected demographic shift since World War II is primarily characterised by two *transitory* changes: an initial, temporary increase in fertility (the baby boom), and a continual decrease in old-age mortality. Besides the baby boom, fertility remains stable from the mid-1930s onwards. Neither change impacts the population growth rate permanently if people have finite lifespans and therefore does not affect the economic long-run trends. By contrast, a standard semi-endogenous growth approach instead sees the decline in population growth over the last half century that results from this transition as a shift in trend. Such a long-run interpretation immediately leads to the conclusion that current demographic developments are detrimental to growth, irrespective of the population age structure or how households are modelled.

Contrary to what a standard long-run analysis suggests, I find that the demographic shift towards an older population is a net positive for output per capita. Between 1950 and 2000, the demographic transition boosts per-capita output by on average 0.35 percent per year. This corresponds to roughly 20 percent of observed US growth over the same period, thus making demographics comparable to the share of growth attributed to human capital accumulation (see Fernald and Jones, 2014). The positive effect dissipates around the turn of the millennium when the baby boom retires, but the growth impact does not turn negative on average during the twenty-first century. Yet, the shift when this happens is sizeable: the growth rate declines by 0.65 percentage points between 1995 and 2025. Demographics therefore account for a large chunk of the recent decline in observed growth rates. It is important to stress though that this decline is not because demographic change is inherently bad for the economy, but rather because of a reversion back from the above-normal growth that it induced in the first place. This contrasts with papers that see the current ageing of the population as a major drag on output per capita.[2]

At the core of these results lies the positive impact via technological progress. The contribution of technical change to output growth is two to three times larger than that of capital deepening, depending on the time period considered. This difference reflects the extra bang for the buck obtained from technology being nonrivalry. Together these two channels more than offset the negative impact of an increasing share of non-productive, retired households. Capital deepening alone cannot accomplish this: removing endogenous technology (which is readily achieved as a special case of the general model) eliminates the positive net effect and generates a decline in output per capita which is quantitatively similar to Krueger and Ludwig (2007), the closest antecedent to my analysis. This importance of technology echoes the empirical results of Cutler *et al.* (1990) and Acemoglu and Restrepo (2017, 2022), who argue that ageing leads to another type of technical change, automation.

A counterfactual simulation in which the baby boom never happens also reveals that the baby boom accounts for around half of the positive impact in the baseline calibration. In general, a temporary boom in fertility leaves the population age structure unchanged once it has passed. Therefore, the growth impact from such an event is only about scale: it generates a larger population which all else equal raises aggregate R&D investment and subsequently the stock of technology, and this benefits all people. Changes in the age structure explain the remaining half, in part due to the nonstationary population already present in 1950 and in part due to the increase in life expectancy over time. Overall, this counterfactual scenario as well as the baseline calibration leaves little reason to worry about future demographic change as far as its impact on output per capita is concerned.

The quantitative analysis constitutes the second main contribution of this paper. It fits within a vast macroeconomic literature that uses large-scale life-cycle models to analyse the impact of demographic change on a variety of topics such as fiscal policy, international capital flows, wealth

[2] Eggertsson, Lancastre and Summers (2019), Eggertsson, Mehrotra and Robbins (2019) and Jones (forthcoming) for instance tie the currently weak macroeconomic performance to secular stagnation and identifies demographic change as a leading cause.

accumulation, and asset returns.[3] Papers that touch upon the issue of growth consider either standard factor accumulation (for instance Krueger and Ludwig, 2007, and Cooley and Henriksen, 2018), human capital accumulation (Ludwig, Schelkle and Vogel, 2012, Vandenbroucke, 2021, and references therein), or labour-replacing automation technology (Heer and Irmen, 2014, and Benzell *et al.*, 2021). To the best of my knowledge, labour-augmenting technological change has yet to receive any attention within this literature.

More broadly, quantitative macroeconomic models with R&D-based growth are somewhat scarce and usually examine business cycles under a representative household; examples include Comin and Gertler (2006), Nuño (2011), Benigno and Fornaro (2018), Anzoategui *et al.* (2019) and Bianchi, Kung and Morales (2019). Two exceptions are Aksoy *et al.* (2019) and Basso and Jimeno (2021), who analyse ageing and growth using versions of Comin and Gertler's model. My paper differs from these in three respects. First, I provide a richer household description by incorporating a state-of-the-art life-cycle problem and a realistic population structure. By contrast, Aksoy *et al.* and Basso and Jimeno utilise a perpetual-youth framework as in Gertler (1999), which effectively boils down to a stylised two-generation model of workers and retirees. Second, I pinpoint analytically the key mechanisms at work. Lastly, I keep the long-run trend fixed whereas Aksoy *et al.* and Basso and Jimeno consider transitions from high-growth steady states to low-growth steady states. As emphasised above, this matters for interpretation: both find growth declines in the early twenty-first century that are quantitatively comparable to here and therefore conclude that ageing leads to lower growth, the opposite conclusion of what my results indicate.

The rest of the paper proceeds as follows. Section 2 outlines the quantitative model and defines its equilibrium. Section 3 simplifies the quantitative model and derives analytical results that highlight the key mechanisms. Sections 4 and 5 outline the computational details and the calibration. Section 6 shows the quantitative results and Section 7 presents sensitivity analyses of the baseline calibration. Section 8 concludes.

2 A quantitative OLG model with endogenous growth

The quantitative framework that I consider is a closed economy populated by overlapping generations of households, production firms, R&D firms, and a government. Time is discrete and a period amounts to one year. The production sector consists of a perfectly competitive final-good firm and monopolistically competitive intermediate-good firms. As in Romer (1990) and Jones (1995), technology grows endogenously from the creation of new intermediate-good firms via innovations in the R&D sector.[4] The main driving force of the model is changes in the demographic structure, which varies exogenously and is then imposed on rest of the model.[5]

[3] A far from exhaustive list of papers that fall into one or several of these areas include De Nardi, İmrohoroğlu and Sargent (1999), Storesletten (2000), Fehr, Jokisch and Kotlikoff (2004), Börsch-Supan, Ludwig and Winter (2006), Domeij and Flodén (2006b), Attanasio, Kitao and Violante (2007), Kotlikoff, Smetters and Walliser (2007), Krueger and Ludwig (2007), İmrohoroğlu and Kitao (2012), Kitao (2014), Carvalho, Ferrero and Nechio (2016), Auclert *et al.* (2021) and Gagnon, Johannsen and López-Salido (2021).

[4] The mechanisms emphasised in this paper are present in any standard endogenous growth model, so the choice between the expanding-variety model of Romer (1990) or a quality-ladder model á la Grossman and Helpman (1991) and Aghion and Howitt (1992) is inconsequential for my purposes.

[5] Clearly technical change reversely affects demographic variables (think of advances in birth control, treatment of diseases, and so on). For simplicity, I abstract from this possibility. This is also the recommendation of Lee

2.1 Demographics

In a given period t, the economy consists of $J+1$ overlapping generations of sizes $N_{0t}, \dots, N_{Jt}$, with total population $N_t = \sum_{j=0}^{J} N_{jt}$. The demographic process is pinned down by age- and time-specific fertility rates, survival rates and net migration rates. At age j, households give birth to f_{jt} children and survive to age $j+1$ in the next period with probability $s_{j+1,t+1}$, with $s_{J+1,t} = 0$ for all t. The net migration rate for age-j households is denoted by m_{jt} and I assume zero net migration for newborns. From an initial population distribution, the demographic structure in subsequent periods are given recursively by

$$N_{0,t+1} = \sum_{j=0}^{J} f_{jt} N_{jt} \qquad \text{and} \qquad N_{j+1,t+1} = \big(s_{j+1,t+1} + m_{j+1,t+1}\big) N_{jt}. \tag{1}$$

Migrants bring their accumulated wealth with them when they move and are economically identical to non-migrants in all respects. This assumption eliminates the need to separate between migrants and non-migrants in the economic model.

2.2 Households

A household consists of a single individual who starts their economic life at age ι with zero assets, are endowed with one unit of time in each period, and live up to a maximum age J. In addition to age, households are heterogeneous with respect to a stochastic labour productivity, with individual realisations denoted by η. This stochastic process follows a time-invariant Markov chain over a state space H with transition kernel $\Pi(\eta, \mathcal{H})$ for each relevant Borel set $\mathcal{H}$. Productivity in the initial age ι is distributed according to the unique invariant distribution Γ associated with Π.

Preferences are defined over consumption c and hours worked h according to a standard time-separable utility function

$$\mathbb{E}\left[\sum_{j=\iota}^{J} \beta^{j-\iota} \left(\prod_{k=\iota+1}^{j} s_k\right) u\big(c_j, h_j\big)\right], \tag{2}$$

where β is the subjective discount factor and expectations are taken over the idiosyncratic labour productivity. (In (2) and in the remainder of this subsection, I leave the time indices t implicit to economise on notation.) The flow utility function takes the form

$$u(c,h) = \frac{c^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h^{1+1/\theta}}{1+1/\theta}, \tag{3}$$

where σ is the inverse of the intertemporal elasticity of substitution for consumption and θ is the Frisch elasticity of labour supply. This functional form belongs to the class of balanced-growth preferences characterised by Boppart and Krusell (2020) that allows for falling hours worked in the long run. Specifically, the income effect of higher wages on leisure dominates the substitution

(2016), who states "While theories are available to relate [fertility, mortality, and health] to individual choices, they have little predictive power and their use might obscure the workings of some better understood mechanisms."

effect whenever $\sigma > 1$, leading to declining hours worked whenever wage growth is positive.

Households supply an amount of efficiency units to the labour market which is the product of three factors: hours worked h, a deterministic age-dependent productivity ε_j, and the idiosyncratic productivity η. Total labour supply at age j is therefore $\ell_j = \varepsilon_j \eta h_j$. As in for instance Aiyagari (1994), the idiosyncratic productivity shocks are not insurable but workers can self-insure by trading a risk-free asset a subject to a strict borrowing constraint. Annuity markets insuring against mortality risk are also absent. Instead assets of households who die prematurely are confiscated by the government and redistributed as lump-sum transfers tr to surviving households. At age j, households are thus faced with a flow budget of the form

$$a_{j+1} + (1+\tau^c)c_j = \big(1 + r(1-\tau^k)\big)a_j + \big(1 - \tau^w(w\ell_j) - \tau^b\big)w\ell_j + tr + b_j(R_j), \tag{4}$$

where r is the rate of return on savings, w is the wage rate, and τ^c, τ^k, τ^w, τ^b denote tax rates where, in particular, τ^w is allowed to vary with labour income. Retired households receive a pension benefit $b_j(R_j)$ which depends on their chosen age of retirement R_j; no pension is paid out before retirement.

All households begin their lives in the labour force and choose consumption, hours worked and the age of retirement following a two-stage process. At the beginning of each period, individuals observe their wealth, idiosyncratic productivity and previous-period retirement status and subsequently make a retirement decision. Consumption and hours are then chosen in a second stage. Retirement is an absorbing state, so the retirement and labour supply choices once retired are trivial and the problem then reduces to a standard consumption-savings choice.

Formally, let the retirement decision be captured by a discrete variable d equal to 1 if working and 0 if choosing to retire. We may think of the retirement age R_j as evolving according to $R_j = R_{j-1} + d_j$ with $R_{\iota-1} = \iota$. With this formulation, working households have $R_j = j + 1$ such that R_j represents their earliest possible retirement age in subsequent ages. Also denote the state vectors before and after the retirement decision by $x'_j = (a_j, \eta, R_{j-1})$ and $x_j = (a_j, \eta, R_j)$, respectively, with corresponding pre- and post-decision value functions $V_j(x'_j)$ and $v_j(x_j)$. The optimal retirement policy is then a function $d_j(x'_j)$ that solves the first-stage problem

$$V_j(x'_j) = \max_{d_j \in D(R_{j-1})} \big\{ v_j(x_j) \big\} \tag{5}$$

subject to $R_j = R_{j-1} + d_j$, with $R_{\iota-1} = \iota$, and

$$D(R_{j-1}) = \begin{cases} \{0,1\} & \text{if } R_{j-1} = j, \\ \{0\} & \text{if } R_{j-1} < j. \end{cases}$$

Optimal policies for consumption, savings and hours worked are functions $c_j(x_j)$, $a_j(x_j)$ and $h_j(x_j)$ that solve the second-stage problem

$$v_j(x_j) = \max_{c_j, h_j} \Big\{ u(c_j, h_j) + \beta s_{j+1} \mathbb{E}\big[V_{j+1}(x'_{j+1}) \bigm| \eta \big] \Big\} \tag{6}$$

subject to the budget constraint (4), the time constraints $h_j \in [0,1]$ if working and $h_j = 0$ if retired, and the borrowing constraint $a_{j+1} \geq 0$.

2.3 Production

There are two production sectors: a perfectly competitive final-goods sector and a monopolistically competitive intermediate-goods sector. The final-goods sector produces a homogenous consumption good Y_t using labour L_t and a CES composite of a continuum of intermediate capital goods k_{it}, $i \in [0, z_t]$, according to the Cobb-Douglas production function

$$Y_t = L_t^{1-\alpha} \left(\int_0^{z_t} k_{it}^{\omega} \, di \right)^{\frac{\alpha}{\omega}}, \qquad 0 < \alpha, \omega < 1. \tag{7}$$

Profit maximisation under perfect competition implies that the wage rate w_t and the price p_{jt} of intermediate j are given by

$$w_t = (1-\alpha) \frac{Y_t}{L_t} \qquad \text{and} \qquad p_{jt} = \alpha \frac{Y_t}{k_{jt}} \frac{k_{jt}^{\omega}}{\int_0^{z_t} k_{it}^{\omega} \, di}. \tag{8}$$

Intermediate firms face a linear production function that converts one unit of capital into one unit of intermediate good. Capital is rented from households at rate $r_t + \delta$, where δ is the capital depreciation rate. Each firm in the intermediate sector has purchased a patent for their particular variety from the R&D sector and subsequently acts as a monopolist. Conditional on having obtained a patent, these firms therefore maximise operating profits $\pi_{it} = (p_{it} - r_t - \delta)k_{it}$ subject to the intermediate-good demand function in (8). Symmetry across firms implies that all firms charge the same price p_t and, subsequently, sell the same quantity k_t and earn the same profits π_t. The resulting monopoly price is a standard mark-up over marginal cost, $p_t = \frac{1}{\omega}(r_t + \delta)$, with corresponding operating profits $\pi_t = (1-\omega)p_t k_t$.

2.4 R&D sector

The perfectly competitive R&D sector develops designs for new intermediate goods used in the production of the final good. An R&D firm that develops a new design in period t sells the patent for that design to an intermediate-good firm at the end of that period for a one-off price $P_{z,t+1}$, who then converts it into usable input in period $t+1$. Innovation is conducted using an amount Q_t of final output, which R&D firms obtain by borrowing from households, with a production function as in Jones (1995):

$$F(Q_t) = \bar{\nu}_t Q_t = \nu Q_t^{\lambda} z_t^{\phi}, \qquad 0 < \lambda \leq 1, \ \phi < 1, \tag{9}$$

where z_t is the measure of intermediate firms at time t. The productivity term $\bar{\nu}_t \equiv \nu Q_t^{\lambda-1} z_t^{\phi}$ captures duplication externalities (via λ) and knowledge spillovers (via ϕ) in the R&D process. While these affect the aggregate development, individual firms take $\bar{\nu}_t$ as given and maximise profits $(P_{z,t+1}\bar{\nu}_t - 1)Q_t$. Free entry into R&D then implies the zero-profit condition

$$Q_t = P_{z,t+1} \cdot \nu Q_t^{\lambda} z_t^{\phi}. \tag{10}$$

As in Comin and Gertler (2006) and subsequent papers, an intermediate firm is not infinitely lived; in each period a share δ_z of intermediate firms become obsolete.[6] The aggregate law of motion of new intermediates is therefore $z_{t+1} = (1-\delta_z)z_t + F(Q_t)$, thus implying a gross growth rate of intermediary firms given by

$$1 + g_{zt} = 1 - \delta_z + \nu Q_t^{\lambda} z_t^{\phi-1}. \tag{11}$$

Lastly, a prospective intermediate firm enters the market only if it is profitable to do so. That is, the firm enters if the sum of expected discounted flow profits π_t exceeds the fixed cost P_z of purchasing a patent. Free entry into the intermediate-good sector drives the profitability of entry to zero. This is equivalent to saying that the following no-arbitrage condition holds in equilibrium:

$$r_t = \frac{\pi_t + \Delta P_{zt} - \delta_z P_{z,t+1}}{P_{zt}}, \tag{12}$$

where $\Delta P_{zt} \equiv P_{z,t+1} - P_{zt}$ is the change in the patent price in period t.

2.5 Public sector

The public sector engages in three activities: (i) it redistributes assets from deceased individuals, (ii) collects taxes on consumption, capital gains and wages via the tax rates τ_t^c, τ_t^k and τ_t^w to finance public consumption G_t, and (iii) maintains a pay-as-you-go social security system. The pension system is financed by the contribution rate τ_t^b on labour earnings. The budget constraint of each public-sector activity is independent of the other two and always balances. In the baseline model, public consumption G_t and the social security contribution rate τ_t^b adjust endogenously to ensure budget balance for the latter two activities.

Pension benefits are independent of earnings history but depend on the age of retirement.[7] Specifically, a household with retirement age R receives a base level benefit that is scaled by a factor $\mu(R)$ relative to some normal retirement age R^{norm} to capture early retirement penalties and delayed retirement credits. The base level benefit is a fraction ζ of average gross labour income $w_t\bar{\ell}_t$, where $\bar{\ell}_t$ is the average number of efficiency units per worker. An age-j household therefore receives a pension transfer

$$b_{jt}(R) = \begin{cases} \mu(R)\,\zeta w_t \bar{\ell}_t & \text{if } j \geq \max\{R^{min}, R\}, \\ 0 & \text{if } j < \max\{R^{min}, R\}, \end{cases} \tag{13}$$

where R^{min} is the earliest age where households are allowed to start collecting pension.

2.6 Definition of competitive and stationary equilibria

Since the state x of a household is determined by its asset wealth, idiosyncratic productivity and retirement status, the household state space is given by $X = \mathbb{R}_+ \times H \times \{\iota, \ldots, J+1\}$. The

[6] This feature is primarily technical to ensure non-zero steady-state R&D investment. We can also set $\delta_z < 0$ to generate exogenous growth, as in the standard neoclassical model, in addition to that created through R&D.

[7] Earnings-dependent pension would introduce another continuous state variable in the household problem and I abstract from this feature to avoid the additional computational complexity that it entails. Similar simplifications are used in several papers, including Krueger and Ludwig (2007) and Heer and Irmen (2014).

aggregate state of the economy is pinned down by the aggregate stock of capital K_t, the measure of intermediary firms z_t, the patent price P_{zt} and probability spaces $(X, \mathcal{B}(X), \Phi_{jt})$, where $\mathcal{B}(X)$ is the Borel σ-algebra on X and $\Phi_{jt} : \mathcal{B}(X) \to [0,1]$ is a probability measure capturing the distribution of age-j households across individual states in period t. We then have:

Definition 1 (Competitive equilibrium)**.** Given a demographic evolution $\{s_{jt}, m_{jt}, N_{jt}\}_{t=0}^{\infty}$ for all ages j and initial conditions K_0, z_0, $P_{z,0}$, $\{\Phi_{j0}\}_{j=\iota}^{J}$, a competitive equilibrium consists of household decision rules $\{d_{jt}(\cdot), c_{jt}(\cdot), a_{jt}(\cdot), h_{jt}(\cdot)\}_{t=0}^{\infty}$, transfers $\{tr_t, b_{jt}(R)\}_{t=0}^{\infty}$ and measures $\{\Phi_{jt}\}_{t=0}^{\infty}$ for all j; factor payments $\{r_t, w_t, \pi_t\}_{t=0}^{\infty}$ and production allocations $\{K_t, L_t\}_{t=0}^{\infty}$; measures of intermediate firms, patent prices and R&D investment $\{z_t, P_{zt}, Q_t\}_{t=0}^{\infty}$; tax rates $\{\tau_t^c, \tau_t^k, \tau_t^b, \tau_t^w(\cdot)\}_{t=0}^{\infty}$ and public consumption $\{G\}_{t=0}^{\infty}$; and aggregates $\{Y_t, C_t, A_t, A_t^M\}_{t=0}^{\infty}$ of output, consumption, household wealth and migrant asset flows, such that:

(i) Household decision rules $d_{jt}(x')$, $c_{jt}(x)$, $a_{jt}(x)$, $h_{jt}(x)$ solve problems (5) and (6).

(ii) Profit-maximising behaviour of final- and intermediate-good firms gives rise to a consolidated production function of the Cobb-Douglas form

$$Y_t = K_t^{\alpha}(Z_t L_t)^{1-\alpha}, \qquad \text{where} \qquad Z_t \equiv z_t^{\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}, \tag{14}$$

with corresponding factor prices and profits

$$r_t = \alpha\omega\frac{Y_t}{K_t} - \delta, \qquad w_t = (1-\alpha)\frac{Y_t}{L_t}, \qquad \pi_t = \alpha(1-\omega)\frac{Y_t}{z_t}. \tag{15}$$

(iii) The measure of intermediary firms z_t, the patent price P_{zt} and R&D investment Q_t satisfy Equations (10) to (12).

(iv) The public sector and social security budgets balance:

$$G_t = \tau_t^c C_t + \tau_t^k r_t A_t + \sum_{j=\iota}^{J} N_{jt} \int_X \tau^w\big(w_t \ell_{jt}(x)\big) w_t \ell_{jt}(x)\, \mathrm{d}\Phi_{jt}, \tag{16}$$

$$\tau_t^b w_t L_t = \sum_{j=\iota}^{J} N_{jt} \int_X b_{jt}(R)\, \mathrm{d}\Phi_{jt}. \tag{17}$$

(v) Bequests are given by

$$tr_{t+1} = \frac{1}{\sum_{j=\iota}^{J} N_{j,t+1}} \left[\Big(1 + r_{t+1}(1-\tau_{t+1}^k)\Big) \sum_{j=\iota}^{J} N_{jt}(1 - s_{j+1,t+1}) \int_X a_{jt}(x)\, \mathrm{d}\Phi_{jt} \right]. \tag{18}$$

(vi) Aggregates of consumption C_t, household wealth A_t and migrant asset flow A_t^M equal the sum of individual variables:

$$C_t = \sum_{j=\iota}^{J} N_{jt} \int_X c_{jt}(x)\, \mathrm{d}\Phi_{jt}, \tag{19}$$

$$A_{t+1} = \sum_{j=\iota}^{J} N_{jt} \int_X a_{jt}(x)\, \mathrm{d}\Phi_{jt} + A^M_{t+1}, \tag{20}$$

$$A^M_{t+1} = \sum_{j=\iota}^{J} m_{j+1,t+1}\, N_{jt} \int_X a_{jt}(x)\, \mathrm{d}\Phi_{jt}. \tag{21}$$

(vii) Markets for labour, assets and goods clear:

$$L_t = \sum_{j=\iota}^{J} N_{jt} \int_X \ell_{jt}(x)\, \mathrm{d}\Phi_{jt}, \tag{22}$$

$$A_t = K_t + P_{zt} z_t, \tag{23}$$

$$Y_t + A^M_{t+1} = C_t + G_t + I_t + Q_t, \tag{24}$$

where capital investment I_t satisfies the usual law of motion $K_{t+1} = (1-\delta)K_t + I_t$.

(viii) For all Borel sets $S = \mathcal{A} \times \mathcal{H} \times \mathcal{R} \in \mathcal{B}(X)$, the distributions Φ_{jt} evolve according to

$$\Phi_{j+1,t+1}(S) = \int_X P_{jt}(x, S)\, \mathrm{d}\Phi_{jt} \qquad \text{for } j = \iota, \dots, J-1, \tag{25}$$

where, for each relevant next-period, pre-retirement decision state $x' = (a_j(x), \eta', R)$ (and dropping time subscripts for simplicity), the transition function $P_j(\cdot)$ is given by

$$P_j(x, S) = \begin{cases} \int_{\mathcal{H}} d_{j+1}(x')\, \Pi(\eta, \mathrm{d}\eta') & \text{if } a_j(x) \in \mathcal{A},\ R \notin \mathcal{R},\ R+1 \in \mathcal{R}, \\ \int_{\mathcal{H}} \big(1 - d_{j+1}(x')\big)\, \Pi(\eta, \mathrm{d}\eta') & \text{if } a_j(x) \in \mathcal{A},\ R \in \mathcal{R},\ R+1 \notin \mathcal{R}, \\ \Pi(\eta, \mathcal{H}) & \text{if } a_j(x) \in \mathcal{A},\ R, R+1 \in \mathcal{R}, \\ 0 & \text{otherwise}, \end{cases} \tag{26}$$

and, for ι-year-olds,

$$\Phi_\iota(S) = \begin{cases} \int_{\mathcal{H}} d_\iota(0, \eta, \iota)\, \mathrm{d}\Gamma & \text{if } 0 \in \mathcal{A},\ \iota \notin \mathcal{R},\ \iota+1 \in \mathcal{R}, \\ \int_{\mathcal{H}} \big(1 - d_\iota(0, \eta, \iota)\big)\, \mathrm{d}\Gamma & \text{if } 0 \in \mathcal{A},\ \iota \in \mathcal{R},\ \iota+1 \notin \mathcal{R}, \\ \Gamma(\mathcal{H}) & \text{if } 0 \in \mathcal{A},\ \iota, \iota+1 \in \mathcal{R}, \\ 0 & \text{otherwise.} \end{cases} \tag{27}$$

◁

As in Romer (1990), the competitive equilibrium features endogenous growth in total factor productivity (TFP), here denoted by Z_t, through changes in the measure of intermediate firms

z_t. Yet, as in Jones (1995) the rate of TFP growth is exogenously determined by the rate of population growth along a balanced growth path:

Definition 2 (Stationary equilibrium)**.** A stationary equilibrium, or steady state, is a competitive equilibrium in which all variables grow at constant rates (possibly zero) and all growth rates are determined by the population growth rate n. In particular, the growth rate of new intermediary firms is

$$1 + g_z = (1+n)^{\chi}, \qquad \text{where} \qquad \chi \equiv \frac{\lambda(1+\theta\sigma)}{(1-\phi)(1+\theta\sigma) - \lambda(1+\theta)\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}, \tag{28}$$

and the growth rates of TFP, output per capita, and hours per worker are, respectively,

$$1 + g_Z = (1+n)^{\gamma_Z}, \qquad 1 + g_y = (1+n)^{\gamma_y}, \qquad 1 + g_h = (1+n)^{\gamma_h},$$

where $\gamma_Z \equiv \chi \frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}$, $\gamma_y \equiv \frac{1+\theta}{1+\theta\sigma}\gamma_Z$ and $\gamma_h \equiv \frac{\theta(1-\sigma)}{1+\theta\sigma}\gamma_Z = \gamma_y - \gamma_Z$. ◁

The growth rates in Definition 2 are derived in Appendix B. Existence of a stationary equilibrium generally requires the parameters in Equation (28) to satisfy $\chi > 0$, and I impose this restriction in what follows.

3 Identifying the mechanisms: Results from a simple model

The observed and projected demographic transition from 1950 and onwards is characterised by two salient features: the temporary surge in fertility between the mid-1940s and mid-1960s that caused the baby boom, and an ageing of the population from rising old-age survival rates. With finite lifespans, neither impacts the long-run population growth rate, so the growth effects from these changes are purely transitional.[8] To understand the main mechanisms through which these transitory changes affect per-capita output, it is instructive to first analyse a simplified version of the model in Section 2. To that end, consider a Solow-like world without a public sector and in which aggregate household behaviour is captured by an exogenous savings rate sr and a labour supply which grows by the rate of the population.[9] If capital fully depreciates after one period ($\delta = 1$) and intermediary firms only survive for one period ($\delta_z = 1$), it is possible to solve for a stationary equilibrium of this economy in closed form. Table 1 summarises the model under these restrictions.

As a start, it turns out convenient to decompose output per capita y_t using the Cobb-Douglas production function (P1) into the TFP level Z_t, the capital intensity as captured by the capital-output ratio K_t/Y_t, and the employment rate L_t/N_t:

$$y_t \equiv \frac{Y_t}{N_t} = Z_t \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_t}{N_t}. \tag{29}$$

[8] This need not hold in models of perpetual youth, in which an increase in the survival rate permanently increases the population growth rate, thus increasing long-run growth (see Prettner, 2013, for such an analysis).

[9] Krueger and Ludwig (2007) include a public sector in a similar analysis and highlight that general equilibrium effects from tax adjustments may dampen or even reverse any direct effects from demographic change (see also Fehr, Jokisch and Kotlikoff, 2004, for a quantitative case of the latter). Their findings also apply here.

TABLE 1. Summary of the simple model.

OUTPUT	$Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha}\,,$	$Z_t \equiv z_t^{\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}$	(P1)
PROFITS	$\pi_t = \alpha(1-\omega)\,Y_t\,/\,z_t$		(P2)
INTEREST RATE	$1+r = \alpha\omega\,Y_t\,/\,K_t$		(P3)
WAGE	$w_t = (1-\alpha)\,Y_t\,/\,L_t$		(P4)
R&D OUTPUT	$z_{t+1} = \nu Q_t^\lambda z_t^\phi$		(RD1)
R&D ZERO PROFITS	$Q_t = P_{z,t+1} z_{t+1}$		(RD2)
NO ARBITRAGE	$P_{zt} = \pi_t\,/\,(1+r_t)$		(RD3)
ASSET MARKET	$sr\,Y_t = K_{t+1} + P_{z,t+1} z_{t+1}$		(M1)
POPULATION GROWTH	$L_{t+1}\,/\,L_t = 1+n$		(G1)
STATIONARY GROWTH	$1+g_z = (1+n)^\chi\,,$	$\chi \equiv \dfrac{\lambda}{1-\phi-\lambda\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}$	(G2)

Notes. Supply side and R&D sector as in Definitions 1 and 2 with $\delta = \delta_z = 1$. Inelastic labour supply implies that either $\sigma \to 1$ or $\theta \to 0$, which gives the definition of χ in (G2).

The effect on output per capita from changes in the demographic structure depends on the impact on each of these decomposing factors. In a stationary equilibrium, tedious but straightforward algebra using Table 1 allows us to write the capital-output ratio as

$$\frac{K_t}{Y_t} = \frac{\omega\,sr}{(1+n)^{\frac{\chi(1-\phi)}{\lambda}}}, \tag{30}$$

which shows that the capital-output ratio is directly proportional to the savings rate, just as in the Solow model. Likewise, the TFP level is given by

$$Z_t = \left[\nu^{\frac{1}{\lambda}}\,\frac{1-\omega}{\omega}\left(\frac{\omega\,sr}{(1+n)^{\frac{\chi(1-\alpha\phi)}{\lambda}}}\right)^{\frac{1}{1-\alpha}}\right]^{\gamma_Z} L_t^{\gamma_Z}. \tag{31}$$

Equation (31) writes TFP as the product of two factors. The first is an *R&D intensity*, which is determined by the savings rate and thus reflects household behaviour and composition. The second, $L_t^{\gamma_Z}$, is the *scale effect* inherent in all semi-endogenous growth models (see Jones, 2005, for a discussion). This effect appears because an increase in the population size raises aggregate investment, even without changes in household behaviour, and this generates larger stocks of capital and TFP. While capital accumulation only improves per-capita output if it also increases the capital intensity, this is not the case for TFP. This reflects the fact that technology, unlike capital, is a nonrival good, meaning that when the R&D process leads to a new discovery, it can be used by *everyone* over and over again at no additional cost. What matters for output per capita then is the stock of TFP rather than TFP per capita, as seen in Equation (29). Output per person therefore rises via TFP simply by virtue of a larger population.

At this point, let us endogenise the two remaining key elements: the employment rate and the savings rate. Consider a three-generation model consisting of young, middle-aged, and old households (indexed by $j = 1, 2, 3$), in which young and middle-aged households supply labour inelastically and old households are retired. If in each period the number of young households grows by rate $1 + n$ and a j-year-old survives till age $j + 1$ with probability s_{j+1}, then the employment rate in a stationary equilibrium is pinned down by

$$\frac{L_t}{N_t} = \frac{N_{1t} + N_{2t}}{N_{1t} + N_{2t} + N_{3t}} = \frac{1 + \frac{s_2}{1+n}}{1 + \frac{s_2}{1+n}\left(1 + \frac{s_3}{1+n}\right)}. \tag{32}$$

To keep things simple, suppose households have logarithmic preferences over consumption, that only middle-aged households are able to save for retirement, and that annuity markets are present. Households then maximise lifetime utility $\log(c_1) + s_2\beta \log(c_2) + s_2 s_3 \beta^2 \log(c_3)$ subject to the budget constraints $c_1 = w$, $c_2 + a = w$, and $c_3 = \frac{1+r}{s_3} a$. This is a special case of the household side in Section 2 and the solution to the household problem is characterised by the savings policy $a = \frac{s_3\beta}{1+s_3\beta} w$. With wages given by (P4), the aggregate savings rate in a stationary equilibrium can then be written as

$$sr = \frac{a_{t+1} N_{2t}}{Y_t} = (1 - \alpha) \frac{s_3\beta}{1 + s_3\beta} \frac{\frac{s_2}{1+n}}{1 + \frac{s_2}{1+n}}, \tag{33}$$

which is a product of the labour share, the share of labour income allocated to savings, and the share of saving, middle-aged workers in the labour force.

3.1 Population ageing via increases in survival rates

Consider now an ageing of the population through an increase in either of the survival rates s_2 and s_3. This naturally pushes down the employment rate through a mechanical increase in the old-age dependency ratio, as seen in Equation (32). This reduces output per capita by (29). Meanwhile, Equations (30), (31) and (33) highlight a counteracting force that improves output per capita: higher survival rates raises the savings rate, which in turn leads to larger capital and R&D intensities, the latter of which improves TFP. Additionally, an increase in the middle-age survival rate s_2 also raises the size of the labour force relative to its previous trend, thereby increasing TFP further through the scale effect present in (31).

The intuition for the effects on the capital and R&D intensities is straightforward. A higher likelihood of middle-age survival raises the share of households that save while a higher likelihood of old-age survival increases the life-cycle savings motive, so households save more intensively. Because capital and intermediate firms only last for one period, and because there is no arbitrage in the asset market, households allocate savings proportionately to capital and R&D investment according to their factor shares of output.[10] The capital stock, however, increases disproportionately more than TFP because the production of capital is linear in investments whereas TFP improvements through R&D features decreasing returns to scale.[11] Therefore,

[10] To see this, substitute the profits and interest rate equations (P2) and (P3) into the no-arbitrage condition (RD3) and rearrange terms to get $P_{zt} z_t = \frac{1-\omega}{\omega} K_t$. From the asset market condition (M1), we then obtain $K_{t+1} = \omega sr Y_t$ and $Q_t = (1 - \omega) sr Y_t$.

[11] By definition of Z_t, the R&D production function (RD1) can be written $Z_{t+1} = \nu^{\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}} Q_t^{\lambda \frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}} Z_t^{\phi}$ and

Table 2. Comparative statics summary of the simple model.

Change	Duration	Output per capita	Technology: R&D intensity	Technology: Scale	Capital intensity	Employment rate
Mid-age mortality: $s_2 \uparrow$	Permanent	Ambiguous	$+$	$+$	$+$	$-$
Old-age mortality: $s_3 \uparrow$	Permanent	Ambiguous	$+$		$+$	$-$
Fertility: $1 + n \uparrow$	Temporary	$+$		$+$		

both K_t/Y_t and Z_t rise.

Although this discussion determines the sign of the effects on K_t/Y_t and Z_t, the model remains ambiguous about their relative importance with respect to output growth. Take for instance an increase in the old-age survival rate s_3. If we denote the savings rates under the old and new survival rates by sr^* and sr^{**}, then the capital intensity contribution to cumulative transitional growth is $(sr^{**}/sr^*)^{\frac{\alpha}{1-\alpha}}$ by the decomposition (29) and the capital-output ratio (30). By Equation (31), the corresponding contribution from TFP is $(sr^{**}/sr^*)^{\frac{\chi}{1-\alpha}\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}$. Equation (33) implies that $sr^{**}/sr^* > 1$ if the old-age survival rate increases, so comparing exponents yields that the former is dominating the latter if $\frac{\chi}{1-\alpha}\frac{1-\omega}{\omega} < 1$, and vice versa.

3.2 A baby boom through a temporary increase in fertility rates

Next consider a temporary increase in the population growth rate $1 + n$, similar to a baby boom. Clearly, this leaves the age structure unchanged in the long run. Thus, neither the employment rate nor the savings rate (and subsequently the capital and R&D intensities) is affected, as shown by Equations (32) and (33). The only long-run impact is through the scale effect in Equation (31): a temporary fertility boom permanently raises the population size, which leads to higher TFP and subsequently higher output per capita. This is also exactly what a standard semi-endogenous growth model with a representative household predicts.

Equations (30) to (33) nevertheless provide some guidance about the transition dynamics for this scenario beyond the scale effect. The savings rate is low when the share of middle-aged households is low, and the employment rate is high when the share of old households is low. The initial boost in the share of young households therefore generates positive growth via the employment rate and reduced growth via the capital and R&D intensities. As this generation reaches middle age and their children enter the labour force, the positive impact on the employment rate remains at the same time as the savings rate recovers, thus reverting the capital and R&D intensities back towards their starting points. When this generation finally reaches retirement, the effect on the employment rate also reverts back. These predictions will be useful to understand the quantitative results.

3.3 Taking stock

The steady-state predictions are summarised in Table 2. This section highlights that the demographic transition deteriorates the employment rate through the old-age dependency ratio,

the exponents on Q_t and Z_t sum to less than 1: the necessary parameter restriction for a stationary equilibrium to exist is $\chi > 0$, which under log preferences holds for feasible values of α, ω, λ and ϕ if and only if $\lambda\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega} + \phi < 1$.

improves the capital intensity via the savings rate, and generates technical change through both the savings rate and the scale effect associated with the nonrivalry of technology. However, the relative strengths of these channels, as well as their net effect on transitional growth, are ambiguous. This motivates the need for a quantitative investigation.

4 Numerical experiment and implementation

The quantitative exercise I consider is an "MIT shock".[12] An initial steady state with a stable population is imposed in the year 1900. In the beginning of 1901, the economy is surprised by the transition driven by exogenous demographic change. After this initial shock, there is perfect foresight of aggregate variables until the new steady state, which is assumed (and verified) to be reached by 2600. The main period of interest is 1950 to 2100, and the choice of an initial steady state already in 1900 is made in order to reduce the impact of the steady-state assumption on these years of the transition. We are interested in the transitional growth effects, so I impose the same population growth rate in the initial and final steady states. This ensures that the long-run growth trend does not change, so all growth in excess of the trend can be attributed to transitional factors.

As is standard, the model is solved numerically by computing initial and final steady states and then by iterating over the entire transition path in between these two periods. The equilibrium algorithm follows the usual procedure, whereby we guess a set of variables and, given this guess, solve the household problem, compute the household distributions, obtain the implied aggregate variables, and verify the equilibrium conditions in Definition 1. I solve the household problem using the discrete-continuous endogenous grid method of Iskhakov *et al.* (2017), which generalises Carroll's (2006) method to allow for discrete choices, and approximate the household distributions by histograms over wealth, idiosyncratic productivity, and retirement status. The set of guessed variables along the transition consists of interest rates, pension contribution rates, average labour supplies, bequests, and intermediary-firm growth rates: $\{r_t, \tau_t^b, \bar{\ell}_t, tr_t, g_{zt}\}_{t=1901}^{2599}$. I update these guesses using Ludwig's (2007) modified quasi-Newton algorithm, in which the final steady-state Jacobian for the equilibrium conditions corresponding to $\{r_t, \tau_t^b, \bar{\ell}_t, tr_t, g_{zt}\}$ is used to approximate the initial Jacobian for the transition algorithm.

5 Calibration

The model is calibrated to match the US economy and stays as close as possible to standard parameter values used in the literature. The resulting choices are summarised in Table 3 and are discussed in detail below.

5.1 Demographics

The demographic process requires survival rates, migration rates, and fertility rates. I construct these from estimates covering the period 1900 to 2100 for the survival rates and 1900 to 2060 for

[12] As explained by Boppart, Krusell and Mitman (2018), the term "MIT shock" originates from Tom Sargent and refers to a shock imposed on a deterministic steady-state equilibrium in order to analyse the subsequent transition dynamics along a perfect-foresight path. That is, in an economy without shocks a shock nevertheless occurs only to never happen again. (The "MIT" modifier refers to the fact that this method was prevalent at MIT at the time.)

Table 3. Calibrated parameters of the baseline model.

Parameter	**Explanation**	**Value**	**Target/source**
Demographics and households			
$\widetilde{n}$	Long-run population growth rate	0.0018	Final steady state
ι	Initial adult age	20	Children between 0–19
J	Maximum age	99	Certain death at 100
β	Discount factor	1.016	Capital/output = 2.8
ψ	Leisure weight	14.080	50 hours/week in 1900
σ	Inverse IES	1.75	Boppart and Krusell (2020)
θ	Frisch elasticity	0.5	Chetty *et al.* (2011)[a]
Individual productivity			
$\{\varepsilon_j\}_{j=\iota}^{J}$	Deterministic productivity	Fig. 3a	PSID
ρ	Persistence shock	0.97	Heathcote *et al.* (2010)
σ_ϵ^2	Variance shock	0.02	Heathcote *et al.* (2010)
Production			
δ	Capital depreciation rate	0.046	Investment/output = 0.136
α	Intermediate goods share	0.36	Labour share = 0.64
ω	Markup/EoS intermediates	0.7143	Profit share = 0.10
R&D			
ν	R&D productivity	0.009	$z = Z = 1$ in initial period
δ_z	Firm obsolescence rate	0.005	R&D/output = 0.014
λ	Duplication externality	0.75	Comin and Gertler (2006)[b]
ϕ	Knowledge spillovers	0.117	$g_Q = 6.73\% \implies g_Z = 1.26\%$
Social security			
ζ	Replacement rate	0.413	Clingman *et al.* (2021)
R^{norm}	Normal retirement age	65	Social Security Administration
R^{min}	Lowest retirement age	62	Social Security Administration
$\mu(R_j)$	Early/delayed scaling	0.8–1.15	Social Security Administration
Taxes			
τ^c	Consumption tax rate	0.080	BEA national accounts
τ^k	Capital gains tax rate	0.368	BEA national accounts
$\tau^w(w\bar{\ell})$	Income tax rate at mean income	0.115	BEA national accounts
κ_0	Asymptotic income tax rate	0.6290	OECD tax database
κ_1	Income tax progressivity	0.5736	OECD tax database
κ_2	Income tax scale parameter	0.5044	OECD tax database
κ_3	Income tax rate at zero income	−0.2053	OECD tax database

[a] Also Domeij and Flodén (2006a).
[b] Also Jones and Williams (2000).

the other two. In subsequent periods, I fix each variable at its last year of observation.

I collect survival probabilities by age and year between 1933 and 2019 from the *Human Mortality Database*. Remaining periods are constructed by interpolation using decennial life tables from Bell and Miller (2005) prior to 1933 and quinquennial life tables from the United Nations (2019) after 2019. Birth rates by age and year between 1917 and 2009 are collected from Heuser (1976) and Hamilton and Cosgrove (2010), and between 2017 and 2060 from the US Census Bureau's (2018) population projection. I keep fertility in prior years fixed at the 1917 levels and interpolate to get the intermediate years 2010 to 2016. Net migration levels by age and year between 2017

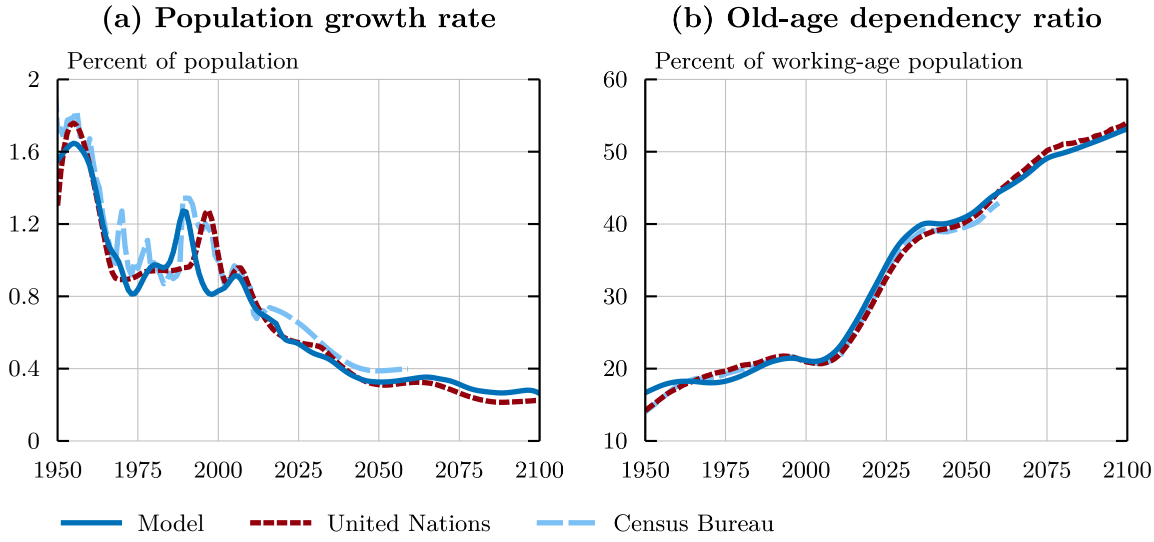


FIGURE 2. Demographic evolution.

and 2060 is also taken from the Census Bureau's population projection. In prior years, I proxy net migration by immigration, for which annual totals are provided by the US Department of Homeland Security (2020). The age distribution of migration flows before 2017 is assumed to be the same as in the population projection. I then convert the implied age-specific migration levels into migration rates using population estimates from the Census Bureau's intercensal tables and population projection.

Holding the demographic variables fixed after 2100, the population converges to a stable growth rate of 0.18 percent in the final steady state. In the initial steady state, I construct a stationary population structure with the same growth rate by the recursion $N_{j+1} = \frac{s_{j+1}+m_{j+1}}{1+n} N_j$, where N_0 is set to match the total population size in the data and where $\{m_j\}_{j=1}^J$ are shifted to ensure consistency with the calibrated fertility rates.[13] Figure 2 plots the implied demographic development and shows that this calibration reasonably matches official estimates and projections from the United Nations and the Census Bureau.

5.2 Preferences and labour productivity

Households start their economic lives at 20 and die with certainty at 100. The discount factor β and the weight on labour supply ψ are calibrated to match, respectively, a capital-output ratio of 2.8 and an average labour supply per worker of 0.45 in the initial steady state. The former yields a discount factor in the vicinity of the estimate by Hurd (1989), who explicitly accounts for mortality risk and finds a β of 1.011. Given a time endowment of 16 hours per day, the latter implies an average of 50 hours worked per week and worker in 1900, which matches the estimates of Ramey and Francis (2009, Figure 1A).

[13] The recursion implies that $N_j = \left(\prod_{k=1}^{j} \frac{s_k+m_k}{1+n}\right) N_0$. Consistency requires $N_0 = \frac{1}{1+n} \sum_{j=0}^{J} f_j N_j$. Combining these yields the condition $1+n = \sum_{j=0}^{J} \left(\prod_{k=1}^{j} \frac{s_k+m_k}{1+n}\right) f_j$. If migration rates are shifted by a common term x, such that $m_j = \widehat{m}_j + x$ where $\widehat{m}_j$ is the rate implied from the data, then this is an equation in one unknown which we can solve numerically.

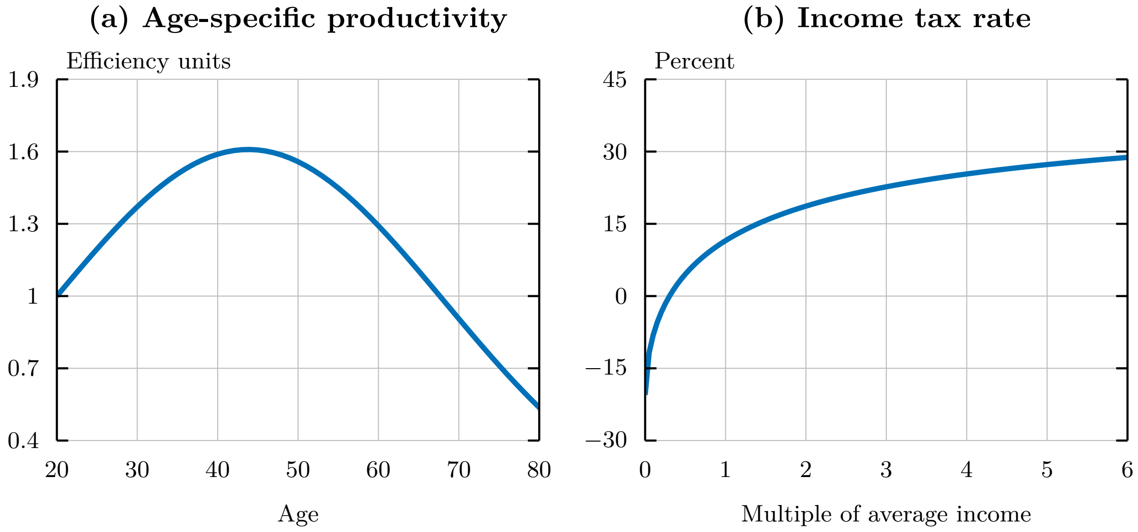


Figure 3. Calibrated efficiency and tax profiles.

I set the Frisch elasticity of labour supply θ to 0.5, as recommended by Chetty *et al.* (2011) along the intensive margin. This value is also consistent with Domeij and Flodén (2006a), who explicitly account for biases arising from uninsurable income risk and borrowing constraints. The inverse of the intertemporal elasticity of substitution σ is calibrated following Boppart and Krusell (2020). Based on long-run macro evidence, Boppart and Krusell argue that 2 percent productivity growth implies a fall in hours worked by roughly 0.4 percent. Recalling from Definition 2 that the steady-state growth rates of hours worked and productivity are linked via $1 + g_h = (1 + g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}}$, this empirical pattern suggests that $\theta(1-\sigma)/(1+\theta\sigma) \approx -0.2$. Given $\theta = 0.5$, this generates a σ of 1.75. Although based on long-run macro evidence, these parameter values are also consistent with micro evidence: Heathcote, Storesletten and Violante (2014) consider an incomplete-markets model with similar preferences and estimate $(\sigma, \theta) = (1.71, 0.46)$ using US earnings and consumption survey data.

I estimate the deterministic age-efficiency profile $\{\varepsilon_j\}_{j=\iota}^{J}$ by a fixed-effects regression of real log wages on a quadratic in age using earnings data from the 1968–2019 family files of the *Panel Study of Income Dynamics* (PSID). This procedure follows the usual steps in the literature and the details are relegated to Appendix D. The resulting profile is shown in Figure 3a and features a standard hump shape which peaks between the ages of 40 and 50. The process for idiosyncratic productivity shocks is assumed to follow an AR(1) process,

$$\log \eta_{j+1} = \rho \log \eta_j + \epsilon_{j+1}, \qquad \epsilon \sim N(0, \sigma_\epsilon^2),$$

with the persistence parameter and error variance set to $\rho = 0.97$ and $\sigma_e^2 = 0.02$, respectively, to match the estimated income process in Heathcote, Storesletten and Violante (2010). This process is discretised into a five-state Markov chain using Rouwenhorst's method (Kopecky and Suen, 2010).

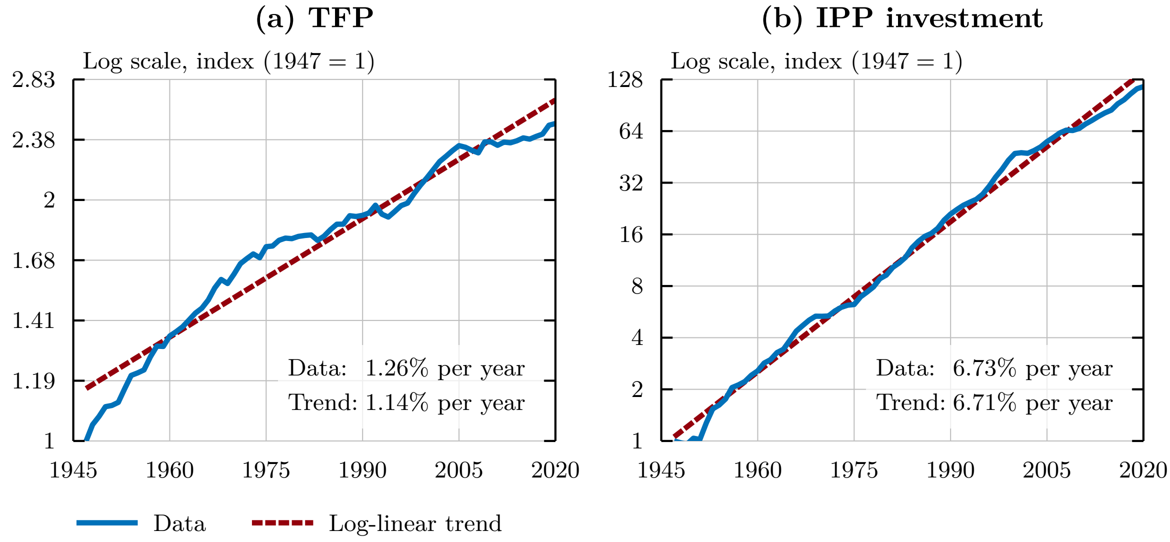


FIGURE 4. TFP and gross IPP investment in the data.

Source. Fernald (2014) and BEA NIPA Table 5.6.3, line 1.

5.3 Production and R&D

The labour share of final output is set to $1 - \alpha = 0.64$, a standard value, while intermediate producers have a markup of $1/\omega = 1.4$. Together with α, the value of ω implies an aggregate profit share of 10 percent, a standard benchmark in much of the business cycle literature. Capital investment as a share of GDP is roughly constant over time at 13.6 percent and I set the capital depreciation rate δ to 0.046 in order to match this value in the initial steady state. Here, capital investment is defined as gross private domestic investment less private fixed investment in intellectual property products (IPP), both of which are available in the national accounts from the Bureau of Economic Analysis (BEA, NIPA Table 1.1.5). Besides formal R&D, IPP investment also includes spending on nonrival goods such as the development of computer software and the creation of entertainment, literary, and artistic originals. I use this measure as a proxy for R&D investment Q in the model.

There are four R&D parameters to calibrate: the productivity parameter ν, the intermediate-firm exit rate δ_z, the duplication externality λ, and the knowledge spillover ϕ. The first one is a scale parameter; I set it so that $z = Z = 1$ in the initial period. The intermediate-firm exit rate is set to 0.5 percent, which yields an aggregate R&D investment equal to 1.4 percent of GDP in the initial steady state. In the data, IPP investment as a share of GDP is stable at 0.7 percent before 1950 (after 1950 it trends upward). I do not match this value exactly because this requires such a low δ_z that the transition paths become infeasibly long. As highlighted by Bloom *et al.* (2020), there is no consensus on the true value of λ. I follow the calibrations in Jones and Williams (2000) and Comin and Gertler (2006) and set it to 0.75. The parameter ϕ is calibrated similarly to Bloom *et al.* and Jones and Williams. Specifically, I use the fact that growth in both TFP and IPP investment have been roughly constant in the United States over the post-war period, as shown in Figure 4. From the R&D growth rate in (11), $1 + g_z = 1 - \delta_z + \nu Q^{\lambda} z^{\phi - 1}$, it follows that the right-hand side must also have been approximately constant. Log-differencing the last term on the right-hand side of Equation (11) then yields that

$$\phi = 1 - \lambda \frac{g_Q}{g_z},$$

where g_x denotes the net growth rate of a variable x. Fernald's (2014) utilisation-adjusted TFP estimates for the United States imply an average annual TFP growth rate of 1.26 percent between 1947 and 2020. The definition of TFP in (14), $Z = z^{\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}$, and the parameter values for α and ω then imply a g_z of 5.72 percent. Real IPP investment (BEA NIPA Table 5.6.3) grows at an approximately constant rate of 6.73 percent over the same time period. The values for λ and g_z together with $g_Q = 0.0673$ then yields $\phi = 0.117$.[14] This calibration yields values of γ_Z and γ_y equal to 0.23 and 0.18, which is similar to the mid point of the range of estimates in Jones (2002). This implies that in a steady state with for example 1.2 percent population growth, TFP and output per capita growth equals 0.27 and 0.22 percent per year, respectively.

5.4 Public sector

I set the gross pension replacement rate ζ to 0.413 based on Clingman, Burkhalter and Chaplain's (2021) estimates for average-income workers who retire at the normal retirement age. The normal retirement age, R^{norm}, is set to 65 and the earliest age to collect retirement benefits, R^{min}, is 62. Early and delayed retirement adjustment via $\mu(R)$ is similar to that of the US social security system. For every year of early retirement, the base level benefit is reduced by 6 ⅔ percent per year for the first three years and 5 percent per any additional year. For every year of delayed retirement, the base level benefit is scaled up by 3 percent up until a maximum age of 70. After the age of 70, no extra benefit is given for delaying retirement.[15]

I compute aggregate tax rates on (i) consumption, (ii) capital, and (iii) labour income for the years 1950 to 2020 using national accounts data from the BEA. In short, these tax rates are computed as aggregate tax revenues divided by their corresponding tax bases; further details are provided in Appendix E. I keep the tax rates τ^c and τ^k fixed over time and set them equal to the temporal averages of (i) and (ii), respectively. The temporal average of the income tax rate (iii) is used to scale the progressive income tax schedule as described below.

The individual income tax rate $\tau^w(e)$ for a household with earnings e is parametrised by Gouveia and Strauss's (1994) functional form:

$$\tau^w(e) = \kappa_0 \left[1 - \left(\kappa_2 \left(\frac{e}{\bar{e}} \right)^{\kappa_1} + 1 \right)^{-\frac{1}{\kappa_1}} \right] + \kappa_3, \qquad \kappa_1 > 0, \tag{34}$$

where the inclusion of average labour earnings $\bar{e}$ makes the tax rate invariant to units of measurement. In Equation (34), κ_0 controls the asymptotic tax rate $\lim_{e\to\infty} \tau^w(e) = \kappa_0 + \kappa_3$,

[14] The positive value of ϕ implies positive knowledge spillovers in R&D: the more we know, the easier it is to discover new ideas. This contrasts with for example Jones (2002) and Bloom *et al.* (2020), who find negative values for ϕ for the aggregate US economy. The calibration methodology here is virtually identical to these papers, but Jones and Bloom *et al.* compute ϕ under the implicit assumption that the intermediate-firm markup ω is exactly equal to the capital share α. If $\omega = \alpha$ is imposed here, I too find a negative value for ϕ.

[15] These are the exact retirement ages and scaling rules used by the Social Security Administration for cohorts born before 1924. Later cohorts have higher normal retirement ages and more generous delayed retirement credits, though I show in Appendix C that the results are insensitive to a more accurate development of R^{norm} and $\mu(R)$.

κ_1 determines the degree of tax progressivity (where $\kappa_1 \to 0$ reduces τ^w to a flat tax), and κ_2 is a scale parameter. The parameter κ_3 (assumed to be zero by Gouveia and Strauss) is the marginal tax rate at zero earnings.

The estimation of $\{\kappa_0, \dots, \kappa_3\}$ is based on the OECD tax database, which provides estimates of average labour income tax rates at incomes equal to 67, 100, 133, and 167 percent of average wage earnings. These estimates are available for each year since 2000 and incorporate central, state and local government taxes and various types of deductions and tax credits. I replicate the OECD's methodology for these years and construct tax rates for a full grid of hypothetical incomes, ranging from 0 to a multiple 20 of average wage earnings, and subsequently fit Equation (34) to these tax rate estimates. I then shift the overall level of the tax function (while maintaining its degree of progressivity) by adjusting the estimated parameters until the tax rate at average earnings, $\tau^w(\bar{e})$, equals the labour income tax rate computed from the national accounts. This ensures (approximate) consistency with the aggregate tax data in the national accounts. Appendix E outlines the details for these steps and Figure 3b plots the resulting income tax schedule.

6 Quantitative results

We are interested in quantifying the transitional effects of the demographic transition on output per capita and the mechanisms through which it operates. The results below therefore focus on per-capita growth rates. Additional figures of other variables are available in Appendix A. Because children do not affect any of the economic variables, I take "per capita" to mean "per adult person", where adults are individuals aged 20 and above. I consider three exercises: (i) a growth accounting analysis of the baseline calibration; (ii) a comparison between the baseline model and a special case without endogenous TFP in order to evaluate the importance of TFP; and (iii) a counterfactual simulation in which the baby boom never occurs in order to investigate the relative importance between changes in fertility and changes in life expectancy. In each scenario I recalibrate the preference parameters β and ψ to match, respectively, a capital-output ratio of 2.8 and an average labour supply of 0.45 in the initial period.

6.1 Accounting for transitional growth

For the purposes of growth accounting, it is useful to first reformulate the expression for output per capita in (29), $y = Z\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} \frac{L}{N}$. First, since aggregate labour supply is the sum of all efficiency units supplied by households, we can decompose aggregate labour into total employment (E), average hours per worker ($\bar{h}$), and average productivity per hour worked ($\bar{\varepsilon}$): $L = E\,\bar{h}\,\bar{\varepsilon}$. Next, output per capita, TFP, and hours per worker are all proportional to the population size raised to some powers in the initial period according to Definition 2. Even in the absence of the demographic transition, these variables would continue to grow with the population. To capture these underlying trends, let $\widetilde{N}$ denote the population *if* the population instead remained on its initial balanced growth path. We may then write output per capita as

$$y \;=\; \frac{Z}{\widetilde{N}^{\gamma_Z}} \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} \frac{E}{N}\,\frac{\bar{h}}{\widetilde{N}^{\gamma_h}}\,\bar{\varepsilon}\,\widetilde{N}^{\gamma_y}, \tag{35}$$

where γ_Z, γ_y, and $\gamma_h = \gamma_y - \gamma_Z$ are the exponents from the steady-state growth rates of TFP, output per capita, and hours per worker in Definition 2. Multiplying and dividing the right-hand side of (35) by $\widetilde{N}^{\gamma_y}$ detrends TFP and hours per worker as well as isolates the trend component of per-capita output in the initial steady state. But the long-run population growth rate remains fixed in the numerical experiment, so $\widetilde{N}^{\gamma_y}$ captures the long-run trend also during the transition and in the final steady state. The *transitional* impact on output per capita is therefore given by the change in per-capita output *relative* to this trend $\widetilde{N}^{\gamma_y}$. The underlying mechanisms are captured by the first five factors on the right-hand side; these are constant in steady state so any changes in these are transitory as well. Log-differencing (35) between two subsequent periods then gives a growth decomposition of the form

$$g_y \;=\; \left(g_Z - \gamma_Z\widetilde{n}\right) + \frac{\alpha}{1-\alpha}\,g_{K/Y} + g_{E/N} + \left(g_h - \gamma_h\widetilde{n}\right) + g_\varepsilon + \gamma_y\widetilde{n}, \tag{36}$$

where g_x denotes the net growth rate of a variable x and $\widetilde{n}$ is the steady-state population growth rate. This separation between transitory changes and long-run forces follows in the spirit of Jones (2002) and makes the quantification of the transitional mechanisms easy.

6.2 Comparative statics

Following the discussion in Section 3, it is instructive to first consider a comparison between the initial period in 1900 and the final period in 2600 for the baseline scenario. Figure 5a shows the overall change in the age structure while Figure 5b displays the cumulative percentage change in output per capita relative to trend between the initial and final periods. The transition from the expansive population pyramid in the initial period to the stationary population pyramid in the final period *raises* output per capita by 42 percent. This finding follows primarily from a substantial increase of 58 percent in TFP relative to trend. The capital intensity also increases by 16 percent while the higher share of old-age households reduces the employment rate by 23 percent. The qualitative changes in the underlying channels are therefore precisely as the simple model in Section 3 predicts.

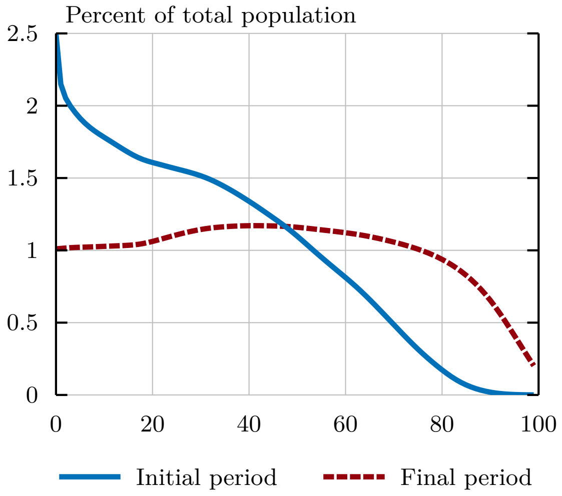
A natural question immediately emerges from Figure 5: why does TFP grow so much more than the capital intensity? It is not because an overwhelming majority of household savings are allocated to R&D rather than to capital investments.[16] Rather, this is a result of the scale effect discussed in Section 3, which occurs when the demographic transition increases the population size relative to the trend $\widetilde{N}$. A simple decomposition of the transitional TFP change illustrates this point. Specifically, we can use the fact that, in steady state, TFP is proportional to the actual population size N raised to γ_Z. The corresponding proportionality constant provides a measure of the R&D intensity. Since the scale effect is pinned down by the actual population relative to the initial trend, $N/\widetilde{N}$, we obtain the following decomposition:

$$\underset{\uparrow\,58\,\%}{\text{Detrended TFP}} \;=\; \frac{Z}{\widetilde{N}^{\gamma_Z}} \;=\; \frac{Z}{N^{\gamma_Z}}\cdot\left(\frac{N}{\widetilde{N}}\right)^{\gamma_Z} \;=\; \underset{\uparrow\,13\,\%}{(\text{R\&D intensity})} \times \underset{\uparrow\,40\,\%}{\begin{array}{c}\text{Transitory}\\ \text{scale effect}\end{array}},$$

where the percentages show the cumulative change in each factor between the initial and the final steady states. The 58 percent rise in TFP is due to a 13 percent increase in the R&D intensity and a 40 percent transitory increase in scale. Changes in household behaviour and composition

[16] Capital investment exceeds R&D investment by a factor of 4 during most of the transition and the growth rate of the capital stock relative to TFP is even higher (see Figure A.3 in Appendix A).

(a) Population age distribution



(b) Cumulative transitory growth

$$\frac{y}{\widetilde{N}^{\gamma_y}} = \frac{Z}{\widetilde{N}^{\gamma_Z}} \cdot \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} \cdot \frac{E}{N} \cdot \frac{\bar{h}}{\widetilde{N}^{\gamma_h}} \cdot \bar{\varepsilon}$$

	TFP	Capital intensity	Emp. rate	Hours	Prod.
42%	58%	16%	−23%	0%	1%
(100%)	(138%)	(38%)	(−55%)	(−1%)	(3%)

Figure 5. Comparative statics: population structure and output per capita.

thus impact output per capita through the R&D intensity much like it does through the capital intensity. The key difference between TFP and the capital intensity instead lies in the fact that technology is nonrival whereas physical capital is not, as seen from the large scale effect.[17]

6.3 Growth dynamics

The comparative statics is informative but says little about the growth dynamics during the transition. We therefore now turn to the main period of interest: 1950 to 2100. Figure 6 illustrates the growth rate net of trend and its corresponding decomposition (that is, the five first terms on the right-hand side of (36)) during this period. Focusing first on the overall growth rate, two key results immediately emerge. First, the demographic transition positively affects output per capita throughout the second half of the twentieth century, with growth rates firmly above the long-run trend. Second, although this effect fades at the turn of the century, the demographic development does not negatively affect output growth. Instead, growth remains around trend throughout the twenty-first century.

Turning to the underlying mechanisms, we again find a qualitative development exactly as predicted in Section 3: the demographic transition leads to positive TFP growth and capital deepening, but lowers the employment rate. Moreover, average hours worked decline in periods of positive growth (and vice versa) while average efficiency per hour worked rises in periods where the share of middle-aged workers is higher. The former is a result of the calibration of the household utility function, where the income effect of higher wages on leisure dominates the substitution effect. The latter follows from the hump shape in workers' productivity.

The dynamics of the baby boom predicted in Section 3 are also clearly visible in Figure 6. The share of young workers increases when the baby boomers enter adulthood in the 1960s and

[17] A problem here is that using the population as the relevant scaling variable is somewhat arbitrary. We could use total employment instead, since it grows in parallel with the population in steady state (recall Equation (31) in the simple model, which states that TFP = (R&D intensity) × (Employment)$^{\gamma_Z}$). Switching to employment yields a slightly smaller scale effect, 30 percent, although this does not change the basic point of the exercise.

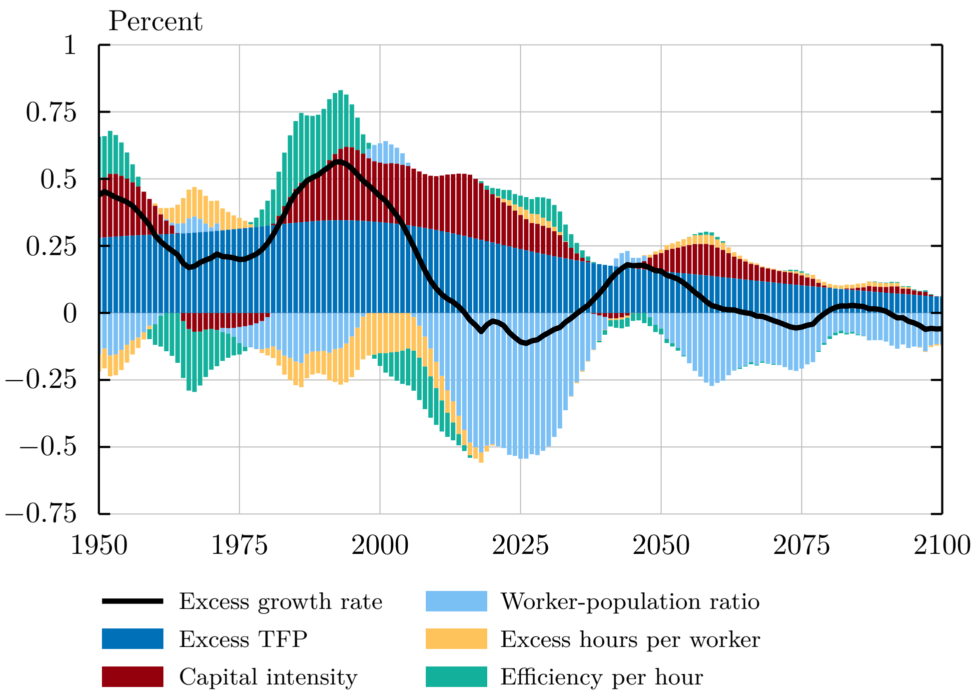


FIGURE 6. Growth accounting of the baseline model.

1970s, which improves the employment rate and worsens the capital intensity. TFP growth hardly changes, which suggests that the scale effect of the baby boom far outweighs any negative impact on the R&D intensity. As this generation becomes middle aged in the 1980s and 1990s, growth through the capital intensity and TFP improves. Finally, the baby boom retires in the early decades of the twenty-first century, thereby causing a negative impact via the employment rate.

Table 4 adds to these observations by summarising the average annual growth rates of output and its decomposition, first for the entire time period considered and then separately for each century. Overall, the demographic transition boosts output per capita by 0.14 percent per year. The effect is almost entirely driven by the twentieth-century development, where the average annual growth rate net of trend is 0.35 percent. By comparison, the observed long-run growth rate of GDP per adult person in the US data is approximately 1.8 percent per year. From this viewpoint, the impact obtained here is quantitatively significant; taken at face value it implies that around 20 percent of actual US post-war growth can be attributed to transitory demographic factors. This contribution makes demographics comparable in importance to human capital accumulation, whose share of US growth over the same period is also around 20 percent (see for instance Fernald and Jones, 2014).

Just as in the steady-state comparison, what stands out quantitatively from both Figure 6 and Table 4 is the importance of TFP for overall growth. Over the full time period, excess TFP grows by 0.22 percent per year, thus accounting for more than 150 percent of the net-of-trend growth rate. Its contribution is more than twice as large as that of capital deepening. The effect is even larger during the latter half of the twentieth century, with excess TFP growing 0.32 percent per year, more than three times faster than the capital intensity, thus driving the bulk

Table 4. Growth accounting of the baseline model.

	Excess output per capita	Excess TFP	Capital intensity	Employment rate	Excess hours per worker	Efficiency per hour
Period	$g_y - \gamma_y \widetilde{n}$	$g_Z - \gamma_Z \widetilde{n}$	$\frac{\alpha}{1-\alpha}\, g_{K/Y}$	$g_{E/N}$	$g_h - \gamma_h \widetilde{n}$	g_ε
1950–2100	0.14 %	0.22 %	0.09 %	−0.15 %	−0.02 %	0.01 %
1950–2000	0.35 %	0.32 %	0.10 %	−0.07 %	−0.03 %	0.04 %
2001–2100	0.04 %	0.17 %	0.08 %	−0.20 %	−0.01 %	−0.01 %
1995	0.54 %	0.35 %	0.27 %	−0.09 %	−0.15 %	0.16 %
2025	−0.11 %	0.24 %	0.13 %	−0.54 %	0.03 %	0.04 %
Difference	−0.65 pp.	−0.11 pp.	−0.15 pp.	−0.46 pp.	0.18 pp.	−0.12 pp.

Notes. The table displays the growth decomposition corresponding to Equation (36) for the baseline calibration. The numbers reported are average annual net growth rates in percentage points.

of excess output growth.

Changes in the share of workers in the adult population constitutes the main drag on output growth, depressing the annual growth rate by on average 0.15 percentage points overall and by an additional 0.05 percentage points during the twenty-first century. This follows because increases in the average retirement age do not keep up with increasing life expectancy; Figure 7 shows that, although there is a shift toward later retirement over time, the increase in retirement age is so marginal that it hardly affects the share of retirees in the 65+ population.

Because of concerns about recent declines in observed growth, it is also of interest to zoom in on the decline in model growth from peak in 1995 to trough in 2025. The decomposition for this period is shown in the lower half of Table 4. The growth rate and most of its components change monotonically between these years, so it suffices to consider snapshots at the endpoints of the period and the differences between the two. Overall, changes in the demographic structure leads to a 0.65 percentage point drop in the growth rate over the last three decades, thus suggesting that demographics explain a significant chunk of the growth decline observed in the data. The decline stems in part from roughly similar declines of 0.1 to 0.2 percentage points in the growth rates of TFP, capital intensity, and average efficiency. The growth rate of hours increases by a similar magnitude, therefore marginally counteracting the overall development. The majority, however, stems from the retirement of the baby boom: growth in the employment rate declines by 0.46 percentage points, accounting for 70 percent of the total decline.

In sum, neither the comparative statics nor the transition dynamics of the baseline model provides any indication that the demographic transition in general and population ageing in particular is detrimental to economic growth. If anything, it is positive. Although demographics account for a significant decline in growth rates over the last three decades due to a rising share of retirees in the population, it is important to stress that this is *not* because population ageing is particularly bad for output growth. In fact, it is hardly a result of population ageing at all. Rather, the temporary *rise* in fertility that caused the baby boom induces a long period of above-normal employment rates during the twentieth century when the baby boomers are

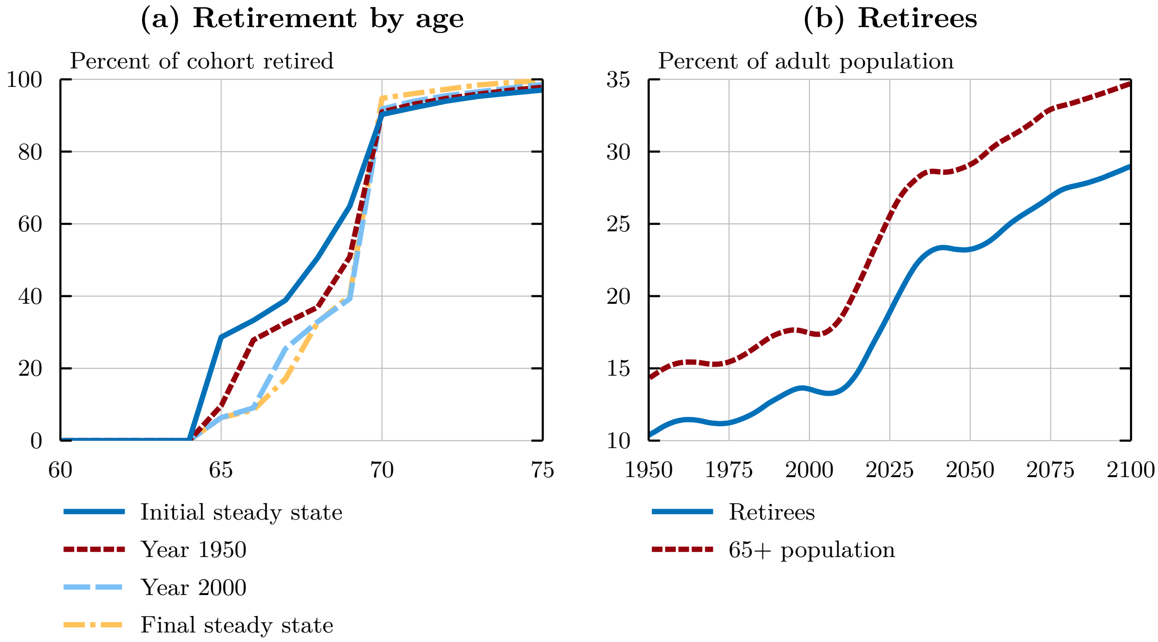


FIGURE 7. Retirement in the baseline model.

of working age. We are just now experiencing the end of that period, with employment rates reverting back to normal as this generation retires. As already stressed in the simple model in Section 3, this employment rate decline would occur even in the absence of improving old-age mortality.

6.4 The effect of ignoring technological change

The finding that the demographic transition does not negatively impact per-capita output contrasts with the general notion of population ageing as a major drag on economic activity. For instance, Krueger and Ludwig (2007), the perhaps closest paper to the analysis here, use a similar quantitative model and find a cumulative drop of 12.6 percent in US output per capita between 2005 and 2080. However, most previous work, including Krueger and Ludwig, tend to ignore the TFP channel that I incorporate. To what extent can the differences be explained by this additional mechanism?

To answer this question, I consider version of the model with exogenous technological change. Specifically, the model above nests a standard model without endogenous growth as the special case with perfect substitution between intermediate firms ($\omega = 1$) and a zero intermediate-firm exit rate ($\delta_z = 0$). The former eliminates profits, thus forcing the patent price and R&D investment to zero, which in turn reduces the intermediate-firm dynamics to $z_{t+1} = (1 - \delta_z)z_t$. The latter ensures that the measure of intermediate firms is constant rather than converging to zero. I also include exogenous TFP that grows at a constant rate equal to the baseline long-run growth rate. This feature ensures that the long-run trends are the same in both scenarios, although it matters little since the long-run growth rate is a negligible 0.03 percent per year.

Figure 8a plots the net-of-trend growth rate under this specification against that of the baseline

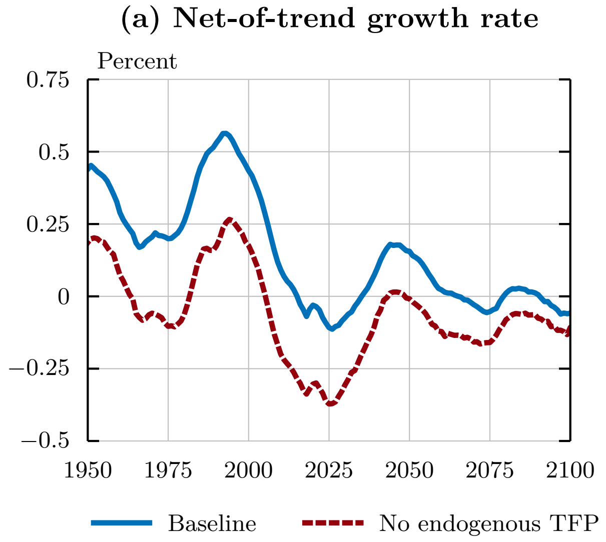


FIGURE 8. Net-of-trend growth rates with and without endogenous TFP.

model. The positive effects on output now disappear, with growth rates consistently around 0.1 to 0.3 percentage points below the baseline. The cumulative decline in output per capita is similar to Krueger and Ludwig (2007): 11.5 percent between 2005 and 2080 and 9.1 percent over the full period. In comparison, over the same time periods the baseline model exhibits *positive* cumulative growth of 2.5 and 24.3 percent, respectively. The difference is almost entirely driven by the lack of TFP in the former, as shown in Figure 8b. The impacts on the capital intensity, the employment rate and the average efficiency are virtually unchanged.[18] There is also a small counteracting effect in hours worked: lower income leads to a rise in hours, and this raises output. But this effect is too small to offset the TFP difference. Thus, whether the demographic transition raises or lowers output depends crucially on whether or not technical change is properly accounted for.

6.5 The impact of the baby boom

The fact that the demographic transition is driven partly by the baby boom and partly by rising life expectancy raises an obvious follow-up question: which one is the main driver of the impact observed in Figure 6? As a final exercise, I therefore consider a counterfactual scenario in which the baby boom never occurs. That is, I replace actual fertility rates between 1935 and 1975 with interpolated values so that fertility exhibits a roughly stable trajectory after 1935 rather than the hump shape corresponding to the baby boom; see Figure 9b. The counterfactual fertility rates affect the cohorts that become adults after 1955 and that die with certainty no later than 2075. The population is thus identical to the baseline in 1950. Likewise, the age structure is practically identical to the baseline in 2100, only the population size differs. Over the full period 1950 to 2100, this exercise consequently removes the scale effect associated with the baby boom and isolates the impact from changes in the population structure.

Figure 9a plots the net-of-trend growth rate under this specification against that of the baseline. The positive effect during the 1980s and 1990s now disappear, with growth rates remaining closer

[18] See Figure A.4 in Appendix A for a comparison of the full growth decompositions.

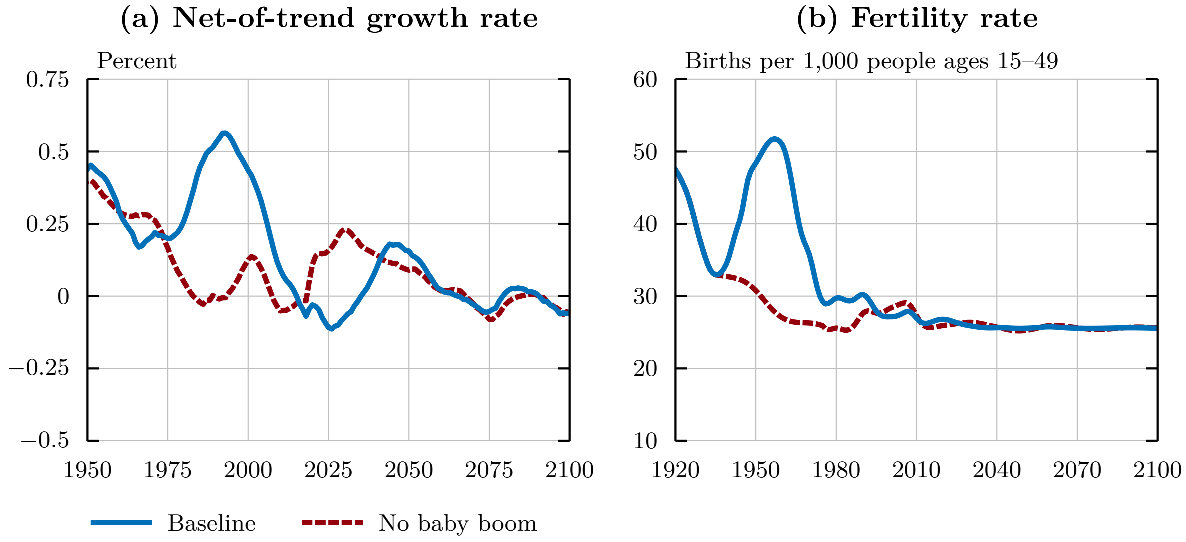


FIGURE 9. Counterfactual simulation: no baby boom.

to trend from the 1980s onward. The no-baby boom counterfactual reduces the average annual growth rate between 1950 and 2000 by 0.19 percentage points, thereby eliminating around half of the positive post-war growth effect. The effect over the full time period is similar, with the average net-of-trend growth rate reduced from 0.14 to 0.09 percent per year. As expected, this decline is driven entirely by lower TFP growth owing to a reduction in scale; the annual average growth rates of the capital intensity and the employment rate between 1950 and 2100 are still 0.09 and −0.15 percent, identical to the baseline.

The scale effect associated with the baby boom and changes in the population age structure therefore account for roughly equal parts of the positive effect on output per capita. For the latter, it is important to keep in mind that the population structure in 1950 is not stationary, so changes in the population structure would occur even in the absence of changing mortality rates. Most growth in the counterfactual scenario takes place in the 1950s and 1960s, which points to the initial nonstationary composition as the key driver at work. If we instead focus on the period after 1980 the average annual growth rate net of trend is a meagre 0.04 percent, resulting in a cumulative 5.4 percent increase in output per capita over 120 years. Evidently, increasing life expectancy alone does not meaningfully alter output per capita. Thus, as far as growth is concerned, these findings leave little reason to worry about predicted future population ageing, which is primarily driven by changes in mortality rates.

7 The importance of the R&D production function

The results in the previous section rest on several assumptions regarding the calibration and parametrisation of the model. Most are either standard in the literature or easily justified in the data (or both). In Appendix C I run a battery of alternative specifications that adjust assumptions about household preferences, inequality, fiscal policy or pension rules and show that these generally do not meaningfully impact the baseline findings. A central nonstandard model element that warrants further treatment, however, is the R&D production function.

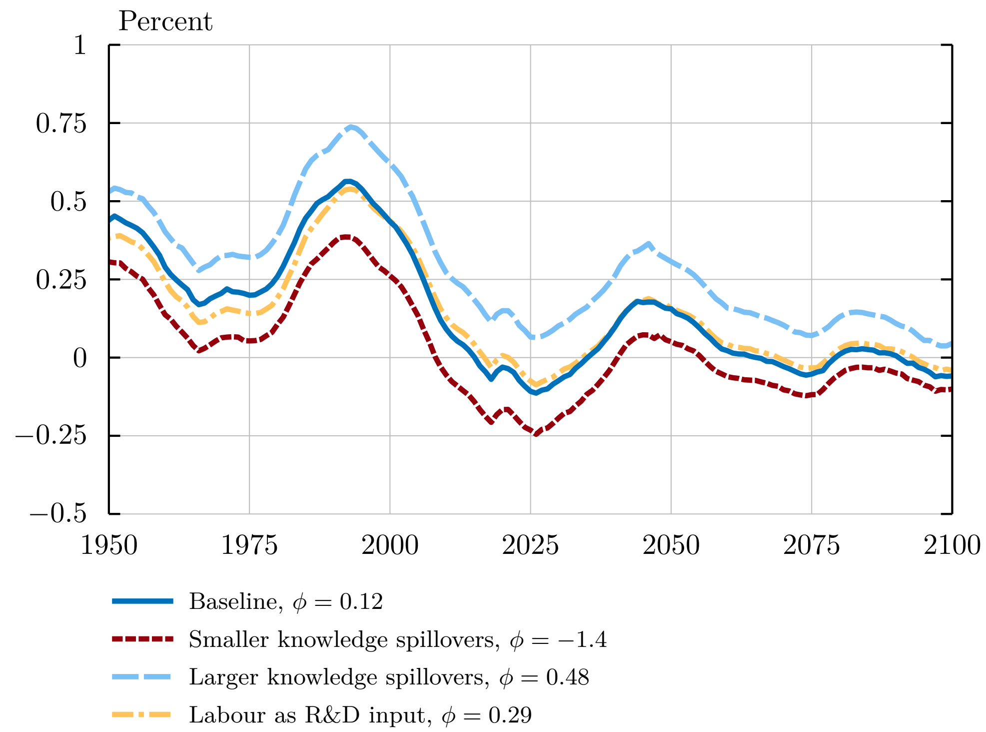


FIGURE 10. Net-of-trend growth rates for different R&D production functions.

This section explores the sensitivity of the results with respect to two different R&D parametrisation choices: the choice of the R&D production function and the calibration of the knowledge spillover parameter ϕ. The latter is particularly important since ϕ governs the degree to which R&D becomes easier or harder over time. In the baseline model, this parameter is calibrated as in Bloom *et al.* (2020) by adjusting ϕ such that if the model produces a constant growth rate of R&D investment equal to the growth rate of gross IPP investment in the data, then model TFP also grows by the same constant rate as TFP in the data. Given the difficulty of accurately measuring productivity growth and R&D investment in the data, however, the resulting value of ϕ is likely surrounded by a lot of uncertainty.

I consider three alternatives to the baseline model: one with smaller knowledge spillovers, one with larger knowledge spillovers, and one where labour is the only productive input into R&D. In the first alternative, I set $\phi = -1.4$ based on the estimate in Bloom *et al.* (2020, Table A1) for the aggregate US economy. Bloom *et al.* obtain this value under the knife-edge condition that the intermediary-firm markup ω is exactly the same as the capital share α, which explains the much lower estimate of ϕ than in the baseline calibration. In the second alternative, I use the Penn World Table to construct a measure of US TFP growth as the Solow residual between output and a Cobb-Douglas combination of the capital stock and total hours worked. Using the same calibration method as in the baseline, I then get $\phi = 0.48$. Recalling that output per capita grows by the gross rate $1 + g_y = (1 + n)^{\gamma_y}$ in steady state, these values of ϕ imply values of γ_y equal to 0.06 and 0.35. Interestingly, this is almost identical to the bounds on γ_y identified by Jones (2002), who finds the range of plausible values of γ_y to lie between 0.05 and 0.33. These two scenarios therefore constitutes a form of lower and upper bounds on the baseline results.

Table 5. Average net-of-trend growth rates for different R&D production functions.

	Baseline	Smaller knowledge spillovers	Larger knowledge spillovers	Labour as R&D input
(ϕ, γ_y)	(0.12, 0.18)	(−1.4, 0.06)	(0.48, 0.35)	(0.29, 0.18)
1950–2100	0.14 %	0.03 %	0.29 %	0.14 %
1950–2000	0.35 %	0.20 %	0.49 %	0.31 %
2001–2100	0.04 %	−0.06 %	0.19 %	0.06 %

Notes. The table reports average annual growth rates net of the long-run trend, $g_y - \gamma_y \widetilde{n}$, under different calibrations of the R&D production function.

The last alternative is common in the growth literature and specifies the R&D process as

$$z_{t+1} = (1 - \delta_z) z_t + \nu L_{zt}^{\lambda} z_t^{\phi},$$

where L_{zt} is total labour devoted to R&D.[19] Here, I set $\phi = 0.29$ such that the long-run growth rate is the same as in the baseline.[20] As before, I also recalibrate the preference parameters β and ψ in each alternative to match the same targets as in the baseline.

The aggregate growth rates from these alternative scenarios are plotted against the baseline in Figure 10. Qualitatively, changing the knowledge spillover is straightforward: the larger the knowledge spillover, the larger the growth impact from R&D since researchers become more productive the more knowledge is created. Figure 10 confirms this prediction. Making labour the only R&D input in the last specification means that there is a higher demand for labour relative to capital from producers, with subsequent general equilibrium effects on wages and interest rates. Since the R&D sector is a small share of the total economy, these effects are small. Moreover, none of the major underlying mechanisms of the model changes, as the cost of R&D (the wage bill of researchers in this case) is still financed via household savings. The impact on growth relative to the baseline is therefore negligible.

Table 5 shows average net-of-trend growth rates over the three time periods considered for the baseline. With larger knowledge spillovers, growth rates increase by about 0.15 percentage points per year. With smaller knowledge spillovers, growth declines by between 0.1 and 0.15 percentage points per year. Using labour as the only R&D input leaves growth rates virtually identical to the baseline. Overall, growth rates are positive across the board. Only in the most pessimistic calibration do we observe a negative impact on per-capita output during the twenty-first century, but even here the impact is small: a negative 0.06 percent per year. Therefore, although reasonable variations of the R&D process impact the quantitative findings, they do not change

[19] The labour and goods market conditions also change to $L_t + L_{zt} = \sum_j N_{jt} \int_X \ell_{jt}(x)\,\mathrm{d}\Phi_{jt}$ and $Y_t + A_{t+1}^M = C_t + G_t + I_t$, respectively, and the long-run growth rate is again pinned down by the population growth rate, $1 + g_z = (1+n)^{\chi}$, but now with $\chi \equiv \frac{\lambda(1+\theta\sigma)}{(1-\phi)(1+\theta\sigma) - \lambda\theta(1-\sigma)\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}$ (see Appendix B).

[20] Alternatively, we could follow Bloom *et al.* (2020) and deflate real IPP investment by the average real wage to obtain a measure of the "effective" number of researchers and then calibrate ϕ as in the baseline. If the real wage is private sector wages (NIPA Table 2.1, line 4) divided by private sector hours (NIPA Table 6.9, line 3) and deflated by the PCE index (NIPA Table 2.3.4, line 1), then the annual growth rate of effective researchers between 1948 and 2020 is 5.10 percent. This gives a similar value: $\phi = 0.33$.

the basic point of this paper: that the demographic transition and the ageing of the population improves output per capita under endogenous technological change.

8 Conclusion

The model presented in this paper allows the population structure to affect output per capita via three channels: through the size of the labour force relative to the population, through capital accumulation, and through technological progress. This framework stands in stark contrast to standard macroeconomic life-cycle models, in which technological change is exogenous. It also contrasts to most models of endogenous growth, which hide the entire population structure in a representative household. A key point throughout this paper is that this matters for how we think about demographic change and its impact on output per capita.

My main findings suggest that current and projected US demographic change since 1950 raises output per capita and that this effect is large at times; at least on par with the growth contribution from US educational attainment over the second half of the twentieth century. I show that this is primarily due to the inclusion of endogenous technological change. Removing this channel completely reverses the positive impact. Overall, these findings challenge a seemingly conventional wisdom that current demographic developments are detrimental to economic activity.

The framework employed in this paper admittedly leaves out several other potentially important channels, such as human capital accumulation, automation, or international technology flows. However, papers that consider endogenous responses in human capital or automation typically find that these extra adjustment margins improve the impact of population ageing on output per capita, much like the inclusion of technological progress in this paper. Moreover, new technologies imported from foreign countries would only serve to raise the productivity of research under my baseline calibration. Thus, if anything, I would expect these mechanisms to only strengthen the main takeaways of this paper.

Lastly, the results above also raise natural questions about the conclusions drawn in other literatures that rely on exogenous growth. For example, the pension system is a key determinant of the savings rate and, since the savings rate is central to technical change here, an immediate question is therefore whether social security reform becomes more or less costly under R&D-driven growth. Other topics include migration policy, other types of fiscal policy reform, and optimal taxation. A recent paper by Jones (2022c) for instance suggests that the optimal progressivity of income taxation is significantly altered by the presence of endogenous technological change. The model considered here could serve as a basis to analyse these questions further.

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Appendices

Appendix A Additional figures

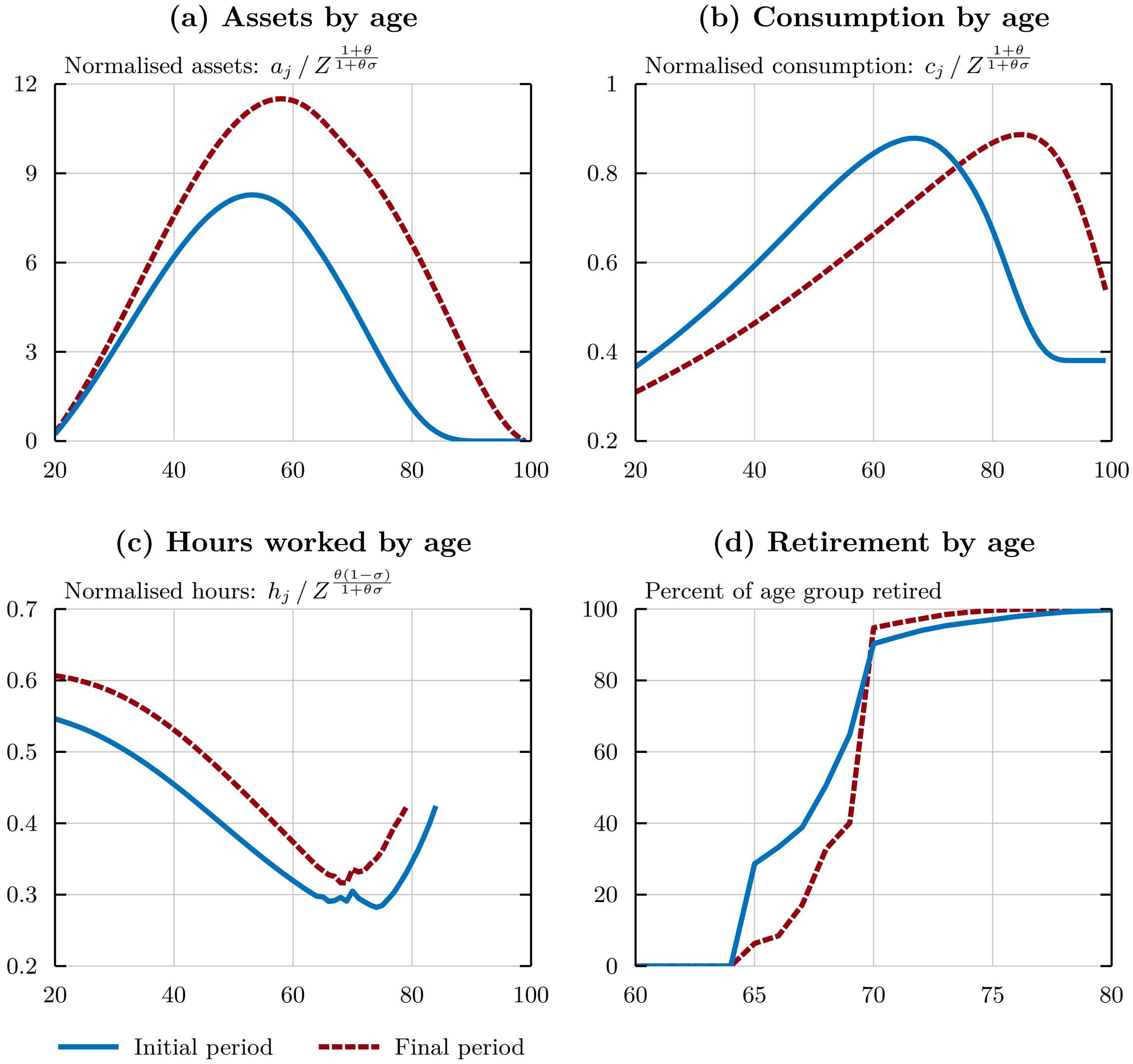

Figure A.1. Average life-cycle profiles in the baseline scenario.

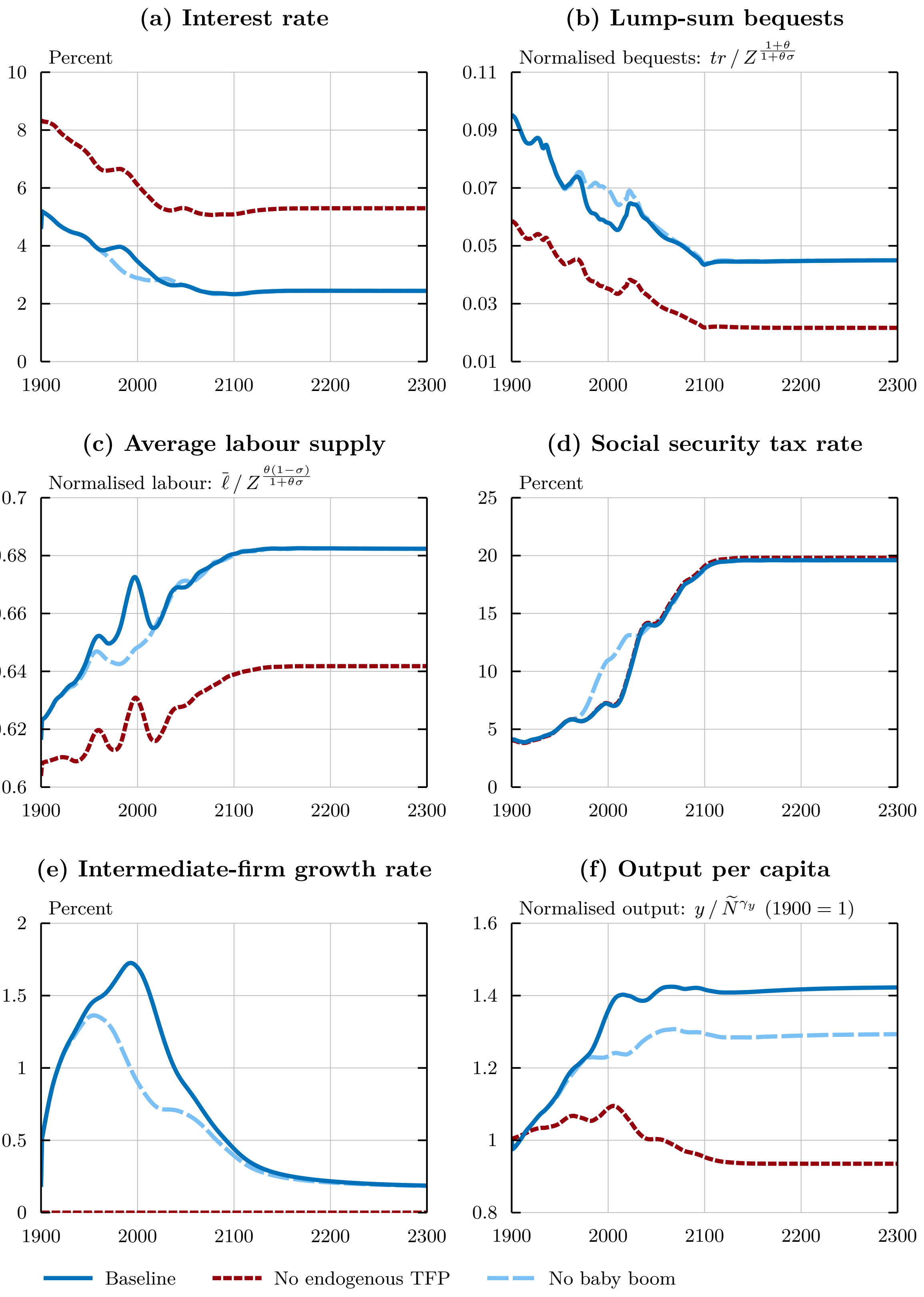

Figure A.2. Transition paths of equilibrium variables and detrended output per capita.

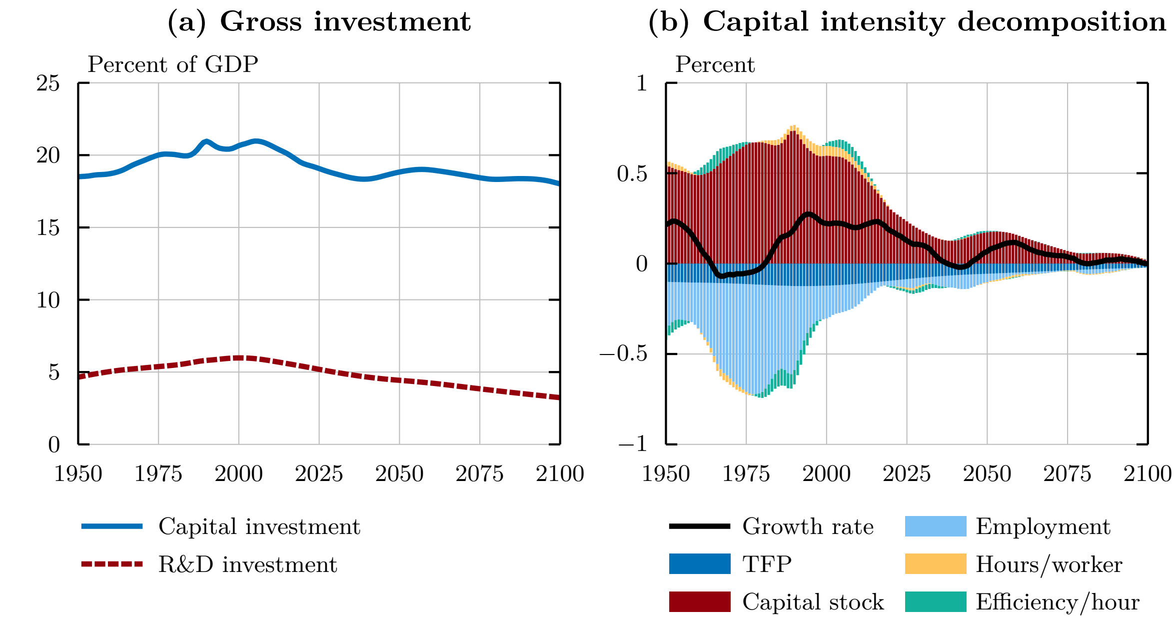

Figure A.3. Investment and capital deepening in the baseline scenario.

Notes. Figure A.3b displays a growth decomposition of the capital intensity as follows: $\frac{\alpha}{1-\alpha} g_{K/Y} = \alpha\Big(g_K - g_Z - g_E - g_{\bar{h}} - g_{\bar{\varepsilon}}\Big)$. This decomposition is obtained by log differencing the capital intensity $\left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{K}{ZL}\right)^{\alpha}$ together with the labour decomposition $L = E\,\bar{h}\,\bar{\varepsilon}$.

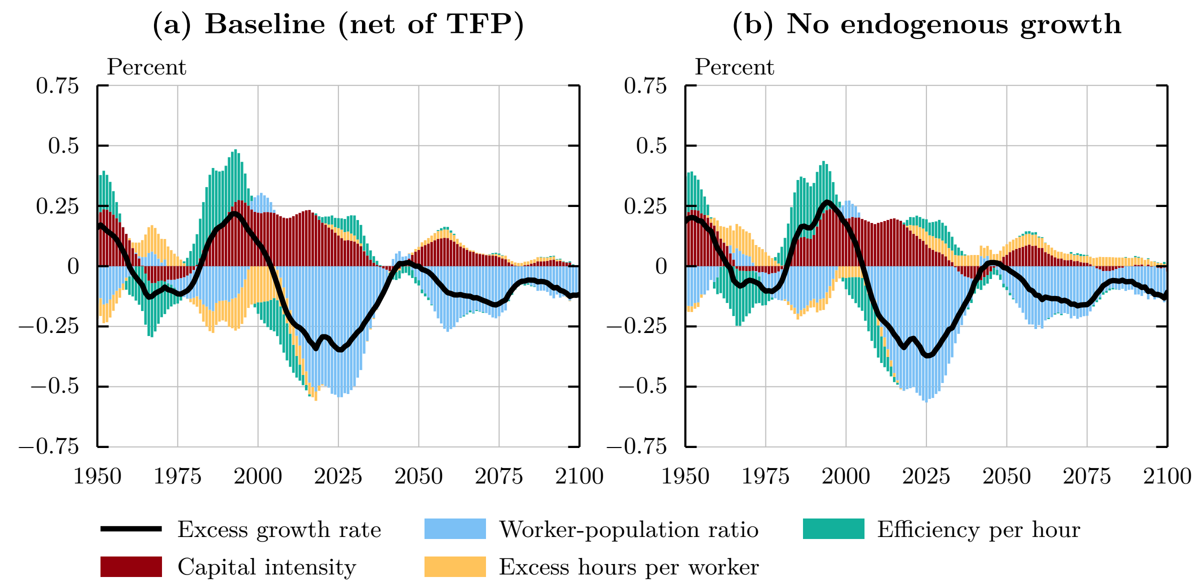

Figure A.4. Growth accounting comparison for the no-endogenous growth scenario.

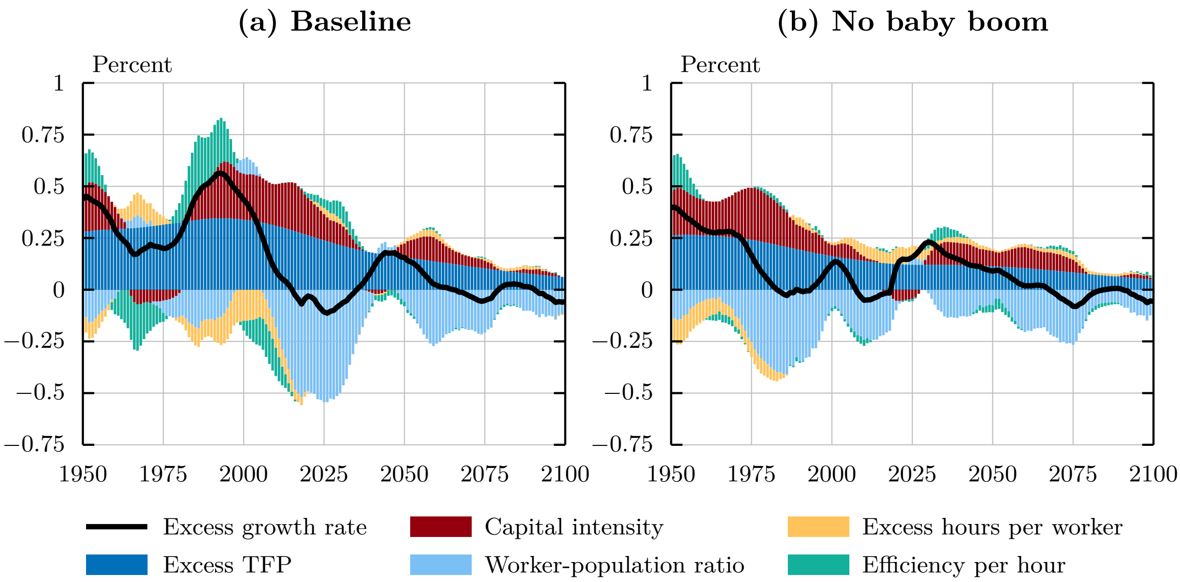

FIGURE A.5. Growth accounting comparison for the no-baby boom scenario.

Appendix B Deriving the stationary growth rates

The stationary growth rates in Definition 2 are straightforwardly derived using the combined insights from Jones (1995) and Boppart and Krusell (2020). These steps are outlined below. In the last subsection I also derive the intermediate firm growth rate when labour is the only R&D input, as considered in Section 7.

B.1 TFP, consumption per capita, hours per worker

In a stationary equilibrium, interior solutions to the household maximisation problem are characterised by the Euler equation, the intratemporal first-order condition, and the budget constraint as follows:

$$c_j^{-\sigma} = \beta s_{j+1}\left(1 + r(1-\tau^k)\right)\mathbb{E}\left[c_{j+1}^{-\sigma} \mid \eta\right], \tag{B.1}$$

$$\psi h_j^{1/\theta} = \frac{\left(1-\tau^{wm}(w\ell_j) - \tau^b\right)w\varepsilon_j\eta}{c_j^{\sigma}(1+\tau^c)}, \tag{B.2}$$

$$a_{j+1} + (1+\tau^c)c_j = \left(1 + r(1-\tau^k)\right)a_j + \left(1-\tau^w(w\ell_j) - \tau^b\right)w\ell_j + tr + b_j(R_j), \tag{B.3}$$

where τ^{wm} denotes the *marginal* income tax rate. Let $1 + g_x$ denote the stationary gross growth rate of a variable x. The Euler equation (B.1) implies a constant interest rate under balanced growth, which in turn only holds for a constant capital-output ratio since $r = \alpha\omega\frac{Y}{K} - \delta$. Cobb-Douglas production implies that

$$Y = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} ZL, \tag{B.4}$$

and aggregate growth therefore equals $(1+g_Z)(1+g_L)$. TFP is just the measure of intermediate firms raised to a power, $Z = z^{\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}$, so its growth rate is similarly the growth rate of intermediate firms raised to the same power. Moreover, since wages are standard neoclassical, $w = (1-\alpha)\frac{Y}{L}$, it follows from (B.4) that wages grow by the rate of TFP. For the first-order condition (B.2) to hold along a balanced growth path, we then necessarily need $(1+g_h)^{1/\theta} = (1+g_Z)(1+g_c)^{-\sigma}$. Meanwhile, the budget constraint (B.3) is only consistent with balanced growth if consumption grows by the same rate as labour income: $1 + g_c = (1+g_Z)(1+g_h)$. Combining these conditions yields that the growth rates of TFP, consumption per capita and hours per worker must satisfy

$$1 + g_Z = (1+g_z)^{\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}, \qquad 1 + g_c = (1+g_Z)^{\frac{1+\theta}{1+\theta\sigma}}, \qquad 1 + g_h = (1+g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}}. \tag{B.5}$$

Substituting in $1 + g_z = (1+n)^{\chi}$ into Equation (B.5) then gives the growth rates as stated in Definition 2.

B.2 Intermediary firm growth rate for the baseline model

It remains to show that the growth rate of intermediate firms is given by $1 + g_z = (1 + n)^{\chi}$. By Equation (11), it holds that $1 + g_z = 1 - \delta_z + \nu Q^{\lambda} z^{\phi - 1}$. Looking at the right-hand side, we get a constant growth rate $1 + g_z$ if and only if

$$(1 + g_z)^{1-\phi} = (1 + g_Q)^{\lambda}. \tag{B.6}$$

The growth rate of R&D investment equals the aggregate growth rate by the goods market condition. Moreover, we can decompose aggregate labour into total employment (E), average hours per worker ($\bar{h}$), and average productivity per hour worked ($\bar{\varepsilon}$): $L = E\,\bar{h}\,\bar{\varepsilon}$. In a stationary equilibrium, employment grows by the rate of the population $1 + n$ while average efficiency per hour is constant under a fixed population structure. It follows that the labour force growth rate is given by $1 + g_L = (1 + g_h)(1 + n)$. Together with Equation (B.5), this allows us to rewrite the growth rate of R&D investment into

$$1 + g_Q \;=\; (1 + g_Z)(1 + g_h)(1 + n) \;=\; (1 + g_z)^{\frac{1+\theta}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}(1 + n).$$

Plugging this into (B.6) and rearranging terms yields the growth rate in Definition 2:

$$1 + g_z = (1 + n)^{\chi}, \qquad \text{where} \qquad \chi \equiv \frac{\lambda(1 + \theta\sigma)}{(1 - \phi)(1 + \theta\sigma) - \lambda(1 + \theta)\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}. \tag{B.7}$$

B.3 Intermediary firm growth rate when R&D uses only labour

If R&D uses only labour, as considered in Section 7, then the R&D process is given by $1 + g_z = 1 - \delta_z + \nu L_z^{\lambda} z^{\phi-1}$. In a stationary equilibrium, R&D labour L_z must grow by the rate of total labour supply according to the labour market condition. Again inspecting the right-hand side, we therefore get a constant growth rate $1 + g_z$ if and only if

$$(1 + g_z)^{1-\phi} = (1 + g_L)^{\lambda}. \tag{B.8}$$

Using Equation (B.5), we can rewrite the labour force growth rate into

$$1 + g_L \;=\; (1 + g_h)(1 + n) \;=\; (1 + g_z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma}\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}(1 + n).$$

Plugging this into (B.8) and rearranging terms yields the growth rate

$$1 + g_z = (1 + n)^{\chi}, \qquad \text{where} \qquad \chi \equiv \frac{\lambda(1 + \theta\sigma)}{(1 - \phi)(1 + \theta\sigma) - \lambda\theta(1 - \sigma)\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}. \tag{B.9}$$

Note from Equations (B.5) and (B.9) that TFP growth collapses to the benchmark growth rate in Jones (1995) if we, like Jones, consider a steady state with constant hours worked (here via log preferences, $\sigma \to 1$) and an intermediary-firm markup which exactly equals the capital share parameter ($\omega = \alpha$).

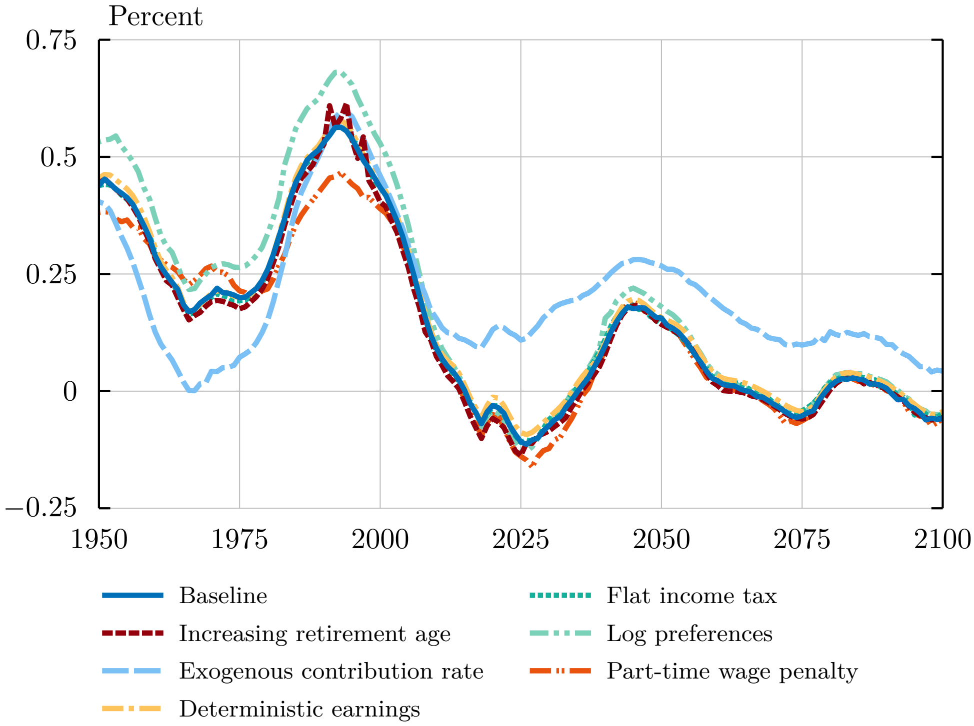

Figure C.1. Robustness checks: growth rates under different model specifications.

Appendix C Additional robustness checks

This appendix complements Section 7 by exploring several additional robustness checks which are outlined discussed below. In each scenario, I recalibrate the discount factor β and the disutility-of-labour weight ψ to match the same calibration targets as in the baseline. The net-of-trend growth rate of each alternative is plotted against the baseline in Figure C.1 while Figure C.2 shows their growth decompositions. Overall, neither alternative changes the qualitative conclusions. Similarly, the quantitative differences are negligible for all cases except the one with an exogenous pension contribution rate.

Increased retirement age. The baseline model assumes a social security system wherein the normal retirement age (NRA) and the early/delayed scaling of pension benefits remain fixed over time. In reality, there have been gradual increases in both the NRA and the delayed retirement credit. Postponed retirement influences all growth channels considered in the baseline results: it impacts TFP and the capital intensity through a reduced life-cycle savings motive, it impacts the worker-population ratio directly, and it alters average efficiency and hours per worker due to the hump shape in workers' productivity. Increasing the statutory retirement age may therefore influence growth in either direction. I consider an alternative in which the NRA and the delayed retirement credit increase as in reality. Specifically, I increase the NRA to 66 for cohorts born between 1940 and 1956 and to 67 for subsequent cohorts. Moreover, the delayed retirement credit is increased by 0.5 percentage points for every other cohort between 1924 and 1943. That is, the delayed retirement credit is 3 percent for cohorts born before 1925, 3.5 percent for the 1925–1926 cohorts, . . . , 7.5 percent for the 1941–1942 cohorts, and 8 percent for all subsequent cohorts. Overall, the difference to the baseline is small because most households in the baseline

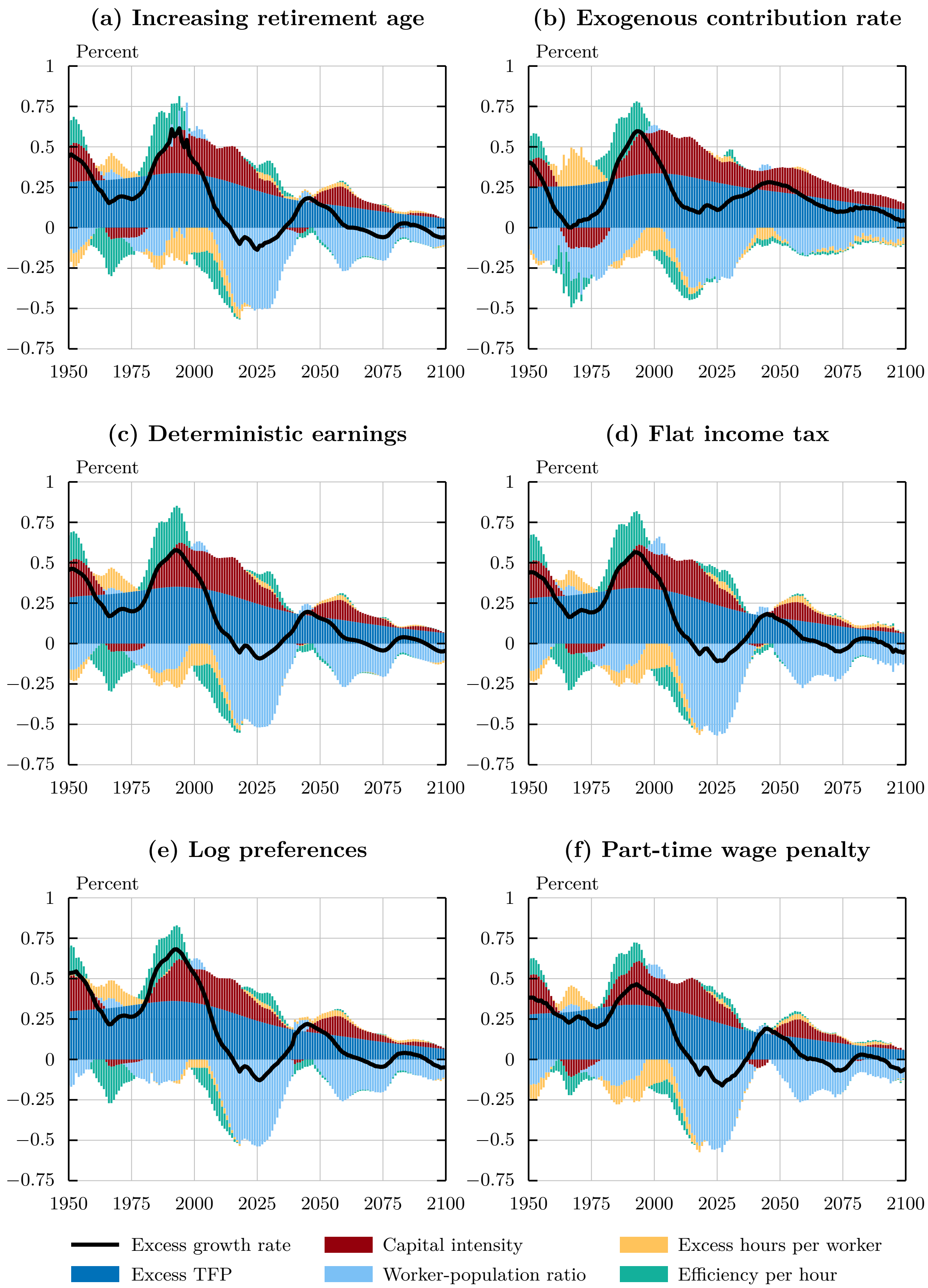

Figure C.2. Growth accounting of robustness checks.

already retire between the ages of 65 and 70 (see Figure 7), so making retirement in these ages more beneficial does not change much at the aggregate level.

Exogenous contribution rate. The baseline assumes an exogenous social security replacement rate and that the pension contribution rate adjusts endogenously to balance the social security budget. The alternative considered here lies on the other side of the spectrum: taking an exogenous contribution rate and letting the replacement rate adjust endogenously. Krueger and Ludwig (2007) stress the importance of this choice. Maintaining benefit levels under population ageing by increasing contribution rates reduces both the incentive (through higher transfer income once retired) and the ability (through higher taxation) to save for retirement. It also increases the opportunity cost of working once eligible for social security, thereby reducing the incentive to postpone retirement. Both effects dampen growth through all three core mechanisms emphasised in Section 3. Holding contribution rates fixed and cutting pension benefits works in the opposite direction by incentivising higher savings for retirement and a later age of retirement. Here, I construct an exogenous contribution rate based on the contributions to social insurance in the national accounts (see Appendix E) and feed this time series into the model. As shown in Figure E.1, the contribution rate rises monotonically throughout the post-war period. In the simulations, the resulting increase in contributions is more than sufficient to offset the increased pension bill of an ageing population, so benefits grow more generous via increases in the replacement rate, peaking at 0.6 around 1990. After 1990, the contribution rate stabilises. Yet, the population continues to grow older, causing the public sector to cut benefits throughout the twenty-first century via reductions in the replacement rate. The increased generosity during the twentieth century reduces growth relative to baseline by 0.09 percentage points per year as households retire earlier and save less. Conversely, the decline in the replacement rates during the twenty-first century increases growth relative to the baseline by 0.13 percentage points per year.

Deterministic earnings. In the baseline, households face uninsurable idiosyncratic productivity shocks. These shocks add a savings motive in addition to standard life-cycle behaviour. In an alternative, I remove this feature by making productivity deterministic. Although this reduces inequality and the overall level of household savings, the difference to the baseline turns out to be negligible.

Flat income tax. Rather than considering a progressive income tax, I analyse an alternative in which all households face a constant marginal (and average) income tax rate equal to that obtained from the national accounts: $\tau^w = 0.115$. Contrary to the deterministic earnings scenario, imposing a flat tax increases inequality and the level of household savings. Again, the difference to the baseline is nevertheless negligible.

Log preferences. Another nonstandard feature of the baseline model is that I consider preferences of the Boppart and Krusell (2020) class that generate declining hours worked along a balanced growth path. By contrast, a large part of the macroeconomic literature restricts itself to a subset of this class, defined by King, Plosser and Rebelo (1988), in which hours worked are constant in the long run. Constant long-run hours in my model are obtained by considering the special case with $\sigma \to 1$ so that flow utility becomes $u(c, h) = \log(c) - \psi \frac{h^{1+1/\theta}}{1+1/\theta}$. With $\sigma > 1$, the income effect of higher wages on hours worked exceeds the corresponding substitution effect.

Therefore, and as shown in Figure 6, households on average reduce hours worked during periods of positive growth, which dampens aggregate growth. When $\sigma \to 1$, the income and substitution effects exactly offset each other, so hours do not fall when growth is positive. Between 1950 and 2000, growth under logarithmic preferences is therefore about 0.1 percentage point higher per year. This difference is explained entirely by the different adjustments in hours worked. Aggregate growth during the twenty-first century, on the other hand, is virtually identical because, with wage growth around zero, the response in hours is similar in both scenarios.

Part-time wage penalty. Several authors stress the importance for nonconvexities in the budget set to generate endogenous retirement (see for instance Rogerson and Wallenius, 2013, and Ljungqvist and Sargent, 2014). Social security plays this role in the baseline model. An additional and often considered nonconvexity is nonlinear wages (see for instance French, 2005, and Kitao, 2014). This feature imposes that wages are an increasing function of hours worked to capture the empirical observation that part-time work does not pay the same hourly wage as full-time work. I consider a specification that follows French (2005) by imposing a total labour income at age j given by $w\varepsilon_j \eta h_j^{1+\xi}$, with $\xi \geq 0$. The labour market condition then changes to $L = \sum_j \int_X \varepsilon_j \eta h_j(x)^{1+\xi} \, \mathrm{d}\Phi_j$ and, by similar derivations as in Appendix B, it can be shown that the long-run growth rate of intermediate firms becomes

$$1 + g_z = (1+n)^{\chi}, \qquad \text{where} \qquad \chi \equiv \frac{\lambda\big(1 + \theta\sigma - \xi\theta(1-\sigma)\big)}{(1-\phi)\big(1+\theta\sigma - \xi\theta(1-\sigma)\big) - \lambda(1+\theta)\frac{\alpha}{1-\alpha}\frac{1-\omega}{\omega}}.$$

The long-run growth rates of consumption per capita and hours per worker become, respectively,

$$1 + g_c = (1+g_Z)^{\frac{1+\theta}{1+\theta\sigma-\xi\theta(1-\sigma)}} \qquad \text{and} \qquad 1 + g_h = (1+g_Z)^{\frac{\theta(1-\sigma)}{1+\theta\sigma-\xi\theta(1-\sigma)}}.$$

For $\xi = 0$, wages are a linear function of hours worked and this reduces the model to the baseline case. Here, I follow French and set the value of ξ to 0.415 based on Aaronson and French's (2004) empirical finding that a 50 percent reduction in hours also leads to a 25 percent reduction in the hourly wage. I recalibrate the model under the assumption that $\xi = 0.415$ is the true value, which yields a lower intertemporal elasticity of substitution, $\sigma = 1.84$, and a flatter age-efficiency profile.[21] The inclusion of the part-time wage penalty nevertheless does little to alter the retirement dynamics of the model. Growth rates are overall similar to the baseline and the contributions of TFP, the capital intensity, the worker-population ratio and average hours worked remain unchanged. The average annual growth rate for the twentieth century is overall similar (0.32 percent versus 0.35 percent in the baseline) but the dynamics over time are more stable. This follows from smaller swings in average productivity due to two reasons. First, average productivity for an age-j worker, $\varepsilon_j \eta h_j^{\xi}$, is now a function of hours worked since full-time workers are more productive. Therefore, declines in hours worked negatively impacts average productivity. Second, the age-efficiency profile is flatter than in the baseline. Changes in the age composition of the labour force, which are more prominent in the twentieth century, therefore leads to smaller changes in average efficiency.

[21] For the baseline, I estimate the age-efficiency profile $\{\varepsilon_j\}_{j=\iota}^{J}$ by constructing a measure of wages in the PSID data by dividing annual labour income with annual hours . Here, I assume that $\xi = 0.415$ holds and instead construct PSID wages as (total annual labour income)/(annual hours worked)$^{1.415}$ (see Appendix D for estimation details).

Appendix D Estimating the life cycle earnings profile

I parametrise the age efficiency profile $\{\varepsilon_j\}_{j=\iota}^{J}$ as the exponential of a quadratic age polynomial: $\varepsilon_j = \exp\left\{\vartheta_0 + \vartheta_1 j + \vartheta_2 j^2\right\}$. In the model, total labour earnings of individual i of age j at time t is $e_{ijt} = w_t \varepsilon_j \eta_{ijt} h_{ijt}$. Dividing both sides by hours h_{ijt} and taking logs yields an individual hourly wage of the form $\ln w_{ijt} = \ln w_t + \ln \eta_{ijt} + \vartheta_0 + \vartheta_1 j + \vartheta_2 j^2$. Here, the overall wage $\ln w_t$ is a common time variable whereas η_{ijt} captures any idiosyncratic earnings differences. This motivates the following fixed effects model:

$$\ln w_{ijt} \;=\; \varrho_t + \varrho_i + \vartheta_0 + \vartheta_1 j + \vartheta_2 j^2 + u_{ijt}, \tag{D.1}$$

where ϱ_t is a time fixed effect, ϱ_i is an individual fixed effect, and u_{ijt} is the error term. Equation (D.1) implicitly captures cohort fixed effects through the individual fixed effects and therefore posits a linear relationship between time, age and cohorts. It is well known that collinearity between the three prohibits simultaneous identification of these effects. To remedy this issue, I follow the approach advocated by Heckman and Robb (1985) and replace the time fixed effect by two macroeconomic variables which plausibly proxy for the underlying unobserved time variables in the context of an earnings regression: the aggregate log real wage level and the percentage point deviation of the unemployment rate from its long-run mean. The former corresponds to $\ln w_t$ above and controls for secular wage growth and the latter (which is also used by French, 2005) controls for fluctuations in the business cycle.

I estimate Equation (D.1) using micro data on earnings from the *Panel Study of Income Dynamics* (PSID) for survey years 1968 to 2019. Since income and employment in the PSID refer to the previous year, the data correspond to calendar years 1967 to 2018. I consider households from the nationally representative SRC sample and construct individual wages as total annual labour income divided by annual hours worked. The aggregate wage used to proxy the time fixed effect is constructed from the national accounts by dividing total private industry wages (BEA NIPA Table 2.3, line 4) by total private industry hours worked (BEA NIPA Table 6.9, line 3). The unemployment rate is obtained from the Bureau of Labor Statistics (BLS, series ID LNS14000000). All nominal variables are deflated into 2012 dollars using the PCE price index (BEA NIPA Table 2.3.4, line 1).

For the benchmark estimation, I impose standard sample restrictions (see for instance French, 2005, Heathcote, Storesletten and Violante, 2010, and Huggett, Ventura and Yaron, 2011): I select male household heads with no inconsistencies in reported age, who work between 300 and 5,840 hours a year (30 percent of part time and twice full time, respectively), and whose hourly wage exceeds \$3 per hour and does not exceed \$100 per hour in 2012 dollars. I consider individuals between the ages of 18 and 75. This goes against the standard practice of excluding ages at the beginning and end of the working life to avoid sample selection issues relating to work-life entry and exit. This choice is motivated by the need for an efficiency profile for all ages above 20, given that retirement in the model is endogenous. The alternative, estimating the age profile on individuals between, say, the ages of 25 and 60, would instead require extrapolation of the age profile to younger and older ages, and it is not clear that this approach is preferable. An upper bound at 75 is nevertheless imposed to ensure there are at least 100 observations in each age group. Extrapolation beyond 75 is inconsequential, since between 95 and 99 percent of model households retire before 75. The final sample consists of 90,832 person-year observations.

Table D.1. Estimation of deterministic age-efficiency profile.

	Benchmark	Robustness checks					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ϑ_0	−1.3734***	0.9857***	1.1419***	−0.9102***	−1.2920***	−1.0902***	−1.1382***
	(0.22496)	(0.01914)	(0.03645)	(0.17826)	(0.20430)	(0.16550)	(0.10446)
ϑ_1	0.0734***	0.0954***	0.0835***	0.0813***	0.0709***	0.0610***	0.0606***
	(0.00202)	(0.00100)	(0.00184)	(0.00183)	(0.00183)	(0.00145)	(0.00095)
ϑ_2	−0.0008***	−0.0010***	−0.0008***	−0.0008***	−0.0008***	−0.0007***	−0.0007***
	(0.00002)	(0.00001)	(0.00002)	(0.00002)	(0.00002)	(0.00002)	(0.00001)
Individual FE	✓		✓	✓	✓	✓	✓
Time controls[a]	✓			✓	✓	✓	✓
Aggregate wage from	BEA			BLS	BEA	BEA	BEA
Female heads					✓	✓	✓
Spouses/partners						✓	✓
Additional controls[b]							✓
Observations	90,832	90,832	90,832	90,832	110,169	165,034	161,012
Adjusted R^2	0.160	0.121	0.156	0.162	0.158	0.152	0.153

Notes. Dependent variable: log real hourly wage. Wages defined as labour earnings/hours. Regressors of interest: quadratic age polynomial with coefficients ϑ_0, ϑ_1, ϑ_2. Robust standard errors in parentheses. *, **, and *** denote statistical significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

[a] Includes controls for the aggregate real wage level (using BEA or BLS data) and business cycle fluctuations (as proxied by the deviation of the unemployment rate from its long-run mean).

[b] Includes controls for education level (eight groups), family size, marital status (married, single, widowed, divorced, separated), and fixed effects for place of residence (50 states and D.C. plus abroad).

Table D.1 shows the benchmark estimation results along with several robustness checks. Column (1) is the benchmark corresponding to the age profile in Figure 3a. Column (2) shows standard OLS estimates and column (3) includes only individual fixed effects. In both of these, any secular wage growth is interpreted as being due to age differences. This inflates the ϑ_1 estimates and provides steeper profiles with higher peaks; peak age productivity is about 115 to 130 percent larger than in the initial age, as opposed to only 70 percent in the benchmark model. This underlines the importance of controlling for time effect. Column (4) changes the aggregate wage variable in the benchmark to average hourly earnings of production and nonsupervisory employees (collected from the BLS, series ID CES0500000008). It is known that this wage measure exhibits lower wage growth over recent decades than the imputed wage measure from the national accounts. The estimation results are therefore similar to column (2) and (3).[22] Columns (5) and (6) expand the sample to include spouses and partners as well as female household heads. Column (7) adds additional individual-level controls which may change over time and are therefore not captured by individual fixed effects. These additions lower the point estimates further, although these samples raise additional concerns for sample selection and also provide worse fits to the data as measured by the adjusted R^2.

Lastly, in one robustness check I consider an earnings schedule where wages are an increasing function of hours worked such that part-time workers have lower hourly wages then full-time workers

[22] I stick with the BEA measure for the aggregate wage as the benchmark since the BLS measure is more limited in scope.

Table D.2. Estimation of deterministic age-efficiency profile with part-time wage penalty.

	Benchmark	Robustness checks					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ϑ_0	-4.1083^{***}	-1.8276^{***}	-1.6727^{***}	-3.6135^{***}	-4.0415^{***}	-3.9382^{***}	-3.9195^{***}
	(0.22905)	(0.01998)	(0.03658)	(0.18088)	(0.20837)	(0.16694)	(0.11210)
ϑ_1	0.0540^{***}	0.0757^{***}	0.0634^{***}	0.0616^{***}	0.0515^{***}	0.0438^{***}	0.0400^{***}
	(0.00203)	(0.00104)	(0.00184)	(0.00184)	(0.00183)	(0.00144)	(0.00101)
ϑ_2	-0.0006^{***}	-0.0008^{***}	-0.0006^{***}	-0.0006^{***}	-0.0006^{***}	-0.0005^{***}	-0.0004^{***}
	(0.00002)	(0.00001)	(0.00002)	(0.00002)	(0.00002)	(0.00002)	(0.00001)
Individual FE	✓		✓	✓	✓	✓	✓
Time controls[a]	✓			✓	✓	✓	✓
Aggregate wage from	BEA			BLS	BEA	BEA	BEA
Female heads					✓	✓	✓
Spouses/partners						✓	✓
Additional controls[b]							✓
Observations	90,832	90,832	90,832	90,832	110,169	165,034	161,012
Adjusted R^2	0.123	0.094	0.120	0.125	0.120	0.110	0.113

Notes. Dependent variable: log real hourly wage. Wages defined as labour earnings/hours$^{1.415}$. Regressors of interest: quadratic age polynomial with coefficients ϑ_0, ϑ_1, ϑ_2. Robust standard errors in parentheses. *, **, and *** denote statistical significance at the 10 percent, 5 percent, and 1 percent levels, respectively.

[a] Includes controls for the aggregate real wage level (using BEA or BLS data) and business cycle fluctuations (as proxied by the deviation of the unemployment rate from its long-run mean).

[b] Includes controls for education level (eight groups), family size, marital status (married, single, widowed, divorced, separated), and fixed effects for place of residence (50 states and D.C. plus abroad).

(see Appendix C). In this case, I estimate the age profile (D.1) identically to above with the only exception that wages are constructed as (total annual labour income)/(annual hours worked)$^{1.415}$ from the PSID data. The corresponding estimation results are shown in Table D.2.

Appendix E Constructing tax rates

Below I provide further details on the calibration of all model taxes described in Section 5.4. First I describe the construction of the aggregate tax rates obtained from the national accounts, then the estimation a progressive income tax function, and finally the calibration of the income tax function used in the model (which builds on the former two).

E.1 Aggregate tax rates

The methodology for computing the aggregate tax rates on consumption, capital and labour income is taken off-the-shelf from Fernández-Villaverde *et al.* (2015), which in turn builds on Mendoza, Razin and Tesar (1994) and Jones (2002), and I restate it here for completeness. In short, each tax rate is constructed by aggregating all relevant tax revenues at the general government level and then dividing by the corresponding tax base. All data for this exercise are available in the national income and product accounts (NIPA) provided by the US Bureau of Economic Analysis (BEA). Table E.1 summarises the variables that I use.

Table E.1. Tax data variables.

Variable	Explanation	Source	
C	Personal consumption expenditures	BEA NIPA Table 1.1.5	line 2
EC	Compensation of employees	BEA NIPA Table 1.12	line 2
W	Wages and salaries	BEA NIPA Table 1.12	line 3
PRI	Proprietors' income[a]	BEA NIPA Table 1.12	line 9
RI	Rental income of persons[a]	BEA NIPA Table 1.12	line 12
CP	Corporate profits[a]	BEA NIPA Table 1.12	line 13
NI	Net interest and miscellaneous payments	BEA NIPA Table 1.12	line 18
PCT	Personal current taxes	BEA NIPA Table 3.1	line 3
TPI	Taxes on production and imports	BEA NIPA Table 3.1	line 4
CT	Taxes on corporate income	BEA NIPA Table 3.1	line 5
CSI	Contributions for government social insurance	BEA NIPA Table 3.1	line 7
PRT	Property taxes	BEA NIPA Table 3.3	line 9

[a] With inventory valuation adjustment and capital consumption adjustment.

The average consumption tax rate τ^c is constructed as

$$\tau^c = \frac{TPI - PRT}{C - (TPI - PRT)}. \tag{E.1}$$

The numerator of (E.1) is the revenue from consumption taxation. I subtract property taxes from total taxes on production because homeowners in the national accounts are treated as businesses that rent their properties to themselves. Property taxes are therefore incorporated as taxes on capital instead. The consumption tax base in the denominator is total personal consumption expenditures net of consumption taxes paid (that is, the pre-tax value of consumption).

The national accounts do not provide a breakdown of personal current taxes into labour and capital income. To construct an estimate of the split into labour and capital income, I construct an average personal income tax rate τ^p as an intermediate step via

$$\tau^p = \frac{PCT}{W + PRI/2 + CI}, \tag{E.2a}$$

$$CI \equiv PRI/2 + RI + CP + NI. \tag{E.2b}$$

The numerator is the sum of personal current taxes at the federal, state and local levels. The tax base is the sum of wages, proprietors' income and capital income (CI), in which we assume that proprietors' income is split evenly between labour and capital income. This assumption follows Jones (2002), who emphasises that any split of proprietor's income into labour and capital income is somewhat arbitrary and therefore chooses the fifty-fifty split as a middle ground.

I then estimate the total revenue from personal taxes on income and capital as $\tau^p(W+PRI/2)$ and $\tau^p CI$, respectively. The average labour income and capital tax rates τ^w and τ^k are subsequently

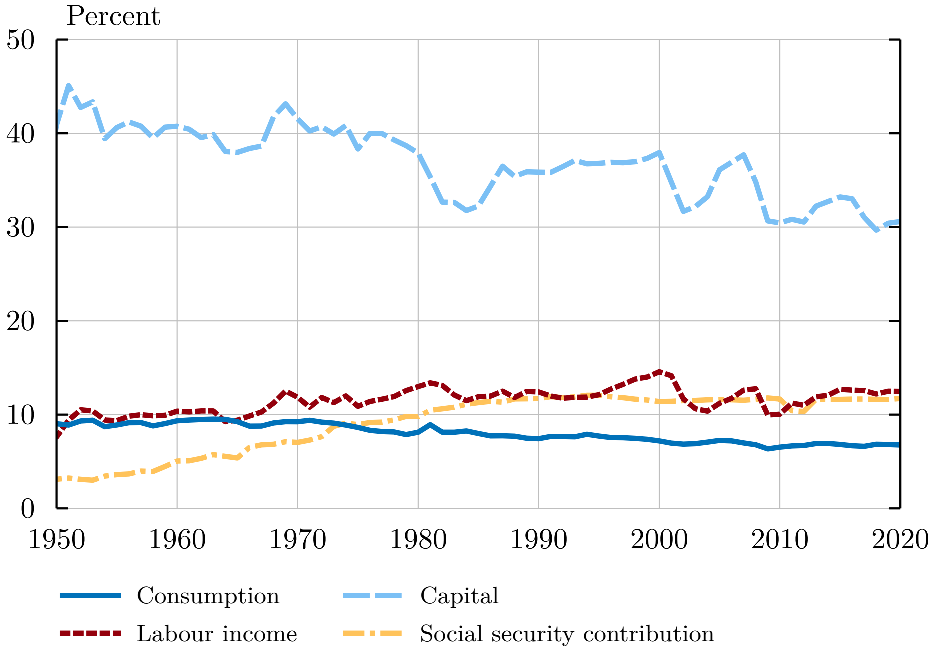

FIGURE E.1. Aggregate tax rate estimates from the national accounts.

computed as

$$\tau^w = \tau^p \frac{W + PRI/2}{EC + PRI/2} \tag{E.3}$$

and

$$\tau^k = \frac{\tau^p CI + CT + PRT}{CI + PRT}, \tag{E.4}$$

respectively. Lastly, in one robustness check I take the social security contribution rate τ^b as exogenous. For this scenario, I construct the social security tax rate as

$$\tau^b = \frac{CSI}{EC + PRI/2}. \tag{E.5}$$

The sum of Equations (E.3) and (E.5) gives the measure of the average labour income tax rate used by Fernández-Villaverde *et al.* (2015). Figure E.1 plots the four estimated tax rates over time, which highlights that imposing constant tax rates in the model for consumption, labour income and capital is a reasonable assumption.

E.2 Estimating the income tax rate function

To estimate the progressive income tax function, I compute effective tax rates at different levels of income and fit the parametrised tax function (34) to these artificial data. The construction of the tax rates follows the methodology used in the OECD tax database. In essence, we start from a given gross labour income and then create a hypothetical effective tax rate based on the applicable tax rules and regulations in the year of interest. The effective tax rate is measured as the total net tax liability divided by gross income. The net tax liability in turn is total taxes paid minus any tax credits received, where total taxes paid is given by applying taxable earnings (gross income less standard deductions) to the relevant marginal tax rates.

The OECD considers three types of households: singles without children, heads of households with children, and married couples filing jointly (with or without children). In addition to regular income taxes, the calculations also include social security contributions from employers and employees. Moreover, taxation at all levels of government (federal, state, local) is considered. At the state and local levels, the OECD therefore assumes a representative worker that lives in Detroit, Michigan.

In line with the primary estimates published in the OECD tax database,[23] I only consider singles without children in my calibration. This choice is of secondary importance since I eventually scale the estimated tax function to match the national accounts. I also abstract from social security contributions since those are modelled separately in my framework. The subsections below outline the formulas, parameters and parameter values used for this particular case as well as the estimation procedure and results.[24]

E.2.1 Taxable earnings

Taxable earnings at government level $\mathtt{x} \in \{\mathtt{fed}, \mathtt{state}, \mathtt{local}\}$ is given by gross income $\mathtt{GI}$ minus a tax allowance $\mathtt{TAXALLOW}_x$, provided that this is positive:

$$e^{\mathtt{x}}(\mathtt{GI}) = \max\{\mathtt{GI} - \mathtt{TAXALLOW}_{\mathtt{x}}, 0\}.$$

At the federal level, the allowance consists of a standard deduction $\mathtt{STDALLOW}$ and a personal exemption $\mathtt{EXEMPT}_{\mathtt{fed}}$. The personal exemption is reduced at a taper rate $\varphi^T_{\mathtt{ex}}$ for every USD 2,500 by which gross income exceeds the threshold $\mathtt{THOLD}_{\mathtt{ex}}$. At the state and local levels, the allowances are fixed personal exemptions $\mathtt{EXEMPT}_{\mathtt{state}}$ and $\mathtt{EXEMPT}_{\mathtt{local}}$, respectively. Thus,

$$\begin{aligned}
\mathtt{TAXALLOW}_{\mathtt{fed}} &= \mathtt{STDALLOW} + \mathtt{EXEMPT}_{\mathtt{fed}}\left(1 - \varphi^T_{\mathtt{ex}}\left\lceil \frac{\max\{\mathtt{GI} - \mathtt{THOLD}_{\mathtt{ex}}, 0\}}{2500}\right\rceil\right), \\
\mathtt{TAXALLOW}_{\mathtt{state}} &= \mathtt{EXEMPT}_{\mathtt{state}}, \\
\mathtt{TAXALLOW}_{\mathtt{local}} &= \mathtt{EXEMPT}_{\mathtt{local}},
\end{aligned}$$

where $\lceil \cdot \rceil$ is the ceiling function: $\lceil x \rceil = \min\{n \in \mathbb{Z} : n \geq x\}$.

E.2.2 Taxes

The federal income tax is progressive, with higher marginal tax rates at higher levels of income. Specifically, consider N federal tax brackets with marginal tax rates $\tau_1^{\mathtt{fed}}, \dots, \tau_N^{\mathtt{fed}}$ starting at

[23] See Table I.5, available for download at OECD.Stat.

[24] The implementation code (available upon request) also incorporates the other cases (with respect to household composition and social security contributions) and can therefore construct tax rates for these cases as well. Supplementary documentation for these cases is given in the annual OECD publication *Taxing Wages* (available in the OECD iLibrary).

earnings thresholds $\bar{e}_1, \ldots, \bar{e}_N$ (where $\bar{e}_1 = 0$). Taxable earnings at the state and local levels are subject to flat tax rates $\tau^{\texttt{state}}$ and $\tau^{\texttt{local}}$, respectively. Given a largest applicable federal tax bracket $I = \max\{i : e^{\texttt{fed}}(\texttt{GI}) > \bar{e}_i\}$, the total tax liability at each level of government is then given by functions $T^{\texttt{x}}(\texttt{GI})$ of gross income as follows:

$$
\begin{aligned}
T^{\texttt{fed}}(\texttt{GI}) &= \sum_{i=1}^{I-1} \tau_i^{\texttt{fed}} \Big(\bar{e}_{i+1} - \bar{e}_i\Big) + \tau_I^{\texttt{fed}} \Big(e^{\texttt{fed}}(\texttt{GI}) - \bar{e}_I\Big), \\
T^{\texttt{state}}(\texttt{GI}) &= \tau^{\texttt{state}}\, e^{\texttt{state}}(\texttt{GI}), \\
T^{\texttt{local}}(\texttt{GI}) &= \tau^{\texttt{local}}\, e^{\texttt{local}}(\texttt{GI}).
\end{aligned}
$$

E.2.3 Tax credits

The OECD considers three types of federal tax credits: the Earned Income Tax Credit (EIC), the Child Tax Credit, and the Making Work Pay tax credit (MWP). Since we only consider households without children, we can discard from the Child Tax Credit. The EIC and the MWP provide refundable tax credits equal to some fractions $\varphi_{\texttt{eic}}$ and $\varphi_{\texttt{mwp}}$ of gross income up to some maximum amounts $\varphi_{\texttt{eic}}\overline{\texttt{eic}}$ and $\overline{\texttt{mwp}}$, respectively. The tax credits are phased down at taper rates $\varphi^T_{\texttt{eic}}$ and $\varphi^T_{\texttt{mwp}}$ once gross income exceeds thresholds $\texttt{THOLD}_{\texttt{eic}}$ and $\texttt{THOLD}_{\texttt{mwp}}$. The total tax credit amount from these programs are therefore given by

$$
\texttt{eic}(\texttt{GI}) = \max\Big\{\varphi_{\texttt{eic}} \min\Big\{\texttt{GI}, \overline{\texttt{eic}}\Big\} - \varphi^T_{\texttt{eic}} \max\Big\{\texttt{GI} - \texttt{THOLD}_{\texttt{eic}}, 0\Big\}, 0\Big\}
$$

and

$$
\texttt{mwp}(\texttt{GI}) = \max\Big\{\min\Big\{\varphi_{\texttt{mwp}}\texttt{GI}, \overline{\texttt{mwp}}\Big\} - \varphi^T_{\texttt{mwp}} \max\Big\{\texttt{GI} - \texttt{THOLD}_{\texttt{mwp}}, 0\Big\}, 0\Big\}.
$$

Total federal tax credits is the sum of EIC and MWP. At the state level, the OECD includes the Michigan Earned Income Tax Credit, which is an additional refundable credit equal to a fraction $\varphi_{\texttt{meic}}$ of the federal EIC amount. The local level incorporates the Michigan City Income Tax Credit (CTC) which is a nonrefundable credit equal to some fraction of the total local tax liability $T^{\texttt{local}}(\texttt{GI})$ up to some maximum amount $\overline{\texttt{ctc}}$. Below this upper bound, the CTC credit rates decline with income. As with the federal income tax, consider N credit rate brackets with marginal credit rates $\varphi_{1,\texttt{ctc}}, \ldots, \varphi_{N,\texttt{ctc}}$ starting at tax liability thresholds $\overline{T}_1, \ldots, \overline{T}_N$ (where $\overline{T}_1 = 0$). Given a largest applicable tax credit bracket $I = \max\{i : T^{\texttt{local}}(\texttt{GI}) > \overline{T}_i\}$, the total tax credit at each level of government is then given by functions $C^{\texttt{x}}(\texttt{GI})$ of gross income as follows:

$$
\begin{aligned}
C^{\texttt{fed}}(\texttt{GI}) &= \texttt{eic}(\texttt{GI}) + \texttt{mwp}(\texttt{GI}), \\
C^{\texttt{state}}(\texttt{GI}) &= \varphi_{\texttt{meic}}\, \texttt{eic}(\texttt{GI}),
\end{aligned}
$$

$$C^{\texttt{local}}(\texttt{GI}) \;=\; \min\left\{\sum_{i=1}^{I-1}\varphi_{i,\texttt{ctc}}\Big(\overline{T}_{i+1}-\overline{T}_i\Big) \;+\; \varphi_{I,\texttt{ctc}}\Big(T^{\texttt{local}}(\texttt{GI})-\overline{T}_I\Big),\; \overline{\texttt{ctc}}\right\}.$$

E.2.4 Effective income tax rate

The effective income tax rate $\tau^w(\texttt{GI})$ at gross income $\texttt{GI}$ is the total tax liability net of tax credits measured as a percentage of gross income:

$$\tau^w(\texttt{GI}) \;=\; \frac{1}{\texttt{GI}}\sum_{\texttt{x}\in X}\Big(T^{\texttt{x}}(\texttt{GI})-C^{\texttt{x}}(\texttt{GI})\Big),$$

where $X=\{\texttt{fed},\texttt{state},\texttt{local}\}$. In the practical implementation of these tax calculations, we consider an average gross income level $\overline{\texttt{GI}}$ and then express all other gross incomes as a percentage of that average.

E.2.5 Estimation

I use the methodology above to create effective income tax rates on a grid of gross incomes for each year from 2000 to 2020. The grid is linearly spaced with 401 points and ranges from 0 to a multiple 20 of average gross income. All the necessary parameter values for this exercise are collected from the OECD and are listed in Table E.2. From these artificial tax rates, I then fit the income tax function $\tau^w(\texttt{GI}) = \kappa_0\left[1-\left(\kappa_2\left(\frac{\texttt{GI}}{\overline{\texttt{GI}}}\right)^{\kappa_1}+1\right)^{-\frac{1}{\kappa_1}}\right]+\kappa_3$ by a nonlinear OLS estimation.

Figure E.2 shows the constructed tax rates for the lower part of the income grid together with the corresponding fit and its estimated coefficients. Interestingly, even though the time period considered saw two major tax reforms (the Economic Growth and Tax Reconciliation Relief Act of 2001 and the Tax Cuts and Jobs Act of 2017) and underwent three economic downturns (the early 2000s recession, the Great Recession, and the COVID-19 recession), effective income tax rates remain largely stable over this time period. Therefore, the estimated tax function provides a close fit of the constructed tax rates, as seen by the high R^2 of 0.97.

E.3 Changing the tax rate level while maintaining progressivity

Once the income tax function is estimated following the steps in Appendix E.2, I adjust its level without changing its progressivity using a similar approach as Guvenen, Kuruscu and Ozkan (2014). To that end, let $\widetilde{\tau}(e)$ be an average tax rate function of the Gouveia and Strauss (1994) form:

$$\widetilde{\tau}(e) \;=\; \widetilde{\kappa}_0\left[1-\left(\widetilde{\kappa}_2\left(\frac{e}{\bar{e}}\right)^{\widetilde{\kappa}_1}+1\right)^{-\frac{1}{\widetilde{\kappa}_1}}\right]+\widetilde{\kappa}_3. \tag{E.6}$$

Denote its corresponding marginal tax rate by $\widetilde{\tau}^m(e)=\frac{\partial}{\partial e}\big(\widetilde{\tau}(e)e\big)$. Suppose we want to change this tax function into a similarly parametrised function $\tau(e)$ with parameters $\kappa_0,\ldots,\kappa_3$ without changing the degree of progressivity. Following Guvenen, Kuruscu and Ozkan (2014), we then need the ratio of net take-home shares at any two earnings levels e and e' to be equal between the two tax systems:

$$\frac{1-\tau^m(e')}{1-\tau^m(e)} \;=\; \frac{1-\widetilde{\tau}^m(e')}{1-\widetilde{\tau}^m(e)}.$$

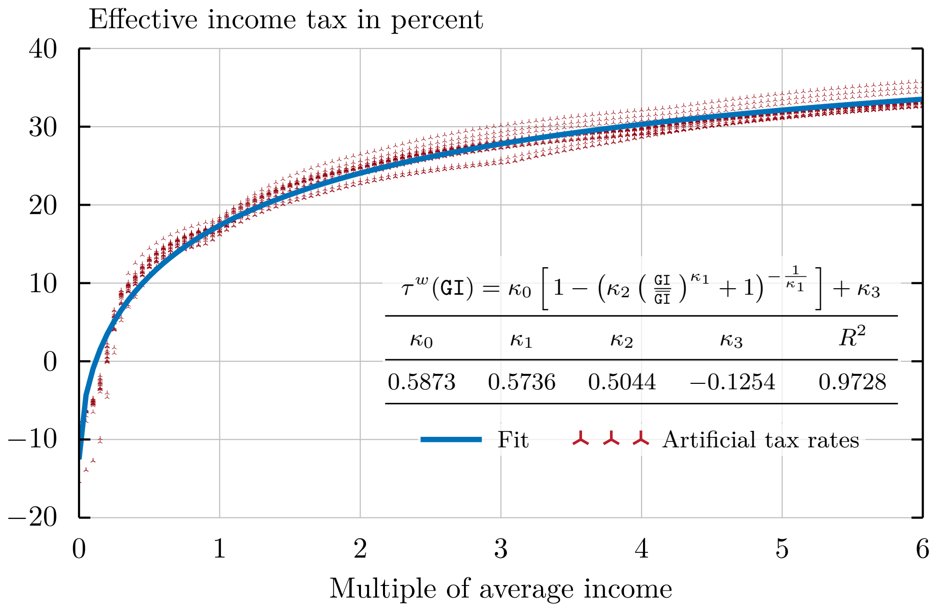

FIGURE E.2. Estimation of the income tax function.

This expression can be rearranged to obtain

$$\tau^m(e) = 1 - \bar{k}\big(1 - \widetilde{\tau}^m(e)\big), \qquad \text{where} \qquad \bar{k} \equiv \frac{1 - \tau^m(e')}{1 - \widetilde{\tau}^m(e')} \tag{E.7}$$

is a level ratio between the two tax systems that we are free to choose. Since $\tau(e)e = \int_0^e \tau^m(x)\,dx$, we can integrate Equation (E.7) to obtain an average tax rate of a similar form:

$$\tau(e) = 1 - \bar{k}\big(1 - \widetilde{\tau}(e)\big). \tag{E.8}$$

Substituting Equation (E.6) into (E.8) and rearranging terms, we finally get

$$\tau(e) = \kappa_0 \left[1 - \left(\kappa_2 \left(\frac{e}{\bar{e}}\right)^{\kappa_1} + 1\right)^{-\frac{1}{\kappa_1}}\right] + \kappa_3,$$

where $\kappa_0 \equiv \bar{k} \cdot \widetilde{\kappa}_0$, $\kappa_1 \equiv \widetilde{\kappa}_1$, $\kappa_2 \equiv \widetilde{\kappa}_2$ and $\kappa_3 \equiv 1 - \bar{k}(1 - \widetilde{\kappa}_3)$. The parameters $\kappa_0, \ldots, \kappa_3$ for the model income tax function in Table 3 use the estimates in Figure E.2 as $\widetilde{\kappa}_0, \ldots, \widetilde{\kappa}_3$ and sets the scale parameter $\bar{k}$ such that the tax rate at average income $\bar{e}$ matches the income tax rate computed from the national accounts. Using Equation (E.8) and denoting the national accounts tax rate by τ^{NA}, this scaling requires that $\tau^{NA} = 1 - \bar{k}\big(1 - \widetilde{\tau}(\bar{e})\big)$. This can be rearranged to give the scale parameter as

$$\bar{k} = \frac{1 - \tau^{NA}}{1 - \widetilde{\tau}(\bar{e})}, \qquad \text{where} \qquad \widetilde{\tau}(\bar{e}) = \widetilde{\kappa}_0 \left[1 - \left(\widetilde{\kappa}_2 + 1\right)^{-\frac{1}{\widetilde{\kappa}_1}}\right] + \widetilde{\kappa}_3$$

is a function of estimated parameters only.

Table E.2. Income tax parameters.

	Average income	Standard deduction	Federal personal exemption			Earned Income Tax Credit				Making Work Pay Tax Credit			
			Amount	Threshold	Taper	Rate	Threshold	Taper	Max	Rate	Threshold	Taper	Max
Year	$\overline{\texttt{GI}}$	$\texttt{STDALLOW}$	$\texttt{EXEMPT}_{\texttt{fed}}$	$\texttt{THOLD}_{\texttt{ex}}$	$\varphi^T_{\texttt{ex}}$	$\varphi_{\texttt{eic}}$	$\texttt{THOLD}_{\texttt{eic}}$	$\varphi^T_{\texttt{eic}}$	$\overline{\texttt{eic}}$	$\varphi_{\texttt{mwp}}$	$\texttt{THOLD}_{\texttt{mwp}}$	$\varphi^T_{\texttt{mwp}}$	$\overline{\texttt{mwp}}$
2000	\$33,129	\$4,400	\$2,800	\$128,950	2%	7.65%	\$5,800	7.65%	\$4,600				
2001	\$33,998	\$4,550	\$2,900	\$132,950	2%	7.65%	\$5,950	7.65%	\$4,750				
2002	\$35,026	\$4,700	\$3,000	\$137,300	2%	7.65%	\$6,100	7.65%	\$4,900				
2003	\$36,084	\$4,750	\$3,050	\$139,500	2%	7.65%	\$6,240	7.65%	\$4,990				
2004	\$36,739	\$4,850	\$3,100	\$142,700	2%	7.65%	\$6,390	7.65%	\$5,100				
2005	\$37,637	\$5,000	\$3,200	\$145,950	2%	7.65%	\$6,530	7.65%	\$5,220				
2006	\$39,377	\$5,150	\$3,300	\$150,500	1.33%	7.65%	\$6,740	7.65%	\$5,380				
2007	\$42,064	\$5,350	\$3,400	\$156,400	1.33%	7.65%	\$7,000	7.65%	\$5,590				
2008	\$43,196	\$5,450	\$3,500	\$159,950	0.67%	7.65%	\$7,160	7.65%	\$5,720				
2009	\$44,295	\$5,700	\$3,650	\$166,800	0.67%	7.65%	\$7,470	7.65%	\$5,970	6.2%	\$75,000	2%	\$400
2010	\$45,665	\$5,700	\$3,650			7.65%	\$7,480	7.65%	\$5,980	6.2%	\$75,000	2%	\$400
2011	\$46,895	\$5,800	\$3,700			7.65%	\$7,590	7.65%	\$6,070				
2012	\$47,746	\$5,950	\$3,800			7.65%	\$7,770	7.65%	\$6,210				
2013	\$48,774	\$6,100	\$3,900	\$250,000	2%	7.65%	\$7,970	7.65%	\$6,370				
2014	\$50,099	\$6,200	\$3,950	\$254,200	2%	7.65%	\$8,110	7.65%	\$6,480				
2015	\$50,963	\$6,300	\$4,000	\$258,250	2%	7.65%	\$8,240	7.65%	\$6,580				
2016	\$51,945	\$6,300	\$4,050	\$259,400	2%	7.65%	\$8,270	7.65%	\$6,610				
2017	\$53,376	\$6,350	\$4,050	\$261,500	2%	7.65%	\$8,340	7.65%	\$6,670				
2018	\$55,058	\$12,000				7.65%	\$8,490	7.65%	\$6,780				
2019	\$56,577	\$12,200				7.65%	\$8,650	7.65%	\$6,920				
2020	\$60,220	\$12,400				7.65%	\$8,790	7.65%	\$7,030				

TABLE E.2. Income tax parameters. *(Cont.)*

	Federal marginal tax rates							Federal income tax brackets						
Year	$\tau_1^{\texttt{fed}}$	$\tau_2^{\texttt{fed}}$	$\tau_3^{\texttt{fed}}$	$\tau_4^{\texttt{fed}}$	$\tau_5^{\texttt{fed}}$	$\tau_6^{\texttt{fed}}$	$\tau_7^{\texttt{fed}}$	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$	$\bar{e}_4$	$\bar{e}_5$	$\bar{e}_6$	$\bar{e}_7$
2000	15%	28%	31%	36%	39.6%			$0	$26,250	$63,550	$132,600	$288,350		
2001	10%	15%	27.5%	30.5%	35.5%	39.1%		$0	$6,000	$27,050	$65,550	$136,750	$297,370	
2002	10%	15%	27%	30%	35%	38.6%		$0	$6,000	$27,950	$67,700	$141,250	$307,050	
2003	10%	15%	25%	28%	33%	35%		$0	$7,000	$28,400	$68,800	$143,500	$311,950	
2004	10%	15%	25%	28%	33%	35%		$0	$7,150	$29,050	$70,350	$146,750	$319,100	
2005	10%	15%	25%	28%	33%	35%		$0	$7,300	$29,700	$71,950	$150,150	$326,450	
2006	10%	15%	25%	28%	33%	35%		$0	$7,550	$30,650	$74,200	$154,800	$336,550	
2007	10%	15%	25%	28%	33%	35%		$0	$7,825	$31,850	$77,100	$160,850	$349,700	
2008	10%	15%	25%	28%	33%	35%		$0	$8,025	$32,550	$78,850	$164,550	$357,700	
2009	10%	15%	25%	28%	33%	35%		$0	$8,350	$33,950	$82,250	$171,550	$372,950	
2010	10%	15%	25%	28%	33%	35%		$0	$8,375	$34,000	$82,400	$171,850	$373,650	
2011	10%	15%	25%	28%	33%	35%		$0	$8,500	$34,500	$83,600	$174,400	$379,150	
2012	10%	15%	25%	28%	33%	35%		$0	$8,700	$35,350	$85,650	$178,650	$388,350	
2013	10%	15%	25%	28%	33%	35%	39.6%	$0	$8,925	$36,250	$87,850	$183,250	$398,350	$400,000
2014	10%	15%	25%	28%	33%	35%	39.6%	$0	$9,075	$36,900	$89,350	$186,350	$405,100	$406,750
2015	10%	15%	25%	28%	33%	35%	39.6%	$0	$9,225	$37,450	$90,750	$189,300	$411,500	$413,200
2016	10%	15%	25%	28%	33%	35%	39.6%	$0	$9,275	$37,650	$91,150	$190,150	$413,350	$415,050
2017	10%	15%	25%	28%	33%	35%	39.6%	$0	$9,325	$37,950	$91,900	$191,650	$416,700	$418,400
2018	10%	12%	22%	24%	32%	35%	37%	$0	$9,525	$38,700	$82,500	$157,500	$200,000	$500,000
2019	10%	12%	22%	24%	32%	35%	37%	$0	$9,700	$39,475	$84,200	$160,725	$204,100	$510,300
2020	10%	12%	22%	24%	32%	35%	37%	$0	$9,875	$40,125	$85,525	$163,300	$207,350	$518,400

Table E.2. Income tax parameters. *(Cont.)*

	Personal exemption		Marginal tax rates		Michigan EIC	Michigan City Income Tax Credit						
	State	Local	State	Local		Rates			Credit brackets			Max
Year	$\texttt{EXEMPT}_{\texttt{state}}$	$\texttt{EXEMPT}_{\texttt{local}}$	$\tau^{\texttt{state}}$	$\tau^{\texttt{local}}$	$\varphi_{\texttt{meic}}$	$\varphi_{1,\texttt{ctc}}$	$\varphi_{2,\texttt{ctc}}$	$\varphi_{3,\texttt{ctc}}$	$\overline{T}_1$	$\overline{T}_2$	$\overline{T}_3$	$\overline{\texttt{ctc}}$
2000	\$2,900	\$750	4.2%	2.85%		20%	10%	5%	\$0	\$100	\$150	\$10,000
2001	\$2,900	\$750	4.2%	2.75%		20%	10%	5%	\$0	\$100	\$150	\$10,000
2002	\$3,000	\$750	4.1%	2.65%		20%	10%	5%	\$0	\$100	\$150	\$10,000
2003	\$3,100	\$750	4%	2.5%		20%	10%	5%	\$0	\$100	\$150	\$10,000
2004	\$3,100	\$750	4%	2.5%		20%	10%	5%	\$0	\$100	\$150	\$10,000
2005	\$3,200	\$600	3.9%	2.5%		20%	10%	5%	\$0	\$100	\$150	\$10,000
2006	\$3,300	\$600	3.9%	2.5%		20%	10%	5%	\$0	\$100	\$150	\$10,000
2007	\$3,300	\$600	3.9%	2.5%		20%	10%	5%	\$0	\$100	\$150	\$10,000
2008	\$3,300	\$600	4.35%	2.5%	10%	20%	10%	5%	\$0	\$100	\$150	\$10,000
2009	\$3,500	\$600	4.35%	2.5%	20%	20%	10%	5%	\$0	\$100	\$150	\$10,000
2010	\$3,600	\$600	4.35%	2.5%	20%	20%	10%	5%	\$0	\$100	\$150	\$10,000
2011	\$3,700	\$600	4.35%	2.5%	20%	20%	10%	5%	\$0	\$100	\$150	\$10,000
2012	\$3,763	\$600	4.33%	2.45%	6%							
2013	\$3,950	\$600	4.25%	2.4%	6%							
2014	\$3,950	\$600	4.25%	2.4%	6%							
2015	\$3,950	\$600	4.25%	2.4%	6%							
2016	\$4,000	\$600	4.25%	2.4%	6%							
2017	\$4,000	\$600	4.25%	2.4%	6%							
2018	\$4,000	\$600	4.25%	2.4%	6%							
2019	\$4,050	\$600	4.25%	2.4%	6%							
2020	\$4,750	\$600	4.25%	2.4%	6%							

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