CS440 : Introduction to Artificial Intelligence **Spring 2014**

Problem Set 3

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1. **Probability**

(a) Suppose:

$$D = \left\{ \begin{array}{ll} d_1 & \text{if disease affects you} \\ d_2 & \text{if disease doesn't affect you} \end{array} \right\}$$

$$T = \left\{ \begin{array}{ll} t_1 & \text{test was correct} \\ t_2 & \text{test was a false positive} \end{array} \right\}$$

Then:

$$P(D = d_1) = \frac{1}{8000}$$

$$P(T|D) = 0.97$$

$$P(\neg T|\neg D) = 0.97 \rightarrow P(T|\neg D) = 1 - P(\neg|\neg D) = 0.03$$

$$P(D = d_1|T = t_1) = \alpha \left\langle 0.97(0.000125), (1 - 0.97)(1 - \frac{1}{8000} \right\rangle$$

$$P(D = d_1|T = t_1) = \alpha \left\langle 0.00012125, 0.02999625 \right\rangle \cdot \frac{1}{0.00012125 + 0.02999625}$$

$$P(D = d_1|T = t_1) = \alpha \left\langle 0.00402589856395783, 0.995974101436042 \right\rangle$$

$$P(D = d_1|T = t_1) = 0.00402589856395783$$

No, you shouldn't have anything to worry about.

(b) Suppose:
We have a random variable M:

$$M = \left\{ \begin{array}{ll} r & \text{if the marble is red} \\ b & \text{if the marble is blue} \end{array} \right\}$$

Given by the information we know that:

$$P(M = r) = 0.30$$

$$P(M = b) = 0.70$$

We know that we have a uniform distribution since $\Sigma_{m \in M} P(M = m) \leq 1$.
Therefore:

$$P(M = r \wedge M = r) = P(M = r)P(M = r) = (0.3)(0.3) = 0.09$$

$$P(M = b \wedge M = b) = P(M = b)P(M = b) = (0.7)(0.7) = 0.49$$

$$P(M = r \wedge M = b) = P(M = r)P(M = b) = (0.3)(0.7) = 0.21$$

$$P(M = b \wedge M = r) = P(M = r)P(M = b) = (0.3)(0.7) = 0.21$$

Suppose you select one of your two marbles at random. What is the probability that it is blue?

$$P(M = b|M = r \wedge M = b) = 0.21$$
$$P(M = b|M = b \wedge M = b) = 0.49$$
$$P(M = b) = 0.7$$

2. **Bayesian Networks**

Special Exhibits:

$$E = \left\{ \begin{array}{ll} g & \text{if Gourmet Food} \\ a & \text{if art on loan} \\ e & \text{if empty} \end{array} \right\}$$

Thieves:

$$T = \left\{ \begin{array}{ll} t & \text{if will steal} \\ f & \text{if won't steal} \end{array} \right\}$$

Weather:

$$W = \left\{ \begin{array}{ll} h & \text{if hot} \\ c & \text{if cold} \\ n & \text{if neutral} \end{array} \right\}$$

Alarm:

$$A = \left\{ \begin{array}{ll} t & \text{if alarm sounded} \\ f & \text{if alarm didn't sound} \end{array} \right\}$$

Rats:

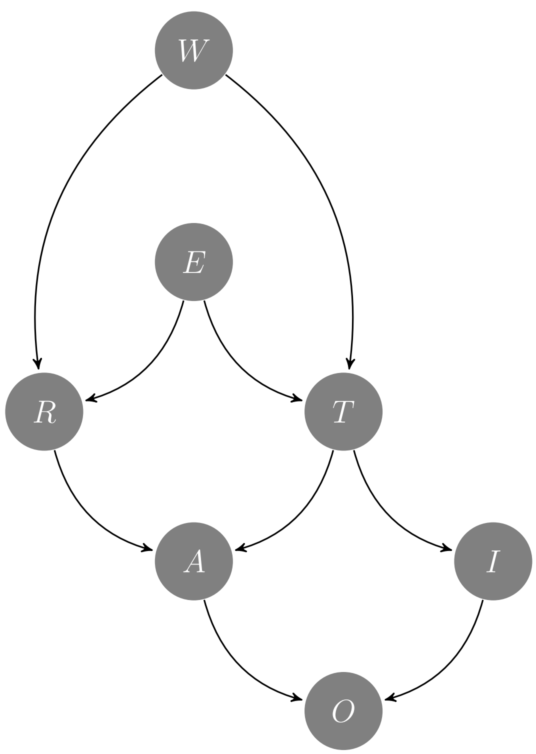
$$R = \left\{ \begin{array}{ll} t & \text{if rats} \\ f & \text{if no rats} \end{array} \right\}$$

Insurance claim:

$$I = \left\{ \begin{array}{ll} t & \text{if insurance claim} \\ f & \text{if no insurance claim} \end{array} \right\}$$

Overtime:

$$O = \left\{ \begin{array}{ll} t & \text{if overtime} \\ f & \text{if no overtime} \end{array} \right\}$$



(a)

(b) This structure is a polytree since it is a directed, acyclic graph. This is because for every vertex there is no way to get back to vertex v.

(c) 287 parameters are needed for the full joint probability.

(d) Baye's net parameters:

W	$P(W)$
h	(a)
c	(b)
n	(c)

E	$P(E)$
g	(d)
a	(e)
e	(f)

E	W	$P(R\|W,E)$
g	h	(g)
g	e	(h)
g	n	(i)
a	h	(j)
a	e	(k)
a	n	(l)
e	h	(m)
e	e	(n)
e	n	(o)

E	W	$P(T\|W,E)$
g	h	(p)
g	e	(q)
g	n	(r)
a	h	(s)
a	e	(t)
a	n	(u)
e	h	(v)
e	e	(w)
e	n	(x)

R	T	$P(A\|R,T)$
t	t	(y)
t	f	(z)
f	t	(aa)
f	f	(ab)

T	$P(I\|T)$
t	(ac)
f	(ad)

A	I	$P(O\|A,I)$
t	t	(ae)
t	f	(af)
f	t	(ag)
f	f	(ah)

34 parameters required for the baye's net
Savings $\rightarrow$ 287 - 34 = 253 less parameters.

(e) Find the markov blankets for each of the 7 variables:
MB(W) = { R, T, E }
MB(E) = { R, T, W }
MB(R) = { W, E, A, T }
MB(T) = { W, E, A, I, R }
MB(A) = { R, T, O, I }
MB(I) = { T, O, A }
MB(O) = { A, I }

(f) No, the alarm going off is not conditionally independent of the insurance claim being submitted, given that there are rats.

This is a result of the random variable, R, having no influence on the random variable, I. The only influencing variable for I is whether or not the thieves did damage.

3. **Probabilities in Bayesian Networks**

(a) $P(A) = 0.00972 + 0.00972 + 0.00054 + 0.00144 + 0.08748 + 0.00648 + 0.00486 + 0.00096 + 0.01458 + 0.00648 + 0.00216 + 0.00036 + 0.13122 + 0.00432 + 0.01944 +$

0.00024
$P(A) = 0.3$

$P(\neg A) = 0.01008 + 0.13608 + 0.00056 + 0.2016 + 0.09072 + 0.09072 + 0.00504 + 0.01344 + 0.01512 + 0.09072 + 0.00224 + 0.00504 + 0.13608 + 0.06048 + 0.02016 + 0.00336$
$p(\neg A) = 0.7$

$P(B) = 0.00972 + 0.00972 + 0.08748 + 0.00648 + 0.01458 + 0.00648 + 0.13122 + 0.00432 + 0.01008 + 0.13608 + 0.09072 + 0.09072 + 0.01512 + 0.09072 + 0.13608 + 0.06048$
$P(B) = 0.9$

$P(C|A) = 0.00972 + 0.08748 + 0.01458 + 0.13122 + 0.00054 + 0.00486 + 0.00216 + 0.1944$
$P(C|A) = 0.27$

$P(C|\neg A) = 0.01008 + 0.09072 + 0.01512 + 0.13608 + 0.00056 + 0.00504 + 0.00224 + 0.02016$
$P(C|\neg A) = 0.28$

$P(D|B, C) = 0.00972 + 0.08748 + 0.01008 + 0.09072$
$P(D|B, C) = 0.198$

$P(D|B, \neg C) = 0.00972 + 0.00648 + 0.13608 + 0.09072$
$P(D|B, \neg C) = 0.243$

$P(D|\neg B, C) = 0.00054 + 0.00486 + 0.00056 + 0.00504$
$P(D|\neg B, C) = 0.011$

$P(D|\neg B, \neg C) = 0.00144 + 0.00096 + 0.02016 + 0.01344$
$P(D|\neg B, \neg C) = 0.036$

$p(E|C) = 0.00972 + 0.00054 + 0.01008 + 0.00056 + 0.01458 + 0.00216 + 0.01512 + 0.00224$

$P(E|C) = 0.055$

$P(E|\neg C) = 0.00972 + 0.00144 + 0.13608 + 0.02016 + 0.00648 + 0.00036 + 0.09072 + 0.00504$
$P(E|\neg C) = 0.27$

(b) Express P(D — B) using the probabilities you calculated

$$\begin{aligned} P(D|B) &= \Sigma_{a\in A}\Sigma_{c\in C}P(A,B,C,D) = \Sigma_{a\in A}\Sigma_{c\in C}P(A)P(B)P(C|A)P(D|B,C) \\ &= P(B)\Sigma_{a\in A}p(A)\Sigma_{c\in C}P(C|A)P(D|B,C) \\ P(D|B) &= P(B)\{P(a)\left[P(c|a)P(D|B,c) + P(\neg c|a)P(D|B,\neg c)\right] \\ &\quad +P(\neg a)\left[P(c|\neg a)P(D|B,c) + P(\neg c|\neg a)P(D|B,\neg c)\right]\} \end{aligned}$$

(c) Use the calculated values from part (a) to obtain the value of P(D — B).

$$\begin{aligned} P(D|B) &= (0.9)\{(0.3)[(0.27)(0.198) + (0.03)(0.243)] + (0.7)[(0.28)(0.198) + (0.42)(0.243)]\} \\ &= (0.9)[(0.3)(0.06075) + (0.7)(0.1575)] \\ P(D|B) &= 0.1156275 \end{aligned}$$

(d) Compute the following probabilities

i. $P(E|A)$

$$\begin{aligned} P(E|A) &= \alpha\Sigma_{c\in C}P(A,E,C) = \alpha\Sigma_{c\in C}P(E|C)P(C|A)P(A) \\ &= \alpha P(A)\Sigma_{c\in C}(P(E|C)P(C|A) \\ &= P(A)[P(E|c)P(c|A) + P(E|\neg c)P(\neg c|A)] \\ P(E|A) &= 0.006885 \end{aligned}$$

ii. $P(A|B)$

$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$

$$
\begin{aligned}
P(D|B) &= \Sigma_{a\in A}\Sigma_{c\in C}P(A,B,C,D)\\
&= \Sigma_{a\in A}\Sigma_{c\in C}P(A)P(B)P(C|A)P(D|B,C)\\
&= P(A)P(B)\Sigma_{c\in C}\Sigma_{d\in D}P(C|A)P(D|B,C)\\
&= P(A)P(B)[P(c|A)P(d|B,c)+P(c|A)P(\neg d|B,c)\\
&\quad +P(\neg c|A)P(d|B,\neg c)+P(\neg c|A)P(\neg d|B,\neg c)]\\
&= (0.3)(0.9)[(0.27)(0.198)+(0.27)(0.43308)+(0.03)(0.243)+(0.03)(0.162)]\\
&= (0.3)(0.9)(0.1825416)\\
P(D|B) &= 0.049286232
\end{aligned}
$$

iii. $P(B,D|C)$

$$P(B,D|C) = \frac{P(C|B,D)P(B,D)}{P(C)} = \frac{P(C|B,D)P(B)P(D|B)}{P(C)}$$

$$
\begin{aligned}
P(C) &= \alpha\Sigma_{a\in A}P(A,C)\\
&= \alpha\Sigma_{a\in A}P(A)P(C|A)\\
&= P(a)P(C|a)+P(\neg a)P(C|\neg a)\\
&= (0.3)(0.27)+(0.7)(0.28)\\
&= 0.277
\end{aligned}
$$

$$P(D|B) = 0.441$$

$$
\begin{aligned}
P(C|D,B) &= \Sigma_{a\in A}\Sigma_{c\in C}P(A,B,C,D)\\
&= \Sigma_{a\in A}P(A)P(B)P(C|A)P(D|B,C)\\
&= P(B)P(D|B,C)\Sigma_{a\in A}P(A)P(C|A)\\
&= P(B)P(D|B,C)[P(a)P(C|a)+P(\neg a)P(C|\neg a)\\
&= P(B)P(D|B,C)P(C)\\
P(C|D,B) &= (0.9)(0.198)(0.277) = 0.0493614
\end{aligned}
$$

$$P(B,D|C) = \frac{P(C|B,D)P(B)P(D|B)}{P(C)} = \frac{(0.0493614)(0.9)(0.441)}{0.277}$$

$$P(B, D|C) = 0.07072758$$

iv. $P(C|D, E)$

$$P(C|D,E) = \frac{P(D,E|C)P(C)}{P(D,E)} = \frac{P(D,E|C)P(C)}{P(D)P(E)}$$

$$\begin{aligned}
P(D,E|C) &= \Sigma_{b\in B}\Sigma_{a\in A}P(A,B,C,D,E) \\
&= \Sigma_{b\in B}\Sigma_{a\in A}P(A)P(B)P(C|A)P(D|B,C)P(E|C) \\
&= P(E|C)\Sigma_{b\in B}P(B)P(D|B,C)\Sigma_{a\in A}P(A)P(C|A) \\
&= P(E|C)\{P(b)P(D|b,C)[P(a)P(C|a) + P(\neg a)P(C|\neg a)] \\
&\quad +P(\neg b)P(D|\neg b,C)[P(a)P(C|a) + P(\neg a)P(C|\neg a)]\} \\
&= (0.055)\{(0.9)(0.198)[(0.3)(0.27) + (0.7)(0.28)] + (0.1)(0.011)[(0.3)(0.27) + (\\
&= (0.055)[(0.9)(0.198)(0.227) + (0.1)(0.011)(0.277)] \\
&= (0.055)(0.0496661) \\
P(D,E|C) &= 0.0027316355
\end{aligned}$$

$$\begin{aligned}
P(D) &= \Sigma_{a\in A}\Sigma_{b\in B}\Sigma_{c\in C}P(A,B,C,D) \\
&= \Sigma_{a\in A}\Sigma_{b\in B}\Sigma_{c\in C}P(A)P(B)P(C|A)P(D|B,C) \\
&= \Sigma_{a\in A}P(A)\Sigma_{c\in C}P(C|A)\Sigma_{b\in B}P(D|B,C)P(B) \\
&= P(a)\{P(c|a)[P(D|b,c)P(b) + P(D|\neg b,c)P(\neg b)] \\
&\quad\ +P(\neg|a)[P(D|b,\neg c)P(b) + P(D|\neg b,\neg c)P(\neg b)]\} \\
&\quad +P(\neg a)\{P(c|\neg a)[P(D|b,c)P(b) + P(D|\neg b,c)P(\neg b)] \\
&\quad\ +P(\neg|\neg a)[P(D|b,\neg c)P(b) + P(D|\neg b,\neg c)P(\neg b)]\} \\
&= (0.3)[(0.27)(0.1793) + (0.03)(0.2223)] + (0.7)[(0.28)(0.1793) + (0.42)(0.2223)] \\
P(D) &= 0.117023
\end{aligned}$$

$$
\begin{aligned}
P(E) &= \Sigma_{c \in C} \Sigma_{a \in A} P(A, C, E) \\
&= \Sigma_{c \in C} \Sigma_{a \in A} P(A) P(C|A) P(E|C) \\
&= P(E|c)[P(a)P(c|a) + P(\neg a)P(c|\neg a)] \\
&\quad + P(E|\neg c)[P(a)P(\neg c|a) + P(\neg a)P(\neg c|\neg a)] \\
&= (0.055)[(0.3)(0.27) + (0.7)(0.28)] + (0.27)[(0.3)(0.03) + (0.7)(0.42)] \\
P(E) &= 0.097045
\end{aligned}
$$

$$
\begin{aligned}
P(C|D, E) &= \tfrac{P(D,E|C)P(C)}{P(D)P(E)} \\
&= \tfrac{(0.0027316355)(0.277)}{0.117023)(0.097045)} \\
P(C|D, E) &\approx 0.0666282068465314
\end{aligned}
$$