Introduction to Intelligent Systems '18/'19
Lab 3

Useful matlab functions: `load, plot, hold on, hist, bar, length, find, rand, char, sum, xor, normpdf`

Instructions

You should hand in a brief report, in which you provide answers to all the problems, with the same structure of the present document (Assignment 1, Assignment 2, etc.). The code developed to solve the problems has to be included in the report, commented, and opportunely referenced.

The plots have to be self-consistent, with clear title, labels and captions. They have to be sufficiently commented in the text.

The grading will be based on the following aspects (ordered acc. to importance):

- completeness of the report and correctness/plausibility of the results
- readability, layout, formatting of the text
- appearance of the plot, e.g. axis labels, quality of the caption
- language/English issues

A grade equal to 10 can be achieved by solving all the problems correctly and matching the above conditions. The bonus assignment 3 can be evaluated to compensate for eventual deficits of above conditions.

Assignment 1

class conditional probabilities, priors, posterior probabilities, Bayes decision rule, minimum error classification.

Let's assume we obtained the class conditional probability of salmon and sea bass, describing the distribution of the length of the two classes. These are given by `p_salmon` and `p_seabass` at length l (in cm), all in the file lab3_1.mat.

1. Given that sea bass is caught 3 times as often as salmon, calculate the posterior probabilities (don't forget to normalize to 1 for all lengths) and plot them.
2. Now a new fish is measured, which turns out to have a length of 8 cm. According to your posterior probabilities, how would you classify this fish? And what if it's length is 20 cm?

Assignment 2

normal distribution.

Consider the following two sets of observed 1D data (feature values) coming from objects of two different classes (read the data from the file lab3_1.mat):

```
S1 = [ -30.8508   23.6509   -15.4613   -0.0466   26.5835   41.1727
  -23.5292   47.8198   29.8043   7.9640   4.1434   3.4718
  0.9068   10.6914   11.5387   34.7819   55.8093   24.0074
  3.7086   28.7557   15.4968   -20.8930   38.4767   19.2119
  14.0552   25.4574   17.8422   -18.2446   5.1299   5.6184]

S2 = [44.6799   66.8210   41.2427   45.1618   42.8800   38.2579
  48.0776   47.2593   65.3007   47.5098   39.3579   66.0346
  62.3468   47.7037   34.9384]
```

and the following test set of data points that need to be classified:

```
T = [5   23   40   70   95]
```

1. Plot the elements of the two sets S1 and S2 on the x-axis of a Fig. 1: the elements of S1 as blue circles (bo) and the elements of the set S2 as red circles (ro). Plot in the same Fig. 1 the points of the test set T as black squares (ks).
2. Let's assume S1 and S2 are drawn from normal distributions. Compute the parameters (mean and standard deviation) of the two distributions, using maximum estimation. Create a Fig.2 in which you plot the two Gaussian functions, one in blue the other in red, together with the points of the two training data sets S1 and S2. The two functions are the class conditional probability densities $p(x|\omega_1)$ and $p(x|\omega_2)$ that the two classes produce a value x of the considered feature. Comment on the use of the functions, in comparison to using the rugged data.
3. Estimate the prior probabilities $P(\omega_1)$ and $P(\omega_2)$.
4. Create a Fig. 3 in which you plot the two products $P(\omega_1)p(x|\omega_1)$ and $P(\omega_2)p(x|\omega_2)$, together with the points of the two training data sets S1 and S2 and the test set T.

5. Substitute the values of the prior probabilities and the class conditional probability densities determined above in the equation $P(\omega_1)p(x|\omega_1) = P(\omega_2)p(x|\omega_2)$ and solve the resulting equation for x in order to determine the value(s) of the decision criterion that we should use for classification.
6. Using the thus obtained value(s) of the decision criterion, determine the classes of the points in the test set [5 23 40 70 95]. Create a Fig. 4 which copies Fig. 3 and adds to it a specification of which domain on the x-axis belongs to class 1 (blue) and which complementary domain corresponds to class 2 (red).
7. Evaluate the misclassification rate of the created classifier for each of the two classes. (For this, you can for instance use the error function erf, familiar from the iris recognition assignment in the first week.)

Assignment 3 (BONUS)

Let us consider two one-dimensional normal distributions with mean $m_1 = 1$ and $m_2 = -3$, with standard deviations $s_1 = s_2 = \sqrt{2}$. The two distributions have same prior probability $P(w_1) = P(w_2) = 0.5$. Compute the value of the decision criterion x*. Briefly comment on the approach that you take and report the main steps of the calculations.