UNIVERSITY OF WATERLOO
FINAL EXAMINATION

Term: Fall Year: 2005

Student Name ______________________________

UW Student ID Number ______________________________

Course Abbreviation and Number	MATH 117
Course Title	Calculus 1 for Electrical & Computer Eng.
Sections(s)	001, 002, 003, 004, and 005
Sections Combined Course(s)	SYDE 111
Section Numbers of Combined Course(s)	001
Instructor(s)	☐ B. Ingalls ☐ D. Harmsworth (EE)
	☐ G. Tenti ☐ D. Harmsworth (CE)
	☐ M. Scott ☐ D.E. Chang

Date of Examination	December 12, 2005	
Time Period	Start time 12:30 p.m..	End time: 3:00 p.m.
Duration of Exam	2.5 hours	
Number of Exam Pages (including this cover sheet)	11	
Exam Type	Special Materials	
Additional Materials Allowed	Formula Sheet.	

Marking Scheme:

Question	1	2	3	4	5	6	7	Totals
Marks	15	15	15	15	15	15	10	100
Score								

1. Evaluate the following integrals:

(a) $\int \frac{dx}{x^2(x-1)}$;

(b) $\int_0^{\pi} \sin^2(5x)dx$;

(c) $\int_0^{\frac{\pi}{2}} x^2 \sin(x)dx$

2. (a) Show that $\int f(x)dx = xf(x) - \int xf'(x)dx$, and use it to evalueate $\int \sin^{-1}(x)dx$.

(b) Evaluate $\int_0^\infty e^{-st} \cdot e^{j\omega t} dt, \quad (s > 0)$.

(c) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin(x)}{\sqrt{\cos(x)}} dx$.

3. (a) A simple pendulum performs a simple harmonic motion with period $P = 2\pi\sqrt{\ell / g}$, where ℓ is the constant length and g is the acceleration of gravity. First the pendulum is made to swing at the Equator ($g = 9.7805\ m \cdot s^{-2}$) and then at the North Pole ($g = 9.8322\ m \cdot s^{-2}$). What is the relative (*i.e.*, percentage) change in P in these two cases?
[You may use g at the North Pole as its nominal value.]

(b) The half-live of C^{14} is about 5,700 years, A piece of charcoal found at Stonehenge turns out to contain 63% as much C^{14} as a present day sample. Find the age of the Stonehenge sample.

4. (a) Gas escapes from a spherical balloon at the rate of $2\ m^3 \cdot min^{-1}$. How fast is the surface area shrinking when the radius is $12\,m$? [Area of a sphere equals $4\pi r^2$.]

(b) Solve the differential equation $x''(t) - x(t) = 0$ subject to the initial conditions $x(0) = 1$, $\frac{dx}{dt}(0) = 0$.

5. In an underwater telephone cable the ratio of the radius of the core to the thickness of the protective sheath is denoted by x. The speed υ at which a signal is transmitted is proportional to $x^2 \ell n\left(\frac{1}{x}\right)$.

Show that

$$\frac{d\upsilon}{dx} = Kx\left[2\ell n\left(\frac{1}{x}\right) - 1\right]. \quad (K > 0 \text{ and constant}).$$

Hence deduce the stationary value of υ (*i.e.*, its extrema), distinguish between them, and show that the speed is greatest when $x = \frac{1}{\sqrt{e}}$.

6. Solve the differential equation

$$x''(t) + 4x'(t) + 5x(t) = 0$$

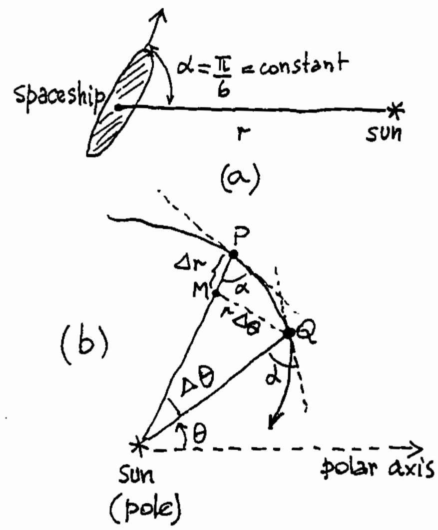
subject to the initial conditions

$$x(0) = 1 \ , \qquad \frac{dx}{dt}(0) = 0 \ ,$$

and express the solution in terms of real functions.

7. Luke Skyrunner has just been knocked out in his spaceship by his archenemy, Captain Nogood. The evil captain has set the controls to send the spaceship into the sun! His perverted mind insists on a slow death; therefore, he sets the controls so that the ship makes a constant angle of $\alpha = \frac{\pi}{6}$ with the sun (See figure (a).) Find the path that Luke's spaceship will follow. [**Hint**: It is convenient to use polar coordinates to find $\frac{\Delta r}{\Delta \theta}$ (see figure (b).) Then le $\Delta\theta \to 0\ldots$

Also note that the path is an open curve; so that θ should range from -∞ to +∞]



From the basic definitions it is possible to deduce the following trigonometric identities relating the functions.

Triangle identities

$$\cos^2 x + \sin^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \operatorname{cosec}^2 x$$

Compound-angle identities

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Sum and product identities

$$\sin x + \sin y = 2 \sin \tfrac{1}{2}(x+y) \cos \tfrac{1}{2}(x-y)$$
$$\sin x - \sin y = 2 \sin \tfrac{1}{2}(x-y) \cos \tfrac{1}{2}(x+y)$$
$$\cos x + \cos y = 2 \cos \tfrac{1}{2}(x+y) \cos \tfrac{1}{2}(x-y)$$
$$\cos x - \cos y = -2 \sin \tfrac{1}{2}(x+y) \sin \tfrac{1}{2}(x-y)$$

$$\sin 2x = 2 \sin x \cos x$$
$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x\end{aligned}$$
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

(Writing $x = \theta/2$ we can obtain similar identities called half-angle formulae.)