Introductory Electricity

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Problem Set 3

This problem set is due **July 27th, 2014** if you would like to receive feedback on your work. The exercises below vary in difficulty. Some are straightforward computational exercises intended to horn essential skills accompanying concepts in the previous lectures. Others may require more critical thinking to develop a deeper appreciation for the topics introduced. If you are unable to make progress on any particular problem and would like to obtain some hints before the solutions are released, please feel free to email us with your request.

1 Lecture Summary:

In lecture on July 20th, 2014, we discussed currents, resistance, and simple circuitry.

1. Resistance is the tendency to disrupt the flow of electrons. Thus, a wire which has a higher resistance can carry a smaller current for a given voltage than a wire which has a smaller resistance. This is the statement of Ohm's Law. Suppose a circuit has a resistance R and is driven by a voltage source V. Ohm's Law states that

$$V = IR,$$

where I is the current.

2. For a given material, the resistance is not a fixed constant, but it varies with temperature according to

$$R(T) = R_0[1 + \alpha(T - T_0)],$$

where T_0 is some reference temperature (often chosen to be room temperature), R_0 is the resistance at that temperature, and α is a material-dependent temperature coefficient of resistance. Just like I is the macroscopic version of $\mathbf{J}$, the resistance R is the macroscopic version of the resistivity ρ.

3. For a parallel circuit, the current is the same through all the resistors, and the equivalent resistance R_{eq} is the sum of all the resistances $R_1, R_2, ..., R_N$

$$R_{\text{eq}} = R_1 + R_2 + ... + R_N.$$

4. For a series circuit, the potential difference across each resistor is the same, the total current through the circuit is the sum of the current through each resistor, and the equivalent resistance R_{eq} is

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N}.$$

2 References:

1. To learn more about elementary circuits, please read chapters 19 and 20 of *Physics* by Raymond A. Serway and Jerry S. Faughn.
2. For some practice with circuits, please visit `http://www.physicsclassroom.com/class/circuits/Lesson-4/Parallel-Circuits`.
3. Those interested in learning more about superconductivity are encouraged to read *Superconductivity: a Very Short Introduction* by Stephen J. Blundell.
4. For some online resources on superconductivity at a beginner's level, please consult
   (a) `http://www.aip.org/history/mod/superconductivity/`
   (b) `http://www.supraconductivite.fr/en/index.php#supra-intro`
   (c) `http://web.ornl.gov/info/reports/m/ornlm3063r1/contents.html`

3 Exercises:

Exercise 1 – Ohm's Law: the Microscopic and Macroscopic Versions

Recall from the previous lecture the following expression derived from the Drude model of conductivity

$$\mathbf{J} = \sigma \mathbf{E},$$

where $\mathbf{J}$ is the current density, σ is the conductivity of the material, and $\mathbf{E}$ is the electric field. Explain how this equation is related to Ohm's Law

$$V = IR,$$

and show that the resistance R of a straight wire of cross sectional area A and length L is given by

$$R = \frac{L}{\sigma A}.$$

Determine the units of $\mathbf{J}$, $\mathbf{E}$, σ, V, I, and R in terms of Coulombs and other fundamental units (i.e. meters, kilograms, and seconds). To what does 1 Ω equal?

Exercise 2 – A Series Circuit

Find the current and potential difference across each of the resistors in the circuit of Fig. 1. Find the equivalent resistance.

Exercise 3 – A Parallel Circuit

Find the current and potential difference across each of the resistors in the circuit of Fig. 2. Find the equivalent resistance.

Exercise 4 – A Combination Circuit, Part I

We reference Fig. 3 in this problem. Let $R_1 = 10\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 20\ \Omega$, and $R_4 = 10\ \Omega$.

1. Find the equivalent resistance of this circuit.
2. If the total current is 10 A, what voltage must be supplied to the circuit?

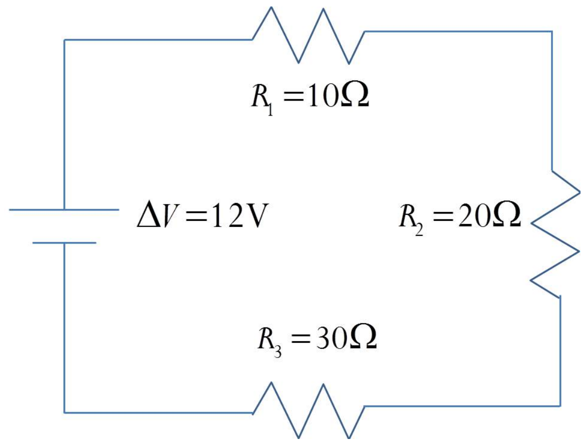

Figure 1: A series circuit.

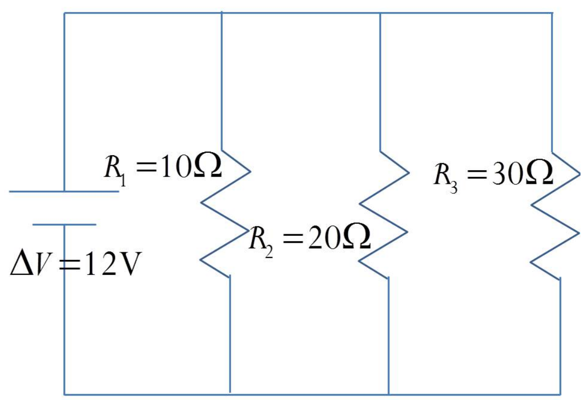

Figure 2: A parallel circuit.

3. The circuit is now connected to a 9-V battery. Find the current and potential difference across each resistor. *Hint:* It may help to redraw the circuit in a more recognizable form.

Exercise 5 – A Combination Circuit, Part II

Suppose you are given two resistors, R_1 and R_2, of unknown resistances and a power source of unknown voltage. You have an ammeter at your disposal with which you can measure current, but no voltmeter to measure the voltage difference. Assume that electrical wires are readily available for use. Determine a way to decipher the specifications of the resistors and power source. Please include any relevant circuit schematics.

Exercise 6 – The Sensitive Resistor

Consider a metal wire with temperature coefficient of resistance α and a room-temperature internal resistance R_0. The wire is connected to a battery of potential V. Assume that the charge-carrier density of the wire n is known. Calculate the drift velocity of the mobile electrons as a function of (any reasonable) temperature.

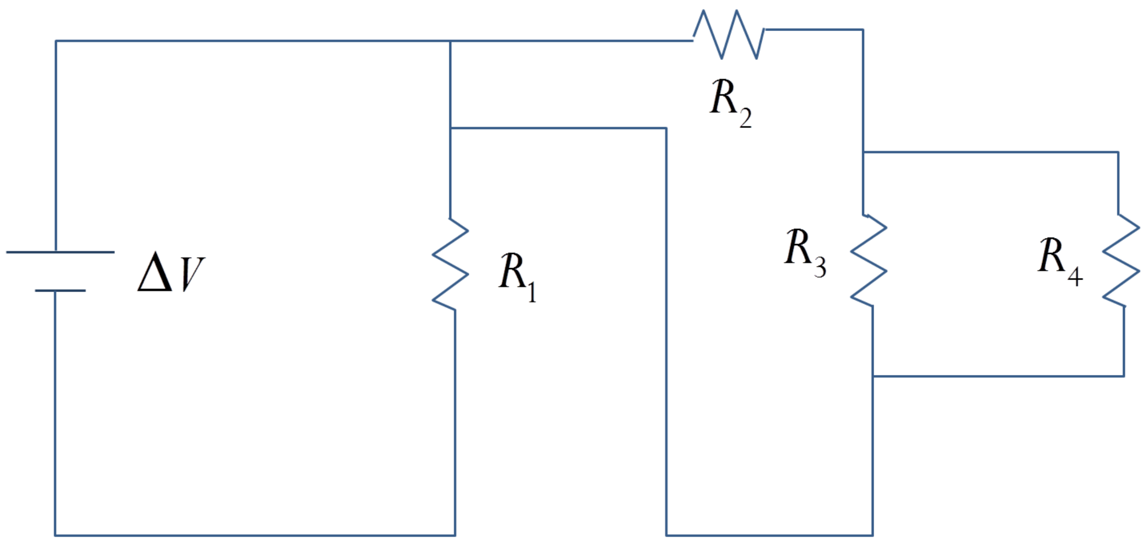

Figure 3: A combination circuit.

Exercise 7 – The Superconductor

Superconductors are conducting materials with zero resistance. Thus, currents flowing through them dissipate no heat and lose no energy. Unfortunately, all known superconductors today only operate at temperatures much lower than room temperature, limiting their potential usage drastically.

1. Recall from the Drude model of electrical conduction the origin of conductivity.
2. How does the BSC theory explain the lack of electrical resistivity in superconductors?
3. Is Ohm's Law valid for superconductors? Why?
4. Zero electrical resistance alone does not characterize superconductivity. Superconductors are also perfect diamagnets. This means that in the presence of a magnetic field, a superconductor will produce its own field to cancel out the applied field as to maintain an internal environment of zero magnetic field. This behavior explains the Meissner Effect of levitating magnets placed on a superconductor. Using these two properties of superconductivity, propose one possible usage of superconductors.