CS421 : hw6 **Summer 2015**

Dan McQuillan *Handed In: July 20, 2015*

1 Theoretical Questions

1.1 Problem 1

In both cases:
let $\Sigma = \{a, b, c\}$

1.1.1 The language where a occurs in every third position

((a ∨ b ∨ c) (a ∨ b ∨ c) a)* (((a ∨ b ∨ c) (a ∨ b ∨ c) a) ∨ ((a ∨ b ∨ c) (a ∨ b ∨ c)) ∨ (a ∨ b ∨ c))?

1.1.2 The language where each string contains exactly 3 c's

(a ∨ b)* c (a ∨ b)* c (a ∨ b)* c (a ∨ b)*

1.2 Problem 2

1. $L_1 = \{w|w$ starts with a symbol 0 and contains the symbol 1 at least once$\}$ where $\Sigma = \{0, 1\}$

   (a) Regular expression
   0 (0 ∨ 1)* 1 (0 ∨ 1)*

   (b) Regular grammar
   $< Language\_1 >::= 0 < ZeroOrOne >$
   $< ZeroOrOne >::= 0 < ZeroOrOne >$
   $< ZeroOrOne >::= 1 < ZeroOrOneOrEmpty >$
   $< ZeroOrOneOrEmpty >::= 0 < ZeroOrOneOrEmpty >$
   $< ZeroOrOneOrEmpty >::= 1 < ZeroOrOneOrEmpty >$
   $< ZeroOrOneOrEmpty >::= \epsilon$

2. $L_2 = \{w|w$ contains an equal number of 0s and 1s$\}$ where $\Sigma = \{0, 1\}$

   Proof: By contradiction; assume the language L_2 is regular.

   Let n be the length guaranteed by the pumping lemma. Suppose we have a string $w = 0^n1^n$. Then $|w| = 2n \geq n$ and $w \in L_2$.

   Therefore, there exists strings x, y, and z such that w = xyz, $|xy| \leq n, y \neq \epsilon$ and for any number i, $xy^iz \in L_2$.

   Since, $|xy| \leq n$ y must consist of only 0s. However $xy^2z = 0^{n+|y|}1^n$, and since $|y| > 0$, we have that $xy^2z \notin L_2$

Therefore, we have a contradiction and our language is not regular. ■

3. $L_3 = \{w|$ the length of w is odd$\}$ where $\Sigma = \{a, b\}$

   (a) Regular expression
   (a ∨ b) ((a ∨ b) (a ∨ b))*

   (b) Regular grammar
   $< Language\_3 >::= a < AOrBOrEmpty >$
   $< Language\_3 >::= b < AOrBOrEmpty >$
   $< AOrBOrEmpty >::= b < Language\_3 >$
   $< AOrBOrEmpty >::= a < AOrC >$
   $< AOrBOrEmpty >::= \epsilon$

4. $L_4 = \{w|w$ does not contain symbol a immediately followed by symbol b$\}$ where $\Sigma = \{a, b, c\}$

   (a) Regular expression
   ((a*c) ∨ b ∨ c)* a*

   (b) Regular grammar
   $< Language\_4 >::= a < AOrC >$
   $< Language\_4 >::= b < Language\_4 >$
   $< Language\_4 >::= c < Language\_4 >$
   $< Language\_4 >::= \epsilon$
   $< AOrC >::= c < Language\_4 >$
   $< AOrC >::= a < AOrC >$
   $< AOrC >::= \epsilon$

5. $L_5 = \{w|$ the length of w is a perfect cube $\}$ where $\Sigma = \{a, b, c\}$

   Proof: By contradiction; assume the language L_5 is regular.

   Let n be the length guaranteed by the pumping lemma.

   Suppose $w = a^{\frac{n^3}{3}} b^{\frac{n^3}{3}} c^{\frac{n^3}{3}}$.

   Since $|w| = \frac{n^3}{3} + \frac{n^3}{3} + \frac{n^3}{3} = n^3$, which is a perfect cube, $w \in L_5$.

   By the pumping lemma we know that we can split $w = xyz$ s.t. the conditions of the pumping lemma hold.

   We know that:

   $$1 \le |y| \le |xy| \le n$$

   Since for the pumping lemma to hold we also require that any amount of y terms in the middle to still let the expression hold. Therefore we know also that:

   $$xy^2z \in L_5$$

Therefore we may assume that $|xy^2z|$ is a perfect cube. However we know that:

$$\begin{aligned}
n^3 &= |w| && (1)\\
&= |xyz| && (2)\\
&< |xy^2z| && (3)\\
&\leq n^3 + n \qquad \text{since, } |y| \leq n && (4)\\
&< n^3 + 3n^2 + 3n + 1 && (5)
\end{aligned}$$

In summary, we now know that:

$$n^3 < |xy^2z| < n^3 + 3n^2 + 3n + 1$$

That is $|xy^2z|$ lies between two subsequent perfect cubes. Therefore, it cannot be a perfect cube itself, and hence we have a contradiction to $xyyz \in L_5$. ■