MATH 531: Project 1

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Problem 1:

Given: $A = \{\emptyset, \{\emptyset\}, \{2\}, 4\}$

A:

$$A \cap \{\emptyset\} = \{\emptyset\}$$

Since $\{\emptyset\} \subseteq A$ the intersection of A and $\{\emptyset\}$ would just be $\{\emptyset\}$.

B:

$$A \cap \{2\} = \emptyset$$

Since $2 \notin A$ the intersection of A and $\{2\}$ is the $\emptyset$.

C:

$$A \cap \{4\} = \{4\}$$

Since $\{4\} \subseteq A$ the intersection of A and $\{4\}$ is just $\{4\}$.

D:

$$A \cup \{2\} = \{\emptyset, \{\emptyset\}, \{2\}, 4, 2\}$$

As $2 \notin A$, the union of A and $\{2\}$ would contain all the elements of A as well as 2.

E:

$$A \cup \{4\} = A$$

Since $4 \in A$ the union of A and $\{4\}$ is just A.

F:

$$A \cup \{\{\emptyset\}\} = A$$

Since $\{\emptyset\} \in A$ the union of A and $\{\{\emptyset\}\}$ is just A.

Problem 2:

Given: $x \in \mathbb{R}$ and $y \in \mathbb{R}$ with $x < y$.

Prove: $x < \frac{x+y}{2} < y$.

Idea:

$x < \frac{x+y}{2}$

$2x < x + y$

$x < y$

So the first part is true.

$\frac{x+y}{2} < y$

$x + y < 2y$

$x < y$

So the other part is also true.

Proof:

Let $x \in \mathbb{R}$ and $y \in \mathbb{R}$ with $x < y$. \$ \$ (Goal: $x < \frac{x+y}{2} < y$)

Case 1:

Since $x < y$, ift

$$x - y < 0.$$

Therefore

$$x - y + 2y < 2y.$$

Ift

$$x + y < 2y.$$

Therefore

$$\frac{x+y}{2} < y.$$

Done Case 1.

Case 2:

Since $x < y$, ift

$$y - x > 0.$$

Therefore

$$y - x + 2x > 2x.$$

Ift

$$x + y > 2x.$$

Therefore

$$\frac{x+y}{2} > x.$$

Done Case 2.

Conclude

$$x < \frac{x+y}{2} < y.$$

Done Proof.

Problem 3:

A:

The sets obey $\emptyset \in A$, $A \subseteq B$, and $A \cap B = \emptyset$.

Not Possible.

Since $A \cap B = \emptyset$ there cannot be any elements in A that are also in B besides the null set. This contradicts the condition that $A \subseteq B$, since this would mean for every element in A, that element would also have to be in B.

B:

The sets obey $A \in B$, $B \in C$, and $B \subseteq C$.

Possible.

$A = \emptyset$

$B = \{\emptyset\}$

$C = \{\emptyset, \{\emptyset\}\}$

Since $A = \emptyset$ in this case it is always true that $A \in B$.

Then $\{\emptyset\} \in C$ so the second condition is met.

Additionally since $\emptyset \in C$ that would mean all elements of B are in A so the final condition is met.

C:

$A \in B$ and $A \cap B = \emptyset$.

Possible.

$A = \{2\}$

$B = \{\{2\}\}$

Since $\{2\} \in B$ the first condition is met.

Additionally since sets A and B have none of the same elements the second condition is met.

Problem 4:

Given: $A_n = [0, n]$ for $n = 1, 2, ...$

Find: $\bigcup_{n=1}^{7} A_n$

Idea:

$\bigcup_{n=1}^{7} A_n = [0, 7]$

Proof:

($\subseteq$) Let $x \in \cup_{n=1}^{7} A_n$. (Goal: $x \in [0, 7]$)

Ift $x \in A_{n*}$ for some $n*$, $1 \leq n* \leq 7$.

Therefore

$$x \in [0, n*].$$

Note that $0 \leq x$ and $x \leq n*$.

Since $x \leq n*$ and $n* \leq 7$, itf

$$x \leq 7.$$

Recall that $x \geq 0$.

Therefore

$$0 \leq x \leq 7$$

or

$$x \in [0, 7].$$

Done ($\subseteq$).

($\supseteq$) Let $x \in [0, 7]$. (Goal: $x \in \bigcup_{n=1}^{7} A_n$)

Since $A_7 = [0, 7]$, ift

$$x \in A_7.$$

Since $x \in A_7$, ift

$$x \in \bigcup_{n=1}^{7} A_n$$

Done ($\supseteq$).

Since $x \in [0, 7]$ and $x \in \bigcup_{n=1}^{7} A_n$, ift

$$\bigcup_{n=1}^{7} A_n = [0, 7].$$

Done Proof.