ECE 438 Lab 5

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3 Design of a Simple FIR Filter

INLAB REPORT: Submit the difference equation, system diagram, and the analytical expression of the impulse response for the filter Hf(z). Also submit the plot of the magnitude response for the three values of θ. Explain how the value of θ affects the magnitude of the filter's frequency response.

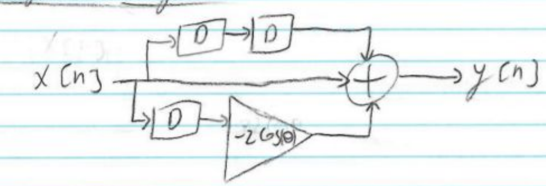
Difference equation

$$H_f(z) = 1 - 2\cos(\theta)z^{-1} + z^{-2}$$

$$Y(z) = X(z)[1 - 2\cos(\theta)z^{-1} + z^{-2}]$$

$$\boxed{Y[n] = x[n] - 2\cos(\theta)x[n-1] + x[n-2]}$$

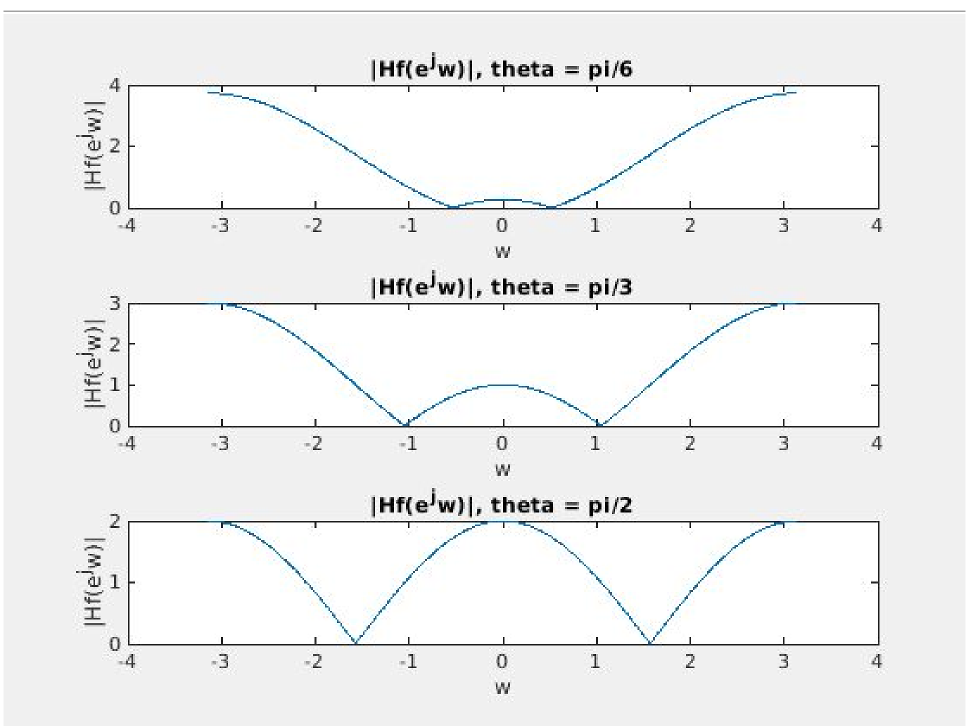
System diagram



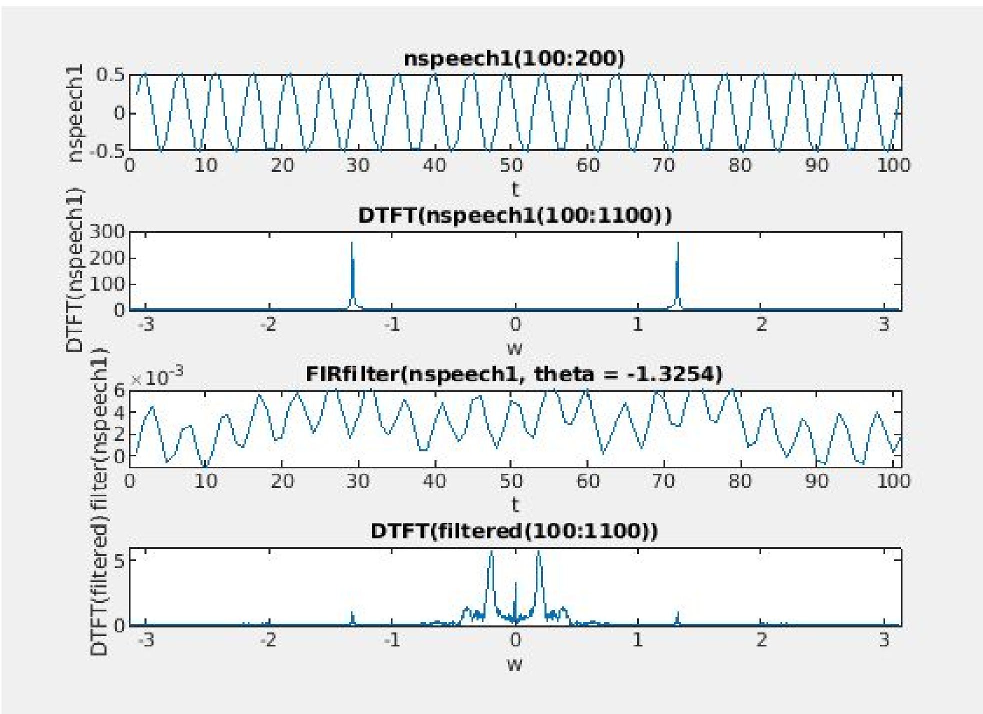
impulse response

$$\boxed{h[n] = \delta[n] - 2\cos(\theta)\delta[n-1] + \delta[n-2]}$$

- As theta increases from pi/6 to pi/2, the magnitude of the filter's frequency response also increases. This is because the -2cos(theta)x[n-1] term shrunk as theta changed from pi/6 to pi/2



INLAB REPORT: For both the original audio signal and the filtered output, hand in the following:
•The time domain plot of 101 samples.
•The plot of the magnitude of the DTFT for 1001 samples.
Also hand in the code for the FIRfilter filtering function. Comment on how the frequency content of the signal changed after filtering. Is the filter we used a lowpass, highpass,bandpass, or a bandstop filter? Comment on how the filtering changed the quality of the audio signal.



- After the signal was filtered, the frequencies beyond absolute value of 1 rad were removed. Hence, the filter is a lowpass filter. With the loud beep cut off from the signal, the speech is now audible.

```
function [ y ] = FIRfilter( x, theta )
%UNTITLED4 Summary of this function goes here
%   Detailed explanation goes here

y = zeros;
y(1) = x(1);
y(2) = x(2) - 2*cos(theta)*x(1);
y(3) = x(3) - 2*cos(theta)*x(2) + x(1);
for i = 4:length(x)
    y(i) = x(i) - 2*cos(theta)*x(i - 1) + x(i - 2);
end

end
```

4 Design of A Simple IIR Filter

INLAB REPORT: Submit the difference equation, system diagram and the analytical expression of the impulse response for Hi(z). (Hint: The frequency response of the system can be obtained by restrict-ing the z-transform to the unit circle. So the DTFT of hi[n] is Hi(ejw). Therefore, to get hi[n], you can take the inverse Fourier transform of Hi(ejw).) Also submit the plot of the magnitude of the frequency response for each value of r. Explain how the value of r affects this magnitude.

Difference equation

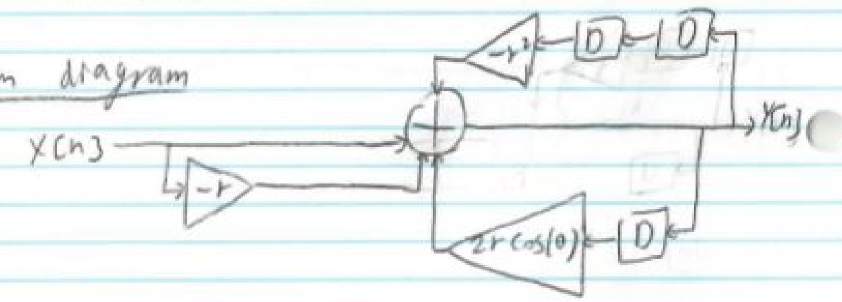
$$H_i(z) = \frac{1-r}{1-2r\cos(\theta)z^{-1}+r^2z^{-2}}$$

$$Y(z)\left(1-2r\cos(\theta)z^{-1}+r^2z^{-2}\right) = X(z)(1-r)$$

$$Y[n] - 2r\cos(\theta)Y[n-1] + r^2Y[n-2] = X[n] - rX[n]$$

$$\boxed{Y[n] = X[n] - rX[n] + 2r\cos(\theta)Y[n-1] - r^2Y[n-2]}$$

System diagram



impulse response

① $$H_i(z) = \frac{1-r}{(1-re^{j\theta}z^{-1})(1-re^{-j\theta}z^{-1})} = \frac{A}{(1-re^{j\theta}z^{-1})} + \frac{B}{(1-re^{-j\theta}z^{-1})}$$

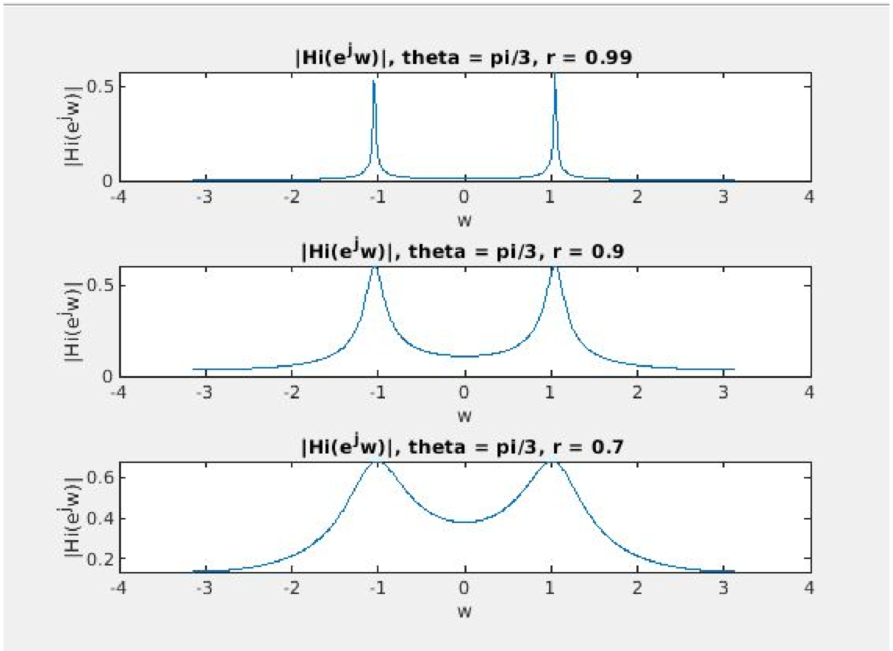
② $$1-r = A - Are^{-j\theta}z^{-1} + B - Bre^{j\theta}z^{-1}$$

③ $$-Are^{-j\theta} - Bre^{j\theta} = 0, \quad A+B = 1-r$$

④ $$A = -Be^{2j\theta}, \quad -Be^{2j\theta} + B = 1-r, \quad B = \frac{1-r}{1-e^{2j\theta}}, \quad A = \left(\frac{-1+r}{1-e^{2j\theta}}\right)e^{2j\theta}$$

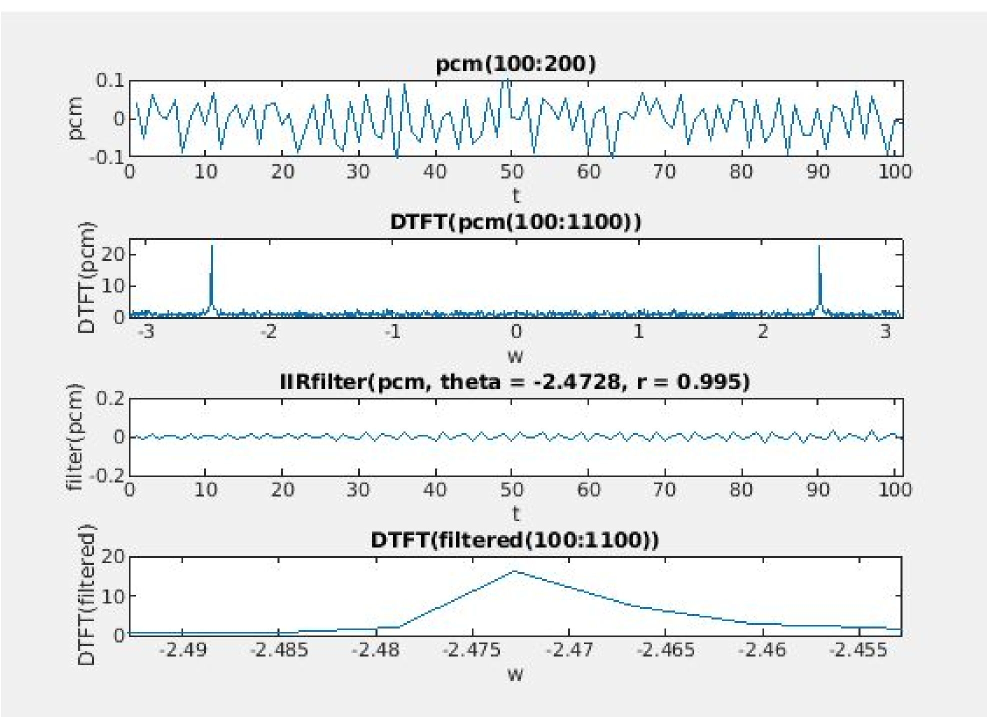
⑤ $$H_i(z) = \frac{\left(\frac{-1+r}{1-e^{2j\theta}}\right)e^{2j\theta}}{(1-re^{j\theta}z^{-1})} + \frac{\left(\frac{1-r}{1-e^{2j\theta}}\right)}{(1-re^{-j\theta}z^{-1})},$$

$$\boxed{h[n] = e^{2j\theta}\left(\frac{-1+r}{1-e^{2j\theta}}\right)\cdot\left(re^{j\theta}\right)^n u[n] + \left[\left(\frac{1-r}{1-e^{2j\theta}}\right)\cdot\left(re^{-j\theta}\right)^n u[n]\right]}$$



- As the value of r decreases, the depth of the trough of the plot between w = -1 & w = 1 at w = 0 becomes shallower.

INLAB REPORT: For both the pcm signal and the filtered output, submit the following:
•The time domain plot of the signal for 101 points.
•The plot of the magnitude of the DTFT computed from 1001 samples of the signal.
•The plot of the magnitude of the DTFT for ω in the range [θ−0.02, θ+ 0.02].
Also hand in the code for the IIRfilter filtering function. Comment on how the signal looks and sounds before and after filtering. How would you expect changes in r to change the filtered output? Would a value of r= 0.9999999 be effective for this application? Why might such a value for r be ill-advised? (Consider the spectrum of the desired signal around ω=θ.)



- Before filtering, the signal was noisy, and with the filter applied, the signal looks cleaner. Decreasing r below 0.995 keeps the noise in the signal. With r = 0.9999999, the frequency response of the filtered output decreases to 0, and the sound becomes less audible.

```
function [ y ] = IIRfilter( x, theta, r )
%UNTITLED4 Summary of this function goes here
%   Detailed explanation goes here

y = zeros;
y(1) = x(1) - r*x(1);
y(2) = x(2) - r*x(2) + 2*r*cos(theta)*y(1);
y(3) = x(3) - r*x(3) + 2*r*cos(theta)*y(2) - (r^2)*y(1);
for i = 4:length(x)
    y(i) = x(i) - r*x(i) + 2*r*cos(theta)*y(i - 1) - (r^2)*y(i - 2);
end

end
```

5.1 Filter Design Using Truncation

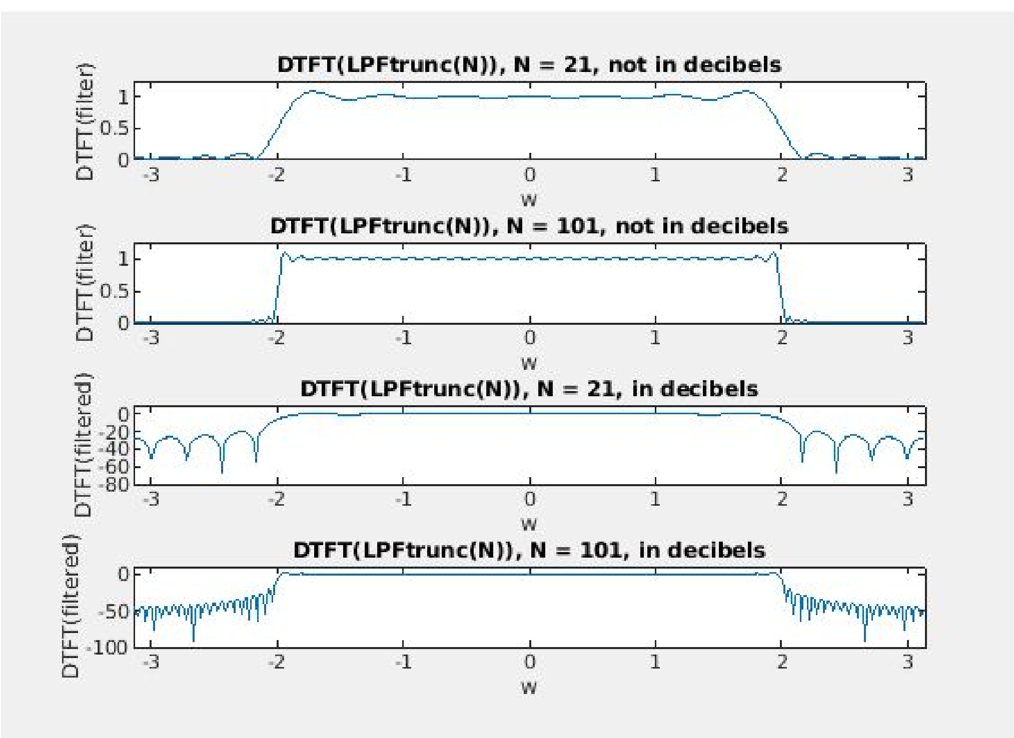
INLAB REPORT:

•Submit the plots of the magnitude response for the two filters(not in decibels). On each of the plots, mark the passband, the transition band and the stopband.

•Submit the plots of the magnitude response in decibels for the two filters.

•Explain how the filter size effects the stopband ripple. Why does it have this effect?

•Comment on the quality of the filtered signals. Does the filter size have a noticeable effect on the audio quality?



- The stopband ripple decreases as the filter size increases. This is because n = 0,1,...,N-1, and with more values of N, we get a better estimation of the filter with less ripple.
- As N increased, the background noise in the audio grew quieter, and the speech signal became clearer