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Physics 530A

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A conducting sphere of radius a, floats in a dielectric fluid with dielectric constant $\frac{\epsilon}{\epsilon_0}$ such that it is half submerged in the fluid. The spehre is brought to potential V.

1. Find the electric field everywhere outside the sphere.
2. Find the surface-charge distribution on the sphere.
3. Find the polarization-charge density induced on the dielectric at r = a.

2 solution

1. Since $\nabla \times E = 0$, we can assume that the field is continuous across the sphere. The electric displacement field, $D = \epsilon D$, across a closed surface enclosing the sphere is equal to the free charge Q.

$$\int D_{air} \cdot dA + \int D_{dielectric} \cdot dA = Q \tag{1}$$

$$2\pi a^2(\epsilon_1 E + \epsilon_0 E) = Q \tag{2}$$

where $\epsilon_1 = \epsilon_0 \frac{\epsilon}{\epsilon_0}$

$$2\pi a^2(\epsilon_0 \frac{\epsilon}{\epsilon_0} E + \epsilon_0 E) = Q \rightarrow E = \frac{Q}{2\pi r^2} \frac{1}{\epsilon + \epsilon_0} \tag{3}$$

$$\therefore \int D_{diaelectric} \cdot dA = \frac{Q}{2\pi r^2} \frac{\epsilon}{\epsilon + \epsilon_0} \tag{4}$$

and

$$\therefore \int D_{air} \cdot dA = \frac{Q}{2\pi r^2} \frac{\epsilon_0}{\epsilon + \epsilon_0} \tag{5}$$

2. The surface charge density/distribution is equal to the displacement on the surface of the sphere therefore r = a. That means that (4) and (5) become

$$\int D_{diaelectric} \cdot dA = \frac{Q}{2\pi r^2} \frac{\epsilon}{\epsilon + \epsilon_0} \tag{6}$$

and

$$\therefore \int D_{air} \cdot dA = \frac{Q}{2\pi a^2} \frac{\epsilon_0}{\epsilon + \epsilon_0} \tag{7}$$

3. The surface polarisation charge exists only on the surface of the dielectric of the sphere.

$$\sigma_p = -P(a) = -(\epsilon - \epsilon_0)E(a) \tag{8}$$

Therefore the sum of the sigmas of the sphere exposed to the air and the other to the dielectric should be constant. That is:

$$\sigma + \sigma_0 = \epsilon E(a) - (\epsilon - \epsilon_0)E(a) \rightarrow \epsilon_0 E(a) = \sigma_0 \tag{9}$$