MATH 135 — Fall 2006

Assignment #7 — Due: Wednesday 08 November 2006, 8:20 a.m.

Hand-In Problems

1. Solve the simultaneous congruences

$$\begin{aligned} 2x &\equiv 13 \pmod{59} \\ 5x &\equiv 42 \pmod{34} \end{aligned}$$

2. Solve the simultaneous congruences

$$\begin{aligned} x &\equiv 3 \pmod{5} \\ x &\equiv 7 \pmod{11} \\ x &\equiv 17 \pmod{19} \end{aligned}$$

3. Solve the following system of equations in $\mathbb{Z}_{17}$:

$$\begin{aligned} [5]\,[x] + [4]\,[y] &= [5] \\ [6]\,[x] + [9]\,[y] &= [1] \end{aligned}$$

4. Determine all solutions to the congruence $x^{13} + 7x + 5 \equiv 0 \pmod{91}$.

5. If p and q are integers that are not divisible by 3 or 5, prove that $p^4 \equiv q^4 \pmod{15}$.

6. Consider the system of simultaneous congruences

$$\begin{aligned} x &\equiv a \pmod{m} \\ x &\equiv b \pmod{n} \end{aligned}$$

where $\gcd(m, n) = d$.

   (a) By following the technique for solving a system of congruences, prove that if this system has a solution, then $d \mid a - b$.

   (b) By modifying the proof of the Chinese Remainder Theorem, prove that if this system has a solution, then it has a unique solution modulo $\frac{mn}{d} = \text{lcm}(m, n)$.

   (c) Solve the system with $m = 52$, $n = 32$, $a = 7$, and $b = 15$.

7. In a sequence of p zeros and q ones, the ith term, is called a *change point* if $t_i \neq t_{i-1}$, for $i = 2, 3, 4, \ldots, p+q$. For example, the sequence $0, 1, 1, 0, 0, 1, 0, 1$ has $p = q = 4$ and five change points t_2, t_4, t_6, t_7, t_8. Consider all possible sequences of p zeros and q ones, with $1 \leq p \leq q$.

   (a) Determine the minimum possible number of change points. Justify your answer.

   (b) Determine the maximum possible number of change points if $p = q$. Justify your answer.

   (c) Determine the maximum possible number of change points if $p < q$. Justify your answer.

   (This problem is not directly related to the course material, but is included to keep your problem solving skills sharp.)

Recommended Problems

1. Text, page 83, #52
2. Text, page 83, #55
3. Text, page 86, #81
4. Text, page 86, #84
5. Text, page 87, #95