Parcial 2

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$x(t) = |A \operatorname{sen}(2\pi F_0 t)|^2 \quad t \in \left[-\frac{1}{2F_0}, \frac{1}{2F_0}\right] \quad A, F_0 \in \mathbb{R}^+$

$T = \frac{1}{2F_0} - \left(-\frac{1}{2F_0}\right) = \frac{2}{2F_0} = \frac{1}{F_0} \Rightarrow T = T_0$

$t \in \left[-\frac{T_0}{2}, \frac{T_0}{2}\right]$

$x(t) = A^2 \operatorname{sen}^2(2\pi F_0 t) = A^2\left[\frac{1}{2} - \frac{1}{2}\cos(2\pi 2F_0 t)\right]$

$x(t) = \frac{A^2}{2} - \frac{A^2}{2}\cos[4\pi F_0 t]$

$x(t) = a_0 + \sum_{n=1}^{N} a_n \cos(n\omega_0 t) + b_n \operatorname{sen}(n\omega_0 t)$

$x(t) = \sum_{n=-N}^{N} c_n e^{jn\omega_0 t} \qquad c_n = \frac{a_n - jb_n}{2}$

$\frac{A^2}{2} - \frac{A^2}{2}\cos(2\omega_0 t) = a_0 + a_2\cos(2\omega_0 t)$

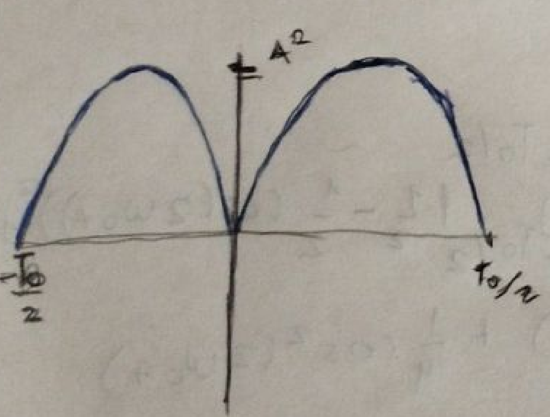
$\omega_0 = 2\pi F_0 \qquad \omega_n = n\omega_0$

$\omega_n = 2\omega_0 \qquad n\omega_0 = 2\omega_0 \qquad n = 2$

$a_0 = c_0 = \frac{A^2}{2} \qquad a_2 = -\frac{A^2}{2} \qquad c_2 = c_{-2} = -\frac{A^2}{4}$

$c_n = 0 \;\forall\; n \neq \{0, -2, 2\}$

$a_n = 0 \;\forall\; n \neq \{0, 2\} \qquad b_n = 0$



Señal par $x(t) = -x(t) = x(-t)$

$b_n = 0$

$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\cos(n\omega_0 t)\,dt$

$= \frac{4}{T_0}\int_{0}^{T_0/2} A^2 \operatorname{sen}^2(2\pi F_0 t)\cos(n\omega_0 t)\,dt$

$= \frac{4A^2}{T_0}\left[\frac{1}{2}\int_0^{T_0/2}\cos(n\omega_0 t)\,dt - \frac{1}{2}\int_0^{T_0/2}\cos(2\omega_0 t\cos(n\omega_0 t)\,dt\right]$

$\cos(2\omega_0 t)\cos(n\omega_0 t) = \frac{1}{2}\left[\cos(2\omega_0 t + n\omega_0 t) + \cos(2\omega_0 t - n\omega_0 t)\right]$

$$a_n = \frac{2A^2}{T_0} \operatorname{sen}(n\omega_0 t)\, n\omega_0 - \frac{2A^2}{2T_0} \operatorname{sen}((2+n)\omega_0 t)(2+n)\omega_0$$

$$- \frac{2A^2}{2T_0} \operatorname{sen}((2-n)\omega_0 t)(2-n)\omega_0 \Big|_0^{T_0/2}$$

$$a_n = \frac{2A^2}{T_0} \operatorname{sen}\left(n \frac{2\pi}{T_0}\frac{T_0}{2}\right) n \frac{2\pi}{T_0} - \frac{2A^2}{2T_0} \operatorname{sen}\left((2+n)\frac{2\pi}{T_0}\frac{T_0}{2}\right)(2+n)\frac{2\pi}{T_0}$$

$$\operatorname{sen}(n\pi) = \operatorname{sen}((2+n)\pi) = \operatorname{sen}((2-n)\pi) = 0$$

$$a_n = 0 \quad \text{si } n \neq 0,2$$

$n=2$ $\quad a_2 = \frac{4A^2}{T_0}\left[-\frac{1}{2}\int_0^{T_0/2} \cos^2(2\omega_0 t)\,dt\right] = \frac{4A^2}{T_0}\left[-\frac{1}{2}\frac{T_0}{4}\right] = -\frac{A^2}{2}$

$n=0$ $\quad a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A^2 \operatorname{sen}^2(\omega_0 t)\,dt = \frac{A^2}{T_0}\int_{-T_0/2}^{T_0/2} \operatorname{sen}^2(\omega_0 t)\,dt$

$$a_0 = \frac{A^2}{T_0}\frac{T_0}{2} = \frac{A^2}{2}$$

2. $e_r = 1 - \dfrac{\sum_{n=-N}^{N} |c_n|^2 P_n}{P_x}$

$$P_n = \frac{1}{T}\int_{-T/2}^{T/2} |\varphi_n(t)|^2\,dt = 1$$

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} (A^2 \operatorname{sen}^2(\omega_0 t))^2\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \left(\frac{1}{2} - \frac{1}{2}\cos(2\omega_0 t)\right)^2 dt$$

$$\left(\frac{1}{2} - \frac{1}{2}\cos(2\omega_0 t)\right)^2 = \frac{1}{4} - \frac{1}{2}\cos(2\omega_0 t) + \frac{1}{4}\cos^2(2\omega_0 t)$$

$$P_x = \frac{1}{T_0}\left[\frac{1}{4}T_0 - \frac{1}{2}\operatorname{sen}(2\omega_0 t)(2\omega_0) + \frac{1}{4}\frac{T_0}{2}\right]\Big|_{-T_0/2}^{T_0/2}$$

$$P_x = \frac{1}{T_0}\left[\frac{1}{4}T_0 + \frac{1}{8}T_0 - \frac{2\pi}{T_0}\underbrace{\operatorname{sen}\left(\frac{4\pi}{T_0}\frac{T_0}{2}\right)}_{\operatorname{sen}(2\pi)=0} + \frac{2\pi}{T_0}\underbrace{\operatorname{sen}\left(\frac{4\pi}{T_0}\left(-\frac{T_0}{2}\right)\right)}_{\operatorname{sen}(-2\pi)=0}\right]$$

$$P_x = \frac{1}{T_0}\left[\frac{1}{4}T_0 + \frac{1}{8}T_0\right] = \frac{1}{4} + \frac{1}{8} = \frac{3}{8} \qquad e_r = 1 - \frac{8}{3}\sum_{n=-N}^{N} |c_n|^2$$

3. $c(t) = A_c \operatorname{sen}(2\pi F_c t)$ $\quad A_c, F_c \in \mathbb{R}$; $m(t) \in \mathbb{R}$

$$y(t) = \left(1 + \frac{m(t)}{A_c}\right) c(t)$$

$$Y(\omega) = \mathcal{F}\{y(t)\} = \mathcal{F}\left\{\left(1 + \frac{m(t)}{A_c}\right) c(t)\right\} = \mathcal{F}\{c(t)\} + \frac{1}{A_c} \mathcal{F}\{m(t)c(t)\}$$

$$\mathcal{F}\{c(t) = C(\omega) = \mathcal{F}\{A_c \operatorname{sen}(2\pi F_c t)\} = A_c \mathcal{F}\left\{\frac{e^{j2\pi F_c t} - e^{-j2\pi F_c t}}{2j}\right\}$$

$$\mathcal{F}^{-1}\{X(\omega \pm \omega_0)\} = x(t) e^{\mp j\omega_0 t}$$

$$\mathcal{F}\{x(t \pm t_0)\} = X(\omega) e^{\pm j\omega_0 t}$$

$$\rightarrow \mathcal{F}\{x(t) e^{\mp j\omega_0 t}\} = X(\omega \pm \omega_0)$$

$\mathcal{F}\{1 \cdot e^{\pm j\omega_0 t}\}$ $\quad x(t) = 1$ $\quad X(\omega) = 2\pi\delta(\omega)$

$\delta(t)$ $\quad D(t) = 1$

$$C(\omega) = \frac{A_c}{2j}\left[2\pi\delta(\omega - 2\pi F_c) - 2\pi\delta(\omega + 2\pi F_c)\right]$$

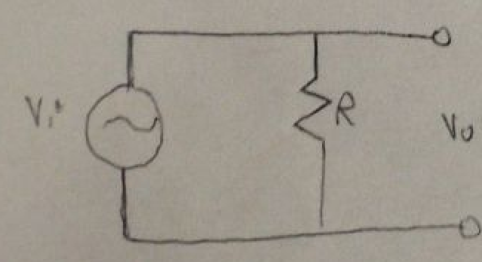
$$m(t)c(t) = m(t) A_c \operatorname{sen}(2\pi F_c t)$$

$$\mathcal{F}\{m(t)c(t)\} = \frac{A_c}{2j}\left[\mathcal{F}\{m(t)e^{j2\pi F_c t}\} - \mathcal{F}\{m(t)e^{-j2\pi F_c t}\}\right]$$

$$= \frac{A_c}{2j}\left[M(\omega - 2\pi F_c) - M(\omega + 2\pi F_c)\right]$$

$$Y(\omega) = \frac{A_c \pi}{j}\left[\delta(\omega - 2\pi F_c) - \delta(\omega + 2\pi F_c)\right] + \frac{1}{2j}\left[M(\omega - 2\pi F_c) - M(\omega + 2\pi F_c)\right]$$

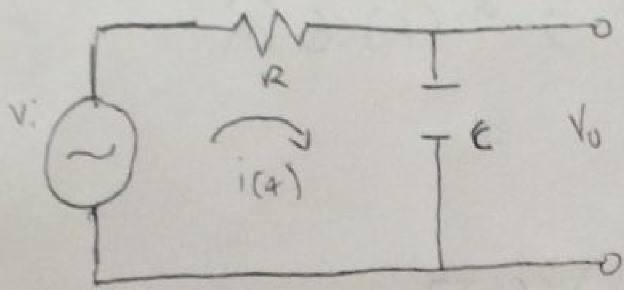
4. Carga resistiva



$$i(t) = i_R(t) = \frac{V_R(t)}{R}$$

$V_i(t) = V_o(t)$ $\quad \frac{V_o(s)}{V_i(s)} = 1$

Carga RC Serie



$$v_i(t) = V_R(t) + V_c(t) \; ; \; V_c(t) = V_o(t)$$

$$V_R(t) = R\, i_R(t) \qquad i_R(t) = i_c(t)$$

$$i(t) = C \frac{dV_c(t)}{dt}$$

$$V_i(s) = V_R(s) + V_c(s)$$

$$V_i(s) = RCs\, V_c(s) + V_c(s)$$

$$\frac{V_c(s)}{V_i(s)} = \frac{1}{RCs + 1}$$