MATH 135 — Fall 2006

Assignment #9 — Due: Wednesday 29 November 2006, 8:20 a.m.

N.B. This is the final regularly scheduled Assignment. There will be an Assignment 10 created and solutions posted. While you should not hand Assignment 10 in, working through the problems will be helpful in learning the material from Chapter 9.

<u>Hand-In Problems</u>

1. Express $(\sqrt{2}-\sqrt{6}i)^{32}$ in standard form.
2. Prove that $(1+i)^n = 2^{\frac{1}{2}n}(\cos(\frac{1}{4}n\pi)+i\sin(\frac{1}{4}n\pi))$ for every $n \in \mathbb{P}$.
3. Determine all $z \in \mathbb{C}$ such that $z^8 = -16i$. Plot your solutions in the complex plane.
4. Determine all $z \in \mathbb{C}$ such that $iz^3 + 1 + i = 0$. Plot your solutions in the complex plane.
5. (a) Prove directly that if $z = r\operatorname{cis}(\theta)$ and $w = s\operatorname{cis}(\phi)$, then $\dfrac{z}{w} = \dfrac{r}{s}\operatorname{cis}(\theta-\phi)$.

   (b) Using this result, evaluate $\dfrac{\sqrt{2}+\sqrt{6}i}{\sqrt{3}-\sqrt{3}i}$. Express your answer in polar form.
6. If $z \in \mathbb{C}$ and $r, s \in \mathbb{P}$, we define $z^{r/s}$ to be an sth root of z^r.
   Determine all possible values of $(\sqrt{3}-i)^{2/5}$.
7. Determine all $z \in \mathbb{C}$ such that $z^9 + z^6 + z^3 + 1 = 0$. Plot your solutions in the complex plane.
8. (a) Write $-4\sqrt{3}+4i$ in the form e^{x+iy} where $x, y \in \mathbb{R}$.

   (b) If $z \in \mathbb{C}$ and $\frac{1}{2}(e^{iz}+e^{-iz}) = 4i$, determine the two possible values of e^{iz}.

   (c) If $z = x+iy$ with $x, y \in \mathbb{R}$, show that $e^{iz} = e^{-y}\operatorname{cis}(x)$.

   (d) If $\frac{1}{2}(e^{iz}+e^{-iz}) = 4i$ and $z = x+iy$ with $x, y \in \mathbb{R}$, determine all possible values of z.
   If $z \in \mathbb{C}$, we define $\cos z = \frac{1}{2}(e^{iz}+e^{-iz})$, so this part solves the equation $\cos z = 4i$.
   In this part, you should explicitly find x and y. Do not try to take the "ln" of a complex number. (If you had $e^{iz} = 4i$ (which you won't), you should not write something like $iz = \ln(4i)$.)
9. Let $f(x) = 3x^3+2x^2+x+2$, $g(x) = 4x^2+x$ and $h(x) = 3x^5+2x^3+x^2+x+2$ be polynomials in $\mathbb{Z}_5[x]$. Determine

   (a) $f(x)+g(x)$

   (b) $f(x)g(x)$

   (c) The quotient and remainder when $g(x)$ is divided by $f(x)$

   (d) The quotient and remainder when $h(x)$ is divided by $g(x)$
10. Determine the number of five-digit positive integers whose digits have a product of 2000.
    (This problem is only partially related to the course material, but is included to keep your problem solving skills sharp.)

Recommended Problems

1. Text, page 219, #65
2. Text, page 219, #67
3. Text, page 220, #74
4. Text, page 222, #123
5. Text, page 222, #129
6. Text, page 262, #22