Math 117 / SyDe 111 Assignment #7 Fall 2006

due November 17th

- MEM, p.523 #38, 39a, 39c
- MEM, p.518 #34
- MEM, p.495 #7
- MEM, p.508 #19
- MEM, p.593 #1a, 2b, 5

Problem I: Carbon Dating

Carbon 14 is a radioactive isotope produced by the interaction of the sun's rays with our atmosphere. The ratio of C14 to normal carbon (C12) is roughly constant in the atmosphere, and this same ratio is present in all living things (since plants absorb it from the air, and animals eat the plants). However, when an organism dies, it stops absorbing C14, so the ratio begins to drop due to radioactive decay. Measuring how much it has dropped allows us to tell how long ago an organism died. Now, let $x(t)$ represent the mass of C14 in a sample of organic matter, with time t measured from the time of death, in years.

i) All radioactive substances are known to decay at a rate proportional to the amount of the substance present. Translate this statement into a differential equation for $x(t)$.

ii) This differential equation is simple enough that you should be able to <u>guess</u> the form of the function $x(t)$.

There will, however, be two unknown constants in it.

iii) For these two constants, we need two pieces of information. One could be the original mass of C14 in the sample. We don't know this, but call it X_0 (i.e. let $x(0) = X_0$).

A second piece of information <u>is</u> known: C14 has a half-life of about 5700 years, meaning that $x(5700) = \frac{1}{2}x(0)$.

Use these to evaluate the constants.

iv) Finally, suppose our sample is found to contain 63% of the atmospheric level of C14. How old is the sample?

Other suggested problems (not to be handed in)

MEM, p. 523 *37, 39b
p. 528 *41ab, 43, 47, 52, 58 } (differentiation (including implicit and logarithmic))
p. 537 *71 (intro to integration)
p. 580 *1,3 (differentiation)
p. 593 *1b, 2ac. (max/min)