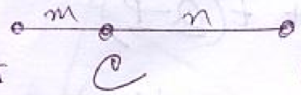
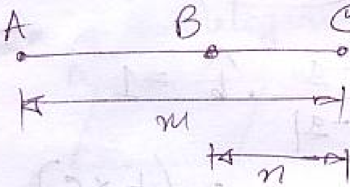
Vector

(*) 

$$C = \frac{mB + nA}{m+n}$$

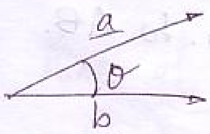
Midpoint of A, B, $M = \frac{A+B}{2}$

(*) 

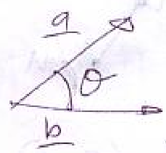
$$C = \frac{mB - nA}{m-n}$$

(*) ~~A, B, C~~ $\underline{a}, \underline{b}, \underline{c}$ coplaner, then, $\underline{c} = p\underline{a} + q\underline{b}$, $p, q \in \mathbb{R}$

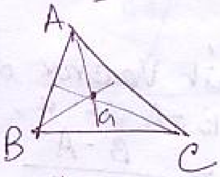
(*) $\underline{a}, \underline{b}$ ~~copt~~ colinear then, $\underline{b} = \lambda \underline{a}$, $\lambda \in \mathbb{R}$

(*) 

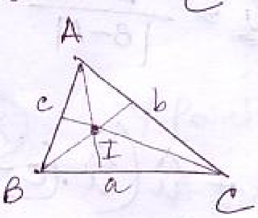
$$\cos\theta = \frac{\underline{a}\cdot\underline{b}}{|\underline{a}||\underline{b}|} = \hat{a}\cdot\hat{b}$$

(*) 

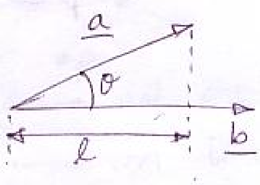
$$\sin\theta = \frac{|\underline{a}\times\underline{b}|}{|\underline{a}||\underline{b}|}$$

(*) 

$$G = \frac{A+B+C}{3}$$

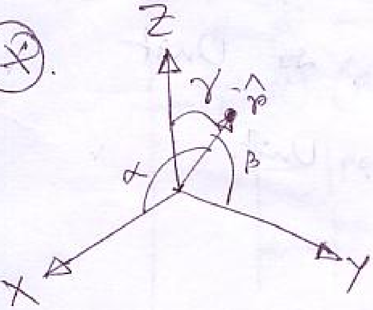
(*) 

$$I = \frac{aA + bB + cC}{a+b+c}$$

(*) লম্ব অভিক্ষেপ 

$$\begin{aligned} l &= a\cos\theta \\ &= a(\hat{a}\cdot\hat{b}) \\ &= \underline{a}\cdot\hat{b} \end{aligned}$$

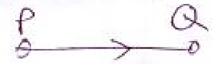
$$\underline{a}_{on\,b} = \hat{b}\,(\underline{a}\cdot\hat{b})$$

(*) 

$\hat{r}$ এর directional cosines,

$$\begin{aligned} l &= \cos\alpha = \hat{r}\cdot\hat{i} \\ m &= \cos\beta = \hat{r}\cdot\hat{j} \\ n &= \cos\gamma = \hat{r}\cdot\hat{k} \end{aligned}$$

(*) Straight Line:



$$\underline{r} = \underline{a} + t\underline{b} \qquad \underline{r} = \underline{P} + t(\underline{Q} - \underline{P})$$

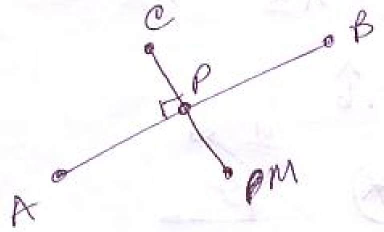
(*) Test to whether, $\underline{c}$ is on $(\underline{r} = \underline{a} + t\underline{b})$

$$(\underline{c} - \underline{a}) \times \underline{b} = 0 \quad \text{or,} \quad \frac{\underline{c} - \underline{a}}{|\underline{c} - \underline{a}|} \cdot \hat{b} = 1$$

(*) $[a\ b\ c] = [b\ c\ a] = [c\ a\ b] = \underline{a} \cdot (\underline{b} \times \underline{c})$

(*) $\underline{A} \times (\underline{B} \times \underline{C}) = (\underline{A} . \underline{C})\underline{B} - (\underline{A} . \underline{B})\underline{C}$

(*)



$CP \perp AB$ @ P

M = mirror(C) w. r. to AB.

Let, $p = A + (B - A)t$

$$\Rightarrow (C - P) \cdot (B - A) = 0$$
$$\Rightarrow C \cdot (B - A) = (A + (B - A)t) \cdot (B - A)$$
$$\Rightarrow C \cdot (B - A) - A \cdot (B - A) = |B - A|^2 t$$
$$\Rightarrow t = \frac{(C - A) \cdot (B - A)}{|B - A|^2}$$

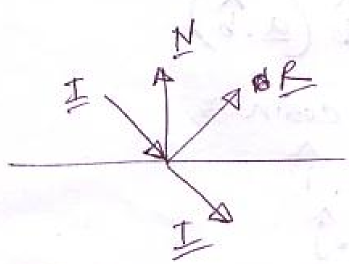
let, Unit Vector of $B - A$ is u, so,

$$\hat{u} = \frac{B - A}{|B - A|}$$

$$\therefore p = A + \hat{u}(\hat{u} \cdot (C - A))$$

$$C - P = P - M$$
$$\Rightarrow \boxed{M = 2P - C}$$

(*)

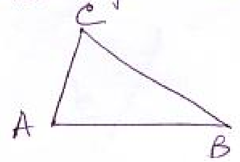


$$\hat{R} = (\hat{I} - 2(\hat{N} . \hat{I})\hat{N}) \text{. to Unit}$$

$$\hat{N} = (\hat{R} + -\hat{I}) \text{. to Unit}$$
$$= (\hat{R} - \hat{I}) \text{ to Unit.}$$

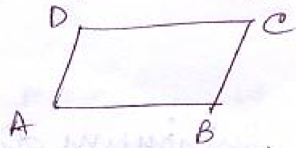
$$\hat{N} . \hat{R} > 0$$

(*) Area of Triangle



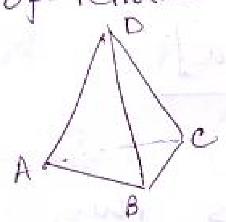
$$\frac{1}{2}\left|(B-A)\times(C-A)\right|$$

(*) Area of Parallelogram



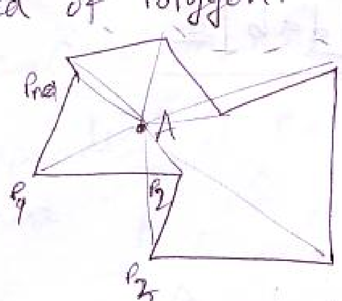
$$\left|(B-A)\times(D-A)\right|$$

(*) Vol^m of Tetrahedron



$$\frac{1}{6}\left|\begin{bmatrix} A-D & B-D & C-D \end{bmatrix}\right|$$

(*). Area of Polygon:



$$\frac{1}{2}\left|\sum_{i=1}^{n}(P_{i+1}-A)\times(P_i-A)\right|$$

$$=\frac{1}{2}\left|\sum_{i=1}^{n}P_{i+1}\times P_i\right|$$

(*) Line/Point Classification:

- $[P\ A\ B] \neq 0 \xrightarrow{op}$ not on line
- $[P\ A\ B] = 0 \longrightarrow$ on line
  - $P = B \longrightarrow$ destination $[t=1]$
  - $P = A \longrightarrow$ src $[t=0]$
  - $(P-A).(P-B) > 0 \longrightarrow$ outside the line $[t<0 \,||\, t>$
    - $|P-B| < |P-A| \longrightarrow$ after B (beyond) $[t>1]$
    - $|P-B| > |P-A| \longrightarrow$ before ~~B~~A $[t<0]$
  - $(P-A).(P-B) < 0 \longrightarrow$ inside AB. $[0<t<1]$

(*) 2 St. lines: [3D]

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} \qquad \mathbf{r}^* = \mathbf{a}' + s\mathbf{b}'$$

they intersect iff,

$$[\mathbf{b}\ \mathbf{b}'\ \mathbf{a}-\mathbf{a}'] = 0$$

$$\Leftrightarrow [\mathbf{a}-\mathbf{a}'\ \mathbf{b}\ \mathbf{b}'] = 0$$

$$\Leftrightarrow [\mathbf{a}\ \mathbf{b}\mathbf{b}'] = [\mathbf{a}'\ \mathbf{b}\mathbf{b}']$$

If they do not intersect, then their minimum distance

$$= (\mathbf{a}'-\mathbf{a}) \cdot \frac{(\mathbf{b}\times\mathbf{b}')}{|\mathbf{b}\times\mathbf{b}'|} = \frac{[\mathbf{b}\ \mathbf{b}'\ \mathbf{a}'-\mathbf{a}]}{|\mathbf{b}\times\mathbf{b}'|}$$

(*) if $\underline{a}, \underline{b}, \underline{c}, \underline{d}$ are 4 vectors then,

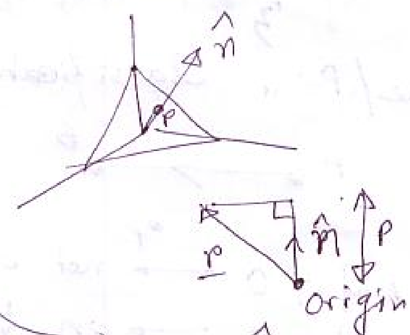
$$\underline{d} = \frac{[d\ b\ c]\underline{a} + [d\ c\ a]\underline{b} + [d\ a\ b]\underline{c}}{[a\ b\ c]}$$

where

$[abc] \neq 0$

(*) Equation of Plane:

$$\boxed{p - \underline{r}\cdot\hat{n} = 0}$$



$p = \underline{r}\cdot\hat{n}$

$\Rightarrow$ ~~p~~ $p -$

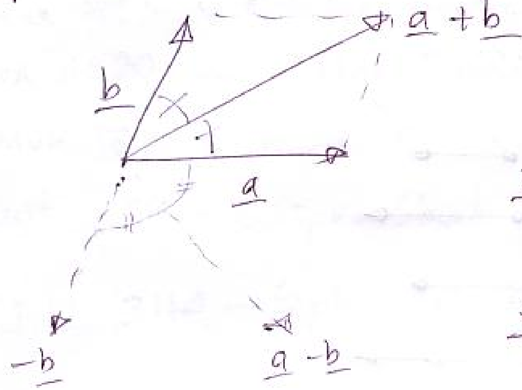
(*) Equation of Bisectors of the angles between 2 planes.

$p - \underline{r}\cdot\hat{n}_1 = 0$

$p' - \underline{r}\cdot\hat{n}_2 = 0$

$$\boxed{p + p' = \mathbf{r}\cdot(\hat{n}_1 + \hat{n}_2)}$$

(*) Eq^n of a bisector of an angle.



$$\underline{r} = t(\underline{a} + \underline{b})$$

or

$$\underline{r} = t(\underline{a} - \underline{b})$$

(*) eq^n of a plane passing through A, B, C.

$$\underline{r} = C + s(A - C) + t(B - C)$$

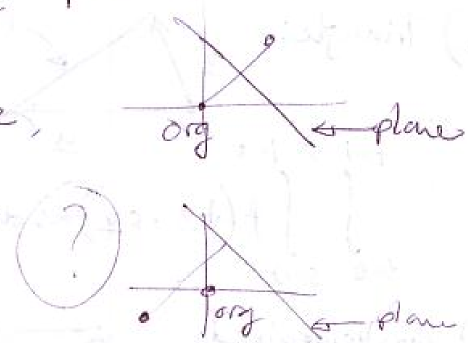
(*) Angles bet^n planes, $p = \underline{r}.\hat{n}_1$; $p = \underline{r}.\hat{n}_2$

$$\cos\theta = \hat{n}_1 . \hat{n}_2$$

(*) Perp. $dist^n$ of ~~p~~ a Point ~~C~~ from the planre $p = \underline{r}.\hat{n}$,

$p - \underline{c}.\hat{n}$ → +ve,

$p - \underline{c}.\hat{n}$ → -ve



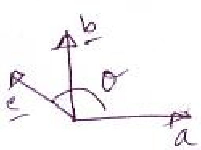
(*) Sphere of Unit Radius.

Normal @ $\underline{A}$, $\hat{N} = \underline{A}$

Tangent Plane @ $\underline{A}$, $p = r.N$

$$\Rightarrow \boxed{\underline{r}.\underline{A} = 1}$$

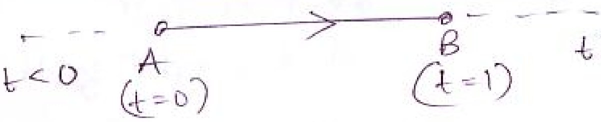
(*) if $\hat{a}$, $\hat{b}$ are coplaner, and $\hat{c}$ is θ ~~deg~~ ccw ~~b~~ from A (and see below) then,



$$\boxed{\hat{c} = (\hat{a}\cos\theta + \hat{b}\sin\theta)}$$

(*) Line Representation.

$$r = A + t(B-A)$$



$t \in [0,1]$: LS 

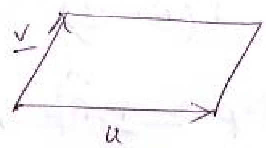
$t \in [0,\infty)$: Ray + 

$t \in (-\infty,1]$: Ray − 

$t \in (-\infty,\infty)$: Line. 

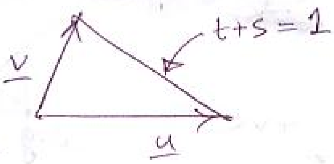
(*). Vector Integration Techniques:

1) Parallelogram:



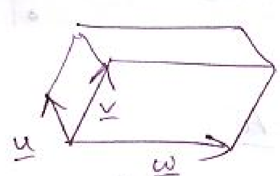
$$\int_{t=0}^{t=1}\int_{s=0}^{s=1} f(t\underline{u}, s\underline{v})\, \cancel{du\,dv}\; ds\,dt$$

2) Triangle:



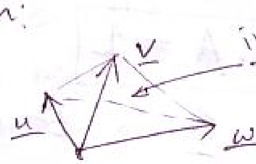
$$\int_{t=0}^{t=1}\int_{s=0}^{s=1-t} f(t\underline{u}, s\underline{v})\, ds\,dt$$

3) Parallelopiped.



$$\int_{a=0}^{a=1}\int_{b=0}^{b=1}\int_{c=0}^{c=1} f(a\underline{u}, b\underline{v}, c\underline{w})\, da\,db\,dc$$

4) Tetrahedron:



in this plane, $a+b+c=1$

$$\int_{a=0}^{a=1}\int_{b=0}^{b=1-a}\int_{c=0}^{c=1-a-b} f(a\underline{u}, b\underline{v}, c\underline{w})\, da\,db\,dc.$$

Vector Problems:

Timus 1703 — Robotic Arm [3D] **

UVA 11580. — Finding Transmitter [3D] ****

Timus 1710 — Boris, You are Wrong [2D] *

TIMUS 1697 — Sniper Shot [3D] *****

TJU 3114 — Spherical Mirrors [3D] ***