Dean Melwban
COMP 417
A4 - Write Up

- In order to find proper A + B matrices for LQR control, we must compute Jacobian Matrices to linearize the system.

First, I expand $\ddot{x}$ and $\ddot{\theta}$, plugging in relevant given constants such as pole length (l), pole mass(m), etc..

$$\ddot{x} = \frac{2(0.5)(0.5)\dot{\theta}^2 \sin\theta + 3(0.5)(9.82)\sin\theta\cos\theta + 4(u-\dot{x})}{4(0.5+0.5) - 3(0.5)\cos^2\theta}$$

$$= \frac{0.5\,\dot{\theta}^2 \sin\theta + 14.73\sin\theta\cos\theta + 4u - 4\dot{x}}{4 - 1.5\cos^2\theta}$$

$$\ddot{\theta} = \frac{-3(0.5)(0.5)\dot{\theta}^2 \sin\theta\cos\theta + 2((0.5+0.5)(9.82)\sin\theta + (u-\dot{x})\cos\theta)}{0.5(4(0.5+0.5) - 3(0.5)\cos^2\theta)}$$

$$= \frac{-3(0.25\dot{\theta}^2\sin\theta\cos\theta + 19.64\sin\theta + 2\cos\theta\, u - 2\cos\theta\,\dot{x})}{2 - .75\cos^2\theta}$$

- A linear expansion is done by taking a Taylor series to the first order.
  ↳ We know goal = $[0,0,0,\pi]^T$, so we must linearize around this point.

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\theta}\\ \ddot{\theta}\end{bmatrix} = \dot{\vec{x}} \quad \Big| \quad \dot{\vec{x}} \approx \nabla_x f(\vec{x},u)[\vec{x},u]\cdot\vec{x} + \nabla_u f(\vec{x},u)[\vec{x},u]\cdot u$$

$$\equiv Ax + Bu$$

Hence,

$$A = \begin{bmatrix} \frac{d\dot{x}}{dx} & \frac{d\dot{x}}{d\dot{x}} & \frac{d\dot{x}}{d\theta} & \frac{d\dot{x}}{d\dot{\theta}} \\ \frac{d\ddot{x}}{dx} & \frac{d\ddot{x}}{d\dot{x}} & \frac{d\ddot{x}}{d\theta} & \frac{d\ddot{x}}{d\dot{\theta}} \\ \frac{d\dot{\theta}}{dx} & \frac{d\dot{\theta}}{d\dot{x}} & \frac{d\dot{\theta}}{d\theta} & \frac{d\dot{\theta}}{d\dot{\theta}} \\ \frac{d\ddot{\theta}}{dx} & \frac{d\ddot{\theta}}{d\dot{x}} & \frac{d\ddot{\theta}}{d\theta} & \frac{d\ddot{\theta}}{d\dot{\theta}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1.6 & 0 & 3.682 \\ 0 & -4.8 & 0 & 29.46 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

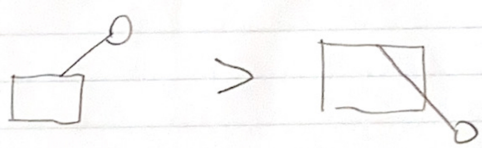
$$B = \begin{bmatrix} \frac{d\dot{x}}{du} & \frac{d\ddot{x}}{du} & \frac{d\dot{\theta}}{du} & \frac{d\ddot{\theta}}{du} \end{bmatrix} = \begin{bmatrix} 0 & 1.6 & 4.8 & 0 \end{bmatrix}$$

- Q and R were hand tuned to get optimal performance for <u>balancing</u>.

- For the Q matrix, we know there is a 1-1 correspondence based on each entry in the diagonal, with respective penalty to element of state.

Ex:
$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \text{All penalties for } \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} \text{ are equal.}$$

- However, I concluded that certain states were less desirable than others.

↳  , Just from intuition.

Hence, I penalized θ more than x. I also wanted to restrict the impact of the end effector moving the cart that much, so $\dot{x}$ was big too.

$$Q = \begin{bmatrix} 13 & 0 & 0 & \\ 0 & 22 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & & 20 \end{bmatrix}$$ is the Q/R I tuned to get good performance for balancing

$R = [60]$.

The control method was straight from LQR formula

$$u = k(x-g) \rightarrow \text{np.matmult}(-k, x-\text{goal})$$

$\square$