UNIVERSITY OF WATERLOO
FINAL EXAMINATION

Term: Spring **Year:** 2006

Student Name	_____
UW Student ID Number	_____

Course Abbreviation and Number	MATH 119
Course Title	Calculus 2 for Electrical & Computer Eng.
Sections(s)	001 and 002
Sections Combined Course(s)	SYDE 112
Section Numbers of Combined Course(s)	001
Instructor(s)	D. Harmsworth, S. Sivaloganathan, J. West

Date of Exam	August 4, 2006
Time Period	Start time: 16:00 End time: 18:30
Duration of Exam	$2\frac{1}{2}$ hours
Number of Exam Pages (including this cover sheet)	11 pages
Exam Type	Closed Book
Additional Materials Allowed	none.

Marking Scheme:

Question	Marks	Score
1	12	
2	10	
3	10	
4	13	
5	10	
6	15	
7	10	
8	10	
9	10	
Totals	100	

[12] 1. (a) Find $P_{3,0}(x)$ for the function $f(x) = \ln(1+x)$.

(b) Use your result to approximate $\ln 2$, by setting $x = \frac{-1}{2}$, and apply Taylor's Inequality to obtain an upper bound on the associated error.

[6] 2. (a) Find the radius of convergence of the power series $\sum_{k=0}^{\infty} \frac{(x-3)^k}{2^k\sqrt{k+1}}$. (You don't need to check the endpoints; we're asking only for the radius.)

[4] (b) Find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{2^{n-1}}$.

(**Hint:** Consider the geometric series $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$.)

[10] 3. Show that, if f is a function of the independent variables x and y, and the latter are changed to independent variables u and v, where $u = e^{y/x}$ and $v = x^2 + y^2$, then

$$x^3 \frac{\partial f}{\partial y} - x^2 y \frac{\partial f}{\partial x} = uv \frac{\partial f}{\partial u}.$$

[13] 4. Find the critical points of the function $z = 12xy - 3xy^2 - x^3$, and classify each point as a local maximum, local minimum, or saddle point.

[10] 5. Find the minimum value of $3x^2 + 2y^2 + 6z^2$, subject to the constraint $x + y + z = 1$.

[7] 6. (a) Evaluate $\int_0^4 \int_{\sqrt{x}}^2 e^{y^3}\, dy\, dx$.

[8] (b) Evaluate $\iiint_G xyz \; dV$, where G is the solid in the first octant bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the coordinate planes.

[10] 7. Let $\vec{F}(x,y) = \left(\frac{x}{y}, -x\right)$, defined for $x > 0,\ y > 0$.

Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the straight line segment from $(1,3)$ to $(3,1)$.

[10] 8. Consider the double integral $\int_0^{\frac{1}{2}} \int_0^{x^2} e^{-x^2} e^{-y} dy\, dx$.

Changing the order of iteration does not make this any easier to evaluate, and changing the coordinate system doesn't help either, so instead, **estimate** the value, and find an upper bound for the associated error. **Hint:** First reduce it to a single integral, and then use your knowledge of Taylor Polynomials -- second order should suffice.

[10] 9. Find the volume removed if a square hole of edge length a is cut vertically through the middle of a horizontal cylinder of radius a.