Problem 1

30 points

Let CFG G be the following grammar.

S → aSb | bY | Y a

Y → bY | aY |

a. Give a simple description of L(G) in English.

Language *L* (*G*) for the G is as follows:

Consider the productions in the grammar

$S \to aSb$

$S \to bY$

$S \to Ya$

$Y \to bY$

$Y \to aY$

$Y \to \varepsilon$

Case 1:

Consider production $S \to Ya$ to derive the language.

Substitute Y with production $Y \to \varepsilon$ then

$S \to \in a$

$S \to a$

Case 2:

Consider production $S \to bY$ to derive the language.

Substitute Y with production $Y \to \varepsilon$ then

$S \to \in b$

$S \to b$

Case 3:

Consider production $S \to aSb$ to derive the language

Substitute S with production $S \to bY$ then

$S \to abYb$

Substitute Y with production $Y \to bY$ then

$S \to abbYb$

Substitute Y with production $Y \to \varepsilon$ then

$S \to abb \in b$

$S \to abbb$

Case 4:

Consider production $S \to bY$ to derive the language.

Substitute Y with production $Y \to bY$ then

$S \to bbY$

Substitute Y with production $Y \to \varepsilon$ then

$S \to bb \in$

$S \to bb$

Therefore form the Case 1, Case 2, Case 3 and Case 4 the language obtained is as follows:

L(*G*) = {*a*, *b*, *abbb*, *bb*…}

Using the grammar *G*, many more strings can be generated.

b. Use that description to give a CFG for L ¯(G) i.e., the complement of L(G).

Description of the *L* (*G*) is as follows:

The grammar G generates a language *L*(*G*) consists of the strings which are described as follows:

- Strings with consecutive number of ***a***'s with a length ranging from 1 to infinity.
- Strings with consecutive number of ***b*'s** with a length ranging from 1 to infinity.
- String with start symbol ***a*** followed by number of ***b*'s**.
- Strings with start symbol ***b*** followed by number of ***a*'s.**
- Strings with ***a*** as start symbol and ***b*** as end symbol.
- Strings with ***b*** as start symbol and ***a*** as end symbol.
- Strings that contains the same start and end symbols. For example, $aba,\ bab$ etc.

From the above description as *L*(*G*) is generating all the possible combination of *a*'s and *b*'s except $a^i b^i \ where\ i \geq 0$. The *L*(*G*) does not produce strings like
$\in,\ ab,\ aabb,\ aaabbb\ \ldots$

The complements of $L(G)$ i.e. $\overline{L(G)}$ = $\{\in,\ ab,\ aabb,\ aaabbb\ \ldots\}$

The grammar for $\overline{L(G)}$ is $a^i b^i \ where\ i \geq 0$.

Therefore, the CFG G' for $\overline{L(G)}$ is as follows:

$S \rightarrow aSb \mid \in$

Give unambiguous CFGs for the following languages.

a. {w| in every prefix of w the number of a's is at least the number of b's}

Let G be the CFG:

$S \rightarrow aS \mid aSAbA \mid a$

$A \rightarrow aA \mid aAbA \mid \varepsilon$

b. {w| the number of a's and the number of b's in w are equal}

Let G be the CFG:

$S \rightarrow aSbS$

$S \rightarrow bSaS$

$S \rightarrow \varepsilon$

This CFG can be also written as:

La's=b's = {w | number of a's in (w) = number of b's in (w)}

$S \rightarrow aSb$

$S \rightarrow bSa$

$S \rightarrow SS$

$S \rightarrow \varepsilon$

c. {w| the number of a’s is at least the number of b’s in w}

Let G be the CFG:

$S \rightarrow aS \mid aSbS \mid \varepsilon$

This CFG can be also written as:

$S \rightarrow aS$

$S \rightarrow aSbS$

$S \rightarrow \varepsilon$