Problem 2: #1

Prove:

For all $n \in \mathbb{Z}^+$: $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ —Answer

Base case: Take $n \cdot 1$.

$L \cdot H \cdot S = \sum_{i=1}^{1} \frac{1}{1(1+1)}$ $\quad R H S = \frac{n}{n+1}$

$= \frac{1}{1(1+1)} = \frac{1}{1 \cdot 2}$ $\quad = \frac{1}{1+1}$

$= \frac{1}{2}$ $\quad = \frac{1}{2}$

$L \cdot H \cdot S = R \cdot H \cdot S$

Base case, $n=1$ is true

Induction Hypothesis:

Suppose, $P(n)$ is true for $n=k$

So, $\sum_{i=1}^{k} \frac{1}{k(k+1)} = \frac{k}{k+1}$

Inductive Step:

Take $n=k+1$

$L \cdot H \cdot S = \sum_{i=1}^{k+1} \frac{1}{(k+1)(k+2)}$ $\quad R \cdot H \cdot S = \frac{k+1}{k+2}$

We need to prove $L \cdot H \cdot S = R \cdot H \cdot S$

$L \cdot H \cdot S = \sum_{i=1}^{k+1} \frac{1}{(k+1)(k+2)}$