CS 370 Winter 2009: Assignment 1

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Lectures: MWF 9:30 MC2038 Office Hours: Tues 2:30-3:30 PM
Office Hours: Thurs 9:30-10:30 AM

Web Site: www.cs.uwaterloo.ca/~glabahn/cs370/

Due Wednesday Jan 28, 5:00 PM, in the Assignment Boxes, 3rd Floor MC

1. bf (10 marks) Consider the following two recurrence relations:

$$p_n + p_{n-1} - \frac{6}{25}p_{n-2} = 0 \quad \text{and} \quad q_n - q_{n-1} + \frac{4}{25}q_{n-2} = 0.$$

   (i) (2 marks) Show that if we are using exact arithmetic and $p_0 = q_0 = 1$ and $p_1 = q_1 = \frac{1}{5}$ then $p_n = \frac{1}{5^n}$ and $q_n = \frac{1}{5^n}$. Thus in exact arithmetic both recurrence relations are valid methods for computing the sequence $\frac{1}{5^n}$.

   (ii) (8 marks) Which recurrence relation should be used with initial values $p_0 = q_0 = 1$ and $p_1 = q_1 = \frac{1}{5}$ when one works in floating point arithmetic. Justify your answer by analysing errors.

2. (10 marks) Let F be a Floating Point Number System with machine epsilon E. In this question you are to analyze the expression $y^2 - 1$ for y a floating point number. Since $y^2 - 1 = (y-1)(y+1)$ there are two ways to do such a computation.

   a) One of the computations is $(y \ominus 1) \otimes (y \oplus 1)$. Show that

$$\frac{|(y \ominus 1) \otimes (y \oplus 1) - (y^2 - 1)|}{|y^2 - 1|} \leq 3E \tag{1}$$

   assuming no arithmetic exceptions occur and we do the algebra of analysing the error to first order (that is, we drop terms of size E^2 and smaller). Note that floating point multiplication is defined as $a \otimes b = fl(a * b)$.

   b) Show that (again up to first order)

$$\frac{|(y \otimes y) \ominus 1 - (y^2 - 1)|}{|y^2 - 1|} \leq \left(\frac{|y^2|}{|y^2 - 1|} + 1\right)E \tag{2}$$

   c) If you knew that $y^2 \approx 1$, which of the expressions $(y \otimes y) \oplus (-1)$ or $(y \ominus 1) \otimes (y \oplus 1)$ would you pick to evaluate $y^2 - 1$? Justify your answer.

3. **(10 marks)** Given 2 parameters, a and b

   (a) Compute the unique cubic polynomial, $p(x)$ such that

$$p(1) = a, \; p(2) = b, \; p'(0) = b, \; p''(1) = a - b$$

   The coefficients of $p(x)$ are functions of a and b.

(b) Show that there is no unique cubic polynomial such that

$$p(0) = u,\ p(1) = 1,\ p'(2) = v,\ p''(\frac{11}{9}) = 0$$

For what values of u and v is there some cubic polynomial satisfying these conditions?

4. **(10 marks)**

   (a) Write down the linear system of equations needed to determine the derivative values s_1, s_2, s_3, s_4 for the cubic spline $S(x)$ going through the points $(0,1)$, $(1,3)$, $(3,-5)$ and $(4,1)$ and having natural boundary conditions.

   (b) What is the resulting cubic spline $S(x)$ from part (a)?

5. (a) **(6 marks)** Determine the values $a_1, \ldots, a_4$ so that

$$S(x) = \begin{cases} a_1 + 25x + 9x^2 + x^3 & x \in [-3,-1] \\ 26 + a_2 x + a_3 x^2 - x^3 & x \in [-1,0] \\ 26 + 19x + a_4 x^2 - x^3 & x \in [0,1] \end{cases}$$

   is a spline with natural boundary conditions.

   (b) **(4 marks)** Compute the values of $S(x)$ at $-3, -1, 0, 1$ and then construct the cubic interpolating polynomial using a Lagrange basis.

6. **(20 marks)**

   (a) Draw a parametric curve representation of the curves generated by the 28 points found in the *points* file found on the course web page. Generate 2 pictures, first using piecewise linear splines and then using cubic splines. The curves with the cubic spline should appear smooth.

   (b) Draw the image on graph paper (as described in the course notes) of your name. You can use your initials (at least three characters) or other images representing the name. Please make sure that all letters are different - give yourself a nickname for a day if need be.

   Construct a parameteric representation of the curves in this image. Use parametric curve interpolation to show the output using piecewise linear splines and cubic splines. More precisely you are to do the following.

   Prepare two Matlab .m files, one for each of the following tasks:

   1 A plot of the symbols in the equation created by joining the original data points with straight lines. (The symbols will not look very smooth). Plot axes and grid lines for this plot. The plot should have a title.

   2 Create two plots showing a smoother spline version of the symbols in the mathematical equation. The two plots should use at least two different spline end conditions for the symbols. For example, if the first plot uses *not-a-knot* conditions for the symbols then you might try natural conditions for the second plot. Do not use any axes or grid lines for these plots. The plots should have titles.

For both parts submit both your graphs and a hard copy of your code.