ECE 438 Digital Signal Processing
Week 11: Discrete-Time Random Processes (Week 2)

Date 4/9/2020

Section 2

Name		Sign	Time spent outside lab
David Dang	[%]	David Dang	18
Benedict Lee	[%]	Benedict Lee	18

Grading Rubric (Spring 2020)

	below expectations	lacks in some respect	meets all expectations
Completeness of the report			
Organization of the report *One-sided, with cover sheet, answers are in the same order as questions in the lab, copies of the questions*			
Quality of figures *Correctly labeled with title, x-axis, y-axis, and name(s)*			
Understanding of correlation coefficient for 2 random variables (30 pts) *Derivation, numerical estimates, scatter plots, questions*			
Understanding of autocorrelation for filtered random processes (35 pts) *Derivation, scatter plots, autocorrelation plots, matlab code, questions*			
Understanding of correlation of two random processes (35 pts) *Cross-correlation plot, matlab code (CorR), signal plots, auto and cross correlation plots of signals, questions*			

ECE 438 Lab 7b
David Dang & Benedict Lee

<u>1.2 Samples of Two Random Variables</u>
INLAB REPORT:
1. Hand in your derivations of the correlation coefficient ρXZ along with your numerical estimates of the correlation coefficient ˆρXZ.
2. Why are ρXZ and ˆρXZ not exactly equal?
3. Hand in your scatter plots of (Xi, Zi) for the four cases. Note the theoretical correlation coefficient ρXZ on each plot.
4. Explain how the scatter plots are related to ρXZ.

1. $E[X]=\mu_X=0,\ E[Y]=\mu_Y=0,\ E[XY]=E[X]E[Y]=0,\ \sigma_X^2=1,\ \sigma_Y^2=1,\ \sigma_X=1,\ \sigma_Y=1$

$E[X^2]=E[(X-0)^2]=E[(X-\mu_X)^2]=\sigma_X^2=1$

Since X and Y are independent R.V.s, $Var(X+Y)=Var(X)+Var(Y)=1+1=2$

$Z=Y$: $\rho_{XZ}=\rho_{XY}=\dfrac{E[XY]-\mu_X\mu_Y}{\sigma_X\sigma_Y}=\dfrac{\mu_X\mu_Y-\mu_X\mu_Y}{1\cdot 1}=0$ //

$Z=(X+Y)/2$. $\sigma_Z^2=Var\left(\frac{X+Y}{2}\right)=\frac{1}{4}(Var(X+Y))=\frac{1}{2}\Rightarrow\sigma_Z=\frac{1}{\sqrt{2}}$

$\rho_{XZ}=\dfrac{E[XZ]-\mu_X\mu_Z}{\sigma_X\sigma_Z}=\dfrac{E\left[X\left(\frac{X+Y}{2}\right)\right]-0}{1\cdot\frac{1}{\sqrt{2}}}=\frac{\sqrt{2}}{2}E[X^2+XY]=\frac{1}{\sqrt{2}}(E(X^2)+E(XY))=\frac{1}{\sqrt{2}}(1+0)=\frac{1}{\sqrt{2}}$
$=0.7071$ (4 d.p.) //

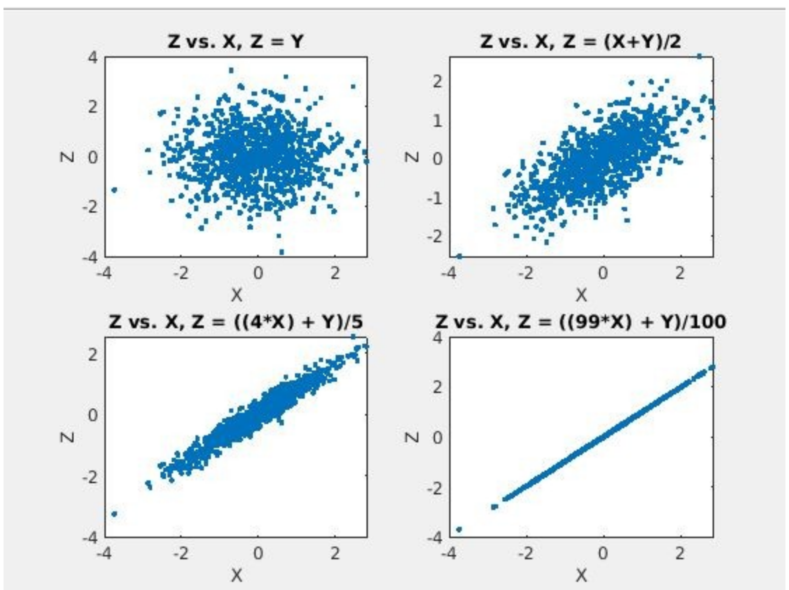
$Z=(4X+Y)/5$: $Var(Z)=\frac{1}{25}[16Var(X)+Var(Y)]=\frac{17}{25}\Rightarrow\sigma_Z=\frac{\sqrt{17}}{5}$

$\rho_{XZ}=\dfrac{E\left[\left(\frac{4X^2+XY}{5}\right)\right]}{1\cdot\frac{\sqrt{17}}{5}}=\dfrac{4E[X^2]+E[XY]}{\sqrt{17}}=\dfrac{4}{\sqrt{17}}=0.9701$ (4 d.p.) //

$Z=\left(\frac{99X+Y}{100}\right)$: $Var(Z)=\frac{1}{10000}[9801Var(X)+Var(Y)]=\frac{4901}{5000}\Rightarrow\sigma_Z=\sqrt{\frac{4901}{5000}}$

$\rho_{XZ}=\dfrac{E\left[X\left(\frac{99X+Y}{100}\right)\right]}{\sqrt{\frac{4901}{5000}}}=\dfrac{99E[X^2]+E[XY]}{100\sqrt{\frac{4901}{5000}}}=\dfrac{99}{100\sqrt{\frac{4901}{5000}}}=0.9999$ (4 d.p.) //

```
PxzEst1 =

   -0.0533

PxzEst2 =

    0.6730

PxzEst3 =

    0.9671

PxzEst4 =

    0.9999
```

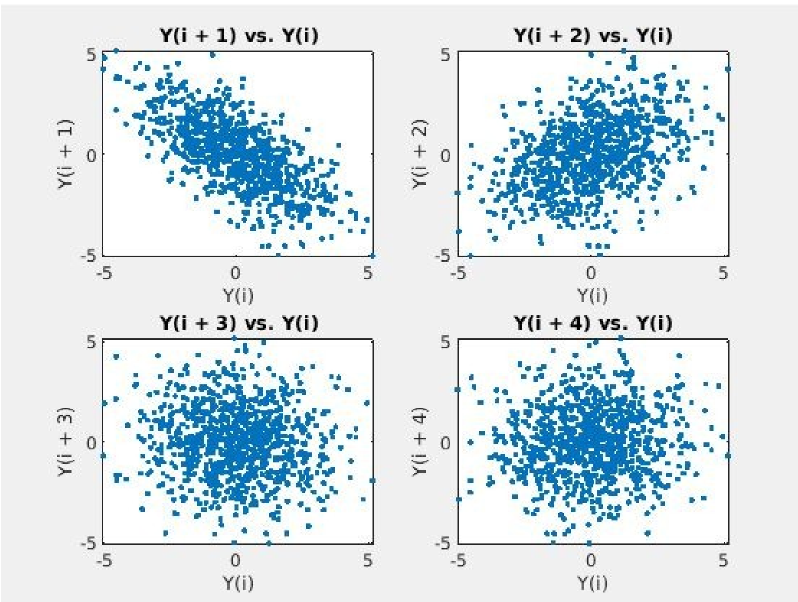


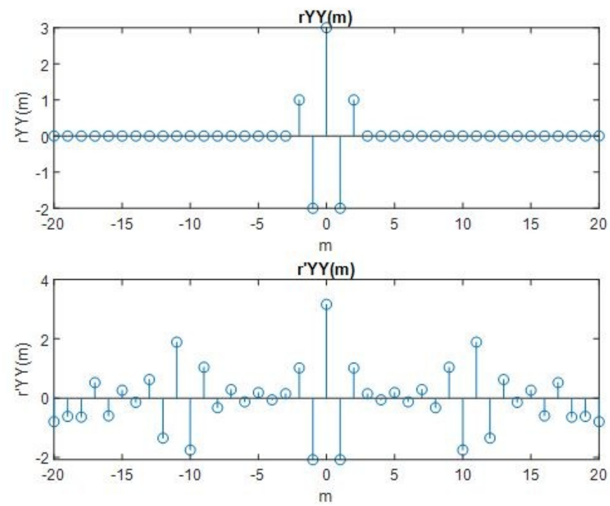
- ρXZ and ˆρXZ are not exactly equal because ˆρXZ only uses a finite number of samples of the random variables X and Z.
- As ρXZ increases, the plot of Z vs. X becomes more linear and the linear correlation between X and Z becomes more distinct and stronger.

2.2 Experiment

INLAB REPORT: For the filter in equation (12),

1. Show your derivation of the theoretical output autocorrelation, rY Y(m).
2. Hand in the four scatter plots. Label each plot with the corresponding theoretical correlation, using rY Y(m). What can you conclude about the output random process from these plots?
3. Hand in your plots of rY Y(m) and r′Y Y(m) versus m. Does equation (13) produce a reasonable approximation of the true autocorrelation? For what value of m does rY Y(m) reach its maximum? For what value of m does r′Y Y(m) reach its maximum?
4. Hand in your Matlab code.





$$y(m) = x(m) - x(m-1) + x(m-2)$$

$$h(m) = \delta(m) - \delta(m-1) + \delta(m-2) = \{1, -1, 1\}$$

$$h(-m) = \delta(-m) - \delta(-m-1) + \delta(-m-2) = \delta(m) - \delta(-(m+1)) + \delta(-(m+2)) = \delta(m) - \delta(m+1) + \delta(m+2)$$

$$r_{xx}(m) = \sigma_x^2 \delta(m) = \delta(m) = \{1\}$$

$$r_{xy}(m) = h(m) * r_{xx}(m) = r_{xx}(m) - r_{xx}(m-1) + r_{xx}(m-2) = \{1\} - \{1\} + \{1\} = \{1, -1, 1\}$$

$$r_{yy}(m) = h(m) * r_{xy}(-m) = r_{xy}(m) - r_{xy}(m+1) + r_{xy}(m+2) = \{1,-1,1\} - \{1,-1,1\} + \{1,-1,1\} = \{1, -2, \underset{m=0}{3}, -2, 1\}$$

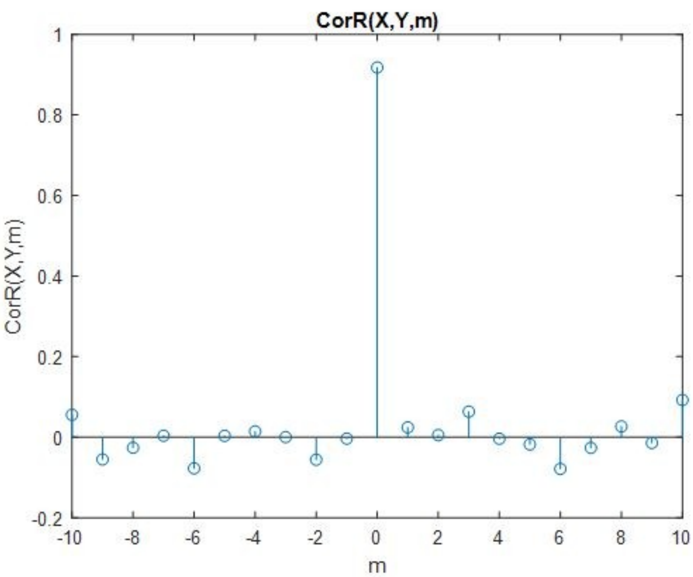
- rYY(0) and rYY(1) have opposite signs but relatively similar magnitudes, hence the 1st plot shows a relatively strong negative linear correlation between Y(i+1) and Y(i). rYY(0) and rYY(2) have the same sign but relatively dissimilar magnitudes, hence the 2nd plot shows a relatively weak positive linear correlation between Y(i+2) and Y(i). rYY(0) and rYY(3) as well as rYY(0) and rYY(4) have vastly different magnitudes, hence the 3rd and 4th plots show no correlation between Y(i+3) and Y(i) as well as Y(i+4) and Y(i) correspondingly; both plots are merely a randomly distributed collection of points.
- Equation (13) produces a reasonable approximation of the true autocorrelation. Both rYY(m) and r’YY(m) reach their maximum value of 3 and 3.1252 respectively at m = 0.

```
X0 = randn(1, 1000);
X1 = [0, X0(1:999)];
X2 = [0, 0, X0(1:998)];
Y = X0-X1+X2;
Y0 = Y(1:900);
Y1 = Y(2:901);
Y2 = Y(3:902);
Y3 = Y(4:903);
Y4 = Y(5:904);
figure(1)
subplot(2,2,1)
plot(Y0, Y1, '.')
title('Yi, Yi+1')
subplot(2,2,2)
plot(Y0, Y2, '.')
title('Yi, Yi+2')
subplot(2,2,3)
plot(Y0, Y3, '.')
title('Yi, Yi+3')
subplot(2,2,4)
plot(Y0, Y4, '.')
title('Yi, Yi+4')
m = -20:20;
rpYY = zeros(1, 41);
for i=1:41
    for x=1:(21-abs(m(i)))
        rpYY(i) = rpYY(i) + (1./(21-abs(m(i)))).*Y(x).*Y(x+abs(m(i)));
    end
end
rYY = zeros(1,41);
rYY(19) = 1;
rYY(20) = -2;
rYY(21) = 3;
rYY(22) = -2;
rYY(23) = 1;
figure(2)
subplot(2,1,1)
stem(m, rYY)
title('rYY(m)')
xlabel('m')
ylabel('rYY(m)')
subplot(2,1,2)
stem(m, rpYY)
title("r'YY(m)")
xlabel('m')
ylabel("r'YY(m)")
```

3.2 Experiment

INLAB REPORT:

1. Submit your plot for the cross-correlation between X and Y. Label the m-axis with the corresponding lag values.
2. Which value of m produces the largest cross-correlation? Why?
3. Is the cross-correlation function an even function of m? Why or why not?
4. Hand in the code for your CorR function.

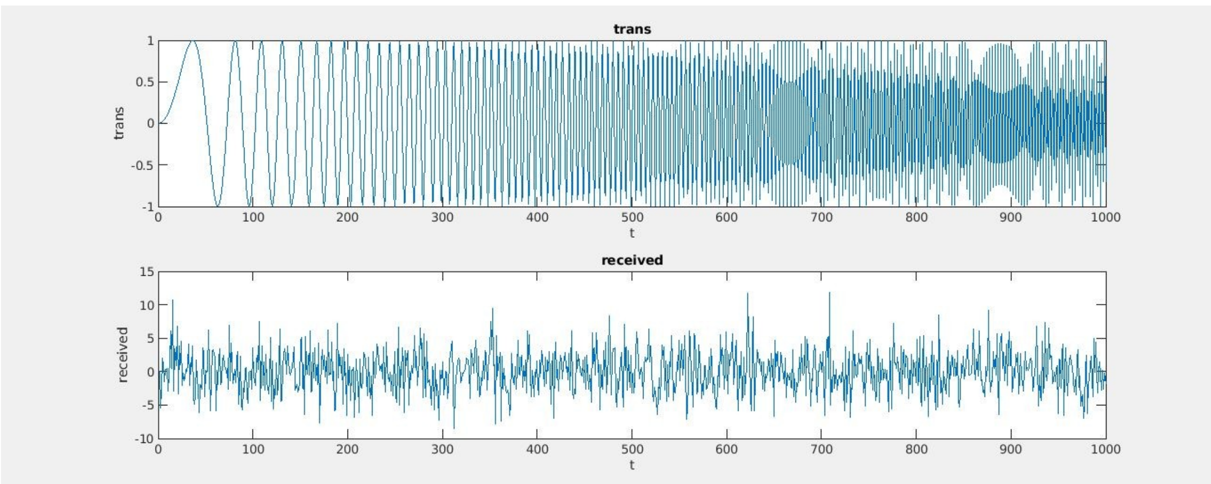



```
function C = CorR(X,Y,m)
N = length(X);
C = zeros(1,length(m));
for i=1:length(m)
   if m(i)>= 0
      for n=1:(N-m(i))
         C(i) = C(i) + (1/(N-m(i)))*X(n)*Y(n+m(i));
      end
   else
      for n=(abs(m(i))+1):N
         C(i) = C(i) + (1/(N-abs(m(i))))*X(n)*Y(n+m(i));
      end
   end
end
end
```

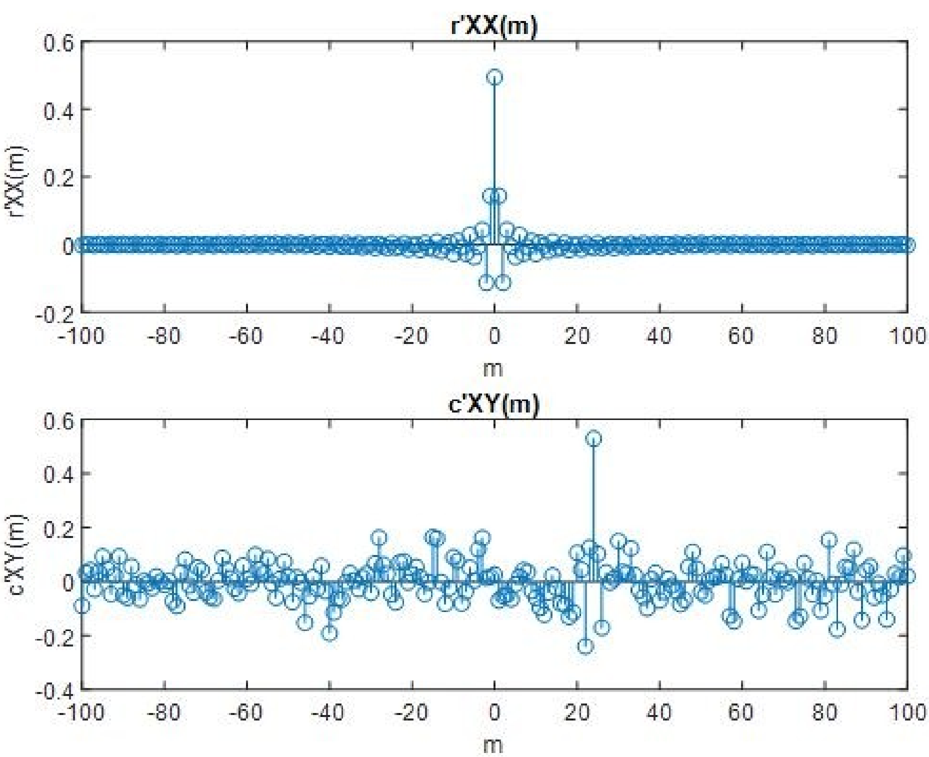
- m = 0 produces the largest cross-correlation, because the transmitted signal X(n) is designed such that rXX(m) has a large peak at m = 0. Equating m - D = m = 0, D = 0. Hence, cXY(0) = α*rXX(0), where the largest cross-correlation occurs at m = 0.
- The cross-correlation function is not an even function of m because it is not symmetrical about m = 0, also seen in the piecewise definition of the function over the 2 regions of m >= 0 and m < 0.

INLAB REPORT:

1. Plot the transmitted signal and the received signal on a single figure using subplot. Can you estimate the delay D by a visual inspection of the received signal?
2. Plot the sample autocorrelation of the transmitted signal, r′XX(m) vs. m for −100≤m≤100.
3. Plot the sample cross-correlation of the transmitted signal and the received signal, c′XY(m) vs. m for −100≤m≤100.
4. Determine the delay D from the sample correlation. How did you determine this?



- It is difficult to estimate D by a visual inspection of the received signal, as it displays as consistently-behaved noise throughout.



- r’XX(m) peaks at m = 0 to 0.4942, while c’XY(m) peaks 24 samples later at m = 24 to 0.5292. Given how isolated these 2 points are in their corresponding plots, both are taken to represent the same point, with the peak point in r’XX(m) at the origin translated by a delay of D = 24 to the right to get the peak point of c’XY(m) at m = 24.