Thermal Conductivity

Dan Marrinan

Sam Stuard

Overview

- Introduce effects of temperature gradients on metals and explain thermocouples
- Show the heat equation and its solution with the given boundary conditions
- Demonstrate the experimental setup and procedure
- Present results
- Discuss interpretation of results

Introduction

- Heat conduction takes place all around us
- Heat exchangers in power generation
- HVAC
- Computers

Metal with Temperature Gradient

- Electrical current density in metal with temperature gradient

$$J = \sigma E - \sigma S \frac{dT}{dx}$$

- “Hot” electrons diffuse faster than “cold” electrons
- Electron concentration gradient forms electric field

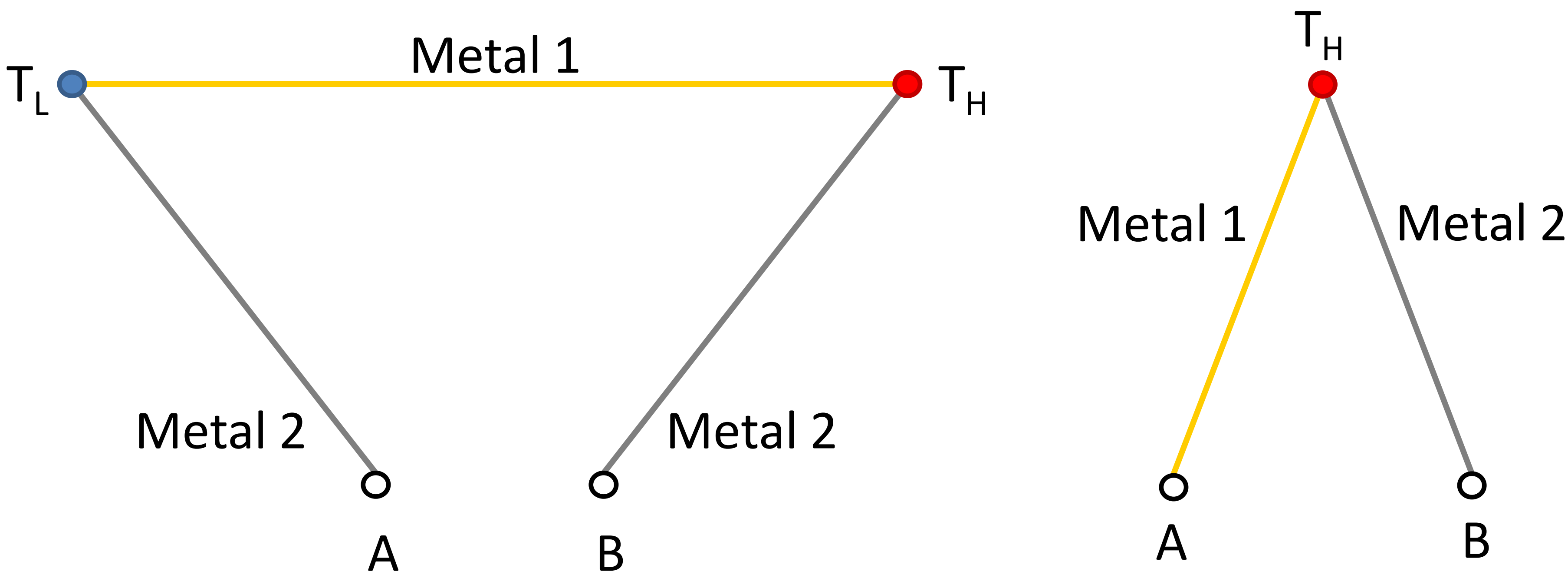
$$V_{HL} = -\int_0^L E dx = \int_{T_L}^{T_H} S dT$$

Thermocouples

- Metals have different Seebeck Coefficients

$$V_{BA} = \int_{T_L}^{T_H} (S_2 - S_1)\, dT$$

- Cold junction compensation

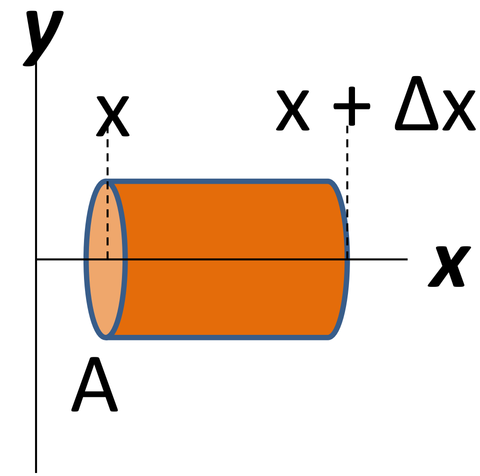


Heat Transfer

- Heat transferred by electrons and phonons
- Diffusion of electrons
- Vibrations in crystal lattice form packets of energy called phonons
- Fourier's Law

$$W = -k \frac{dT}{dx}$$

Heat Equation



- Apply conservation of energy

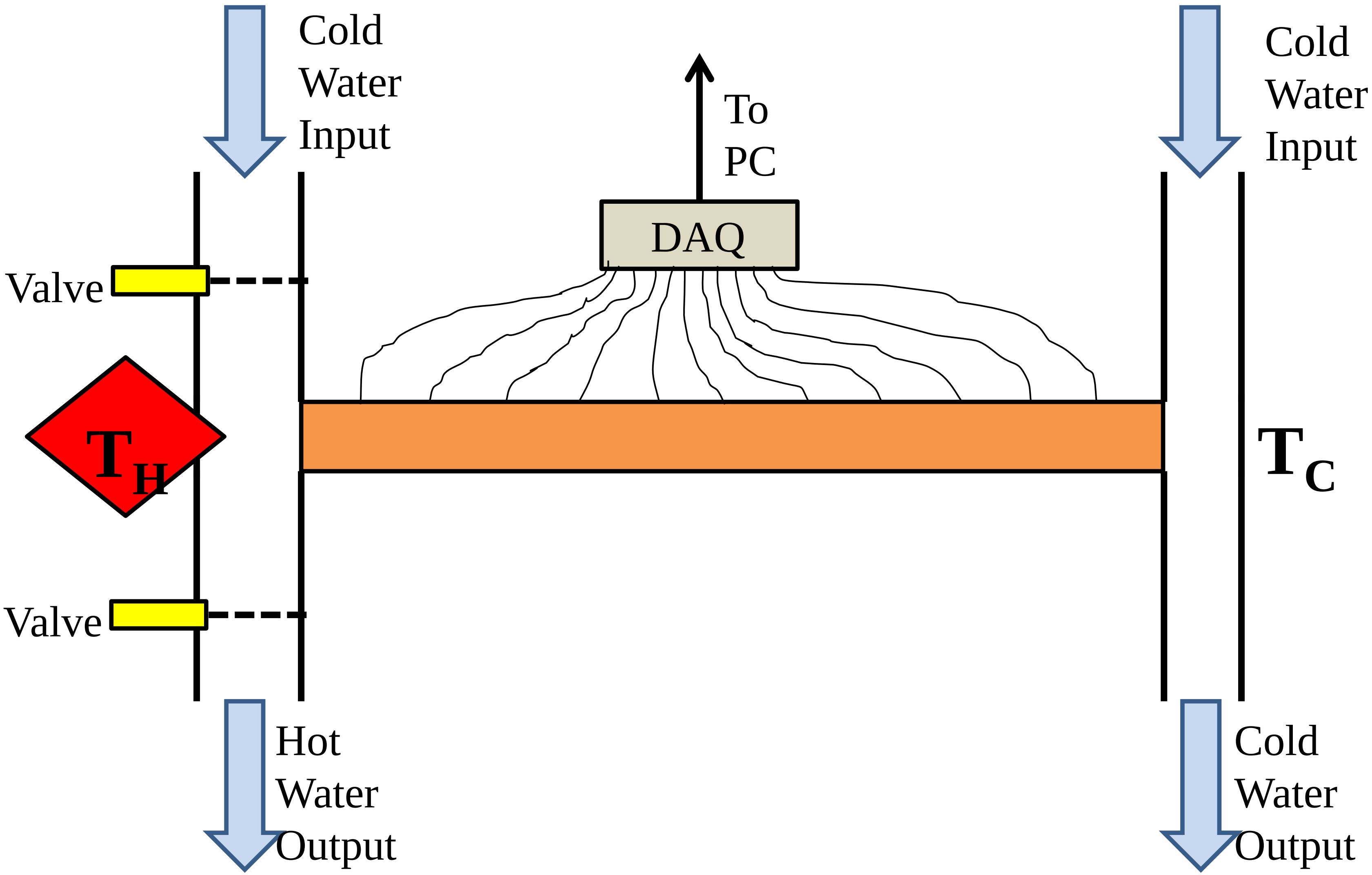
$$\frac{\delta U(x,t)}{\delta t} = \alpha^2 \frac{\delta^2 U(x,t)}{\delta x^2}$$

- Boundary conditions: U(0,t) = U(L,t) = 0
- Initial Condition: U(x,0) = kx

$$U(x,t) = -\frac{2KL}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{\mathrm{n}}}{\mathrm{n}}\sin\left(\frac{n\pi x}{L}\right)e^{-\left(\frac{n^2\pi^2\alpha^2}{L^2}\right)t}$$

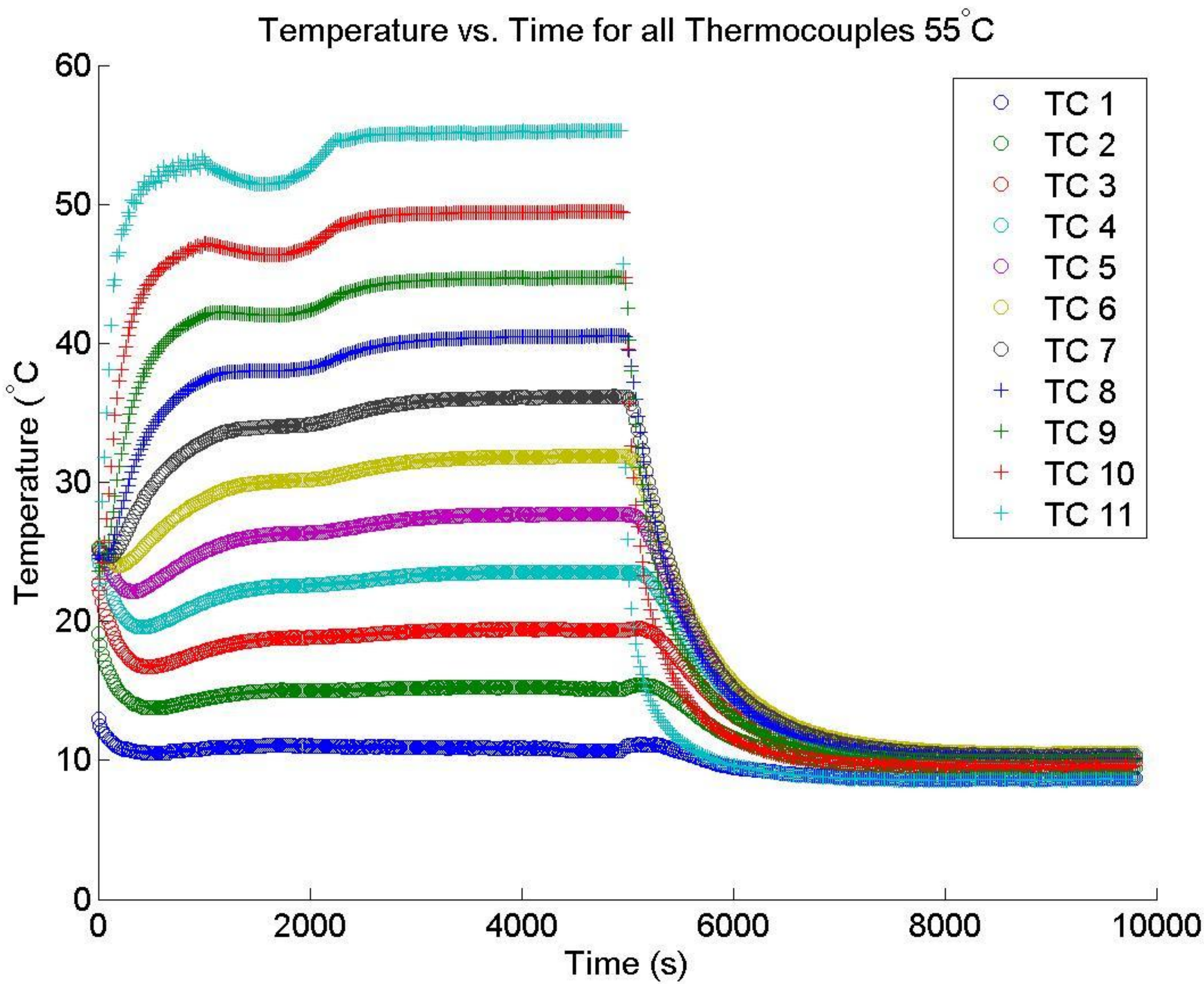
- Purpose of this experiment was to test U(x,t)

Experimental Setup

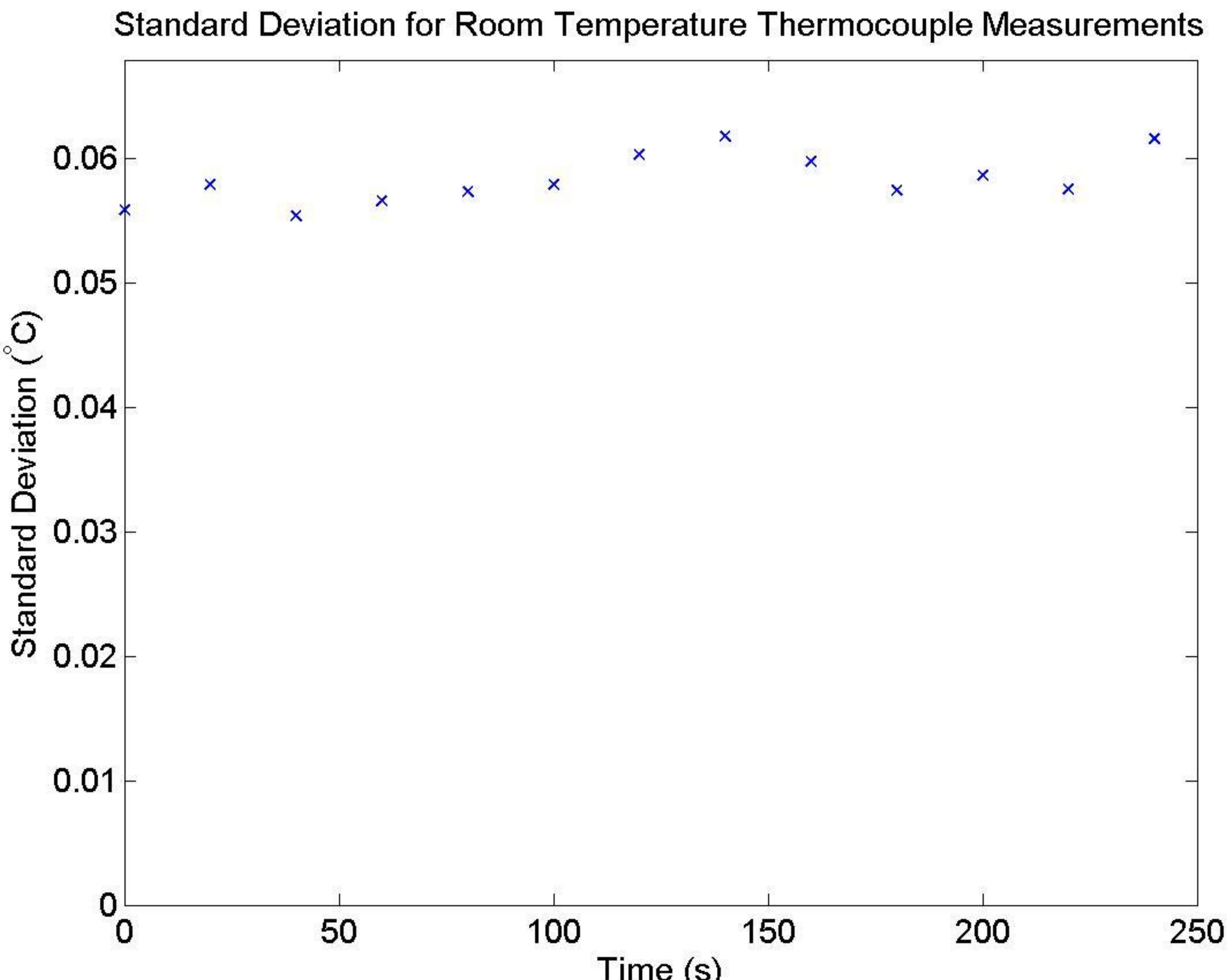


Experimental Procedure

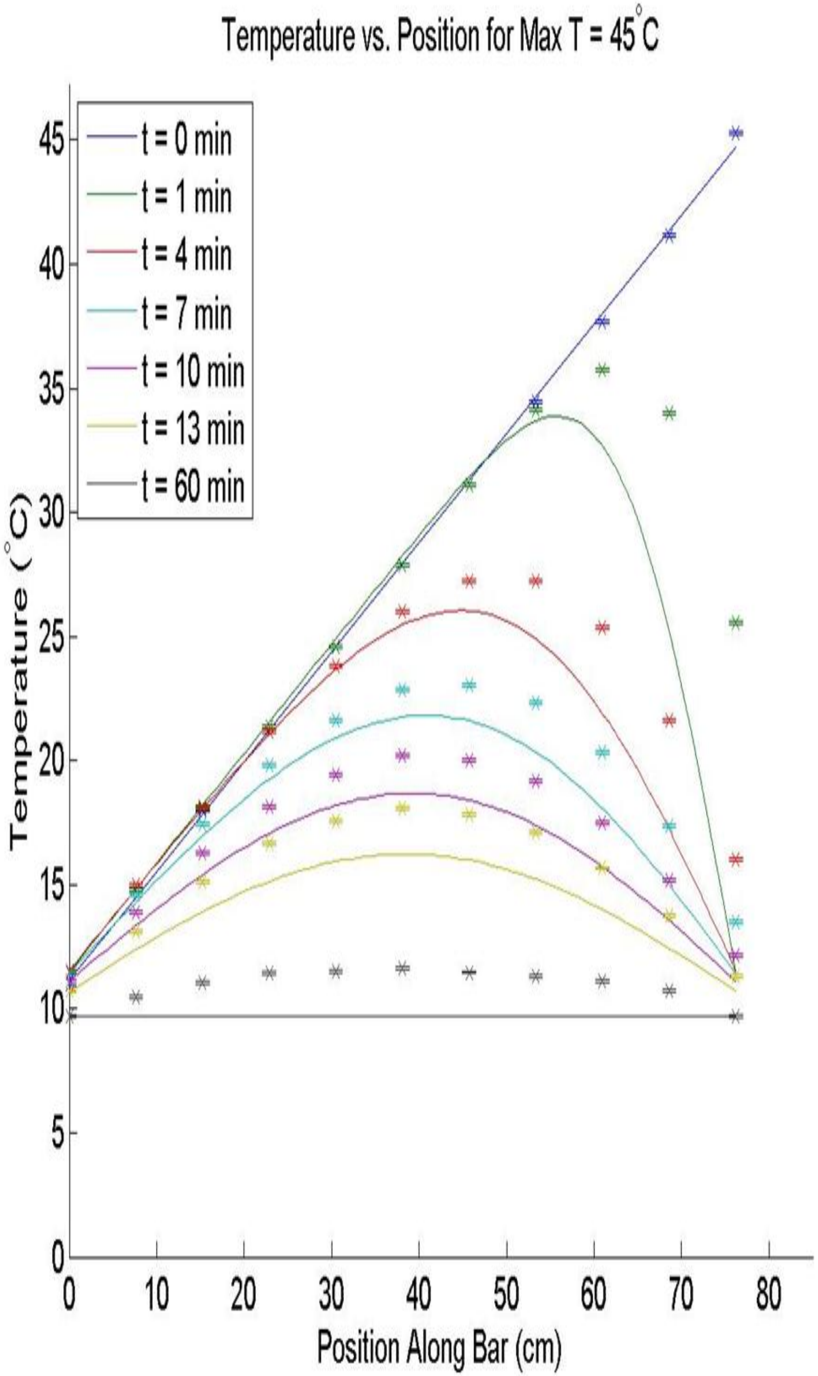
- Turn on DAQ
- Turn on cold water
- Trap hot water
- Let system reach steady state
- Let cold water flow through both sides
- Wait!

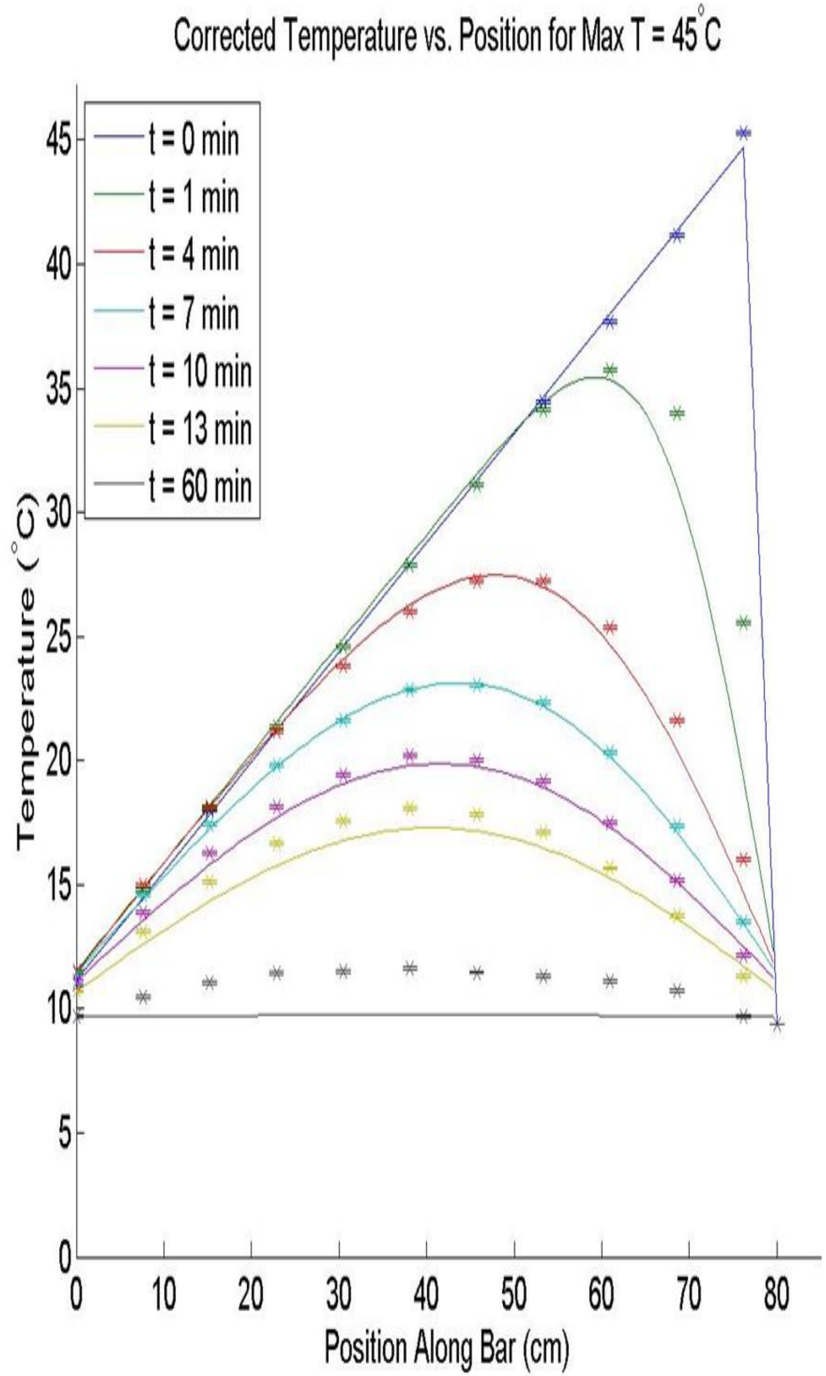


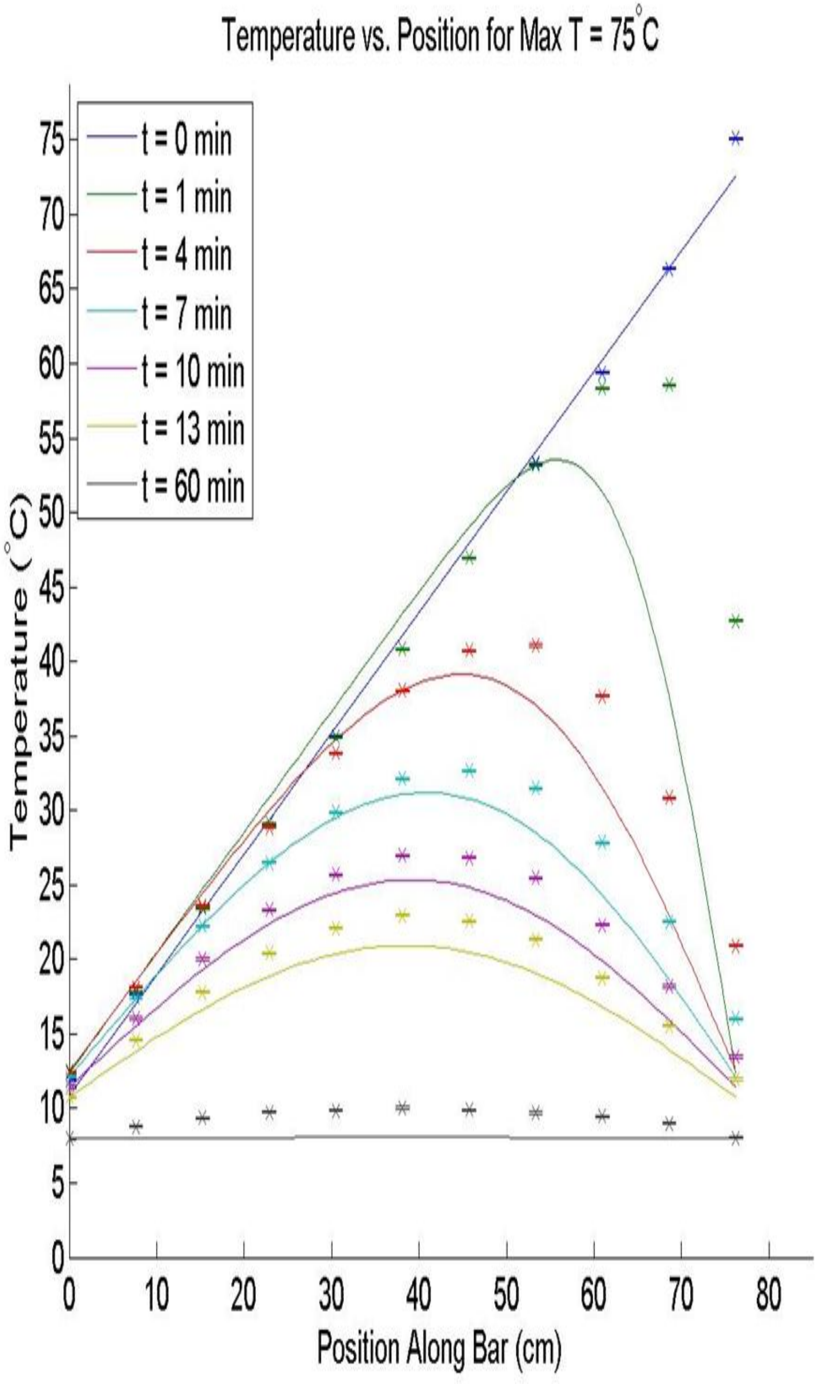
Temperature vs. Time for all Thermocouples 55°C
Temperature (°C)
Time (s)
0
10
20
30
40
50
60
0
2000
4000
6000
8000
10000
TC 1
TC 2
TC 3
TC 4
TC 5
TC 6
TC 7
TC 8
TC 9
TC 10
TC 11



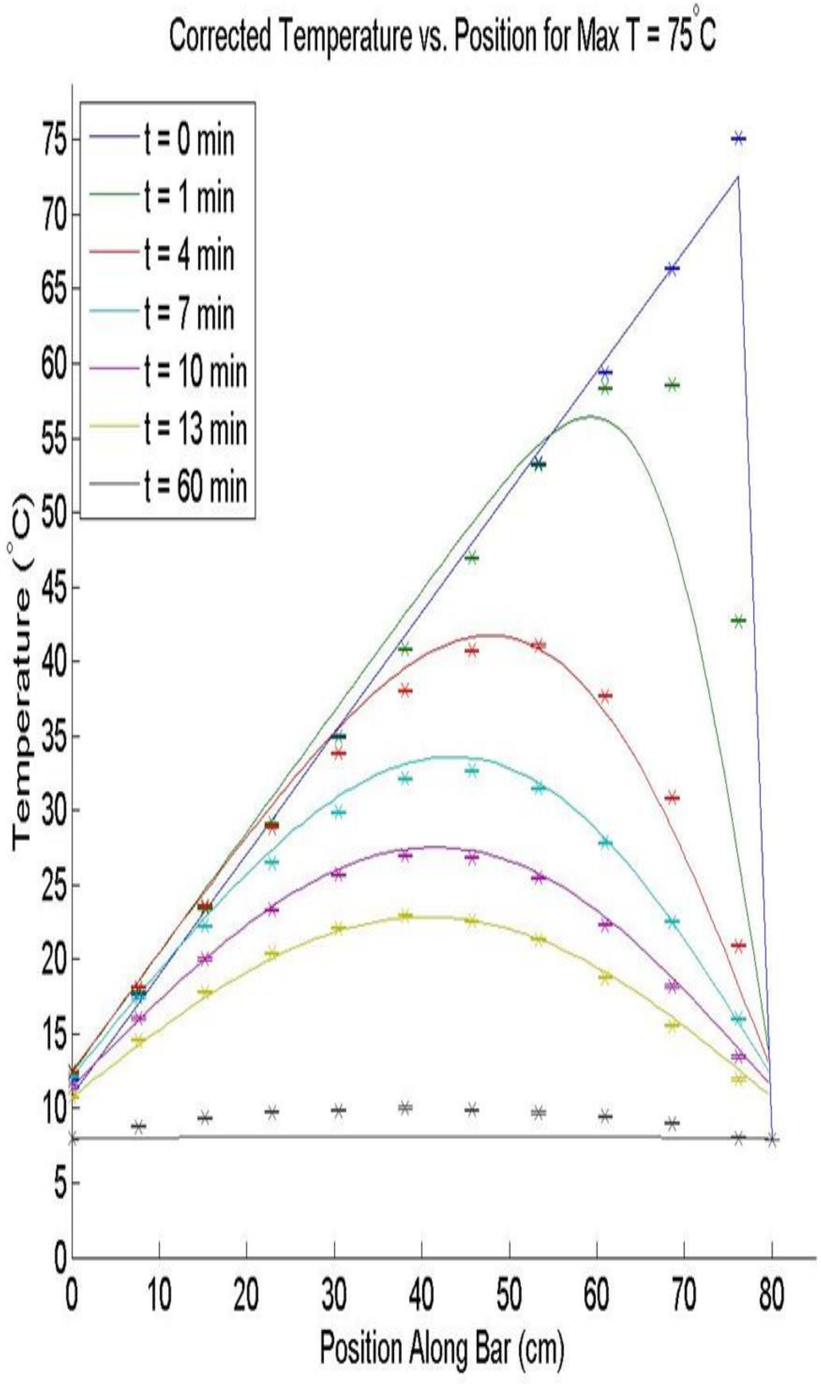
Standard Deviation for Room Temperature Thermocouple Measurements
Standard Deviation (°C)
Time (s)
0
0.01
0.02
0.03
0.04
0.05
0.06
0
50
100
150
200
250


Temperature vs. Position for Max T = 45°C
t = 0 min
t = 1 min
t = 4 min
t = 7 min
t = 10 min
t = 13 min
t = 60 min
Temperature (°C)
Position Along Bar (cm)
0
5
10
15
20
25
30
35
40
45
0
10
20
30
40
50
60
70
80


Corrected Temperature vs. Position for Max T = 45°C
t = 0 min
t = 1 min
t = 4 min
t = 7 min
t = 10 min
t = 13 min
t = 60 min
Temperature (°C)
Position Along Bar (cm)
0
5
10
15
20
25
30
35
40
45
0
10
20
30
40
50
60
70
80



Temperature vs. Position for Max T = 75°C
t = 0 min
t = 1 min
t = 4 min
t = 7 min
t = 10 min
t = 13 min
t = 60 min
Temperature (°C)
Position Along Bar (cm)



Corrected Temperature vs. Position for Max T = 75°C
t = 0 min
t = 1 min
t = 4 min
t = 7 min
t = 10 min
t = 13 min
t = 60 min
Temperature (°C)
Position Along Bar (cm)

Corrections to Experimental Data

- Added 1.5" piece of copper after "hot" thermocouple

	45° C	50° C	55° C	75° C
Original χ^2 for t = 1 min	20.7	8.7	46.7	87.7
Corrected χ^2 for t = 1 min	2.3	0.1	6.8	12.4

- Estimate heat transfer due to radial conduction
- Estimate heat transfer due to radiation
  - Explains nonlinearity at t=0
  - Explains why theory and data don't converge for large times

Conclusions

- Solution to heat equation was good but incomplete
- Including other methods of heat transfer give a better solution, but also make differential equation more difficult to solve.