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Solid State Assignment 5
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Problem 2: Critical Field of a Thin Superconducting Film

a) Taking result: $\mu_0 M(x) \approx -B_a\left(\frac{1}{8\lambda_L^2}\right)\left(\delta^2 - 4x^2\right)$; $\delta \ll \lambda_L$; $-\delta/2 < x < \delta/2$

Work density to bring a superconductor into $\underline{B}$ field

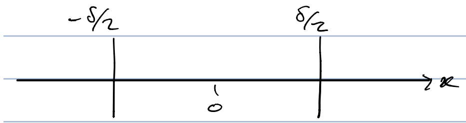
$$W = -\int_0^{B_a} \underline{M}\cdot d\underline{B}$$

$$= -\int_0^{B_a} -\frac{B_a}{\mu_0}\left(\frac{1}{8\lambda_L^2}\right)\left(\delta^2 - 4x^2\right)\cdot dB \qquad \text{both } \hat{\underline{x}}$$

$$= \frac{(\delta^2 - 4x^2)}{8\mu_0\lambda_L^2}\int_0^{B_a} B \, dB = \frac{B_a^2(\delta^2-4x^2)}{16\mu_0\lambda_L^2}$$

The free energy density inside the superconductor at field strength B_a is the initial free energy density at 0 field strength ; $F_s(0)$ plus the additional amount gained from the work density to bring the superconductor into the field of strength B_A

$$\therefore \; F_s(x, B_a) \approx F_s(B=0) + \frac{B_a^2(\delta^2-4x^2)}{16\mu_0\lambda_L^2} \qquad \text{when } \delta \ll \lambda_L \text{ from } M(x) \text{ sol.}$$



where x is the distance from the centre of the SC along $\hat{\underline{x}}$.

b) $F_{SM}(x, B_a) = \frac{(\delta^2 - 4x^2)B_a^2}{16\mu_0\lambda_L^2}$

$$\frac{\int_{-\delta/2}^{\delta/2} F_{SM}(x, B_a)\, dx}{\delta}$$ is the average (over x) magnetic contribution to the free energy density in the SC

$$= \frac{1}{\delta}\int_{-\delta/2}^{\delta/2} \frac{(\delta^2-4x^2)B_a^2}{16\mu_0\lambda_L^2}\, dx$$

$$= \frac{B_a^2}{16\mu_0\lambda_L^2}\left(\underbrace{\Big[\delta x\Big]_{-\delta/2}^{\delta/2}}_{\frac{\delta^2}{2}+\frac{\delta^2}{2} = \delta^2} - 4\underbrace{\left[\frac{x^3}{\delta 3}\right]_{-\delta/2}^{\delta/2}}_{\frac{\delta^2}{24}+\frac{\delta^2}{24} = \frac{\delta^2}{12}}\right)$$

$$= \frac{B_a^2}{16\mu_0\lambda_c^2}\left(\delta^2 - \frac{\delta^2}{3}\right) = \frac{B_a^2}{24\mu_0}\left(\frac{\delta}{\lambda_c}\right)^2$$

c) The normal state has $F_N(x, B_{ac}) \approx$ const.

At the critical field B_{ac}; $F_S(B_{ac}) = F_N(B_{ac})$

We can write F_S using the average magnetic contribution, instead of the x dependent contribution, as:

$$F_S(B_{ac}) = F_S(0) + \frac{B_{ac}^2}{24\mu_0}\left(\frac{\delta}{\lambda_c}\right)^2$$

We can write $F_N(B_{ac}) = F_N(0)$ as it is $\sim$ constant

And so the condensation energy is:

$$F_c = F_S(B_{ac}) - F_S(0) = F_N(0) - F_S(0)$$

$$= \frac{B_{ac}^2}{24\mu_0}\left(\frac{\delta}{\lambda_c}\right)^2$$

$$F_c = \frac{1}{2\mu_0} H_c^2$$

$$\therefore \frac{1}{2\mu_0} H_c^2 = \frac{B_{ac}^2}{24\mu_0}\left(\frac{\delta}{\lambda_c}\right)^2$$

$$B_{ac} = 2\sqrt{3}\left(\frac{\lambda_c}{\delta}\right) H_c$$

$F_c = \frac{1}{2\mu_0} H_c^2$ comes from considering a bulk type-I SC

$\hookrightarrow$ $M \propto H$

$\hookrightarrow$ $W = -\int_0^{B_a} \underline{M} \cdot d\underline{B} \propto H^2$

and then considering only the magnetic component of $F \propto W \propto H^2$

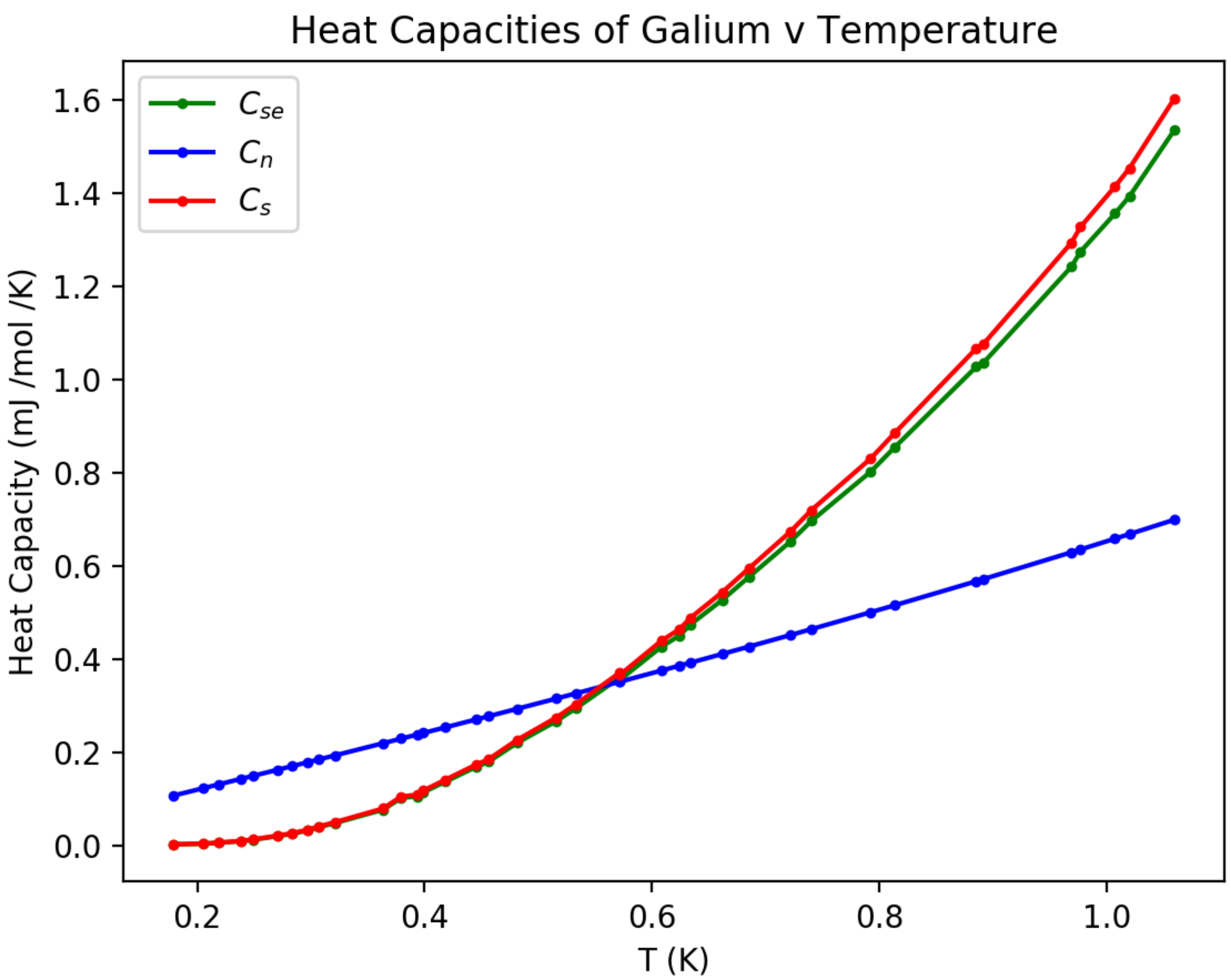
$\rightarrow$ the $\frac{1}{2\mu_0}$ are constants of portionality for a bulk type I SC

d) $\delta = 10\,nm$ $\lambda_c = 37\,nm$; Pb. $\Rightarrow$ $B_{ac} = 2\sqrt{3}\left(\frac{37}{10}\right) H_c$

$\therefore$ increase from bulk $\rightarrow$ $\frac{B_{ac}}{H_c} = 2\sqrt{3}\left(\frac{37}{10}\right) = 12.82$ factor increase.

Problem 1: Excitation Gap in Superconducting Galium

d) The calculated heat capacities from (a), (b) and (c) are plotted versus temperature below.



The data for these plots is included in the table titled 'Solid State Problem 1d Data'.

- C_s was found by multiplying the given C data by T
- C_{se} was found by getting the phonon contribution, C_{ph}, by taking $\beta = 0.0568$ (from figure 1) and multiplying by T^3, then subtracting C_{ph} from C_s.
- C_n was found by $\gamma T + C_{ph}$ where $\gamma = 0.596$ from figure 1.

e) $C_{ne}(T_c) = \gamma T_c$ where $\gamma = 0.596$ mJ mol^{-1} K^{-2} from the graph in Figure 1

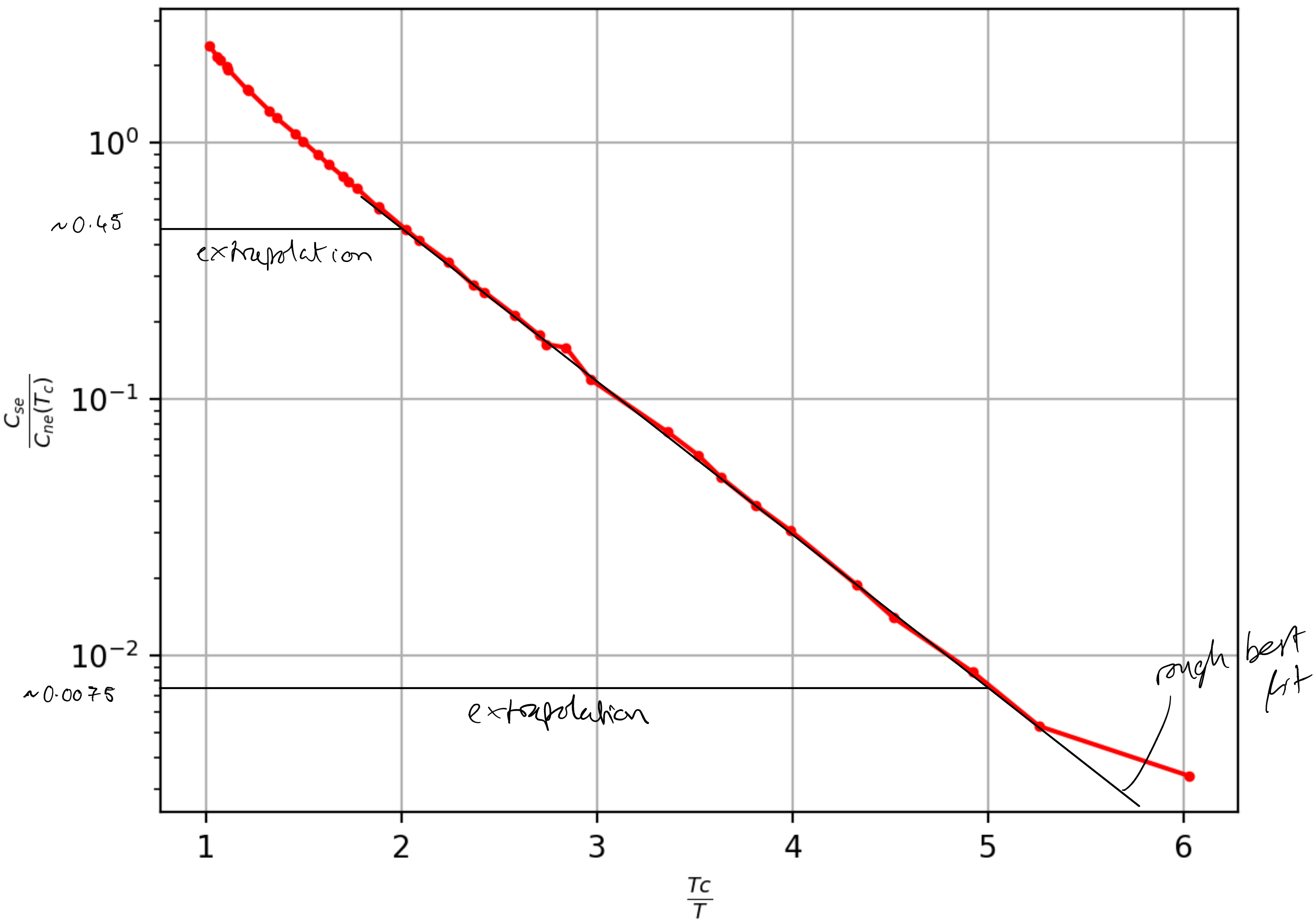
$= 0.596 \cdot 1.078$

$C_{ne}(T_c) = 0.642$ mJ mol^{-1} K^{-1}

Solid State Problem 1d Data

T (K)	C_n (mJ /mol /K)	C_s (mJ /mol /K)	C_se (mJ /mol /K)
2.05E-01	1.23E-01	3.89E-03	3.40E-03
2.19E-01	1.31E-01	6.13E-03	5.54E-03
2.39E-01	1.43E-01	9.79E-03	9.02E-03
2.49E-01	1.49E-01	1.29E-02	1.21E-02
2.70E-01	1.62E-01	2.08E-02	1.97E-02
2.83E-01	1.70E-01	2.60E-02	2.47E-02
2.97E-01	1.78E-01	3.32E-02	3.17E-02
3.07E-01	1.84E-01	4.05E-02	3.88E-02
3.21E-01	1.93E-01	4.97E-02	4.79E-02
3.63E-01	2.19E-01	7.92E-02	7.65E-02
3.79E-01	2.29E-01	1.05E-01	1.02E-01
3.94E-01	2.38E-01	1.09E-01	1.05E-01
3.99E-01	2.41E-01	1.18E-01	1.14E-01
4.18E-01	2.53E-01	1.41E-01	1.37E-01
4.45E-01	2.70E-01	1.73E-01	1.68E-01
4.56E-01	2.77E-01	1.85E-01	1.79E-01
4.82E-01	2.93E-01	2.27E-01	2.21E-01
5.16E-01	3.15E-01	2.74E-01	2.67E-01
5.33E-01	3.26E-01	3.03E-01	2.94E-01
5.72E-01	3.51E-01	3.71E-01	3.61E-01
5.72E-01	3.51E-01	3.66E-01	3.55E-01
6.08E-01	3.75E-01	4.39E-01	4.26E-01
6.24E-01	3.86E-01	4.65E-01	4.51E-01
6.34E-01	3.92E-01	4.89E-01	4.74E-01
6.62E-01	4.11E-01	5.43E-01	5.27E-01
6.86E-01	4.27E-01	5.95E-01	5.77E-01
7.22E-01	4.52E-01	6.73E-01	6.52E-01
7.40E-01	4.64E-01	7.20E-01	6.97E-01
7.92E-01	5.01E-01	8.31E-01	8.02E-01
8.14E-01	5.16E-01	8.85E-01	8.55E-01
8.85E-01	5.67E-01	1.07E+00	1.03E+00
8.92E-01	5.72E-01	1.08E+00	1.04E+00
9.69E-01	6.29E-01	1.29E+00	1.24E+00
9.77E-01	6.35E-01	1.33E+00	1.28E+00
1.01E+00	6.59E-01	1.41E+00	1.36E+00
1.02E+00	6.68E-01	1.45E+00	1.39E+00
1.06E+00	6.99E-01	1.60E+00	1.54E+00

f) $\frac{C_{se}(T)}{C_{ne}(T_c)}$ versus $\frac{T_c}{T}$ with a log scaled y axis.



g) $C_{ne}(T_c)$ and T_c provide a scaling factor. Looking at the proportionality:

$$C_{se} \propto e^{-\frac{E}{k_B T}}$$

$\therefore$ on a log graph against $\frac{1}{T}$ this will produce a straight line at gradient $-\frac{E}{k_B}$

This means there is an energy difference between different T values which corresponds to an energy gap. This is due to the known quantisation of energy and so this is the gap between the states.

h) $T < T_c/2 \rightarrow \frac{T_c}{T} > 2$

From g) we see that the gradient between two points, $m = -\frac{E_g}{k_B}$ where E_g is the energy gap,

From the plot in (f); $E_g = -k_B m = 0.118\,meV$ using marked extrapolations

The energy gap determined from (f) is the energy to excite one electron from a Cooper pair; $\Delta(0) = E_g$

BCS theory relates this. $2\Delta(0) \approx 3.5\, k_B T_c$.

From BCS theory; $\Delta(0) \approx \frac{7}{2} k_B T_c = 0.163$ meV

therefore the result from BCS theory is roughly the same as derived here.

Note; E_g could be found using $m = \frac{-E_g}{2k_B}$, which then defines E_g as the energy to break and excite a Cooper pair, where $E_g = 2\Delta(0)$.
→ I have used $m = \frac{-E_g}{k_B T}$ as this was given on the lecture slides.

This method works when ignoring the temperature dependence as it introduces a very small difference when $T < T_c/2$. The gap also decreases as T increases and so this gives an estimate at the max gap.

$\Delta(0)$ is the energy per electron (at T=0K) to break a Cooper pair and excite a single electron to a higher energy state