CS 245 / SE 112
Natural Deduction for Predicate Logic

To the rules for propositional logic, we add the following:

	introduction	elimination
$=$	$\dfrac{}{t = t}\,{=}\mathsf{i}$	$\dfrac{t_1 = t_2 \quad \phi[t_1/x]}{\phi[t_2/x]}\,{=}\mathsf{e}$
$\forall$	$\dfrac{\forall x \phi}{\phi[t/x]}\,\forall x\,\mathsf{e}$	$\dfrac{\boxed{\begin{array}{l} x_0 \\ \vdots \\ \phi[x_0/x] \end{array}}}{\forall x \phi}\,\forall x \mathsf{i}$
$\exists$	$\dfrac{\phi[t/x]}{\exists x \phi}\,\exists x \mathsf{i}$	$\dfrac{\exists x \phi \qquad \boxed{\begin{array}{ll} x_0 & \phi[x_0/x] \\ & \vdots \\ & \chi \end{array}}}{\chi}\,\exists x \mathsf{e}$

In all substitutions $\phi[t/x]$, t must be free for x in ϕ, that is, no free occurrence of x in ϕ is in the scope of any $\forall y$ or $\exists y$ for any variable y in t. The introduced variables x_0 must be fresh (not occurring outside their boxes).

For convenience, we add one derived rule to the above rules from the textbook.

$$\frac{t_1 = t_2 \quad \phi[t_2/x]}{\phi[t_1/x]}\,{=}\mathsf{e}_R$$