

## 5. Challenges and Tools in Empirical Work

PhD Applied Methods

Duncan Webb  
NovaSBE

Spring 2026

## Empirical work in practice

- In the previous lectures, we focused on the **identification** of causal effects
  - Potential outcomes framework
  - Randomized controlled trials
  - Selection bias and how to avoid it

## Empirical work in practice

- In the previous lectures, we focused on the **identification** of causal effects
  - Potential outcomes framework
  - Randomized controlled trials
  - Selection bias and how to avoid it
- But implementing empirical research involves many additional **practical challenges**
- Today: focus on a set of tools and techniques for dealing with these challenges

## Why focus on RCTs?

- We'll use **randomized controlled trials** as our main example throughout
- **Why?** Because with RCTs we can abstract away from identification concerns

## Why focus on RCTs?

- We'll use **randomized controlled trials** as our main example throughout
- **Why?** Because with RCTs we can abstract away from identification concerns
- This allows us to focus on the **empirical difficulties** that arise even when identification is clean
- **Important:** Most of these tools apply to other research designs too (DiD, IV, RDD, etc.)

# Overview of topics

- ① **Clustering:** When and how to randomize at the group level
- ② **Power:** Designing experiments with sufficient statistical power
- ③ **Choosing controls:** Which variables should we control for?
- ④ **Heterogeneous treatment effects:** How do effects vary across groups?
- ⑤ **Attrition:** What to do when participants drop out?
- ⑥ **Multiple hypothesis testing:** How to avoid false discoveries?
- ⑦ **Robustness and replication:** How reliable are empirical results?
- ⑧ **Standard errors:** How to correctly compute uncertainty?
- ⑨ **Spillovers:** What if SUTVA is violated?
- ⑩ **Measurement:** How to deal with measurement error?

## Beyond individual randomization

- So far: randomization at the **individual level**
- Sometimes randomizing individuals is:
  - **Impractical**: Hard to treat some students in a school but not others
  - **Unethical**: Perceived as unfair within communities
  - **Contaminated**: Treatment spills over between individuals

## Beyond individual randomization

- So far: randomization at the **individual level**
- Sometimes randomizing individuals is:
  - **Impractical**: Hard to treat some students in a school but not others
  - **Unethical**: Perceived as unfair within communities
  - **Contaminated**: Treatment spills over between individuals
- **Solution**: Randomize at a higher level - **cluster randomization**
- Treat entire groups (clusters) as units: schools, villages, clinics, firms



## Examples of clustered randomization

- **Education:** Randomize schools (not students)
  - Teacher training programs
  - School infrastructure improvements
- **Health:** Randomize clinics or villages
  - Deworming programs (Miguel & Kremer 2004)
  - Community health worker programs
- **Development:** Randomize villages or districts
  - Microfinance expansion
  - Infrastructure projects (roads, electricity)

## Examples of clustered randomization

- **Education:** Randomize schools (not students)
  - Teacher training programs
  - School infrastructure improvements
- **Health:** Randomize clinics or villages
  - Deworming programs (Miguel & Kremer 2004)
  - Community health worker programs
- **Development:** Randomize villages or districts
  - Microfinance expansion
  - Infrastructure projects (roads, electricity)

**Key insight:** The unit of randomization  $\neq$  unit of analysis

## Spillovers and SUTVA violations

Recall our SUTVA assumption:  $Y_i$  depends only on own treatment  $D_i$

**With spillovers, potential outcomes become:**

$$Y_i(D_i, \mathbf{D}_{-i}) \quad (1)$$

where  $D_{-i}$  is the treatment status of others

**With spillovers, potential outcomes become:**

$$Y_i(D_i, \mathbf{D}_{-i}) \quad (1)$$

where  $\mathbf{D}_{-i}$  is the treatment status of others

### Example: Deworming

- Direct effect: Health benefits to treated children
- **Spillover**: Reduced disease transmission to untreated children
- Individual randomization would **underestimate** total effect

## Spillovers and SUTVA violations

Recall our SUTVA assumption:  $Y_i$  depends only on own treatment  $D_i$

**With spillovers**, potential outcomes become:

$$Y_i(D_i, \mathbf{D}_{-i}) \quad (1)$$

where  $\mathbf{D}_{-i}$  is the treatment status of others

**Example:** Deworming

- Direct effect: Health benefits to treated children
- **Spillover:** Reduced disease transmission to untreated children
- Individual randomization would **underestimate** total effect

**Cluster randomization** partially solves this:

- Captures within-cluster spillovers
- But still misses cross-cluster spillovers

## Notation for clustered designs

Let  $c = 1, \dots, C$  index clusters,  $i = 1, \dots, N_c$  index individuals within cluster  $c$

- $D_c \in \{0, 1\}$ : treatment status of cluster  $c$
- $Y_{ic}$ : outcome for individual  $i$  in cluster  $c$
- All individuals in cluster  $c$  receive same treatment

## Notation for clustered designs

Let  $c = 1, \dots, C$  index clusters,  $i = 1, \dots, N_c$  index individuals within cluster  $c$

- $D_c \in \{0, 1\}$ : treatment status of cluster  $c$
- $Y_{ic}$ : outcome for individual  $i$  in cluster  $c$
- All individuals in cluster  $c$  receive same treatment

**Potential outcomes:**

$$Y_{ic}(1) = \text{outcome if cluster } c \text{ is treated} \quad (2)$$

$$Y_{ic}(0) = \text{outcome if cluster } c \text{ is not treated} \quad (3)$$

## Notation for clustered designs

Let  $c = 1, \dots, C$  index clusters,  $i = 1, \dots, N_c$  index individuals within cluster  $c$

- $D_c \in \{0, 1\}$ : treatment status of cluster  $c$
- $Y_{ic}$ : outcome for individual  $i$  in cluster  $c$
- All individuals in cluster  $c$  receive same treatment

**Potential outcomes:**

$$Y_{ic}(1) = \text{outcome if cluster } c \text{ is treated} \quad (2)$$

$$Y_{ic}(0) = \text{outcome if cluster } c \text{ is not treated} \quad (3)$$

**Observed outcome:**

$$Y_{jc} = D_c Y_{jc}(1) + (1 - D_c) Y_{jc}(0) \quad (4)$$

Note: Everyone in the cluster has the same  $D_C$ !



## Estimation with clustering

Simple comparison of means still works:  $\widehat{ATE} = \bar{Y}_{treated} - \bar{Y}_{control}$

## Estimation with clustering

Simple comparison of means still works:  $\widehat{ATE} = \bar{Y}_{treated} - \bar{Y}_{control}$

**BUT:** Standard errors must account for clustering!

**Why?** Outcomes within clusters are correlated:

- Students in same school face same teachers, facilities
- This reduces **effective sample size**

## Estimation with clustering

Simple comparison of means still works:  $\widehat{ATE} = \bar{Y}_{treated} - \bar{Y}_{control}$

**BUT:** Standard errors must account for clustering!

**Why?** Outcomes within clusters are correlated:

- Students in same school face same teachers, facilities
- This reduces **effective sample size**

**Intuition:** 1000 students in 10 schools provides **less information** than 1000 randomly selected students

⇒ Use **cluster-robust standard errors**

## Design trade-offs: Number vs size of clusters

For a fixed total sample size  $N$ , how to allocate across clusters?

## Design trade-offs: Number vs size of clusters

For a fixed total sample size  $N$ , how to allocate across clusters?

**Key parameter:** Intra-cluster correlation (ICC) =  $\rho$

- $\rho$  = correlation between outcomes of individuals in same cluster
- $\rho = 0$ : no clustering, like individual randomization
- $\rho = 1$ : everyone in cluster identical

## Design trade-offs: Number vs size of clusters

For a fixed total sample size  $N$ , how to allocate across clusters?

**Key parameter:** Intra-cluster correlation (ICC) =  $\rho$

- $\rho$  = correlation between outcomes of individuals in same cluster
- $\rho = 0$ : no clustering, like individual randomization
- $\rho = 1$ : everyone in cluster identical

**Design effect** (variance inflation):

$$DE = 1 + (n - 1)\rho \quad (5)$$

where  $n$  = average cluster size

## Design trade-offs: Number vs size of clusters

For a fixed total sample size  $N$ , how to allocate across clusters?

**Key parameter:** Intra-cluster correlation (ICC) =  $\rho$

- $\rho$  = correlation between outcomes of individuals in same cluster
- $\rho = 0$ : no clustering, like individual randomization
- $\rho = 1$ : everyone in cluster identical

**Design effect** (variance inflation):

$$DE = 1 + (n - 1)\rho \quad (5)$$

where  $n$  = average cluster size

**Implications:**

- More clusters > bigger clusters (for statistical power)
- If  $\rho = 0.05$  and  $n = 20$ : need  $\approx 2\times$  the sample size!
- Rules of thumb: Need at least 20-30 clusters for reliable inference

# Advantages and disadvantages

## Advantages of cluster randomization:



# Advantages and disadvantages

## Advantages of cluster randomization:

- Captures within-cluster spillovers
- Administratively simpler
- More acceptable to communities
- Can study cluster-level interventions

# Advantages and disadvantages

## Advantages of cluster randomization:

- Captures within-cluster spillovers
- Administratively simpler
- More acceptable to communities
- Can study cluster-level interventions

## Disadvantages:

# Advantages and disadvantages

## Advantages of cluster randomization:

- Captures within-cluster spillovers
- Administratively simpler
- More acceptable to communities
- Can study cluster-level interventions

## Disadvantages:

- Lower statistical power
- Requires more clusters for balance
- Still misses cross-cluster spillovers
- Harder to study heterogeneous effects

# Advantages and disadvantages

## Advantages of cluster randomization:

- Captures within-cluster spillovers
- Administratively simpler
- More acceptable to communities
- Can study cluster-level interventions

## Disadvantages:

- Lower statistical power
- Requires more clusters for balance
- Still misses cross-cluster spillovers
- Harder to study heterogeneous effects

**Bottom line:** Use clustered randomization when spillovers matter or individual randomization is infeasible

## Outline

# Designing an experiment

Two main questions when designing an experiment:

- ① Who to randomize, how, etc.
- ② Sample size (and share treated)

Experiments are an unusual case where you have great control over sample size

The last thing you want: go through the whole burden and have insignificant effects because you have high standard errors

## Finite sample and inference

So far, we have always considered the asymptotic values of the estimator

For instance:

$$\mathbb{E}[Y_i | D_i = 1]$$

is the asymptotic value of:

$$\frac{1}{N_1} \sum_{i \in D_1} y_i$$

which, inversely, is the empirical counterpart to  $\mathbb{E}[Y_j|D_j = 1]$

This is because we have been interested in **identification** (what we would learn in infinite samples)

## Finite sample and inference

Random experiment:  $T$  and  $C$  are similar for  $N = \infty$

In finite samples,  $T$  and  $C$  always *somewhat* different, e.g. by chance my treatment group has slightly older students than the control group

This **imbalance** could be confounded with treatment effect

### Inference is accounting for that:

With finite sample, can I consider that the difference  $T$  vs.  $C$  is high enough to indicate more than unavoidable imbalance?

Yes, if statistically “significant”

Imbalance is not a source of bias; the standard error is there to account for that



## Reminder: significance tests

Estimator  $\hat{\beta}$  asymptotically normal with mean  $\beta$  and variance  $V(\hat{\beta}) = \sigma_{\beta}^2$

If  $\beta = 0$ , then, for a risk  $\alpha$  (e.g. 5%) we can define  $t_{\alpha/2}$  such that:

$$P\left(-t_{\alpha/2} < \frac{\hat{\beta}}{\sigma_{\beta}} < t_{\alpha/2}\right) = 1 - \alpha$$

Thus

$$2\Phi(t_{\alpha/2}) - 1 = 1 - \alpha$$

and we can read  $t_{\alpha/2}$  for the normal distribution table

For  $\alpha = 0.05$ ,  $\Phi(1.96) = 0.975$

If  $|\hat{\beta}/\sigma_{\beta}| > 1.96$ , we can reject the null  $\beta = 0$

# Balance table

**Table A3: Baseline balance: covariates**

Variable	(1)	(2)	(3)	(4)	(3)-(2)	(4)-(2)
	Total Mean/(SD)	Control Mean/(SD)	Base + YGL Mean/(SD)	Base Only Mean/(SD)	Pairwise t-test P-value	Pairwise t-test P-value
Girl's age (years)	14.000 (6.798)	13.741 (6.904)	14.104 (6.535)	14.033 (7.356)	0.292	0.483
Girl has a brother (=1)	0.548 (0.548)	0.574 (0.531)	0.556 (0.567)	0.508 (0.511)	0.512	0.033**
Mother passed away (=1)	0.049 (0.210)	0.040 (0.183)	0.050 (0.223)	0.053 (0.209)	0.331	0.282
Mother in household (=1)	0.816 (0.450)	0.835 (0.469)	0.805 (0.439)	0.820 (0.459)	0.208	0.595
Guardian knows how to read and write (=1)	0.828 (0.485)	0.829 (0.479)	0.836 (0.514)	0.810 (0.428)	0.799	0.474
Guardian has no education (=1)	0.095 (0.365)	0.085 (0.255)	0.096 (0.418)	0.102 (0.342)	0.465	0.310
Guardian attended secondary or higher education (=1)	0.303 (0.648)	0.308 (0.681)	0.293 (0.623)	0.318 (0.685)	0.648	0.794
Guardian occupation: Agriculture (=1)	0.773 (0.666)	0.768 (0.716)	0.781 (0.632)	0.762 (0.697)	0.696	0.899
Observations	2390	568	1216	606		
Schools	140	35	70	35		

Notes: Sample includes all girls in baseline. Columns (1)-(4) show means and standard deviations of covariates from the girls' baseline survey. Columns (5)-(6) show the p-value of a pairwise test comparing *Base Only* and *Base + YGL with control* , respectively. Standard errors cluster at the school level. \*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

## Overall balance

Often, because we are doing **multiple hypothesis tests** we will get a few significant imbalances when looking across multiple outcomes

How can we test for **overall** imbalance?

## Overall balance

Often, because we are doing **multiple hypothesis tests** we will get a few significant imbalances when looking across multiple outcomes

How can we test for **overall** imbalance?

- 1 Run regression:

$$\text{Treated}_i = \beta_0 + \beta_1 \text{Outcome1}_i + \beta_2 \text{Outcome2}_i + \dots + \beta_K \text{OutcomeK}_i \quad (6)$$

- 2 Use an F-test (joint test) of  $\beta_1 = \beta_2 = \dots = \beta_K = 0$

## The power of the experiment

If the policy has an impact, we want to be able to see it

Unless the effect is very small, we want to reject the null

But if there is a lot of imprecision (large estimator variance), we may fail to do so

Type II error is the probability of a **false negative**, i.e.,  $\beta > 0$ , but we fail to reject the null ( $\hat{\beta}/\sigma_{\beta} < 1.96$ ). In other words, we fail to detect an effect that is really there.

This will happen sometimes, for some samples

**Power** =  $1 - P(\text{Type II error})$ , i.e. the probability that we detect an effect if there really is one.

# The power of the experiment

**Usual approach:** set an acceptable power (typically 80%), and then:

- ① Set a reasonable  $\beta$  that you feel you should be able to “see” (the **minimum detectable effect** you want)
- ② And figure out the sample size that ensures that power for a true effect  $\beta$

## Computing the power

Let's calculate the power, where  $(\hat{\beta}/\sigma_{\beta} < 1.96)$  and  $\beta$  is random:

$$P\left(\frac{\hat{\beta}}{\sigma_{\beta}} > t_{\alpha/2} | \beta\right) = \kappa$$

where  $\kappa$  is the power.

$$P\left(\frac{\hat{\beta} - \beta}{\sigma_{\beta}} > t_{\alpha/2} - \frac{\beta}{\sigma_{\beta}} | \beta\right) = \kappa$$

$$\Phi\left(\frac{\beta}{\sigma_{\beta}} - t_{\alpha/2}\right) = \kappa$$

Thus:

$$\frac{\beta}{\sigma_{\beta}} - t_{\alpha/2} = t_{1-\kappa}$$

## Minimum detectable effect

The  $\beta$  that will be “significant” 80% of the time (at 5% level) is such that:

$$\frac{\beta}{\sigma_{\beta}} - t_{\alpha/2} = t_{1-\kappa}$$

or

$$\beta = (t_{\alpha/2} + t_{1-\kappa})\sigma_{\beta}$$

with  $t_{\alpha/2} = 1.96$  if  $\alpha = 0.05$  and  $t_{1-\kappa} = 0.84$  if  $\kappa = 0.80$

$(t_{\alpha/2} + t_{1-\kappa})\sigma_{\beta}$  is the **minimum detectable effect (MDE)**



## MDE and sample size

Consider the model:

$$y = c + \beta D_i + u$$

Remember that:

$$\sigma_{\beta}^2 = \frac{1}{\bar{D}(1 - \bar{D})} \frac{V(u)}{N}$$

Thus:

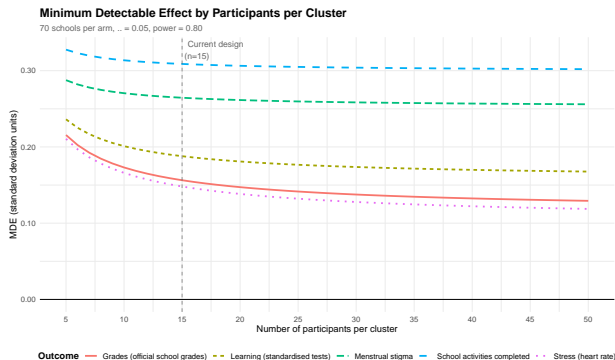
$$\text{MDE} = (t_{\alpha/2} + t_{1-\kappa}) \sqrt{\frac{1}{\bar{D}(1 - \bar{D})} \frac{V(u)}{N}}$$

Interpret each of those terms... (think in terms of finite sample imbalance)

How does MDE increase with sample size?

## MDE and cluster size

**Key insight:** With clustered randomization, increasing cluster size doesn't decrease MDE much



What matters most is the **number of clusters**, not individuals per cluster

## With instrumental variables

$$y = c + \beta T + u$$

where treatment  $T$  is instrumented by some random assignment  $D_i$

Reminder:

$$V(\hat{\beta}_{IV}) = \frac{1}{\bar{D}(1 - \bar{D})} \frac{V(u)}{N} \frac{1}{\pi_1^2}$$

The precision decreases linearly with the (net) take-up

So does the MDE

If take-up is 50%, implies more than doubling sample size.

# Outline

1. Clustering
2. Power
- 3. Choosing controls**
4. Heterogeneous treatment effects
5. Attrition
6. Multiple Hypothesis Testing
7. Robustness and Replication
8. Standard Errors in Regressions
9. Spillovers
10. Measurement

# Why add control variables?

Two main reasons to add control variables in a regression:

## 1. For identification of the causal effect

- Control for confounders
- Make the conditional independence assumption more plausible

# Why add control variables?

Two main reasons to add control variables in a regression:

## 1. For identification of the causal effect

- Control for confounders
- Make the conditional independence assumption more plausible

## 2. To reduce variance of the error term

- Increase precision of estimates
- Reduce standard errors
- Increase statistical power

# Why add control variables?

Two main reasons to add control variables in a regression:

## 1. For identification of the causal effect

- Control for confounders
- Make the conditional independence assumption more plausible

## 2. To reduce variance of the error term

- Increase precision of estimates
- Reduce standard errors
- Increase statistical power

With a large set of potential controls, **which should we choose?**

## Identification through DAGs

- **Directed Acyclic Graphs (DAGs):**  
graphical representation of causal relationships
- We can encode the relationship between  $D$  and  $Y$  using an arrow
- Direction emphasizes that  $D$  causes  $Y$ , not vice versa
- More intuitive than equations for complex settings

$$D \longrightarrow Y$$

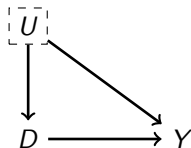


# Confounders

- $U$  affects both treatment  $D$  and outcome  $Y$
- $U$  is a **confounder**
- Creates association between  $D$  and  $Y$  that is *not* the causal effect
- Why?  $D$  and  $Y$  correlated through their common cause  $U$

# Confounders

- $U$  affects both treatment  $D$  and outcome  $Y$
- $U$  is a **confounder**
- Creates association between  $D$  and  $Y$  that is *not* the causal effect
- Why?  $D$  and  $Y$  correlated through their common cause  $U$



**Example:** Education ( $D$ ) and wages ( $Y$ )

Ability ( $U$ ) affects both choices

High ability  $\implies$  more education *and* higher wages

$\implies$  Observe  $D$  and  $Y$  correlated even without causal effect

## Controlling for confounders

- If  $X$  is observable confounder, can control for it
- Compare  $D = 1$  vs.  $D = 0$  *within* values of  $X$
- Removes spurious association through  $X$
- Recovers causal effect of  $D$  on  $Y$

## Controlling for confounders

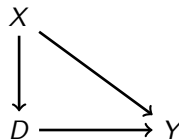
- If  $X$  is observable confounder, can control for it
- Compare  $D = 1$  vs.  $D = 0$  *within* values of  $X$
- Removes spurious association through  $X$
- Recovers causal effect of  $D$  on  $Y$

### Example continued:

Control for ability (or test scores as proxy)  
 Compare high-ability people with/without education

Compare low-ability people with/without education

⇒ Removes confounding by ability



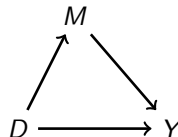
Conditioning on  $X$  makes  
 $D$  and  $Y$  independent  
 (conditional on  $X$ )

## Bad controls: Post-treatment variables

- $M$  is caused by treatment  $D$
- Also called “mediator”
- Controlling for  $M$  blocks part of causal effect
- Shuts down mechanism:  $D \rightarrow M \rightarrow Y$

## Bad controls: Post-treatment variables

- $M$  is caused by treatment  $D$
- Also called “mediator”
- Controlling for  $M$  blocks part of causal effect
- Shuts down mechanism:  $D \rightarrow M \rightarrow Y$



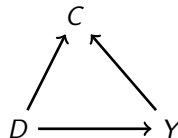
**Example:** Job training  $\rightarrow$  skills  $\rightarrow$  earnings  
 Controlling for skills misses indirect effect

## Bad controls: Colliders

- $C$  is caused by *both*  $D$  and  $Y$
- Called a “collider”
- $D$  and  $Y$  are independent
- But if you condition on  $C$ , creates spurious correlation

## Bad controls: Colliders

- $C$  is caused by *both*  $D$  and  $Y$
- Called a “collider”
- $D$  and  $Y$  are independent
- But if you condition on  $C$ , creates spurious correlation



### Intuition:

Knowing  $C$  gives information about  $D$  and  $Y$

If  $D$  high but  $C$  low, can infer  $Y$  must be low

$\implies$  Conditioning on  $C$  makes  $D$  and  $Y$  correlated

Even though no causal relationship!



## Collider example: Beauty and talent

**Setting:** Study beauty ( $D$ ) and talent ( $Y$ ) among actors

- Beauty increases probability of becoming actor
- Talent increases probability of becoming actor
- Sample: only people who became actors

## Collider example: Beauty and talent

**Setting:** Study beauty ( $D$ ) and talent ( $Y$ ) among actors

- Beauty increases probability of becoming actor
- Talent increases probability of becoming actor
- Sample: only people who became actors

**Problem:** “Becoming an actor” is a collider

- Both beauty and talent cause it
- By conditioning on actors, we create spurious correlation

## Collider example: Beauty and talent

**Setting:** Study beauty ( $D$ ) and talent ( $Y$ ) among actors

- Beauty increases probability of becoming actor
- Talent increases probability of becoming actor
- Sample: only people who became actors

**Problem:** “Becoming an actor” is a collider

- Both beauty and talent cause it
- By conditioning on actors, we create spurious correlation

**Result:** Among actors, beauty and talent negatively correlated!

- Very beautiful actors succeed with low talent
- Less beautiful actors need high talent to succeed
- This correlation is *not* causal - it's an artifact of sample selection

## Summary: Which controls?

Type	Control?	Why?
Confounder	Yes	Removes confounding
Post-treatment	No	Blocks causal mechanism
Collider	No	Creates spurious correlation

**Key takeaway:** Not all controls are good! Need to think carefully about the causal structure

## Controls for precision

Even in RCTs (where identification is clean), controls increase precision

Recall:  $V(\hat{\beta}) = \sigma^2(X'X)^{-1}$  where  $\sigma^2$  is residual variance

If  $X_i$  predicts  $Y_i$ , then including controls reduces  $\sigma^2$

⇒ Lower standard errors, higher power

TABLE V  
OLS AND REDUCED-FORM ESTIMATES OF EFFECT OF CLASS-SIZE ASSIGNMENT ON  
AVERAGE PERCENTILE OF STANFORD ACHIEVEMENT TEST

Explanatory variable	Reduced form: initial class size			
	(5)	(6)	(7)	(8)
Small class	4.82 (2.19)	5.37 (1.25)	5.36 (1.21)	5.37 (1.19)
Regular/aide class	.12 (2.23)	.29 (1.13)	.53 (1.09)	.31 (1.07)
White/Asian (1 = yes)	—	—	8.35 (1.35)	8.44 (1.36)
Girl (1 = yes)	—	—	4.48 (.63)	4.39 (.63)
Free lunch (1 = yes)	—	—	-13.15 (.77)	-13.07 (.77)
White teacher	—	—	—	-.57 (2.10)
Teacher experience	—	—	—	.26 (.10)
Master's degree	—	—	—	-.51 (1.06)
School fixed effects	No	Yes	Yes	Yes
$R^2$	.01	.25	.31	.31

**STAR experiment:** Controls reduce SEs, point estimates similar

## Which controls for precision?

### Ideal controls:

- Strongly correlated with outcome  $Y_i$
- Pre-treatment (measured before randomization)
- Not affected by treatment

## Which controls for precision?

**Ideal controls:**

- Strongly correlated with outcome  $Y_i$
- Pre-treatment (measured before randomization)
- Not affected by treatment

### Common choices:

- Baseline outcome (e.g., pre-treatment earnings)
- Demographics (age, gender, education)
- Stratification variables

## Which controls for precision?

### Ideal controls:

- Strongly correlated with outcome  $Y_i$
- Pre-treatment (measured before randomization)
- Not affected by treatment

### Common choices:

- Baseline outcome (e.g., pre-treatment earnings)
- Demographics (age, gender, education)
- Stratification variables

With many potential controls, how to choose?



## Control selection with LASSO (for RCTs)

### Post-Double-Selection LASSO (Belloni et al., 2014)

#### Procedure:

- ① Run LASSO of  $Y_i$  on all controls (select predictors of outcome)
- ② Run LASSO of  $D_i$  on all controls (select predictors of treatment)
- ③ Final regression: union of variables from steps 1 and 2

## Control selection with LASSO (for RCTs)

### Post-Double-Selection LASSO (Belloni et al., 2014)

#### Procedure:

- ① Run LASSO of  $Y_i$  on all controls (select predictors of outcome)
- ② Run LASSO of  $D_i$  on all controls (select predictors of treatment)
- ③ Final regression: union of variables from steps 1 and 2

**LASSO:** penalized regression that automatically sets some coefficients to zero

$$\min_{\beta, \gamma} \sum_{i=1}^n (Y_i - \beta D_i - X_i' \gamma)^2 + \lambda \sum_{j=1}^p |\gamma_j| \quad (7)$$

## Control selection with LASSO (for RCTs)

### Post-Double-Selection LASSO (Belloni et al., 2014)

#### Procedure:

- ① Run LASSO of  $Y_i$  on all controls (select predictors of outcome)
- ② Run LASSO of  $D_i$  on all controls (select predictors of treatment)
- ③ Final regression: union of variables from steps 1 and 2

**LASSO:** penalized regression that automatically sets some coefficients to zero

$$\min_{\beta, \gamma} \sum_{i=1}^n (Y_i - \beta D_i - X_i' \gamma)^2 + \lambda \sum_{j=1}^p |\gamma_j| \quad (7)$$

**Advantages:** Data-driven, reduces researcher discretion, helps avoid p-hacking

## Outline

1. Clustering
2. Power
3. Choosing controls
4. Heterogeneous treatment effects
5. Attrition
6. Multiple Hypothesis Testing
7. Robustness and Replication
8. Standard Errors in Regressions
9. Spillovers
10. Measurement

## Beyond average treatment effects

So far: focused on **average treatment effect** (ATE)

$$\tau_{ATE} = \mathbb{E}[Y_i(1) - Y_i(0)] \quad (8)$$

## Beyond average treatment effects

So far: focused on **average treatment effect (ATE)**

$$\tau_{ATE} = \mathbb{E}[Y_i(1) - Y_i(0)] \quad (8)$$

But effects likely vary across individuals/groups

**Conditional Average Treatment Effect (CATE):**

$$\tau(X_i) = \mathbb{E}[Y_i(1) - Y_i(0)|X_i] \quad (9)$$

where  $X_i$  are observable characteristics

## Beyond average treatment effects

So far: focused on **average treatment effect** (ATE)

$$\tau_{ATE} = \mathbb{E}[Y_i(1) - Y_i(0)] \quad (8)$$

But effects likely vary across individuals/groups

**Conditional Average Treatment Effect** (CATE):

$$\tau(X_i) = \mathbb{E}[Y_i(1) - Y_i(0)|X_i] \quad (9)$$

where  $X_i$  are observable characteristics

**Question:** Why care about heterogeneity?

# Why study heterogeneous effects?

## 1. Policy targeting

- Which subgroups benefit most from the treatment?
- Can improve cost-effectiveness



## Why study heterogeneous effects?

## 1. Policy targeting

- Which subgroups benefit most from the treatment?
- Can improve cost-effectiveness

## 2. Understanding mechanisms

- How does the treatment work?
- Who is most affected and why?

# Why study heterogeneous effects?

## 1. Policy targeting

- Which subgroups benefit most from the treatment?
- Can improve cost-effectiveness

## 2. Understanding mechanisms

- How does the treatment work?
- Who is most affected and why?

## 3. External validity

- Will treatment work in different contexts?
- What characteristics predict larger effects?

## Basic approach: Interaction terms

Simple regression with interactions:

$$Y_i = \alpha + \beta D_i + \gamma X_i + \delta(D_i \times X_i) + u_i \quad (10)$$

where  $X_i$  is characteristic of interest

## Basic approach: Interaction terms

Simple regression with interactions:

$$Y_i = \alpha + \beta D_i + \gamma X_i + \delta(D_i \times X_i) + u_i \quad (10)$$

where  $X_i$  is characteristic of interest

**Interpretation:**

- $\beta$ : effect for group with  $X_i = 0$
- $\beta + \delta$ : effect for group with  $X_i = 1$
- $\delta$ : *differential* effect across groups

## Basic approach: Interaction terms

Simple regression with interactions:

$$Y_i = \alpha + \beta D_i + \gamma X_i + \delta(D_i \times X_i) + u_i \quad (10)$$

where  $X_i$  is characteristic of interest

**Interpretation:**

- $\beta$ : effect for group with  $X_i = 0$
- $\beta + \delta$ : effect for group with  $X_i = 1$
- $\delta$ : *differential* effect across groups

Can test:  $H_0 : \delta = 0$  (no heterogeneity)

## Example: Cash transfers and suicide (Roth et al. 2019)

**Setting:** Rollout of unconditional cash transfer in Indonesia

**Treatment:** ~10% of household expenditure

**Outcome:** Suicide rate per 100,000 people

## Example: Cash transfers and suicide (Roth et al. 2019)

**Setting:** Rollout of unconditional cash transfer in Indonesia

**Treatment:** ~10% of household expenditure

**Outcome:** Suicide rate per 100,000 people

**Main result:** Transfer reduced suicides by 0.36 per 100,000 (18% reduction)

## Example: Cash transfers and suicide (Roth et al. 2019)

**Setting:** Rollout of unconditional cash transfer in Indonesia

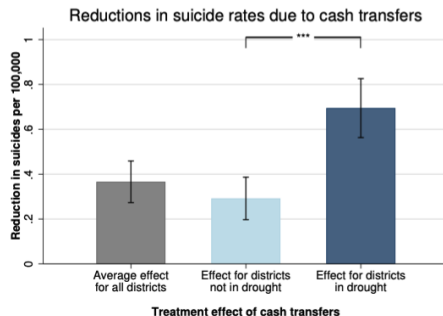
**Treatment:** ~10% of household expenditure

**Outcome:** Suicide rate per 100,000 people

**Main result:** Transfer reduced suicides by 0.36 per 100,000 (18% reduction)

### Heterogeneity:

- Larger reductions in areas with economic distress
- Larger effects in areas experiencing drought
- Benefits greatest among most vulnerable





## Challenges with interaction terms

### Problem 1: Many potential dimensions of heterogeneity

- Age, gender, income, education, location, etc.
- Which interactions to test?

## Challenges with interaction terms

### Problem 1: Many potential dimensions of heterogeneity

- Age, gender, income, education, location, etc.
- Which interactions to test?

## Problem 2: Multiple hypothesis testing

- Test many interactions  $\Rightarrow$  some significant by chance
- Risk of p-hacking / false discoveries

## Challenges with interaction terms

### Problem 1: Many potential dimensions of heterogeneity

- Age, gender, income, education, location, etc.
- Which interactions to test?

### Problem 2: Multiple hypothesis testing

- Test many interactions  $\implies$  some significant by chance
- Risk of p-hacking / false discoveries

$\implies$  Need more disciplined, data-driven approach

# Machine learning for CATEs

**Goal:** Estimate  $\tau(x) = \mathbb{E}[Y_i(1) - Y_i(0)|X_i = x]$  in data-driven way

## Two fundamental challenges:

# Machine learning for CATEs

**Goal:** Estimate  $\tau(x) = \mathbb{E}[Y_i(1) - Y_i(0)|X_i = x]$  in data-driven way

**Two fundamental challenges:**

## 1. The “ground truth” problem

- Standard ML: Cross-validate by comparing  $\hat{Y}_i$  to observed  $Y_i$
- Causal inference: We *never* observe  $\tau_i = Y_i(1) - Y_i(0)$  for any unit
- $\implies$  Cannot directly cross-validate treatment effect predictions

# Machine learning for CATEs

**Goal:** Estimate  $\tau(x) = \mathbb{E}[Y_i(1) - Y_i(0)|X_i = x]$  in data-driven way

**Two fundamental challenges:**

## 1. The “ground truth” problem

- Standard ML: Cross-validate by comparing  $\hat{Y}_i$  to observed  $Y_i$
- Causal inference: We *never* observe  $\tau_i = Y_i(1) - Y_i(0)$  for any unit
- $\implies$  Cannot directly cross-validate treatment effect predictions

## 2. The “adaptive bias” problem

- Standard CART uses same data to: (1) find splits, (2) estimate effects
- You split *because*  $\hat{\tau}$  looked large  $\implies$  estimates biased upward
- Like picking the best-performing stock, then reporting its past returns as expected future returns

## Causal trees (Athey & Imbens 2016)

**Idea:** Partition covariate space to maximize treatment effect heterogeneity

**“Honest” estimation** (key innovation):

- ① Split sample in half: training + estimation
- ② Use training data to build tree (decide where to split)
- ③ Use estimation data to compute treatment effects within each leaf

## Causal trees (Athey & Imbens 2016)

**Idea:** Partition covariate space to maximize treatment effect heterogeneity

**“Honest” estimation** (key innovation):

- ① Split sample in half: training + estimation
- ② Use training data to build tree (decide where to split)
- ③ Use estimation data to compute treatment effects within each leaf

**Why this works:** Estimation sample wasn't used to choose splits

⇒ No cherry-picking bias, valid confidence intervals



## Causal trees (Athey & Imbens 2016)

**Idea:** Partition covariate space to maximize treatment effect heterogeneity

**“Honest” estimation (key innovation):**

- 1 Split sample in half: training + estimation
- 2 Use training data to build tree (decide where to split)
- 3 Use estimation data to compute treatment effects within each leaf

### Why this works: Estimation sample wasn't used to choose splits

⇒ No cherry-picking bias, valid confidence intervals

**Splitting criterion:** Find partitions where treatment effects differ most across leaves, but penalize tiny subgroups (noisy estimates)

## Causal trees (Athey & Imbens 2016)

**Idea:** Partition covariate space to maximize treatment effect heterogeneity

**“Honest” estimation** (key innovation):

- ① Split sample in half: training + estimation
- ② Use training data to build tree (decide where to split)
- ③ Use estimation data to compute treatment effects within each leaf

**Why this works:** Estimation sample wasn't used to choose splits

⇒ No cherry-picking bias, valid confidence intervals

**Splitting criterion:** Find partitions where treatment effects differ most across leaves, but penalize tiny subgroups (noisy estimates)

**Causal forests** (Wager & Athey 2018): Average many honest trees for smoother estimates

## GATES: Group Average Treatment Effects

**The problem:** Many covariates, don't know *ex ante* which matter for heterogeneity

**Key insight** (Chernozhukov et al. 2020): Don't estimate the full CATE—estimate **features** of it

## GATES: Group Average Treatment Effects

**The problem:** Many covariates, don't know *ex ante* which matter for heterogeneity

**Key insight** (Chernozhukov et al. 2020): Don't estimate the full CATE—estimate **features** of it

**GATES procedure:**

- ① Use *any* ML to predict  $\hat{\tau}(X_i)$
- ② Sort by predicted effect
- ③ Group into quintiles
- ④ Estimate actual ATE within each group

# GATES: Group Average Treatment Effects

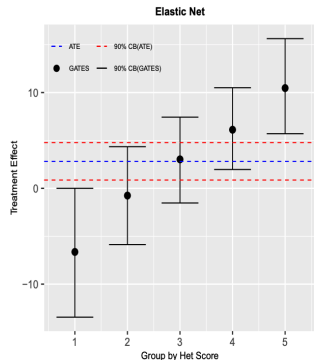
**The problem:** Many covariates, don't know *ex ante* which matter for heterogeneity

**Key insight** (Chernozhukov et al. 2020): Don't estimate the full CATE—estimate **features** of it

**GATES procedure:**

- ① Use *any* ML to predict  $\hat{\tau}(X_i)$
- ② Sort by predicted effect
- ③ Group into quintiles
- ④ Estimate actual ATE within each group

**Interpretation:** If ML detects real heterogeneity, Group 5 should have larger effects than Group 1



Upward slope  $\Rightarrow$  ML captures real heterogeneity

## CLAN: Who are the high and low responders?

**CLAN** (Classification Analysis):

Once GATES reveals heterogeneity, **who** are high vs. low responders?

**Method:** Compare characteristics of “most affected” ( $G_K$ ) vs. “least affected” ( $G_1$ ) groups

## CLAN: Who are the high and low responders?

**CLAN** (Classification Analysis):

Once GATES reveals heterogeneity, **who** are high vs. low responders?

**Method:** Compare characteristics of “most affected” ( $G_K$ ) vs. “least affected” ( $G_1$ ) groups

**Example:** Immunization in Haryana, India  
(Banerjee et al. 2019)

- RCT of SMS + incentives to boost vaccination
- Policy question: *where* to target expensive intervention?

## CLAN: Who are the high and low responders?

**CLAN** (Classification Analysis):

Once GATES reveals heterogeneity, **who** are high vs. low responders?

**Method:** Compare characteristics of “most affected” ( $G_K$ ) vs. “least affected” ( $G_1$ ) groups

**Example:** Immunization in Haryana, India (Banerjee et al. 2019)

- RCT of SMS + incentives to boost vaccination
- Policy question: *where* to target expensive intervention?

**CLAN finding:** Villages with low baseline rates benefited most  $\implies$  target rollout there

TABLE 5. CLAN of Immunization Incentives

	20% Most ( $\delta_5$ )	Elastic Net 20% Least ( $\delta_1$ )	Difference ( $\delta_5 - \delta_1$ )
Number of vaccines to pregnant mother	2.161 (2.110,2.212)	2.288 (2.237,2.337)	-0.128 (-0.200,-0.055) [0.001]
Number of vaccines to child since birth	4.230 (4.100,4.369)	4.714 (4.573,4.860)	-0.513 (-0.710,-0.311) [0.000]
Fraction of children received polio drops	1.000 (1.000,1.000)	1.000 (1.000,1.000)	0.000 (0.000,0.000) [0.000]
Number of polio drops to child	2.964 (2.954,2.975)	2.998 (2.987,3.007)	-0.033 (-0.047,-0.019) [0.000]
Fraction of children received immunization card	0.899 (0.878,0.922)	0.932 (0.908,0.956)	-0.036 (-0.065,-0.004) [0.000]
Fraction of children received Measles vaccine by 15 months of age	0.127 (0.100,0.155)	0.255 (0.230,0.282)	-0.131 (-0.167,-0.094) [0.052]
Fraction of children received Measles at credible locations	0.290 (0.252,0.327)	0.435 (0.400,0.470)	-0.152 (-0.198,-0.097) [0.000]

Notes: Medians over 100 splits. 90% confidence interval in parenthesis.

Notes: P-values for the hypothesis that the parameter is equal to zero in brackets.

Top 20% vs. bottom 20% responders



## Outline

# The problem of attrition

**Attrition:** Participants drop out between treatment assignment and outcome measurement

Common in:

- Surveys with multiple waves
- RCTs
- Any study with follow-up measurement

# The problem of attrition

**Attrition:** Participants drop out between treatment assignment and outcome measurement

Common in:

- Surveys with multiple waves
- RCTs
- Any study with follow-up measurement

**Why is this a problem?**

Even with perfect randomization, attrition can create selection bias

## Example: Job training RCT

**Setup:** Randomize 1,000 people to training vs. control

Measure employment 1 year later



## Example: Job training RCT

**Setup:** Randomize 1,000 people to training vs. control

Measure employment 1 year later

**Problem:** 20% of treatment group, 10% of control don't respond

**Issue:** If attrition related to outcomes, estimates biased

- Maybe successful people in treatment don't respond (too busy working)
- Maybe unsuccessful people in control don't respond (discouraged)
- $\implies$  Comparing non-random samples

## Testing for differential attrition

### First step: Test whether attrition differs by treatment status

Regress attrition indicator on treatment:

$$\text{Attrited}_i = \alpha + \beta D_i + u_i \quad (11)$$

Test  $H_0 : \beta = 0$

Regress attrition indicator on treatment:

$$\text{Attrited}_i = \alpha + \beta D_i + u_i \quad (11)$$

Test  $H_0 : \beta = 0$

If reject  $H_0$ : differential attrition

## Treatment affects who stays in sample



## Testing for differential attrition

**First step:** Test whether attrition differs by treatment status

Regress attrition indicator on treatment:

$$\text{Attrited}_i = \alpha + \beta D_i + u_i \quad (11)$$

Test  $H_0 : \beta = 0$

If reject  $H_0$ : **differential attrition**

Treatment affects who stays in sample

**But:** Even if no differential attrition, can still have bias if attrition related to potential outcomes

## Testing attrition on baseline characteristics

**Additional test:** Compare baseline characteristics of attriters vs. non-attriters

For each baseline covariate  $X_i$ :

$$X_i = \alpha + \beta \text{Attrited}_i + \gamma D_i + \delta(\text{Attrited}_i \times D_i) + u_i \quad (12)$$

## Testing attrition on baseline characteristics

**Additional test:** Compare baseline characteristics of attriters vs. non-attriters

For each baseline covariate  $X_i$ :

$$X_i = \alpha + \beta \text{Attrited}_i + \gamma D_i + \delta (\text{Attrited}_i \times D_i) + u_i \quad (12)$$

**Interpretation:**

- $\beta \neq 0$ : Attriters differ from non-attriters (potential bias)
- $\delta \neq 0$ : **Type** of attritor differs by treatment  $\implies$  confounds treatment estimates

## Testing attrition on baseline characteristics

**Additional test:** Compare baseline characteristics of attriters vs. non-attriters

For each baseline covariate  $X_i$ :

$$X_i = \alpha + \beta \text{Attrited}_i + \gamma D_i + \delta (\text{Attrited}_i \times D_i) + u_i \quad (12)$$

**Interpretation:**

- $\beta \neq 0$ : Attriters differ from non-attriters (potential bias)
- $\delta \neq 0$ : **Type** of attritor differs by treatment  $\implies$  confounds treatment estimates

**Important:** These are diagnostic tests, not solutions

## Solutions to attrition

### Three main approaches:

## 1. Prevention (best option)

- Minimize attrition through study design
- Multiple contact methods, incentives, tracking

# Solutions to attrition

Three main approaches:

## 1. Prevention (best option)

- Minimize attrition through study design
- Multiple contact methods, incentives, tracking

## 2. Bounding (Lee bounds)

- Worst-case scenario analysis
- No parametric assumptions

# Solutions to attrition

Three main approaches:

## 1. Prevention (best option)

- Minimize attrition through study design
- Multiple contact methods, incentives, tracking

## 2. Bounding (Lee bounds)

- Worst-case scenario analysis
- No parametric assumptions

## 3. Modeling (e.g., inverse probability weighting)

- Reweight to correct for selection
- Requires strong assumptions

## Lee bounds: Intuition

**Idea** (Lee 2009): Bound treatment effect without knowing why people attrite

**Key assumption:** Monotonicity

- Treatment affects attrition in only one direction
- E.g., treatment only increases attrition (or only decreases it)



## Lee bounds: Intuition

**Idea** (Lee 2009): Bound treatment effect without knowing why people attrite

**Key assumption:** Monotonicity

- Treatment affects attrition in only one direction
- E.g., treatment only increases attrition (or only decreases it)

**Logic:**

- Suppose treatment group has 80% response, control has 100%
- 20% of treatment group are “extra attriters” caused by treatment
- Worst case: these 20% had highest (or lowest) outcomes
- $\implies$  Trim top/bottom 20% of treatment group to get bounds

## Lee bounds: Method

**Setup:** Let  $S_i(d) \in \{0, 1\}$  indicate whether observed if assigned treatment  $d$

**Monotonicity assumption:**  $S_i(1) \geq S_i(0)$  for all  $i$  (or vice versa)

## Lee bounds: Method

**Setup:** Let  $S_i(d) \in \{0, 1\}$  indicate whether observed if assigned treatment  $d$

**Monotonicity assumption:**  $S_i(1) \geq S_i(0)$  for all  $i$  (or vice versa)

**Construction** (Example: 80% response in treatment, 100% in control):

- 1 Identify excess proportion: 20% are treatment-induced attriters

○○○○○○○

○○○○○○○○○○○○○○○○

○○○○○○○○○○○○○○

○○○○○○○○○○

○○○○○○○○●○○

● ○○○○○○○○○○○○○○○○

○○○○○○○

○○○○○○○○○○○○○○

○○○○○○○

**Monotonicity assumption:**  $S_i(1) \geq S_i(0)$  for all  $i$  (or vice versa)

① Identify excess proportion: 20% are treatment-induced attriters

2 Construct bounds by trimming:

- **Upper bound:** Drop bottom 20% of treatment outcomes
- **Lower bound:** Drop top 20% of treatment outcomes

## Lee bounds: Method

**Setup:** Let  $S_i(d) \in \{0, 1\}$  indicate whether observed if assigned treatment  $d$

**Monotonicity assumption:**  $S_i(1) \geq S_i(0)$  for all  $i$  (or vice versa)

**Construction** (Example: 80% response in treatment, 100% in control):

- ① Identify excess proportion: 20% are treatment-induced attriters
- ② Construct bounds by trimming:
  - **Upper bound:** Drop bottom 20% of treatment outcomes
  - **Lower bound:** Drop top 20% of treatment outcomes
- ③ Compare trimmed treatment mean to control mean

## Lee bounds: Method

**Setup:** Let  $S_i(d) \in \{0, 1\}$  indicate whether observed if assigned treatment  $d$

**Monotonicity assumption:**  $S_i(1) \geq S_i(0)$  for all  $i$  (or vice versa)

**Construction** (Example: 80% response in treatment, 100% in control):

- ① Identify excess proportion: 20% are treatment-induced attriters
- ② Construct bounds by trimming:
  - **Upper bound:** Drop bottom 20% of treatment outcomes
  - **Lower bound:** Drop top 20% of treatment outcomes
- ③ Compare trimmed treatment mean to control mean

**Intuition:** We don't know which 20% would have attrited, so consider worst cases

# Manski bounds

**Alternative approach** (Manski 1990): Even weaker assumptions

For attriters, assume worst-case outcomes within support  $[y_{\min}, y_{\max}]$ :

# Manski bounds

**Alternative approach (Manski 1990):** Even weaker assumptions

For attriters, assume worst-case outcomes within support  $[y_{\min}, y_{\max}]$ :

- **Upper bound:** Treatment attriters have  $Y = y_{\max}$ , control attriters have  $Y = y_{\min}$
- **Lower bound:** Treatment attriters have  $Y = y_{\min}$ , control attriters have  $Y = y_{\max}$

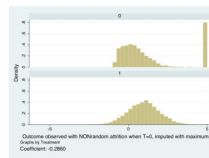
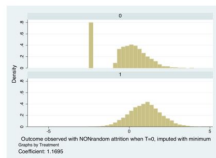


# Manski bounds

## Alternative approach (Manski 1990): Even weaker assumptions

For attriters, assume worst-case outcomes within support  $[y_{\min}, y_{\max}]$ :

- **Upper bound:** Treatment attriters have  $Y = y_{\max}$ , control attriters have  $Y = y_{\min}$
- **Lower bound:** Treatment attriters have  $Y = y_{\min}$ , control attriters have  $Y = y_{\max}$



# Inverse probability weighting

Alternative approach: Model attrition probability

**Basic intuition:** Reweight observed sample to look like full sample (before attrition)

## Inverse probability weighting

Alternative approach: Model attrition probability

**Basic intuition:** Reweight observed sample to look like full sample (before attrition)

**Step 1:** Estimate probability of non-attrition:  $\hat{p}_i = \hat{\Pr}(S_i = 1|X_i, D_i)$

using logit/probit with baseline covariates  $X_i$

## Inverse probability weighting

Alternative approach: Model attrition probability

**Basic intuition:** Reweight observed sample to look like full sample (before attrition)

**Step 1:** Estimate probability of non-attrition:  $\hat{p}_i = \hat{\Pr}(S_i = 1|X_i, D_i)$

using logit/probit with baseline covariates  $X_i$

**Step 2:** Use sample weight  $w_i = 1/\hat{p}_i$  in main regression

$$Y_i = \alpha + \tau D_i + X_i' \beta + (X_i \times D_i)' \gamma + \varepsilon_i \quad (13)$$

## Inverse probability weighting

Alternative approach: Model attrition probability

**Basic intuition:** Reweight observed sample to look like full sample (before attrition)

**Step 1:** Estimate probability of non-attrition:  $\hat{p}_i = \hat{\Pr}(S_i = 1|X_i, D_i)$

using logit/probit with baseline covariates  $X_i$

**Step 2:** Use sample weight  $w_i = 1/\hat{p}_i$  in main regression

$$Y_i = \alpha + \tau D_i + X_i' \beta + (X_i \times D_i)' \gamma + \varepsilon_i \quad (13)$$

**Assumption:** Attrition independent of outcomes conditional on  $(X_i, D_i)$  – Much stronger than Lee bounds!

# Outline

1. Clustering
2. Power
3. Choosing controls
4. Heterogeneous treatment effects
5. Attrition
- 6. Multiple Hypothesis Testing**
7. Robustness and Replication
8. Standard Errors in Regressions
9. Spillovers
10. Measurement

# What is a p-value?

**Question:** What is the definition of a p-value?

# What is a p-value?

**Question:** What is the definition of a p-value?

**Answer:** The probability of observing a result at least as extreme as what we got, *if there were no true effect*

Standard hypothesis testing assumes: **We only do one test**



# What is a p-value?

**Question:** What is the definition of a p-value?

**Answer:** The probability of observing a result at least as extreme as what we got, *if there were no true effect*

Standard hypothesis testing assumes: **We only do one test**

**Problem:** What if we run multiple tests?

# The multiple testing problem

**Thought experiment:** Run 1,000 regressions on completely random data with  $Y_i \perp X_i$

With  $\alpha = 0.05$ , how many will look “significant”?

With  $\alpha = 0.05$ , how many will look “significant”?

**Answer:** About 50 (i.e., 5% of 1,000)

# The multiple testing problem

**Thought experiment:** Run 1,000 regressions on completely random data with  $Y_i \perp X_i$

With  $\alpha = 0.05$ , how many will look “significant”?

**Answer:** About 50 (i.e., 5% of 1,000)

**Even worse:** What if we run 1,000 tests and *only report* the significant ones?



## Why this matters for empirical work

Multiple testing arises naturally in many contexts:

- Testing treatment effects on **many outcomes**
  - Health study: blood pressure, cholesterol, weight, ...
  - Education: test scores, attendance, graduation, ...

## Why this matters for empirical work

Multiple testing arises naturally in many contexts:

- Testing treatment effects on **many outcomes**
  - Health study: blood pressure, cholesterol, weight, ...
  - Education: test scores, attendance, graduation, ...
- Testing for **heterogeneous effects**
  - By gender, age, income, region, ...

## Why this matters for empirical work

Multiple testing arises naturally in many contexts:

- Testing treatment effects on **many outcomes**
  - Health study: blood pressure, cholesterol, weight, ...
  - Education: test scores, attendance, graduation, ...
- Testing for **heterogeneous effects**
  - By gender, age, income, region, ...
- Testing **many specifications**
  - Different control variables
  - Different samples
  - Different functional forms



## Solutions: Pre-analysis plans

**Pre-analysis plan (PAP):** Document research design before seeing data

Specify in advance: primary outcomes, subgroup analyses, specifications

## Solutions: Pre-analysis plans

**Pre-analysis plan (PAP):** Document research design before seeing data

Specify in advance: primary outcomes, subgroup analyses, specifications

**Benefits:** Prevents p-hacking, makes researcher choices transparent, now standard for RCTs

## Solutions: Pre-analysis plans

**Pre-analysis plan (PAP):** Document research design before seeing data

Specify in advance: primary outcomes, subgroup analyses, specifications

**Benefits:** Prevents p-hacking, makes researcher choices transparent, now standard for RCTs

**Limitation:** Doesn't solve multiple testing itself—still need to adjust p-values

## Solutions: Adjust p-values

**Idea:** Adjust significance thresholds to account for multiple testing

**Bonferroni correction** (classic method):

- Testing  $m$  hypotheses
- Reject if  $p < \alpha/m$
- Example: 20 tests, want  $\text{FWER} \leq 0.05 \implies$  use threshold  $0.05/20 = 0.0025$

# Solutions: Adjust p-values

**Idea:** Adjust significance thresholds to account for multiple testing

**Bonferroni correction** (classic method):

- Testing  $m$  hypotheses
- Reject if  $p < \alpha/m$
- Example: 20 tests, want  $\text{FWER} \leq 0.05 \implies$  use threshold  $0.05/20 = 0.0025$

**Problem:** Very conservative

- Low power to detect true effects
- Especially with many tests

## Family-Wise Error Rate (FWER)

**Definition:** Probability of making *at least one* false rejection

$$\text{FWER} = \Pr(\text{At least one false positive}) \quad (14)$$

Bonferroni controls FWER at level  $\alpha$

## Family-Wise Error Rate (FWER)

**Definition:** Probability of making *at least one* false rejection

$$\text{FWER} = \Pr(\text{At least one false positive}) \quad (14)$$

Bonferroni controls FWER at level  $\alpha$

**Issue:** Very stringent criterion

- In exploratory research, may accept some false positives
- Want to balance false positives vs. false negatives

## False Discovery Rate (FDR)

**Definition:** Expected proportion of false rejections among all rejections

$$\text{FDR} = \mathbb{E} \left[ \frac{\text{False positives}}{\text{Total rejections}} \right] \quad (15)$$

**Interpretation:** Among all tests you reject, what fraction are false positives?



$$\text{FDR} = \mathbb{E} \left[ \frac{\text{False positives}}{\text{Total rejections}} \right] \quad (15)$$

- FWER: Probability of *any* false positive
- FDR: *Rate* of false positives among rejections
- FDR is less stringent  $\implies$  more power

# Benjamini-Hochberg procedure (1995)

**Goal:** Control FDR at level  $q$  (e.g.,  $q = 0.10$ )

**Procedure** for  $m$  hypothesis tests:

- ① **Sort p-values:**  $p_{(1)} \leq p_{(2)} \leq \dots \leq p_{(m)}$

# Benjamini-Hochberg procedure (1995)

**Goal:** Control FDR at level  $q$  (e.g.,  $q = 0.10$ )

**Procedure** for  $m$  hypothesis tests:

- ① **Sort p-values:**  $p_{(1)} \leq p_{(2)} \leq \dots \leq p_{(m)}$
- ② **Find largest  $k$  such that:**

$$p_{(k)} \leq \frac{k}{m} \cdot q \tag{16}$$

### Procedure for $m$ hypothesis tests:

- $$p_{(k)} \leq \frac{k}{m} \cdot q \quad (16)$$

- 66 / 111

# Benjamini-Hochberg procedure (1995)

**Goal:** Control FDR at level  $q$  (e.g.,  $q = 0.10$ )

**Procedure** for  $m$  hypothesis tests:

- ① **Sort p-values:**  $p_{(1)} \leq p_{(2)} \leq \dots \leq p_{(m)}$
- ② **Find largest  $k$  such that:**

$$p_{(k)} \leq \frac{k}{m} \cdot q \quad (16)$$

- ③ **Reject all hypotheses**  $H_{(1)}, \dots, H_{(k)}$

**Result:**  $\text{FDR} \leq q$  under independence

## Benjamini-Hochberg: Intuition

**Key insight:** Look for p-values that are “too small” to be false positives

**Under null:** p-values uniformly distributed on  $[0,1]$

**Under alternative:** p-values concentrated near 0

## Benjamini-Hochberg: Intuition

**Key insight:** Look for p-values that are “too small” to be false positives

**Under null:** p-values uniformly distributed on  $[0,1]$

**Under alternative:** p-values concentrated near 0

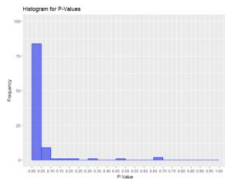
BH procedure finds cutoff where p-values deviate from uniform distribution



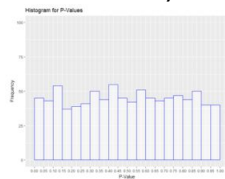


# Benjamini-Hochberg: Visual intuition

**True rejections: Concentrated near zero**

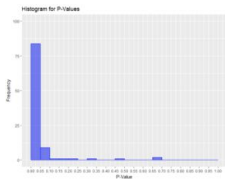


**False rejections: Uniformly distributed**

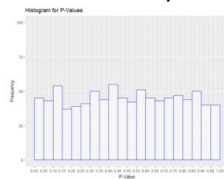


# Benjamini-Hochberg: Visual intuition

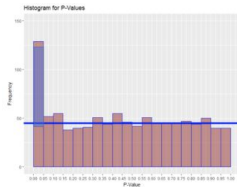
**True rejections: Concentrated near zero**



**False rejections: Uniformly distributed**



**BH isolates p-values above uniform cutoff:**



## Example: Testing 10 hypotheses

Suppose we have 10 p-values, want  $FDR \leq 0.10$ :

Rank $k$	$p_{(k)}$	$\frac{k}{m} \cdot q = \frac{k}{10} \cdot 0.10$	Reject?
1	0.001	0.010	✓
2	0.008	0.020	✓
3	0.015	0.030	✓
4	0.042	0.040	✓
5	0.056	0.050	×
6	0.120	0.060	×
⋮	⋮	⋮	×

Largest  $k$  with  $p_{(k)} \leq k \cdot 0.01$  is  $k = 4 \implies$  Reject first 4 hypotheses

## Other approaches to reduce multiple testing

Beyond adjusting p-values:

### 1. Reduce number of tests / Use summary indices

- Pre-specify primary outcome(s)
- Use F-tests for joint hypotheses
- Combine outcomes into summary index (e.g., Anderson 2008)
  - Average standardized effects across related outcomes
  - Example: “Cognitive index” = average of test scores
  - Reduces  $m$  tests to 1 test

## Other approaches to reduce multiple testing

Beyond adjusting p-values:

### 1. Reduce number of tests / Use summary indices

- Pre-specify primary outcome(s)
- Use F-tests for joint hypotheses
- Combine outcomes into summary index (e.g., Anderson 2008)
  - Average standardized effects across related outcomes
  - Example: “Cognitive index” = average of test scores
  - Reduces  $m$  tests to 1 test

### 2. Report all tests transparently

- Show results for all outcomes (not just significant ones)
- Let readers judge robustness

## Practical recommendations

### What should you do?

- **Pre-specify** main hypotheses when possible
- **Distinguish** confirmatory vs. exploratory analyses
- **Adjust p-values** when testing multiple outcomes
  - BH procedure for FDR control
  - Bonferroni for FWER control (if very conservative)
- **Report transparently**
  - Show all outcomes tested
  - Report both adjusted and unadjusted p-values
- **Implementation:** Available in R (`p.adjust()`) and Stata (`rwolf`, `wyoung`)

# Outline

1. Clustering
2. Power
3. Choosing controls
4. Heterogeneous treatment effects
5. Attrition
6. Multiple Hypothesis Testing
7. Robustness and Replication
8. Standard Errors in Regressions
9. Spillovers
10. Measurement

## The replication crisis

## Mounting evidence that empirical results are less robust than we thought

### Problem 1: Low replication rates

- Psychology replication project: only 39% of studies replicate
- Economics: similar concerns emerging



# The replication crisis

Mounting evidence that empirical results are less robust than we thought

## Problem 1: Low replication rates

- Psychology replication project: only 39% of studies replicate
- Economics: similar concerns emerging

## Problem 2: Researcher degrees of freedom

- Many choices in data analysis: sample, controls, specification, etc.
- Different choices  $\implies$  different results
- Risk of p-hacking (intentional or not)

# The replication crisis

Mounting evidence that empirical results are less robust than we thought

## Problem 1: Low replication rates

- Psychology replication project: only 39% of studies replicate
- Economics: similar concerns emerging

## Problem 2: Researcher degrees of freedom

- Many choices in data analysis: sample, controls, specification, etc.
- Different choices  $\implies$  different results
- Risk of p-hacking (intentional or not)

**Goal:** Get to the truth! But what went wrong?

## Evidence: Garden of forking paths

**Study:** Breznau, Rinke, Wuttke (PNAS 2022)

**Design:** 73 research teams test same hypothesis with same data

### Does immigration reduce support for social policies?

## Evidence: Garden of forking paths

**Study:** Breznau, Rinke, Wuttke (PNAS 2022)

**Design:** 73 research teams test same hypothesis with same data

## Does immigration reduce support for social policies?

## Results: Massive variance in estimates

- Estimates ranged from strongly negative to strongly positive
- Driven by: controls, fixed effects, clustering, sample choices
- Much variance unexplained even controlling for observables

## Evidence: Researcher-driven variation

**Study:** Huntington-Klein et al.

**Design:** 146 economist teams estimate same effect with progressive restrictions

- ① No restrictions (same data)
- ② Specify DiD design
- ③ Pre-cleaned data

# Evidence: Researcher-driven variation

**Study:** Huntington-Klein et al.

**Design:** 146 economist teams estimate same effect with progressive restrictions

- ① No restrictions (same data)
- ② Specify DiD design
- ③ Pre-cleaned data

**Results:**

- IQR of 3-4pp (avg effect: 4pp) with freedom
- IQR still 2pp with pre-cleaned data
- Both data and analytical choices matter

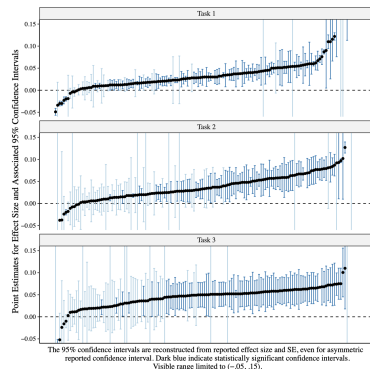


Figure 3: Specification Curve for All Reported Estimates by Task with Estimates Ordered From Smallest to Largest

# What went wrong?

Three distinct but related issues:

## 1. Researcher degrees of freedom

- Many choices: sample definition, controls, specifications, outliers
- Different reasonable choices  $\implies$  different results
- Creates flexibility to find “significant” results (even unintentionally)

# What went wrong?

Three distinct but related issues:

## 1. Researcher degrees of freedom

- Many choices: sample definition, controls, specifications, outliers
- Different reasonable choices  $\implies$  different results
- Creates flexibility to find “significant” results (even unintentionally)

## 2. Publication bias (editorial decisions)

- Journals prefer novel, significant, “clean” results
- Null results, replications rarely published (file drawer problem)
- $\implies$  Published literature overrepresents significant findings



## Solutions

## How to improve reliability of empirical research?

## 1. Transparency

- Pre-register analysis plans (PAPs)
- Post data and code (replication packages)
- Report all specifications, not just significant ones

# Solutions

How to improve reliability of empirical research?

## 1. Transparency

- Pre-register analysis plans (PAPs)
- Post data and code (replication packages)
- Report all specifications, not just significant ones

## 2. Robustness checks

- Show results not driven by one specific choice
- Vary sample, controls, functional forms
- Put extensive checks in appendix

# Solutions

How to improve reliability of empirical research?

## 1. Transparency

- Pre-register analysis plans (PAPs)
- Post data and code (replication packages)
- Report all specifications, not just significant ones

## 2. Robustness checks

- Show results not driven by one specific choice
- Vary sample, controls, functional forms
- Put extensive checks in appendix

## 3. Design for power

- Choose designs with high statistical power
- Reduces false negatives and winner's curse
- Better to detect true effects reliably

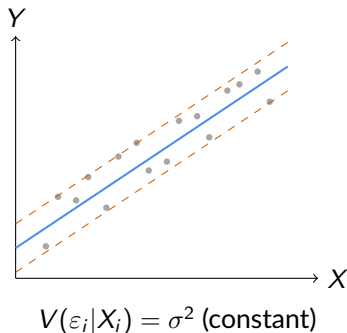
# Outline

1. Clustering
2. Power
3. Choosing controls
4. Heterogeneous treatment effects
5. Attrition
6. Multiple Hypothesis Testing
7. Robustness and Replication
8. Standard Errors in Regressions
9. Spillovers
10. Measurement

# Homoskedasticity vs. Heteroskedasticity

**Basic OLS assumption:** Constant error variance (homoskedasticity)

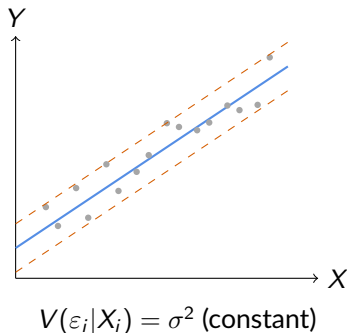
## Homoskedasticity



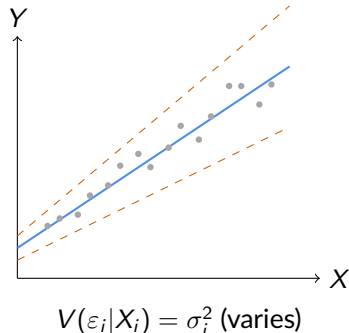
# Homoskedasticity vs. Heteroskedasticity

**Basic OLS assumption:** Constant error variance (homoskedasticity)

**Homoskedasticity**



**Heteroskedasticity**



## Variance of OLS estimator

**Under homoskedasticity:**

$$V(\hat{\beta}) = \sigma^2(X'X)^{-1} \quad (17)$$

## Variance of OLS estimator

**Under homoskedasticity:**

$$V(\hat{\beta}) = \sigma^2(X'X)^{-1} \quad (17)$$

**Under heteroskedasticity:**

$$V(\hat{\beta}) = (X'X)^{-1}X'\Omega X(X'X)^{-1} \quad (18)$$

where  $\Omega = \text{diag}(\sigma_1^2, \dots, \sigma_n^2)$



## Variance of OLS estimator

**Under homoskedasticity:**

$$V(\hat{\beta}) = \sigma^2(X'X)^{-1} \quad (17)$$

**Under heteroskedasticity:**

$$V(\hat{\beta}) = (X'X)^{-1}X'\Omega X(X'X)^{-1} \quad (18)$$

where  $\Omega = \text{diag}(\sigma_1^2, \dots, \sigma_n^2)$

**Problem:** If we use homoskedastic formula when heteroskedasticity present:

- Standard errors are wrong
- Confidence intervals and hypothesis tests invalid

## Practical recommendation

**Key point:** There is almost never a good justification for assuming homoskedasticity

⇒ **Always use robust (heteroskedasticity-consistent) standard errors**

## Practical recommendation

**Key point:** There is almost never a good justification for assuming homoskedasticity

⇒ **Always use robust (heteroskedasticity-consistent) standard errors**

**Implementation:**

- **Stata:** Add `, robust` option to regression
  - `reg y x, robust`

## Practical recommendation

**Key point:** There is almost never a good justification for assuming homoskedasticity

⇒ **Always use robust (heteroskedasticity-consistent) standard errors**

**Implementation:**

- **Stata:** Add `, robust` option to regression
  - `reg y x, robust`
- **R:** Use `vcovHC()` from `sandwich` package
  - `coeftest(model, vcov = vcovHC(model, type="HC1"))`

This should be your *default*, not an exception!

## Clustering: Why does it matter?

**Standard OLS assumption:** Observations are independent (IID)

**Problem:** Often observations are correlated within groups



## Clustering: Why does it matter?

**Standard OLS assumption:** Observations are independent (IID)

**Problem:** Often observations are correlated within groups

**Common examples:**

- **Panel data:** Same individual over time
- **Schools/classrooms:** Students in same school
- **Geographic/spatial:** Units in same region
- **Families:** Siblings, households

**Key distinction:** This is about *correlation in residuals*, not a SUTVA violation

Treatment of one unit doesn't affect others, but their errors are correlated

## Clustered errors: Covariance structure

**Example:** 2 groups, 3 people each

**Without clustering (IID)**

$$\Omega = \begin{pmatrix} \sigma^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2 \end{pmatrix}$$

Diagonal: all errors independent



## Clustered errors: Covariance structure

**Example:** 2 groups, 3 people each

**Without clustering (IID)**

$$\Omega = \begin{pmatrix} \sigma^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2 \end{pmatrix}$$

Diagonal: all errors independent

**With clustering**

$$\Omega = \begin{pmatrix} \sigma^2 & \sigma_{12} & \sigma_{13} & 0 & 0 & 0 \\ \sigma_{12} & \sigma^2 & \sigma_{23} & 0 & 0 & 0 \\ \sigma_{13} & \sigma_{23} & \sigma^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2 & \sigma_{45} & \sigma_{46} \\ 0 & 0 & 0 & \sigma_{45} & \sigma^2 & \sigma_{56} \\ 0 & 0 & 0 & \sigma_{46} & \sigma_{56} & \sigma^2 \end{pmatrix}$$

Red: within-cluster correlations

## Why clustering increases standard errors

**Intuition:** Correlation reduces effective sample size

Each observation provides *less independent information* when correlated

## Why clustering increases standard errors

**Intuition:** Correlation reduces effective sample size

Each observation provides *less independent information* when correlated

**Consequence:** Variance of OLS estimator is actually higher:

$$V(\hat{\beta}) = (X'X)^{-1}X'\Omega X(X'X)^{-1} \quad (19)$$

where  $\Omega$  has off-diagonal elements (covariances)

## Why clustering increases standard errors

**Intuition:** Correlation reduces effective sample size

Each observation provides *less independent information* when correlated

**Consequence:** Variance of OLS estimator is actually higher:

$$V(\hat{\beta}) = (X'X)^{-1}X'\Omega X(X'X)^{-1} \quad (19)$$

where  $\Omega$  has off-diagonal elements (covariances)

If you ignore clustering:

- Standard errors too small
- Over-rejection of null hypotheses
- False confidence in results

## Modeling cluster structure: Random effects

**Approach 1:** Assume random group effects (panel data structure)

Error decomposes:  $\varepsilon_{gi} = u_g + v_{gi}$  where  $u_g$  is group shock,  $v_{gi}$  is idiosyncratic

## Modeling cluster structure: Random effects

**Approach 1:** Assume random group effects (panel data structure)

Error decomposes:  $\varepsilon_{gi} = u_g + v_{gi}$  where  $u_g$  is group shock,  $v_{gi}$  is idiosyncratic

Then the covariance matrix  $\Omega_g$  is:

$$\Omega_g = \begin{pmatrix} \sigma_u^2 + \sigma_v^2 & \sigma_u^2 & \cdots & \sigma_u^2 \\ \sigma_u^2 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \sigma_u^2 \\ \sigma_u^2 & \cdots & \sigma_u^2 & \sigma_u^2 + \sigma_v^2 \end{pmatrix}$$

with  $V(u_g) = \sigma_u^2$  and  $V(v_{gi}) = \sigma_v^2$

## Modeling cluster structure: Random effects

**Approach 1:** Assume random group effects (panel data structure)

Error decomposes:  $\varepsilon_{gi} = u_g + v_{gi}$  where  $u_g$  is group shock,  $v_{gi}$  is idiosyncratic

Then the covariance matrix  $\Omega_g$  is:

$$\Omega_g = \begin{pmatrix} \sigma_u^2 + \sigma_v^2 & \sigma_u^2 & \cdots & \sigma_u^2 \\ \sigma_u^2 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \sigma_u^2 \\ \sigma_u^2 & \cdots & \sigma_u^2 & \sigma_u^2 + \sigma_v^2 \end{pmatrix}$$

with  $V(u_g) = \sigma_u^2$  and  $V(v_{gi}) = \sigma_v^2$

Note:  $\forall i \neq j : \text{cov}(\varepsilon_{gi}, \varepsilon_{gj}) = \sigma_u^2$

## Modeling cluster structure: Moulton factor

**Approach 2:** Constant within-group correlation (Moulton 1986)

Assumes within-group correlation is constant:  $\text{cor}(\varepsilon_i, \varepsilon_j) = \rho$  if  $G_i = G_j$ , else 0



## Modeling cluster structure: Moulton factor

### Approach 2: Constant within-group correlation (Moulton 1986)

Assumes within-group correlation is constant:  $\text{cor}(\varepsilon_i, \varepsilon_j) = \rho$  if  $G_i = G_j$ , else 0

Leading to covariance matrix:  $\Omega_g = \sigma_e^2 \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \rho \\ \rho & \cdots & \rho & 1 \end{pmatrix}$

## Modeling cluster structure: No assumptions

### Approach 3: Don't assume specific structure—just estimate

$$\hat{\Omega}_g = \begin{bmatrix} \hat{u}_{g1}^2 & \hat{u}_{g1}\hat{u}_{g2} & \hat{u}_{g1}\hat{u}_{g3} \\ \hat{u}_{g2}\hat{u}_{g1} & \hat{u}_{g2}^2 & \hat{u}_{g2}\hat{u}_{g3} \\ \hat{u}_{g3}\hat{u}_{g1} & \hat{u}_{g3}\hat{u}_{g2} & \hat{u}_{g3}^2 \end{bmatrix}$$

This is what **cluster-robust standard errors** do

Most flexible approach—recommended!

## Practical recommendations: Clustering

## When to cluster?

Whenever observations might be correlated within groups

## How to choose cluster level?

- Cluster at level where treatment is assigned (if applicable)
- Cluster at highest level of potential correlation
- When in doubt: cluster at higher level (more conservative)

# Practical recommendations: Clustering

## When to cluster?

Whenever observations might be correlated within groups

## How to choose cluster level?

- Cluster at level where treatment is assigned (if applicable)
- Cluster at highest level of potential correlation
- When in doubt: cluster at higher level (more conservative)

## Implementation:

- **Stata:** `reg y x, cluster(groupvar)`
- **R:** `coeftest(model, vcov = vcovCL(model, cluster = ~groupvar))`

**Result:** Accounting for clustering *almost always* increases SEs

## Outline

## Spillovers as SUTVA violations

**Recall SUTVA** (Stable Unit Treatment Value Assumption):

Treatment of unit  $i$  doesn't affect outcomes of unit  $j$



## Spillovers as SUTVA violations

**Recall SUTVA** (Stable Unit Treatment Value Assumption):

Treatment of unit  $i$  doesn't affect outcomes of unit  $j$

**Spillovers:** This assumption is violated!

**Definition:** Person  $i$  being treated affects outcome  $Y_j$  of person  $j$

**Examples:**

- Vaccines: My vaccination protects you (positive spillover)
- Deworming: Treating some kids reduces transmission to others
- Information: Treated individuals share knowledge with control group
- Market effects: Training increases labor supply, affects wages



## Indirect treatment effect

**Object of interest:** How much does treatment of others affect me?

**Indirect Treatment Effect (ITE):** Compare untreated in treated clusters vs. untreated in control clusters

$$\text{ITE} = \mathbb{E}[Y_i(0, D_{-i} = 1) - Y_i(0, D_{-i} = 0) | D_i = 0] \quad (20)$$

where:

- $Y_i(0, D_{-i} = 1)$ : Outcome for untreated  $i$  when others in cluster are treated
- $Y_i(0, D_{-i} = 0)$ : Outcome for untreated  $i$  when others in cluster are untreated

If  $\text{ITE} \neq 0$ , SUTVA is violated

## Measuring spillovers: Two-step clustered RCT

### Design:

- 1 Step 1:** Randomize which clusters get treatment (e.g., schools, villages)

## Measuring spillovers: Two-step clustered RCT

## Design:

- 1 **Step 1:** Randomize which clusters get treatment (e.g., schools, villages)
- 2 **Step 2:** Randomize which individuals within treated clusters get treatment

## Measuring spillovers: Two-step clustered RCT

### Design:

- 1 **Step 1:** Randomize which clusters get treatment (e.g., schools, villages)
- 2 **Step 2:** Randomize which individuals within treated clusters get treatment

**This allows estimation of:**

- **Direct effect:** Treated vs. untreated in same cluster
- **Indirect effect:** Untreated in treated cluster vs. untreated in control cluster
- **Total effect:** Treated in treated cluster vs. untreated in control cluster

## Measuring spillovers: Two-step clustered RCT

**Design:**

- 1 **Step 1:** Randomize which clusters get treatment (e.g., schools, villages)
- 2 **Step 2:** Randomize which individuals within treated clusters get treatment

**This allows estimation of:**

- **Direct effect:** Treated vs. untreated in same cluster
- **Indirect effect:** Untreated in treated cluster vs. untreated in control cluster
- **Total effect:** Treated in treated cluster vs. untreated in control cluster

**Extension:** Can vary proportion treated in each cluster (e.g., {0%, 30%, 60%})

Estimates how spillovers change with treatment intensity (but requires high power)

## Example: Miguel & Kremer (2004) deworming study

**Setting:** Deworming treatment in Kenyan schools

**Design:** Phased randomization of schools

- Group 1 schools treated in 1998
- Group 2 schools treated in 1999
- Group 3 schools treated in 2000 (control)

## Example: Miguel & Kremer (2004) deworming study

**Setting:** Deworming treatment in Kenyan schools

**Design:** Phased randomization of schools

- Group 1 schools treated in 1998
- Group 2 schools treated in 1999
- Group 3 schools treated in 2000 (control)

**Key finding:** Large positive spillovers

- Direct effect: Treated students healthier
- Spillover effect: Untreated students in nearby treated schools also healthier
- Mechanism: Reduced disease transmission

## Example: Miguel & Kremer (2004) deworming study

**Setting:** Deworming treatment in Kenyan schools

**Design:** Phased randomization of schools

- Group 1 schools treated in 1998
- Group 2 schools treated in 1999
- Group 3 schools treated in 2000 (control)

**Key finding:** Large positive spillovers

- Direct effect: Treated students healthier
- Spillover effect: Untreated students in nearby treated schools also healthier
- Mechanism: Reduced disease transmission

**Implication:** Standard RCT would *underestimate* total benefits

(Control group also benefits from nearby treatment)



## Testing for spillovers: The naive approach

**Idea:** Test if treatment effects vary by exposure to treated individuals

## Basic regression:

$$Y_i = \alpha + \beta D_i + \gamma \text{Exposure}_i + \varepsilon_i \quad (21)$$

where  $\text{Exposure}_i$  = number of treated individuals near  $i$

## Problem with naive approach

**Issue:** Exposure is not exogenous!

**Example problems:**

- Urban vs. rural: People in dense cities more exposed to treated units
- Social networks: Popular people more exposed
- Geography: People near population centers more exposed

## Problem with naive approach

**Issue:** Exposure is not exogenous!

**Example problems:**

- Urban vs. rural: People in dense cities more exposed to treated units
- Social networks: Popular people more exposed
- Geography: People near population centers more exposed

⇒ Exposure<sub>i</sub> correlated with unobservables (e.g., density, centrality)

## Problem with naive approach

**Issue:** Exposure is not exogenous!

**Example problems:**

- Urban vs. rural: People in dense cities more exposed to treated units
- Social networks: Popular people more exposed
- Geography: People near population centers more exposed

⇒ Exposure<sub>i</sub> correlated with unobservables (e.g., density, centrality)

⇒  $\gamma$  is **biased** (OVB problem)

Cannot distinguish true spillovers from confounding by exposure-related factors

## Solution: Recentering approach (Borusrak & Hull 2023)

**Key idea:** Control for *expected* exposure under random assignment

### Steps:

- 1 Simulate random assignments many times (e.g., 10,000)
- 2 Calculate expected exposure:  $\mathbb{E}[\text{Exposure}_i]$  for each unit

## Solution: Recentering approach (Borusyak & Hull 2023)

**Key idea:** Control for *expected* exposure under random assignment

**Steps:**

- ① Simulate random assignments many times (e.g., 10,000)
- ② Calculate expected exposure:  $\mathbb{E}[\text{Exposure}_i]$  for each unit
- ③ Regress outcome on actual exposure, controlling for expected exposure:

$$Y_i = \beta D_i + \gamma \text{Exposure}_i + \delta \mathbb{E}[\text{Exposure}_i] + \varepsilon_i \quad (22)$$

# Recentering: Intuition

## Why does this work?

- $\mathbb{E}[\text{Exposure}_i]$  captures confounders (density, centrality, etc.)

## Recentering: Intuition

### Why does this work?

- $\mathbb{E}[\text{Exposure}_i]$  captures confounders (density, centrality, etc.)
- These confounders affect *expected* exposure, not random deviations



## Recentering: Intuition

## Why does this work?

- $E[\text{Exposure}_i]$  captures confounders (density, centrality, etc.)
- These confounders affect *expected* exposure, not random deviations
- Actual exposure – expected exposure = exogenous variation from randomization

## Recentering: Intuition

## Why does this work?

- $\mathbb{E}[\text{Exposure}_i]$  captures confounders (density, centrality, etc.)
- These confounders affect *expected* exposure, not random deviations
- Actual exposure – expected exposure = exogenous variation from randomization
- $\gamma$  identifies causal spillover effect from this exogenous variation

## Summary: Spillovers

### Key takeaways:

- Spillovers violate SUTVA—treatment of  $i$  affects  $j$
- Can be positive or negative, and quantitatively important
- **In RCTs:** Use two-step clustered design to measure indirect effects
- **Testing for spillovers:**
  - Naive exposure regressions are biased
  - Use recentering approach (Borusyak & Hull 2023)
  - Control for expected exposure to remove confounding
- Applies beyond RCTs: Any setting with spatial/network structure
- **Practical advice:** Always consider whether spillovers plausible in your setting

# Outline

1. Clustering
2. Power
3. Choosing controls
4. Heterogeneous treatment effects
5. Attrition
6. Multiple Hypothesis Testing
7. Robustness and Replication
8. Standard Errors in Regressions
9. Spillovers
10. Measurement

# Measurement matters

**Often overlooked:** Measurement is critical but undervalued in econometrics courses

**Key insight:** Regardless of causal identification, if we don't measure the right thing, we can't even know  $Y_i$ !

# Measurement matters

**Often overlooked:** Measurement is critical but undervalued in econometrics courses

**Key insight:** Regardless of causal identification, if we don't measure the right thing, we can't even know  $Y_i$ !

**Two types of measurement error:**

① **Classical measurement error:** Random noise, independent of true value

- $X_i^{\text{observed}} = X_i^{\text{true}} + \varepsilon_i$  where  $\mathbb{E}[\varepsilon_i] = 0$ ,  $\text{Cov}(X_i^{\text{true}}, \varepsilon_i) = 0$

# Measurement matters

**Often overlooked:** Measurement is critical but undervalued in econometrics courses

**Key insight:** Regardless of causal identification, if we don't measure the right thing, we can't even know  $Y_i$ !

**Two types of measurement error:**

- ① **Classical measurement error:** Random noise, independent of true value
  - $X_i^{\text{observed}} = X_i^{\text{true}} + \varepsilon_i$  where  $\mathbb{E}[\varepsilon_i] = 0$ ,  $\text{Cov}(X_i^{\text{true}}, \varepsilon_i) = 0$
- ② **Non-classical measurement error:** Systematic, correlated with truth:  $\text{Cov}(X_i^{\text{true}}, \varepsilon_i) \neq 0$ 
  - Example: Rich people overreport income, poor people underreport
  - Can bias estimates in *any* direction

## Classical measurement error: Attenuation bias

**Setup:** True model:  $Y_i = \alpha + \beta X_i^{\text{true}} + u_i$ ; observe  $X_i^{\text{obs}} = X_i^{\text{true}} + \varepsilon_i$



## Classical measurement error: Attenuation bias

**Setup:** True model:  $Y_i = \alpha + \beta X_i^{\text{true}} + u_i$ ; observe  $X_i^{\text{obs}} = X_i^{\text{true}} + \varepsilon_i$

**Regressing  $Y_i$  on  $X_i^{\text{obs}}$ :**

## Classical measurement error: Attenuation bias

**Setup:** True model:  $Y_i = \alpha + \beta X_i^{\text{true}} + u_i$ ; observe  $X_i^{\text{obs}} = X_i^{\text{true}} + \varepsilon_i$

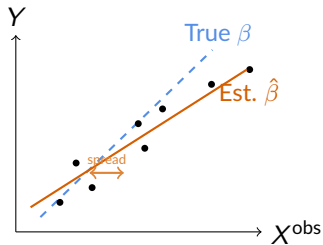
**Regressing  $Y_i$  on  $X_i^{\text{obs}}$ :**

$$\hat{\beta} = \frac{\text{Cov}(Y_i, X_i^{\text{obs}})}{\text{Var}(X_i^{\text{obs}})} = \frac{\beta \text{Var}(X_i^{\text{true}})}{\text{Var}(X_i^{\text{true}}) + \text{Var}(\varepsilon_i)} = \beta \cdot \underbrace{\frac{\text{Var}(X^{\text{true}})}{\text{Var}(X^{\text{true}}) + \text{Var}(\varepsilon)}}_{\text{attenuation factor} < 1} \quad (23)$$

$\Rightarrow$  **Attenuation bias:** Estimates biased toward zero

# Visualizing measurement error

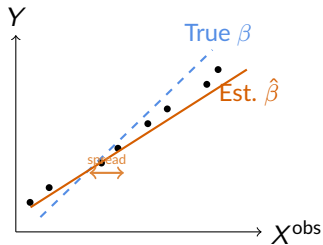
## Noise in $X$ (attenuation)



Noise **spreads out**  $X$  (horizontally)  
 $\Rightarrow$  **Flatter slope**, bias toward zero

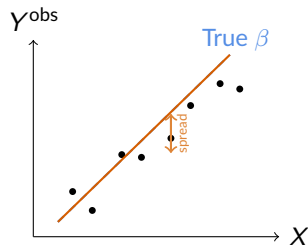
## Visualizing measurement error

Noise in  $X$  (attenuation)



Noise **spreads out**  $X$  (horizontally)  
 $\Rightarrow$  **Flatter slope**, bias toward zero

Noise in  $Y$  (no bias)



Noise **spreads out**  $Y$  **conditional on**  $X$   
 (vertically)  
 $\Rightarrow$  **Same slope**, no bias, just less precision

## Sources of bias: Self-reported data (Part 1)

Self-reports are common but problematic. Key sources of bias:

### 1. Recall bias

- People misremember past events/behaviors
- **Example:** Arthi et al. (2018, Tanzania): People report 4× more farm work hours when recalling vs. real-time measurement
- $\implies$  Large understatement of agricultural labor productivity

## Sources of bias: Self-reported data (Part 1)

Self-reports are common but problematic. Key sources of bias:

### 1. Recall bias

- People misremember past events/behaviors
- **Example:** Arthi et al. (2018, Tanzania): People report 4× more farm work hours when recalling vs. real-time measurement
- $\implies$  Large understatement of agricultural labor productivity

### 2. Social desirability bias / Experimenter demand

- Respondents give “socially acceptable” answers
- Want to please researchers or conform to norms
- Can measure with Crowne-Marlowe social desirability scale
- Adjust responses or use indirect questioning methods

## Sources of bias: Self-reported data (Part 1)

Self-reports are common but problematic. Key sources of bias:

### 1. Recall bias

- People misremember past events/behaviors
- **Example:** Arthi et al. (2018, Tanzania): People report 4× more farm work hours when recalling vs. real-time measurement
- $\implies$  Large understatement of agricultural labor productivity

### 2. Social desirability bias / Experimenter demand

- Respondents give “socially acceptable” answers
- Want to please researchers or conform to norms
- Can measure with Crowne-Marlowe social desirability scale
- Adjust responses or use indirect questioning methods

## Sources of bias: Self-reported data (Part 2)

### 3. Question order effects

- Earlier questions influence responses to later ones
- Priming, anchoring, consistency effects



## Sources of bias: Self-reported data (Part 2)

### 3. Question order effects

- Earlier questions influence responses to later ones
- Priming, anchoring, consistency effects

### 4. Framing effects

- Same information, different presentation  $\implies$  different responses
- **Examples:**
  - “90% survival rate” vs. “10% mortality rate”
  - “Tax relief” vs. “tax cuts” vs. “tax decreases”

## Sources of bias: Self-reported data (Part 3)

## Sources of bias: Self-reported data (Part 3)

### 5. Selection bias

- Who answers the survey?
- Non-response often non-random
- **Example:** US political polling often has  $<5\%$  response rate (Pew)
- Those who respond differ systematically from non-responders

## Sources of bias: Self-reported data (Part 3)

### 5. Selection bias

- Who answers the survey?
- Non-response often non-random
- **Example:** US political polling often has  $<5\%$  response rate (Pew)
- Those who respond differ systematically from non-responders

### 6. Hypothetical vs. incentivized responses

- Hypothetical: “Would you buy this product at \$10?”
- Incentivized: Actually buying with real money
- **Recommendation:** Use incentive-compatible elicitation when possible
- Revealed preferences  $>$  stated preferences

## Partial solution to classical error: Creating indices

**Idea:** Combine multiple noisy measures to reduce measurement error

**Simple case:** Average two measures  $X_1$  and  $X_2$  of same construct:  $X^{\text{index}} = (X_1 + X_2)/2$

If both have independent noise:  $\text{Var}(\text{noise in index}) = \frac{1}{2}\text{Var}(\text{noise in each})$

## Partial solution to classical error: Creating indices

**Idea:** Combine multiple noisy measures to reduce measurement error

**Simple case:** Average two measures  $X_1$  and  $X_2$  of same construct:  $X^{\text{index}} = (X_1 + X_2)/2$

If both have independent noise:  $\text{Var}(\text{noise in index}) = \frac{1}{2}\text{Var}(\text{noise in each})$

**Derivation:** Let  $X_j = X^{\text{true}} + \varepsilon_j$  with  $\varepsilon_1 \perp \varepsilon_2$ . Then:  $X^{\text{index}} = X^{\text{true}} + \frac{\varepsilon_1 + \varepsilon_2}{2}$  and  $\text{Var}\left(\frac{\varepsilon_1 + \varepsilon_2}{2}\right) = \frac{1}{4} \cdot 2\sigma_\varepsilon^2 = \frac{1}{2}\sigma_\varepsilon^2$

## Partial solution to classical error: Creating indices

**Idea:** Combine multiple noisy measures to reduce measurement error

**Simple case:** Average two measures  $X_1$  and  $X_2$  of same construct:  $X^{\text{index}} = (X_1 + X_2)/2$

If both have independent noise:  $\text{Var}(\text{noise in index}) = \frac{1}{2}\text{Var}(\text{noise in each})$

**Derivation:** Let  $X_j = X^{\text{true}} + \varepsilon_j$  with  $\varepsilon_1 \perp \varepsilon_2$ . Then:  $X^{\text{index}} = X^{\text{true}} + \frac{\varepsilon_1 + \varepsilon_2}{2}$  and  $\text{Var}\left(\frac{\varepsilon_1 + \varepsilon_2}{2}\right) = \frac{1}{4} \cdot 2\sigma_\varepsilon^2 = \frac{1}{2}\sigma_\varepsilon^2$

**More generally:**  $n$  measures  $\implies$  noise variance reduced by factor of  $1/n$

## Partial solution to classical error: Creating indices

**Idea:** Combine multiple noisy measures to reduce measurement error

**Simple case:** Average two measures  $X_1$  and  $X_2$  of same construct:  $X^{\text{index}} = (X_1 + X_2)/2$

If both have independent noise:  $\text{Var}(\text{noise in index}) = \frac{1}{2}\text{Var}(\text{noise in each})$

**Derivation:** Let  $X_j = X^{\text{true}} + \varepsilon_j$  with  $\varepsilon_1 \perp \varepsilon_2$ . Then:  $X^{\text{index}} = X^{\text{true}} + \frac{\varepsilon_1 + \varepsilon_2}{2}$  and  $\text{Var}\left(\frac{\varepsilon_1 + \varepsilon_2}{2}\right) = \frac{1}{4} \cdot 2\sigma_\varepsilon^2 = \frac{1}{2}\sigma_\varepsilon^2$

**More generally:**  $n$  measures  $\implies$  noise variance reduced by factor of  $1/n$

**Key assumption:** Measurement errors independent (if correlated, gains smaller)



## Weighted indices

Can do better than simple averages by weighting measures differently:

### 1. Sum of standardized scores (Kling et al. 2007)

- Convert to z-scores  $(X_j - \bar{X}_j)/SD(X_j)$ , then average

## Weighted indices

Can do better than simple averages by weighting measures differently:

### 1. Sum of standardized scores (Kling et al. 2007)

- Convert to z-scores  $(X_j - \bar{X}_j)/SD(X_j)$ , then average

### 2. Factor analysis

- Extract common factor; variables uncorrelated with others = mostly noise  $\implies$  weight less

## Weighted indices

Can do better than simple averages by weighting measures differently:

### 1. Sum of standardized scores (Kling et al. 2007)

- Convert to z-scores  $(X_j - \bar{X}_j)/SD(X_j)$ , then average

### 2. Factor analysis

- Extract common factor; variables uncorrelated with others = mostly noise  $\implies$  weight less

### 3. Inverse-covariance weighting

- Variables uncorrelated with others provide independent info  $\implies$  weight more
- Opposite logic to factor analysis—context determines which is better

# Dealing with outliers and missing values

**Outliers:** Extreme values can drive results

**Solutions:**

- **Winsorization:** Cap extreme values at percentile (e.g., 1st/99th)
- **Trimming:** Drop extreme observations entirely
- **Transformation:** Log, inverse hyperbolic sine (for zeros)
- **Robust regression:** Median regression (LAD), Huber M-estimator

## Dealing with outliers and missing values

**Outliers:** Extreme values can drive results

**Solutions:**

- **Winsorization:** Cap extreme values at percentile (e.g., 1st/99th)
- **Trimming:** Drop extreme observations entirely
- **Transformation:** Log, inverse hyperbolic sine (for zeros)
- **Robust regression:** Median regression (LAD), Huber M-estimator

**Missing values:** Often not missing at random

**Solutions:**

- **Imputation:** Replace with median, mean, or predicted values
- **Bounds / IPW:** As with attrition (Lee bounds, inverse probability weighting)
- **Missing indicator:** Include dummy for missing, set value to 0
- **Leave as missing:** Report sample size, acknowledge limitation

## Best practices: No single “right” answer

**Reality:** For many measurement issues, no obviously “best” approach

**The strategy:**

### ① Be clear and justify your choices

- Explain why you chose specific method
- Acknowledge limitations and alternatives

## Best practices: No single “right” answer

**Reality:** For many measurement issues, no obviously “best” approach

**The strategy:**

### ① Be clear and justify your choices

- Explain why you chose specific method
- Acknowledge limitations and alternatives

### ② Show robustness to alternative approaches

- Report results under different measurement choices
- Include in main text or appendix
- If results fragile to measurement choices, that’s important!

## Best practices: No single “right” answer

**Reality:** For many measurement issues, no obviously “best” approach

**The strategy:**

### ① Be clear and justify your choices

- Explain why you chose specific method
- Acknowledge limitations and alternatives

### ② Show robustness to alternative approaches

- Report results under different measurement choices
- Include in main text or appendix
- If results fragile to measurement choices, that’s important!

**Remember:** Good measurement is as important as good identification

Can have perfect RCT but meaningless results if measuring wrong thing



## Key Takeaways

### Main lessons from today:

- 1 **Choosing controls:** Not all controls are good—think causally (DAGs). Bad controls can bias estimates or block mechanisms.

# Key Takeaways

## Main lessons from today:

- ① **Choosing controls:** Not all controls are good—think causally (DAGs). Bad controls can bias estimates or block mechanisms.
- ② **Heterogeneous effects:** Average effects may mask important variation. Use modern ML methods (GATES, causal forests) to study heterogeneity rigorously.

# Key Takeaways

## Main lessons from today:

- ① **Choosing controls:** Not all controls are good—think causally (DAGs). Bad controls can bias estimates or block mechanisms.
- ② **Heterogeneous effects:** Average effects may mask important variation. Use modern ML methods (GATES, causal forests) to study heterogeneity rigorously.
- ③ **Attrition:** Can create selection bias even in RCTs. Test for differential attrition and use bounds (Lee, Manski) or IPW.

## Key Takeaways

### Main lessons from today:

- 1 **Choosing controls:** Not all controls are good—think causally (DAGs). Bad controls can bias estimates or block mechanisms.
- 2 **Heterogeneous effects:** Average effects may mask important variation. Use modern ML methods (GATES, causal forests) to study heterogeneity rigorously.
- 3 **Attrition:** Can create selection bias even in RCTs. Test for differential attrition and use bounds (Lee, Manski) or IPW.
- 4 **Multiple hypothesis testing:** Testing many hypotheses inflates false positives. Control FDR (Benjamini-Hochberg) or FWER (Bonferroni). Pre-specify analyses.

# Key Takeaways

## Main lessons from today:

- ① **Choosing controls:** Not all controls are good—think causally (DAGs). Bad controls can bias estimates or block mechanisms.
- ② **Heterogeneous effects:** Average effects may mask important variation. Use modern ML methods (GATES, causal forests) to study heterogeneity rigorously.
- ③ **Attrition:** Can create selection bias even in RCTs. Test for differential attrition and use bounds (Lee, Manski) or IPW.
- ④ **Multiple hypothesis testing:** Testing many hypotheses inflates false positives. Control FDR (Benjamini-Hochberg) or FWER (Bonferroni). Pre-specify analyses.
- ⑤ **Replication:** Results less robust than we thought. Researcher degrees of freedom, publication bias, and winner's curse all contribute. Transparency and robustness checks are essential.

## Key Takeaways (cont.)

- 6 **Standard errors:** Always use heteroskedasticity-robust SEs. Cluster when observations correlated within groups (panel, spatial, etc.). Most common mistake in applied work.

## Key Takeaways (cont.)

- ⑥ **Standard errors:** Always use heteroskedasticity-robust SEs. Cluster when observations correlated within groups (panel, spatial, etc.). Most common mistake in applied work.
- ⑦ **Spillovers:** SUTVA violations matter. Design two-step clustered RCTs to measure indirect effects. Use recentering approach (Borusyak & Hull) to test for spillovers.

## Key Takeaways (cont.)

- ⑥ **Standard errors:** Always use heteroskedasticity-robust SEs. Cluster when observations correlated within groups (panel, spatial, etc.). Most common mistake in applied work.
- ⑦ **Spillovers:** SUTVA violations matter. Design two-step clustered RCTs to measure indirect effects. Use recentering approach (Borusyak & Hull) to test for spillovers.
- ⑧ **Measurement error:** Classical error in  $X$  causes attenuation bias. Self-reports have many biases (recall, social desirability, framing). Combine multiple measures to reduce noise.



## Key Takeaways (cont.)

- ⑥ **Standard errors:** Always use heteroskedasticity-robust SEs. Cluster when observations correlated within groups (panel, spatial, etc.). Most common mistake in applied work.
- ⑦ **Spillovers:** SUTVA violations matter. Design two-step clustered RCTs to measure indirect effects. Use recentering approach (Borusyak & Hull) to test for spillovers.
- ⑧ **Measurement error:** Classical error in  $X$  causes attenuation bias. Self-reports have many biases (recall, social desirability, framing). Combine multiple measures to reduce noise.

**Overall theme:** Even with perfect identification, empirical work requires careful attention to practical challenges. Good research design anticipates and addresses these issues proactively.

# GATES/CLAN: Why sample splitting matters

**Overfitting problem:** Using *same* data to train ML and estimate GATES  $\implies$  biased estimates

**Solution:** Sample splitting

- **Auxiliary sample:** Train ML predictor
- **Main sample:** Estimate GATES/CLAN (not used to choose groups)

**Aggregation:** Results may depend on the particular random split

- Repeat with many splits, report **median** estimates
- More robust than any single split

**Implementation:** R package `GenericML`