



Homework #2

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COMP 310

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CFGs

1. $\{a^i b^j c^k : i + j = 2k\}$

$$S \rightarrow aScc \mid A \mid \lambda$$

$$A \rightarrow bAcc \mid \lambda$$

2. $\{w \in \{a, b\}^* : w \text{ contains } abaab\}$

$$S \rightarrow AabaabA$$

$$A \rightarrow aA \mid bA \mid \lambda$$

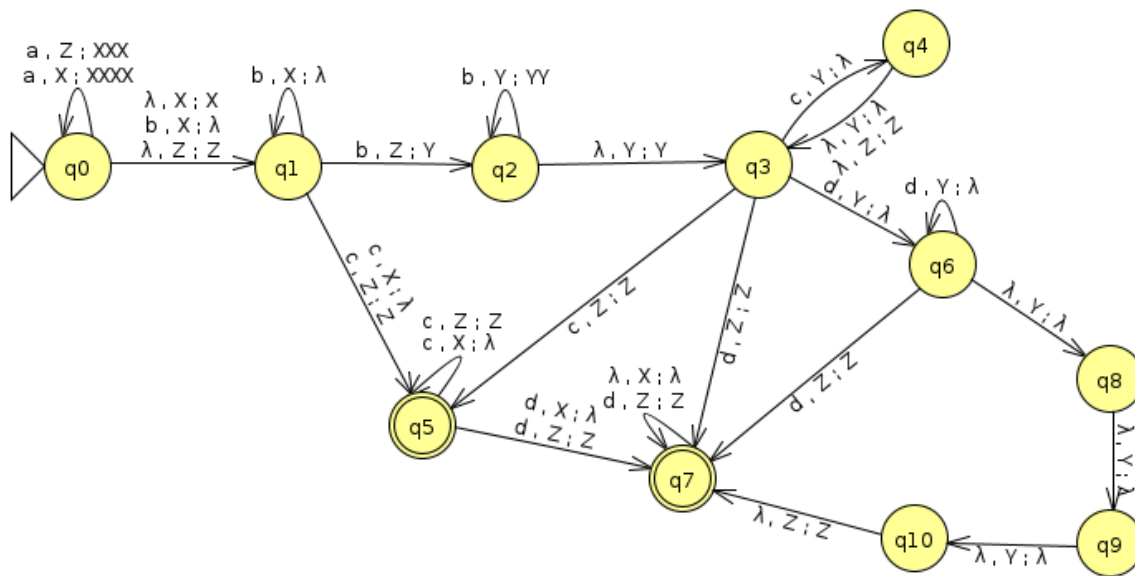
3. $\{a^i b^j : 2i < j + 2 < 3i\}$

$$S \rightarrow aaAbbb$$

$$A \rightarrow aAbb \mid aAbbb \mid \lambda$$

NPDAs

4. $\{a^i b^j c^k d^l : 3i + 2k > j - l - 3\}$



5. $\{w \in \{a, b, c\}^* : 2n_a + 3n_b = n_c + 2\}$

Reductions

6. Useless (unreachable or non-terminating) productions are crossed out:

$$\begin{aligned}
 S &\rightarrow \cancel{CaBF} \mid AA \mid \cancel{EAB} \\
 A &\rightarrow \cancel{CaE} \mid \cancel{CabE} \mid aB \\
 B &\rightarrow DbDb \mid aA \mid aS \mid a \\
 C &\rightarrow \cancel{AaA} \mid \cancel{BbB} \mid \cancel{a} \mid \cancel{b} \\
 D &\rightarrow AA \mid SS \mid a \\
 E &\rightarrow \cancel{EE} \mid \cancel{Fa} \mid \cancel{AaBF} \\
 F &\rightarrow \cancel{Ea} \mid \cancel{Fa} \mid \cancel{EF}
 \end{aligned}$$

Because productions E and F do not include any terminating variables and mostly recurse either between or upon themselves, any production which uses E and F variables is non-terminating and can be eliminated.

7.

$$\begin{aligned}
S &\rightarrow Aa \mid aA \mid Bb \mid bB \mid a \mid b \\
A &\rightarrow Sa \mid B \mid aS \\
B &\rightarrow B \mid Ba \mid aB \mid Ca \mid a \\
C &\rightarrow A \mid AA \mid B \mid AB \mid BA \mid a
\end{aligned}$$

8.

$$\begin{aligned}
S &\rightarrow SS \mid Aa \mid aAA \mid Sa \mid AaS \mid ab \\
A &\rightarrow AA \mid bA \mid b
\end{aligned}$$

Chomsky Normal Form

9.

$$\begin{aligned}
C &\rightarrow BB \mid AF \mid NE \\
S &\rightarrow BB \mid AF \mid NE \\
A &\rightarrow BI \mid AJ \mid OG \\
B &\rightarrow OO \mid O \mid OM \\
D &\rightarrow NO \\
E &\rightarrow SD \\
F &\rightarrow OO \\
G &\rightarrow ND \\
H &\rightarrow BB \\
I &\rightarrow AH \\
J &\rightarrow NA \\
K &\rightarrow BN \\
L &\rightarrow NK \\
M &\rightarrow AL \\
N &\rightarrow a \\
O &\rightarrow b
\end{aligned}$$

CYK Algorithm

10.

abaaba:

a	b	a	a	b	a
S, A	B, C	S, A	S, A	B, C	S, A
null	B, E	S, A, D, E	null	B, E	
D	S, B, E	null	D		
S, A, C, D	A, C	C, D			
S, B, D, E	S, A, B, C, E				
S, A, B, C, D, E, C					

aaba:

a	a	b	a
S, A	S, A	B, C	S, A
S, A, D, E	null	B, E	
null	D		
C, D			

baab:

b	a	a	b
B, C	S, A	S, A	B, C
B, E	S, A, D, E	null	
S, B, E	null		
A, C			

Proofs of non-CF

11. $\{w \in \{a^i, b^j, c^k\}^* : i = 2j = 3k\}$

Let $w = a^p b^{p/2} c^{p/3}$ be a string in the language of length at least p . We can write $w = xyz$ where $x = a^p$, $y = b^{p/2}$, and $z = c^{p/3}$.

Now let's consider the string $xy^2z = a^p (b^{p/2})^2 c^{p/3}$. This string is not in the language because it violates the condition $i = 2j = 3k$. Therefore, the language is not context-free by the pumping lemma.

12. $\{a^{n!}\}$

Let $w = a^{p!}$ be a string in the language of length at least p . We can write $w = xyz$ where $x = a^k$, $y = a^l$, and $z = a^{p!-k-l}$ for some k, l such that $0 \leq k, l \leq p$ and $k + l \leq p$.

Now let's consider the string $xy^2z = a^k (a^l)^2 a^{p!-k-l}$. This string is not in the language because it has fewer than $p!$ a 's. Therefore, the language is not context-free by the pumping lemma.