UID - 118551722 ENPM662 HW5
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All dimensions have been changed to metres in code & in report

From Euler Lagrange Dynamics Equation —

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + J^T(q)F$$

As it is given $\ddot{q} = 0$; $\dot{q} = 0$
Since the movement of the robot is quasi static

$$g(q) = \tau + J^T(q)F$$

$$\tau = g(q) - J^T(q)F$$

$$F = \begin{bmatrix} F_x \\ F_y \\ F_z \\ T_x \\ T_y \\ T_z \end{bmatrix}$$

$T_x = T_y = T_z = 0$
in our case
(given)

To find $g(q)$

→ We need to find Centre of gravity of each link in local frame

→ Tranform COG_{local} coordinates in base frame $\{0\}$ coordinates

→ Since $g(q) = m\, g^T \cdot r_{ci}$ } Dot product between g & r_{ci} vectors

where the r_{ci} is the coordinates of CG

Here for symplicity in code

$$g = 9.8\, m/s^2$$

& $r_{ci} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ where only $r_{ci}[2]$ is multiplied by 'g'

Since the gravity acts in Z direction in world/base frame

Final is formed

$$\cancel{g(q) = \begin{bmatrix} m_1 g r_{c_1}[2] & m_2 g r_{c_2}[2] \\ m_4 g r_{c_4}[2] & m_5 g r_{c_5}[2] \\ m_6 g r_{c_6}[2] & m_7 g r_{c_7}[2] \end{bmatrix}}$$

Jacobian calculation & position update is done the same way as last homework

Potential energy can be calculated as:-

$$P = m_1 g r_{c_1}[2] + m_2 g r_{c_2}[2] + m_4 g r_{c_4}[2] + m_5 g r_{c_5}[2] + m_6 g r_{c_6}[2] + m_7 g r_{c_7}[2]$$

$$\cancel{g(q) = \begin{bmatrix} \frac{\partial P}{\partial \theta_1} \\ \frac{\partial P}{\partial \theta_2} \\ \frac{\partial P}{\partial \theta_4} \end{bmatrix}}$$

$$L = K - P$$

$$\underbrace{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)}_{\text{First term}} - \frac{\partial L}{\partial q} = \tau$$

First term will be zero since $\dot{q}$ term is not there

$$
g(q) = \begin{bmatrix} \dfrac{\partial P}{\partial \theta_1} \\ \dfrac{\partial P}{\partial \theta_2} \\ \dfrac{\partial P}{\partial \theta_4} \\ \dfrac{\partial P}{\partial \theta_5} \\ \dfrac{\partial P}{\partial \theta_6} \\ \dfrac{\partial P}{\partial \theta_7} \end{bmatrix}
$$

This is than calculated for each iteration & updated value of torque is shown using

$$
\tau = g(q) - J^T(q)F
$$

~~All COM values were taken from~~

All COM values were taken from the paper quoted in Readme. (except link 2 & link 3)

Link 2 & Link 3

Composite COG of Link 2 & 3 in Link 2 frame

$$COG_{Link2\ \&\ Link3} = \begin{bmatrix} 0 \\ 0 \\ 0.316/2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0.158 \end{bmatrix}$$

Considered uniform mass distribution for this case.
Rest of COG values taken from reference quoted below

Reference of paper from which COG values were taken:
https://hal.inria.fr/hal-02265294/document

Next pages contain the same report for HW4 as an appendix. Note that this time in code the Time period of 200 seconds was considered. It is updated in the code.

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(All dimensions taken in mm)

Homework-04

• **Updated DH Table :-**

Link, i	a	Alpha α	Theta θ	d
1	0	θ_1^*	$\pi/2$	d_1
2	0	θ_2^*	$-\pi/2$	0
3	a_3	θ_3^+	$-\pi/2$	d_3
4	$-a_3$	θ_4^*	$\pi/2$	0
5	0	θ_5^*	$\pi/2$	d_5
6	a_3	θ_6^*	$-\pi/2$	0
n (or 7)	0	θ_7^*	0	$-(d_7+100)$

* - variable

Changes to DH Table :- • $^+\theta_3 = 0$; (Since the joint will be locked)

• For n, d is mentioned as $-d_7-100$ to include end-effector i.e., the pen length.

• Remaining table remains the same

Used the following equation to find Homogeneous transformation between Links 'i' & 'i-1'

Homogeneous Transformation Matrix between Links 'i' and 'i-1' :

$$T_{i-1}^{i} = \begin{bmatrix} cos\theta_i & -cos\alpha_i sin\theta_i & sin\alpha_i sin\theta_i & a_i cos\theta_i \\ sin\theta_i & cos\alpha_i cos\theta_i & -sin\alpha_i cos\theta_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From the above matrix, we can use method 2 for finding the Jacobian

Refer Jupyter terminal for the Jacobian Matrix

For method 2 the partial derivatives need to be taken for the fourth row of 'n' frame matrix wrt base frame.

The matrix required can be calculated as below :-

$${}^{0}_{1}T = {}^{0}_{1}T$$

$${}^{0}_{2}T = {}^{0}_{1}T \, {}^{1}_{2}T$$

$${}^{0}_{3}T = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}_{3}T$$

$${}^{0}_{4}T = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}_{3}T \, {}^{3}_{4}T$$

$${}^{0}_{5}T = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}_{3}T \, {}^{3}_{4}T \, {}^{4}_{5}T$$

$${}^{0}_{6}T = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}_{3}T \, {}^{3}_{4}T \, {}^{4}_{5}T \, {}^{5}_{6}T$$

$${}^{0}_{7}T = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}_{3}T \, {}^{3}_{4}T \, {}^{4}_{5}T \, {}^{5}_{6}T \, {}^{6}_{7}T$$

${}^{0}_{1}T, {}^{1}_{2}T, {}^{2}_{3}T, {}^{3}_{4}T, {}^{4}_{5}T, {}^{5}_{6}T, {}^{6}_{7}T$ are obtained from D-H table using the formula given earlier.

The 4th row of ${}^{0}_{7}T$ will be equal to column vector ${}^{0}X_{p}$

The 3rd row of ${}^{0}_{1}T$, ${}^{0}_{2}T$, ${}^{0}_{3}T$, ${}^{0}_{4}T$, ${}^{0}_{5}T$, ${}^{0}_{6}T$, ${}^{0}_{7}T$ will be

${}^{0}Z_1$, ${}^{0}Z_2$, ${}^{0}Z_3$, ${}^{0}Z_4$, ${}^{0}Z_5$, ${}^{0}Z_6$, ${}^{0}Z_7$ column vectors

Note :- Padding of '1' is removed above column vectors

The manipulator Jacobian will be

$$
{}^{0}J = \begin{bmatrix} \frac{\partial {}^{0}X_p}{\partial \theta_1} & \frac{\partial {}^{0}X_p}{\partial \theta_2} & \frac{\partial {}^{0}X_p}{\partial \theta_3} & \frac{\partial {}^{0}X_p}{\partial \theta_4} & \frac{\partial {}^{0}X_p}{\partial \theta_5} & \frac{\partial {}^{0}X_p}{\partial \theta_6} & \frac{\partial {}^{0}X_p}{\partial \theta_7} \\ {}^{0}Z_1 & {}^{0}Z_2 & {}^{0}Z_3 & {}^{0}Z_4 & {}^{0}Z_5 & {}^{0}Z_6 & {}^{0}Z_7 \end{bmatrix}
$$

This is a 6×7 matrix

Since Joint 3 is locked we will remove the 3rd row from above Jacobian to make it square

$$
{}^{0}J_{\text{Joint3 locked}} = \begin{bmatrix} \frac{\partial {}^{0}X_p}{\partial \theta_1} & \frac{\partial {}^{0}X_p}{\partial \theta_2} & \frac{\partial {}^{0}X_p}{\partial \theta_4} & \frac{\partial {}^{0}X_p}{\partial \theta_5} & \frac{\partial {}^{0}X_p}{\partial \theta_6} & \frac{\partial {}^{0}X_p}{\partial \theta_7} \\ {}^{0}Z_1 & {}^{0}Z_2 & {}^{0}Z_4 & {}^{0}Z_5 & {}^{0}Z_6 & {}^{0}Z_7 \end{bmatrix}
$$

The element's mentioned in above matrix are as below:-

$$^0Z_1 = \begin{bmatrix} \sin(\theta_1) \\ -\cos(\theta_1) \\ 0 \end{bmatrix}$$

$$^0Z_2 = \begin{bmatrix} -\sin(\theta_2)\cos(\theta_1) \\ -\sin(\theta_1)\sin(\theta_2) \\ \cos(\theta_2) \end{bmatrix}$$

$$^0Z_4 = \begin{bmatrix} -\sin(\theta_2)\cos(\theta_1)\cos(\theta_4) + \sin(\theta_4)\cos(\theta_1)\cos(\theta_2) \\ -\sin(\theta_1)\sin(\theta_2)\cos(\theta_4) + \sin(\theta_1)\sin(\theta_4)\cos(\theta_2) \\ \sin(\theta_2)\sin(\theta_4) + \cos(\theta_2)\cos(\theta_4) \end{bmatrix}$$

$$^0Z_5 = \begin{bmatrix} (\sin(\theta_2)\sin(\theta_4)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_4))\sin(\theta_5) + \sin(\theta_1)\cos(\theta_5) \\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_4) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_4))\sin(\theta_5) - \cos(\theta_1)\cos(\theta_5) \\ (\sin(\theta_2)\cos(\theta_4) - \sin(\theta_4)\cos(\theta_2))\sin(\theta_5) \end{bmatrix}$$

$$^0Z_6 = \begin{bmatrix} -((\sin(\theta_2)\sin(\theta_4)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_4))\cos(\theta_5) - \sin(\theta_1)\sin(\theta_5))\sin(\theta_6) + (-\sin(\theta_2)\cos(\theta_1)\cos(\theta_4) + \sin(\theta_4)\cos(\theta_1)\cos(\theta_2))\cos(\theta_6) \\ -((\sin(\theta_1)\sin(\theta_2)\sin(\theta_4) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_4))\cos(\theta_5) + \sin(\theta_5)\cos(\theta_1))\sin(\theta_6) + (-\sin(\theta_1)\sin(\theta_2)\cos(\theta_4) + \sin(\theta_1)\sin(\theta_4)\cos(\theta_2))\cos(\theta_6) \\ (\sin(\theta_2)\sin(\theta_4) + \cos(\theta_2)\cos(\theta_4))\cos(\theta_6) - (\sin(\theta_2)\cos(\theta_4) - \sin(\theta_4)\cos(\theta_2))\sin(\theta_6)\cos(\theta_5) \end{bmatrix}$$

$^0Z_7 =$

$$\begin{bmatrix} -\left(\left(\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)+\cos(\theta_1)\cos(\theta_2)\cos(\theta_4)\right)\cos(\theta_5)-\sin(\theta_1)\sin(\theta_5)\right)\sin(\theta_6)+\left(-\sin(\theta_2)\cos(\theta_1)\cos(\theta_4)+\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\right)\cos(\theta_6) \\ -\left(\left(\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)+\sin(\theta_1)\cos(\theta_2)\cos(\theta_4)\right)\cos(\theta_5)+\sin(\theta_5)\cos(\theta_1)\right)\sin(\theta_6)+\left(-\sin(\theta_1)\sin(\theta_2)\cos(\theta_4)+\sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\right)\cos(\theta_6) \\ \left(\sin(\theta_2)\sin(\theta_4)+\cos(\theta_2)\cos(\theta_4)\right)\cos(\theta_6)-\left(\sin(\theta_2)\cos(\theta_4)-\sin(\theta_4)\cos(\theta_2)\right)\sin(\theta_6)\cos(\theta_5) \end{bmatrix}$$

Recommended to check in code since the matrix is too large to paste a screenshot here

$^0X_p =$

```
⎡a₃·((sin(θ₂)·sin(θ₄)·cos(θ₁) + cos(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ₅) - sin(θ₁)·sin
⎢
⎢a₃·((sin(θ₁)·sin(θ₂)·sin(θ₄) + sin(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ₅) + sin(θ₅)·cos
⎢
⎣                                                                            a₃
(θ₅))·cos(θ₆) + a₃·(-sin(θ₂)·cos(θ₁)·cos(θ₄) + sin(θ₄)·cos(θ₁)·cos(θ₂))·sin(θ₆
(θ₁))·cos(θ₆) + a₃·(-sin(θ₁)·sin(θ₂)·cos(θ₄) + sin(θ₁)·sin(θ₄)·cos(θ₂))·sin(θ₆
·(sin(θ₂)·sin(θ₄) + cos(θ₂)·cos(θ₄))·sin(θ₆) + a₃·(sin(θ₂)·cos(θ₄) - sin(θ₄)·c
) - a₃·sin(θ₂)·sin(θ₄)·cos(θ₁) - a₃·cos(θ₁)·cos(θ₂)·cos(θ₄) + a₃·cos(θ₁)·cos(θ
) - a₃·sin(θ₁)·sin(θ₂)·sin(θ₄) - a₃·sin(θ₁)·cos(θ₂)·cos(θ₄) + a₃·sin(θ₁)·cos(θ
os(θ₂))·cos(θ₅)·cos(θ₆) - a₃·sin(θ₂)·cos(θ₄) + a₃·sin(θ₂) + a₃·sin(θ₄)·cos(θ₂)
₂) - d₃·sin(θ₂)·cos(θ₁) + d₅·(-sin(θ₂)·cos(θ₁)·cos(θ₄) + sin(θ₄)·cos(θ₁)·cos(θ
₂) - d₃·sin(θ₁)·sin(θ₂) + d₅·(-sin(θ₁)·sin(θ₂)·cos(θ₄) + sin(θ₁)·sin(θ₄)·cos(θ
 + d₁ + d₃·cos(θ₂) + d₅·(sin(θ₂)·sin(θ₄) + cos(θ₂)·cos(θ₄)) + (-d₇ - 100)·((si
₂)) + (-d₇ - 100)·(-((sin(θ₂)·sin(θ₄)·cos(θ₁) + cos(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ
₂)) + (-d₇ - 100)·(-((sin(θ₁)·sin(θ₂)·sin(θ₄) + sin(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ
n(θ₂)·sin(θ₄) + cos(θ₂)·cos(θ₄))·cos(θ₆) - (sin(θ₂)·cos(θ₄) - sin(θ₄)·cos(θ₂))
₅) - sin(θ₁)·sin(θ₅))·sin(θ₆) + (-sin(θ₂)·cos(θ₁)·cos(θ₄) + sin(θ₄)·cos(θ₁)·co
₅) + sin(θ₅)·cos(θ₁))·sin(θ₆) + (-sin(θ₁)·sin(θ₂)·cos(θ₄) + sin(θ₁)·sin(θ₄)·co
·sin(θ₆)·cos(θ₅))
s(θ₂))·cos(θ₆))⎤
               ⎥
s(θ₂))·cos(θ₆))⎥
               ⎥
               ⎦
```

$\frac{\partial\, ^0X_p}{\partial \theta_1} =$

```
⎡a₃·((-sin(θ₁)·sin(θ₂)·sin(θ₄) - sin(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ₅) - sin(θ₅)·co
⎢
⎢a₃·((sin(θ₂)·sin(θ₄)·cos(θ₁) + cos(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ₅) - sin(θ₁)·sin
⎢
⎣
s(θ₁))·cos(θ₆) + a₃·(sin(θ₁)·sin(θ₂)·cos(θ₄) - sin(θ₁)·sin(θ₄)·cos(θ₂))·sin(θ₆
(θ₅))·cos(θ₆) + a₃·(-sin(θ₂)·cos(θ₁)·cos(θ₄) + sin(θ₄)·cos(θ₁)·cos(θ₂))·sin(θ₆

) + a₃·sin(θ₁)·sin(θ₂)·sin(θ₄) + a₃·sin(θ₁)·cos(θ₂)·cos(θ₄) - a₃·sin(θ₁)·cos(θ
) - a₃·sin(θ₂)·sin(θ₄)·cos(θ₁) - a₃·cos(θ₁)·cos(θ₂)·cos(θ₄) + a₃·cos(θ₁)·cos(θ

₂) + d₃·sin(θ₁)·sin(θ₂) + d₅·(sin(θ₁)·sin(θ₂)·cos(θ₄) - sin(θ₁)·sin(θ₄)·cos(θ₂
₂) - d₃·sin(θ₂)·cos(θ₁) + d₅·(-sin(θ₂)·cos(θ₁)·cos(θ₄) + sin(θ₄)·cos(θ₁)·cos(θ
      0
)) + (-d₇ - 100)·(-((-sin(θ₁)·sin(θ₂)·sin(θ₄) - sin(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ
₂)) + (-d₇ - 100)·(-((sin(θ₂)·sin(θ₄)·cos(θ₁) + cos(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ

₅) - sin(θ₅)·cos(θ₁))·sin(θ₆) + (sin(θ₁)·sin(θ₂)·cos(θ₄) - sin(θ₁)·sin(θ₄)·cos
₅) - sin(θ₁)·sin(θ₅))·sin(θ₆) + (-sin(θ₂)·cos(θ₁)·cos(θ₄) + sin(θ₄)·cos(θ₁)·co

(θ₂))·cos(θ₆)) ⎤
s(θ₂))·cos(θ₆))⎥
               ⎥
               ⎦
```

$\frac{\partial\, ^0X_p}{\partial \theta_2} =$

Recommended to check in code since the matrix is too large to paste a screenshot here

```
⎡a₃·(-sin(θ₂)·sin(θ₄)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₄))·sin(θ₅) + a₃·(-sin(θ₂
⎢
⎢a₃·(-sin(θ₁)·sin(θ₂)·sin(θ₄) - sin(θ₁)·cos(θ₂)·cos(θ₄))·sin(θ₅) + a₃·(-sin(θ₁
⎢
⎣                                                           a₃·(sin(θ₂)·sin(θ₄) +
)·cos(θ₁)·cos(θ₄) + sin(θ₄)·cos(θ₁)·cos(θ₂))·cos(θ₅)·cos(θ₆) + a₃·sin(θ₂)·cos(
)·sin(θ₂)·cos(θ₄) + sin(θ₁)·sin(θ₄)·cos(θ₂))·cos(θ₅)·cos(θ₆) + a₃·sin(θ₁)·sin(
 cos(θ₂)·cos(θ₄))·cos(θ₅)·cos(θ₆) + a₃·(-sin(θ₂)·cos(θ₄) + sin(θ₄)·cos(θ₂))·si
θ₁)·cos(θ₄) - a₃·sin(θ₂)·cos(θ₁) - a₃·sin(θ₄)·cos(θ₁)·cos(θ₂) - d₃·cos(θ₁)·cos
θ₂)·cos(θ₄) - a₃·sin(θ₁)·sin(θ₂) - a₃·sin(θ₁)·sin(θ₄)·cos(θ₂) - d₃·sin(θ₁)·cos
n(θ₆) - a₃·sin(θ₂)·sin(θ₄) - a₃·cos(θ₂)·cos(θ₄) + a₃·cos(θ₂) - d₃·sin(θ₂) + d₅
(θ₂) + d₅·(-sin(θ₂)·sin(θ₄)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₄)) + (-d₇ - 100)·(
(θ₂) + d₅·(-sin(θ₁)·sin(θ₂)·sin(θ₄) - sin(θ₁)·cos(θ₂)·cos(θ₄)) + (-d₇ - 100)·(
·(-sin(θ₂)·cos(θ₄) + sin(θ₄)·cos(θ₂)) + (-d₇ - 100)·(-(sin(θ₂)·sin(θ₄) + cos(θ
(-sin(θ₂)·sin(θ₄)·cos(θ₁) - cos(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ₆) - (-sin(θ₂)·cos(θ
(-sin(θ₁)·sin(θ₂)·sin(θ₄) - sin(θ₁)·cos(θ₂)·cos(θ₄))·cos(θ₆) - (-sin(θ₁)·sin(θ
₂)·cos(θ₄))·sin(θ₆)·cos(θ₅) + (-sin(θ₂)·cos(θ₄) + sin(θ₄)·cos(θ₂))·cos(θ₆))
₁)·cos(θ₄) + sin(θ₄)·cos(θ₁)·cos(θ₂))·sin(θ₆)·cos(θ₅))⎤
                                                      ⎥
₂)·cos(θ₄) + sin(θ₁)·sin(θ₄)·cos(θ₂))·sin(θ₆)·cos(θ₅))⎥
                                                      ⎥
                                                      ⎦
```

$$\frac{\partial x_p}{\partial \theta_4} =$$

Recommended to check in code since the matrix is too large to paste a screenshot here

```
⎡a3·(sin(θ2)·sin(θ4)·cos(θ1) + cos(θ1)·cos(θ2)·cos(θ4))·sin(θ6) + a3·(sin(θ2)·
⎢
⎢a3·(sin(θ1)·sin(θ2)·sin(θ4) + sin(θ1)·cos(θ2)·cos(θ4))·sin(θ6) + a3·(sin(θ1)·
⎢
⎣                                                 a3·(-sin(θ2)·sin(θ4) - cos(θ2)·

cos(θ1)·cos(θ4) - sin(θ4)·cos(θ1)·cos(θ2))·cos(θ5)·cos(θ6) - a3·sin(θ2)·cos(θ1

sin(θ2)·cos(θ4) - sin(θ1)·sin(θ4)·cos(θ2))·cos(θ5)·cos(θ6) - a3·sin(θ1)·sin(θ2

cos(θ4))·cos(θ5)·cos(θ6) + a3·(sin(θ2)·cos(θ4) - sin(θ4)·cos(θ2))·sin(θ6) + a3

)·cos(θ4) + a3·sin(θ4)·cos(θ1)·cos(θ2) + d5·(sin(θ2)·sin(θ4)·cos(θ1) + cos(θ1)

)·cos(θ4) + a3·sin(θ1)·sin(θ4)·cos(θ2) + d5·(sin(θ1)·sin(θ2)·sin(θ4) + sin(θ1)

·sin(θ2)·sin(θ4) + a3·cos(θ2)·cos(θ4) + d5·(sin(θ2)·cos(θ4) - sin(θ4)·cos(θ2))

·cos(θ2)·cos(θ4)) + (-d7 - 100)·((sin(θ2)·sin(θ4)·cos(θ1) + cos(θ1)·cos(θ2)·co

·cos(θ2)·cos(θ4)) + (-d7 - 100)·((sin(θ1)·sin(θ2)·sin(θ4) + sin(θ1)·cos(θ2)·co

 + (-d7 - 100)·(-(-sin(θ2)·sin(θ4) - cos(θ2)·cos(θ4))·sin(θ6)·cos(θ5) + (sin(θ

s(θ4))·cos(θ6) - (sin(θ2)·cos(θ1)·cos(θ4) - sin(θ4)·cos(θ1)·cos(θ2))·sin(θ6)·c

s(θ4))·cos(θ6) - (sin(θ1)·sin(θ2)·cos(θ4) - sin(θ1)·sin(θ4)·cos(θ2))·sin(θ6)·c

2)·cos(θ4) - sin(θ4)·cos(θ2))·cos(θ6))

os(θ5))⎤
       ⎥
os(θ5))⎥
       ⎥
       ⎦
```

$$\frac{\partial x_p}{\partial \theta_5} =$$

Recommended to check in code since the matrix is too large to paste a screenshot here

$$\left[\begin{matrix} a_3 \left(- \left(\sin(\theta_2) \sin(\theta_4) \cos(\theta_1) + \cos(\theta_1) \cos(\theta_2) \cos(\theta_4)\right) \sin(\theta_5) - \sin(\theta_1) \cos(\theta_5)\right) \cos(\theta_6) - \left(- d_7 - 100\right) \left(- \left(\sin(\theta_2) \sin(\theta_4) \cos(\theta_1) + \cos(\theta_1) \cos(\theta_2) \cos(\theta_4)\right) \sin(\theta_5) - \sin(\theta_1) \cos(\theta_5)\right) \sin(\theta_6) \\ a_3 \left(- \left(\sin(\theta_1) \sin(\theta_2) \sin(\theta_4) + \sin(\theta_1) \cos(\theta_2) \cos(\theta_4)\right) \sin(\theta_5) + \cos(\theta_1) \cos(\theta_5)\right) \cos(\theta_6) - \left(- d_7 - 100\right) \left(- \left(\sin(\theta_1) \sin(\theta_2) \sin(\theta_4) + \sin(\theta_1) \cos(\theta_2) \cos(\theta_4)\right) \sin(\theta_5) + \cos(\theta_1) \cos(\theta_5)\right) \sin(\theta_6) \\ - a_3 \left(\sin(\theta_2) \cos(\theta_4) - \sin(\theta_4) \cos(\theta_2)\right) \sin(\theta_5) \cos(\theta_6) + \left(- d_7 - 100\right) \left(\sin(\theta_2) \cos(\theta_4) - \sin(\theta_4) \cos(\theta_2)\right) \sin(\theta_5) \sin(\theta_6) \end{matrix}\right]$$

$\dfrac{\partial X_P}{\partial \theta_6} =$ Recommended to check in code since the matrix is too large to paste a screenshot here

```
⎡-a₃⋅((sin(θ₂)⋅sin(θ₄)⋅cos(θ₁) + cos(θ₁)⋅cos(θ₂)⋅cos(θ₄))⋅cos(θ₅) - sin(θ₁)⋅si
⎢
⎢-a₃⋅((sin(θ₁)⋅sin(θ₂)⋅sin(θ₄) + sin(θ₁)⋅cos(θ₂)⋅cos(θ₄))⋅cos(θ₅) + sin(θ₅)⋅co
⎢
⎣                                                      a₃⋅(sin(θ₂)⋅sin(θ₄) + co

n(θ₅))⋅sin(θ₆) + a₃⋅(-sin(θ₂)⋅cos(θ₁)⋅cos(θ₄) + sin(θ₄)⋅cos(θ₁)⋅cos(θ₂))⋅cos(θ

s(θ₁))⋅sin(θ₆) + a₃⋅(-sin(θ₁)⋅sin(θ₂)⋅cos(θ₄) + sin(θ₁)⋅sin(θ₄)⋅cos(θ₂))⋅cos(θ

s(θ₂)⋅cos(θ₄))⋅cos(θ₆) - a₃⋅(sin(θ₂)⋅cos(θ₄) - sin(θ₄)⋅cos(θ₂))⋅sin(θ₆)⋅cos(θ₅

₆) + (-d₇ - 100)⋅(-((sin(θ₂)⋅sin(θ₄)⋅cos(θ₁) + cos(θ₁)⋅cos(θ₂)⋅cos(θ₄))⋅cos(θ₅

₆) + (-d₇ - 100)⋅(-((sin(θ₁)⋅sin(θ₂)⋅sin(θ₄) + sin(θ₁)⋅cos(θ₂)⋅cos(θ₄))⋅cos(θ₅

) + (-d₇ - 100)⋅(-(sin(θ₂)⋅sin(θ₄) + cos(θ₂)⋅cos(θ₄))⋅sin(θ₆) - (sin(θ₂)⋅cos(θ

) - sin(θ₁)⋅sin(θ₅))⋅cos(θ₆) - (-sin(θ₂)⋅cos(θ₁)⋅cos(θ₄) + sin(θ₄)⋅cos(θ₁)⋅cos

) + sin(θ₅)⋅cos(θ₁))⋅cos(θ₆) - (-sin(θ₁)⋅sin(θ₂)⋅cos(θ₄) + sin(θ₁)⋅sin(θ₄)⋅cos

₄) - sin(θ₄)⋅cos(θ₂))⋅cos(θ₅)⋅cos(θ₆))

(θ₂))⋅sin(θ₆))⎤
              ⎥
(θ₂))⋅sin(θ₆))⎥
              ⎥
              ⎦
```

$\dfrac{\partial X_P}{\partial \theta_7} =$

```
⎡0⎤
⎢0⎥
⎣0⎦
```

Points S of circle in parametric form can be written as; wrt base frame

$$x = 0.679 \times 1000 = 679 \text{ mm}$$
$$y = 0.1 \sin\theta \times 1000 = 100 \sin\theta$$
$$z = (0.725 + 0.1 \cos\theta) \times 1000$$
$$= 725 + 100 \cos\theta$$
$$\dot{x} = 0$$
$$\dot{y} = 0.1 \dot{\theta} \cos\theta$$
$$\dot{z} = 0 - 0.1 \dot{\theta} \sin\theta = -0.1 \dot{\theta} \sin\theta$$

$\dot{\theta} = \frac{2\pi}{T}$; T = Total time to complete one 360° revolution

Given T = 5 seconds

$$\dot{\theta} = \frac{2\pi}{5}$$

$$\dot{x} = 0$$
$$\dot{y} = \frac{0.1 \times 2\pi}{5} \cos\theta = 0.04\pi \cos\theta \times 1000$$
$$\dot{y} = 40\pi \cos\theta$$
$$\dot{z} = -\frac{0.1 \times 2\pi}{5} \sin\theta = -0.04\pi \sin\theta \times 1000$$
$$\dot{z} = -40\pi \sin\theta$$

x dot column matrix can be written as

$$x\text{ dot} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix}$$

$\dot{\psi} = \dot{\theta} = \dot{\phi} = 0$ (Given)

$$x\text{ dot} = \begin{bmatrix} 0 \\ 0.04\pi \cos\theta \times 1000 \\ -0.04\pi \sin\theta \times 1000 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 40\pi\cos\theta \\ -40\pi\sin\theta \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

qdot, qinitial column vector is given

$$q_{initial} = \begin{bmatrix} 0 \\ 0 \\ \pi/2 \\ 0 \\ \pi \\ 0 \end{bmatrix}$$

$$qdot = \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \\ \dot{q}_7 \end{bmatrix}$$

For running the integration loop

Given:-

$$\dot{q} = J^{-1} \cdot (xdot)$$

$$q_{next} = q_{current} + \dot{q}_{next} \Delta t$$

We know the $\dot{x}$ & $\dot{y}$ & $\dot{z}$ at various points of time throughout the circle.

so I will divide the rotation of the ballpoint θ/theta in 1000 parts.

so

$$\dot{x} = 0$$

$$\dot{y} = 40\pi \cos\left(\frac{2\pi}{1000} * i\right)$$

$$\dot{z} = -40\pi \sin\left(\frac{2\pi}{1000} * i\right)$$

i will be varied from 0 to 1000 for 0 to 2π rotation

$$\theta = \frac{2\pi * i}{1000}$$

For updating the values of q or Joint angles

$$q_{next} = q_{current} + \dot{q}_{next} \Delta t$$

$$\Delta t = \frac{T}{N} = \frac{5}{N}$$

N is the number of iterations

$$\Delta t = \frac{5}{1000} = 0.005$$

Integrator loop