ECE 438 Lab 3 Report

David Dang & Benedict Lee

2 Background Exercises

INLAB REPORT: Submit these background exercises with the lab report.

1. $T_0 = 2, \quad t \in [0,2]$

$$s(t) = rect(t - \tfrac{1}{2})$$

① $a_0 = \frac{1}{2}\int_0^2 rect(t-\frac{1}{2})\, dt = \frac{1}{2}$

② $a_k = \frac{1}{T}\int_T s(t)\, e^{-jk\omega_0 t}\, dt$

$$a_k = \frac{1}{2}\int_0^2 \cancel{rect(t-\tfrac{1}{2})}^{\,1}\, e^{-jk\pi t}\, dt$$

$$= \frac{1}{2}\int_0^1 e^{-jk\pi t}\, dt$$

$$= \frac{1}{2}\left[\frac{1}{-jk\pi}\left[e^{-jk\pi t}\right]_0^1 \right]$$

$$= \frac{1}{2jk\pi}\left(1 - e^{-jk\pi}\right)$$

$$a_k = \frac{1-(-1)^k}{2jk\pi}$$

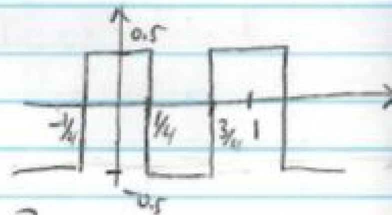
③ $s(t) = \frac{1}{2} + \sum_{k=-\infty}^{\infty} \left(\frac{1}{jk\pi}\right) e^{jk\pi t}$

$$s(t) = \frac{1}{2} + \sum_{k=-\infty}^{-1}\left(\frac{1}{jk\pi}\right) e^{jk\pi t} + \sum_{k=k}^{\infty}\left(\frac{1}{jk\pi}\right) e^{jk\pi t}$$

$$= \frac{1}{2} + \sum_{k=1}^{\infty}\left(\frac{-1}{jk\pi}\right) e^{-jk\pi t} + \left(\frac{1}{jk\pi}\right) e^{jk\pi t}$$

$$= \frac{1}{2} + \sum_{k=1}^{\infty}\left(\frac{1}{jk\pi}\right)\left(e^{jk\pi t} - e^{-jk\pi t}\right) = \boxed{\frac{1}{2} + \sum_{k=1}^{\infty}\left(\frac{2}{k\pi}\right)\sin(k\pi t)}$$

2. $T_0 = 1$, $t \in [-\frac{1}{2}, \frac{1}{2}]$



$$s(t) = rect(2t) - \frac{1}{2}$$

① $a_0 = \int_{-1/2}^{1/2} [rect(2t) - \frac{1}{2}]\, dt = 0$

② $a_k = \frac{1}{T}\int_T s(t) e^{-jkW_0 t}\, dt$

$$a_k = \frac{1}{2}\int_{-1/4}^{1/4} e^{-jk2\pi t} - \frac{1}{2}\int_{1/4}^{3/4} e^{-jk2\pi t}$$

$$= \frac{1}{2}\left(\frac{1}{-jk2\pi}\right)\left[e^{-jk2\pi t}\right]_{-1/4}^{1/4} - \frac{1}{2}\left(\frac{1}{-jk2\pi}\right)\left[e^{-jk2\pi t}\right]_{1/4}^{3/4}$$

$$= \frac{1}{2}\left(\frac{1}{-jk2\pi}\right)\left(e^{-jk\frac{\pi}{2}} - e^{jk\frac{\pi}{2}}\right) - \frac{1}{2}\left(\frac{1}{-jk2\pi}\right)\left(e^{-jk\frac{3\pi}{2}} - e^{-jk\frac{\pi}{2}}\right)$$

$$= \left(\frac{1}{k2\pi}\right)\left(\frac{e^{jk\frac{\pi}{2}} - e^{-jk\frac{\pi}{2}}}{2j}\right) + \frac{1}{k2\pi}\left(\frac{e^{jk\frac{\pi}{2}} - e^{-jk\frac{\pi}{2}}}{2j}\right)$$

$$a_k = \frac{1}{k\pi}\sin\left(\frac{k\pi}{2}\right)$$

③ $s(t) = \sum_{k=-\infty}^{\infty} \frac{1}{k\pi}\sin\left(\frac{k\pi}{2}\right)e^{jk2\pi t}$

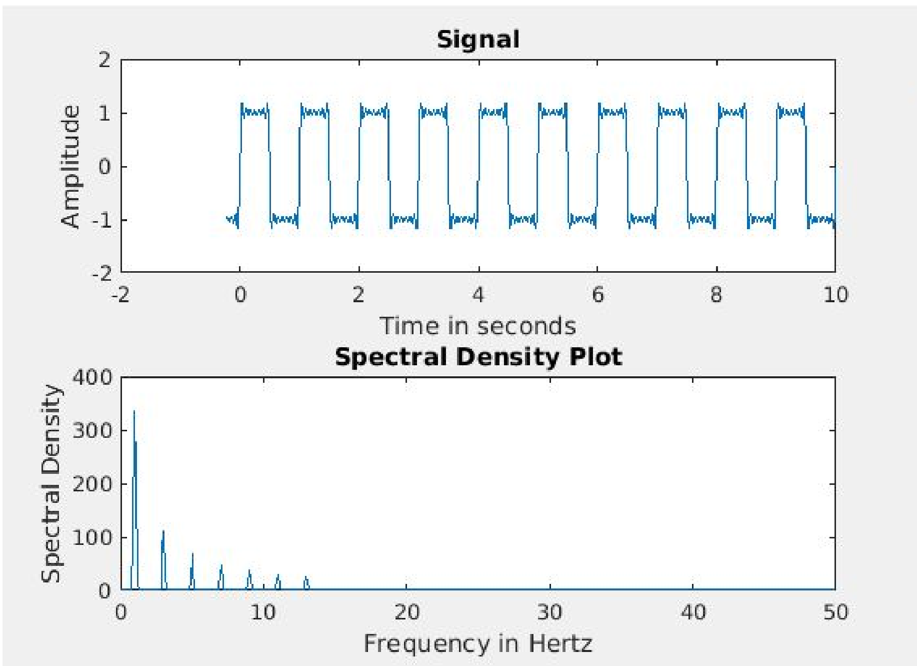
$$s(t) = \sum_{k=-\infty}^{-1} \frac{1}{k\pi}\sin\left(\frac{k\pi}{2}\right)e^{jk2\pi t} + \sum_{k=1}^{\infty}\frac{1}{k\pi}\sin\left(\frac{k\pi}{2}\right)e^{jk2\pi t}$$

$$\sum_{k=1}^{\infty}\left(-\frac{1}{k\pi}\right)\sin\left(-\frac{k\pi}{2}\right)e^{-jk2\pi t} + \left(\frac{1}{k\pi}\right)\sin\left(\frac{k\pi}{2}\right)e^{jk2\pi t}$$

$$\sum_{k=1}^{\infty}\left(\frac{1}{k\pi}\right)\sin\left(\frac{k\pi}{2}\right)\left[e^{-jk2\pi t} + e^{jk2\pi t}\right] = \sum_{k=1}^{\infty}\left(\frac{2}{k\pi}\right)\sin\left(\frac{k\pi}{2}\right)\sin\left(2\pi k t + \frac{\pi}{2}\right)$$

4.1 Synthesis of Periodic Signals

INLAB REPORT: Hand in plots of the Spectrum Analyzer output for each of the three synthesized waveforms.For each case, comment on how the synthesized waveform differs from the desired signal,and on the structure of the spectral density.



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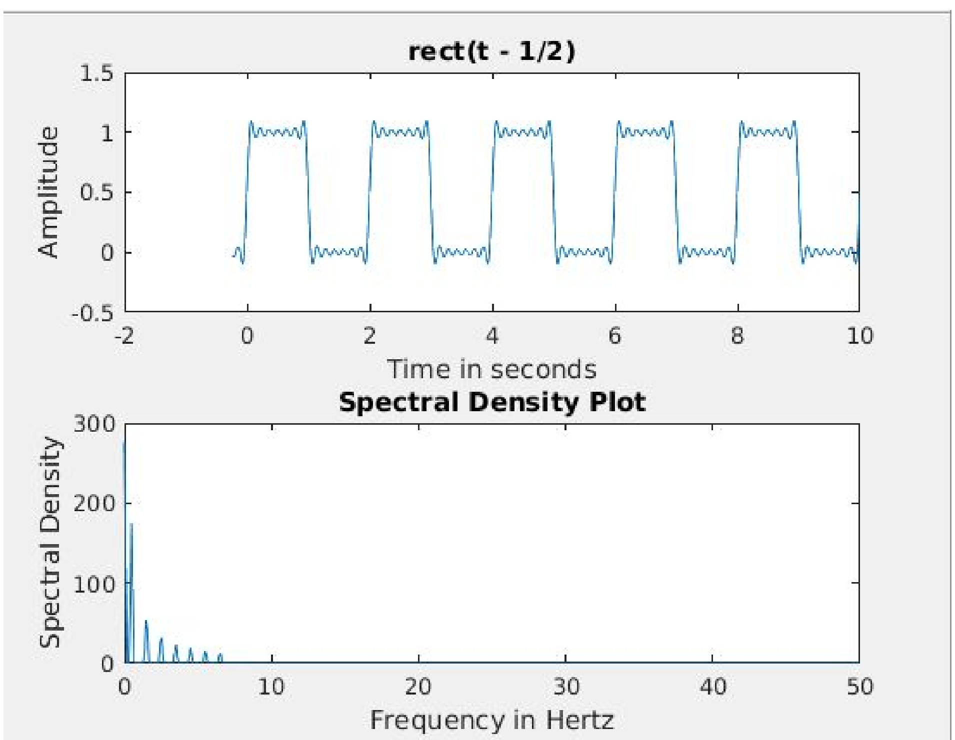

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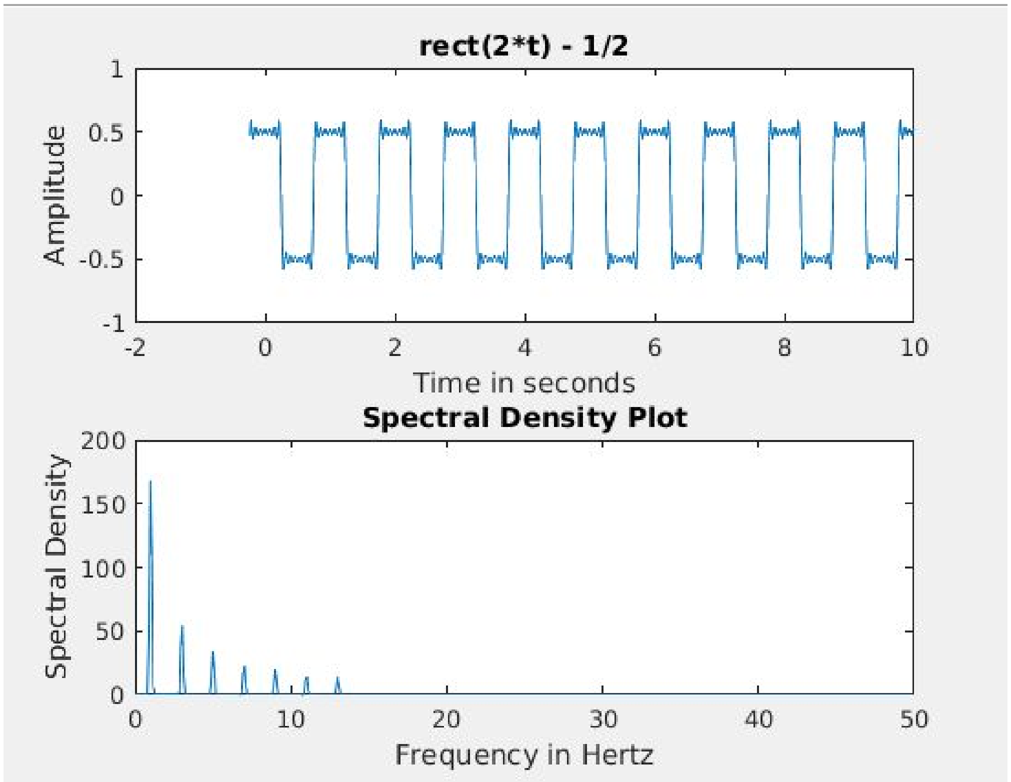

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- rect(t - ½) is a rect function shifted to the right by ½ and has an amplitude of 1. rect(2*t) - ½ is a rect function shifted down by ½ and compressed by a factor of 2. The spectral densities of rect(t - ½) and rect(2*t) - ½ are less than the spectral density of the desired signal

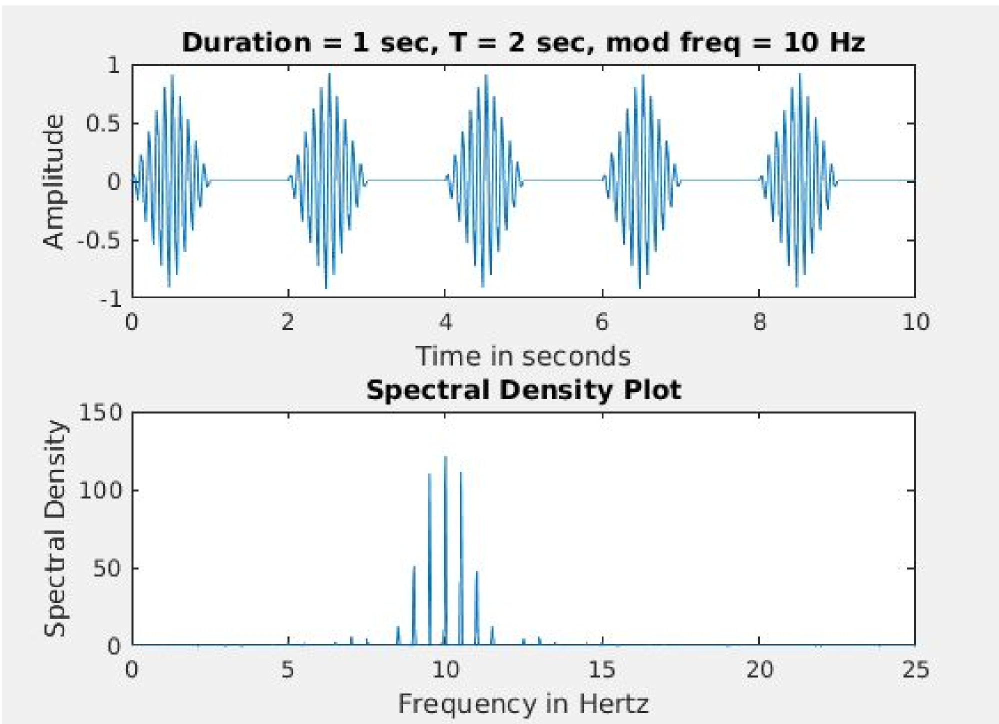
4.2 Modulation Property

INLAB REPORT: Hand in plots of the output of the Spectrum Analyzer for each signal. Answer following questions:

•What effect does changing the modulating frequency have on the spectral density?

•Why does the spectrum have a comb structure and what is the spectral distance between impulses? Why?

•What would happen to the spectral density if the period of the triangle pulse were to increase toward infinity? (in the limit)



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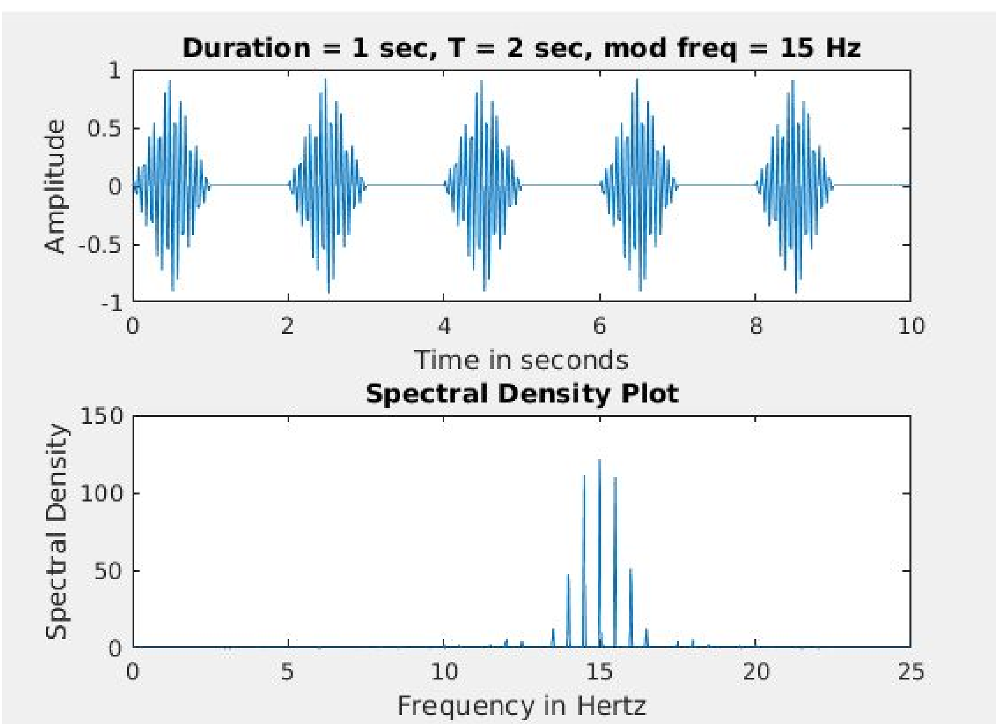

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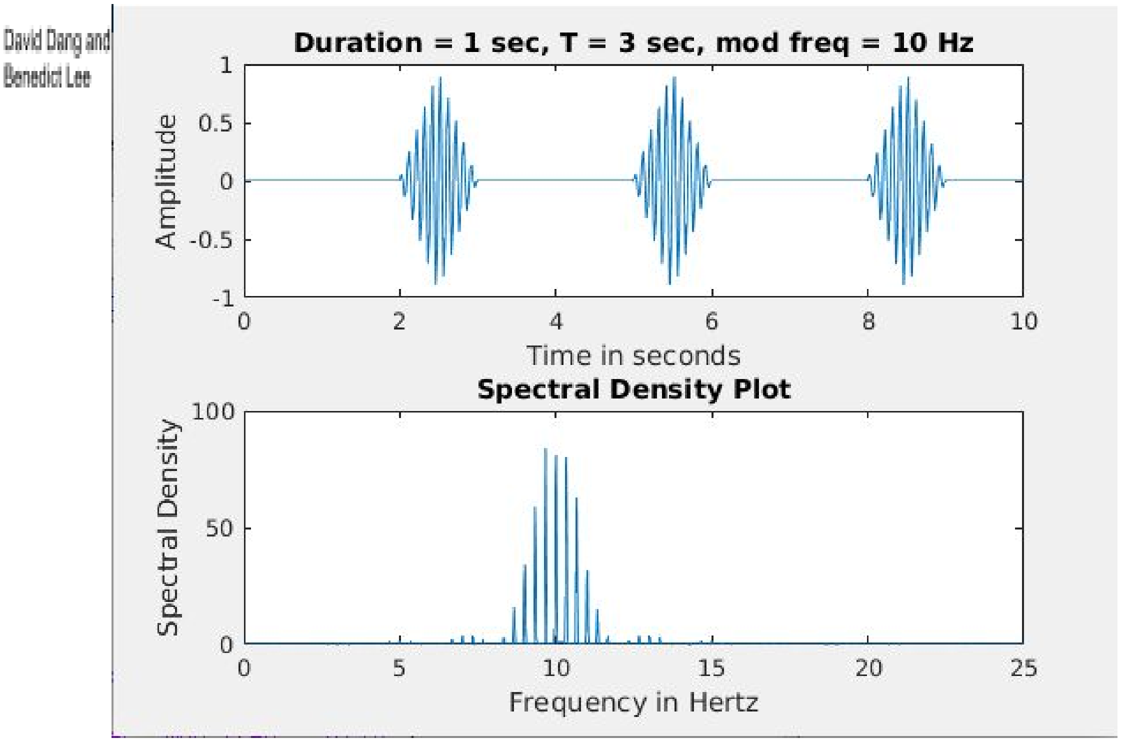
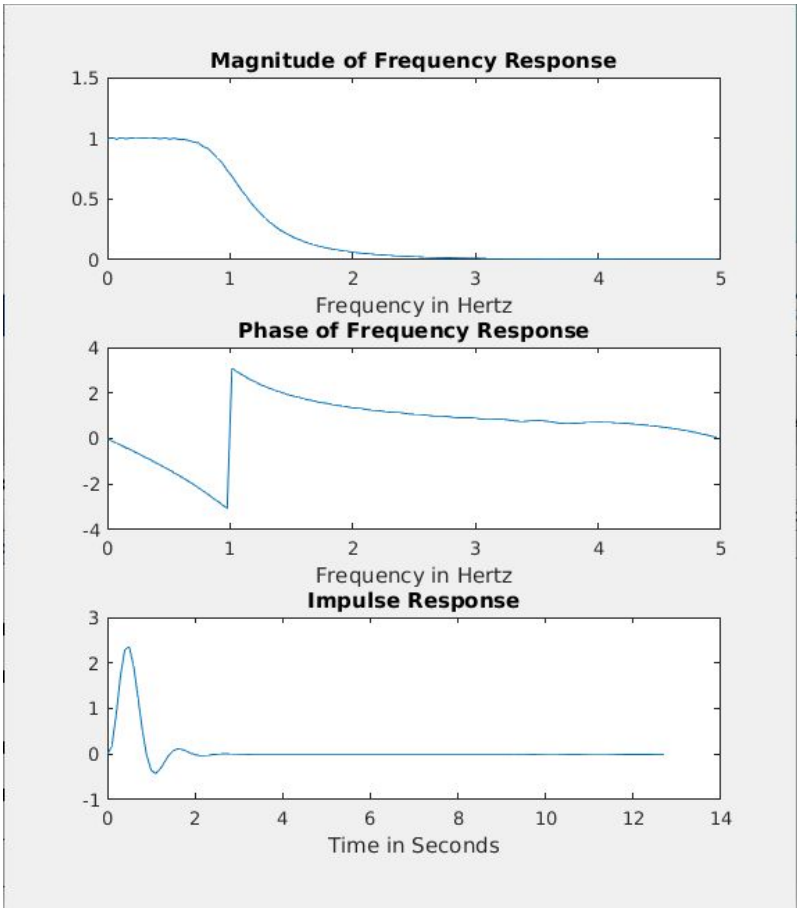


- increasing the modulating frequency by 5 Hz shifts the spectral density to the right by 5 Hz
- The spectrum has a comb structure because it describes the distribution of power in the signal. The spectral distance between impulses is 0.5 Hz because power is distributed differently at intervals of 0.5 Hz
- The spectral density would go to zero as the period T increases to infinity because frequency is proportional to power and the frequency decreases as the period increases

4.3 System Analysis

INLAB REPORT: Hand in the printout of the output of the Network Analyzer (magnitude and phase of the frequency response, and the impulse response) and the plot of the impulse response obtained using a unit step. What are the advantages and disadvantages of each method?



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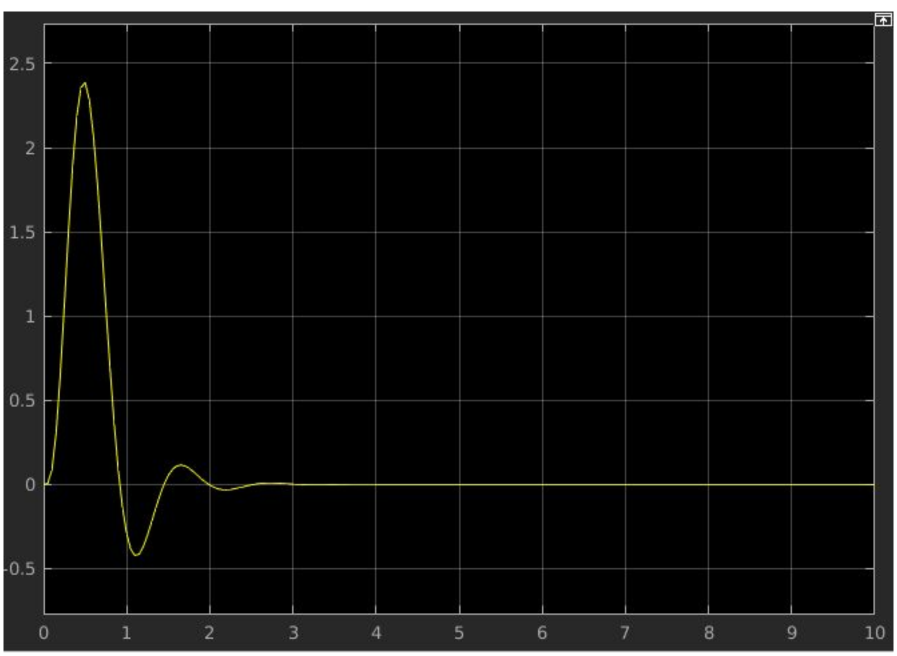
- Obtaining the impulse response from the unit step input using simulink has fewer steps involved than obtaining the impulse response from the network analyzer. However, the network analyzer gives more information about the transfer function than the unit step input.



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5.1 Discrete-Time Fourier Transform

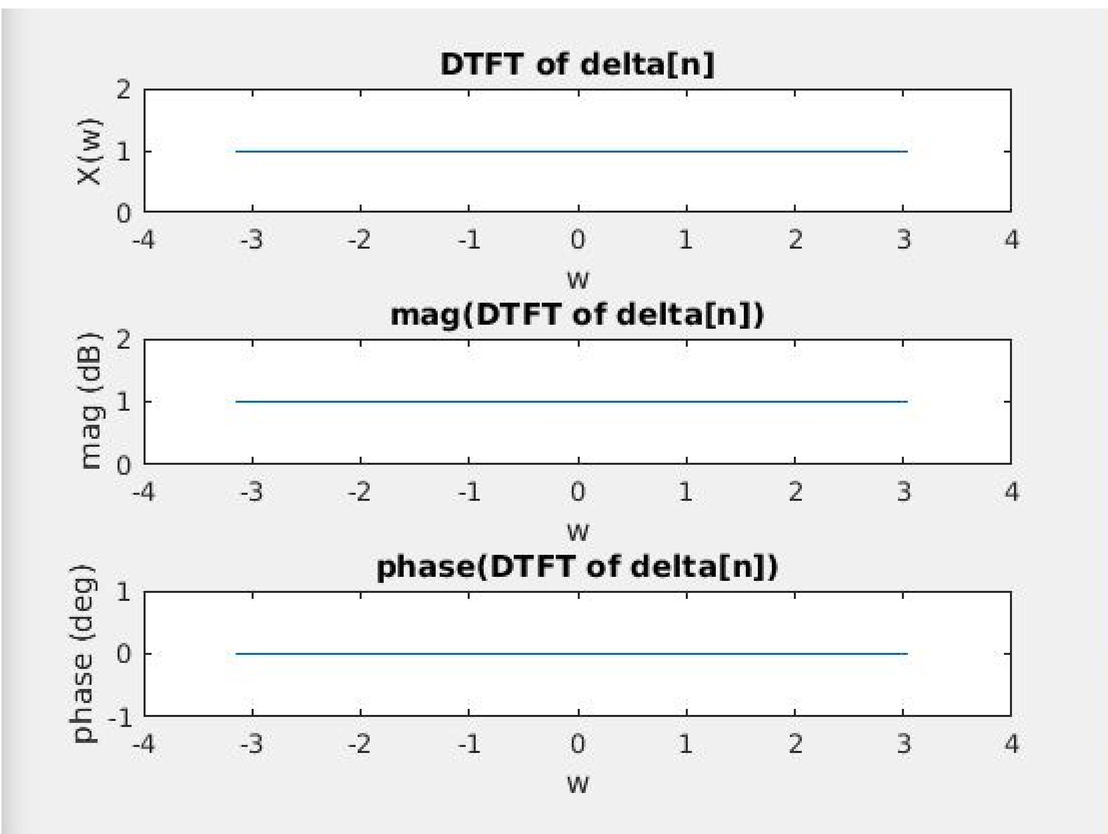
INLAB REPORT: Hand in a printout of your Matlab function. Also hand in plots of the DTFT's magnitude and phase for each of the three signals.

```
function [ X ] = DTFT( x, n0, dw )
%UNTITLED Summary of this function goes here
%   Detailed explanation goes here

i = sqrt(-1);
N = length(x);
w = -pi:dw:pi;
X = zeros(length(w), 1);

for k = 1:length(w)
    for n = 1:N
        X(k) = X(k) + (x(n) * exp(-i*w(k)*(n+n0-1)));
    end
end

end
```

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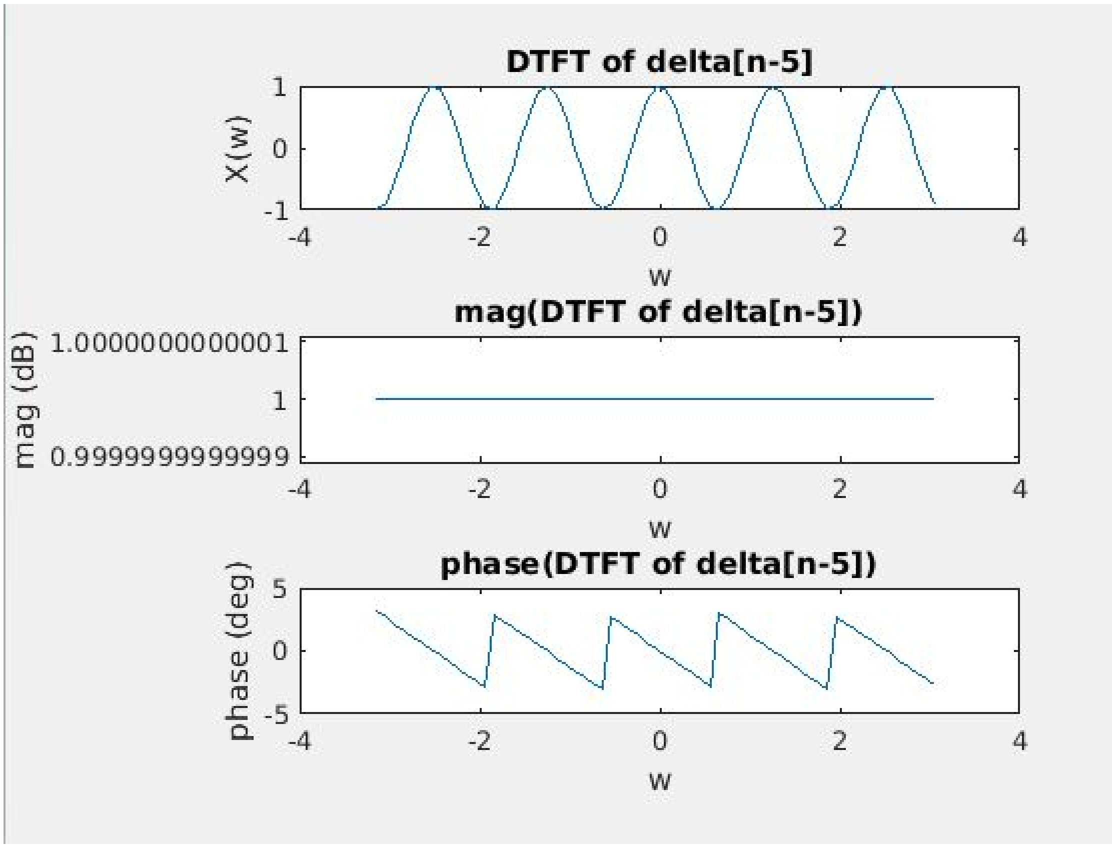

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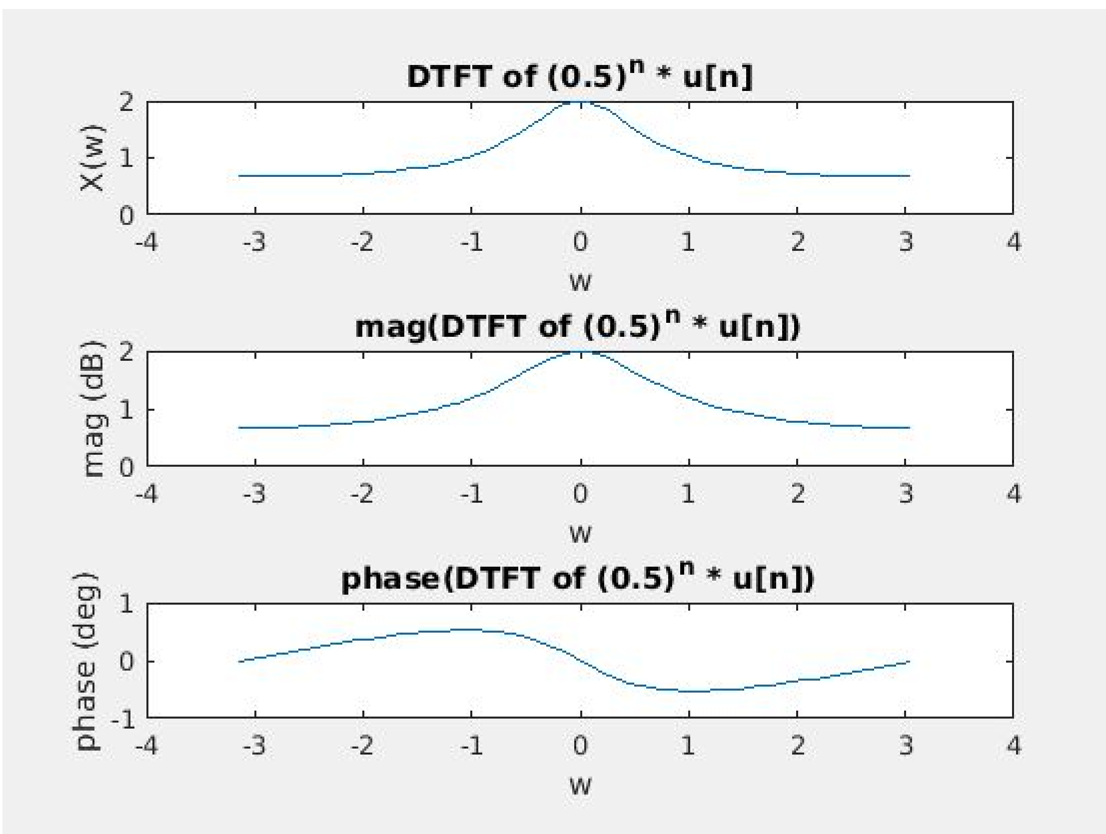

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5.2 Magnitude and Phase of the Frequency Response of a Discrete-Time Systems

INLAB REPORT: Submit these exercises with the lab report.

5.2 $y[n] = 0.9\, y[n-1] + 0.3\, x[n] + 0.24\, x[n-1]$

i.

ii. $h[n] = 0.9\, h[n-1] + 0.3\, \delta[n] + 0.24\, \delta[n-1]$

$h[0] = 0.9\, h[-1] + 0.3$

↳ since causal, $h[0] = 0.3$

$h[1] = 0.9\, h[0] + 0.3\, \delta[1] + 0.24\, \delta[0]$

$= (0.9)(0.3) + 0.24$

$= 0.51$

$h[2] = 0.9\, h[1] + 0.3\, \delta[2] + 0.24\, \delta[1]$

$= (0.9)(0.51)$

$= 0.459$

$h[3] = 0.9\, h[2]$

$= (0.9)(0.459) = 0.4131$

$h[4] = 0.9\, h[3]$

$= (0.9)(0.4131) = 0.37179$

$h[5] = 0.9\, h[4]$

$= (0.9)(0.37179) = 0.334611$

$h[6] = (0.9)\, h[5]$

$= (0.9)(0.334611) = 0.3011499$

iii.

$$H(w) = \sum_{n=-\infty}^{\infty} h[n] e^{-jwn} = \sum_{n=0}^{\infty} h[n] e^{-jwn}$$

$$\boxed{H(w) = 0.3 + 0.51 e^{-jw} + 0.459 e^{-j2w} + 0.4131 e^{-j3w} + 0.37179 e^{-j4w} + 0.334611 e^{-j5w} + 0.3011499 e^{-j6w} \ldots}$$

iv.

$$y[n] = 0.9\, y[n-1] + 0.3\, x[n] + 0.24\, x[n-1]$$

$$Y(w) = 0.9 e^{-jw} Y(w) + 0.3 X(w) + 0.24 e^{-jw} X(w)$$

$$Y(w)\left[1 - 0.9 e^{-jw}\right] = X(w)\left[0.3 + 0.24 e^{-jw}\right]$$

$$\boxed{H(w) = \frac{Y(w)}{X(w)} = \frac{0.3 + 0.24 e^{-jw}}{1 - 0.9 e^{-jw}}}$$

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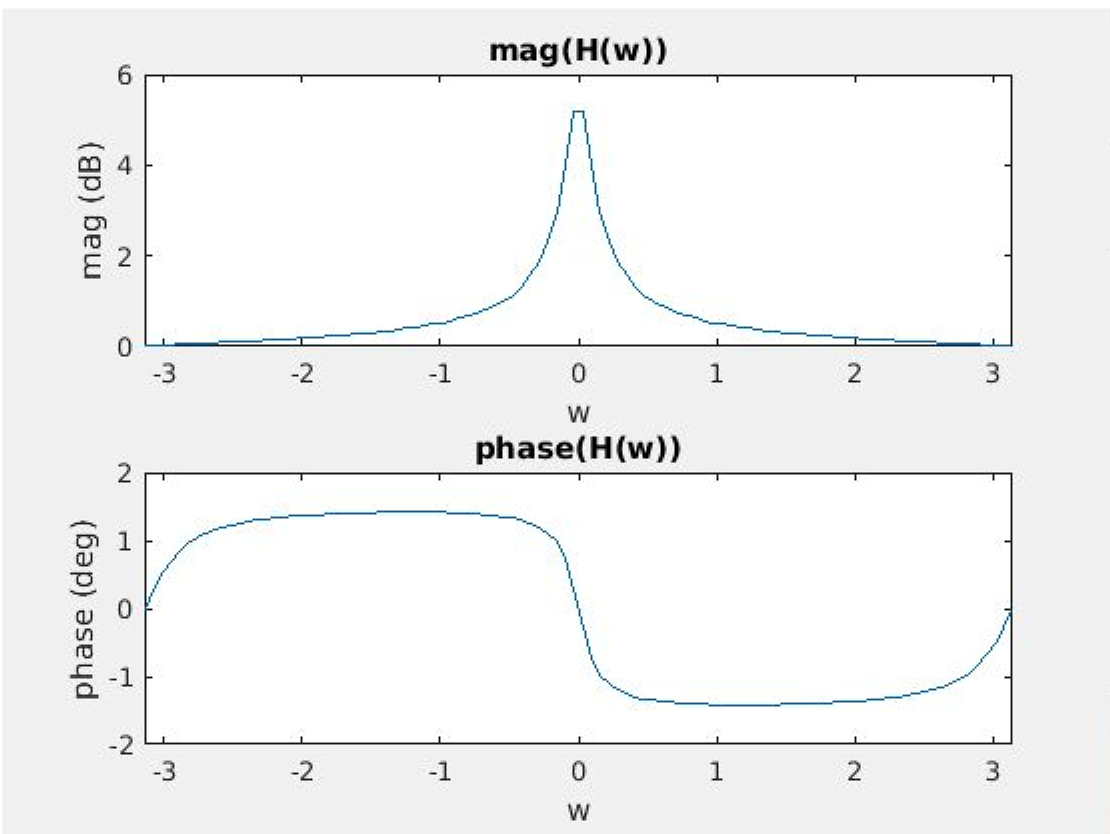

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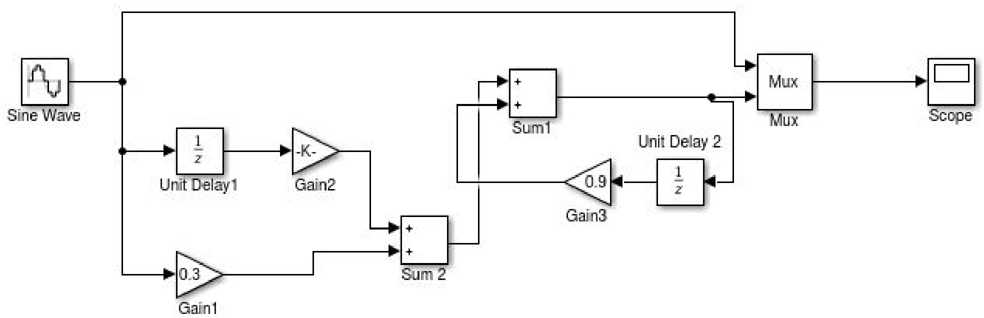
5.3 System Analysis

INLAB REPORT: Hand in the following:

•Printout of your completed block diagram

•Table of both the amplitude measurements you made and their theoretical values.

•Printout of the figure with the impulse response, and the magnitude and phase of the frequency response.



mag(H(w)) = mag(Y(w)) / mag(X(w))		
Pi/4	Pi/8	Pi/16
Calculated: 0.5	Calculated: 1.5	Calculated: 2.75
Theoretical: 0.676	Theoretical: 1.4	Theoretical: 2.66