ECE 438 Digital Signal Processing
Week 9: Discrete-Time Random Processes (Week 1)

Date ______________________

Section ___

Name		Sign	Time spent outside lab
David Dang	[%]	David Dang	7 hrs.
Benedict Lee	[%]	Benedict Lee	7 hrs.

Grading Rubric (Spring 2020)

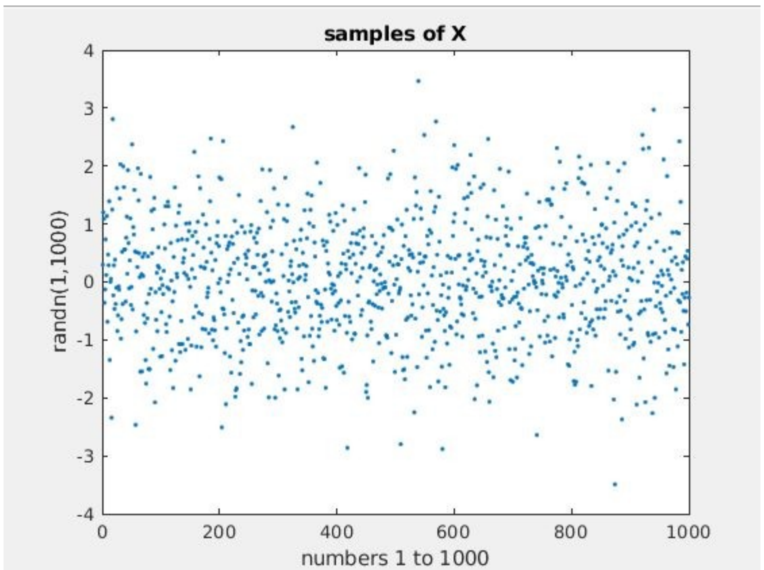
	below expectations	lacks in some respect	meets all expectations
Completeness of the report			
Organization of the report *One-sided, with cover sheet, answers are in the same order as questions in the lab, copies of the questions*			
Quality of figures *Correctly labeled with title, x-axis, y-axis, and name(s)*			
Understanding of random variables and linear transformations (35 pts) *Plots, sample means and variances of X and Y, derivation of mean and variance of Y, transformation and pdf of Y, matlab code, questions*			
Understanding of CDF estimation (20 pts) *Matlab code and plots*			
Understanding of generating samples from a given distribution (20 pts) *Derivation of transformation, matlab code, plots*			
Understanding of PDF estimation (25 pts) *Plots, questions*			

ECE 438 Lab 7a
David Dang & Benedict Lee

2.2 Samples of a Random Variable

INLAB REPORT:

1. Submit the plot of samples of X.
2. Submit the sample mean and the sample variance that you calculated. Why are they not equal to the true mean and true variance?



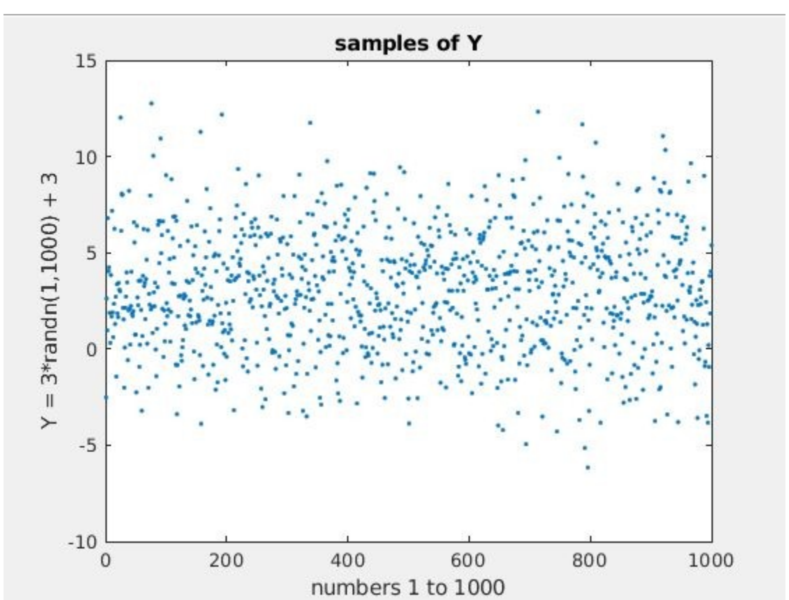

```
>> randMeanVar

m =

    0.0383


v =

    1.0110
```

- The mean and variance don't match the true mean and variance because they represent only a sample of X

2.3 Linear Transformation of a Random Variable

INLAB REPORT:

1. Submit your derivation of the mean and variance of Y.
2. Submit the transformation you used, and the probability density function for Y.
3. Submit the plot of samples of Y and the Matlab code used to generate Y. Include the calculated sample mean and sample variance for Y.




```
>> pdfOfY

m2 =

    3.0424


v2 =

    9.3656
```

```
m = 3;
x = 1:1000;
v = 9;
X = randn(1,1000);
Y = 3*X + 3;
pdfY = (1/(sqrt(2*pi)*sqrt(v))) * exp((-1/(2*v))*((Y - m).^2));
plot(Y, '.')
title('samples of Y')
xlabel('numbers 1 to 1000')
ylabel('Y = 3*randn(1,1000) + 3')
[m2, v2] = meanvar(Y)
```

$Y = aX + b$

$\mu_Y = E[Y] = aE[X] + b$

$\sigma_Y^2 = E[Y^2] - (E[Y])^2 = a^2 \underbrace{E[X^2]} + 2ab\,E[X] + b^2 - (a\mu_x + b)^2$

$\sigma_x^2 = E[x^2] - (E[x])^2$

$\sigma_x^2 + \mu_x^2 = E[x^2]$

$\sigma_Y^2 = a^2(\sigma_x^2 + \cancel{\mu_x^2}) \cancel{+2ab\mu_x} \cancel{+b^2} \cancel{-a^2\mu_x^2} \cancel{-2ab\mu_x} \cancel{-b^2}$

$\boxed{\sigma_Y^2 = a^2 \sigma_x^2}$

$\boxed{\mu_Y = a\mu_x + b}$

w/ $m=3$ & $v=9$,

$3 = a\mu_x + b$

$9 = a^2 \sigma_x^2$

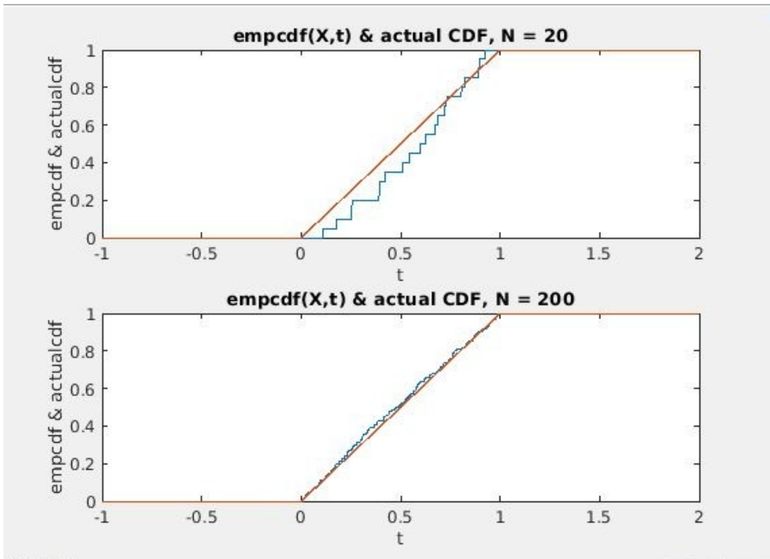
$\mu_x = 0$, $\sigma_x^2 = 1$, $\boxed{a = 3,\ b = 3}$

Pdf of Y

$$\boxed{f_Y(Y) = \frac{1}{\sqrt{2\pi}\,\sigma_y} \exp\left(-\frac{1}{2\sigma_y^2}(Y - \mu_y)^2\right),\quad Y = 3X + 3}$$

3.1 Exercise

INLAB REPORT: Hand in your empcdf function and the two plots.

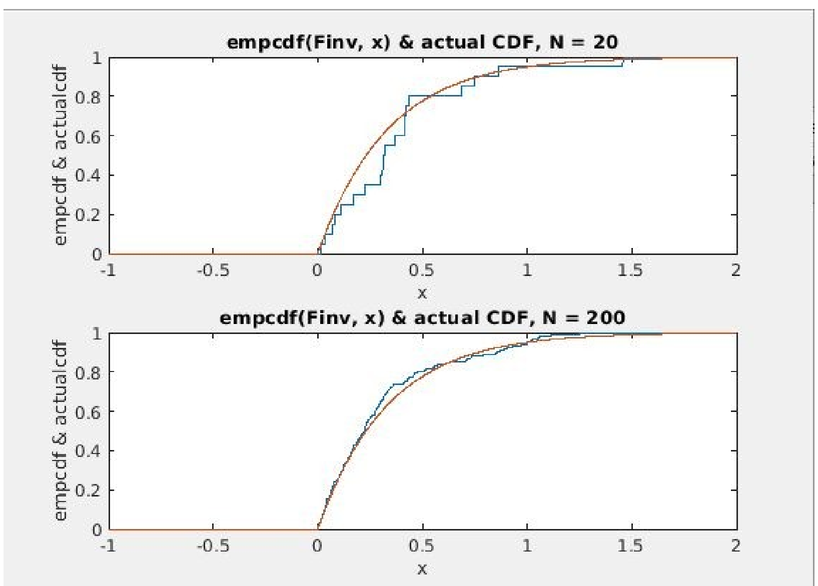



```
function [F] = empcdf(X, t)
%UNTITLED4 Summary of this function goes here
%   Detailed explanation goes here

N = length(X);
F = zeros(N,1);
for i = 1:length(t)
    F(i) = (1/N)*sum(X<=t(i));
end

end
```

4.1 Exercise

INLAB REPORT:

- Hand in the derivation of the required transformation, and your Matlab code.
- Hand in the two plots.




```
subplot(2,1,1)
x = [-1:0.001:2];
N = 20;
U = rand(1,N);
truedist = (1 - exp(-3*x)).* (x>=0);
Finv = (log(-U+1) / -3);
X = empcdf(Finv, x);
plot(x,X)
hold on
plot(x,truedist)
hold off
title('empcdf(Finv, x) & actual CDF, N = 20')
xlabel('x')
ylabel('empcdf & actualcdf')

subplot(2,1,2)
N = 200;
U = rand(1,N);
Finv = (log(-U+1) / -3);
X = empcdf(Finv, x);
plot(x,X)
hold on
truedist = (1 - exp(-3*x)).*(x>=0);
plot(x, truedist)
hold off
title('empcdf(Finv, x) & actual CDF, N = 200')
xlabel('x')
ylabel('empcdf & actualcdf')
```

$$F_x(x) = (1 - e^{-3x})u(x)$$

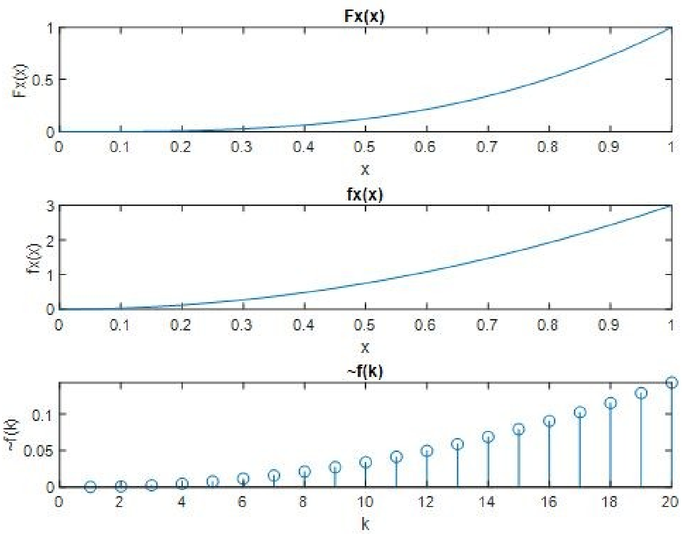
$$X = (1 - e^{-3F_x(x)})$$

$$\frac{\ln(-X+1)}{-3} = F_x(x)$$

5.2 Exercise

INLAB REPORT:

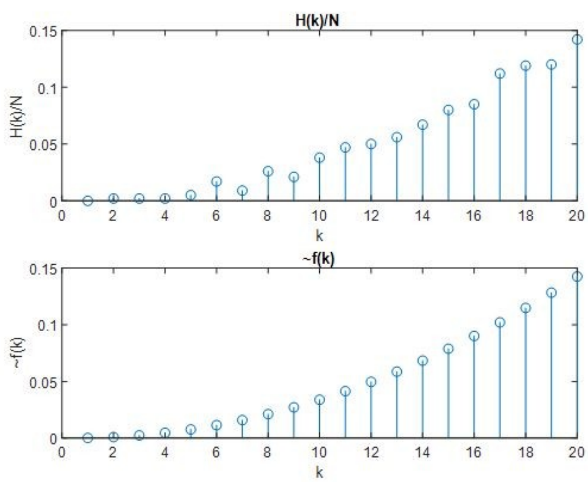
1. Submit your plots of FX(x), fX(x) and ˜f(k). Use stem to plot ˜f(k), and put all three plots on a single figure using subplot.
2. Show (mathematically) how fX(x) and ˜f(k) are related.



- ~f(k) = fx(x)*((xL - x0)/L)
  ~f(k) = fx(x)*(1/20)

INLAB REPORT:

1. Submit your two stem plots of H(k)/N and ˜f(k). How do these plots compare?
2. Discuss the tradeoffs (advantages and the disadvantages) between selecting a very large or very small bin-width.



- The two stem plots are pretty similar, except H(k)/N has lower frequencies for some bins than ~f(k)
- When a large bin width is used, it is more convenient since many points will fall into a few number of bins, but it doesn't give good information about the variability in the data. When a small bin width is used, you get more detail about the data, but then the underlying pattern in the data may be suppressed, especially when the number of bins is large relative to the number of data points.