CS421 : hw8 **Summer 2015**

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1. show: $(\lambda y.xy)(\lambda x.\lambda y.yx) \overset{\alpha}{\sim} (\lambda z.xz)(\lambda y.\lambda x.xy)$

   Proof:
   $\lambda y.xy \overset{\alpha}{\to} \lambda z.xz$ so:
   $\lambda y.xy \overset{\alpha}{\to} \lambda z.xz$ so:
   $(\lambda y.xy)(\lambda x.\lambda y.yx) \overset{\alpha}{\sim} (\lambda z.xz)(\lambda x.\lambda x.\lambda y.yx)$

   $\lambda x.\lambda y.yx \overset{\alpha}{\to} \lambda w.\lambda y.yw$ so:
   $\lambda x.\lambda y.yx \overset{\alpha}{\sim} \lambda w.\lambda y.yw$ so:
   $(\lambda z.xz)(\lambda x.\lambda x.\lambda y.yx) \overset{\alpha}{\sim} (\lambda z.xz)(\lambda w.\lambda y.yw)$

   $\lambda w.\lambda y.yw \overset{\alpha}{\to} \lambda w.\lambda x.xw$ so:
   $\lambda w.\lambda y.yw \overset{\alpha}{\sim} \lambda w.\lambda x.xw$ so:
   $(\lambda z.xz)(\lambda w.\lambda y.yw) \overset{\alpha}{\sim} (\lambda z.xz)(\lambda w.\lambda x.xw)$

   $\lambda w.\lambda x.xw \overset{\alpha}{\to} \lambda y.\lambda x.xy$ so:
   $\lambda w.\lambda x.xw \overset{\alpha}{\sim} \lambda y.\lambda x.xy$ so:
   $(\lambda z.xz)(\lambda w.\lambda x.xw) \overset{\alpha}{\sim} (\lambda z.xz)(\lambda y.\lambda x.xy)$

   Given the congruences above we know that:

   $$(\lambda y.xy)(\lambda x.\lambda y.yx) \overset{\alpha}{\sim} (\lambda z.xz)(\lambda x.\lambda x.\lambda y.yx) \quad (1)$$
   $$\overset{\alpha}{\sim} (\lambda z.xz)(\lambda w.\lambda y.yw) \quad (2)$$
   $$\overset{\alpha}{\sim} (\lambda z.xz)(\lambda w.\lambda x.xw) \quad (3)$$
   $$\overset{\alpha}{\sim} (\lambda z.xz)(\lambda y.\lambda x.xy) \quad (4)$$

   By the transitive property we therefore know:

   $$(\lambda y.xy)(\lambda x.\lambda y.yx) \overset{\alpha}{\sim} (\lambda z.xz)(\lambda y.\lambda x.xy)$$

   ■

2. Evaluate $(\lambda x.x(\lambda y.xy))((\lambda u.u)(\lambda w.w))$

   (a) Eager Evaluation

   $$(\lambda x.x(\lambda y.xy))((\lambda u.u)(\lambda w.w)) \to (\lambda x.x(\lambda y.xy))(\lambda w.w) \quad (5)$$
   $$\to (\lambda w.w)(\lambda y.(\lambda w.w)y) \quad (6)$$
   $$\to (\lambda y.(\lambda w.w)y) \quad (7)$$
   $$\to (\lambda y.y) \quad (8)$$

(b) Lazy Evaluation

$$(\lambda x.x(\lambda y.xy))((\lambda u.u)(\lambda w.w)) \rightarrow ((\lambda u.u)(\lambda w.w))(\lambda y.xy) \quad (9)$$
$$\rightarrow (\lambda w.w)(\lambda y.xy) \quad (10)$$
$$\rightarrow (\lambda y.xy) \quad (11)$$

(c) unrestricted $\alpha\beta$-reduction

$$(\lambda x.x(\lambda y.xy))((\lambda u.u)(\lambda w.w)) \xrightarrow{\beta} (\lambda x.x(\lambda y.xy))(\lambda w.w) \quad (12)$$
$$\xrightarrow{\beta} (\lambda w.w)(\lambda y.(\lambda w.w)y) \quad (13)$$
$$\xrightarrow{\beta} (\lambda y.(\lambda w.w)y) \quad (14)$$
$$\xrightarrow{\beta} (\lambda y.y) \quad (15)$$