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Extending the Kalman Filter to Accelerometers with a DC Offset

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November 18, 2013

1 FILTER FORMULATION

The problem is to modify the Kalman filtered from *Bock et al.* (2011) to a filter that can ingest accelerometer data with a constant DC offset. That is the measured or observed acceleration a^{obs} is related to the true acceleration, a^{true} by

$$a_k^{true} = a_k^{obs} - \Omega_k + \epsilon_a$$

where Ω_k is the DC offset at epoch k and ϵ_a is the accelerometer noise. If we define our system states as the displacement d, the velocity v and the DC offset Ω then following *Lewis et al.* (2008) section 2.4 we can write the continuous difference equation for this system:

$$\frac{d}{dt}\begin{bmatrix} d(t) \\ v(t) \\ \Omega(t) \end{bmatrix} = \frac{d}{dt}\mathbf{x}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)u(t) + \epsilon(t)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} ; \mathbf{B} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} ; u = a^{obs} ; \epsilon = \begin{bmatrix} 0 \\ \epsilon_a \\ \epsilon_\Omega \end{bmatrix} ,$$

where ϵ_a is the accelerometer noise and ϵ_Ω is the DC offset noise. A small value of ϵ_Ω will allow the DC offset to vary slowly through time. This means that the noise vector ϵ is Gaussian, such that $\epsilon \sim (0, \mathbf{Q})$ where the covariance $\mathbf{Q}$ depends on the accelerometer and DC offset noise variances σ_a and σ_Ω like

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_a & 0 \\ 0 & 0 & \sigma_\Omega \end{bmatrix}$$

You can expand the last two equations to verify the system:

$$\frac{d}{dt}\begin{bmatrix} d(t) \\ v(t) \\ \Omega(t) \end{bmatrix} = \begin{bmatrix} \dot{d} \\ \dot{v} \\ \dot{\Omega} \end{bmatrix} = \begin{bmatrix} v \\ -\Omega + a^{obs} + \epsilon_a \\ \epsilon_\Omega \end{bmatrix} .$$

If we are measuring noisy displacements we define the measurement process:

$$z(t) = d^{obs}(t) = \mathbf{H}(t)\mathbf{x}(t) + \eta_d = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}(t) + \eta_d$$

where η_d is the displacement observation noise, which when assumed gaussian has a distribution $\eta_\mathbf{d} \sim (0, \mathbf{R})$ with $\mathbf{R} = \sigma_d$ the displacement noise variance. The continuous system can now be discretized. Again following *Lewis et al.*(2008), we write the discrete system:

$$\mathbf{x}_{k+1} = \mathbf{A}^s\mathbf{x}_k + \mathbf{B}^s a_k^{obs} + \epsilon_k$$

where the discretized noise vector is Gaussian and distributed like $\epsilon_k \sim (0, \mathbf{Q}^s)$. If we discretize the continuous system at the sampling rate τ_a of the accelerometer we must redefine the system transition matrices. These are obtained (*Lewis et al.*, 2008) from the MacLaurin series expansion of the integral forms as:

$$\mathbf{A}^s = \mathbf{I} + \mathbf{A}\tau_a + \frac{\mathbf{A}^2\tau^2}{2} = \begin{bmatrix} 1 & \tau_a & -\tau_a^2/2 \\ 0 & 1 & -\tau_a \\ 0 & 0 & 1 \end{bmatrix} ;$$

$$\mathbf{B}^s = \mathbf{B}\tau_a + \frac{\mathbf{AB}\tau_a^2}{2} = \begin{bmatrix} \tau_a^2/2 \\ \tau_a \\ 0 \end{bmatrix} ;$$

$$\mathbf{Q}^s = \mathbf{Q}\tau_a + \frac{1}{2}(\mathbf{AQ} + \mathbf{QA}^T)\tau_a^2 + \frac{1}{3}\mathbf{AQA}^T\tau_a^3 = \begin{bmatrix} \sigma_a\tau_a^3/3 & \sigma_a\tau_a^2/2 & 0 \\ \sigma_a\tau_a^2/2 & \sigma_a\tau_a + \sigma_\Omega\tau_a^3/3 & -\sigma_\Omega\tau_a^2/2 \\ 0 & -\sigma_\Omega\tau_a^2/2 & \sigma_\Omega\tau_a \end{bmatrix} .$$

The sampled version of the continuous measurement is written as:

$$z_k = d_k^{obs} = \mathbf{H}^s\mathbf{x}_k + \eta_d ,$$

where the noise is is white and Gaussian such that $\eta_d \sim (0, R^s)$. Sampling at the rate of the GPS, τ_d the discretized matrices are simply:

$$\mathbf{H}^s = \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$R^s = \sigma_d/\tau_d .$$

With these definitions the traditional system update and measurement update stages of the Kalman filter can be carried out.