Center Column Equalities in Elementary Cellular Automata

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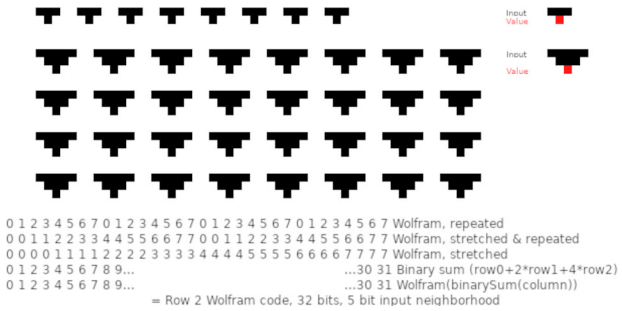
1 Introduction

Elementary cellular automata (ECA) are extensions of logic gate truth tables done linearly iteratively in parallel [2]. Any given center column of an input neighborhood surrounded by zeros has multiple alternate input neighborhoods of larger sizes producing the same center column with a predictable phase offset. The brute force method of attempting all possible neighborhoods for equivalent center columns is trimmed to ??? via Wolfram code extensions beyond row 1. Output for this algorithm is tested via brute force and several kinds of behavior for various classes of rules are explored. The algorithm is applied to Wolfram's prime number cellular automata [2] and all Java source code is available at [1]

2 Main Algorithm



<table>
<tr><td colspan="3">(outRow,sizeNeighborhood), xx is unused</td><td colspan="16">Stretched Wolfram for Row N, 2*N+1 neighborhood size</td></tr>
<tr><td rowspan="4">xx
xx
xx
xx
xx
xx
xx</td><td rowspan="2">xx
xx
xx</td><td rowspan="4">(3, 7)</td><td colspan="16">(index/(2^row)) mod WolframLength, 0 <= index < (2^maxRowNeighborhood)</td></tr>
<tr><td colspan="8">00000000..77777777</td><td colspan="8">00000000...77777777</td></tr>
<tr><td rowspan="3">(2, 5)</td><td colspan="4">0000..7777</td><td colspan="4">0000..7777</td><td colspan="4">0000..7777</td><td colspan="4">0000..7777</td></tr>
<tr><td colspan="2">00..77</td><td colspan="2">00..77</td><td colspan="2">00..77</td><td colspan="2">00..77</td><td colspan="2">00..77</td><td colspan="2">00..77</td><td colspan="2">00..77</td><td colspan="2">00..77</td></tr>
<tr><td>(1, 3)</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td><td>0..7</td></tr>
<tr><td colspan="3">Neighborhood sum</td><td colspan="16">Power sum of Wolfram[(column/(2^row)) mod Wolfram[1].length]</td></tr>
<tr><td colspan="3">Result</td><td colspan="16">Recursion to Wolfram[row-1,sum], base case row = 1</td></tr>
</table>

Initial	0
Pad	000
Pad	00000
Pad & Feedback into row - 1	0000000
Wolfram[feedbackNeighborhood]	00000
Pad & Feedback	0000000
Wolfram[feedbackNeighborhood]	00000
Pad & Feedback	0000000
Wolfram[feedbackNeighborhood]	00000
Repeat until there is a repeated Wolfram[feedbackNeighborhood]	

3 Properties

3.1 Row offset

3.2 Distribution within all possible neighborhoods

3.2.1 30, Class 4

3.2.2 XOR additive

3.3 ECA O(n)

3.4 Applied to Prime Automata

[2]

References

[1] Daniel McKinley. github.com/dmcki23/, 2024.

[2] Stephen Wolfram. *A New Kind of Science*. Wolfram Media, 2002.