Q1

Steps:

① select particle $x_0^{[3]} = 0.9$.

② sample $x_1^{[3]} = 1.95$ (drawn from normal $N(1.9, 0.2^2)$).

③ compute weight $w^{[3]} = 0.1$ because $p(z_1 = 0 \mid x_t = 1.9) = 0.1$ (in front of door).

④ sample $x_2^{[3]} = 2.92$ (drawn from normal $N(2.95, 0.2^2)$).

⑤ compute weight $w^{[3]} = 0.9$ because $p(z_2 = 0 \mid x_t = 2.95) = 0.9$ (in front of wall).

Final answer $x = 2.95$
$w = 0.9$.

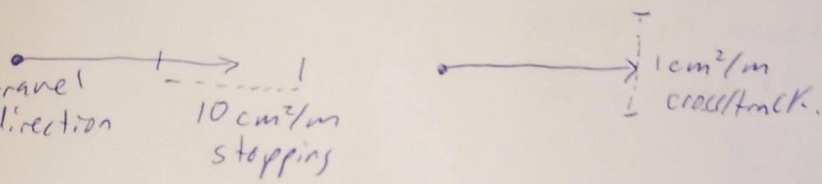
Options:

- $w^{[3]}$ can be multiplied (better, but doesn't follow pseudo-code): $w^{[3]} = 0.09$.

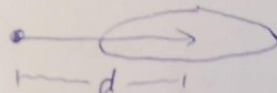
- can perform re-sampling twice. Just state which new x is drawn.
- Can normalize weights $w_{norm}^{[i]} = \frac{w^{[i]}}{\sum_i w^{[i]}}$.

Q2:

Interprettation of motion model

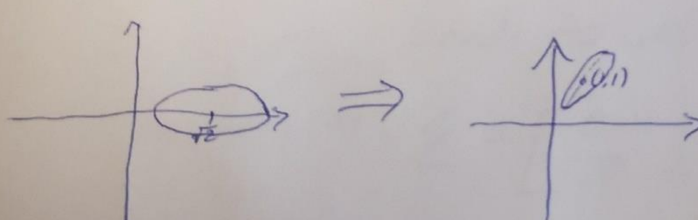


So Gaussian with mean at intended goal,



Variance along $= 0.1 \cdot d$
Variance across $= 0.01 \cdot d$.

a) We travel $d = \sqrt{2}$ along line $y = x$, same statistics as moving along x-axis, but for rotated vectors.



$\mu = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ since we have no "bias" in our target.

Σ-terms:

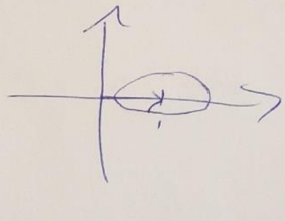
Var[X] vs var[Y] → equal since the two error sources act symmetrically.

cov[X,Y] +/-? → positive since the stopping error gives more change in XY together.

One reasonable set of values is

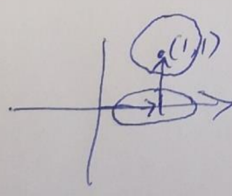
$$\Sigma = \begin{bmatrix} 0.08 & 0.065 \\ 0.065 & 0.08 \end{bmatrix}.$$ We are REALLY not going to be picky in marking. It's about the explanation.

b) First move along x-axis:



mean now $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ Σ is $\begin{bmatrix} 0.1 & 0 \\ 0 & 0.01 \end{bmatrix}$.

A sensible lower bound is to assume an un-related, 2nd step, adding the variances:



Leads to $\mu = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 0.11 & 0 \\ 0 & 0.11 \end{bmatrix}$. Some folks have had nice ideas about how the errors are coupled, how turning matters, etc. As long as those are well explained, that's great!

Q3:

Bayes Filter gives us one time-step recursion. start from $p(x_0)$, get $p(x_1|\ldots)$

$$p(x_1|u_0,z_1) = \eta\, p(z_1|x_1) \int p(x_1|x_0,u_0)\cdot p(x_0)\, dx_0.$$

Repeat the same to get from $p(x_1|\ldots) \rightarrow p(x_2|\ldots)$ but plus in the previous.

$$bel(x_2) = p(x_2|u_{0:1}, z_{1:2}) = \eta \cdot p(z_2|x_2) \cdot \int p(x_2|x_1,u_1) \cdot \left[p(z_1|x_1) \int p(x_1|x_0,u_0) \cdot p(x_0)\, dx_0\right] dx_1.$$