Problem 2: #2

Prove:

answer: Suppose, $S(n) = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

We need to prove all $n \in Z^+$ $S(n)$ is true

Base Step:

take $n = 1$

$L.H.S = 1^3$

$= 1$

$R.H.S = \left[\frac{1(1+1)}{2}\right]^2 \cdot \left[\frac{1 \times \cancel{2}}{\cancel{2}}\right]^2$

$= 1^2 = 1$

$L.H.S = R.H.S$

So, Base Step is True (for $n=1$)

Induction Hypothesis:

Assume that $S(n)$ is true for all $n = k$

So, $1^3 + 2^3 + 3^3 + \dots + k^3 = \left[\frac{k(k+1)}{2}\right]^2$

Inductive Step:

Let us take $n = k+1$

we need to prove

$1^3 + 2^3 + 3^3 + \dots + (k+1)^3 \cdot \left[\frac{(k+1)(k+2)}{2}\right]^2$

$L.H.S = 1^3 + 2^3 + 3^3 + \dots + (k+1)^3$

$= 1^3 + 2^3 + 3^3 + \dots k^3 + (k+1)^3$

$= \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3$