CS421 : hw5	Summer 2015
Dan McQuillan	*Handed In: July 11, 2015*

1 Problems

1.1 Problem 1

Give a most general unifier for the following set of equations (unification problem). The uppercase letters A, B, C, and D denote variables of unification. The lowercase letters f, g, and h are term constructors of arity 2, 3, and 1 respectively (i.e. take two, three or one argument(s), respectively). Show all your work by listing the operations performed in each step of the unification and the result of that step.

$$\text{Unify}\{(f(A, g(B, C, h(D))) = f(g(C, B, C), A))\}$$

1.1.1 Steps to solution

1. Pick a pair: $(f(A, g(B, C, h(D))) = f(g(C, B, C), A))$

   Decompose:
   becomes: $\{A = g(C, B, C); g(B, C, h(D)) = A\}$
   $= \text{Unify}\{A = g(C, B, C); g(B, C, h(D)) = A\}$

2. Pick a pair: $(g(B, C, h(D)) = A)$

   Orient:
   $= \text{Unify}\{A = g(C, B, C); A = g(B, C, h(D))\}$

3. Pick a pair: $(A = g(C, B, C)$

   Eliminate: A with substitution $\{A \to g(C, B, C)\}$
   $= \text{Unify}\{g(C, B, C) = g(B, C, h(D))\}$ o $\{A \to g(C, B, C)\}$

4. Pick a pair: $(g(C, B, C) = g(B, C, h(D)))$

   Decompose:
   becomes: $\{C = B; B = C; C = h(D)\}$
   $= \text{Unify}\{C = B; B = C; C = h(D)\}$ o $\{A \to g(C, B, C)\}$

5. Pick a pair: $(B = C)$

   Orient:
   $= \text{Unify}\{C = B; C = B; C = h(D)\}$ o $\{A \to g(C, B, C)\}$

6. Pick a pair: $(C = B)$

   6.1 Eliminate: C with substitution $\{C \to B\}$
   $= \text{Unify}\{B = B; B = h(D)\}$ o $\{C \to B\}$ o $\{A \to g(C, B, C)\}$

6.2 Compose Substitutions:
$= \text{Unify}\{B = B; B = h(D)\} \text{ o } \{C \to B; A \to g(B, B, B)\}$

7. Pick a pair: $(B = B)$

   Delete
   $= \text{Unify}\{B = h(D)\} \text{ o } \{C \to B; A \to g(B, B, B)\}$

8. Pick a pair: $(B = h(D))$

   8.1. Eliminate: B with substitution $\{B \to h(D)\}$
   $= \text{Unify}\{\} \text{ o } \{B \to h(D)\} \text{ o } \{C \to B; A \to g(B, B, B)\}$

   8.2. Compose Substitutions:
   $= \text{Unify}\{\} \text{ o } \{B \to h(D); C \to h(D); A \to g(h(D), h(D), h(D))\}$

9. Unify is evaluating identity substitution
   $= \{B \to h(D); C \to h(D); A \to g(h(D), h(D), h(D))\}$

1.1.2 Solution

$\text{Unify}\{(f(A, g(B, C, h(D))) = f(g(C, B, C), A))\} = \{B \to h(D); C \to h(D); A \to g(h(D), h(D), h(D))\}$

1.1.3 Check Solution

$$f(A, g(B, C, h(D))) = f(g(C, B, C), A) \tag{1}$$

$$\to f(g(h(D), h(D), h(D)), g(h(D), h(D), h(D))) = f(g(h(D), h(D), h(D)), g(h(D), h(D), h(D))) \tag{2}$$

By applying a simultaneous substitution we verify that the equality holds when the substitution is applied to the constraint in the original unification problem.