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COMP417 - A3

① $p(z_t|x_t) = \begin{cases} p(z==1)=0.9 \text{ \& } p(z==0)=0.1 \text{ if } x_t \text{ in front of door} \\ p(z==1)=0.1 \text{ \& } p(z==0)=0.9 \text{ if } x_t \text{ in front of wall} \end{cases}$

Motion Model: $x_t = x_{t-1} + u_{t-1} + \varepsilon$ where $\varepsilon \sim N(0, 0.2^2)$

*↳ EXAMPLE DONE WITHOUT RESAMPLING STEP! *

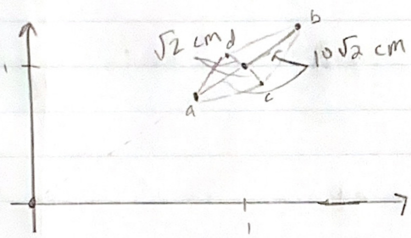
- Particle we will follow is the particle that sits on $x=1$ in the $bel(x_0)$ estimate
- Robot does $u_0=1$. Hence, new position is $x_1 = 1+1+\varepsilon$ where $\varepsilon \sim N(0, 0.2^2)$
$= 2 \pm 0.2$
- Here, to have discrete value for state, we choose ε to be .1, a possible noise value produced by epsilon. Hence, $x_1 = 2.1$ (in front of a door)
- Robot takes a measurement $z_1=0$. Hence $p(z_1|x_1) = 0.1$ b/c robot is in front of a door. Particle at $bel(x_1)$ sits at $x=2.1$ w/ weight 0.1.
- Robot does $u_1 = 1$. Hence, new position is $x_2 = 2.1+1+\varepsilon$ where $\varepsilon \sim N(0, 0.2^2)$
$x_2 = 3.1 \pm 0.2$
- Here, to have discrete value for state, we choose ε to be = 0.1, a possible noise value produced by epsilon. Hence, $x_2 = 3.2$ (in front of wall)
- Robot takes a measurement $z_2=0$. Hence, $p(z_2|x_2) = 0.9$ b/c robot is in front of a wall. $\boxed{\text{Particle at } bel(x_2) \text{ sits at } 3.2 \text{ w/ weight } 0.9.}$
- Possible factors of the result is due to possible noise from the sensor, values chosen from $\varepsilon \sim N(0, 0.2^2)$ and faulty sensor measurements.

2) Likelihood for a state given:

$$p(x) \propto e^{-(x-\mu)^T \Sigma^{-1} (x-\mu)}$$

Cross-track error: 1 cm/m
Dist. Control error: 10 cm/m

a) Straight line



- The robot goes from (0,0) to (1,1) in a straight line trajectory.
  - ↳ Travels $\sqrt{2}$ m
- The distribution is shown by the ellipse drawn

- The robot can fall $10\sqrt{2}$ cm before or after the target (designated by points a and b) due to distance controll error.

- The robot can fall $\sqrt{2}$ cm to the left or right of the target (designated by points c + d).

- The distribution is the same as one that has $\Sigma = \begin{bmatrix} .141 & 0 \\ 0 & .0141 \end{bmatrix}$ and rotated to the right 45°.

Hence, we use the rotation matrix $R = \begin{bmatrix} \cos(45°) & -\sin(45°) \\ \sin(45°) & \cos(45°) \end{bmatrix}$ yielding

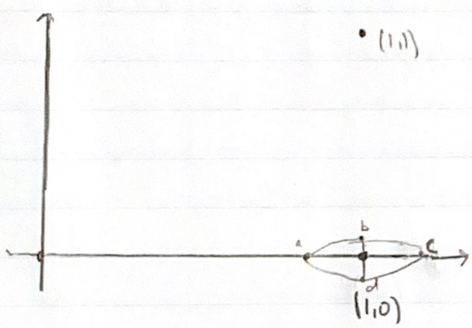
$$\Sigma_{final} = R \Sigma R^T = \begin{bmatrix} .077 & .063 \\ .063 & .077 \end{bmatrix}$$

$$\mu = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

→

b) L- Shape

This example will be demonstrated using an intermediate step for the distribution at (1,0)

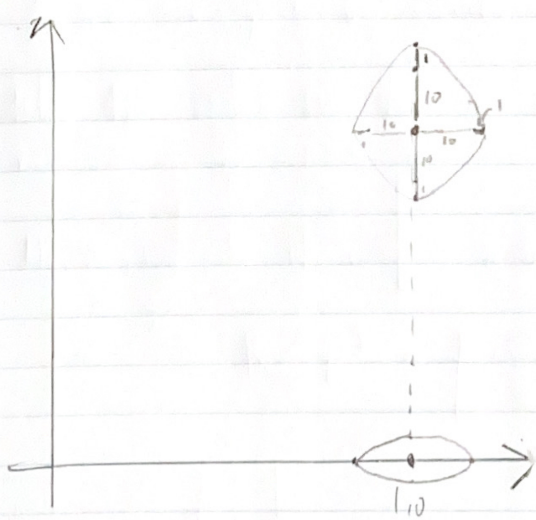


- The robot travels from point (0,0) to (1,0).

- Due to this control, a distribution is formed around (1,0).

- Along the x-axis, the distance control error is the cause of the error. These occur 10 cm before & after (1,0). (Points a,c

- For the y-axis, the error comes from the cross-track error. Points b and d sit 1 cm above and below (1,0). The ellipse now is a distribution of the robot's state after the first control.



- After the second control ((1,0) → (1,1)), we must consider the edge cases of the distribution.

- Because we are going vertically after going horizontally, we append the values for cross-track to initial distance-control error & vice-versa. The result is a circle distribution. Hence,

$$\mu = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \Sigma = \begin{bmatrix} .11 & 0 \\ 0 & .11 \end{bmatrix}$$

3) We know:

$$bel(x_t) = \eta\, p(z_t|x_t) \int p(x_t|u_{t-1}, x_{t-1})\, bel(x_{t-1})\, dx_{t-1}$$

① $bel(x_2) = \eta_x\, p(z_2|x_2) \int p(x_2|u_1, x_1) \underbrace{bel(x_1)}_{\text{expand this term in ②}} dx_1$

② $bel(x_1) = \eta_y\, p(z_1|x_1) \int p(x_1|u_0, x_0)\, bel(x_0)\, dx_0$

Substitute ② into ①:

$$\boxed{bel(x_2) = \eta\, p(z_2|x_2) \int p(x_2|u_1, x_1) \left(\eta\, p(z_1|x_1) \int p(x_1|u_0, x_0)\, bel(x_0)\, dx_0\right) dx_1}$$