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Physics 530A

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Two extremely long, cylindrical conductors of radii a_1 and a_2 are paralle and separated by a distance d which is very large compare to a_1 and a_2, but small compared to the length (so you can ignore end effects).

1. Show that the two-conductor capacitance per capacitance per unit length is given approximately by

$$C \simeq \pi\epsilon_0 (ln\frac{d}{a})^{-1} \tag{1}$$

where $a = \sqrt{a_1 a_2}$. Note: the condition $d \gg a_1, a2$ allows you to use superposition to determine the total field/ potential produced by the combination of the two capacitors (they are far enough apart to have negligable effectsr on each other's charge distributions). Note 2: If we put $+Q$ and $-Q$ on the two conductors and the resulting potential difference between the two is ΔV, then the capacitance is $C = \frac{Q}{\Delta V}$

2 solutions

$$E_{tot} = E_{a_1} + E_{a_2} = \frac{Q}{2\pi\epsilon_0 L} + \frac{Q}{2\pi\epsilon_0 (d-L)} = \frac{Q}{2\pi\epsilon_0}(\frac{1}{L} + \frac{1}{d-L}) \tag{2}$$

$$V = \int_{a_1}^{d-a_2} E dl \tag{3}$$

This basically becomes (via mathematica):

$$V = \frac{Q}{2\pi\epsilon_0} ln(\frac{(d-a_1)(d-a_2)}{a_1 a_2}) \Rightarrow \frac{Q}{2\pi\epsilon_0} 2ln(\frac{d}{a_1 a_2}) \Rightarrow \frac{Q}{\pi\epsilon_0} 2n(\frac{d}{a_1 a_2}) \tag{4}$$

Since $a = a_1 a_2$, we can say that

$$\frac{Q}{\pi\epsilon_0} ln(\frac{d}{a_1 a_2}) \Rightarrow \frac{Q}{\pi\epsilon_0} ln(\frac{d}{a}) \tag{5}$$

Since we know by definition that $C = \frac{Q}{V}$, this equation then becomes:

$$C \simeq \frac{2\pi\epsilon_0}{ln(\frac{d}{a})} \tag{6}$$

QED