Quiz #2

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Problem 1:

a.

$$\sum_{j=12}^{15} 52^{3j-7}$$
$$= 52^{3(12)-7} + 52^{3(13)-7} + 52^{3(14-7} + 52^{3(15)-7}$$
$$= 52^{36-7} + 52^{39-7} + 52^{42-7} + 52^{45-7}$$
$$= 52^{29} + 52^{32} + 52^{35} + 52^{38}$$

b.

$$\sum_{i=25}^{27} \frac{i^2}{(i-29)}$$
$$= \frac{25^2}{(25-29)} + \frac{26^2}{(26-29)} + \frac{27^2}{(27-29)}$$
$$= \frac{25^2}{(-4)} + \frac{26^2}{(-3)} + \frac{27^2}{(-2)}$$

c.

$$\sum_{i=4}^{6} \sum_{j=1}^{3} i(3j-2j)^{j-1}$$
$$= \sum_{i=4}^{6} i \sum_{j=1}^{3} (3j-2j)^{j-1}$$
$$= \sum_{i=4}^{6} i[(3(1)-2(1))^{1-1} + (3(2)-2(2))^{2-1} + (3(3)-2(3))^{3-1}]$$

$$= \sum_{i=4}^{6} i[(3-2)^0 + (6-4)^1 + (9-6)^2]$$
$$= \sum_{i=4}^{6} i(1^0 + 2^1 + 3^2)$$
$$= \sum_{i=4}^{6} i(1 + 2 + 9)$$
$$= \sum_{i=4}^{6} i(12)$$
$$= 12 \sum_{i=4}^{6} i$$
$$= 12(4 + 5 + 6)$$
$$= 12(15)$$
$$= 180$$

d.

$$\sum_{k=1}^{7} \sum_{j=0}^{39} 4kj$$
$$= \sum_{k=1}^{7} 4k \sum_{j=0}^{39} j$$
$$= 4 \sum_{k=1}^{7} k \sum_{j=0}^{39} j$$
$$= 4 \sum_{k=1}^{7} k \frac{39(39+1)}{2}$$
$$= (4) \frac{39(39+1)}{2} \sum_{k=1}^{7} k$$
$$= (4) \frac{39(39+1)}{2} \frac{7(7+1)}{2}$$
$$= (2)(39)(40)(7)(8)$$
$$= 174720$$

e.

$$\sum_{j=1}^{4}\sum_{i=0}^{j} 20i$$

$$= \sum_{j=1}^{4} 20 \sum_{i=0}^{j} i$$

$$= \sum_{j=1}^{4} 20\frac{j(j+1)}{2}$$

$$= \frac{20}{2}\sum_{j=1}^{4} j^2 + j$$

$$= 10(\sum_{j=1}^{4} j^2 + \sum_{j=1}^{4} j)$$

$$= 10[\frac{4(4+1)[2(4)+1]}{2} + \frac{4(4+1)}{2}]$$

$$= 10[\frac{4(5)(8+1)+4(5)}{2}]$$

$$= 5[4(5)(9)+4(5)]$$
$$= 5[180+20]$$
$$= 5[200]$$
$$= 1000$$

f.

$$\sum_{k\in S}(5+\sum_{j\in S} j), S = \{3,7,6,14,15\}$$

$$= (5+\sum_{j\in S} j)\sum_{k\in S} 1$$

$$= [5+(3+7+6+14+15)]\sum_{k\in S} 1$$

$$= (5+45)\sum_{k=0}^{5} 1$$

$$= 50(5)$$
$$= 250$$

g.

$$\sum_{j=51}^{119} j - \sum_{i=0}^{1274} i$$

$$= (\sum_{j=0}^{119} j - \sum_{j=0}^{50} j) - \sum_{i=0}^{1274} i$$

$$= \frac{119(119+1)}{2} - \frac{50(50+1)}{2} - \frac{1274(1274+1)}{2}$$

$$= \frac{119(120)}{2} - \frac{50(51)}{2} - \frac{1274(1275)}{2}$$

$$= \frac{14280}{2} - \frac{2550}{2} - \frac{1624350}{2}$$

$$= \frac{(-1612620)}{2}$$

$$= (-806310)$$

Problem 2:

a.

```
for (k= 0; k ≤ 2n 3+5n+7; k++){
    print “Hello”;}
for (j = 0; j ≤ 7n 3+13; j++){
    print “Hello”;}
for (i = 1; i ≤ (3n-8)(n+2); i++){
    print “Hello”;}
```

$$\sum_{k=0}^{2n^3+5n+7+1} 1 + \sum_{j=0}^{7n^3+13+1} 1 + \sum_{i=0}^{(3n-8)(n+2)} 1$$

$$= 2n^3 + 5n + 8 + 7n^3 + 14 + (3n-8)(n+2)$$

$$= 9n^3 + 5n + 22 + (3n^2 + 6n - 8n - 16)$$

$$= 9n^3 + 3n^2 + 5n - 2n + 22 - 16$$

$$= 9n^3 + 3n^2 + 3n + 6$$

b.

```
k = 0;
while (k ≤ n){
    for (i = 0; i ≤ k-1; i++){
        print “hello”;
        print “hello”;
        print “hello”;}
    k = 5k;
    print “hello”;
    print “hello”;}
```

$$\sum_{k=0}^{\log_5 n} 2 \sum_{i=0}^{k-1} 3$$
$$= (2)(3) \sum_{k=0}^{\log_5 n} 1 \sum_{i=0}^{k-1} 1$$
$$= 6(\sum_{k=0}^{\log_5 n} k - 1)$$
$$= 6(\sum_{k=0}^{\log_5 n} k - \sum_{k=0}^{\log_5 n} 1)$$

$$= 6(\frac{\log_5 n(\log_5 n + 1)}{2} - \log_5 n)$$
$$= 6(\frac{(\log_5 n)^2 + \log_5 n)}{2} - \log_5 n)$$
$$= 3(\log_5 n)^2 + 3\log_5 n - \log_5 n$$
$$= 3(\log_5 n)^2 + 2\log_5 n$$

c.

```
for (i = 1; i ≤ 7n+5; i++){
    for (j = 0; j ≤ i-1; j++){
        for (k = 0; k ≤ 3; k++){
            print “Hello”;}}}
```

$$\sum_{i=1}^{7n+5} \sum_{j=0}^{i-1+1} \sum_{k=0}^{3+1} 1$$
$$= \sum_{i=1}^{7n+5} \sum_{j=0}^{i-1+1} 4$$

$$= \sum_{i=1}^{7n+5} 4 \sum_{j=0}^{i-1+1} 1$$

$$= \sum_{i=1}^{7n+5} 4i$$

$$= 4 \sum_{i=1}^{7n+5} i$$

$$= 4\frac{(7n+5)(7n+5+1)}{2}$$

$$= 2(7n+5)(7n+6)$$
$$= 2(49n^2 + 42n + 35n + 30)$$
$$= 98n^2 + 77n + 60$$

d.

```
for (i = 0; i ≤ n; i = i + 9){
    for (k = 0; k ≤ n; k++){
        print “Hello”;}
    print “Hello”;
    print “Hello”;
    for (j = 1; j ≤ n; j++){
        print “Hello”;
        print “Hello”;
        print “Hello”;}}
```

$$\sum_{i=0}^{n/9} 2(\sum_{k=0}^{n} 1 + \sum_{j=1}^{n} 3)$$

$$= 2\sum_{i=0}^{n/9} 1(n + 3n)$$

$$= 2(n + 3n)\sum_{i=0}^{n/9} 1$$

$$= (2n + 6n)(n/9)$$

$$= \frac{8n^2}{9}$$

e.

```
for (j = 0; j ≤ 7n-3; j++){
    for (i = 1, i ≤ n; i = 5i){
        print “Hello”;
        print “Hello”;
        print “Hello”;
        for (k = 0; k ≤ log! 16; k++){
            print “Hello”;
            print “Hello”;}}}
```

$$\begin{aligned}
&\sum_{j=0}^{7n-3+1} \sum_{i=1}^{\log_5 n} 3 \sum_{k=0}^{\log_2 16} 2 \\
&= \sum_{j=0}^{7n-3+1} 3 \sum_{i=1}^{\log_5 n} 1(2) \sum_{k=0}^{4} 1 \\
&= \sum_{j=0}^{7n-3+1} 3 \sum_{i=1}^{\log_5 n} (2)(4) \\
&= \sum_{j=0}^{7n-3+1} 3(8) \sum_{i=1}^{\log_5 n} 1 \\
&= 24 \sum_{j=0}^{7n-2} \sum_{i=1}^{\log_5 n} 1 \\
&= 24 \sum_{j=0}^{7n-2} \log_5 n \\
&= 24 \log_5 n \sum_{j=0}^{7n-2} 1 \\
&= 24(\log_5 n)(7n-2)
\end{aligned}$$

f.

```
for (k = 1; k ≤ 7n3; k = 4k){
    print “Hello”;
    for (j = 1; j ≤ n; j = j + 4){
        print “Hello”;}
    print “Hello”;}
```

$$\begin{aligned}
&\sum_{k=1}^{\log_4 7n^3} 2 \sum_{j=1}^{n/4} 1 \\
&= 2 \sum_{k=1}^{\log_4 7n^3} n/4 \\
&= (n/2) \log_4 7n^3
\end{aligned}$$