Problem 1:

0.1f. If n is an integer than n = n + 1 is an empty set.

0.1e. If w is a string of 0’s and 1’s and w equals the reverse of w. Then w would be the set of palindromes over a binary alphabet.

0.6d.The set of all possible dependent values and outputs of a function is Range. The set of all possible inputs to a function is Domain.

Therefore, Range of (g) is:{6,7,8,9,10}

the Domain of g(i,j)

X={1,2,3,4,5}

Y={6,7,8,9,10}

Domain D= X*Y

={(1,6),(1,7),(1,8),(1,9),(1,10),(2,6),(2,7),(2,8),

(2,9),(2,10),(3,6),(3,7),(3,8),(3,9),(3,10),(4,6),

(4,7),(4,8),(4,9),(4,10,(5,6),(5,7),(5,8),(5,9),(5,10)}

0.6e.If f is a function in the form of f(a)=b, then b is the output value when the input value is a.

The value of the function f at n = 4 is 7, which is f(4) = 7

The value of function g(4,f(4)) = g(4,7)

=8

Problem 2: #1

Prove:

For all $n \in \mathbb{Z}^+$: $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ — Answer

Base case: Take $n \cdot 1$.

$$L.H.S = \sum_{i=1}^{1} \frac{1}{1(1+1)} \qquad R.H.S = \frac{n}{n+1}$$

$$= \frac{1}{1(1+1)} = \frac{1}{1 \cdot 2} \qquad = \frac{1}{1+1}$$

$$= \frac{1}{2} \qquad = \frac{1}{2}$$

$L.H.S = R.H.S$

Base case, $n=1$ is true

Induction Hypothesis:

Suppose, $P(n)$ is true for $n=K$

$$\text{So, } \sum_{i=1}^{K} \frac{1}{K(K+1)} = \frac{K}{K+1}$$

Inductive Step:

Take $n = K+1$

$$L.H.S = \sum_{i=1}^{K+1} \frac{1}{(K+1)(K+2)} \qquad R.H.S = \frac{K+1}{K+2}$$

We need to prove $L.H.S = R.H.S$

$$L.H.S = \sum_{i=1}^{K+1} \frac{1}{(K+1)(K+2)}$$

Inductive Step continued

$$= \underbrace{\sum_{i=1}^{k} \frac{1}{k(k+1)}} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k}{(k+1)} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k(k+2) + 1}{(k+1)(k+2)}$$

$$= \frac{k^2 + 2k + 1}{(k+1)(k+2)} = \frac{k^2 + k + k + 1}{(k+1)(k+2)}$$

$$= \frac{k(k+1) + 1(k+1)}{(k+1)(k+2)} = \frac{\cancel{(k+1)}(k+1)}{\cancel{(k+1)}(k+2)}$$

$$= \frac{k+1}{k+2} = R.H.S$$

Therefore, proved, It is true for all $n \in Z^+$

Problem 2: #2

Prove:

answer: Suppose, $S(n) = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

We need to prove all $n \in Z^+$ $S(n)$ is true

Base Step:

take $n = 1$

$L.H.S = 1^3$ $\quad$ $R.H.S = \left[\frac{1(1+1)}{2}\right]^2 \cdot \left[\frac{1 \times \cancel{2}}{\cancel{2}}\right]^2$

$= 1$ $\quad$ $= 1^2 = 1$

$L.H.S = R.H.S$

So, Base step is True (for $n=1$)

Induction Hypothesis:

Assume that $S(n)$ is true for all $n = k$

So, $1^3 + 2^3 + 3^3 + \dots + k^3 = \left[\frac{k(k+1)}{2}\right]^2$

Inductive Step:

Let us take $n = k+1$

We need to prove

$1^3 + 2^3 + 3^3 + \dots + (k+1)^3 \cdot \left[\frac{(k+1)(k+2)}{2}\right]^2$

$$
\begin{aligned}
L.H.S &= 1^3 + 2^3 + 3^3 + \dots + (k+1)^3 \\
&= 1^3 + 2^3 + 3^3 + \dots k^3 + (k+1)^3 \\
&= \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3
\end{aligned}
$$

Inductive Step Continued

$$= \frac{K^2(K+1)^2}{4} + (K+1)^3$$

$$= (K+1)^2\left[\frac{K^2+4(K+1)}{4}\right]$$

$$= (K+1)^2\left[\frac{K^2+4K+4}{4}\right.$$

$$= (K+1)^2\left[\frac{K^2+2K+2K+4}{4}\right]$$

$$= (K+1)^2\left[\frac{K(K+2)+2(K+2)}{4}\right]$$

$$= (K+1)^2\left[\frac{(K+2)(K+2)}{4}\right] \cdot (K+1)^2\left(\frac{K+2}{2}\right)^2$$

$$= \left[\frac{(K+1)(K+2)}{2}\right]^2 = R \cdot H \cdot S$$ Hence $\boxed{\because}$