1 Theoretical Questions (25 points)

Problem 1: (25 points)

Give a most general unifier for the following set of equations (unification problem). The uppercase letters A, B, C, and D denote variables of unification. The lowercase letters f, g, and h are term constructors of arity 2, 3, and 1 respectively (*i.e.* take two, three or one argument(s), respectively). Show all your work by listing the operations performed in each step of the unification and the result of that step.

$$\mathsf{Unify}\{(f(A, g(B, C, h(D))) = f(g(C, B, C), A))\}$$

Solution:

Given
$\quad \mathsf{Unify}\{(f(A, g(B, C, h(D))) = f(g(C, B, C), A))\}$
by Decompose $(f(A, g(B, C, h(D))) = f(g(C, B, C), A))$
$\quad = \mathsf{Unify}\{(A = g(C, B, C)); (g(B, C, h(D)) = A)\}$
by Eliminate $(A = g(C, B, C))$
$\quad = \mathsf{Unify}\{(g(B, C, h(D)) = g(C, B, C))\} \circ \{A \mapsto g(C, B, C)\}$
by Decompose $(g(B, C, h(D)) = g(C, B, C))$
$\quad = \mathsf{Unify}\{(B = C); (C = B); (h(D) = C)\} \circ \{A \mapsto g(C, B, C)\}$
by Eliminate $(B = C)$
$\quad = \mathsf{Unify}\{(C = C); (h(D) = C)\} \circ \{B \mapsto C\} \circ \{A \mapsto g(C, B, C)\}$
$\quad = \mathsf{Unify}\{(C = C); (h(D) = C)\} \circ \{A \mapsto g(C, C, C); B \mapsto C\}$
by Delete $(C = C)$
$\quad = \mathsf{Unify}\{(h(D) = C)\} \circ \{A \mapsto g(C, C, C); B \mapsto C\}$
by Orient $(h(D) = C)$
$\quad = \mathsf{Unify}\{(C = h(D))\} \circ \{A \mapsto g(C, C, C); B \mapsto C\}$
by Eliminate $(C = h(D))$
$\quad = \mathsf{Unify}\{\} \circ \{C \mapsto h(D)\} \circ \{A \mapsto g(C, C, C); B \mapsto C\}$
$\quad = \{A \mapsto g(h(D), h(D), h(D)); B \mapsto h(D); C \mapsto h(D)\}$