~~Lec 0~~

① Structure first

② Doubles EPS

③ Distance $(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2+\dots$

③ Rounding

Point

```
struct Point{
    int x,y;
}A;
```

accessing

A.x
A.y

pointer

A->x
A->y

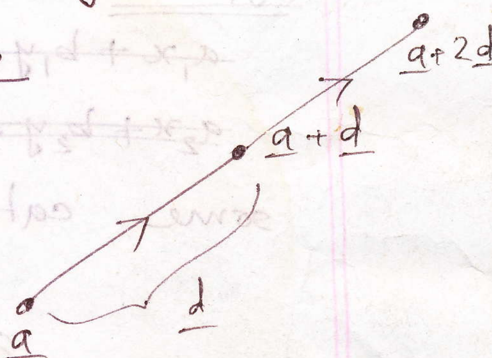
Line:

① $ax+by+c=0 \Rightarrow a,b,c$

② ~~A(t)B~~ $\underline{a} + \underline{d}\,t$

↑ ↑

Point type (x,y)



from 2 points: Line Segment:

P Q

①

$a = Q.x - P.x$

$b = P.y - Q.y$

$c = P.x * Q.y - Q.x * P.y$

```
struct Line{
    Point P, Q;
    int a,b,c;
};
```

```
struct Line{
    Point a,d;
};
```

②

$\underline{a} = \underline{P}$

$\underline{d} = \underline{Q} - \underline{P}$

or

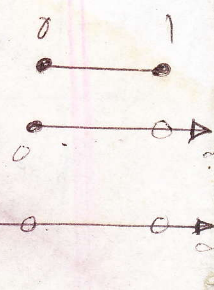
$\underline{a} = \underline{Q}$

$\underline{d} = \underline{P} - \underline{Q}$

then → Line segment $t = [0,1]$

→ ray $t = [0,\infty)$

→ line $t = (-\infty,\infty)$



Line (Perpen to $\underline{a}+\underline{d}t$) passing through $\underline{b}$.

$$[\underline{b} - (\underline{a} + \underline{d}t)] \cdot \underline{d} = 0 \Rightarrow t = \frac{\underline{a}\cdot\underline{d} - \underline{b}\cdot\underline{d}}{d^2}$$

$$\underline{c} = \cancel{b}\ \underline{a} + \underline{d}t$$

Line :: $\underline{b} + (\underline{c} - \underline{b})t$

Circle:

- x, y, r

or,

- Point c;
  int r;

Line / L.S. intersection:

Type 1:

$\cancel{a_1x + b_1y + c_1 = 0}$

$\cancel{a_2x + b_2y + c_2 = 0}$

some catch:

$ax + by + c > 0$

the line :: $\rightarrow ax + by + c = 0$

$ax + by + c < 0$

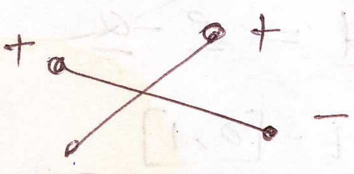
Line 1: (P_1, Q_1)

Line 2: (P_2, Q_2)

$a_1 = Q_1.x - P_1.x \qquad b_1 = P_1.y - Q_1.y$

$c_1 = P_1.x * Q_1.y - Q_1.x * P_1.y$

$a_2 = \dots \qquad b_2 = \dots \qquad c_2 = \dots$



$f_{P_2} = a_1(P_2.x) + b_1(P_2.y) + c_1$

$f_{Q_2} = a_1(Q_2.x) + b_1(Q_2.y) + c_1$

$f_{P_1} = a_2(P_1.x) + b_2(P_1.y) + c_2$

$f_{Q_1} = a_2(Q_1.x) + b_2(Q_1.y) + c_2$

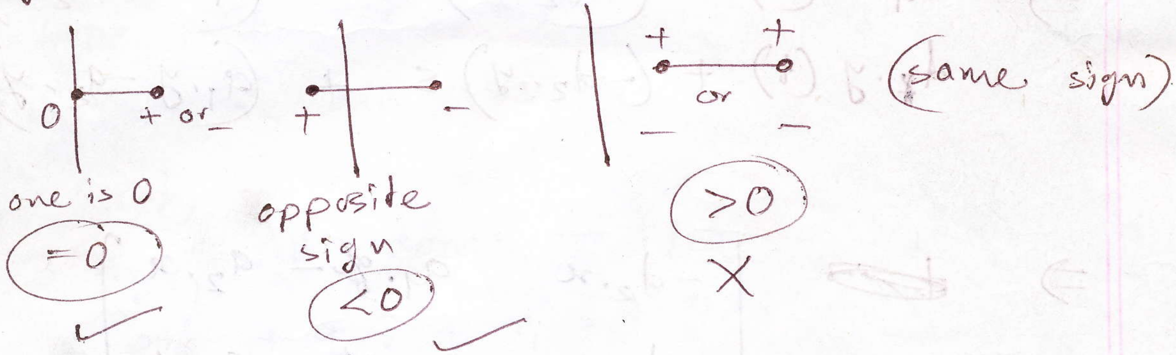
if P_1Q_1 is a L.S. then,

$$f_{P_1} * f_{Q_1} \leq 0$$

if P_2Q_2 is a L.S. then

$$f_{P_2} * f_{Q_2} \leq 0$$

why is so?



if $a_1 * b_2 - a_2 * b_1 = 0$ then {

parallel

if $a_1 * c_2 - a_2 * c_1 = 0$ then

coincident

}

if not parallel, intersection is I

$$I.x = \frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \qquad I.y = \frac{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Type 2:

Line 1: $\underline{a_1} + \underline{d_1}(t)$

Line 2: $\underline{a_2} + \underline{d_2}(s)$

for isech,

$$\underline{a_1} + \underline{d_1}\,t = \underline{a_2} + \underline{d_2}\,s$$

$$\Rightarrow \underline{d_1}\,t - \underline{d_2}\,s + (\underline{a_1} - \underline{a_2}) = 0$$

$$\Rightarrow d_1.x\,(t) + (-d_2.x)\,s + (a_1.x - a_2.x) = 0$$

$$d_1.y\,(t) + (-d_2.y)\,s + (a_1.y - a_2.y) = 0$$

$$\Rightarrow t = \frac{\begin{vmatrix} -d_2.x & a_1.x - a_2.x \\ -d_2.y & a_1.y - a_2.y \end{vmatrix}}{\begin{vmatrix} d_1.x & -d_2.x \\ d_1.y & -d_2.y \end{vmatrix}}$$

$$s = \frac{\begin{vmatrix} a_1.x - a_2.x & d_1.x \\ a_1.y - a_2.y & d_1.y \end{vmatrix}}{\begin{vmatrix} d_1.x & -d_2.x \\ d_1.y & -d_2.y \end{vmatrix}}$$

if $\begin{vmatrix} d_1.x & -d_2.x \\ d_1.y & -d_2.y \end{vmatrix} = 0$ then, parallel.

Now, t, s must satisfy the line/ray/LS criteria. Then ans is:

$$\underline{I} = \underline{q_1} + \underline{d_1}\, t$$

Line in 3D:

Type 1 is not possible

Type 2 remain same.

for isect,

3 equ will come like,

$$p_1 t + q_1 s + r_1 = 0$$
$$p_2 t + q_2 s + r_2 = 0$$
$$p_3 t + q_3 s + r_3 = 0$$

step 1:

Solve with those 2 equations for which

$$p_i q_j \neq p_j q_i$$

~~then use t, s wi~~

if ~~all are~~ no 2 are found then, no solution

step 2:

s, t must satisfy the 3rd equ

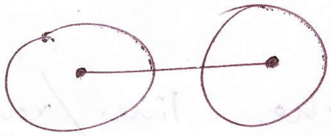
if not, no solⁿ.

Co-linear Test: $(x_1, y_1), (x_2, y_2), (x_3, y_3)$

$$\frac{x_1 - x_2}{y_1 - y_2} = \frac{x_1 - x_3}{y_1 - y_3} \quad \text{or} \quad (x_1 - x_2)(y_1 - y_3) = (x_1 - x_3)(y_1 - y_2)$$

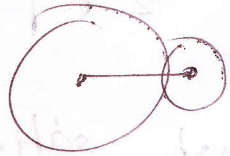
Circle-Circle thing

5 cases:



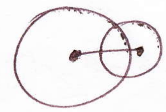
$d > r_1 + r_2$

~~nothing~~ (outside)



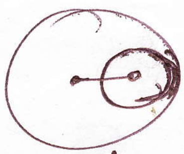
$d = r_1 + r_2$

(touch ex)



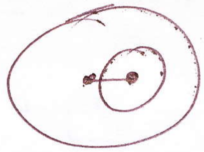
$|r_1 - r_2| < d < r_1 + r_2$

(isect)



$d = |r_1 - r_2|$

(touch internally)



$0 \leq d < |r_1 - r_2|$

(inside)

Finding angle

"acos ($\cos^{-1}$) is better than asin ($\sin^{-1}$)"

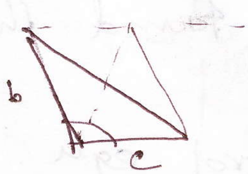
acos $\to (0, \pi)$ asin $\to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

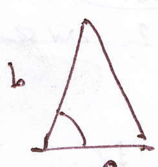
you know a, b, c, how would you angle A?

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2} bc \sin C$$

so, $C = \operatorname{asin}(2\Delta / bc)$

but





will have same Δ but different A

(as well as a)

Line ~~B~~ Divison :

$$C = \frac{m(B) + n(A)}{m+n}$$

A —m— C —n— B

$C.x = (m\,B.x + n\,A.x)/(m+n)$

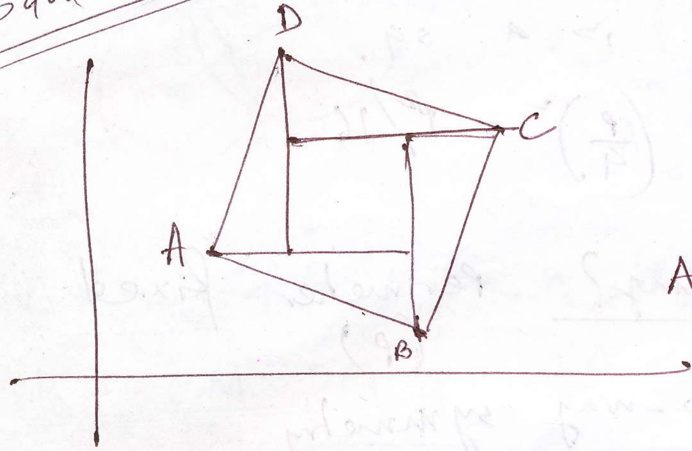
$C.y = (m\,B.y + n\,A.y)/(m+n)$

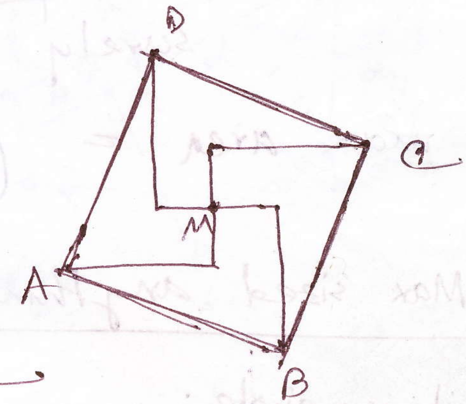
so use,

$$A = acos\left(\frac{b^2+c^2-a^2}{2bc}\right)$$

[cosine rule]

Square





If 2 consecutive points given (A, B)

$dx = B.x - A.x$

$dy = B.y - A.y$

$C.x = B.x - dy$

$C.y = B.y + dx$

$B.x = A.x + dx$

$B.y = A.y + dy$

$D.x = A.x - dy$

$D.y = A.y + dx$

If 2 opposite points are given (A, C)

$M.x = (A.x + C.x)/2$; $M.y = (A.y + C.y)/2$

$dx = M.x - A.x$

$dy = M.y - A.y$

$B.x = M.x + dy$

$B.y = M.y - dx$

$D.x = M.x - dy$

$D.y = M.y + dx$

Max-sized Rect? Perimeter fixed. (P)

4 way symetry.

surely is a sq.

max area $= \left(\frac{P}{4}\right)^2 = P^2/16$.

Max sized anything? Perimeter fixed (P)

it is circle: ∞-way symmetry,

$$2\pi r = P$$

$$\Rightarrow r = \frac{P}{2\pi}$$

$$A = \pi r^2 = \pi \cdot \left(\frac{P}{2\pi}\right)^2 = \frac{P^2}{4\pi}$$

Now,

$$\pi < 4$$

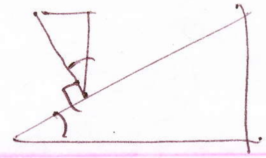
$$\Rightarrow \frac{1}{4} < \frac{1}{\pi}$$

$$\Rightarrow \frac{1}{16} < \frac{1}{4\pi}$$

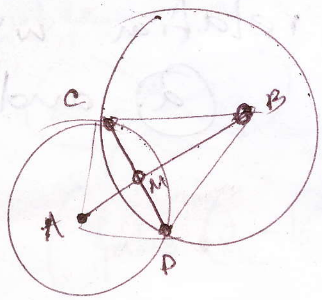
$$\Rightarrow \frac{P^2}{16} < \frac{P^2}{4\pi}$$

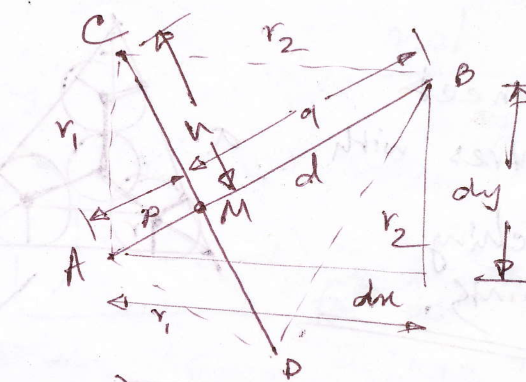
so, clearly circle has greater area

"More symmetry more optimal"



Cir-Cir Intersection → calcing:





Δ = area of tri. of (r_1, r_2, d)

$h = CM = DM = \frac{2\Delta}{d}$

$p = AM = \sqrt{r_1^2 - h^2}$

$q = BM = \sqrt{r_2^2 - h^2}$

$M.x = \frac{p(B.x) + q(A.x)}{p+q}$

$M.y = \frac{p(B.y) + q(A.y)}{p+q}$

$dx = B.x - A.x$

$dy = B.y - A.y$

$C.x = M.x - dy * h/d$

$C.y = M.y + dx * h/d$

dy
dx

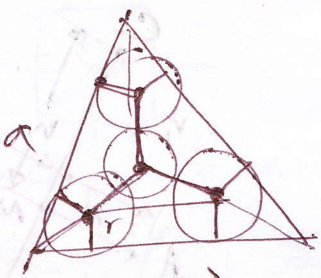
$D.x = M.x + dy * h/d$

$D.y = M.y - dx * h/d$

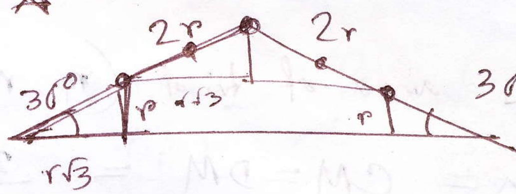
dy
dx

A circle Related Prob:

connect centres with touching points.

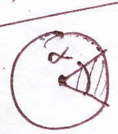


relation with d and r



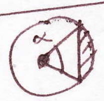
$$a = 4r\sqrt{3}$$

area of sector



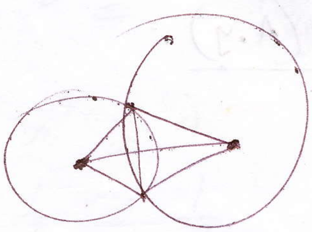
$\frac{1}{2}\alpha r^2$

area of

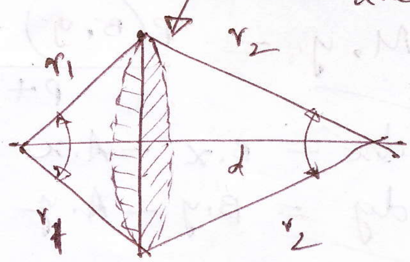


$\frac{1}{2}\alpha r^2 - \frac{1}{2}r^2\sin\alpha = \boxed{\frac{1}{2}r^2(\alpha - \sin\alpha)}$

Overlapped area of circles:

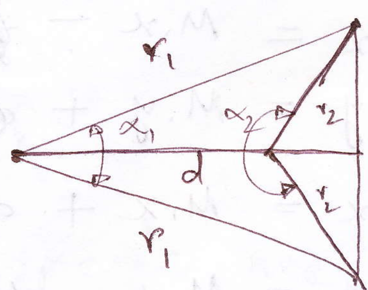


common area



$$\alpha_1 = 2\,acos\left(\left(r_1^2 + d^2 - r_2^2\right) / 2r_1 d\right)$$

$$\alpha_2 = 2\,acos\left(\left(r_2^2 + d^2 - r_1^2\right) / 2r_2 d\right)$$



common area,

$$= \frac{1}{2}r_1^2(\alpha_1 - \sin\alpha_1) + \frac{1}{2}r_2^2(\alpha_2 - \sin\alpha_2)$$

We must use acos because > 180° is possible.

How to handle doubles ? EPS ?

doubles are **not precise**

double a, b;

if ($\cancel{a == b}$) wrong.

if (fabs (a − b) < EPS) → $1e^{-8} \sim 1e^{-12}$ is reasonable.

depends on problem sometimes.

① Firstly equality check.

② then less/greater check.

Rounding :

say, to 3 digits:

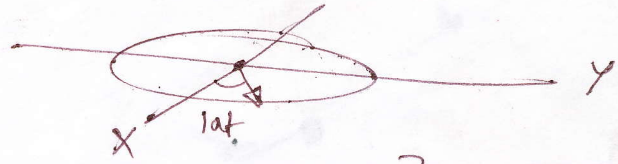
```
double round (double x){
    return floor(x * 1000 + 0.5) / 1000. ;
}
```

+ 0.5 → for rounding.

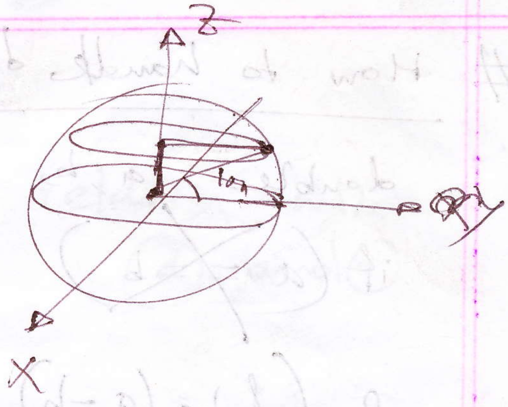
না থাকলে truncate

$\text{lon} \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\text{lat} \rightarrow (-\pi, \pi]$



Geodisic Distance



$$x = R\cos(\text{lat})\cos(\text{lon})$$

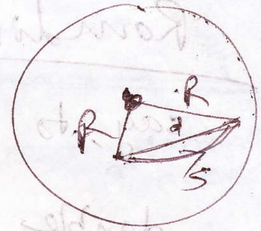
$$y = R\sin(\text{lat})\cos(\text{lon})$$

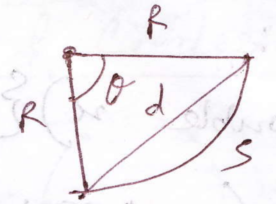
$$z = R\sin(\text{lon})$$

$$A(\text{lat}1, \text{lon}2) \rightarrow A(x_1, y_1, z_1)$$

$$B(\text{lat}2, \text{lon}2) \rightarrow B(x_2, y_2, z_2)$$

$$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2}$$





$$\cos\theta = \frac{R^2+R^2-d^2}{2R.R} = 1 - d^2/2R^2$$

$$\Rightarrow \theta = \text{acos}(1 - d^2/2R^2)$$

$$\boxed{s = R*\theta}$$

Polygon:

area:

$A = 0$

for $(i = 0; i < n; i{+}{+})$ {

$j = (i+1) \% n$

$A += p[i].x * p[j].y - p[j].y * p[i].x$

}

$A /= 2$;

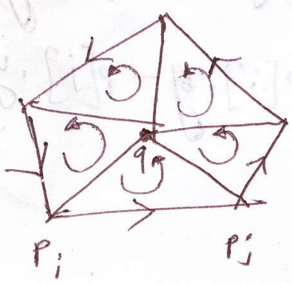
cw / ccw ?

*) $A > 0$ ccw

*) $A < 0$ cw

* $A = 0$??

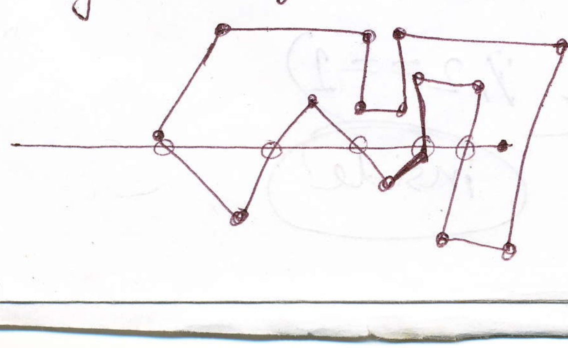
Point in Poly? (Convex)



so, for all i [and $j = (i+1) \% n$] q, p_i, p_j will form a ccw triangle if q lies inside Poly.

Point in Poly (Any)

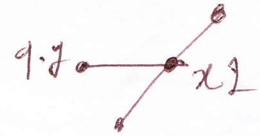
Ray shooting method



~~জোড়~~

বিজোড় সংখ্যক বাহুর ~~[illegible]~~ ছেদ করলে

inside



$$\frac{x1 - \cancel{q}\,p_i.x}{p_i.x - p_j.x} = \frac{q.y - p_i.y}{\quad}$$

(1) check if ~~they are~~ it is online with any ~~[illegible]~~ → decide according to problem.

(2) c = 0;

for (i=0; i<n; i++){

j = (i+1) % n;

if ((p[i].y <= q.y && q.y < p[j].y) ||

(p[j].y <= q.y && q.y < p[i].y)) {

~~x1 = p[i].x + p[i].~~

x1 = p[i].x + (p[i].x − p[j].x)

*(q.y − p[i].y)

/(p[i].y − p[j].y)

if (x1 < q.x)

c++;

}

}

~~return c~~ if (c % 2 == 1)

inside