MATH 135 Fall 2006

Assignment #6 Due: Wednesday 01 November 2006, 8:20 a.m.

Hand-In Problems

1. Determine the remainder when $10^{2006} + 2006^{10}$ is divided by 7.

2. In each part, determine if the congruence has solutions. If it does, determine the complete solution.

   (a) $1713x \equiv 851 \pmod{2000}$

   (b) $1426x \equiv 851 \pmod{2000}$

   (c) $x^2 \equiv 2x \pmod{12}$

   (d) $8x \equiv 12 \pmod{52}$

3. (a) Determine the inverse of $[23]$ in $\mathbb{Z}_{71}$.

   (b) Determine the integer a with $0 \leq a < 71$ such that $[a] = [5]^{-1}[3] + [23]^{-1}[10]$.

4. (a) Prove that $3n^7 + 7n^3 + 11n \equiv 0 \pmod{3}$ for all $n \in \mathbb{P}$.

   (b) Prove that $21 \mid 3n^7 + 7n^3 + 11n$ for all $n \in \mathbb{P}$

5. In this problem, we prove that p is prime if and only if $(p-1)! \equiv -1 \pmod{p}$.

   (a) Prove that if p is not prime, then $(p-1)! \not\equiv -1 \pmod{p}$.

   (b) Prove that if $p = 2$, then $(p-1)! \equiv -1 \pmod{p}$.

   (c) Prove that if p is an odd prime and $a^2 \equiv 1 \pmod{p}$, then $a \equiv \pm 1 \pmod{p}$.

   (d) If p is an odd prime, explain why the set $\{[2], [3], \ldots, [p-2]\}$ in $\mathbb{Z}_p$ contains an even number of elements.

   (e) If p is an odd prime, prove that $[2][3]\cdots[p-2] = [1]$ in $\mathbb{Z}_p$.

   (f) If p is an odd prime, prove that $(p-1)! \equiv -1 \pmod{p}$.

6. In the annual Oktoberfest parade, Pounce de Lion walks at a constant rate of 1.6 m/s along the edge of a rectangular platform with sides 6 m long and ends 2 m long. The platform is moving in a straight line at a rate of 1 m/s. At time $t = 0$ seconds, Pounce starts from the midpoint of one end of the platform. He throws one candy to the crowd at each of the times $t = 1, 2, 3, 4, \ldots$ seconds, but only if he is walking along a side (not an end) at the time. If the platform travels 6 km, how many candies does Pounce throw to the crowd? Explain how you got your answer.
   (This problem is not directly related to the course material, but is included to keep your problem solving skills sharp.)

Recommended Problems

1. Text, page 83, #35

2. Text, page 83, #39

3. Text, page 83, #48

4. Text, page 84, #61

5. Text, page 83, #7

6. Let p be an odd prime number.

   (a) Prove that $x^2 + ab \equiv (a+b)x \pmod{p}$ has exactly two solutions modulo p.

   (b) Solve the linear congruence $2x \equiv 3 \pmod{p}$.

   (c) Solve the congruence $4x^3 \equiv 9x \pmod{p}$.