CP1

Numerical Ordinary Differential Equations

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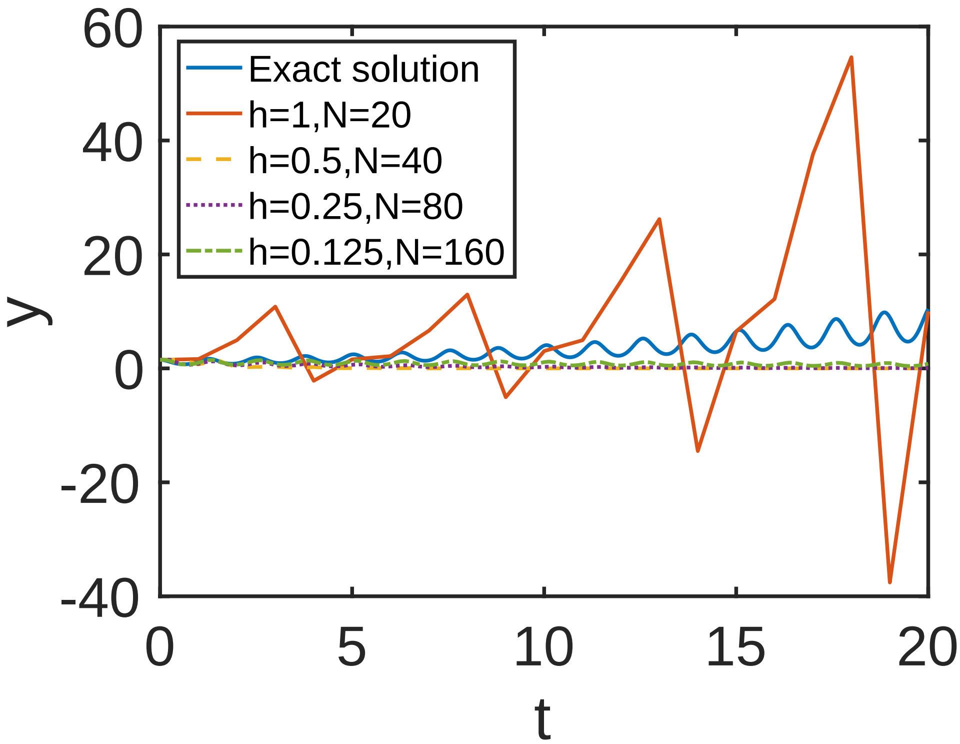

Figure 1: This graph shows solutions, y, against the dependent variable, t, determined exactly and by numerical estimation using Euler's method for varying the number of steps, N, and stepsize, h, values. The three lowest stepsizes are quite overlapping and zooming may be required.

This graph shows the exact solution of the given equation and 4 numerical approximations using Euler's method for varying stepsize(h) and number of steps(N). Over the period of 20 seconds it shows none of the values give a good approximation of the solution. For h=1 it drastically overshoots and undershoots at almost every step, while for the other h values they undershoot and oscillate slightly near 0 in a similar manor. Lower h values appear to give a better approximation at the start of the period but become less close later on. However, a smaller h gives a closer answer for longer to the exact solution. It appears that smaller h values give a better approximation of the solution, for a longer period the lower the value of h.

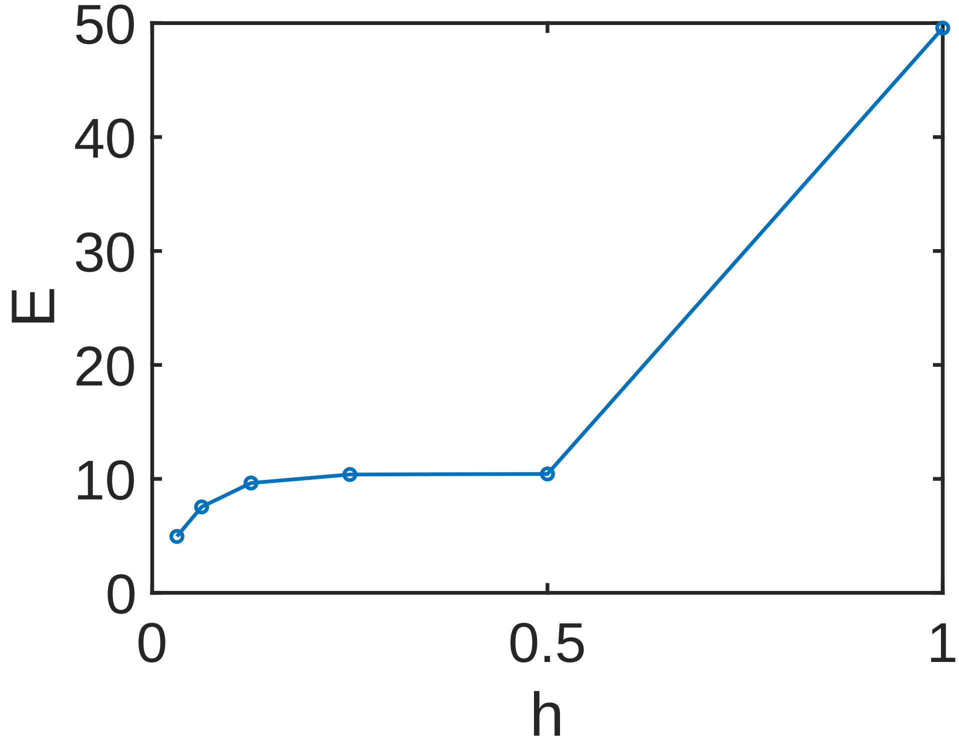

Figure 2: This graph shows the maximum absolute error, E, against stepsize, h.

This graph shows the maximum absolute error of Euler's method for this function when h is varied(and therefore so is N). At the first two values of h(when it is at the lower end of the range) a linear relation is noticed. At higher values of h there is no discernible relation. This is as expected as Euler's method only holds for small enough values of h, and has a linear relation when small enough. Above a certain value of h the linear relation is no longer observed. If this was to be repeated for smaller values of h then I would expect a more linear relation to be shown.

For the relative error to be less than 1% in the region of t $= [0, 20]$, a value of h $< 4.858 \times 10^{-4}$ (to 4 significant figures) is required. I obtained this by using a for loop for different h values to determine the solution using a numerical approximation over N steps(determined by the value of h). The exact solution was found and the maximum of the absolute difference between the two was calculated for the maximum absolute error, E. The maximum of the absolute value of the exact solution was then found and the maximum absolute error was divided by this to normalise it and find the relative error(which was multiplied by 100 for a percentage). I then plotted the values of the relative error against the stepsize and repeated for lower h values until 1% was found by eye. I then began narrowing the search range around the expected value of h that gave 1% and displaying the values. I did this until I found the value of h at which the relative error fell below 1%.