Principles of Statistics

Chapter Two Exercises

Ex 1 Given 2 discrete random variables X & Y, the marginal probability of X is computed by averaging the conditional probability of X given Y, for all values of Y. (You can think of it as ignoring the information from Y.)

$P(R=3 \mid W=6) = \frac{629}{3932} \approx .15997 \approx .160$

The marginal probability is $P(R=3) = .159$

$P(W=5 \mid R=4) = \frac{509}{2916} \approx .175$

$P(W=5) = .182$

Ex 2 These four possible cases. Let O be odd & E represent even. Then the 4 cases are rolling (E, E), (E, O), (O, E), (O, O), which are all equally probable. Each case results in E, O, O, E respectively. Thus the probability of the sum being E is $\frac{2}{4} = 1/2$.

For part b just start adding the even sum probabilities

Ex 3 This problem can be easily solved with a triplely-nested for loop. I used python & in about 10 lines of code got

$\frac{1}{216}(1, 3, 6, 10, 15, 21, 25, 27, 27, 25, 21, 15, 10, 6, 3, 1)$

Ex 4 The author means what's the probability of getting at least 1 "six". This is ~~the probabilit~~ equal to 1 - (probability of getting 0 "six"'s. So we get

$1 - \left(\frac{5}{6}\right)^4 \approx .5177.$

The probability of getting double sixes on 24 throws is

$1 - \left(\frac{35}{36}\right)^{24} \approx .4914$

Ex 5 a) Probability none have same birthday $\left(\frac{364}{365}\right)\left(\frac{363}{365}\right)$

$= \frac{363 \cdot 364}{365^2}$

b) Probability 2 have the same birthday

$$\underbrace{3}_{\text{\# of ways to choose who has the different birthday}} \cdot \underbrace{\left(\frac{1}{365}\right)}_{\text{Prob of 2 having same birthday}}\underbrace{\left(\frac{364}{365}\right)}_{\text{prob of having different birthday}} = 3 \cdot \frac{364}{365^2}$$

c) Probability of 3 having same birthday

$$\left(\frac{1}{365}\right)^2$$

Ex 6 8 fume cupboards would be adequate 100% of the time. $100\% \geq 95\%$ so 8 is correct, but perhaps we can find a tighter bound.

On average 4 workers will use 2 fume cupboards at a time.

$P(c=8) = (.1)^4 = \cancel{.0012}\ .0001$

$P(c=7) = 4(.1)^3(.3) = .0012$

$\cancel{P(C=6) = 4(.6)(.1)^3 + 12(.6)(.3)(.1)^2 = .0252\ .0168}$

$\cancel{P(C=5) = 12(.6)(.3)(.1)^2 + 4(.1)(.3)^3 = .0228\ .0156}$

$P(C=6) = 4(.6)(.1)^3 + 6(.3)^2(.1)^2 = .0078$

$P(C=5) = 12(.6)(.3)(.1)^2 + 4(.3)^3(.1) = .0324$

Thus you need 5 or more fume cupboards only 4.15% of the time. Thus 4 cupboards are sufficient 95.85% of the time.

Ex 7 The probability of a yarborough at whist is

$$\left(\frac{32}{52}\right)\left(\frac{31}{51}\right)\left(\frac{30}{50}\right)\cdots\left(\frac{19}{39}\right) \approx .00054703 < .001$$

so he has a good bet.