DSP HW2

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Task 1

Design a low pass FIR filter with parameters: passband Fpass = 5 MHz; stopband Fstop = 6MHz; attenuation at least 60 dB in the stopband (out-of-band attenuation).
Let the sampling frequency be Fs = 50 MHz.
Determine the design with the lowest computational complexity.
Provide code.

Solution:

- **Fir1 filter**
  First of all, I tried to implement fir1 filter. Because I need to guarantee out-of-band attenuation at least 60 dB I used Blackman window (I tried to implement Hamming window but its attenuation was not sufficient enough). I just ran multiple times the function **fir1()** with different parameter n and choose **n** that gave me results that is specified in the problem statement. Finally, I designed fir1 filter with **n = 300**, which satisfies requirements.

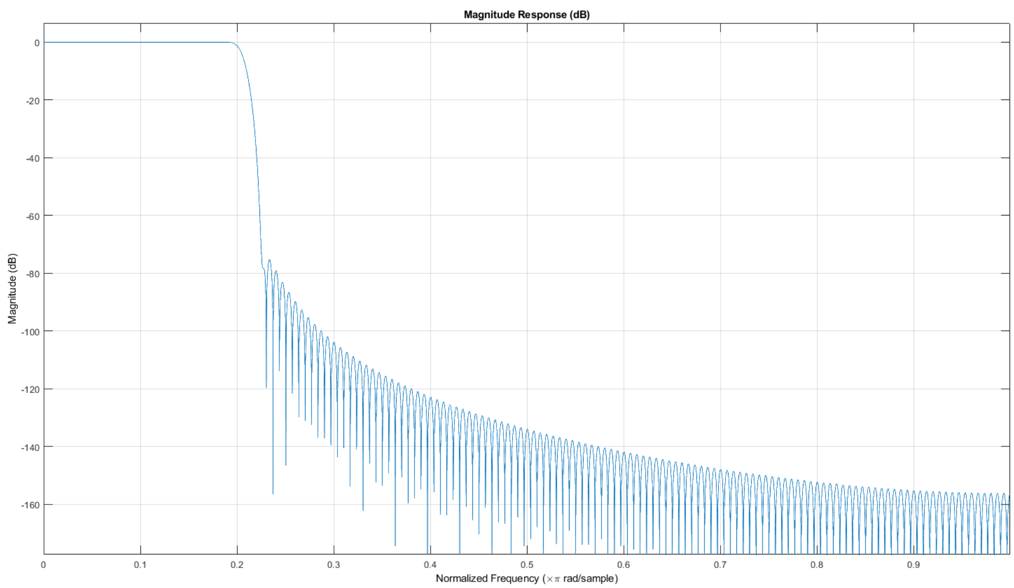

Figure 1: Magnitude response of fir1 filter with blackman window and n = 300.

- **Fir2 filter** Then I designed fir2 filter. I used function **fir2()** and ran it multiple with different parameter **n** and chose **n=210**, because fir2 with this parameter satisfies requirements.

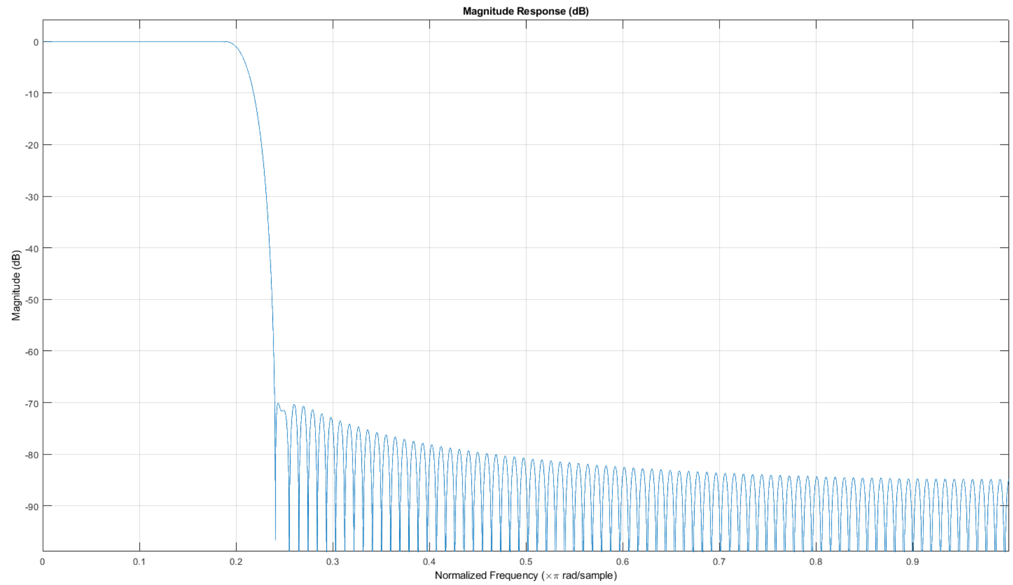

Figure 2: Magnitude response of fir2 filter with n = 210.

- **Firls filter**
  Then I designed firls filter. I used function **firls()** and ran it multiple times with different parameter **n** and chose **n=185**, because firls with this parameter satisfies requirements.

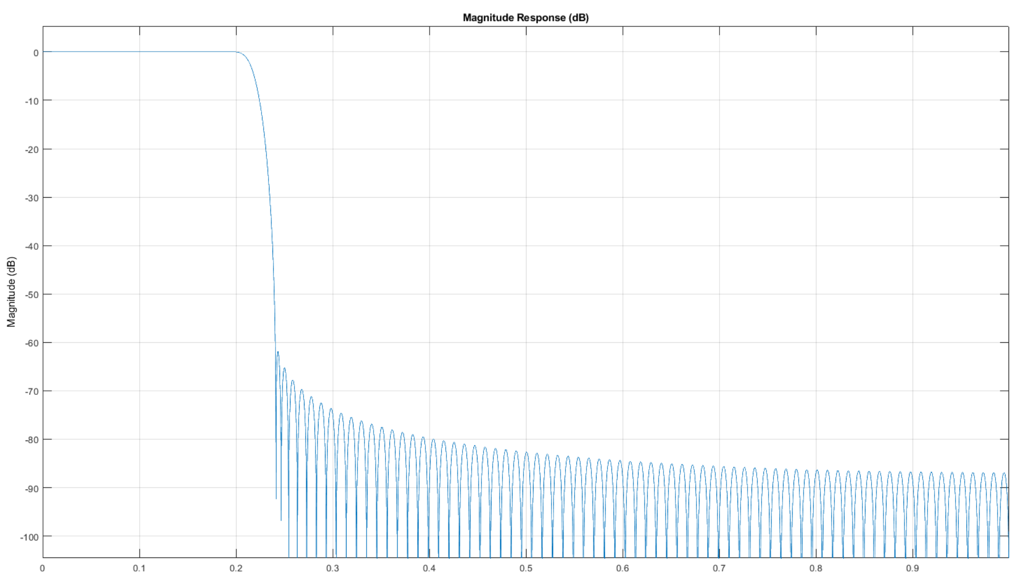

Figure 3: Magnitude response of firls filter with n = 185.

```
Fpass = 5e6;
Fstop = 6e6;
Fs = 50e6;
%fir1
n = 300;
delta = 0.007; % because default attenuation in wpass = -6dB, so I need to
%shift magn. resp.
f1 = fir1(n, 2 * Fpass / Fs + delta, blackman(n + 1));
%fvtool(f1);
%fir2
delta = 0.01;
freq = [0 2*Fpass/Fs (2*Fstop/Fs - delta) 1]; % substruction delta is necessary
%for the proper filter design
mag = [1 1 0 0];
n = 210;
f2 = fir2(n, freq, mag);
%fvtool(f2);
%fir ls
freq = [0 2*Fpass/Fs 2*Fstop/Fs  1];
mag = [1 1 0 0];
n = 185;
f3 = firls(n, freq, mag);
fvtool(f3);
```

So the filter with the lowest complexity is the firls filter, because it has the least order (**n = 185**).

Task 2

Using the impulse invariance method for analog to digital filter conversion, calculate the Cebyshev lowpass digital filter with parametrers: passband 20MHz; passband ripple 0.2 dB; stopband (out-of-band) attenuation 60 dB; sampling frequency Fs = 100 MHz.
a) Plot the impulse response for both analog and digital systems.
b) Plot the magnitude response for analog and digital systems in the frequency domain.
Provide code.

Solution:
First of all, I need to design Chebyshev type I analog filter, then using impulse invariance approximation design the digital prototype of the analog Chebyshev type I filter.

Matlab has a lot of built-in functions for digital filter designing. For this task I will use **cheby1(n, Rp, Fpass, 's')** function, which requires these parameters:

- n - the order of the filter;
- Rp - passband ripple;
- Wp - passband;

Before using this function I need to calculate the order of the filter which I want to design. I will use a built-in Matlab function **cheb1ord(Wp,Ws,Rp,Rs)**, which returns the order of the filter n.

- Wp - passband frequency;
- Ws - stopband frquency;
- Rp - passband ripple;

- Rs - stopband attenuation;

Note that stopband frequency is not specified in this task, so I will specify it by myself. Of course, this parameter affects on the complexity on the filter: the narrower the transition band, the more complex the filter. I ran the function **cheb1ord(Wp,Ws,Rp,Rs)** with different parameter **Ws** and received this results:

- Ws = 21 MHz, n = 25;
- Ws = 25 MHz, n = 11;
- Ws = 30 MHz, n = 8;

Indeed, the narrower the transition band, the higher n. So, let's choose Ws = 25 MHz. Then n = 11. The next computation will be done for the parameters **Ws = 25 MHz**, **n = 11**. Then by writing simple Matlab code I received:

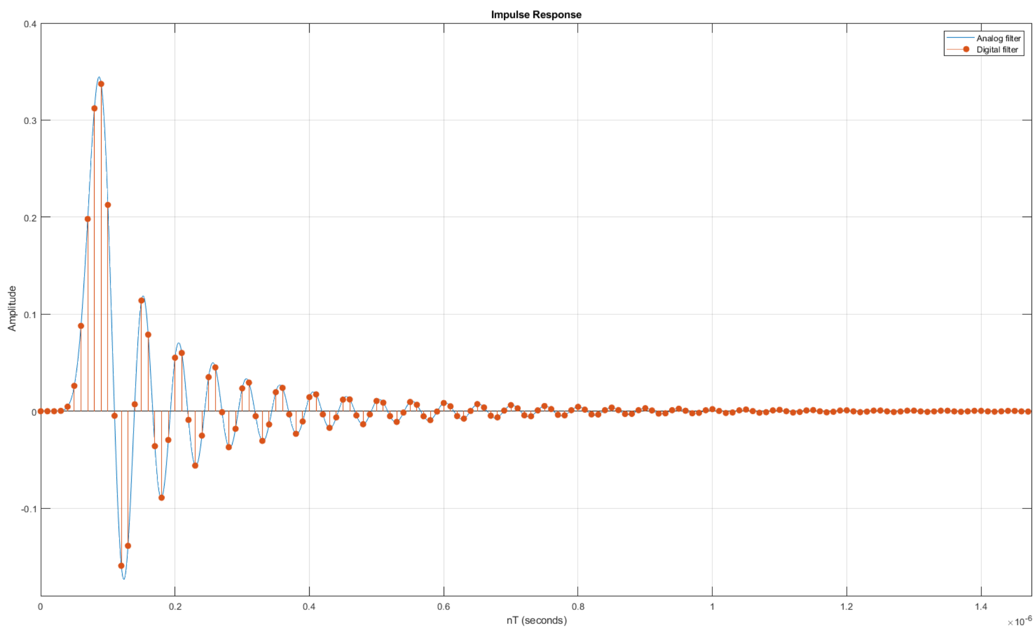

Figure 4: Impulse response of the analog and the digital filters.

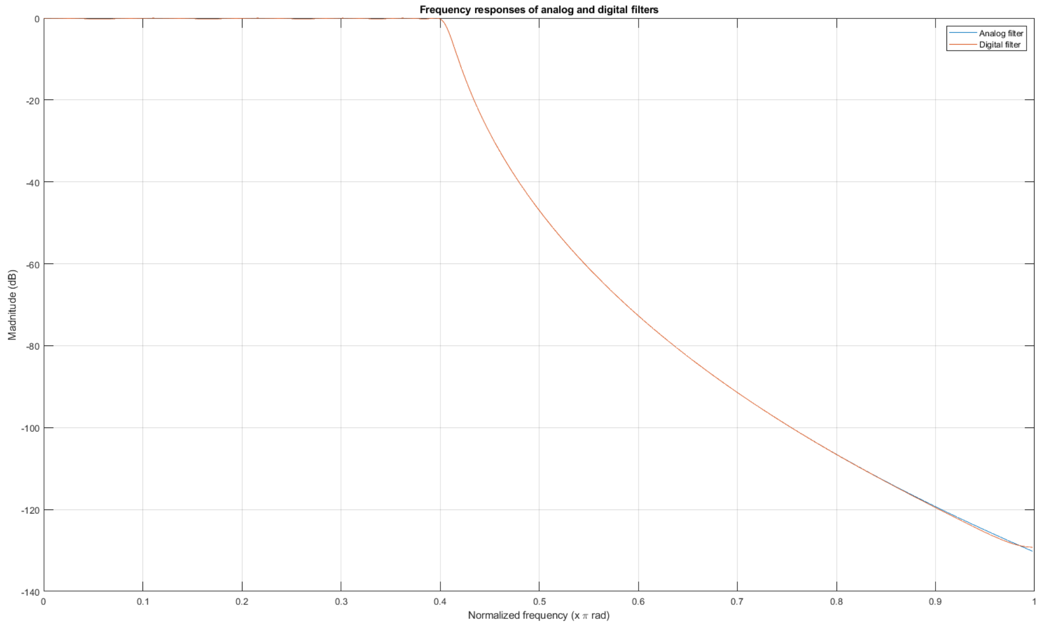

Figure 5: Frequency response of the analog and the digital filters.

```
Fpass = 20e6;
Fstop = 25e6;
Fs = 100e6;
Ripple = 0.2;
Attenuation = 60;
Ws = 2 * Fstop / Fs; % normilize the frequency
Wp = 2 * Fpass / Fs; % normilize the frequency
[n, Wp] = cheb1ord(Wp, Ws, Ripple, Attenuation); % compute the order of
% designing filter
[b,a] = cheby1(n, Ripple, 2 * pi * Fpass, 's'); % analog filter design B(s)/A(s)
figure(1);
[bz, az] = impinvar(b,a,Fs); % design a digital prototype of the analog
% filter B(z)/A(z)
% the following steps are similar to the seminar
[r, p] = residue(b, a); % find direct term of a Partial Fraction Expansion of the
%ratio of two polynomials
t = linspace(0, 100 / Fs, 1000);
h = real(r.'*exp(p.*t) / Fs); % analog filter impulse response
plot(t, h)
hold on;
impz(bz, az, [], Fs); % digital filter impulse invariance
legend('Analog filter', 'Digital filter')
grid on;
hold off;

figure(2);
[H, W] = freqz(bz, az);
[H_an] = freqs(b, a, W * Fs);
```

```
H_dig_db = 20 * log10(abs(H)); % convert magnitude to dB
H_an_db = 20 * log10(abs(H_an));
plot(W / pi, H_an_db);
hold on;
plot(W / pi, H_dig_db);
legend('Analog filter', 'Digital filter')
title('Frequency responses of analog and digital filters')
ylabel('Madnitude (dB)')
xlabel('Normalized frequency (x \pi rad)')
grid on;
hold off;
```

According to the figure N!, digital filter approximates analog one with good accuracy.

Task 3

Implement a digital prototype of the analog filter with the transfer function:

$$H(s) = \frac{s + 2.5}{s^2 + 2.5s + 4}$$

using the Bilinear transformation. The sample clock frequency is Fs = 20 Hz.
a) Determine the linear Difference Equation of the digital filter.
b) Plot impulse and frequency responses for digital and analog filters.
Provide code.

Solution: Bilinear transformation equivalent to the substitution

$$s = \frac{2Fs(z-1)}{(z+1)}$$

the the transfer function of the analog filter $H(s)$

$$H(z) = \frac{\frac{2Fs(z-1)}{(z+1)} + 2.5}{(\frac{2Fs(z-1)}{(z+1)})^2 + 2.5(\frac{2Fs(z-1)}{(z+1)}) + 4} = \frac{2Fs(z-1) + 2.5(z+1)}{(z+1)(\frac{4Fs^2(z-1)^2}{(z+1)^2}) + 2.5(\frac{2Fs(z^2-1)}{(z+1)^2}) + \frac{4(z+1)^2}{(z+1)^2}} =$$

$$= \frac{(z+1)(2Fs(z-1) + 2.5(z+1))}{4Fs^2(z-1)^2 + 5Fs(z^2-1) + 4(z+1)^2} = \frac{(z+1)((2Fs+2.5)z + (2.5-2Fs))}{(4Fs^2+5Fs+4)z^2 + (8-8Fs^2)z + (4+4Fs^2-5Fs)} =$$

$$= \frac{2Fs+2.5}{4Fs^2+5Fs+4} \frac{(z+1)(z+\frac{2.5-2Fs}{2Fs+2.5})}{z^2 + \frac{8-8Fs^2}{4Fs^2+5Fs+4}z + \frac{4+4Fs^2-5Fs}{4Fs^2+5Fs+4}} = \frac{40+2.5}{1600+100+4} \frac{(z+1)(z+\frac{2.5-40}{40+2.5})}{z^2 + \frac{8-3200}{1600+100+4} + \frac{4+1600-100}{1600+100+4}} =$$

$$= 0.0249 \frac{(z+1)(z-0.8824)}{z^2 - 1.8732z + 0.8826} = 0.0249 \frac{z^2(1 + 0.1176^{-1} + 0.8824z^{-2})}{z^2(1 - 1.8732z^{-1} + 0.8826z^{-2})} =$$

$$= 0.0249 \frac{(1 + 0.1176^{-1} + 0.8824z^{-2})}{(1 - 1.8732z^{-1} + 0.8826z^{-2})}$$

$$H(z) = \frac{Y(z)}{X(z)} = 0.0249 \frac{(1 + 0.1176^{-1} + 0.8824z^{-2})}{(1 - 1.8732z^{-1} + 0.8826z^{-2})}$$

$$0.0249X(z)(1 + 0.1176z^{-1} + 0.8824z^{-2}) = Y(z)(1 - 1.8732z^{-1} + 0.8826z^{-2})$$

$$X(z)(0.0249 + 0.0029z^{-1} + 0.0220z^{-2}) = Y(z)(1 - 1.8732z^{-1} + 0.8826z^{-2})$$

Using the time-shifting property I received the difference equation:

$$0.0249x[n] + 0.0029x[n-1] + 0.0220x[n-2] = y[n] - 1.8732y[n-1] + 0.8826y[n-2]$$

Then I just wrote a simple Matlab code, which is similar to the code on seminar and plotted impulse and frequency responses for digital and analog filters.

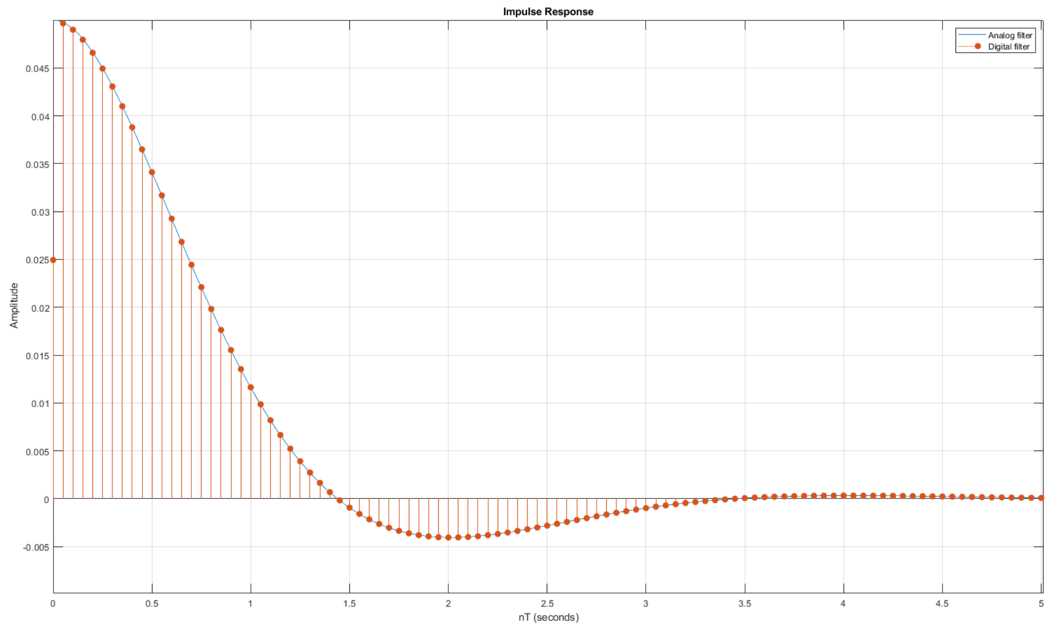

Figure 6: Impulse response of the analog and the digital filters.

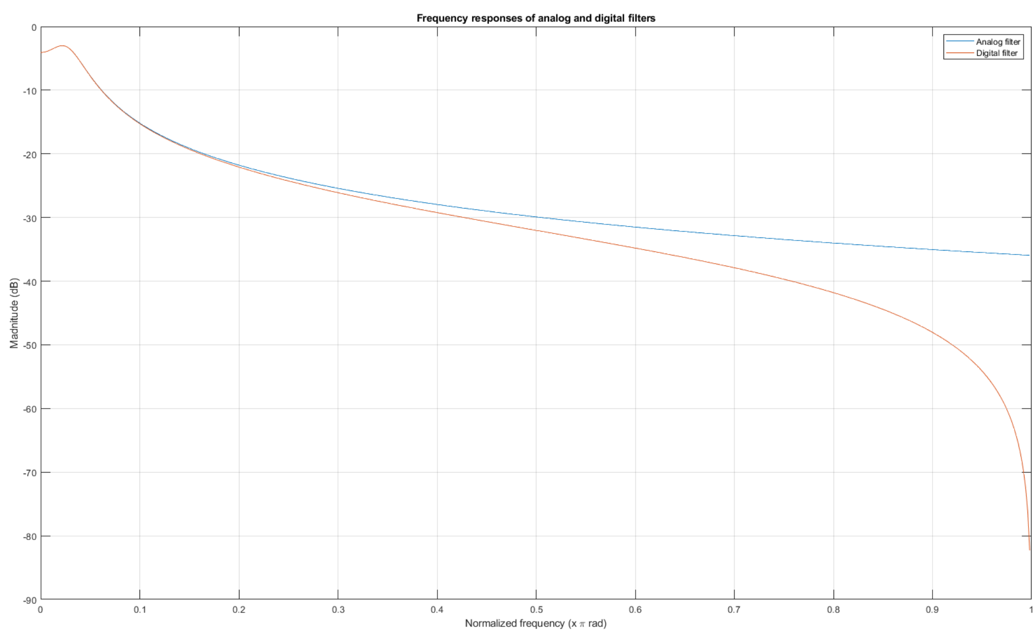

Figure 7: Frequency response of the analog and the digital filters.

```
Fs = 20;
b = [0 1 2.5];
a = [1 2.5 4];
[bz, az] = bilinear(b,a,Fs);
[r, p] = residue(b, a); % find direct term of a Partial Fraction Expansion of the
%ratio of two polynomials
t = linspace(0, 100 / Fs, 1000);
h = real(r.'*exp(p.*t) / Fs); % analog filter impulse response
figure(1);
plot(t, h);
hold on;
impz(bz, az, [], Fs);
legend('Analog filter', 'Digital filter')
grid on;
hold off;

figure(2);
[H, W] = freqz(bz, az);
[H_an] = freqs(b, a, W * Fs);
H_dig_db = 20 * log10(abs(H)); % convert magnitude to dB
H_an_db = 20 * log10(abs(H_an));
plot(W / pi, H_an_db);
hold on;
plot(W / pi, H_dig_db);
legend('Analog filter', 'Digital filter')
title('Frequency responses of analog and digital filters')
ylabel('Madnitude (dB)')
xlabel('Normalized frequency (x \pi rad)')
grid on;
```

According to the figure with frequency response, digital prototype of the analog filter approximates the latter with good accuracy only when $f \leq 0.4Fs \rightarrow f \leq 8MHz$.

Task 4

A filter has the transfer function:

$$H(z) = 3 + 4z^{-1} + 6z^{-2} + 8z^{-3}$$

Determine the impulse response of the filter with the modified frequency response:

$$F(\omega) = H(\omega - \frac{\pi}{4})$$

Solution:
First of all, let's move from Z-domain to frequency domain. For this substitute $z = re^{j\omega}$ to the expressions for the $H(z)$. Note: for simplicity I defined $r = 1$ so $z = e^{j\omega}$. I can do it, because r is just a constant (like a scaling coefficient for each summond).

$$H(\omega) = 3 + 4e^{-j\omega} + 6e^{-j2\omega} + 8e^{-j3\omega}$$

Then for our task substitute modified frequency: $\omega^* = \omega - \frac{\pi}{4}$

$$H(\omega) = 3 + 4e^{-j(\omega - \frac{\pi}{4})} + 6e^{-j(2\omega - \frac{\pi}{2})} + 8e^{-j(3\omega - \frac{3\pi}{4})}$$

$$H(\omega) = 3 + 4e^{\frac{j\pi}{4}}e^{-j\omega} + 6e^{\frac{j\pi}{2}}e^{-j2\omega} + 8e^{\frac{j3\pi}{4}}e^{-j3\omega}$$

So then I need to calculate inverse DTFT:

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\omega)e^{j\omega n}d\omega$$

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} (3 + 4e^{\frac{j\pi}{4}}e^{-j\omega} + 6e^{\frac{j\pi}{2}}e^{-j2\omega} + 8e^{\frac{j3\pi}{4}}e^{-j3\omega})e^{j\omega n}d\omega$$

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} (3e^{j} + 4e^{\frac{j\pi}{4}}e^{j\omega(n-1)} + 6e^{\frac{j\pi}{2}}e^{j\omega(n-2)} + 8e^{\frac{j3\pi}{4}}e^{j\omega(n-3)})d\omega$$

$$h[n] = \frac{1}{2\pi}(\frac{3}{jn}e^{j\omega n}|_{-\pi}^{\pi} + 4e^{\frac{j\pi}{4}}\frac{e^{j\omega(n-1)}}{j(n-1)}|_{-\pi}^{\pi} + 6e^{\frac{j\pi}{2}}\frac{e^{j\omega(n-2)}}{j(n-2)}|_{-\pi}^{\pi} + 8e^{\frac{j3\pi}{4}}\frac{e^{j\omega(n-3)}}{j(n-3)}|_{-\pi}^{\pi})$$

$$h[n] = \frac{1}{2\pi}(\frac{6}{n}(\frac{e^{j\pi n} - e^{-j\pi n}}{2j}) + \frac{8e^{\frac{j\pi}{4}}}{n-1}(\frac{e^{j\pi(n-1)} - e^{-j\pi(n-1)}}{2j}) + \frac{12e^{\frac{j\pi}{2}}}{n-2}(\frac{e^{j\pi(n-2)} - e^{-j\pi(n-2)}}{2j}) +$$

$$+\frac{16e^{\frac{j3\pi}{4}}}{n-3}(\frac{e^{j\pi(n-3)} - e^{-j\pi(n-3)}}{2j})$$

$$h[n] = \frac{3sin(\pi n)}{\pi n} + \frac{4e^{\frac{j\pi}{4}}sin(\pi(n-1))}{\pi(n-1)} + \frac{6e^{\frac{j\pi}{2}}sin(\pi(n-2))}{\pi(n-2)} + \frac{8e^{\frac{j3\pi}{4}}sin(\pi(n-3))}{\pi(n-3)}$$

$$h[n] = 3\delta[n] + 4e^{\frac{j\pi}{4}}\delta[n-1] + 6e^{\frac{j\pi}{2}}\delta[n-2] + 8e^{\frac{j3\pi}{4}}\delta[n-3]$$

$$h[n] = \begin{cases} 3 & ,n = 0 \\ 4e^{\frac{j\pi}{4}} & ,n = 1 \\ 6e^{\frac{j\pi}{2}} & ,n = 2 \\ 8e^{\frac{3j\pi}{4}} & ,n = 3 \\ 0 & ,\text{otherwise} \end{cases}$$

Task 5

For a linear system with the transfer function:

$$H(z) = \frac{1z + 2}{3z^3 + 4z^2 + 5z + 6}$$

a) Calculate the difference equation relating the input x[n] to the output y[n].
b) Design block diagram realization (Direct-Form 1 and Direct-Form 2).
c) Plot impulse and frequency responses.

Solution:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1z + 2}{3z^3 + 4z^2 + 5z + 6} = \frac{z^3(z^{-2} + 2z^{-3})}{z^3(3 + 4z^{-1} + 5z^{-2} + 6z^{-3})}$$

$$X(z)(z^{-2} + 2z^{-3}) = Y(z)(3 + 4z^{-1} + 5z^{-2} + 6z^{-3})$$

Using the time-shifting property I received the difference equation:

$$x[n-2] + 2x[x-3] = 3y[n] + 4y[n-1] + 5y[n-2] + 6y[n-3]$$

$$y[n] = \frac{1}{3}x[n-2] + \frac{2}{3}x[n-3] - \frac{4}{3}y[n-1] - \frac{5}{3}y[n-2] - 2y[n-3]$$

Then I designed block diagram realization in Direct-Form I and Direct-Form II: And then I plotted impulse and frequency responses for this linear system:

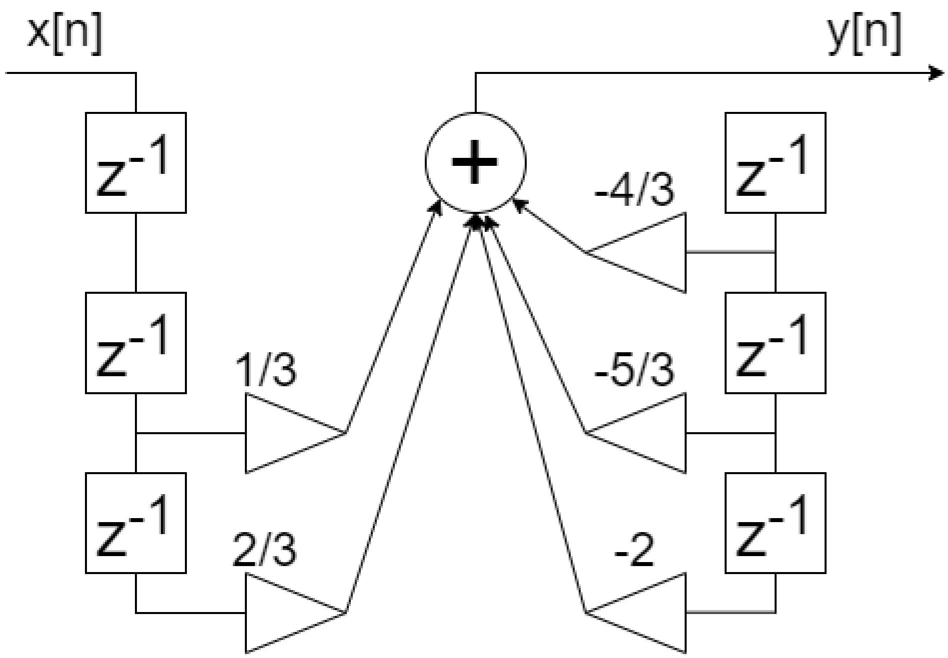

Figure 8: Direct-Form I.

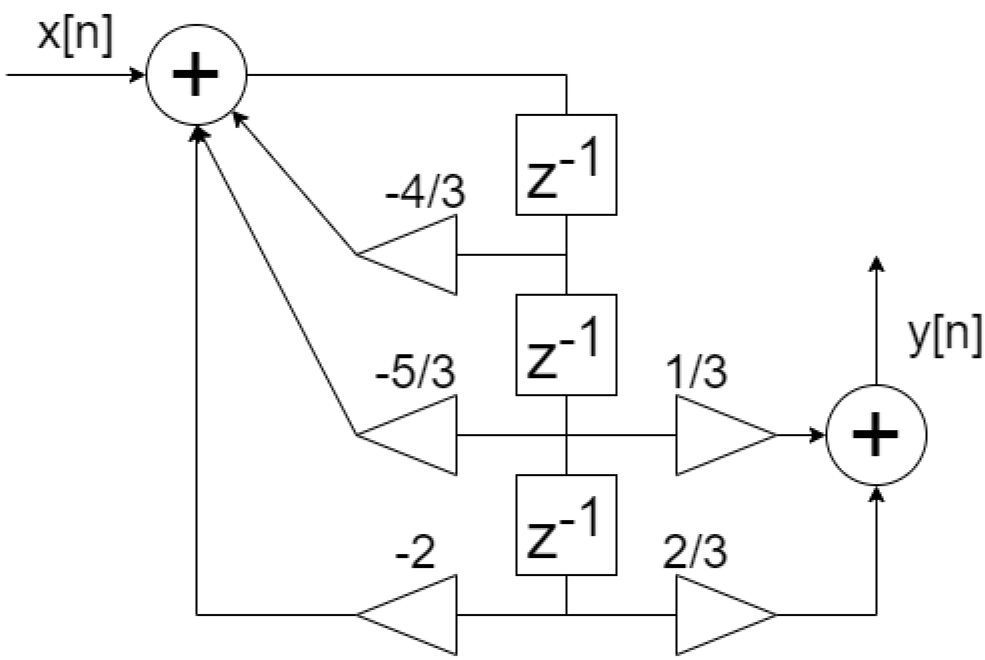

Figure 9: Direct-Form II.

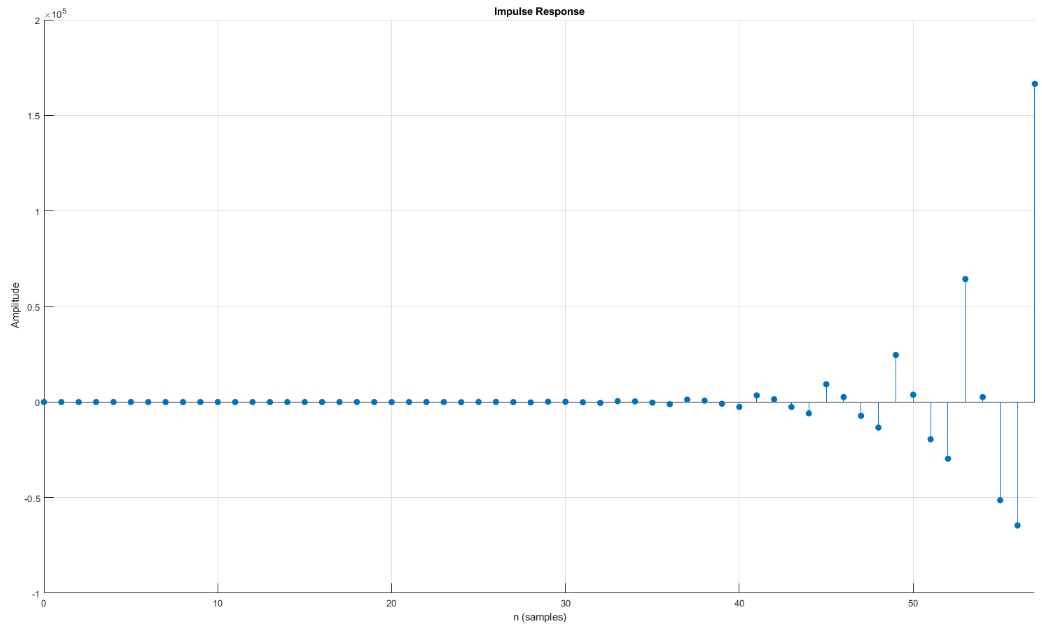

Figure 10: Impulse response of the linear system.

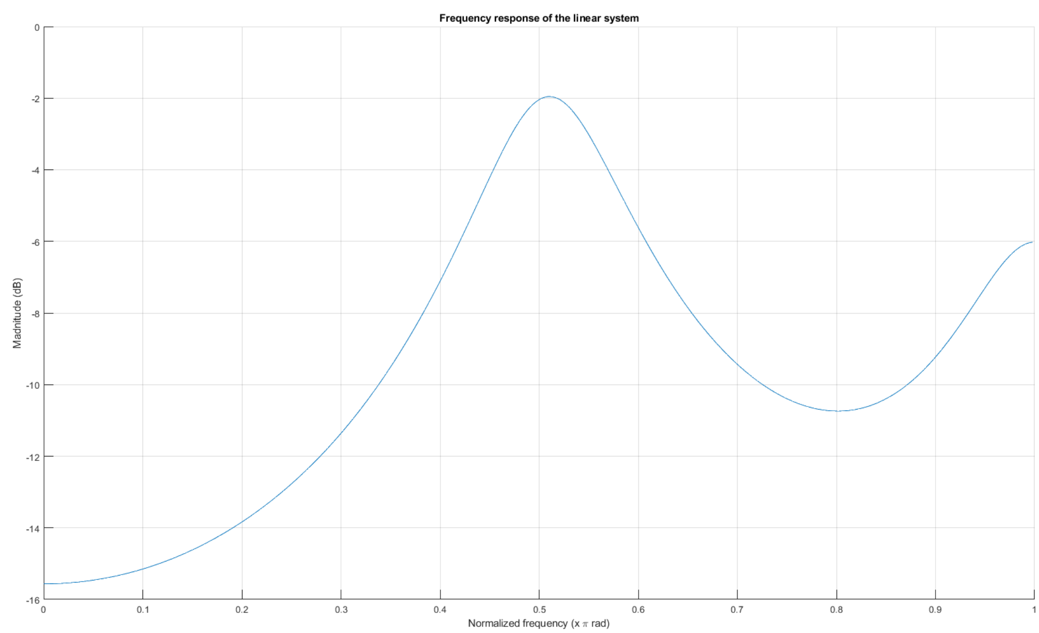

Figure 11: Frequency response of the linear system.

```
bz = [0 0 1 2];
az = [3 4 5 6];

figure(1);
hold on;
impz(bz, az);
grid on;
hold off;

figure(2);
[H, W] = freqz(bz, az);
H_dig_db = 20 * log10(abs(H)); % convert magnitude to dB
hold on;
plot(W / pi, H_dig_db);
title('Frequency response of the linear system')
ylabel('Madnitude (dB)')
xlabel('Normalized frequency (x \pi rad)')
grid on;
hold off;
```