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Modeling the Relation between Adaptation Investment and Public Debt

Modélisation de la Relation entre Investissement d'Adaptation et Dette Publique

Mémoire d'Initiation à la Recherche (MIR)
Sous la tutelle de Michel Guillard

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All the errors are mine.

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MÉMOIRE D'INITIATION À LA RECHERCHE

Master of Economics - Ecole Normale Supérieure Paris-Saclay

Titre: Modélisation des relations entre investissement d'adaptation et dette publique

Mots clés: Capital d'adaptation, dette publique, processus de construction

Résumé: Ce papier étudie l'impact du changement climatique sur la soutenabilité de la dette publique, à travers l'impact d'un désastre naturel lié au changement climatique sur la décision du gouvernement d'investir en capital d'adaptation. Je construis un modèle RBC avec un choc climatique sur la productivité du travail et avec des dépenses gouvernementales. Celles-ci permettent d'accroître la résilience de l'économie en construisant du capital d'adaptation. Cela permet d'aider à réduire l'impact du choc sur l'économie. Je montre que l'investissement optimal du gouvernement dépend des hypothèses faites sur la construction du capital d'adaptation. Je documente l'importance du timing de l'ajustement du capital en réaction au choc avec des frictions et du temps de construction. Je relie cela à l'endettement publique à l'aide d'une règle *ad hoc* portant sur les taxes forfaitaires, pour montrer que la vitesse d'ajustement explique la magnitude de l'investissement et le comportement de la dette.

Title: Modeling the relation between adaptation investment and public debt

Keywords: Adaptation capital, public debt, process of construction

Abstract: This paper studies the impact of climate change on the sustainability of public debt through the impact of a climate-related natural disaster and the decision of the government to invest in adaptation capital. I build a RBC model with disaster shocks on worker productivity and government expenditures. The latter aim to increase adaptation by building adaptation capital. This helps reducing the impact of the shock on the economy. I show that the optimal investment of the government depends on the assumptions on how to build the adaptation capital. I document the importance of the timing of adjustment of adaptation capital both with frictions and time-to-build. I relate it to public indebtedness with an *ad hoc* rule on government lump-sum taxes to show that the speed explains the magnitude of investment and the behavior of indebtedness.

Contents

1 Introduction

As the last report of the Intergovernmental Panel on Climate Change states, the frequency of natural disasters related to climate change (floods, heatwaves, wildfire, storms, droughts mainly) will increase for disasters with higher intensities in the future. In particular, even if the exact intensity of future climatic disasters is difficult to predict, it is still possible to estimate the proportion of extreme events attributable to human activity as in Fischer and Knutti (2015). They find for instance that for, a 2°C warming, the fraction of precipitation extremes attributable to human influence rises to about 40%. This increasing trend is already observable: the frequency of natural disasters has increased over the past 60 years. Using EM-DAT data, the reference in terms of collect of natural disasters data, Figure 1 illustrates that the number of natural disasters has increased since 1970 for almost all regions in the world. Figures 2 and 3 show that this increase is mainly driven by weather related disasters, among which the meteorological and the hydrological disasters have an important role. We chose to start the graphics in 1970 because the Center for research on the Epidemiology of Disasters (CRED) was established in 1973 and although they have data for previous periods, it is likely that they suffer from two bias. First, as Felbermayr and Gröschl (2014) points out, there is a correlation between the development of the country where a disaster hit and the probability that the disaster is recorded since a country records all the more disasters than it has developed insurance markets. Thus, the development of insurance can explain some variability in the annual number of disasters. Second, it is hard to determine whether the low number of disasters before 1960 is due to relevant conditions about the climate or to the lack of measurement. Our graphs show that all continents seem to be affected, although not equally. When we consider the European continent, we observe that the increase in natural disasters occurrence is also driven by weather related disasters.

As defined by CRED-EM-DAT, a natural disaster is an event meeting at least one of the following requirements:

- 10 or more people are reported killed
- 100 or more people are reported affected, that is requiring immediate assistance such as aid to face the basic needs
- there is a declaration of a state of emergency
- there is a call for international assistance

Two distinctions are worth being made. The literature often distinguishes between direct and indirect effects of natural disasters (Pelling et al. (2002)). Direct impacts refer to the damages to fixed assets and capital, human losses and destruction. Indirect impacts refer to the damages on the economic activity: for instance, impact on consumption, investment, indebtedness, productivity. This paper will be interested in the latter.

The second distinction is the following. In the aftermath of a natural disaster, the intervention of the government consists of two parts. First, the government provides funding to

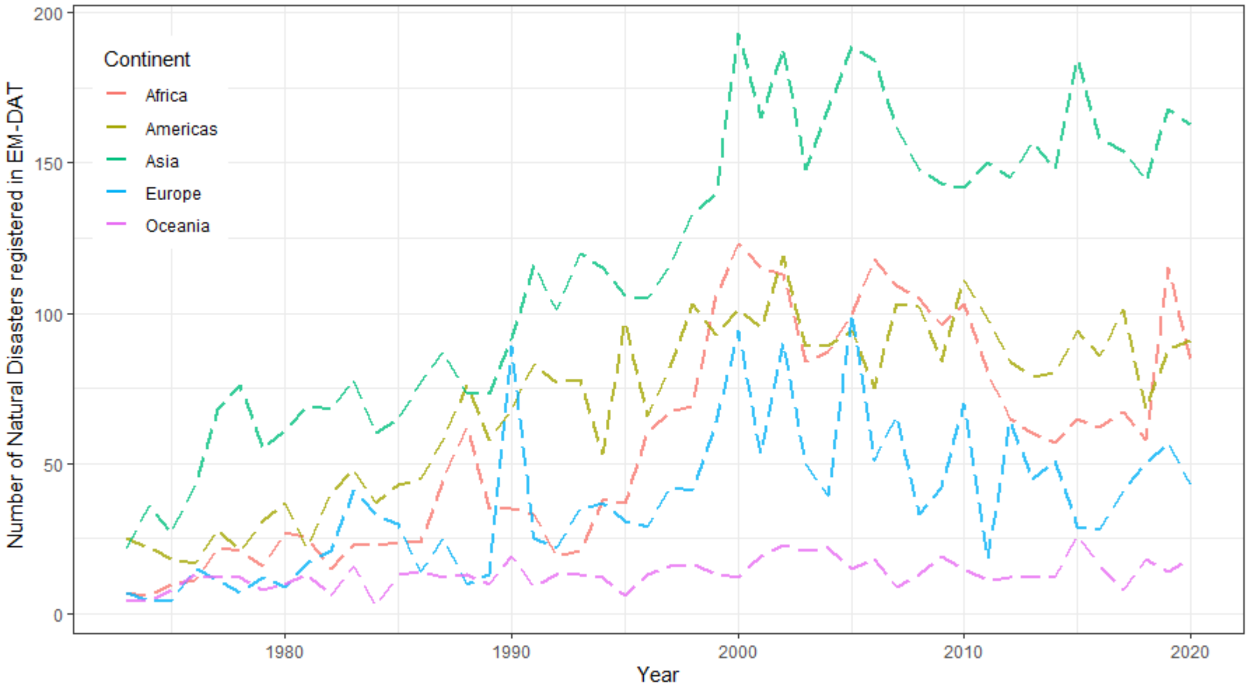

Figure 1: Annual number of natural disasters per continents since 1970
CRED-EM-DAT data, author's computation

help the affected population recover (in the form of medical aid, food supply, institutional organization to relocate the population or tax relief). This help may last for several periods and may be completed by the activity of insurance. Additionally, the government has to invest to rebuild the infrastructures that have been damaged by the disaster: houses, public buildings. It also has to organize transfers in favor of the affected population. However, this overlooks the fact that governments might also be incited to invest in adaptation capital, which may include regular infrastructures, but not necessarily. This investment is all the more important than the probability that a new disaster occurs within a short period of time is high. As a consequence, if we study an economy that has to face climate change, this incentive to invest in adaptation capital might be more relevant.

This paper will focus on the decision to invest in public capital. However, after a natural disaster, the need for government intervention raises the concern of the financing of this intervention. This financing may be rendered more difficult because of the contraction of the economy that a disaster can induce in the short term. This can cause a country to resort to international grants or more often public indebtedness and households' insurance for developed country when its own funds are not sufficient.
As mentioned in Melecky and Raddatz (2011), the mechanism of public indebtedness after a natural disaster can be the following. The government needs funding, like we previously mentioned, to finance reconstruction and aid interventions. It can rely on its accumulated funding only if it had fiscal buffer. It cannot resort to national taxation precisely because, depending on the scale of the disaster, the government cannot tax the households when they are poorer. If the government were to tax the households, it would be likely to worsen the impact of the

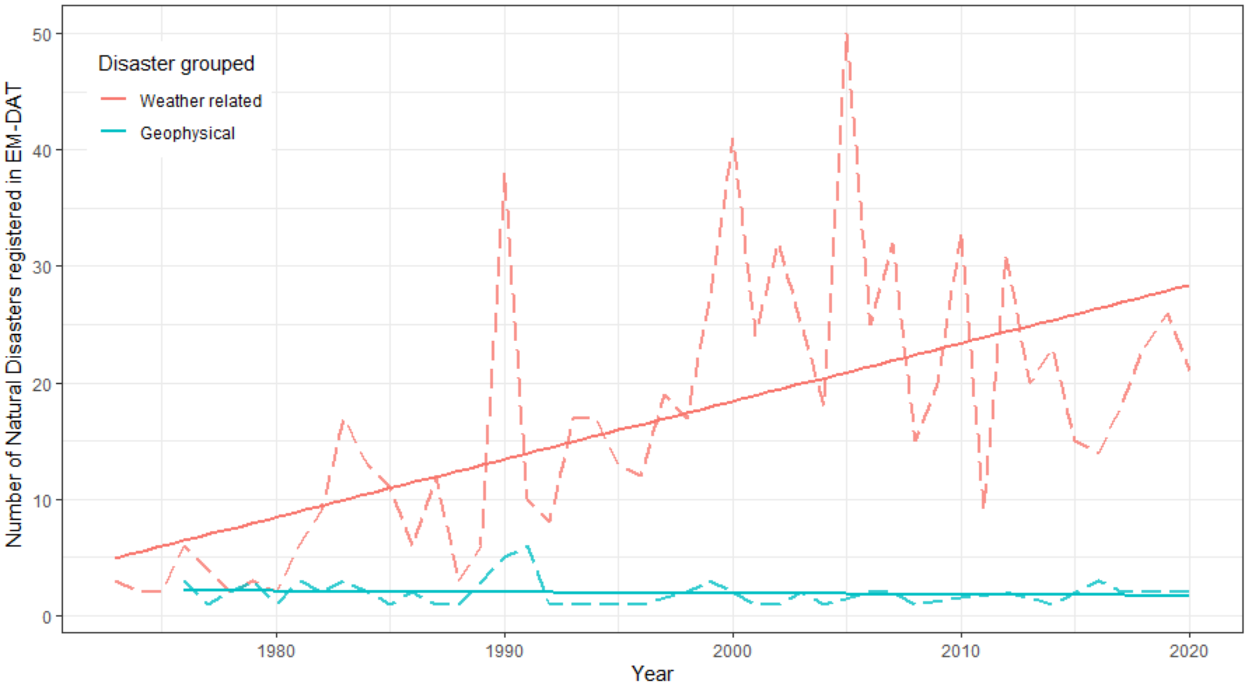

Figure 2: Occurrences of weather related disasters in Europe since 1970
CRED-EM-DAT data, author's computation

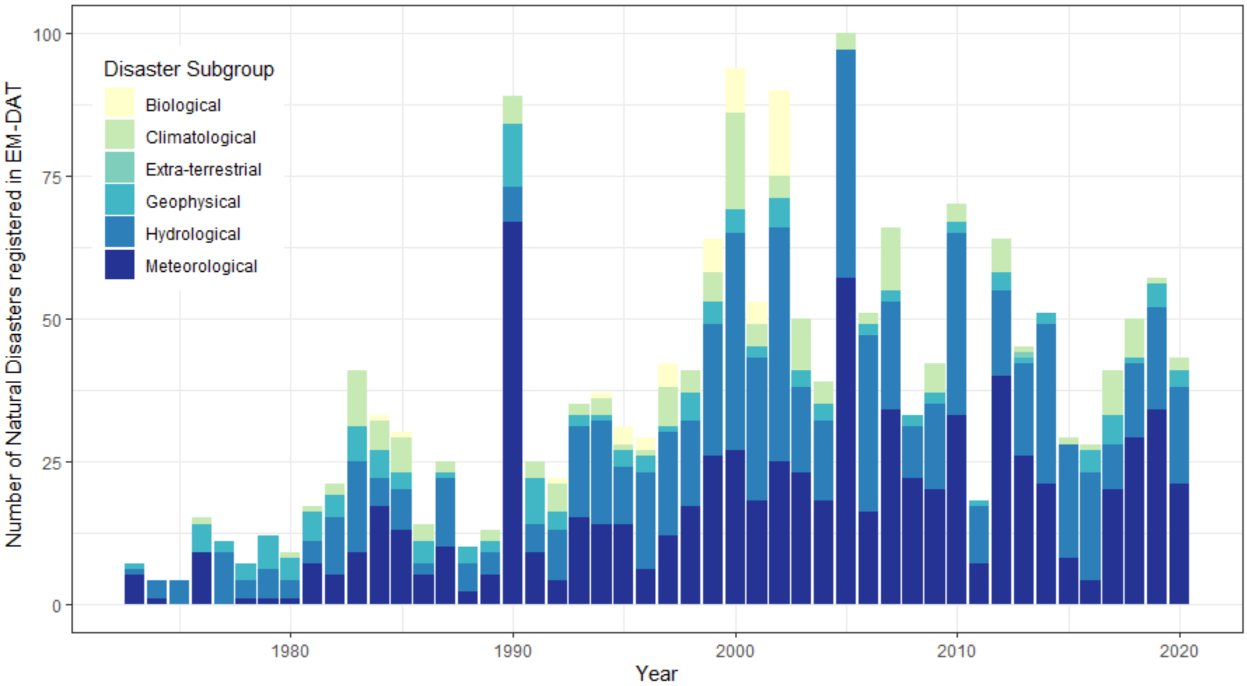

Figure 3: Annual number of natural disasters per type in Europe since 1970
CRED-EM-DAT data, author's computation

disaster on consumption (Marto et al. (2018)). As a consequence, it is often considered more optimal that the government resorts to national debt (or aid, for the developing countries). We will not focus on external indebtedness and will leave it for further researches.
Additionally, since we will be concerned with climate change related natural disasters, we need to account somehow for global warming. Taking into account the climate change in a model means most of the time that we add a damage function for the weather in the model that is related to temperature (see Nordhaus and Boyer (2003) for instance). We will account for it by computing our damage function in derivation to the optimal temperature.

Even though the literature on the relations between natural disasters and public debt is very large (see below for a review), little attention has been drawn yet on the relation between public investment in adaptation capital decision and public indebtedness, a relation that this paper intends to investigate. We will thus introduce a decision of the government to invest in adaptation capital. Because climate disasters affect both the households (and thus the firms) and the government, we introduce public productive capital that represents public infrastructures, which contributes to GDP (Aschauer (1989)). Because investment does not usually respond immediately, but progressively, we will introduce capital frictions on public and private capitals and time-to-build on public capital (as in Leeper et al. (2010), Bouakez et al. (2020)). The latter hypothesis allows to delay the effective creation of public capital, after 1 quarter (baseline scenario), 1 year or 3 years. These assumptions on frictions and time-to-build are all the more important, that we are dealing with a temporary shock that will eventually fade away. The speed of the reaction will play an important role in determining the possibilities of adaptation investment. Finally, we introduce an ad hoc rule on public indebtedness and lump-sum taxes. We will thus be able to study the mechanism through which adaptation capital influences the economy and how its combination with productive capital affects public indebtedness.

Additionally, the main focus will be on the short-term adaptation to climate change, even though the attention is usually set on the long-term costs of adaptation. We emphasize the description of the channel of adaptation over its estimation at horizon 2100. There are advantages and drawbacks to focusing on the short term. Most of the models that study climate change, like Nordhaus and Boyer (2003), do it on the long term. They usually implement different potential optimal trajectories of mitigation (that is policies aiming at reducing carbon emissions) under different scenarios (the Representative Concentration Pathways (RCP) and the Social-shared pathways (SSP)) and horizons. As a consequence, few attention has been given to the short and medium-term of adaptation. One possible issue with the medium-term is of course that it neglects some key issues: it ignores inter-generational issues or long-term shift in societies upon what the carbon transition is built. However, it is more adequate to draw attention on potential policy responses and to advise policy decision concerning adaptation. In fact, adaptation has to be decided in a shorter term than mitigation. Mitigation allows for instance to establish project of reduction of carbon emission at longer horizons, which is why the Paris Agreement of 2015 set some goals for 2050. On the contrary, adaptation also takes place when it is already too late and is more optimal when taken gradually, adapting at the pace of climate change rather than anticipating it (Millner and Dietz (2015)). Furthermore, focusing on the short term allows to study the concrete path of adaptation, that is the fact that adaptation decision come with frictions. This sheds light on the mechanism through which the economy goes when it faces a climate shock. Eventually, focusing on adaptation rather than mitigation takes into account the slow path of our current mitigation as denounced by the IPCC, which may end by requiring additional adaptation when the first major effects of climate change will be felt.

The focus on adaptation investment with regard to public debt raises awareness on the optimal reaction of a social planner after the economy is hit by a climate disaster. The financing by indebtedness if not free, and the government cannot get into debt forever. At the same time, it is not optimal to only invest in adaptation capital and stop investing in infrastructures. Also, investing too much in adaptation capital would crowd households' consumption and investment out. There are thus different trade-off when deciding to invest in such a public externality. To study optimal decision and for simplicity, we will resort to the resolution of our problem with a social planner. This is what justifies the ad hoc rule on public indebtedness and lump-sum taxes: they will not have any backward effect on the economy and will mostly be measures of how the public debt might react in reality. More precise specifications in following researches are welcome.

The remainder of this paper is structured as follows. Section II proceeds to a review of the different strands of literature to which this article pertains. Section III then presents the RBC model with public investment in productive and adaptation capitals and public debt. Section IV presents the calibration that is used to simulate the model. Section V examines the results of our different specifications. Section VI concludes.

2 Review of literature

This paper is related to two distinct strands of literature: the impact of natural disasters on public indebtedness and the investment in adaptation capital to face climate change (and thus natural disasters).

2.1 Public debt sustainability and natural disasters

It is first related to the literature on the estimation of the impact of natural disaster on public debt sustainability. The literature investigating this link is quite abundant.

Feyen et al. (2020) find evidence that the ratio debt-GDP is 2.4% higher one year after a disaster. Lis and Nickel (2009) explore the empirical impact of large-scale extreme weather events on changes in budget balance and they find that the budgetary impact of extreme weather events on a panel data of 138 country from 1985 to 2007 ranges between 0.23 and 1.1 percent of GDP depending on the country group and the measure for extreme events. They do not find any statistically significant effect on European country but find that developing countries are least resilient. Melecky and Raddatz (2011) examine the consequences of a natural disaster for middle and high-income countries over the period 1975 through 2008 on government expenditures and revenues. They divide the disasters into geological, climate, and a residual category, and first show that European countries are more affected in terms of budget by climatic disasters. As for lower-middle-income countries, the increase in deficits is noticeable for all kind of events. Second, they relate the response of a country facing a natural disaster to its ability to borrow and thus to its financial development. They find that a higher financial development is negatively correlated with the economic consequences of a disaster but that the budgetary deficit is larger. Deficit is however less important for countries with a high insurance penetration. Koetsier (2017) compares the path of government debt to a counterfactual and find that the maximum increase in government debt is 11.3 % of GDP and the average damages are around 9.8% of GDP with the maximum being attained between two and four years after the extreme weather event. It finds that the deadliest disasters increase the government debt between 12.4% and 19.4% for all countries and it can amount to 30% for the low-income country. The article explains that the modest negative effect that other studies have found on GDP is due to the government intervention increasing sovereign debt substantially through the deterioration of the public finances. Eventually, Melecky and Raddatz (2015), as in Melecky and Raddatz (2011), find that the deficit expansion of a country following a natural disaster is all the more important than the debt market of the country is developed. Hence, this suggests that the use of public deficit as a measure of the impact of disasters is problematic, as public deficit expansion might be correlated with a country's development.

Concerning the impact of natural disasters on the sovereign ratings, Feyen et al. (2020) find that sovereign ratings are lower during three years after the disaster for extreme weather events due to the rise of public debt after one year. Kraemer et al. (2015) simulate the impact of natural disasters on sovereign ratings and find a significant effect especially for earthquakes and tropical storms. Cevik and Jalles (2020) find the same results for the impact of climate change on sovereign credit ratings: countries that are more vulnerable to risks linked to climate change and natural disasters have higher rates.

Natural disasters can under some circumstances cause a country to default on its debt. Among several examples that have been studied, Sturzenegger and Zettelmeyer (2007) give the example of Ecuador that defaulted in 1997 a few months after floods caused major power shortages and van Dijck et al. (2000) report from the default of Suriname in 1998 following severe droughts that weakened the production of agricultural export goods. Klomp (2015) finds evidence that natural disasters increase the probability of default using the credit default premium that investors charge to a government as a measure of the perceived probability of default. He finds that the credit risk is at its highest directly after the disasters and can reach 15% one month after the disaster. Klomp (2017) uses a logit model to estimate the probability of sovereign default following a disaster. The larger natural disasters are found to increase the probability of default by 3%. Cevik and Jalles (2021) find the same increasing risk for climate change and its impact on default premium.

These empirical evidences have motivated the need for a modeling of the relation between natural disasters and public indebtedness. The literature remains however scarce.

Mallucci (2020) and Phan and Schwartzman (2021) introduce hurricane shocks in the Eaton and Gersovitz (1981) debt model of strategic default. More precisely, Mallucci (2020) studies the impact of extreme weather and climate change on the price of government bonds and governments' borrowing and the strategic decision to default that a government can consider while facing a natural disaster. He uses a framework a la Arellano (2008) to model this decision. They find that weather shocks decrease the ability of a government to issue debt. While few interest has been given yet to the estimation of the sustainability of a country's public debt as such, Pigato (2019) try to stimulate a regular model for debt sustainability for two countries: Jamaica and Dominican Republic. They use different policy scenarios, based on assumptions on the fiscal behavior of the two countries. They associate the unsustainability of public deficit with the reversal of their trend, i.e with a continuous increasing trajectory and they conclude that their countries are likely to default without contingency fund. Eventually, Marto et al. (2018) use a small open economy to assess damages to capital and productivity and the inefficiencies during the reconstruction process and the damages to the sovereign's creditworthiness. To model the possibility of a new shock facing the economy, they add a constraint on the government to return to its ex-ante situation in a given number of year to be able to face a possible new shock. They show how different policies can help diminishing the impact of a natural disaster: international donor grants, more resilient capital, external concessional debt, tax policy. They find that the amount of grants that would be required is impossible to achieve and that tax policy increases the magnitude of the shock on the households and external commercial financing can lead to a default.

Eventually, very few article have tried yet to estimate the link between future climate change and public debt, be it through sovereign ratings and probability of default. The paper by Agarwala et al. (2021) has tried to forecast the additional costs on debt paid by each country depending on the Representative Concentration Pathways (RCP) at the horizon 2100 and found that they could be of a magnitude above 5 between RCP 2.6 and RCP 8.5. One last remark: even though the concern of debt sustainability to face natural disasters has been more studied in the developing countries and especially in Caribbean Countries Rasmussen (2004), there are few papers that study the same question exclusively for European countries (Zenios (2021), Agarwala et al. (2021)).

2.2 Public investments

On the second hand, my paper is related to the literature on adaptation to climate change, and more precisely on the literature on protection public investment.
Adaptation can generally by defined as 'any adjustment in natural or human systems in re-

sponse to actual or expected climatic stimuli or their effects, which moderates harm or exploits beneficial opportunities' (IPCC). That is to say that there are several ways a country can adapt to climate change: more resistant capital, sensitivity of the economy (agriculture is more sensitive to climate change), availability of financial resources to finance the adaptation, good governance, information and so on. The literature is for this reason connected to the economics of development.
The developed countries are naturally already more adapted to climate change through their geography that is usually more favorable. Again, this doesn't mean that the more developed countries will not be affected at all.

The literature is however rather scarce when it comes to study adaptation as a control variable to see how adaptation decision may be taken optimally with other decision, that is considering adaptation decision as a control variable in a model. Hope et al. (1993) include a discrete adaptation decision in the PAGE model with a choice between no adaptation and aggressive adaptation. They model adaptation to make it decrease the "tolerable" climate change ... But the results they find are quite unrealistic, calling for a study of optimal decision. Tol (2007) and Fankhauser (1995) modeled a continuous choice of adaptation based on sea level rise and coastal protection but they do not take any other form of adaptation into account. De Bruin et al. (2009) introduce adaptation in a DICE model to make the trade-off between adaptation and mitigation more visible than in regular DICE model. They find that their AD-DICE model can reproduce the performance of the baseline DICE99 model under optimal adaptation. However, they consider only flow adaptation since they consider that each decade chooses the level of protection for the ten periods, but that protection is a one time-period, meaning each period within the ten periods, the government pays and receives the given level of protection: "this implies that both the costs and benefits of adaptation are instantaneous". Agrawala et al. (2010) introduce adaptation as a policy variable in three Integrated Assessment models (IAM): the DICE, RICE, and WITCH models. However, they confide mostly in numerical simulations. Millner and Dietz (2015) study the optimal balance between investment in traditional productive capital and adaptation capital and they find that in Africa, it will be optimal, in the coming decades, to grow the stock of adaptation capital more but that the adaptation capital will never exceed 1% of the economy.

3 Model

To shed light on the mechanisms relating adaptation investment to public indebtedness, I will focus on the problem of a central planner in a short to medium-term RBC model with both public productive and adaptation capitals.

3.1 Households

The economy is populated with an infinitely lived representative household who maximizes the following lifetime objective function:

$$\max_{C_t,N_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t, N_t)\right] \tag{1}$$

where $\beta \in [0,1]$ is the discount factor, C_t is the consumption at period t, N_t is the labour supply at period t and $U(\cdot)$ is the utility function. We have that $U(.)$ is increase and concave in C_t, that is $\dfrac{\partial U(C_t,N_t)}{\partial C_t} \geq 0$ and $\dfrac{\partial^2 U(C_t,N_t)}{\partial C_t^2} \leq 0$ and is decreasing and concave in N_t.

We will use preferences of the following form:

$$U(C_t, N_t) = \frac{\left[C_t^{\gamma}(1-N_t)^{1-\gamma}\right]^{1-\sigma}}{1-\sigma} \tag{2}$$

with $\sigma > 0, \sigma \neq 1$ and $\gamma \in [0,1]$, and if $\sigma = 1$:

$$U(C_t, N_t) = \gamma C_t + (1-\gamma)(1-N_t) \tag{3}$$

Additionally, the household invests in new capital with respect to the following law of motion :

$$K_t = (1-\delta)K_{t-1} + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) \cdot I_t \tag{4}$$

where δ is the depreciation rate and $S(.)$ is the adjustment cost function a la Christiano et al. (2010), with $S'(.) \geq 0, S''(.) \geq 0$.

Eventually, we have the household's budget constraint:

$$C_t + B_t + I_t \leq W_t N_t + (1 + R_{B,t-1})B_{t-1} \tag{5}$$

with $R_{B,t}$ the net return on nominal bonds purchased at time t, verifying:

$$1 = \beta(1 + R_{B,t})\frac{\mathbb{E}_t\left[U_{C,t+1}\right]}{U_{C,t}} \tag{6}$$

3.2 Firms

Firms are supposed to be in perfect competition and to produce the output Y_t with the following technology:

$$Y_t = F(N_t, K_{t-1}, K_{G,I,t-1}, K_{G,P,t-1}, \tau^W) = \left(\Omega(K_{G,P,t-1}, \tau_t^W) \cdot N_t\right)^{\alpha} (K_{t-1})^{1-\alpha} (K_{G,I,t-1})^{\alpha_G} \tag{7}$$

where $K_{G,I,t-1}$ is the pre-determined stock of public productive capital as in Leeper et al. (2010) and Bouakez et al. (2020), $\Omega(\cdot)$ is the damage function, $K_{G,P,t-1}$ is the pre-determined stock of adaptation capital and where τ_t^W is the current deviation of the temperature with respect to its mean.

τ_t^W evolves according to the following process:

$$\tau_t^W = \rho_\tau \tau_{t-1}^W + \varepsilon_t^\tau,\ \varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau) \tag{8}$$

3.3 Government

Adaptation can be thought both as flow and as stock (Millner and Dietz (2015)). As a flow, we can consider all the adaptation efforts for which costs and benefits happen at the same period, for instance a change in an agricultural fertilizer. As a stock, we can consider all efforts for which the costs are endured currently but for which the benefits will arise in the future, for instance the building of a dam. We will focus on the latter.
The government can choose between productive investment $K_{G,I,t-1}$ that contributes to the production (Leeper et al. (2010), Bouakez et al. (2020)) and adaptation capital $K_{G,P,t-1}$ (De Bruin et al. (2009), Millner and Dietz (2015)) that decreases the impact of a future climate shock.

According to Leeper et al. (2010), Bouakez et al. (2020), it is more accurate to model public investment with implementation delays, leading to a time to build hypothesis. The law of motion of public investment is thus:

$$K_{G,j,t+N-1} = (1-\delta_j)K_{G,j,t+N-2} + \left(1 - S\left(\frac{A_t^j}{A_{t-1}^j}\right)\right) A_t^j,\ \text{for } j = I, P \tag{9}$$

where N is the number of periods a project requires to be achieved, A_t^j is the authorized government investment and δ_j the depreciation rate of the corresponding public capital.

In the interest of realism, I will introduce the staggering of expenditures as in Leeper et al. (2010) with:

$$G_t^j = \sum_{n=0}^{N-1} \phi_n A_{t-n}^j,\ \text{for } j = I, P \tag{10}$$

and

$$\sum_{n=0}^{N-1} \phi_n = 1$$

where G_t^j is the effective implemented government investment at time t. The dynamic of public investment is thus the following: the government votes a budget A_t^j to invest at each period t and the total amount of A_t will be spread over N period, weighted at each period by the ϕ_n corresponding. Hence, at each period t, the total spending is a weighted sum of the N last decisions to invest that are still being staggered. The interest of introducing this time to build hypothesis is to be closer to reality in showing how a decision to invest in adaptation capital can be taken. For instance, it is not optimal to invest in the same amount of adaptation

capital depending on how long we will invest. This is due to the residual importance of the shock when adaptation capital will be realized. Equation (42) shows that the current marginal value of adaptation capital will depend on the value of the shock when adaptation capital will be built. As such, with frictions, time-to-build induces some realistic constraints on the investment decision that will then generate different analysis on the reaction of indebtedness.
One remaining precision: we will not use the friction function in the specification of Leeper et al. (2010), because it is not used in the original model.

Eventually, we must consider the indebtedness behavior of the government. To model this, we will use an ad hoc rule for debt indebtedness. To include a debt behavior of the government, we need to include the related interest rate and lump-sum taxes with a sufficient reaction to debt to avoid a potential explosion of debt. We define the primary ratio surplus s_t as:

$$s_t = \frac{T_t - G_t^I - G_t^P}{Y_t} \tag{11}$$

with T_t the lump-sum taxes, with the following policy rule on T_t to ensure the convergence of debt and for realism purposes:

$$\frac{T_t}{Y_t} = \lambda b_{t-1} + \mu - \frac{GI_t + GP_t}{Y_t} \tag{12}$$

where λ and μ have to be calibrated to ensure the non-explosion of debt (λ) and the steady-state value of the debt over GDP ratio $b_t = \frac{B_t}{Y_t}$. Eventually, the intertemporal government constraint is:

$$(1 + R_{B,t-1})B_{t-1} + G_t^I + G_t^P \leq B_t + T_t \tag{13}$$

We leave for Appendix A the derivation of the conditions on λ ensuring an non-explosion of debt.

3.4 The climate change related natural disaster

As the introduction pointed out, there are different types of climate change related natural disasters that can occur. We can mainly distinguish between the disasters that affect worker productivity like heatwaves and the disasters that affect the stock of capital like landslides and floods. We will focus only on heatwaves in this paper.

3.4.1 Temperature shock

We will henceforth assume that the temperature shock affects the economy mainly through its impact on worker productivity. That is, we will have a function $\Omega(K_{G,P,t}, \tau_t^W)$ of both public adaptation capital stock $K_{G,P,t}$ and of the deviation of temperature τ_t^W with respect to its mean with :

$$\tau_t^W = \rho_\tau \left(\tau_{t-1}^W - \tau^{opt}\right) + \varepsilon_\tau \tag{14}$$

where we have thus $\tau_{SS}^W = \tau^m$ at the steady state: the deviation at the steady-state means that even at the steady-state, the temperature affects the economy. The shock ε_τ of temperature is assumed to affect the economy through its impact on the production function, as in Equation (7).

Our damage function $\Omega(K_{G,P,t-1}, \tau_t^W)$ must therefore satisfy some requirements. We want the function to be increasing in adaptation capital, $\Omega_{K_G}(K_{G,P,t-1}, \tau_t^W) \geq 0$ since more adaptation capital decreases the damage caused by the shock. We also want the marginal utility of adaptation capital to be zero when the temperature is at its mean : $\Omega_{K_G}(K_{G,P,t-1}, \tau^m) = 0$. Moreover, we want $\Omega(K_{G,P,t-1}, \tau_t^W)$ to be decreasing if the temperature is above its mean to model the impact of extreme temperatures on productivity: $\Omega_{\tau^W}(K_{G,P,t-1}, \tau_t^W) < 0$, which we could enrich by saying that the temperature impacts productivity only above a certain level. The interpretation of $\Omega(\cdot)$ is thus by how much the productivity will be decreased with a rise in temperature and how much adaptation investment will reduce this decrease by increasing the value of the productivity after the same shock. Damages therefore happen when $\Omega(\cdot)$ decreases.

Millner and Dietz (2015) use the following damage function with adaptation capital:

$$\Omega(K_{G,P,t-1}, \tau_t^W) = \frac{1 + g(K_{G,P,t-1})}{1 + g(K_{G,P,t-1}) + \tau_t^W} \tag{15}$$

where

$$g(K_{G,P,t-1}) = \beta_1 K_{G,P,t-1}^{\beta_2} \tag{16}$$

with $\beta_1 > 0$ and $\beta_2 \in [0, 1]$, and

$$f(\tau_t^W) = \alpha_1 \tau_t^W + \alpha_2 \tau_t^{W^2}$$

which is the usual climate change function used in IAM, here adapted to represent how the changes in temperature will affect the damage function, with $\alpha_1 \geq 0, \alpha_2 \geq 0$. We let the proof that the damage function corresponds to our conditions in Appendix B.

It is often agreed upon the fact that the optimal way to adapt to climate change is to start early Drudi et al. (2021). In our baseline stochastic framework, we will keep a value of mean temperature different from what we call the "optimal value of temperature", that is the value under which there is no damage on the economy at the steady-state. This has some modeling purposes: with β^2, the marginal benefit of investment in adaptation capital would be infinite at the steady-state in the case where the steady-state value of temperature, and thus adaptation capital, would be zero. This comes from the first-order condition on $K_{G,P,t}$. Our model will thus assume that the temperature represented by τ^W is the mean deviation with respect to the optimal temperature, implying that the investment in adaptation capital is optimal at the steady-state, allowing to simulate an optimal reaction to a given temperature shock.

As explained in Appendix C, we will set β_2 free to calibrate it so as to have different scenarioss of the steady-state share of adaptation capital over the total of public capital.

3.4.2 Other climate related shocks

Although a certain part of the literature on weather shock has focused on the impact on agricultural outputs and its spread to the rest of the economy in two sector models (Gallic and Vermandel (2020), Milivojevic (2021)), it is unlikely that the government invest in adaptation capital against this type of mechanism where subsidies and private protection is more adapted. To remain close to the idea of climate disasters, we fill focus here mainly on weather shocks that can be related to climate change and that impact the economy in such a way that the main adaptation in adaptation capital is due to the government.

4 Calibration

4.1 Calibrated parameters

4.1.1 Spending process

For the delay in the investment hypothesis, we use the calibration by Leeper et al. (2010). When the number N of periods to build is 12 (that is when the project requires 12 quarters in order to be built), when we are dealing with large infrastructure projects, like for instance the building of protective dikes, the parameters are the following : 25% of the budget is spent on the four first periods with $\phi_0 = 0$ and $\phi_1 = \phi_2 = \phi_3 = \frac{1}{12}$ and the rest is spread equally over the remaining periods : $\phi_4 = \cdots = \phi_{11} = \frac{3}{32}$. For projects of smaller magnitude, they can last only one year with $N = 4$ and $\phi_0 = 0, \phi_1 = \phi_2 = \phi_3 = \frac{1}{3}$ or one quarter with $N = 1$. These smaller magnitude projects represent for instance the investment in air conditioning or the implementation of a prevention policy like some awareness-raising against the danger of heatwaves.

It is here relevant to note again that the different types of shocks might allow for different speeds of adaptation and then for better resilience in the future. Also, the past history of shocks might decrease the implementation delays, a mechanism for which we do not account. For instance, it is likely that the heatwave of 2003 has showed the different government the importance of communicating about health in such situations, especially concerning the health of older people, leading to a lesser impact of the following heatwaves.

4.1.2 The discounting of the future

There is a still ongoing debate as to how much to price future benefits compared to current costs, that has started with the debate between Stern (2006) and Nordhaus (2007). Our paper is concerned by this debate, in such an extent that, even though the decisions that we want to model doesn't exceed the range of ten years, there is still an intertemporal discounting of future utility.

We will therefore calibrate β and R so that we have a interest rate of 2% per year. Even if the interest rate is of 3% in Nordhaus and Boyer (2003), the current interest rates are much lower than 3% and 2% is a compromise between literature and current state of the economy.

4.1.3 Adaptation parameters

As explained in Appendix C, we will use the calibration from De Bruin et al. (2009) and leave β_2 free. This allows the model to have a more reactive damage function with the value of adaptation capital than with the calibrations used in Millner and Dietz (2015). In their paper, their calibrations give approximately one value of the damage function depending on the temperature, whatever the value of adaptation capital is, because their parameters are low. This is the same as saying that adaptation capital is useless and does not influence the resilience of the economy. However, we did not use the function form of De Bruin et al. (2009) since they include both the benefits and the costs of adaptation capital. In our model, the costs are already given in the first order conditions, so it would be misleading to add some others.

4.2 Summary - parameters

Parameter	*Interpretation*	*Base case value*
σ	Preference parameter	1
γ	Preference parameter	$\in [0.37; 0.43]$
β	Discount parameter	0.98
α	Elasticity of labour	0.4
α_G	Elasticity of public productive capital	0.1
δ	Depreciation rate for private capital	0.06
δ_I	Depreciation rate for public productive capital	0.06
δ_P	Depreciation rate for adaptation capital	0.02
share	Share of adaptation capital over total public capital	$\in \{5\%, 15\%, 30\%\}$
ω	Friction parameter for private capital	2
ψ	Friction parameter for public capital	2
α_1	Gross damage multiplier parameter	0.0012
α_2	Gross damage multiplier parameter	0.023
β_1	Effectiveness of adaptation	0.115
β_2	Effectiveness of adaptation	$\in [0.5; 2]$
τ^{opt}	Optimal mean temperature	15
τ^{m}	Effective mean temperature	19
λ	Reaction parameter for lump-sum taxes ratio to debt ratio	0.05
μ	Reaction parameter for lump-sum taxes ratio to calibrate $b = 0.6$	$\in [0.13; 0.16]$

Table 1: Parameter values

5 Presentation of the mechanisms: the relation between adaptation investment and public debt

For the optimal reaction in a simple RBC model, we let the derivation of the model in Appendix D. We will set $\tau_{ss} = 4$, that is a mean deviation of 3 degrees from the optimal temperature, as the baseline. This is not particularly meaningful, since our damage function parameters are calibrated for the absolute temperature, and not for such a deviation.

A first general remark can be made. It is never optimal, for the government, to invest in adaptation capital up to setting a steady-state value of the damage function $D = 1$: the marginal net reduction of damages in the production function of capital will be zero in the end. This has some policy implications, as this means that it will not be optimal, once in a situation of changed climate, to try to adapt so as to keep the same living standard. From our different specifications, with different hypotheses on the shape of the staggering of public expenditures, on the share of adaptation capital and the value of β_2, we have found several values for D ranging approximately in the interval $[0.84; 0.98]$.

We now turn to our modelling results. We study, as our baseline, the heatwave shock on labour productivity with no productive public capital and no time-to-build.

5.1 The mechanisms that result from the introduction of adaptation capital

5.1.1 The most basic model

To start from scratch, we will first explain the impact of adding frictions on private investment in the simplest model, without even adaptation capital. This explanation will help us to understand the differentiated impact of our different specifications afterward. The impulse-response functions are displayed in Figure 14 in Appendix E. Without friction, the decrease in output leads the household to reduce its investment and its consumption. The decrease in investment is related to the decrease in output and in profitability of capital. Concerning labour supply, we mainly observe the domination of the substitution effect between labour and leisure: when the household becomes poorer, it substitutes labour for leisure.
However, for labour supply, when we add frictions, we observe that the substitution effect is first observably counterbalanced by a strong income effect: the household wants to work more initially. The decrease in consumption induce the household to need to supply less labour. The precise mechanism is the following. Since the decrease in investment only adjusts slowly, it is not optimal to reach it immediately. Hence, the investment will decrease over several periods. Consumption is, in the simplest economy, what is not used by investment and therefore consumption decreases more when the shock hit, as investment decreases less. This larger decrease in consumption creates the income effect, leading households to work more. Whether the household will indeed start by supplying more labour than the steady-state value or not

will only depend on the initial value of $\frac{I_1}{Y_1}$ and whether this will set $\frac{C_1}{Y_1}$ below or above the steady-state. This thus depends on the magnitude of frictions.

More precisely, our specification induces the intratemporal choice between labour and consumption given by equation (73). This allows to illustrate the channel of the income and substitution effects. When wage increases marginally, labour supply is increased by $\frac{\partial N_t}{\partial W_t} = \frac{1-\gamma}{\gamma}\frac{C_t}{W_t^2}$. The effect of income acts through a marginal variation in consumption, giving $\frac{\partial N_t}{\partial C_t} = -\frac{1-\gamma}{\gamma}W_t$. We have thus: $dN_t = \frac{1-\gamma}{\gamma}\frac{C_t}{W_t^2}dW_t - \left(\frac{1-\gamma}{\gamma}W_t\right)dC_t$. When output decreases, W_t decreases as well, so $\frac{1-\gamma}{\gamma}\frac{C_t}{W_t^2}dW_t < 0$ (negative substitution effect) and thus, labour supply will depend on how consumption is affected (channel through which the income effect affects N_t). Since $-\frac{1-\gamma}{\gamma}W_t < 0$, if consumption is decreased sufficiently by the shock, then labour supply can be increased by it. Thus, with frictions, consumption will be more decreased, implying a large negative income effect, and then increases steeply, implying a positive income effect. Finally, equation (5) gives us the fact that the reaction of investment will determine the one of consumption. Hence the first income effect with frictions on investment, since consumption's decrease is larger and the second income effect afterward since consumption increases steeply due to the frictions on investment.
It is important to recall this mechanism, as it lays the basic explanation that we will resort to when explaining how optimal investment decision on adaptation capital influences the economy when using frictions and time-to-build. Also, these explanations lay the basis for further researches that will implement the distorting effect of taxes and debt on the economy.

5.1.2 With adaptation capital

We now include adaptation capital in the economy. Figure 15 in Appendix E displays the reaction of the economy to a labour-productivity shock and the decision of public adaptation investment without friction, that is by setting $\psi = 0$ and $\omega = 0$. The shock hits the economy in period 1, reducing output and thus private consumption and investment. The reduction in output also induces a decrease in the marginal productivity of capital and of labour, causing a reduction in private investment. Here, we observe that the labour supply displays a similar behavior than in the simplest case with frictions as in Figure 14. However, without friction on capital, the cause of this behavior lies in the introduction of adaptation capital and is due to the substitution effect caused by wage and indirectly by output.

The mechanism is the following. The investment in adaptation investment is done in the first period, up to the equalization of marginal productivity and marginal cost of adaptation capital when it will be realized in the second period. Then, in the second period, the optimal investment in adaptation capital decreases below its steady-state value and adaptation capital is built. This is due to the reduction of damage induced by adaptation capital and thus the decrease of the stock of adaptation capital required. This is equivalent to saying that the damage function has a first rapid movement toward its steady-state value before going more smoothly.

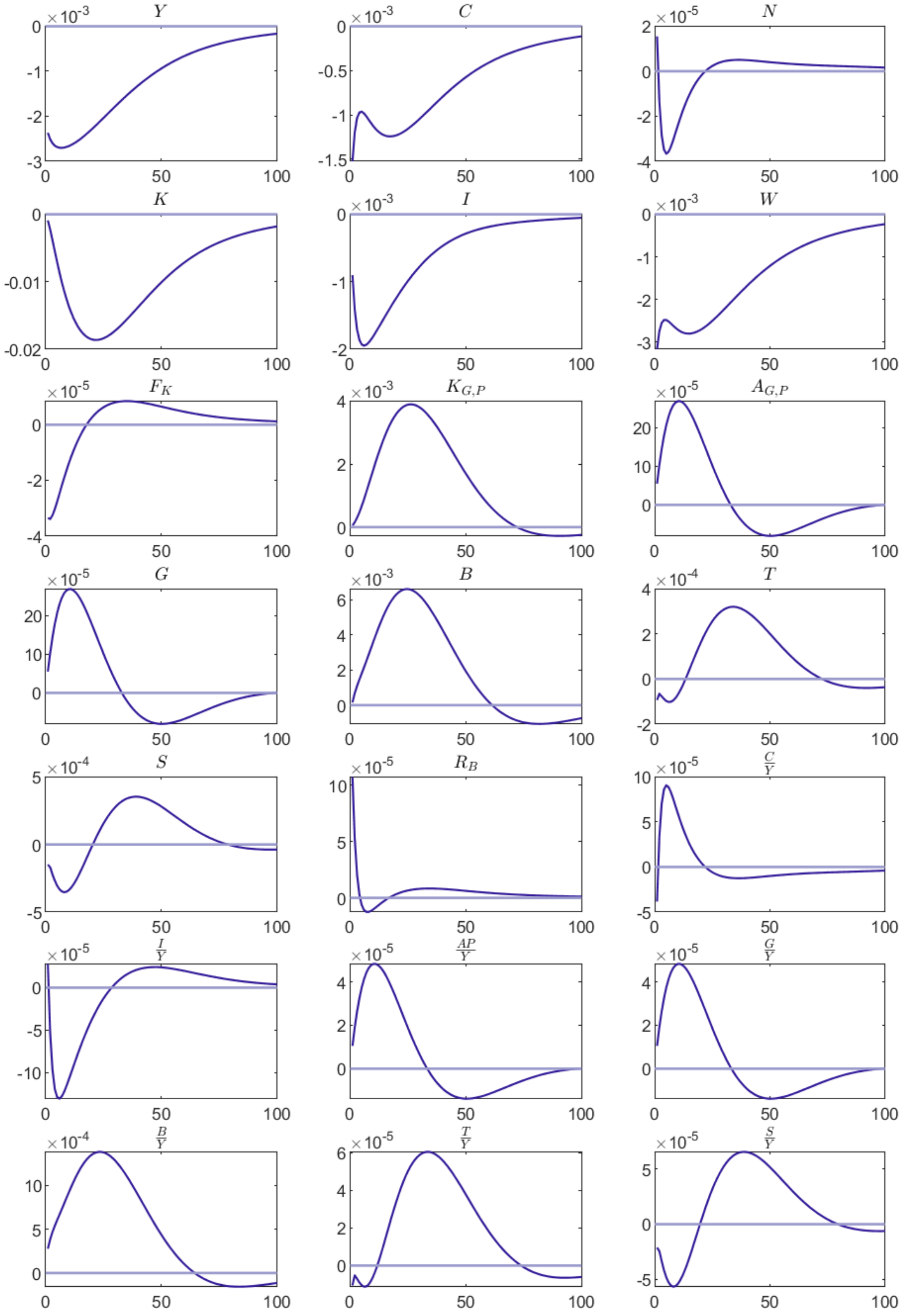

Figure 4: Reaction of the economy to a heatwave in an economy featuring public adaptation capital

However, the important magnitude displayed in adaptation investment results in an almost equivalent important decrease in private investment I_t and thus in private capital K_t for the following periods. This results from the equilibrium of the economy $Y_t = I_t + G_t^P + C_t$. This induces the subsequent steep decrease in output. Labour supply first displays the same counterbalancing effect of income effect because of the decrease in consumption, but then displays a substitution effect due to the impact of output on wages and thus on the cost of labour. The simulated values give approximately a substitution effect of 10%, that is: a movement of wage of scale 1 will induce a corresponding movement of labour supply of scale 0.1.

The substitution effect is initially stronger because of the very important magnitude of the adjustment of private investment and thus capital due to the very important adjustment of adaption investment (the value of adaptation investment almost doubles !). This contributes to the steep decrease of output that we observe in the first periods and thus the dominance of the substitution effect on labour over income effect.
We can observe that, when we introduce frictions as displayed in Figure 4, that is when it is costly to adjust adaptation and private investment as suddenly as in the previous case, we observe the more regular effects that we can also observe in the case without adaptation capital and with frictions on private capital. This induces thus a similar behavior on private consumption and thus the initial dominance of income effect on labour supply. We can explain this by the fact that adaptation capital adjust with a less important magnitude than before, as the maximal utility of adaptation capital is early, during the highest effects of the shock, and frictions only allow capital to be built when the shock has already started to vanish.

5.1.3 The mechanisms on public debt

We now turn to the interpretation of the mechanism on indebtedness. First, the interest rate on nominal government bond is determined by the evolution of the marginal utility of consumption. Since the household consumes less due to the fall of output, the marginal utility of consumption is thus higher when the shock hits than it steady-state value, explaining the increasing in R_B. Then, after the realisation of investment up to the marginal benefit of private capital due to delay, consumption decreases again, explaining the decrease in R_B. Thus, this means that, since consumption increases in the few first periods, the return on nominal government bond must be very high for the household to buy bonds rather than to consume, as in a standard Euler equation, so that the government has to set it high to borrow from the household in the few first periods.

In our specification, the increase in government spending ($G = A^P$ here) impacts the intertemporal government budget constraint (13) and the policy rule (12) determining lump-sum taxes. Using both, we can rewrite:

$$B_t = (1 + R_{B,t-1} - \lambda)B_{t-1} + 2G_t + \mu Y_t \tag{17}$$

with $G_t = G_t^I + G_t^P$. Thus, government debt increases with public expenditures, with past period debt (as $\lambda = 0.05$ and $R > 0$) and with output depending on μ. Since in Equation (23), we calibrate $b = 0.6$, $\mu > 0 \iff 2g > -0,01776$, we can assume $\mu > 0$. Thus, indebtedness varies positively with output.
On the contrary, the behavior of lump-sum taxes ratio t_t may vary, depending on:

- whether b_{t-1} increases or not
- whether the public expenditures over GDP g_t increases or not
- and is increasing with output Y_t

Let's recall that in our model, public debt reacts to public expenditures but do not finance it, as don't do lump-sum taxes. Lump-sum taxes T_t will thus vary depending on the one of the both effect that is more important: either the decrease induced by an increase in public expenditures, or an increase due to the increase in output and public debt. However, the effect of public expenditures is larger, given that the increase in b_{t-1} is dampened by the low value of the λ and that the same stands for Y_t and μ. The idea is that lump-sum taxes adjust more to finance public expenditures that it does to ensure non-explosion of debt. As public expenditures will also adjust by themselves, this will act as an automatic stabiliser.

We can now explain our impulse-response functions. Concerning indebtedness, as debt is more sensible to public expenditures (2 to 1) than to output variations (μ to 1), even if public expenditures adjust with less magnitude than output, it can be enough to induce an increase in indebtedness.
As for lump-sum taxes, the initial decrease in T_t is explained by the more important increase in magnitude of G_t because of the need to invest in adaptation capital and the decrease in μY_t than the increase in λB_{t-1}. This specification is justified by the fact that lump-sum taxes increase is not intuitively optimal when the primary concern is to expand public expenditures, here to protect the economy, but is when the value of debt is too high compared to public expenditures: for instance, when adaptation investment is below its steady-state value because it is not optimal anymore to invest in additional investment capital, the decrease in public expenditures is not enough to counterbalance the dynamic of public debt, that keeps increasing. Thus, when public expenditures are not as necessary as previously, it is the moment where it is more optimal to increase lump-sum taxation to cool the indebtedness.
Eventually, the first kink in lump-sum taxes is due to the fact that public expenditures G have an immediate reaction to the shock due to the reaction of expenditures but with a lower slope than B that depends positively on G but negatively on past output but with a higher slope.

5.2 Introduction of public productive capital

We now introduce public productive capital in the economy. As Figure 5 displays it, most of the economy has the same reactions, except for the parts related to indebtedness.

The first-order conditions of public productive and private capital beings the same, it is logical that they feature a similar behavior in their reaction to the productivity shock, with only

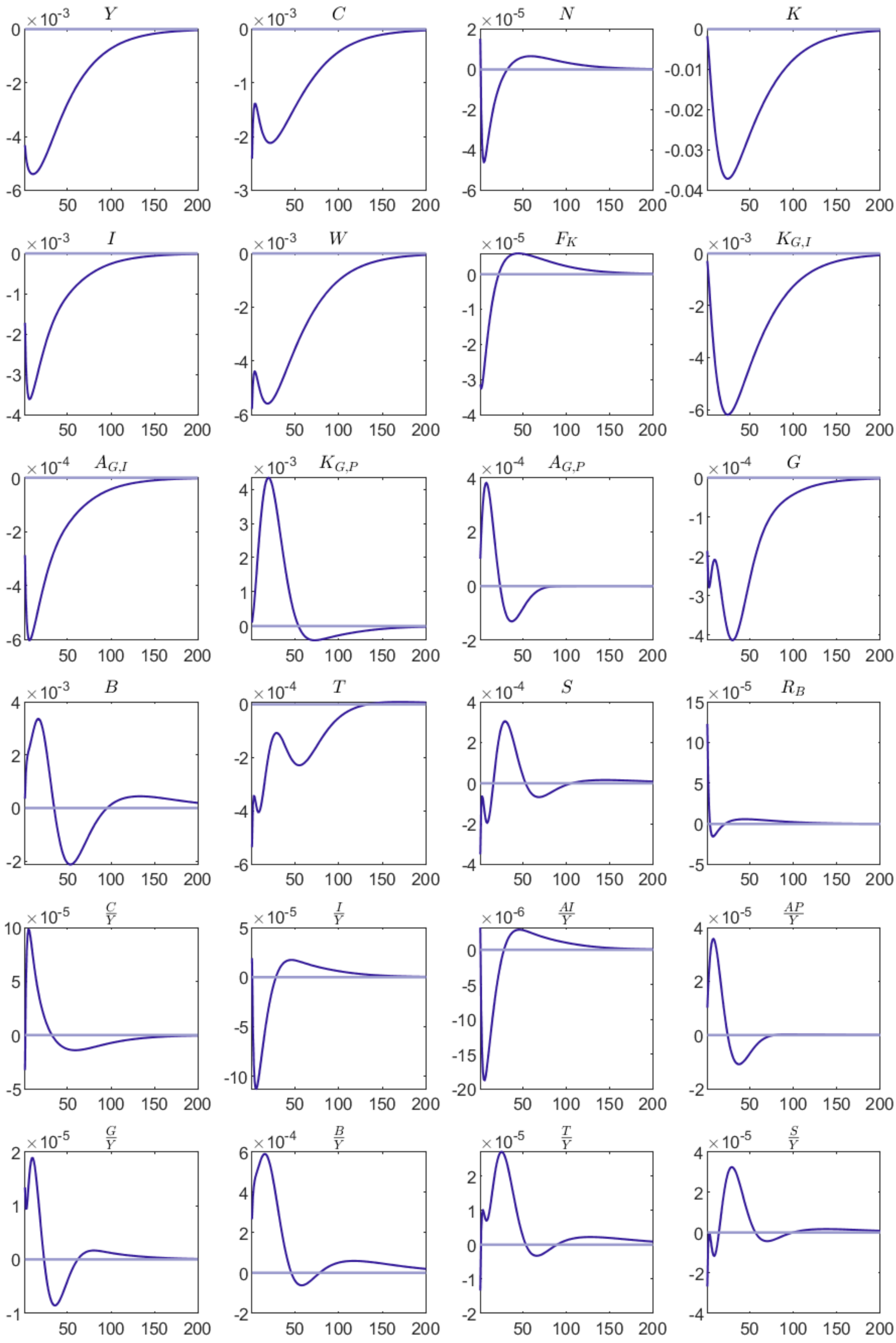

Figure 5: Reaction of the economy to a heatwave in an economy featuring public productive capital

the magnitude being different. In Figure 5, since we include frictions on capital investment, we can observe that the decline in output induces public productive investment to decrease with a progressive adjustment. Here, we observe that the respective magnitudes of both public capital investments induce total public expenditures to be negative: this comes from the fact that public productive investment will only be below its steady-state value with a first decrease that has a more important magnitude than the initial increase in public adaptation investment, and the fact that public adaptation investment will decrease when the optimal stock of adaptation capital will be attained. Again, this is due to the frictions on adaptation investment, that diminish its benefits by delaying it. In the impulse-response function for G, the first decline is related to the initial movement of both investments, the first increase to the comparison between the trends, and then the rest is related mostly to the movement of public adaptation investment.

Accordingly, this induces a different behavior of our public debt rule. Like previously, the movement of B and T are related to public expenditures, output and the value of λ. Initially, when the shock occurs, the steep decrease in public expenditures provokes a steep increase of debt over GDP and both combined for the steep increase in tax over GDP. The rest is then a combination of public expenditures over GDP and past public debt for B and T.

However, this special behavior is largely attributable to the share of public adaptation capital over total public capital, on what we come back below.

5.3 The share of adaptation capital in public capital: different cases

In this subsection, we consider the different behaviors of the model for different values of the share of adaptation capital over total public capital. We set the time-to-build to one period (which is the baseline case) and we study the behavior of the economy with the optimal reaction in adaptation capital investment by the government. We will study three different scenarios for the share $\frac{K_{G,P}}{K_{G,P}+K_{G,I}}$ of adaptation capital over total public capital at the steady-state: 5%, 15% and 30%. 30% might seem a bit excessive and unlikely. However, in the hypothesis that we don't abide by the IPCC recommendations, it is possible that, for the most exposed countries, maybe not in Europe but in warmer regions like India, Sahara aso..., adaptation capital will have to be present in such share of total public infrastructures at some point. Also, this will allow us to highlight the differences between the different reactions.

We have to be very careful while interpreting Figure 19, since the impulse-response functions displayed belong to different economies, with different steady-state values since β_2 is different from each of them (ranging from 0,4573 to 1,9489 through 0,7976). The value of β_2 for 30% share is not even close to the one found in De Bruin et al. (2009) where $\beta_2 = 3.6$. It is also obvious that the economy with the higher share of adaptation capital will have a steady-state value of its damage function higher, reducing less output at the steady-state and thus reducing the reaction once hit by a shock.

For the economy, the reactions have similar shapes in most cases, except in the reaction of

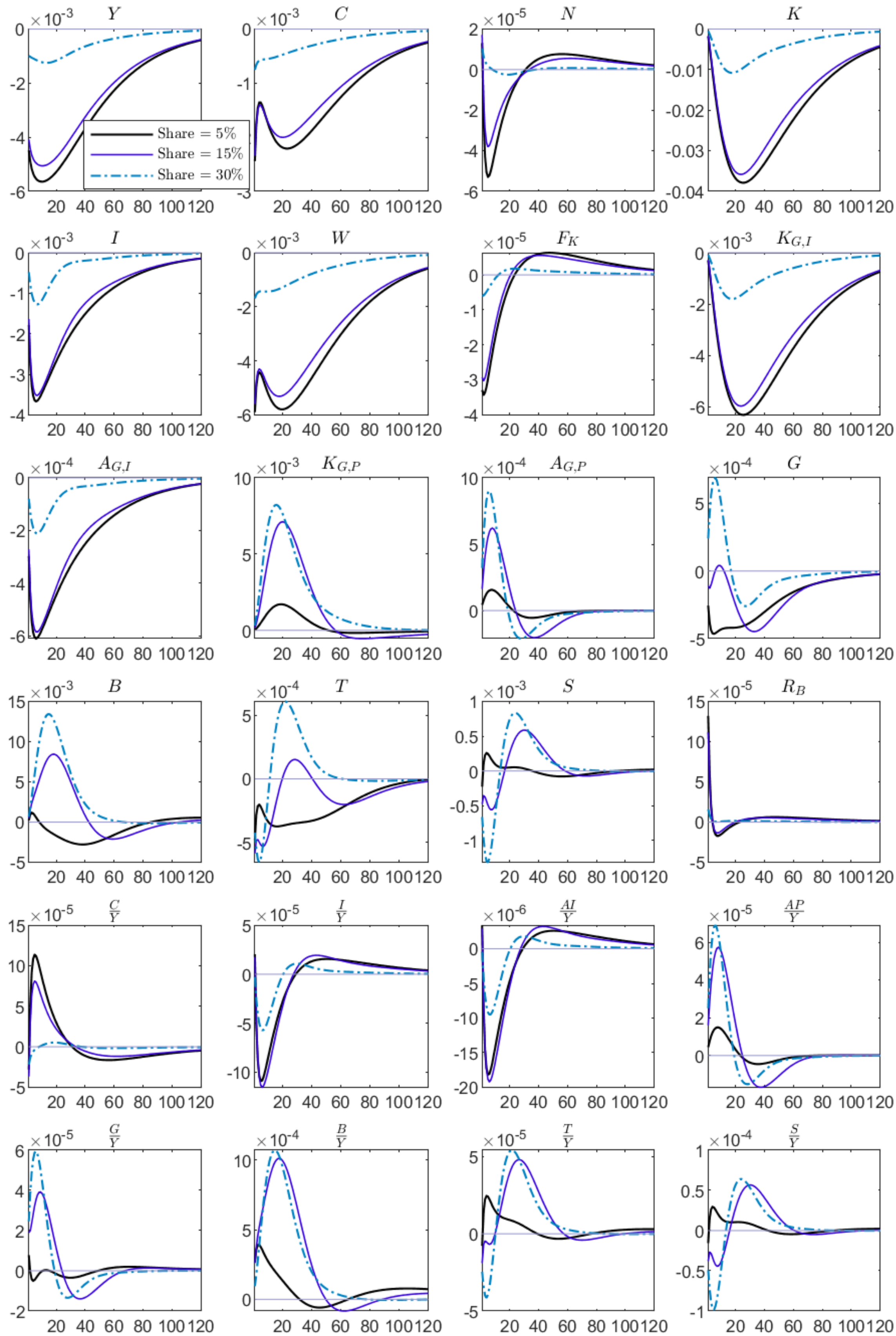

Figure 6: The economy's response to a climate-related labour productivity shock depending on the share of adaptation capital over total public capital

public expenditures (related to the magnitude of reaction of adaptation investment) and thus on debt. In terms of the changes in steady-state values, setting a higher share first increases the steady-state value of adaptation capital, and then allows the other variables to reach higher values through the decrease of the damage function. There is also the backward effect from higher public productive capital stock due to higher output toward higher adaptation capital because of the share. Since the reduction in output is decreased with higher share of adaptation capital, it is thus logical that there will be more room to invest in additional adaptation capital. This result in a shift in public investment toward adaptation capital, as displayed in Figure 19 in Appendix E.

We can thus observe that the reaction of debt that we highlighted previously is sensitive to the specification on the share of adaptation capital. This is by part due to the simplicity of our debt rule, as increasing the share of adaptation capital allows simply to have a higher magnitude of the reaction of adaptation investment. This induce indebtedness to evolve accordingly, which will both increase and delay the reaction of the ratio of lump-sum taxes over GDP. This is also due to the lower reaction of output, that will induce more public expenditures and thus more indebtedness, which is however not very realistic.

5.4 The time-to-build of public capital: different cases

We will now consider the introduction of time-to-build a la Bouakez et al. (2020) in public investment. We set the share of adaptation capital to 15%,

We also have to be careful when comparing the different hypotheses on time-to-build. This allows to interpret the optimal response for each type of adaptation capital (depending on their time-to-build). However, it does not straightforwardly allow to conclude what kind of adaptation capital is more optimal relative to its impact on household consumption and welfare, depending on its time-to-build. Indeed, it only tells us what is the optimal management of adaptation capital, given the time that is required to implement it. For instance, in the simpler case, without time-to-build and without public productive capital but with frictions on adaptation investment, a shock whose auto-correlation parameter $\rho_\tau = 0$ would induce a negative reaction of adaptation capital. This is just to say that the implementation cost with frictions would be too high for a shock whose effects would have already vanished when the adaptation capital would be realized. Thus, the marginal benefit is at most null.

In our specifications, it could seem at first sight that adaptation capital is not very effective, since adding time-to-build to public investment does only decrease the reaction of most variables of the economy, like output, household's consumption aso... However, this conclusion would disregard the fact that our simulated shock is not very important given the necessity to have a temperature shock that allows Dynare to display non-explosive IRFs. With a shock on ε equal to 0.1, this would raise the deviation value of temperature with respect to its output to only 4.1 instead of 4 for an initial variation of 4. Higher values do not permit Dynare to

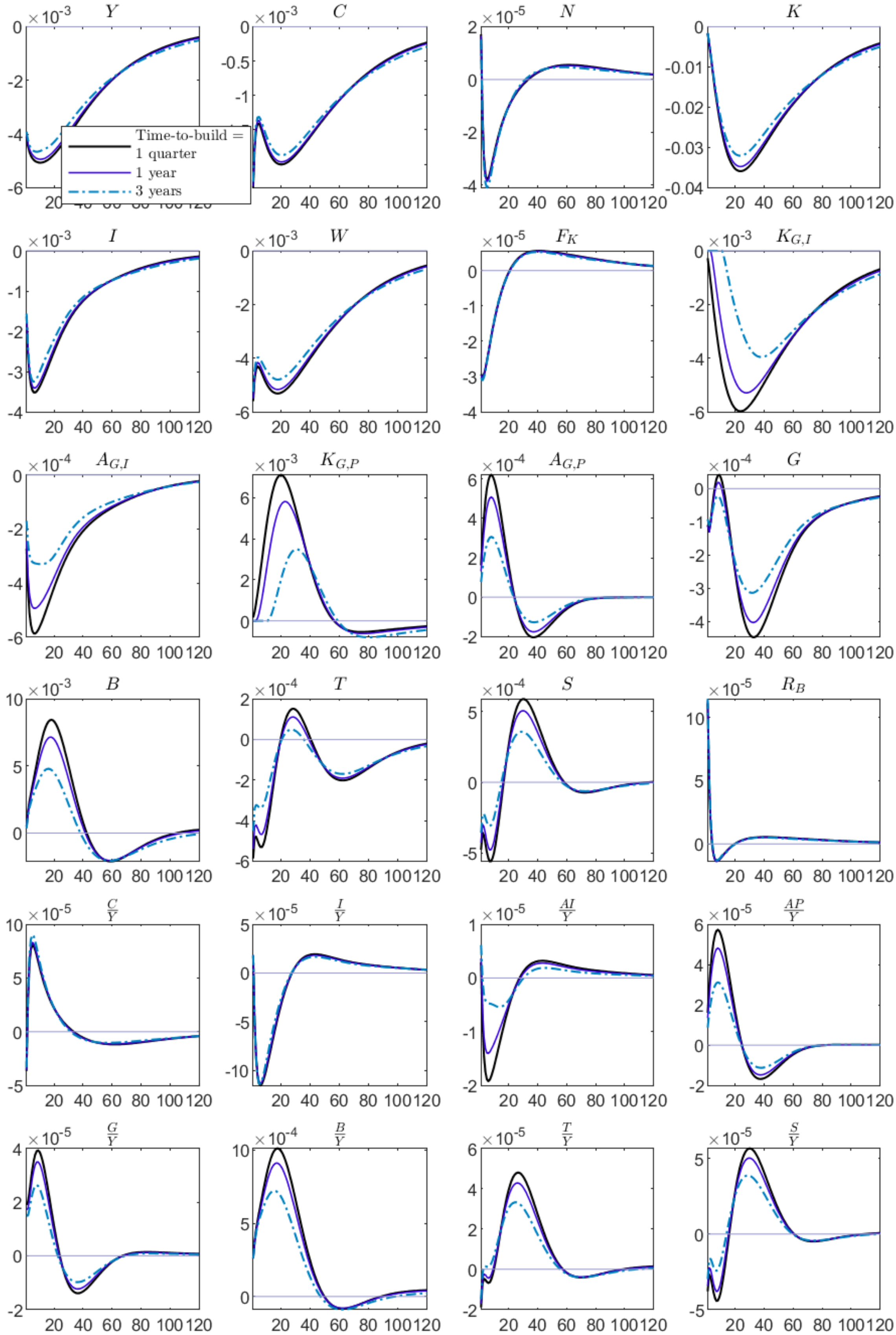

Figure 7: Comparison of the optimal response of adaptation capital and its link to public debt in the cases of different time-to-build

compute IRFs here. What would be interesting to see is in the case of a simulation that would allow the model to change its steady-state to find another one, allowing for more powerful shocks.

Here, the main results of our simulation is that with time-to-build, that is an additional form of frictions that delays the realization of investment, the magnitude of investment in adaptation capital, as well as the magnitude of the decrease in investment in public productive capital, are reduced. Indeed, the decision on actual investment depends on the state of the economy in one or three years, that is at some moments where the economy will have already recovered. Thus, the reaction is less important, as well as the total reaction of public expenditures that displays less magnitude. Additionally, since there are frictions, this adds a delay in the adjustment on investment. Consequently, with our rule on debt, the magnitude of the reaction of indebtedness and of lump-sum taxes is also lower and they follow the same inflexions than before since the time-to-build does only affect the magnitude of public investment and not its temporality.

We leave the graphs of the impulse-response functions with the staggering of expenditures as in Leeper et al. (2010) for Appendix E. Indeed, we did not introduce enough frictions in our model for these graphs to be realistic and smoothed. The model in Leeper et al. (2010) features also consumption habits and capital utilization but no friction on government spending (the role of staggering of expenditures does not play the role of frictions: they only change the Tobin's q for public capitals). Thus, we have to set the private capital friction parameter ω equal to zero to have meaningful IRFs, but they are not realistic yet. For instance, the behavior of the real economy changes very abruptly with the realisation of adaptation investment: with three years to build, labour supply is first substituted with leisure but steeply increases when adaptation capital is realized and when it is thus more profitable to work. In the one year case, there is just an income effect preceding the substitution effect due to the decrease in consumption over GDP, but the same holds. The decrease in consumption over GDP in the one year case, compared to the three years case, is similar to the impact of frictions on private capital in a model: by delaying public investment, time-to-build allows the share of output that is consumed to be increased.
However, they allow for comparisons. Again, we observe that the investment in adaptation capital is less important when the time-to-build is three years rather than one year, which is due to the fact that adaptation capital will be invested when the damage function will have returned closer to its steady-state value. Public productive investment reacts with less magnitude for the same reasons and with adaptation investment, they cause a less important increase in total public expenditures for three years than for one year. Hence the similar reactions of public indebtedness and lump-sum taxes than with the specification of Bouakez et al. (2020).

More generally, the introduction of time-to-build shows that contrarily to what we could think, large infrastructures such as dams should not weight as much on public indebtedness as smaller infrastructures. However, this result disregards the fact that it is also possible that the household self-invests in smaller infrastructures and that with multiple shocks, the investment in large infrastructures will be more important, because their actualized benefits would be more important as well.

5.5 Comparing the optimal reaction on adaptation investment with other different alternatives

We now consider different alternatives to the optimal response to a shock. We compare the optimal scenario (the reaction of adaptation capital given by the first order conditions) to two alternatives. In scenario 2, we assume that the stock of capital remains unchanged, that is stays at its steady-state value when the shock hits the economy. Thus, the investment also remains at its steady-state and only the ratio investment over GDP evolves. In scenario 3, we fix the ratio $\dfrac{A^P}{Y}$ at its steady-state value, so that $A_t^P = \frac{A^P}{Y} \cdot Y_t$. We set the share of adaptation capital over total public capital to 15%, as previously.

Figure 22 in Appendix (E) shows the economy's reaction under the different scenarios. Scenarios 2 and 3 displays some similar impulse-response functions, especially on the seven firsts variables displayed. This comes from the fact that both scenarios induce a lot of differences in terms of adaptation investment compared to the baseline scenario, but do not have large differences between each other. Furthermore, we have already seen that the introduction of time-to-build does affect output only marginally, for reasons explained above. Thus, even if the difference seems marginal, they can be significant.
Where the IRFs shows larger differences is on the behavior of public expenditures and thus on debt, as we can expect for the reasons linked to the investment in adaptation. This results highlights the fact that the expansion of public debt because of climate change will result from the optimal management of adaptation. As such, it will not be an index of a inadequate public management.
This also tells us that a without the possibility given to finance adaptation with debt, the governments may be constrained to resort to sub-optimal policies to face climate change. This may result in important costs on the populations' welfare, that could be avoided otherwise. This sub-optimality could be included in further researches, with constraints on public indebtedness. In addition, this could incite governments to either build fiscal buffers or adaptation capital early to face climate change.

6 Conclusion and further research

6.1 Conclusions

This paper has shown, that adaptation capital induces a higher reaction of indebtedness, the more adjustable the stock of capital is. Without friction on investment and without time-to-build, the reaction of debt is the most important. On the contrary, with time-to-build and frictions, the optimal stock of capital to be built will be lower, thus requiring less investment and weighting less on public finances. However, frictions and time-to-build also reduces the possibility of the economy to protect itself and induces more damaging effects of a climate shock.

We have also shown that with public productive capital, that is productive infrastructures, the optimal composition of public expenditures will be to shift initially toward adaptation capital, while the investment in productive infrastructures will be decreased. The impact on public debt depends on the steady-state share of adaptation capital over total public capital. This is due to the higher reaction of adaptation investment and thus of public infrastructures, although it is optimal that private investment reacts more than public infrastructures.

6.2 Further researches

We leave for further researches the concern for more realism that will help to understand more precisely the mechanisms relating adaptation investment and public debt. A first idea would be to implement the same model in a risky-steady-state a la Coeurdacier et al. (2011) or a la Juillard (2011). A risky steady-state relates to a steady-state where the agents take into account the possibility of shocks even at the steady-state. It would be thus very interesting to compute and simulate it, as the steady-state value of adaptation capital would be higher. This would change the reaction of adaptation investment and thus influence the magnitude of the mechanism through which adaptation investment affects public indebtedness. Another possibility to increase realism is to work on more precise calibrations of the model, especially concerning the damage function. For the latter, it is also to be considered to change the form of the function to allow for easier calibrations.

Another possibility would be to implement our model in a Ramsey planner, to allow to decentralize the economy: the central planner would take into account the optimality conditions of the household and the firms, but would be able to consider additional rules. This would allow the introduction a rule on debt limit that would allow to model more precisely the sustainability of public debt, introducing thus new mechanisms from adaptation investment to public debt.

Eventually, it will be interesting to enrich the relations onward and backward of the economy with debt, for instance by considering an open economy, allowing not only for international indebtedness but also for the additional risk on government's bonds that the shock could provoke, as in Marto et al. (2018) or Diarra et al. (2022). Further researches could also allow for more precise analysis of the sustainability of public debt and the impact of constrained public debt that would result in under-investment in adaptation capital by governments.

6.3 Supplementary materials

Supplementary materials can be found on my github.

7 Appendix

A Conditions for a non-explosive debt

We derive here the condition on λ to have a non-explosive debt, and in particular at the steady-state.

From the government budget constraint, we have:

$$(1 + R_{B,t-1}) \times B_{t-1} = T_t - G_t^I - G_t^P + B_t \tag{18}$$

and in terms of ratio over output:

$$\frac{Y_{t-1}}{Y_t}(1 + R_{B,t-1})b_{t-1} = \frac{T_t - G_t^I - G_t^P}{Y_t} + b_t \tag{19}$$

with the adjustment:

$$\frac{Y_{t-1}}{Y_t}(1 + R_{B,t-1})b_{t-1} = \lambda b_{t-1} + \mu - 2 \cdot g_t + b_t$$

with $g_t = \dfrac{G_t^I + G_t^P}{Y_t}$.

Rearranging, we obtain:

$$\begin{aligned} b_{t-1} &= \frac{\mu - 2g_t + b_t}{1 + R_{B,t-1} - \lambda} \frac{Y_t}{Y_{t-1}} \\ b_t &= b_{t-1} \left[\frac{Y_{t-1}}{Y_t}(1 + R_{B,t-1}) - \lambda \right] - \mu + 2g_t \end{aligned} \tag{20}$$

To ensure the convergence of debt, we need the condition on λ:

$$\begin{aligned} &\frac{Y_{t-1}}{Y_t}(1 + R_{B,t-1}) - \lambda < 1 \\ \iff &\frac{Y_{t-1}}{Y_t}(1 + R_{B,t-1}) - 1 < \lambda \end{aligned} \tag{21}$$

At about the steady-state value of debt, to calibrate μ, we get:

$$\begin{aligned} b &= (1 + R - \lambda)\, b - \mu + 2g \\ &= \frac{2g - \mu}{\lambda - R} \\ &= \frac{\mu - 2g}{R - \lambda} \\ &= \frac{\mu - 2g}{\frac{1}{\beta} - 1 - \lambda} \end{aligned} \tag{22}$$

which gives:

$$\mu = \left(\frac{1}{\beta} - 1 - \lambda\right) \cdot b + 2g \tag{23}$$

Since our condition on convergence of debt imposes, at the steady-state, $\frac{1}{\beta} - 1 < \lambda$, we might have $\mu > 0$ for a positive equilibrium ratio of debt over GDP. The choice of the steady-state value of b will determine whether μ is positive or negative. We will calibrate $b = 0.6$ to abide by the European Stability and Growth Pact.
More generally, we have that λ, the reaction of the lump-sum taxes ratio over GDP to the debt ratio of last period, has to be superior to the individual rate of discounting.

B Proof of the derivatives for the damage function

We had

$$\Omega(K_{G,P,t}, \tau_t^W) = \frac{1 + g(K_{G,P,t-1})}{1 + g(K_{G,P,t-1}) + f(\tau_t^W)}$$

where

$$g(K_{G,P,t-1}) = \beta_1 K_{G,P,t-1}^{\beta_2}$$
$$f(\tau_t^W) = \alpha_1 \tau_t^W + \alpha_2 \tau_t^{W^2}$$

Hence, we have the following derivatives:

$$\begin{aligned}
\frac{\partial \Omega(K_{G,P,t-1}, f(\tau_t^W))}{\partial K_{G,P,t-1}} &= \frac{g_K(K_{G,P,t-1})(1 + g(K_{G,P,t-1}) + f(\tau_t^W)) - g_K(K_{G,P,t-1})(1 + g(K_{G,P,t-1}))}{(1 + g(K_{G,P,t-1}) + f(\tau_t^W))^2} \\
&= \frac{g_K(K_{G,P,t-1}) f(\tau_t)}{(1 + g(K_{G,P,t-1}) + f(\tau_t^W))^2} \\
&= \frac{\beta_1 \beta_2 K_{G,P,t-1}^{\beta_2 - 1} f(\tau_t)}{(1 + g(K_{G,P,t-1}) + f(\tau_t^W))^2} \geq 0 \text{ for } K_{G,P,t-1} \geq 0, f(\tau_t) \geq 0
\end{aligned}$$

$$\frac{\partial \Omega(K_{G,P,t-1}, \tau_t^W)}{\partial \tau_t^W} = -\frac{1 + g(K_{G,P,t-1})}{(1 + g(K_{G,P,t-1}) + f(\tau_t^W))^2} f_\tau(\tau_t^W) < 0$$

C Climate and adaptation capital

C.1 Our choice of specification for climate and adaptation capital

In the article, we very briefly mentioned that we had encountered some troubles while choosing our specification for our damage function. In our review of literature, we found two main specifications, which are described below.

From Millner and Dietz (2015), we have:

$$\Omega(K_{G,P,t-1}, \tau_t^W) = \frac{1 + \nu(K_{G,P,t-1})}{1 + \nu(K_{G,P,t-1}) + \omega(\tau_t^W)} \tag{24}$$

With

$$\nu(K_{G,P,t-1}) = \beta_1 K_{G,P,t-1}^{\beta_2}$$

with $\beta_1 > 0$ and $\beta_2 \in [0, 1]$, and

$$\omega(\tau_t^W) = \alpha_1 \tau_t^W + \alpha_2 \tau_t^{W^2}$$

And from the AD-DICE model in De Bruin et al. (2009), we have the following specification:

$$\frac{D_t(\tau^W, K_{G,P,t-1})}{Y_t} = (\alpha_1 \tau^W + \alpha_2 \tau^{W\alpha_3})(1 - K_{G,P,t-1}) + \gamma_1 K_{G,P,t-1}^{\gamma_2} \tag{25}$$

Since in the DICE model, the damage coefficient of the production function Ω includes the damage function in the following way:

$$\Omega(\cdot) = \frac{1}{1 + D_t(\tau^W)} \tag{26}$$

We consider the specification from Millner and Dietz (2015) to be less restrictive in terms of the value of the optimal adaptation capital. Indeed, when we look at the behavior of the function from the AD-DICE model of De Bruin et al. (2009), we observe the following behavior:

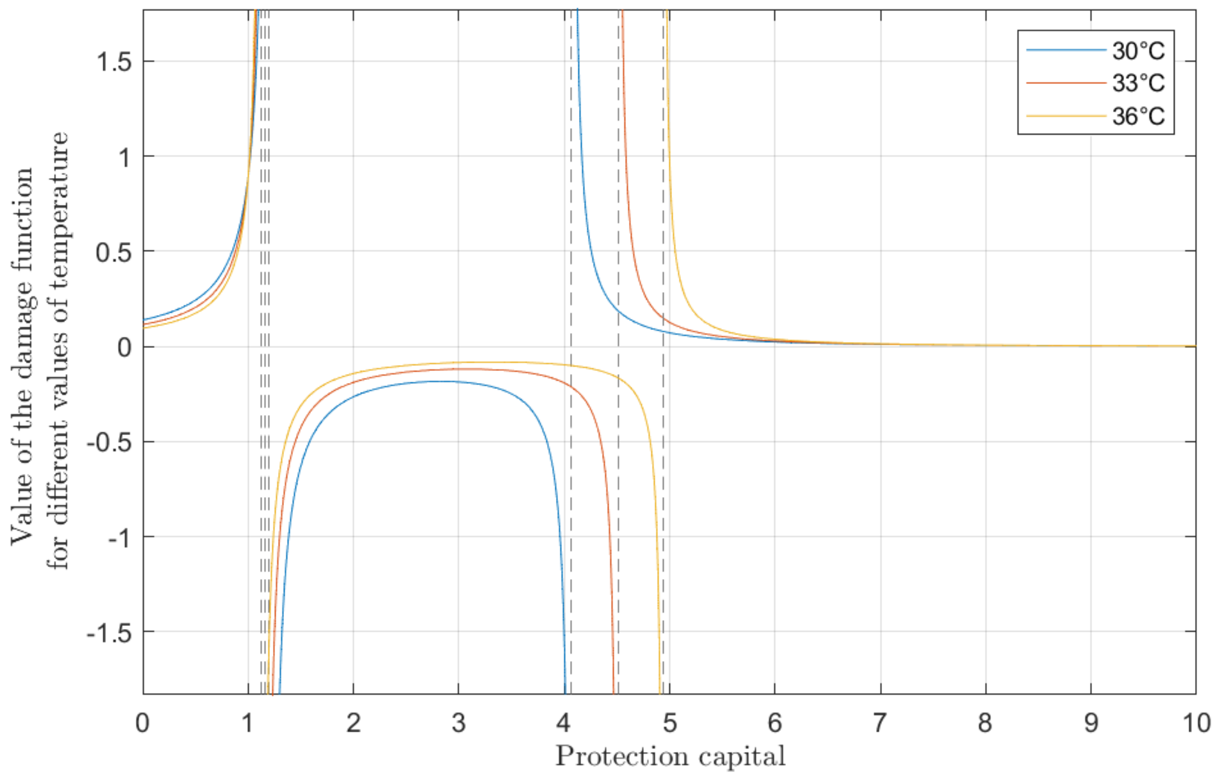

Figure 8: Behavior of the AD-DICE specification for the damage function for different values of temperature

These discontinuities can be problematic in a RBC model that is sensitive to parametrization and for which the optimal value of adaptation capital may vary a lot. Furthermore, another criterion for excluding this specification was the observation, that our computations, on which we will come back below, could give some complex values of adaptation capital at the steady-state. Eventually, this function aims to represent the net cost of temperature in addition with the cost of investing in adaptation capital to fit in intertemporal general equilibrium models computed with GAMS, but we already have the cost of capital specified in the first order conditions of our model.

However, it is interesting to study the three-dimensional behavior of the function for values of adaptation capital that range from 0 to 1, as displayed in Figure 9.

What this figure shows is, that for low values of adaptation capital, the damage function will be convex, meaning an increasing marginal utility whereas it becomes flat and concave for higher values, with decreasing marginal utility.

On the contrary, the specification used in Millner and Dietz (2015) didn't show such behavior when considered in the computation of the steady-state value for adaptation capital. However, when we consider the value of the damage function for different values of temperature, with the calibration given in Millner and Dietz (2015), we observe that for a given value of temperature, there is more or less one value of the damage function and that adaptation capital investment is useless, since more capital does affect the value of the damage function in only a really marginal way:

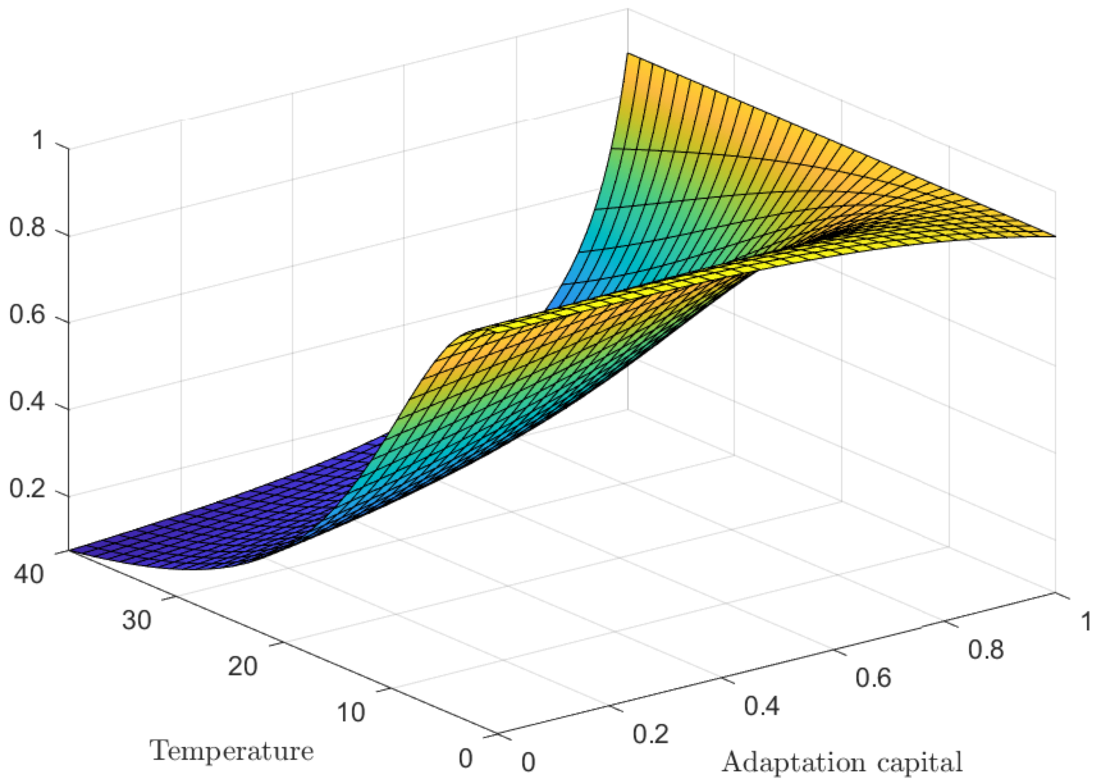

Figure 9: Behavior of the AD-DICE damage model for values of adaptation capital in [0; 1]

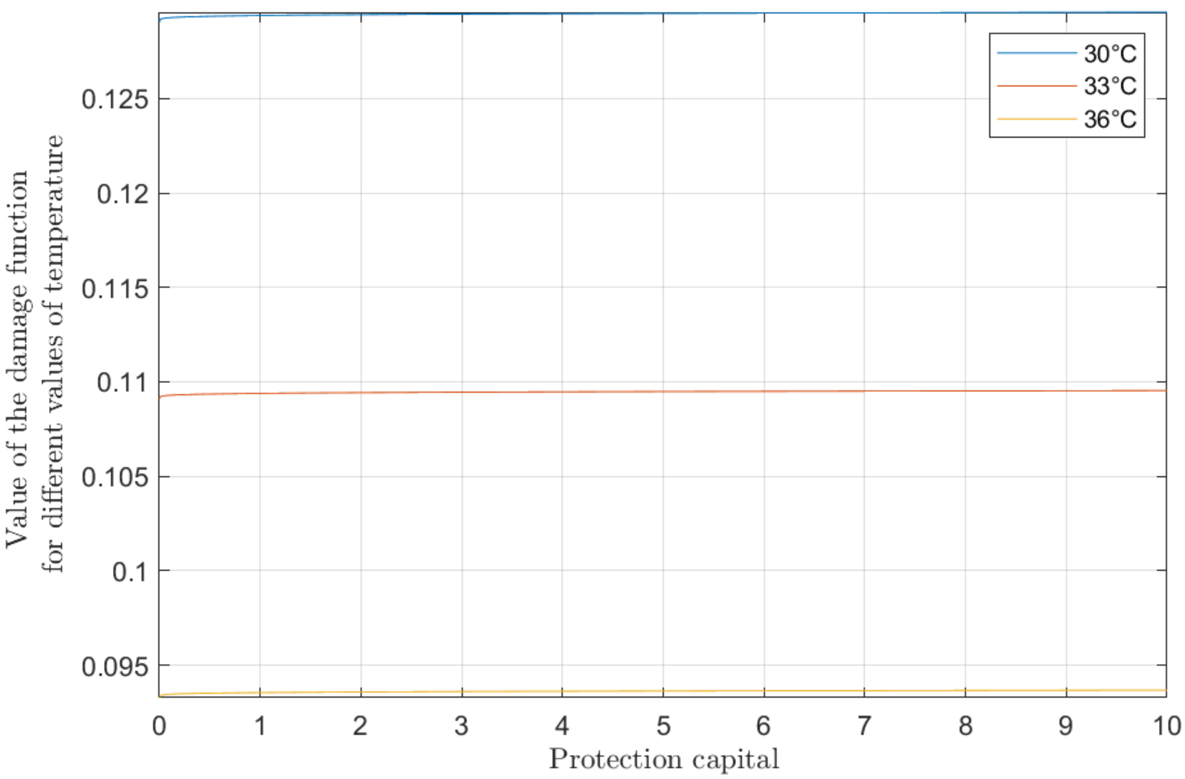

Figure 10: Behavior of the damage function from Millner and Dietz (2015) for given values of temperature and with the calibration given in the article

When we consider the three-dimensional behavior of the function, we observe that it is only evolving depending on the level of temperature:

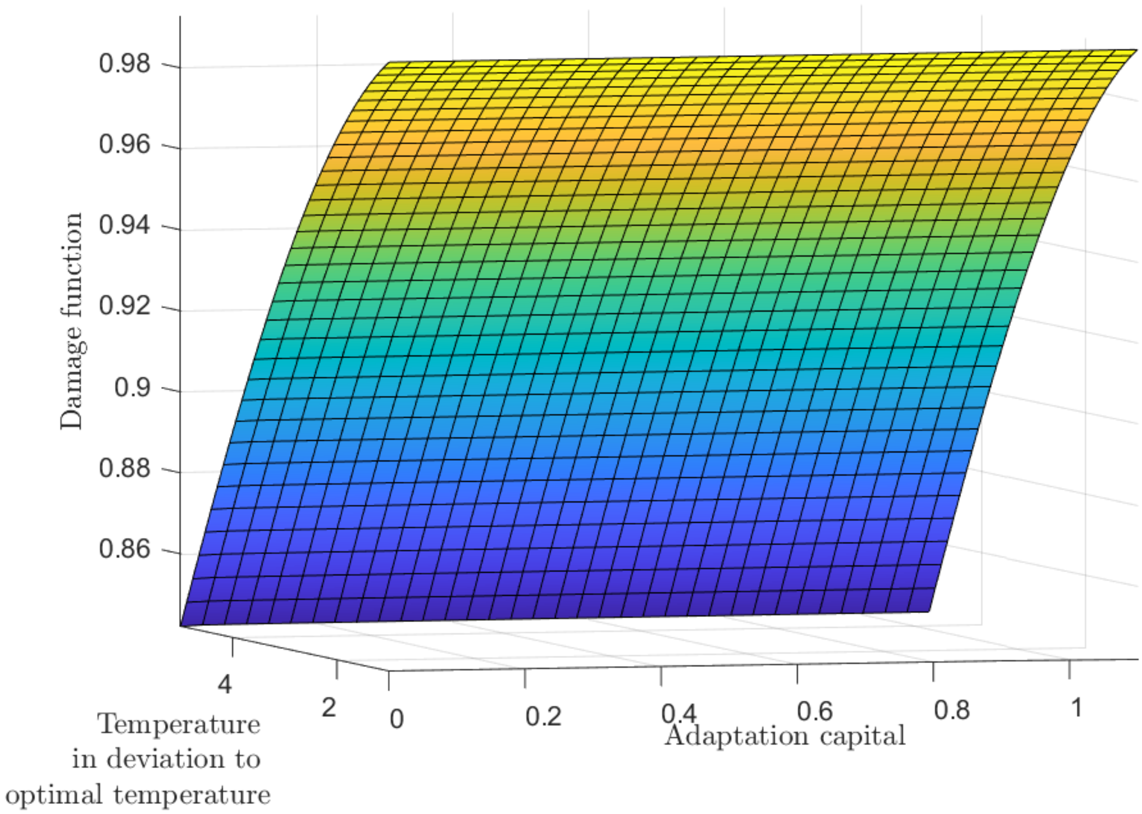

Figure 11: Behavior of the damage function from Millner and Dietz (2015) for given values of temperature and with the calibration given in the article

Hence the conclusion that neither of this specifications (given their calibrations) would be useful for us. In order to get a more meaningful result, we used the calibration of the quadratic function of temperature $\alpha_1\tau^W + \alpha_2\tau^{2,W}$ from the AD-DICE model of De Bruin et al. (2009), since they are more general and more used in the IAM literature. Also, we took their calibration for the parameter β_1 that intervenes in the utility of adaptation capital, but we used that to calibrate β_2 so that it would be meaningful to our problem.

To have a meaningful β_2, we calibrate it so that it can allow for different values of the share s of adaptation capital over total public capital:

$$s = \frac{kP}{kP + kI} \tag{27}$$

This allows us to compare the scenario (and thus the government responses) for values of s ranging from $0.05 = 5\%$ to $0.3 = 30\%$. These shares allow us to keep $\beta_2 \in [0.5; 2]$, which remains plausible. With $\beta_2 = 0.7$, the function evolves with the dynamics displayed in Figure 12, which is much more meaningful. Interestingly, when we increase arbitrarily high the parameters, the behavior of the function resembles the function in AD-DICE, as shown in Figure **??**, and since

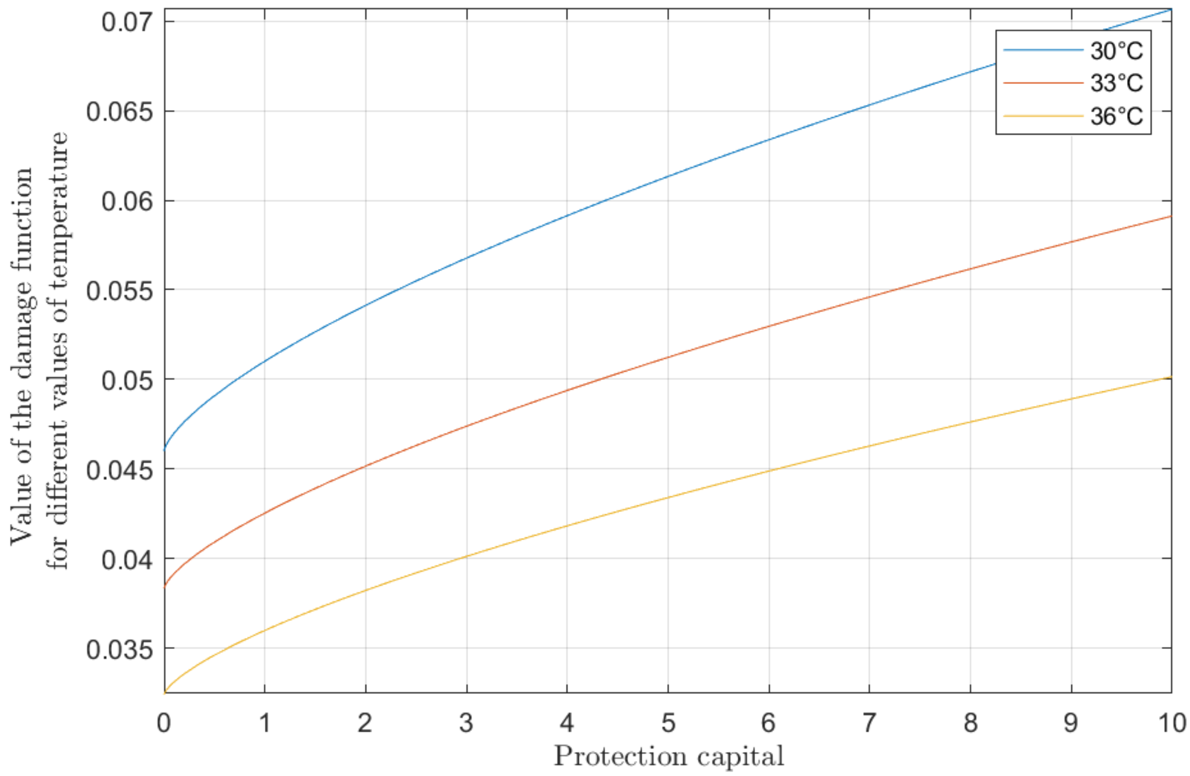

Figure 12: Behavior of the damage function from Millner and Dietz (2015) for given values of temperature and with the calibration given in De Bruin et al. (2009)

we are talking about periods where the average temperature would be above the goal of Paris, it could be meaningful for further researches to try to calibrate more properly this specification and implement it.

C.2 Taking the temperature in deviation from its optimum

Now, for modelling purposes, we have to introduce a long-term temperature, that is the level of τ^W at the steady-state, different from the optimal level of temperature, at which the temperature doesn't affect the economy. This will allow the steady-state level of adaptation capital $K_{G,P}$ to be different from zero, allowing the computations (especially given that our β_2 is inferior to 1 and can represent the fact that the economies are already prepared for smaller sized natural disasters. Hence, in these cases, we will rewrite the process of the temperature in the following way:

$$\tau_t^W = \rho_\tau \tau_{t-1}^W + (1 - \rho_\tau)\tau_{SS} + \varepsilon_\tau \tag{28}$$

and in the equations of impact, we will replace τ_t^W with $\tau_t^W - \tau^*$.

C.3 Solving for the different specifications

At first, we considered trying to solve the steady-state analytically. From the Millner specification, this would expect from us to simplify the value for β, to equalize it to an integer value, making us loose a huge amount of information, maniability and interpretability. Again, when

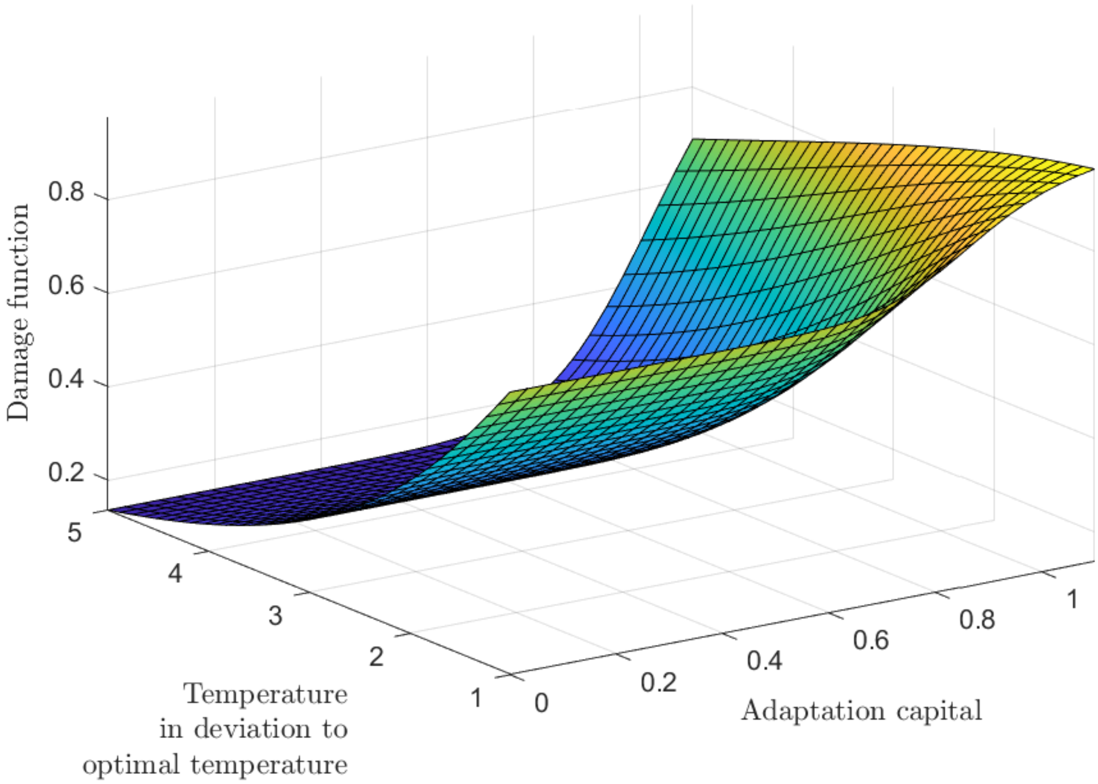

Figure 13: Behavior of the damage function from Millner and Dietz (2015) for given values of temperature and with arbitrary high calibrations

trying to solve the version of De Bruin et al. (2009), the value of $\beta_2 = 3.6$ would require to simplify it to $\beta_2 = 4$, leading to a fourth degree polynomial and to use the Ferrari and Cardan's methods. Again, we would loose a lot of practicability and the solutions would be very hard to compute because of their analytical expressions that are very complicated.

We thus resorted to symbolic computations in Matlab. The program would solve one or several equation(s) for a given variable or set of variables. This is what we used in order to calibrate our β_2, since including the equation for the share of adaptation capital gave us some freedom that we used to set β_2 as a variable.

D Central Planner - First best - With time-to-build a la Bouakez et al. (2020)

In the specification a la Bouakez et al. (2020), there is no staggering of expenditures. Thus, $G_t^j = A_t^j$.We can consequently remove the variable G^j. The central planner thus solves the following maximisation problem:

$$
\begin{aligned}
\mathcal{L}_t =& \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \\
&+ \lambda_{1,t}[F(N_t, K_{t-1}, K_{G,I,t-1}, K_{G,P,t-1}) - C_t - A_t^I - A_t^P - I_t] \\
&+ \lambda_{2,t}\left[(1-\delta)K_{t-1} + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t - K_t\right] \\
&+ \lambda_{3,t}\left[(1-\delta)K_{G,I,t+N-2} + \left(1 - S\left(\frac{A_t^I}{A_{t-1}^I}\right)\right) A_t^I - K_{G,I,t+N-1}\right] \\
&+ \lambda_{4,t}\left[(1-\delta)K_{G,P,t+N-2} + \left(1 - S\left(\frac{A_{t+s}^P}{A_{t-1}^P}\right)\right) A_t^P - K_{G,P,t+N-1}\right]
\end{aligned} \tag{29}
$$

First order conditions :

$$\frac{\partial \mathcal{L}_t}{\partial C_t} = 0 \iff U_{C,t} = \lambda_{1,t} \tag{30}$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial N_t} = 0 &\iff U_{N,t} = -\lambda_{1,t} F_N(N_t, K_{t-1}, K_{G,I,t-1}, K_{G,P,t-1}) \\
&\iff -\frac{U_{N,t}}{U_{C,t}} = F_N(N_t, K_{t-1}, K_{G,I,t-1}, K_{G,P,t-1}, \tau_t^W)
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial K_t} = 0 &\iff \beta \mathbb{E}_t \lambda_{1,t+1} F_K(N_{t+1}, K_t, K_{G,I,t}, K_{G,P,t}) + \beta \mathbb{E}_t \lambda_{2,t+1}(1-\delta) = \lambda_{2,t} \\
&\iff \beta \mathbb{E}_t F_K(N_{t+1}, K_t, K_{G,I,t}, K_{G,P,t}) = \frac{\lambda_{2,t}}{\mathbb{E}_t \lambda_{1,t+1}} \frac{\lambda_{1,t}}{\lambda_{1,t}} - \beta \mathbb{E}_t \frac{\lambda_{2,t+1}}{\lambda_{1,t+1}}(1-\delta)
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial K_{G,I,t+N-1}} = 0 &\iff \beta^N \mathbb{E}_t \lambda_{1,t+N} F_{K_{G,I}}(\cdot) - \lambda_{3,t} + \beta \lambda_{3,t+1}(1-\delta) = 0 \\
&\iff \beta^N \mathbb{E}_t F_{K_{G,I}}(\cdot) = \frac{\lambda_{3,t}}{\mathbb{E}_t \lambda_{1,t+N}} \frac{\lambda_{1,t}}{\lambda_{1,t}} - \beta \mathbb{E}_t (1-\delta) \frac{\lambda_{3,t+1}}{\lambda_{1,t+N}} \frac{\lambda_{1,t+1}}{\lambda_{1,t+1}}
\end{aligned} \tag{33}
$$

$$\frac{\partial \mathcal{L}_t}{\partial K_{G,P,t+N-1}} = 0 \iff \beta^N \mathbb{E}_t \lambda_{1,t+N} F_{K_{G,I}}(\cdot) = \lambda_{4,t} - \beta \mathbb{E}_t \lambda_{4,t+1}(1-\delta) \tag{34}$$

$$\frac{\partial \mathcal{L}_t}{\partial I_t} = 0 \iff 1 = \frac{\lambda_{2,t}}{U_{C,t}}\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta \mathbb{E}_t \frac{\lambda_{2,t+1}}{U_{C,t}}\left[\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)\right] \tag{35}$$

$$\frac{\partial \mathcal{L}_t}{\partial A_t^I} = 0 \iff 1 = \frac{\lambda_{2,t}}{U_{C,t}}\left[1 - S\left(\frac{A_t^I}{A_{t-1}^I}\right) - \frac{A_t^I}{A_{t-1}^I} S'\left(\frac{A_t^I}{A_{t-1}^I}\right)\right] + \beta \mathbb{E}_t \frac{\lambda_{2,t+1}}{U_{C,t}}\left[\left(\frac{A_{t+1}^I}{A_t^I}\right)^2 S'\left(\frac{A_{t+1}^I}{A_t^I}\right)\right] \tag{36}$$

$$\frac{\partial \mathcal{L}_t}{\partial A^P_t} = 0 \iff 1 = \frac{\lambda_{2,t}}{U_{C,t}}\left[1 - S\left(\frac{A^P_t}{A^P_{t-1}}\right) - \frac{A^P_t}{A^P_{t-1}}S'\left(\frac{A^P_t}{A^P_{t-1}}\right)\right] + \beta\mathbb{E}_t\frac{\lambda_{2,t+1}}{U_{C,t}}\left[\left(\frac{A^P_{t+1}}{A^P_t}\right)^2 S'\left(\frac{A^P_{t+1}}{A^P_t}\right)\right] \tag{37}$$

D.1 Central planner - simplification of the FOC

$$U_{C,t} = \lambda_{1,t} \tag{38}$$

$$-\frac{U_{N,t}}{U_{C,t}} = F_N(N_t, K_{t-1}, K_{G,I,t-1}, K_{G,P,t-1}, \tau^W_t) \tag{39}$$

$$\begin{aligned}\beta\mathbb{E}_t F_K(N_{t+1}, K_t, K_{G,I,t}, K_{G,P,t}, \tau^W_t) &= \frac{\lambda_{2,t}}{\lambda_{1,t+1}}\frac{\lambda_{1,t}}{\lambda_{1,t}} - \beta\mathbb{E}_t\frac{\lambda_{2,t+1}}{\lambda_{1,t+1}}(1-\delta) \\ &= \mathbb{E}_t q_t\frac{U_{C,t}}{U_{C,t+1}} - \beta\mathbb{E}_t(1-\delta)q_{t+1}\end{aligned} \tag{40}$$

with $q_t = \frac{\lambda_{1,t}}{\lambda_{2,t}}$

$$\begin{aligned}\beta^N\mathbb{E}_t F_{K_{G,I}}(N_{t+N}, K_{t+N-1}, K_{G,I,t+N-1}, K_{G,P,t+N-1}, \tau^W_{t+N}) &= \mathbb{E}_t\frac{\lambda_{3,t}}{\lambda_{1,t+N}}\frac{\lambda_{1,t}}{\lambda_{1,t}} - \beta\mathbb{E}_t(1-\delta)\frac{\lambda_{3,t+1}}{\lambda_{1,t+N}}\frac{\lambda_{1,t+1}}{\lambda_{1,t+1}} \\ &= \mathbb{E}_t q_{I,t}\frac{U_{C,t}}{U_{C,t+N}} - \beta\mathbb{E}_t(1-\delta)q_{I,t+1}\frac{U_{C,t+1}}{U_{C,t+N}}\end{aligned} \tag{41}$$

with $q_{I,t} = \frac{\lambda_{1,t}}{\lambda_{3,t}}$

$$\begin{aligned}\beta^N\mathbb{E}_t F_{K_{G,P}}(N_{t+N}, K_{t+N-1}, K_{G,I,t+N-1}, K_{G,P,t+N-1}, \tau^W_{t+N}) &= \mathbb{E}_t\frac{\lambda_{4,t}}{\lambda_{1,t+N}} - \beta\mathbb{E}_t\frac{\lambda_{4,t+1}}{\lambda_{1,t+N}}(1-\delta) \\ &= \mathbb{E}_t q_{P,t}\frac{U_{C,t}}{U_{C,t+N}} - \beta\mathbb{E}_t(1-\delta)q_{P,t+1}\frac{U_{C,t+1}}{U_{C,t+N}}\end{aligned} \tag{42}$$

with $q_{P,t} = \frac{\lambda_{1,t}}{\lambda_{4,t}}$

$$1 = q_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}}S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta\mathbb{E}_t q_t\frac{U_{C,t+1}}{U_{C,t}}\left[\left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)\right] \tag{43}$$

$$1 = q_{I,t}\left[1 - S\left(\frac{A^I_t}{A^I_{t-1}}\right) - \frac{A^I_t}{A^I_{t-1}}S'\left(\frac{A^I_t}{A^I_{t-1}}\right)\right] + \beta\mathbb{E}_t q_t\frac{U_{C,t+1}}{U_{C,t}}\left[\left(\frac{A^I_{t+1}}{A^I_t}\right)^2 S'\left(\frac{A^I_{t+1}}{A^I_t}\right)\right] \tag{44}$$

$$1 = q_{P,t}\left[1 - S\left(\frac{A^P_t}{A^P_{t-1}}\right) - \frac{A^P_t}{A^P_{t-1}}S'\left(\frac{A^P_t}{A^P_{t-1}}\right)\right] + \beta\mathbb{E}_t q_t\frac{U_{C,t+1}}{U_{C,t}}\left[\left(\frac{A^P_{t+1}}{A^P_t}\right)^2 S'\left(\frac{A^P_{t+1}}{A^P_t}\right)\right] \tag{45}$$

D.2 Central Planner steady-state

$$-\frac{U_N}{U_C} = F_N(N, K, K_G, K_{G,P}, \tau^W) \tag{46}$$

$$F_K(N, K, K_{G,I}, K_{G,P}, \tau^W) = \frac{q(1-\beta(1-\delta))}{\beta} \tag{47}$$

$$F_{K_{G,I}}(N, K, K_{G,I}, K_{G,P},, \tau^W) = \frac{q_I(1-\beta(1-\delta))}{\beta^N} \tag{48}$$

$$F_{K_{G,P}}(N, K, K_{G,I}, K_{G,P}, \tau^W) = \frac{q_P(1-\beta(1-\delta))}{\beta^N} \tag{49}$$

$$q = 1 \tag{50}$$

$$q_I = 1 \tag{51}$$

$$q_P = 1 \tag{52}$$

$$Y = C + G^I + G^P + I \iff C = Y - G^I - G^P - I \tag{53}$$

$$I = \delta K \tag{54}$$

$$A^I = \delta K_{G,I} \tag{55}$$

$$A^P = \delta K_{G,P} \tag{56}$$

$$Y = F(N, K, K_{G,I}, K_{G,P}, \tau^W) \tag{57}$$

D.3 Solution simplified

We still have (46) :

$$-\frac{U_N}{U_C} = F_N(N, K, K_G, K_{G,P}, \tau^W) \tag{58}$$

With (55), (56) and (54) in (53), we get :

$$C = F(N, K, K_{G,I}, K_{G,P}, \tau^W) - \delta K_{G,I} - \delta K_{G,P} - \delta K \tag{59}$$

With (50), (82) and (83) in respectively (47), (48) and (49), we get :

$$F_K(N, K, K_{G,I}, K_{G,P}, \tau^W) = \frac{(1-\beta(1-\delta))}{\beta} \tag{60}$$

$$F_{K_{G,I}}(N, K, K_{G,I}, K_{G,P}, \tau^W) = \frac{(1-\beta(1-\delta))}{\beta^N} \tag{61}$$

$$F_{K_{G,P}}(N, K, K_{G,I}, K_{G,P}, \tau^W) = \frac{(1-\beta(1-\delta))}{\beta^N} \tag{62}$$

D.4 Specifications

We let the discussion about the specification for adaptation capital for below in Appendix 3.

We use the forms:

$$U(C_t, N_t) = \frac{[C_t^{\gamma}(1-N_t)^{1-\gamma}]^{1-\sigma}}{1-\sigma} \tag{63}$$

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{64}$$

$$F(N_t, K_{t-1}, K_{G,I,t-1}, K_{G,P,t-1}, \tau_t^W) = (\Theta(\tau_t^W, K_{G,P,t-1})P \cdot N_t)^{\alpha}(K_{t-1})^{1-\alpha}(K_{G,I,t-1})^{\alpha_G} \tag{65}$$

$$\Omega(K_{G,P,t}, \tau_t^W) = \frac{1+g(K_{G,P,t-1})}{1+g(K_{G,P,t})+f(\tau_t^W)} \tag{66}$$

where

$$g(K_{G,P,t}) = \beta_1 K_{G,P,t-1}^{\beta_2}$$

$$f(\tau_t^W) = \alpha_1 \tau_t^W + \alpha_2 \tau_t^{W^2}$$

So we have :

$$U_C = \gamma C_t^{\gamma(1-\sigma)-1}(1-N_t)^{(1-\gamma)(1-\sigma)} \tag{67}$$

$$U_N = -\left[(1-\gamma)C^{\gamma(1-\sigma)}(1-N)^{(1-\gamma)(1-\sigma)-1}\right] \tag{68}$$

$$F_N(N_t, K_{t-1}, K_{G,I,t-1}, K_{G,P,t-1}, \tau^W) = \frac{\alpha}{N_t} Y_t \tag{69}$$

$$F_K(N_t, K_{t-1}, K_{G,I,t-1}, K_{G,P,t-1}, \tau^W) = \frac{1-\alpha}{K_{t-1}} Y_t \tag{70}$$

$$F_{K_{G,I}}(N_t, K_{t-1}, K_{G,I,t-1}, K_{G,P,t-1}, \tau^W) = \frac{\alpha_G}{K_{G,I,t-1}} Y_t \tag{71}$$

$$F_{K_{G,P}}(N_t, K_{t-1}, K_{G,I,t-1}, K_{G,P,t-1}, \tau^W) = \frac{\beta_1\beta_2 K_{G,P,t-1}^{\beta_2-1}\left(\alpha_1\tau_t^W + \alpha_2\tau_t^{W^2}\right)}{\left(1+\beta_1 K_{G,P,t-1}^{\beta_2} + \alpha_1\tau_t^W + \alpha_2\tau_t^{W^2}\right)^2} Y_t \tag{72}$$

D.5 Central Planner steady-state specified

$$\frac{1-\gamma}{\gamma}\frac{C}{1-N} = W \iff N = 1 - \frac{1-\gamma}{\gamma}\frac{C}{W} \tag{73}$$

with $W_t = \dfrac{\alpha Y_t}{N_t}$ in the competitive equilibrium.

$$\begin{aligned} F_K(N, K, K_{G,I}, K_{G,P}, \tau^W) = \frac{(1-\beta(1-\delta))}{\beta} &\iff \frac{1-\alpha}{K} Y = \frac{(1-\beta(1-\delta))}{\beta} \\ &\iff K = \frac{\beta(1-\alpha)}{(1-\beta(1-\delta))} Y \end{aligned} \tag{74}$$

$$\begin{aligned} &F_{K_{G,I}}(N, K, K_{G,I}, K_{G,P}, \tau^W) = \frac{(1-\beta(1-\delta))}{\beta^N} \\ &\iff \frac{\alpha_G}{K_{G,I}} Y = \frac{(1-\beta(1-\delta))}{\beta^N} \\ &\iff K_{G,I} = \frac{\beta^N \alpha_G}{(1-\beta(1-\delta))} Y \end{aligned} \tag{75}$$

$$C = F(N, K, K_{G,I}, K_{G,P}, \tau^W) - \delta K_{G,I} - \delta K_{G,P} - \delta K \tag{76}$$

D.6 With time-to-build a la Leeper et al. (2010)

Here, we will add the staggering of expenditures. Thus, the central planner solves the following maximisation problem:

$$\begin{aligned} \mathcal{L}_t =& \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \\ &+ \lambda_{1,t} \left[F(N_t, K_{t-1}, K_{G,I,t-1}, K_{G,P,t-1}) - C_t - \sum_{n=0}^{N-1} \phi_n A^I_{t-n} - \sum_{n=0}^{N-1} \phi_n A^P_{t-n} - I_t \right] \\ &+ \lambda_{2,t} \left[(1-\delta) K_{t-1} + \left(1 - S\left(\frac{I_t}{I_{t-1}} \right) \right) I_t - K_t \right] \\ &+ \lambda_{3,t} [(1-\delta) K_{G,I,t+N-2} + A^I_t - K_{G,I,t+N-1}] \\ &+ \lambda_{4,t} [(1-\delta) K_{G,P,t+N-2} + A^P_t - K_{G,P,t+N-1}] \end{aligned} \tag{77}$$

We only focus on the First-order conditions that are different from the previous resolution, that is the ones for public investment:

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial A^I_t} = 0 &\iff -\mathbb{E}_t \sum_{s=0}^{N-1} \lambda_{1,t} \beta^s \phi_{I,s} + \lambda_{3,t} = 0 \\ &\iff \mathbb{E}_t \sum_{s=0}^{N-1} U_{C,t+s} \beta^s \phi_{I,s} = \lambda_{3,t} \end{aligned} \tag{78}$$

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial A^P_t} = 0 &\iff -\mathbb{E}_t \sum_{s=0}^{N-1} \lambda_{1,t+s} \beta^s \phi_{P,s} + \lambda_{4,t} = 0 \\ &\iff \mathbb{E}_t \sum_{s=0}^{N-1} U_{C,t+s} \beta^s \phi_{I,s} = \lambda_{4,t} \end{aligned} \tag{79}$$

When simplified, they give:

$$\mathbb{E}_t \sum_{s=0}^{N-1} U_{C,t+s} \beta^s \phi_{I,s} = \lambda_{1,t} q_{I,t} = U_{C,t} q_{I,t} \tag{80}$$

$$\mathbb{E}_t \sum_{s=0}^{N-1} U_{C,t+s} \beta^s \phi_{I,s} = \lambda_{1,t} q_{P,t} = U_{C,t} q_{P,t} \tag{81}$$

This allows us to compute the steady-state values for the Tobin's q of public capital:

$$q_I = \sum_{s=0}^{N-1} \beta^s \phi_{I,s} \tag{82}$$

$$q_P = \sum_{s=0}^{N-1} \beta^s \phi_{I,s} \tag{83}$$

Note that, at the steady-state, we have:

$$G^j = \sum_{n=0}^{N-1} \phi_n A^j = A^j \tag{84}$$

E Additional graphs

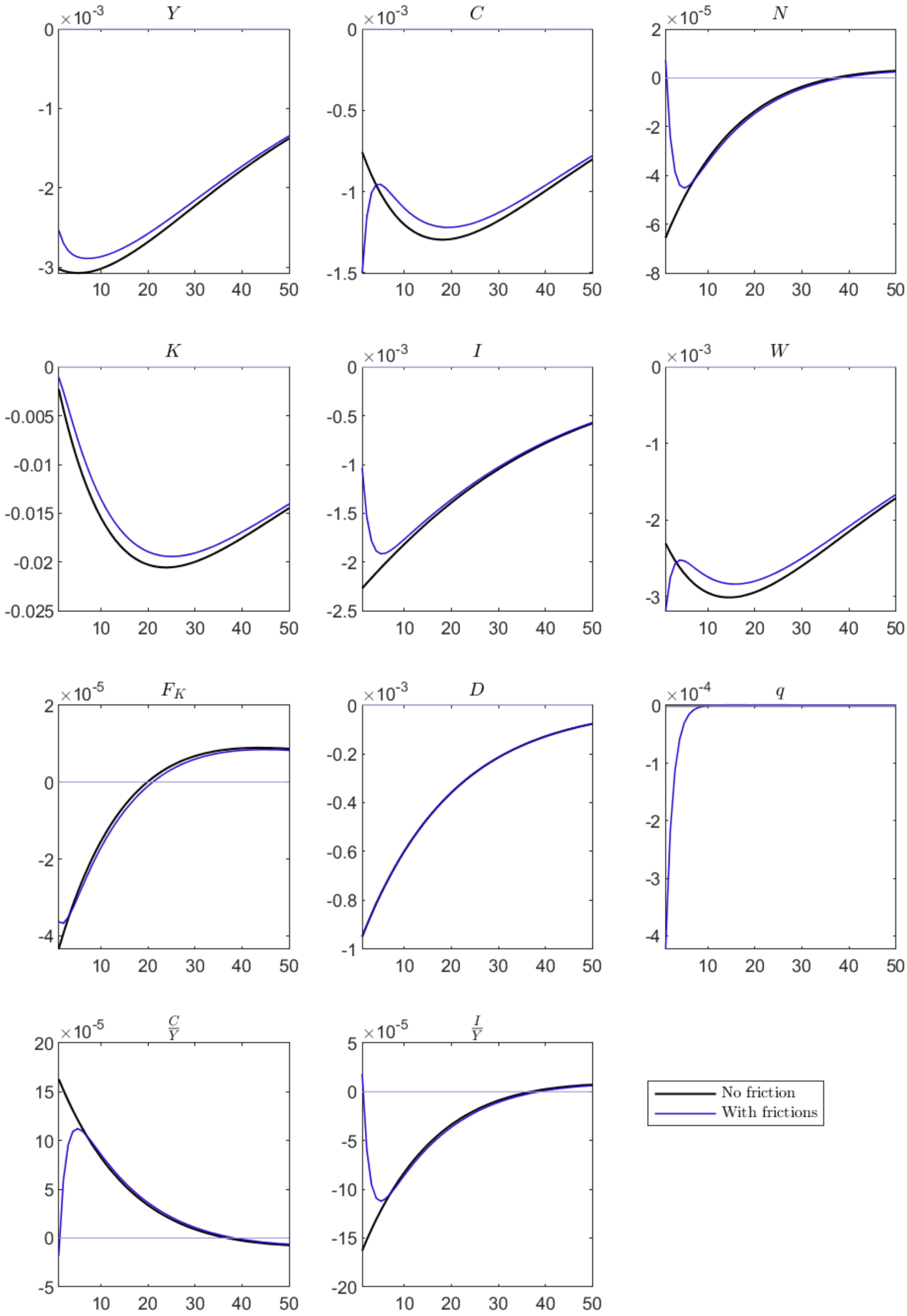

Figure 14: Reaction of the simplest model to a negative productivity shock with and without frictions

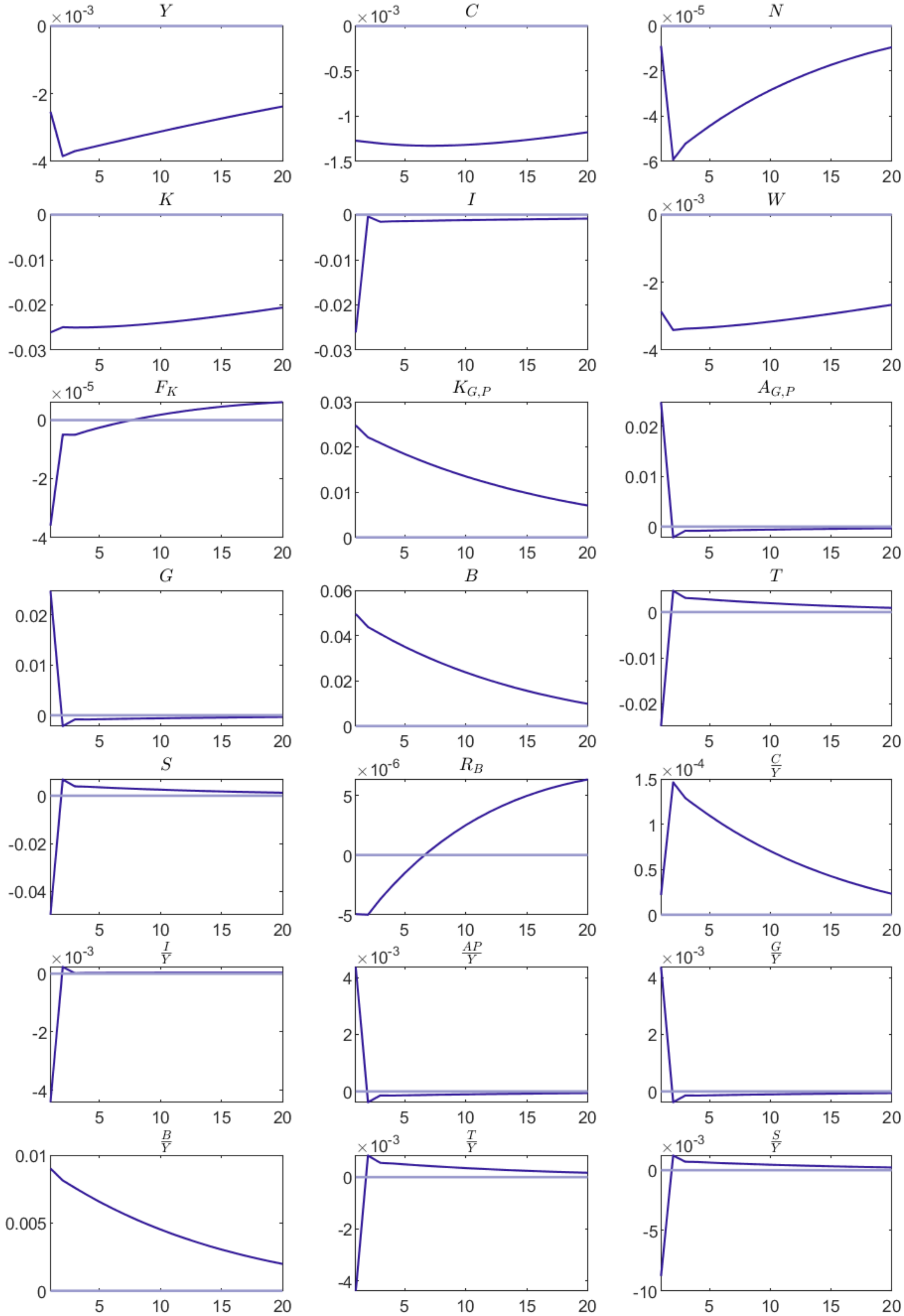

Figure 15: Introduction of adaptation capital in the economy, without frictions on investment - Zooming on the first periods

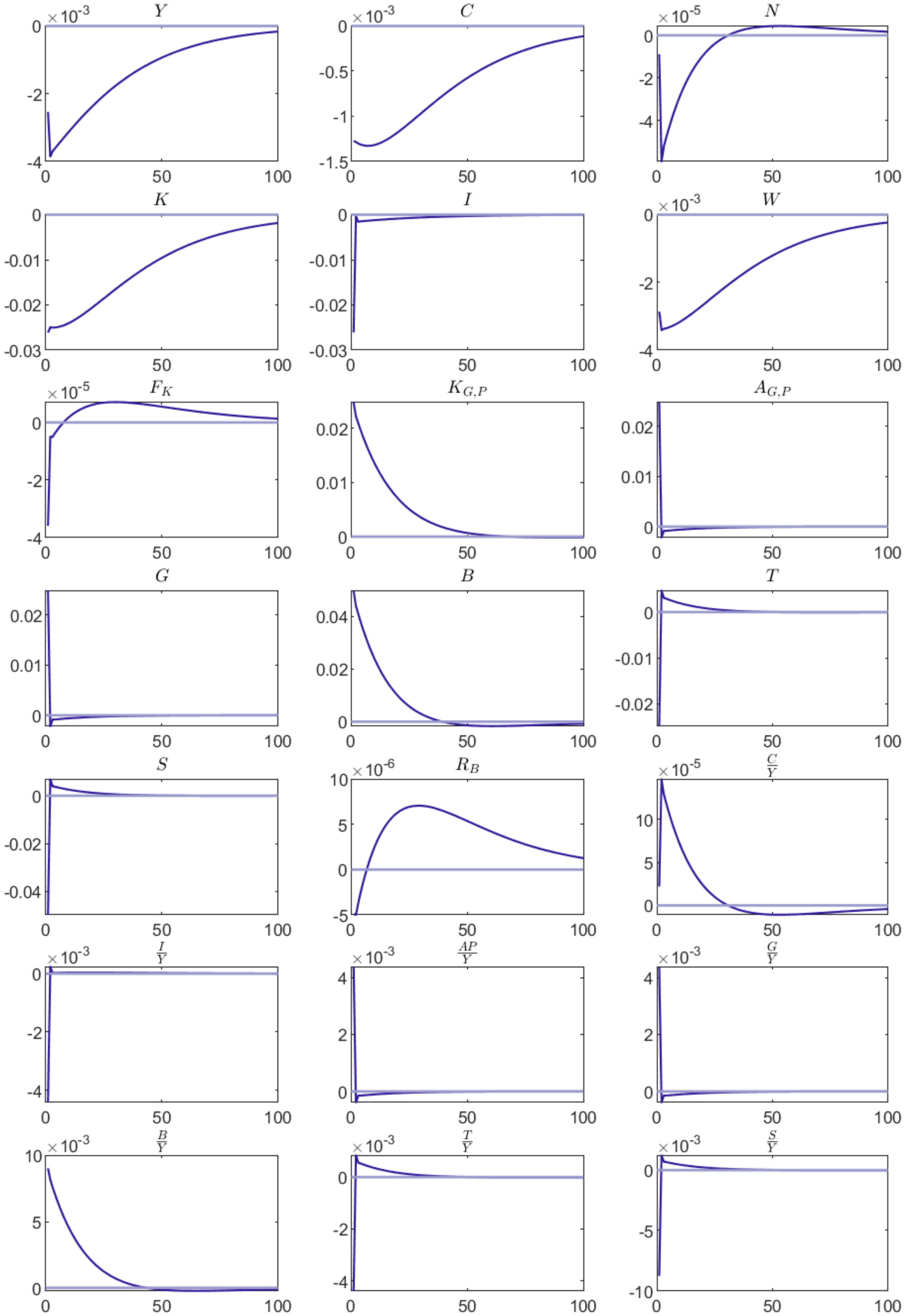

Figure 16: Introduction of adaptation capital in the economy, without frictions on investment

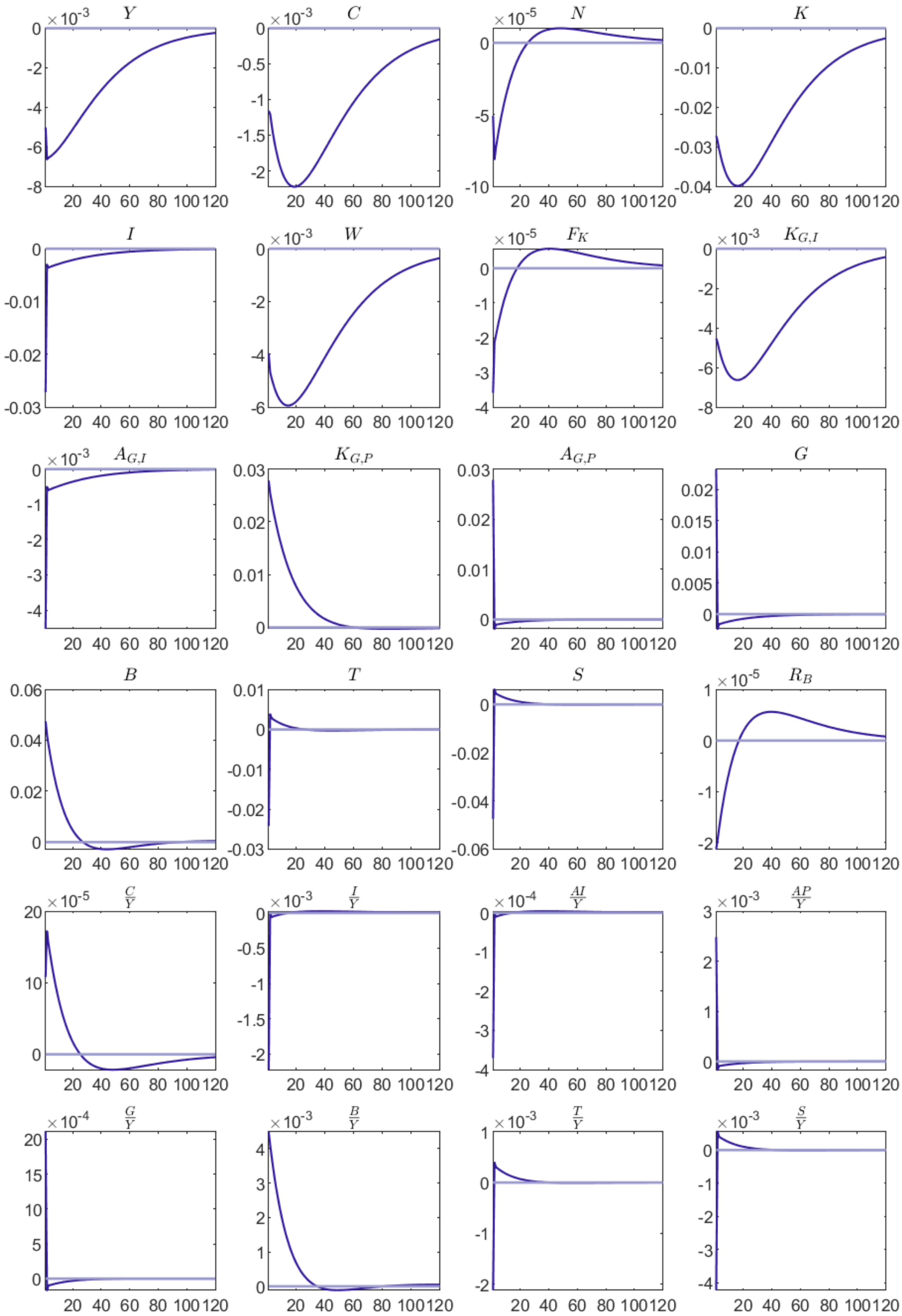

Figure 17: Introduction of public productive capital without frictions

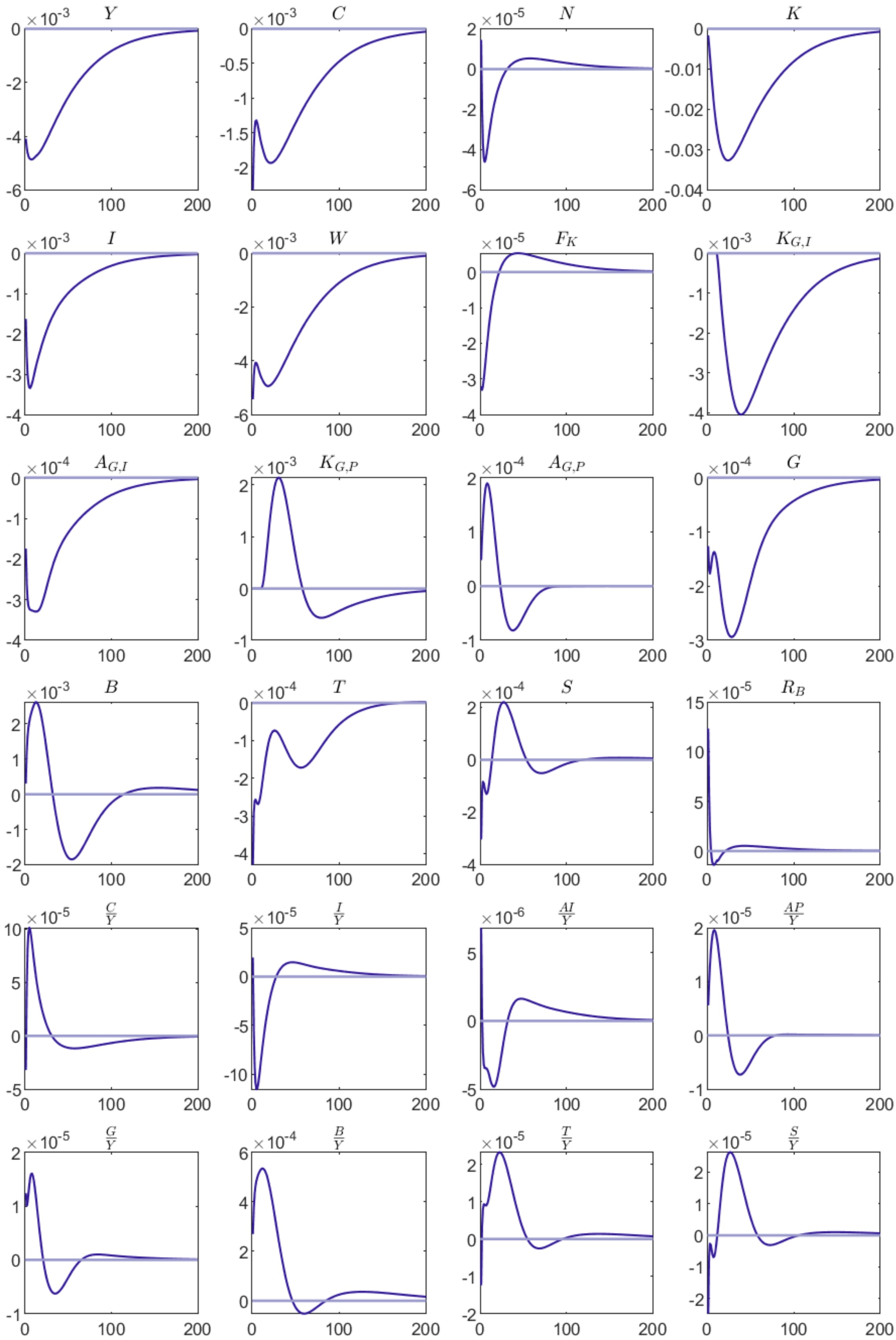

Figure 18: Full model with frictions

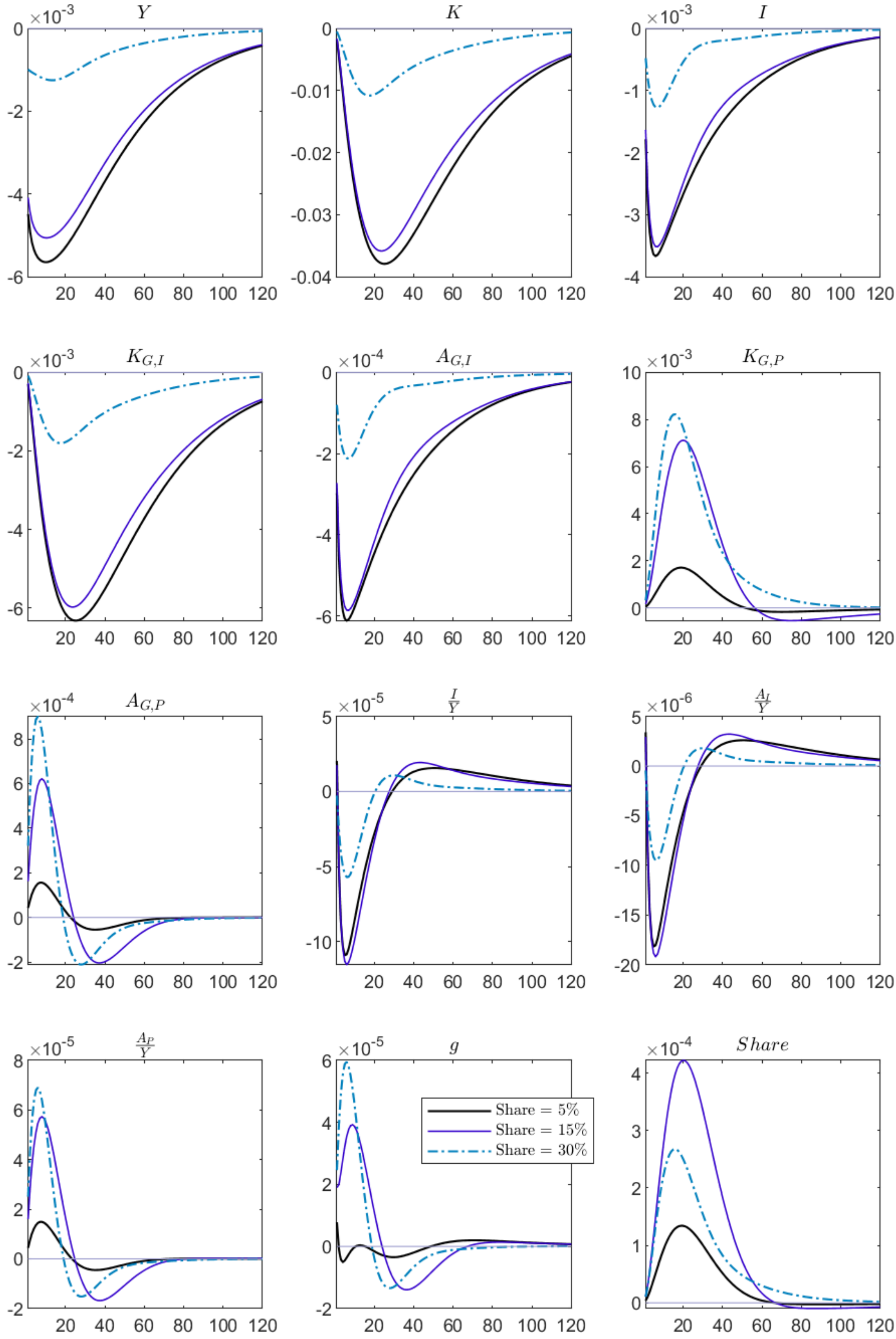

Figure 19: Focus on capital related variables for several share of adaptation capital over total public capital

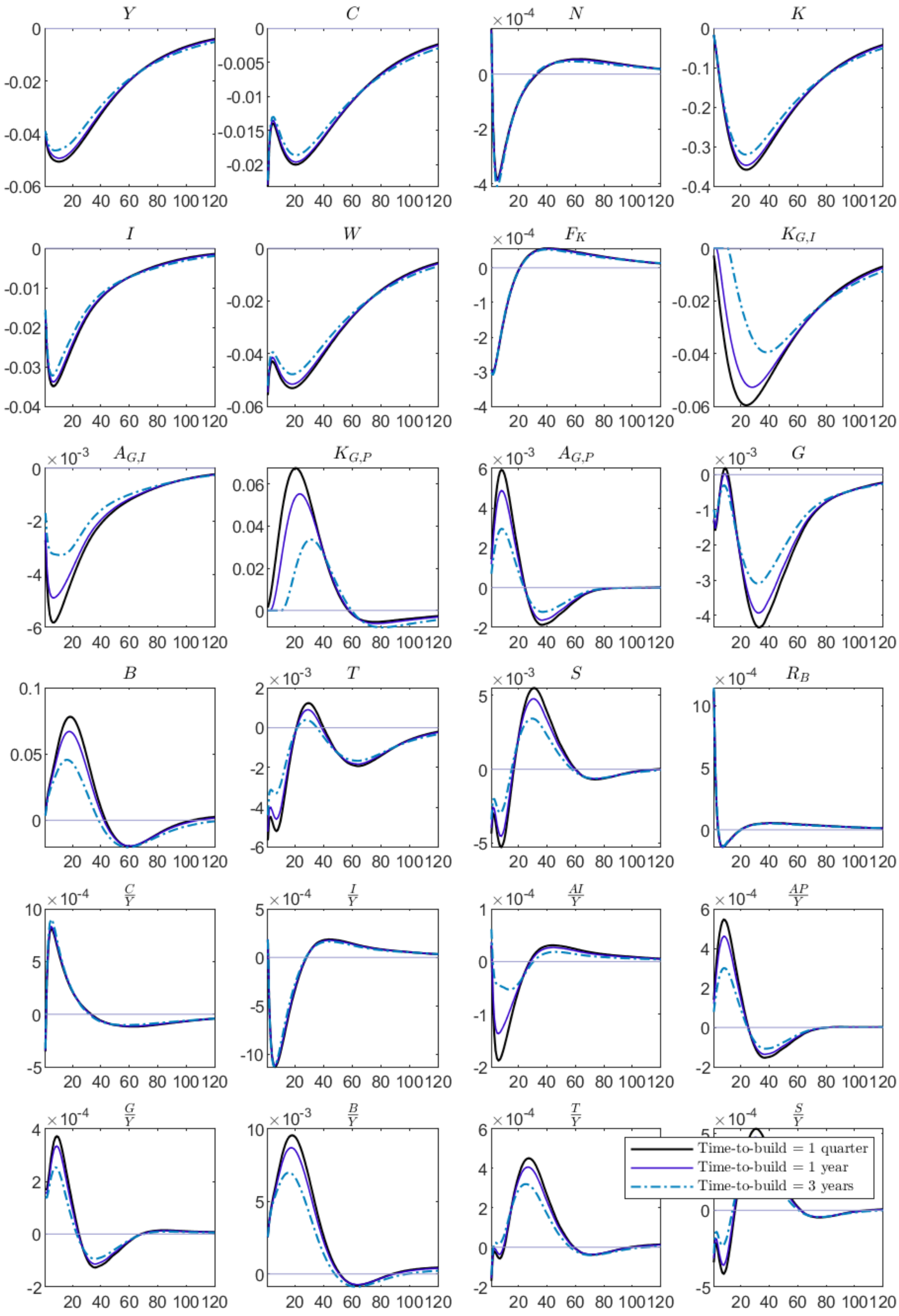

Figure 20: Reaction of the economy under different scenario of time-to-build for public investment and under a temperature shock of 10%

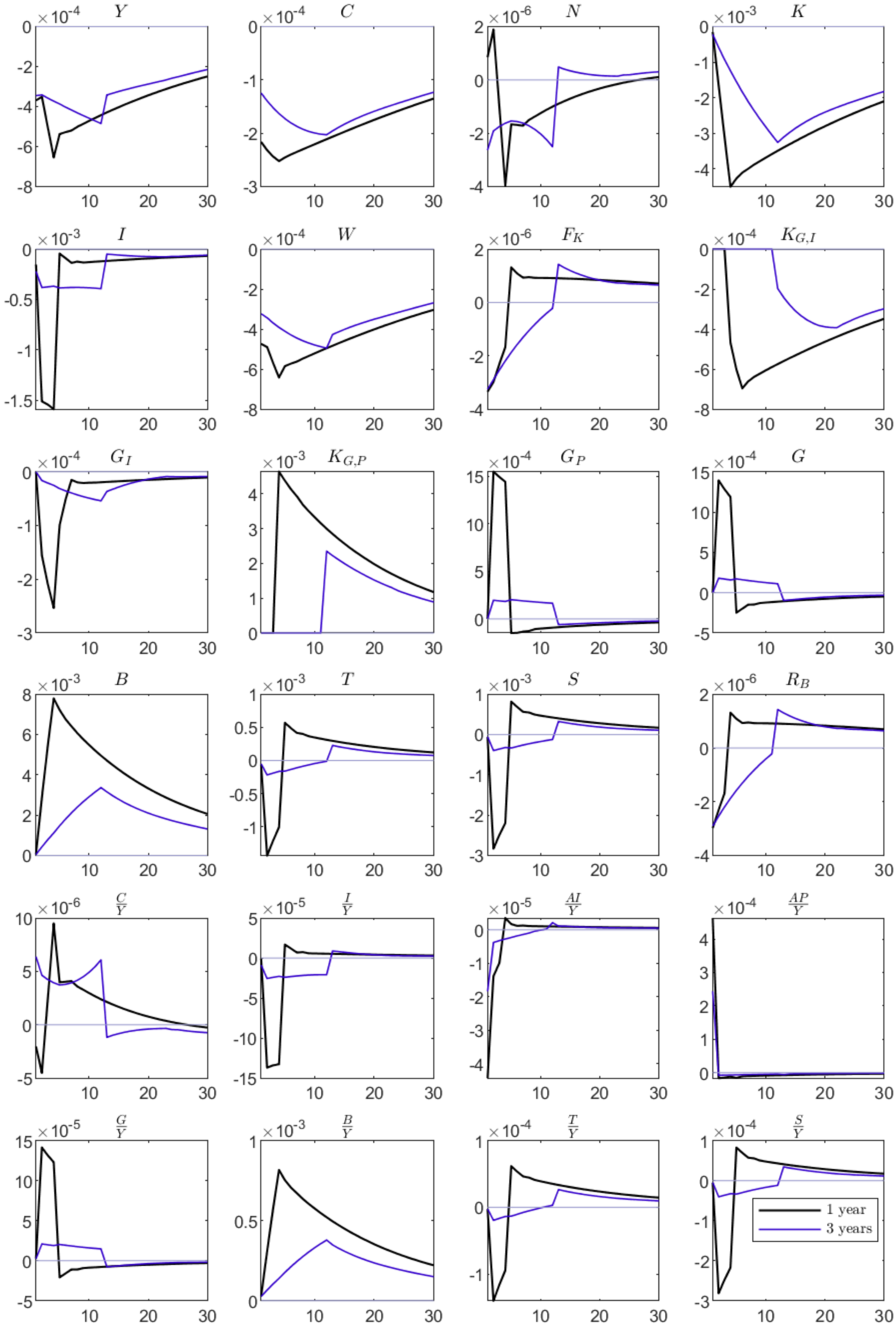

Figure 21: Introduction of the staggering of expenditures in the time-to-build a la Leeper et al. (2010)

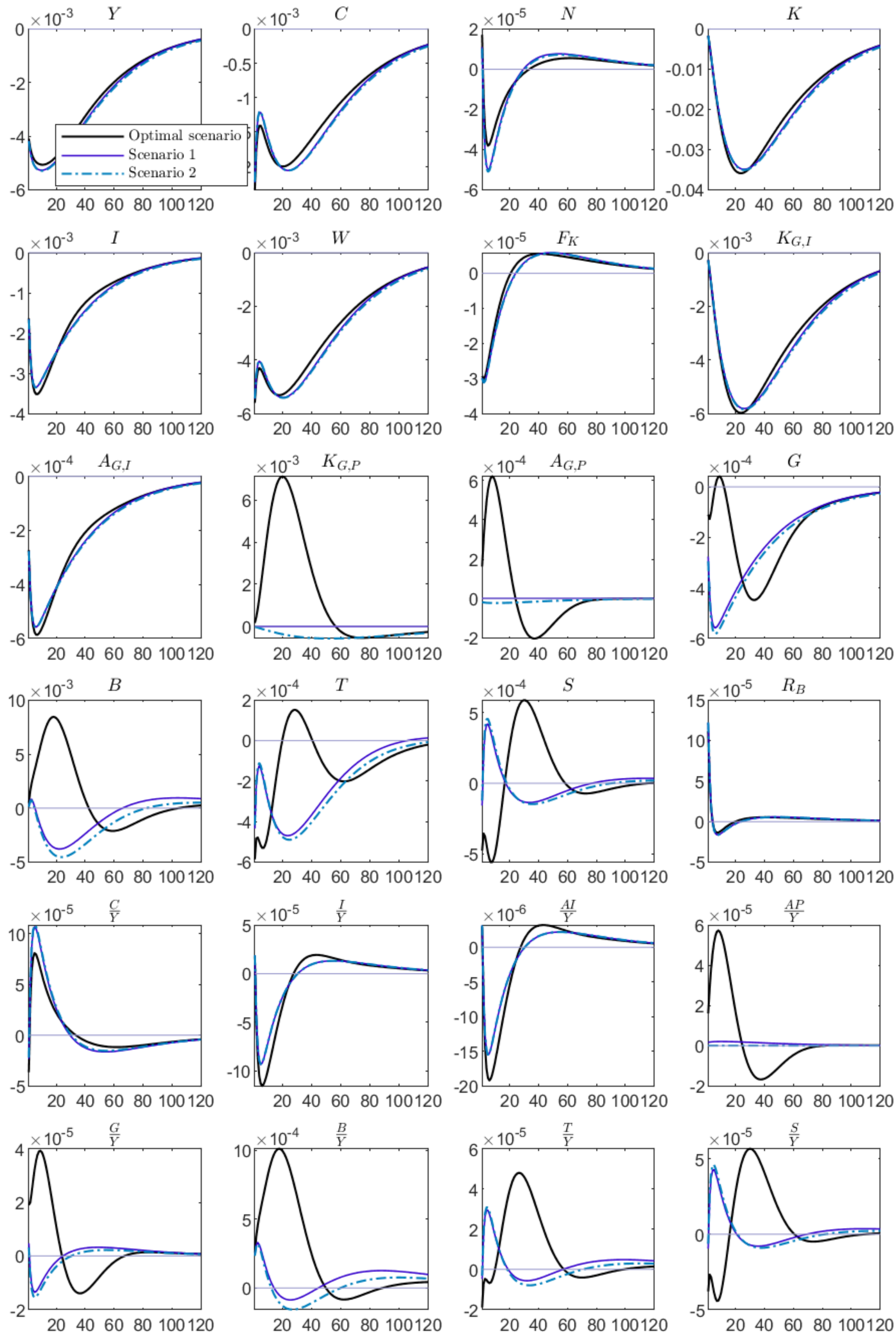

Figure 22: Comparing optimal versus non-optimal reaction of adaptation investment with share = 15%

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