MATH 531: HW1

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Problem 1:

(idea):

$4x^2 + 13 > 12x$

$4x^2 - 12x + 13 > 0$

$4[x^2 - 3x + \frac{13}{4}] > 0$

$4[(x - \frac{3}{2})^2 + \frac{9}{4}] > 0$

$(x - \frac{3}{2})^2 \geq 0$

Proof:

Let x be real.

Note that,

$$(x - \frac{3}{2})^2 \geq 0.$$

Ift,

$$4[(x - \frac{3}{2})^2 + \frac{9}{4}] \geq 0.$$

Therefore,

$$4[(x - \frac{3}{2})^2 + \frac{9}{4}] > 0.$$

Bac,

$$4[(x - \frac{3}{2})^2 + \frac{9}{4}] = 4[x^2 - 3x + \frac{13}{4}].$$

$$= 4x^2 - 12x + 13.$$

Therefore,

$$4x^2 - 12x + 13 > 0.$$

Therefore,

$$4x^2 + 13 > 12x.$$

Done.

Problem 2:

(idea):

$5x + y^2$

$x = \frac{m}{n}$

$y = \frac{a}{b}$

$5(\frac{m}{n}) + (\frac{a}{b})^2$

$\frac{5m}{n} + \frac{a^2}{b^2}$

$\frac{5mb^2+na^2}{nb^2}$

Proof

Let,

$$x = \frac{m}{n},$$

$$y = \frac{a}{b},$$

where $m,\ n,\ a,\ b$ are integers and $n \neq 0, b \neq 0$.

Bac,

$$5x + y^2 = 5(\frac{m}{n}) + (\frac{a}{b})^2$$

$$= \frac{5mb^2 + na^2}{nb^2}.$$

Remember that $5mb^2 + na^2$ and nb^2 are integers.

Additionally $nb^2 \neq 0$.

Therefore $5x + y^2$ is rational.

Done.

Problem 3:

(idea):

$n = 2k + 1$

$5n + 23$

$5(2k + 1) + 23$

$10k + 28$

$2(5k + 14)$

$L = 5k + 14$

$2L$

Proof

Let,

$$n = 2k + 1$$

where k is an integer.

Bac,

$$5n + 23 = 10k + 28$$

$$= 2(5k + 14).$$

Let,

$$L = 5k + 14.$$

Remember that L is an integer.

Therefore,

$$5n + 23 = 2L.$$

Remember a number m is even if,

$$m = 2k.$$

Therefore $5n + 23$ is even.

Done.