Calculating Max Speed with Headwind

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Assume:
- effort is constant
- the wind velocity remains constant
- air resistance is only drag force

$$P = F_{effort} \cdot v$$

Since at constant speed air resistance is equal to the forward force,

$$P = F_{drag} \cdot v$$

$$F_{drag} = \frac{1}{2} \rho v^2 C_D A$$

Since power output is constant:

$$\frac{1}{2} \rho C_D A v_0^3 = \frac{1}{2} \rho C_D A (v_1 - v_w)^2 v_1$$

$$v_0^3 = (v_1 - v_w)^2 v_1$$

$$\frac{(8.94)^3}{11.18} = (v_1 - v_w)^2$$

$$(v_1 - v_w)^2 = 63.91$$

$$v_1^2 + v_w^2 - 2v_1 v_w = 63.91$$

$$v_w^2 - 2v_1 v_w = 63.91 - (11.18)^2$$

$$v_w(v_w - 2v_1) = -61.08$$

$$v_w(v_w - 22.36) + 61.08 = 0$$

$$v_w^2 - 22.36 v_w + 61.08 = 0$$

$$v_w = \frac{22.36 \pm \sqrt{(-22.36)^2 - 4(1)(61.08)}}{2}$$

$$v_w = 3.19,\ 19.17 \quad m\,s^{-1}$$

$$7.14,\ 42.88 \quad mph$$

Using $v_w = 7.14$ mph ($3.19\ m\,s^{-1}$):

$$\frac{1}{2} \rho C_D A v_0^3 = \frac{1}{2} \rho C_D A (v_2 + v_w)^2 v_2$$

$$v_0^3 = (v_2 + v_w)^2 v_2$$

$$(8.94)^3 = (v_2^2 + v_w^2 + 2v_2 v_w) v_2$$

$$714.52 = v_2^3 + (3.19)^2 v_2 \quad 2(3.19) v_2^2$$

$$v_2^3 + 6.38 v_2^2 + 10.18 v_2 - 714.52 = 0$$

$$v_2 = 6.94 \quad m\,s^{-1}$$

$$= \underline{15.52 \ mph}$$