2020 - 2021 Project Continuous Data Analysis/Statistical Modelling

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Introduction

United States Presidential election is widely concerned throughout the world. The republican candidate Trump Donald defeated the democratic candidate Hillary Clinton and won the election in 2016, which is far away from the result of the public opinion poll. In this study, we would like to investigate the factors, like socio-economic and demographic characteristics, which might impact the vote rate to Trump in 2016 election.

The dataset we used in this study is from the Kaggle website. In this dataset, we have 3141 observation indicating the election result and status of different areas of America. We implemented the regression approach which constituted a linear regression and a categorical regression of selected variables on the percentage of votes for the Republican party (GOP), to gain some insight in the association of certain variables with the vote rate of Trump.

Protocol

Research Question

For this project we will analyse part of the "2016 US Presidential Election Dataset" to examine the effect of the per capital money income obtained in the past 12 months on the amount of the percentage of GOP votes during the 2016 presidential elections, while accounting for other potential covariables, using a multiple linear regression model.

Did the economic characteristic (per capital money income obtained in the past 12 months) influence the percentage of GOP votes during the 2016 presidential elections?

What is the impact of socio-economic and demographic characteristics including race, gender, education, household, veterans and number of firms on the percentage of GOP votes during the 2016 presidential elections?

Moreover, we would like to find a way to predict the undecided counties (vote rate: 48%-52%) where our candidate may put more effort to on next election.

Study Method

A new dataset will be created based on the variables we choose to study. And then the distribution of each individual variable will be examined by univariate procedure, if it is necessary the missing value will be removed and the skewed data will be transformed. Bivariate relationships between the continuous variables will be examined with correlation and scatterplot matrix. Then we will split the dataset to training dataset and testing dataset (50:50) and normalize them respectively. The interactions between two variables will be created as new columns.

After the data preprocessing, the forward stepwise selection will be applied on the training dataset to find our "final model". After all the available variables access to the model, we will check whether the "final model" can fulfill all the assumptions.

In order to build logistic regression model, we will categorize the vote rate into 2 level: win (>50%) and lose(<50%). Then Compare your results/conclusions with those of the linear model to adapt the final model.

Fit the final linear model on the "testing" dataset to evaluate the model performance.

Data Exploration

We choose the income (per capital money income obtained in the past 12 months) as our key variable and other variables are white race, household, gender, education, firms and veterans. The distribution of all individual variables is examined by the univariate procedure. Since the firms and veterans are heavily right skewed (skewness: 16.15, 8.27 respectively), the log-transform is applied to those two variables. Missing value is checked by the mean procedure and there is no missing value in our dataset. Bivariate relationships between the continuous variables will be examined with correlation and scatterplot matrix.

We split the whole dataset data into training (1571 observations) and test (1570 observations) datasets with a proportion of 50:50. Both datasets were then standardized independently to have uniformity in interpreting our coefficients.

Simple linear regression

Simple linear regression (SLR) was used to assess the performance of key predictor (income) affecting the percentage of votes obtained by GOP candidate, the formula is as follows:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$
 $\varepsilon_i \sim N(0, \delta^2), i = 1, ..., n$

 $y_i = vote \, rate \, (Percentage \, of \, votes \, for \, Republican \, Party)$

 x_i = income (Per capita money income in past 12 months (dollars), 2009-2013)

The results showed that income affect the votes obtained by GOP candidate marginally

significant (p-value <0.0001), our model then can be written as

$$y_i = 0.767 - 0.0000055x_i + \varepsilon_i$$
 $\varepsilon_i \sim N(0, \delta^2)$, $i = 1, ..., n$

which means income increase one unit will result in the decrease of GOP support of 0.0000055%.

Multiple linear regression

We used forward stepwise regression method to build our model, absolute t-value, F-value and p-value were considered as the access criterions when we judge whether a new variable could be included or removed. Accordingly, newly added variable is of lowest residual sum of squares (SSR) and of highest R square, absolute t-values, F-values and p-value at 0.05 significance level in each step. Multicollinearity and confounding effect were checked by VIFs and p-value after each selection. With the addition of a new relevant variable included into our model, the already fitted variables did not change much in the significance and magnitude of the coefficients, hence no confounding effects. Variables which did not fulfill those criteria were omitted from the model.

According to the criterion we set, our model was built with main effects of income, white race, veterans, and firms. A similar approach was implemented to include influential two-way interactions terms into the model. The following interactions were included: household*education, white*education, income*firms, white*household, income*household, income*gender, education*veterans, white*firms and firms*veterans.

Checking assumptions

Multicollinearity: There is no multicollinearity amongst the predictors according to the VIF method, where we set VIF < 10. Normality: We used the QQ-plot of (studentized) residuals to check for the normality. There were slight deviations at the tails observed. However, normality can be assumed. Linearity: It was verified through the plot of (studentized) residuals versus predicted values. Points look randomly scattered around 0. No evidence of nonlinear pattern, hence linearity could be assumed. Furthermore, equality of variance (Homoscedasticity) was checked by examining the plot of (squared) residuals versus predicted values. Plot indicates that points equally randomly scattered around 0 point and therefore no evidence of heteroscedasticity.

Checking for outliers

Using the Cook's Distance threshold (4/n = 0.0025, n denotes the number of observations) to remove the outliers till convergence. The t-value, F-value and VIF value are the basis of our judgement. There were 93 outliers detected and removed in first time. Then the model was refitted by the new dataset without outliers. When checking the estimate coeffecients' significance, the p-value of white*household and firms*veterans are 0.4253 and 0.5833. These two variables were then removed from

our model. Checking the outlier again and we found only 2 outliers which are slight beyond the threshold. The assumptions for linear regression were assessed once more and arrived at the conclusion that, all assumptions seem to be fulfilled (Appendix II).

Interpretation

A comparation across all statistics presented in Table 1 indicated that race (white) was the strongest direct predictor of gop across mutiple indices, followed by income, Intwhiteeducation, and Veterans, Intincomehousehold, Inteducationveterans, Intincomefirms, firms, Inthouseholdeducation, Intincomegender, Intwhitefirms. Race obtained the largest T-value (t-value = 18.73, p < .0001), demonstrating that it made the largest contribution to the regression equation, while

Table 1 Statistical results of final model

Parameter Estimates								
Variable	DF	Parameter Standard Error		t Value	Pr > t	Variance Inflation		
Intercept	1	0.66781	0.00292	228.53	<.0001	0		
(x ₁) Income	1	-0.00000585	5.656001E-7	-10.35	<.0001	1.43514		
(x ₂) White	1	0.00365	0.00019506	18.73	<.0001	1.48792		
(x ₃) Veterans	1	-0.02913	0.00300	-9.71	<.0001	3.06874		
(x ₄) Firms	1	-0.00793	0.00191	-4.15	<.0001	2.99856		
(x ₅) Household*Education	1	0.00798	0.00202	3.94	<.0001	1.65466		
(x ₆) White*Education	1	-0.00026287	0.00002623	-10.02	<.0001	1.43653		
(x ₇) Income*Firms	1	-0.00000121	2.679091E-7	-4.51	<.0001	1.94733		
(x ₈) Income*Household	1	0.00001319	0.00000265	4.98	<.0001	1.74923		
(x ₉) Income*Gender	1	6.895835E-7	2.030597E-7	3.40	0.0007	1.26568		
(x ₁₀) Education*Veterans	1	-0.00152	0.00034197	-4.46	<.0001	1.58034		
(x ₁₁) White*Firms	1	0.00028353	0.00009679	2.93	0.0034	1.39653		

holding all other predictor variables constant. The squared structure coefficient (R^2 = .2755) demonstrated that race (white) explained the largest amount (27.55%) of variance in y, the predicted values of gop.

A good and standard statistical package has been used in this model, the R-squared value of the final multiple linear model is .5716 with all p < 0.01, which is quite an excellent, highly statistically significant result. About 58% (R^2 %) of variations in y can be attributed to variations in x. Therefore y is reliably predictably with the multiple linear regression model:

$$Y_i = 0.67 - 5.85 * 10^{-6}x_1 + 3.65 * 10^{-3}x_2 - 0.03x_3$$

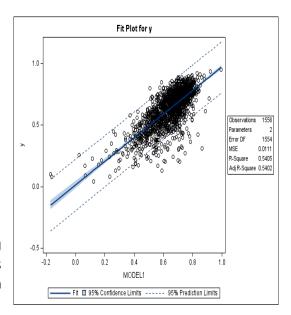
$$- 0.01x_4 + 0.01x_5 - 2.63 * 10^{-4}x_6$$

$$- 1.21 * 10^{-6}x_7 + 1.32 * 10^{-5}x_8 + 6.9$$

$$* 10^{-7}x_9 - 1.52 * 10^{-3}x_{10} + 2.84$$

$$* 10^{-4}x_{11}$$

By applying the final model we built on the testing dataset, the results (R^2 =.5405, MSE = .0111) still shows its robust and reliance. The corresponding plot is shown as below.



The overall findings supported how both race (white) was the most significant direct contributor and income was the second most important direct contributor to predicting variance in gop, as reflected across different T-values, F-values and VIF. This might be because the white people are currently the major communities of voters, and income has a negative contribution on y, the predicted values of gop, which is corresponding to the reasearch that indicates, in poor counties, income is associated

with Republican voting, while in many rich states, the relation between income and vote choice is nearly zero¹⁻⁵.

General Linear Model (GLM)

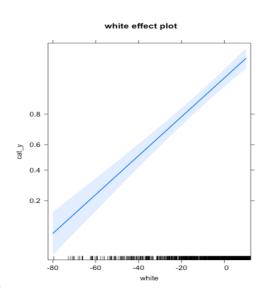
We dichotomized the response variable into two categories: win (vote rate greater than 50%) and lose (vote rate less than 50%), representing by 1 and 0 respectively. The GLM is based on the final model we have found in continuous part, fitting by the dichotomized training dataset. The formula of logistic regression can be written as following:

$$P(Y=1) = rac{1}{1 + e^{-(eta_0 + eta_1 x_1 + eta_2 x_2 + ... + eta_p x_p)}}$$

For easier understanding, it also can be transformed as following equation:

$$log(rac{P(Y=1)}{1-P(Y=1)}) = log(rac{P(Y=1)}{P(y=0)}) = eta_0 + beta_1x_1 + \ldots + eta_px_p$$

Obviously, the left part of the equation is the log odds. We choose white race as an example to interpret our GLM. The effect of white race has linear relationship with the cat_y, displayed as right figure, in which the slope is the estimate coefficient of white race (0.04949103). The conditional odds of the white race would be exp(0.04949103) = 1.050736, indicating that when other variables keep constants the odds would increase 5.0736% if the white race increase one unit. What should be noticed here is that the firms and veterans had been log transformed in previous data process, so when discussing about those two variables, the conditional odds ratios would be the estimate coefficients directly. Using the testing dataset for



prediction, the accuracy of our model is 87.66% and the precision of win and lose are 88.8% and 79% respectively.

Based on the estimate coeffecients of our fitted model, the interaction of househould and education (estimate coeffecient 0.219) shows highest positive effect on the vote rate to our candidate in 2016, while the log-transformed firms (estimate coefficient - 0.8772) shows the highest negative effect on it. Moreover, the income and log-transformed veterans do not show significant effects to the vote rate in our model.

Discussion

Our study performed on county level data on the US elections 2016. In our study, we fitted the final model with training dataset and then applied it to test dataset for prediction. Overlap, t-test and variance test were used to test the similarity and difference. Overlap ranges from 0 (no overlap) to 1 (complete overlap). Since our sample size is greater than 75, we use the Dhat4 = 0.87 of overlap coefficient which implies, the boundaries of both response variables from actual and predicted dataset greatly coincide with one another. Significant difference was found when conducting t- and var- test, which means the prediction is marginally different from observations. Nevertheness, we may improve our model by including more relevant variables in the future.

In order to get the "undecided" counties which we need to focus for the next election, the linear regression model was applied to train the whole dataset with the variables already introduced. We then performed function of 'predict' to get perdicted values corresponding to each observation. Afterwards, we categoried the predicted values and defined interval located between 48%-52% as "undecided" situations. According to those criterion, we got 145 counties among 1738 counties, where the candiate may put more effort in the next election (see Appendix III).

Reference

- 1. Why do we need to re-use training parameters to transform test data? https://sebastianraschka.com/fag/docs/scale-training-test.html
- 2. Jim Frost (2019), Using Confidence Intervals to Compare Means. https://statisticsbyjim.com/hypothesis-testing/confidence-intervals-compare-means/
- 3. Dr. Frank Wood (2010), Inference in Regression Analysis. http://www.stat.columbia.edu/~fwood/Teaching/w4315/Spring2010/lecture_4.pdf
- 4. Nathans, Laura L., Oswald, Frederick L. and Nimon, Kim. "Interpreting Multiple Linear Regression: A Guidebook of Variable Importance." *Practical Assessment, Research & Evaluation*, 17, no. 9 (2012) Practical Assessment, Research & Evaluation: https://hdl.handle.net/1911/71096
- 5. Andrew Gelman, Lane Kenworthy, Yu-Sung Su(2010). Income Inequality and Partisan Voting in the United States. 91(5):1204-1219.

Schedule

The distribution of tasks of the whole program is as follow:

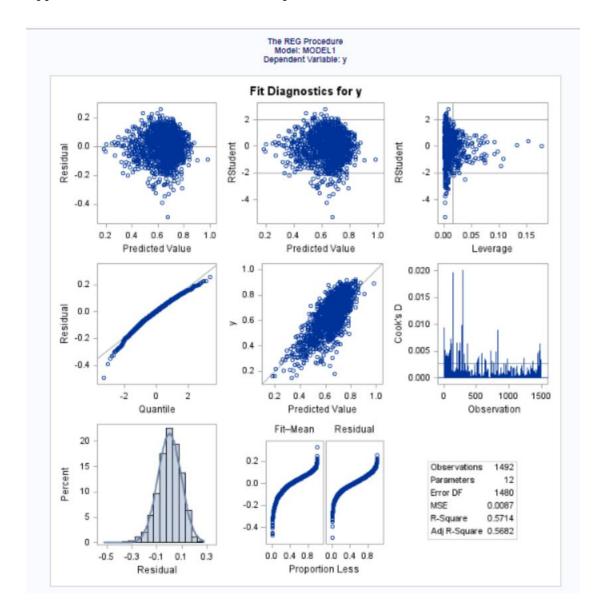
- 2020.11.28 Get an overall understanding of the research topic and work for the protocol together.
- 2020.11.28 2020.12.6 Feihong Du, Lin Tang and Shengmin Zhang work for the code part.
- 2020.12.7 Discuss and solve problems for the code part and work together to form the final version.
- 2020.12.7 2020.12.10 Dereje Mengist Belete, Echo Capwell Forbang and Serge Martin Nkoumnga work for the manuscript.
- 2020.12.10 Feihong Du, Lin Tang and Shengmin Zhang work together to revise the manuscript.
- 2020.12.11 All members work together to form the final report.

APPENDIX

Appendix I: Variables selected and their descriptions

Variable		Codes	Description
Outcome	GOP	Per vote gop	"Percentage of votes for Republican party"
Covariates (X)	Income	INC910213	"Per capita money income in past 12 months (2013 dollars), 2009-2013"
	Household	HSD310213	"Household, 2009-2013"
	Gender	SEX255214	"Female persons, percent, 2014"
	Education	EDU635213	"High school graduate or higher, percent of persons age 25+, 2009-2013"
Firms		SBO001207	"Total number of firms, 2007"
	Veterans	VET605213	"Veterans, 2009-2013"
	White	RHI125214	"White alone, percent, 2014"

Appendix II: Plots of linear model assumptions



Appendix III: Counties need to be treated seriously for the next election

Adams	Brazor	Clevel	Greenv	Indian	Linn C	Nantuc	Richmo	St. He	Warren
Aiken	Calcas	Cumber	Grenad	Island	Lownde	Nash C	Roosev	St. Ja	Washin
Alachu	Calhou	Danvil	Hamilt	Jasper	Lucas	Nevada	Russel	St. Ma	Washoe
Alaska	Cape M	Darlin	Hampsh	Jeffer	Macomb	Ocean	San Ju	St. Ta	Wayne
Albema	Cass C	Delawa	Hardee	Kern C	Madera	Okaloo	Sangam	Talbot	Westmo
Anoka	Champa	Early	Harris	Kings	Madiso	Olmste	Santa	Talbot	Will C
Anson	Chicka	El Dor	Haywoo	Knox C	Manate	Orange	Scotla	Taliaf	Willia
Atkins	Chicot	Escamb	Hendry	La Sal	Mareng	Ouachi	Sedgwi	Terreb	Wilson
Atlant	Chitte	Fairfa	Hidalg	Lake C	Martin	Passai	Semino	Thurst	York C
Attala	Cibola	Fayett	Hoke C	Lancas	Meriwe	Peoria	Shelby	Tollan	
Bell C	Clacka	Frankl	Holmes	Larime	Minera	Pima C	Sonoma	Troup	
Berkel	Claibo	Galves	Hopewe	Lauder	Monroe	Pitt C	Spaldi	Tulare	
Bernal	Claren	Genese	Housto	Lee Co	Monter	Platte	Spokan	Tuscal	
Bexar	Clarke	Glouce	Iberia	Lenoir	Montgo	Provid	St. Ch	Ventur	
Bossie	Clear	Greene	Imperi	Lexing	Moreho	Rapide	St. Fr	Waltha	

```
/*Statistic Molding Project 2020*/
 2
 3
 4
   %let dir='/folders/myfolders/StatmodProject2020/';
 5
   libname moldproj &dir;
 6
 7
   proc import out=moldproj.county
 8
           datafile="/folders/myfolders/StatmodProject2020/county_facts_dictionary.csv" dbms=csv replace;
 9
       getnames=yes;
10
       datarow=2;
   run;
11
12
   proc import out=moldproj.election
13
           datafile="/folders/myfolders/StatmodProject2020/US_County_Level_Presidential_Results_12-16_.csv"
14
           dbms=csv replace;
15
       getnames=yes;
16
       datarow=2;
17
   run;
18
   proc print data=moldproj.election (obs=5);
19
20
21
   /*Choose relevant variables and create a new dataset*/
22
23
24
25
   %let x = white income household gender education firms veterans;
26
   data election (keep=county y &x);
27
       set moldproj.election;
28
       rename county name=county per gop 2016=y RHI125214=white INC910213=income
29
           HSD310213=household SEX255214=gender EDU635213=education SB0001207=firms
30
           VET605213=veterans;
31
   run;
32
33
   proc print data=election (obs=10);
34
   run;
35
   36
   *### STEP 1: Descriptives Analysis ###;
37
   *###############;
38
39
    /*1. Univariate exploration of the data*/
40
   proc univariate plot data=election;
41
       var y &x;
42
43
   /* Log-transform the right skewed data: */
44
    * firms (skewness: 16.1494055) */
45
    /* veterans (skewness: 8.26233385) */
46
   data election;
47
       set election;
48
49
       if firms ne 0 then
50
           firms=log(firms);
       veterans=log(veterans);
51
   run:
52
53
   proc means data=election nmiss;
54
       var y &x;
55
   run;
56
57
   proc univariate plot data=election;
58
      var &x;
   run:
59
60
   /*2. Bivariate relationships*/
61
   title "Correlation matrix and scatter-plots";
62
   proc sgscatter data=election;
64
       matrix &x / diagonal=(histogram normal);
65
66
   title;
67
68
   data election;
69
       set election;
70
       deel=firms/veterans;
71
   run;
72
```

```
73
    proc sgscatter data=election;
 74
        matrix &x deel/ diagonal=(histogram normal);
 75
 76
    title;
 77
 78
    proc corr data=election nosimple;
 79
        var &x;
 80
 81
     *###########;
 82
     *### STEP 2: Data spliting 50%:50% ###;
 83
     *############;
 84
 85
    proc surveyselect data=election out=train_test_split method=srs samprate=0.5
 86
             outall seed=123 noprint;
 87
         samplingunit y;
 88
    run;
 89
    data train_set;
 90
         set train_test_split;
 91
        where Selected=1;
 92
    run;
 93
 94
    data test_set;
 95
         set train_test_split;
 96
        where Selected=0;
 97
    run:
 98
     /* Standardize data! */
    proc stdize data=train_set method=mean out=cent_train;
100
101
    run:
102
103
    proc stdize data=test set method=mean out=cent test;
104
105
106
    proc print data=cent_test;
107
    run;
108
109
     data cent_train;
110
         set cent_train;
111
         intwhiteincome=white*income;
112
         intwhitehousehold=white*household;
113
         intwhitegender=white*gender;
114
         intwhiteeducation=white*education;
         intwhitefirms=white*firms;
115
         intwhiteveterans=white*veterans;
116
         intincomehousehold=income*household;
117
         intincomegender=income*gender;
118
         intincomeeducation=income*education;
119
         intincomefirms=income*firms;
120
         intincomeveterans=income*veterans;
121
         inthouseholdgender=household*gender;
         inthouseholdeducation=household*education;
122
         inthouseholdfirms=household*firms;
123
         inthouseholdveterans=household*veterans;
124
         intgendereducation=gender*education;
125
         intgenderfirms=gender*firms;
126
         intgenderveterans=gender*veterans;
127
         inteducationfirms=education*firms;
128
         inteducationveterans=education*veterans;
129
         intfirmsveterans=firms*veterans;
130
    run:
131
    data cent_test;
132
         set cent test;
133
         intwhiteincome=white*income;
134
         intwhitehousehold=white*household;
135
         intwhitegender=white*gender;
136
         intwhiteeducation=white*education;
137
         intwhitefirms=white*firms;
138
         intwhiteveterans=white*veterans;
139
         intincomehousehold=income*household;
         intincomegender=income*gender;
140
         intincomeeducation=income*education;
141
         intincomefirms=income*firms;
142
         intincomeveterans=income*veterans;
143
         inthouseholdgender=household*gender;
144
         inthouseholdeducation=household*education;
         inthouseholdfirms=household*firms;
145
```

```
146
        inthouseholdveterans=household*veterans;
        intgendereducation=gender*education;
147
         intgenderfirms=gender*firms;
148
         intgenderveterans=gender*veterans;
149
        inteducationfirms=education*firms;
150
         inteducationveterans=education*veterans;
151
        intfirmsveterans=firms*veterans;
152
    run;
153
154
    %let int_x = intwhiteincome intwhitehousehold intwhitegender intwhiteeducation intwhitefirms intwhiteveterans
155
            intincomehousehold intincomegender intincomeeducation intincomefirms intincomeveterans
156
157
            inthouseholdgender inthouseholdeducation inthouseholdfirms inthouseholdveterans
158
159
            intgendereducation intgenderfirms intgenderveterans
160
161
            inteducationfirms inteducationveterans
162
           intfirmsveterans;
163
     *###########;
164
     *### STEP 3: Forward selection
                                       ###:
165
     *############;
166
167
    /* 3.1 Simple Linear regression */
168
     /* t:0.0376 F:61.83 */
169
    proc reg data=train_set;
170
        model y=income / vif;
        title "Simple linear regression";
171
        run;
172
    quit;
173
174
    title;
175
176
    /* First extra predictor*/
177
     /* income need to stay in the model */
178
    /* 3.2 determine the second fixed variable via t-value and p-value */
    /* t:28.40 F:449.92 */
179
    proc reg data=cent_train;
180
        model y=income white;
181
        run;
182
    quit;
183
184
     /* t:-9.36 F:76.41 */
185
    proc reg data=cent_train;
186
        model y=income household;
187
    auit:
188
189
     /* t:-5.10 F:44.42 */
190
    proc reg data=cent_train;
191
        model y=income gender;
192
        run;
193
    quit;
194
     /* t:3.22 F:36.28 */
195
    proc reg data=cent_train;
196
        model y=income education;
197
        run;
198
    quit;
199
200
     /* t:-15.92 F:162.64 */
201
    proc reg data=cent_train;
202
        model y=income firms;
203
        run;
    quit;
204
205
     /* t:-18.27 F:204.34 */
206
    proc reg data=cent_train;
207
        model y=income veterans;
208
        run;
209
    quit;
210
211
    /st we choose white as a candidate varible, now check for it st/
212
     /* r_square:0.3626 vif < 10*/
    proc reg data=cent_train;
213
        model y=income white / vif;
214
        run;
215
    quit;
216
217
     /* 3.3 Add white to model, determine the third fixed variable via t-value and p-value */
    /*t:-2.28 F:302.47 */
218
```

```
219
    proc reg data=cent_train;
220
         model y=income white household;
221
    quit:
222
223
     /*t:-5.05 F:315.56 */
224
    proc reg data=cent_train;
225
         model y=income white gender;
226
         run:
227
    quit;
228
     /*t:-1.25 F:300.57 */
229
    proc reg data=cent_train;
230
        model y=income white education;
231
         run;
232
     quit;
233
234
     /*t:-14.67 F:412.37 */
    proc reg data=cent train;
         model y=income white firms;
236
237
    quit;
238
239
     /*t:-17.04 F:451.66 */
240
    proc reg data=cent train;
241
         model y=income white veterans;
242
         run;
243
    quit;
244
    /st we choose veterans as a candidate varible, now check for it st/
     /* r_square:0.4605 vif<10*/
246
    proc reg data=cent_train;
247
         model y=income white veterans / vif;
248
249
    quit;
250
251
    /* 3.3 Add veterans to model, determine the fourth fixed variable via t-value and p-value */
252
    /*t:0.09 F:338.54 p:0.9249*/
    proc reg data=cent_train;
253
        model y=income white veterans household;
254
         run;
255
    quit;
256
257
     /*t:-1.15 F:339.15 p:0.2485*/
258
    proc reg data=cent_train;
259
         model y=income white veterans gender;
260
    quit;
261
262
    /*t:0.52 F:338.66 p:0.6029*/
263
    proc reg data=cent_train;
264
         model y=income white veterans education;
265
         run;
266
    quit;
267
268
    /*t:-3.33 F:343.69 */
    proc reg data=cent_train;
269
         model y=income white veterans firms;
270
         run;
271
    quit;
272
273
     /st we choose firms as a candidate varible, now check for it st/
274
     /* r square:0.4639 vif<10*/
275
    proc reg data=cent_train;
276
         model y=income white veterans firms/vif;
         run:
277
    quit;
278
279
    /* 3.5 Add firms, determine the fifth fixed variable via t-value f-value and p-value (reject */
280
     /*r:0.22 F:274.79 p:0.8236*/
281
    proc reg data=cent_train;
282
         model y=income white veterans firms household;
283
         run;
284
    quit;
285
    /*r:-0.94 F:275.11 p:0.3458*/
286
    proc reg data=cent_train;
287
         model y=income white veterans firms gender;
288
         run;
289
    quit;
290
     /*r:-0.78 F:275.01 p:0.4330*/
291
```

```
292
    proc reg data=cent_train;
293
        model y=income white veterans firms education;
294
    quit:
295
296
    /* 3.6 fix model with income white veterans and firms, r_square:0.4639 vif < 10 */
297
     /* Add one interaction variable */
    proc reg data=cent_train;
299
        model y=income white veterans firms intwhiteincome;
300
        run;
    quit;
301
302
    proc reg data=cent_train;
303
        model y=income white veterans firms intwhitehousehold;
304
        run;
305
    quit;
306
307
    proc reg data=cent_train;
308
        model y=income white veterans firms intwhitegender;
309
    quit;
310
311
    proc reg data=cent_train;
312
        model y=income white veterans firms intwhiteeducation;
313
        run:
314
    quit;
315
316
    proc reg data=cent_train;
        model y=income white veterans firms intwhitefirms;
317
        run;
318
    quit;
319
320
    proc reg data=cent_train;
321
        model y=income white veterans firms intwhiteveterans;
322
        run;
323
    quit;
324
325
    proc reg data=cent_train;
        model y=income white veterans firms intincomehousehold;
326
        run;
327
    quit;
328
329
    proc reg data=cent_train;
330
        model y=income white veterans firms intincomegender;
331
        run;
332
    quit;
333
    proc reg data=cent_train;
334
        model y=income white veterans firms intincomeeducation;
335
        run;
336
    quit;
337
338
    proc reg data=cent_train;
339
        model y=income white veterans firms intincomefirms;
340
        run;
341
    quit;
342
    proc reg data=cent_train;
343
        model y=income white veterans firms intincomeveterans;
344
        run;
345
    quit;
346
347
    proc reg data=cent train;
348
        model y=income white veterans firms inthouseholdgender;
349
        run;
    quit;
350
351
    proc reg data=cent_train;
352
        model y=white income veterans firms inthouseholdeducation;
353
         run;
354
    quit;
355
356
    proc reg data=cent train;
357
        model y=income white veterans firms inthouseholdfirms;
358
        run;
    quit;
359
360
    proc reg data=cent_train;
361
        model y=income white veterans firms inthouseholdveterans;
362
363
    quit:
364
```

```
365
    proc reg data=cent train;
366
         model y=income white veterans firms intgendereducation;
367
    quit;
368
369
    proc reg data=cent_train;
370
         model y=income white veterans firms intgenderfirms;
371
372
    quit;
373
    proc reg data=cent_train;
374
         model y=income white veterans firms intgenderveterans;
375
376
    quit;
377
378
     proc reg data=cent_train;
379
         model y=income white veterans firms inteducationfirms;
380
381
    quit;
382
    proc reg data=cent_train;
383
         model y=income white veterans firms inteducationveterans;
384
         run;
385
    quit;
386
387
    proc reg data=cent_train;
388
         model y=income white veterans firms intfirmsveterans;
389
         run;
    quit;
390
391
     /* we choose inthouseholdeducation as a candidate interaction varible, now check for it */
392
      * r square:0.5173 vif<10 */
393
     proc reg data=cent_train;
394
         model y=income white veterans firms inthouseholdeducation/ vif;
395
         run:
396
397
         /* 3.7 add inthouseholdeducation, determine the second interaction variable*/
398
    proc reg data=cent_train;
         model y=income white veterans firms inthouseholdeducation intwhiteincome;
399
         run;
400
    quit;
401
402
     proc reg data=cent_train;
403
         model y=income white veterans firms inthouseholdeducation intwhitehousehold;
404
         run;
405
    quit;
406
    proc reg data=cent_train;
407
         model y=income white veterans firms inthouseholdeducation intwhitegender;
408
         run;
409
    quit;
410
411
    proc reg data=cent_train;
412
         model y=income white veterans firms inthouseholdeducation intwhiteeducation;
413
         run;
414
    quit;
415
    proc reg data=cent_train;
416
         model y=income white veterans firms inthouseholdeducation intwhitefirms;
417
         run;
418
    quit;
419
420
    proc reg data=cent train;
421
         model y=income white veterans firms inthouseholdeducation intwhiteveterans;
422
         run;
    quit;
423
424
    proc reg data=cent train;
425
         model y=income white veterans firms inthouseholdeducation intincomehousehold;
426
         run;
427
    quit;
428
429
    proc reg data=cent train;
430
         model y=income white veterans firms inthouseholdeducation intincomegender;
         run;
431
    quit;
432
433
    proc reg data=cent_train;
434
         model y=income white veterans firms inthouseholdeducation intincomeeducation;
435
436
    quit;
437
```

```
438
    proc reg data=cent train;
439
        model y=income white veterans firms inthouseholdeducation intincomefirms;
440
    quit:
441
442
    proc reg data=cent_train;
443
        model y=income white veterans firms inthouseholdeducation intincomeveterans;
444
445
    quit;
446
    proc reg data=cent_train;
447
        model y=income white veterans firms inthouseholdeducation inthouseholdgender;
448
449
    quit;
450
451
     proc reg data=cent_train;
452
        model y=income white veterans firms inthouseholdeducation inthouseholdfirms;
453
454
    quit;
455
    proc reg data=cent_train;
456
        model y=income white veterans firms inthouseholdeducation inthouseholdveterans;
457
        run;
458
    quit;
459
460
    proc reg data=cent_train;
461
        model y=income white veterans firms inthouseholdeducation intgendereducation;
462
        run;
463
    quit;
464
    proc reg data=cent_train;
465
        model y=income white veterans firms inthouseholdeducation intgenderfirms;
466
        run;
467
    quit;
468
469
     proc reg data=cent_train;
470
        model y=income white veterans firms inthouseholdeducation intgenderveterans;
471
        run;
    quit;
472
473
    proc reg data=cent_train;
474
        model y=income white veterans firms inthouseholdeducation inteducationfirms;
475
        run;
476
    quit;
477
478
    proc reg data=cent_train;
479
        model y=income white veterans firms inthouseholdeducation inteducationveterans;
480
        run;
    quit;
481
482
    proc reg data=cent_train;
483
        model y=income white veterans firms inthouseholdeducation intfirmsveterans;
484
        run:
485
    quit;
486
487
     proc reg data=cent_train;
         model y=income white veterans Firms inthouseholdeducation intincomeeducation;
488
         run;
489
490
    proc reg data=cent_train;
491
        model y=income white veterans firms inthouseholdeducation intincomeeducation;
492
493
494
         /* we choose intwhiteeducation as a candidate interaction varible, now check for it */
         /*r:0.5300 vif<10*/
495
    proc reg data=cent_train;
496
         model y=income white veterans firms inthouseholdeducation intwhiteeducation /
497
            vif;
498
         run:
499
500
         /* 3.8 add intwhiteeducation, determine the third interaction variable*/
501
    proc reg data=cent_train;
502
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
503
            intwhiteincome;
         run;
504
    quit;
505
506
     proc reg data=cent_train;
507
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
508
             intwhitehousehold;
509
         run;
    quit;
510
```

```
511
512
     proc reg data=cent_train;
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
513
             intwhitegender;
514
         run;
515
     quit;
516
517
     proc reg data=cent_train;
518
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
519
             intwhitefirms;
         run:
520
     quit;
521
522
     proc reg data=cent_train;
523
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
524
             intwhiteveterans;
525
         run:
526
     quit;
527
     proc reg data=cent_train;
528
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
529
             intincomehousehold:
530
         run;
531
     quit;
532
533
     proc reg data=cent_train;
534
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
535
             intincomegender;
536
         run:
     quit;
537
538
     proc reg data=cent_train;
539
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
540
             intincomeeducation;
541
         run:
542
     quit;
543
544
     proc reg data=cent_train;
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
545
             intincomefirms;
546
         run:
547
     quit;
548
549
     proc reg data=cent_train;
550
         model v=income white veterans firms inthouseholdeducation intwhiteeducation
551
             intincomeveterans;
552
         run:
     quit;
553
554
     proc reg data=cent train;
555
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
556
             inthouseholdgender;
557
         run:
558
     quit;
559
560
     proc reg data=cent_train;
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
561
             inthouseholdfirms;
562
         run;
563
     quit;
564
565
     proc reg data=cent_train;
566
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
567
             inthouseholdveterans;
568
         run;
     quit;
569
570
     proc reg data=cent_train;
571
         \begin{tabular}{lll} \textbf{model} & \textbf{y} = \textbf{income} & \textbf{white} & \textbf{veterans} & \textbf{firms} & \textbf{inthouseholdeducation} & \textbf{intwhiteeducation} \\ \end{tabular}
572
             intgendereducation;
573
         run:
574
     quit;
575
576
     proc reg data=cent_train;
577
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
             intgenderfirms;
578
         run;
579
     quit;
580
581
     proc reg data=cent_train;
582
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
             intgenderveterans;
583
```

```
584
         run;
585
    quit;
586
    proc reg data=cent train;
587
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
588
             inteducationfirms;
589
         run:
590
    quit;
591
592
    proc reg data=cent_train;
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
593
             inteducationveterans:
594
         run;
595
    quit;
596
597
    proc reg data=cent_train;
598
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
599
             intfirmsveterans;
600
         run:
    quit;
601
602
    /st we choose intincomefirms as a candidate interaction varible, now check for it st/
603
     /*r_square:0.5345 vif<10*/
604
    proc reg data=cent_train;
605
         model y=white income veterans firms inthouseholdeducation intwhiteeducation
606
            intincomefirms / vif;
607
608
         /st 3.9 add intincomefirms, determine the fourth interaction variable ^st/
609
    proc reg data=cent_train;
610
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
611
            intincomefirms intwhiteincome;
612
         run:
613
    quit;
614
615
    proc reg data=cent_train;
616
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
617
             intincomefirms intwhitehousehold;
618
    quit;
619
620
    proc reg data=cent train;
621
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
622
             intincomefirms intwhitegender;
623
        run:
624
    quit;
625
    proc reg data=cent_train;
626
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
627
            intincomefirms intwhitefirms;
628
         run;
629
    quit;
630
631
    proc reg data=cent_train;
632
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
            intincomefirms intwhiteveterans;
633
         run;
634
    quit:
635
636
    proc reg data=cent_train;
637
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
638
            intincomefirms intincomehousehold;
639
        run;
640
    quit;
641
    proc reg data=cent_train;
642
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
643
            intincomefirms intincomegender;
644
        run:
645
    quit;
646
647
    proc reg data=cent_train;
648
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
649
            intincomefirms intincomeeducation;
650
    quit;
651
652
    proc reg data=cent_train;
653
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
654
             intincomefirms intincomeveterans;
655
         run;
    quit;
656
```

```
657
658
          proc reg data=cent_train;
                    model y=income white veterans firms inthouseholdeducation intwhiteeducation
659
                            intincomefirms inthouseholdgender;
660
                    run;
661
          quit;
662
663
          proc reg data=cent_train;
664
                   model y=income white veterans firms inthouseholdeducation intwhiteeducation
665
                            intincomefirms inthouseholdfirms;
666
          quit;
667
668
          proc reg data=cent_train;
669
                    model y=income white veterans firms inthouseholdeducation intwhiteeducation
670
                            intincomefirms inthouseholdveterans;
671
                   run:
672
          quit;
673
          proc reg data=cent_train;
674
                    model y=income white veterans firms inthouseholdeducation intwhiteeducation
675
                            intincomefirms intgendereducation;
676
                    run:
677
          quit;
678
679
          proc reg data=cent_train;
680
                    model y=income white veterans firms inthouseholdeducation intwhiteeducation
681
                            intincomefirms intgenderfirms;
682
          quit;
683
684
          proc reg data=cent_train;
685
                    \begin{tabular}{lll} \textbf{model} & \textbf{y=} \textbf{income} & \textbf{white} & \textbf{veterans} & \textbf{firms} & \textbf{inthouseholdeducation} & \textbf{intwhitee} \\ \textbf{ducation} & \textbf{one} & \textbf{one} \\ \textbf{ducation} & \textbf{one} \\ \textbf{du
686
                            intincomefirms intgenderveterans;
687
                    run:
688
          quit;
689
690
          proc reg data=cent_train;
                    model y=income white veterans firms inthouseholdeducation intwhiteeducation
691
                            intincomefirms inteducationfirms;
692
                    run:
693
          quit;
694
695
          proc reg data=cent_train;
696
                    model y=income white veterans firms inthouseholdeducation intwhiteeducation
697
                             intincomefirms inteducation veterans;
698
                    run:
          quit;
699
700
          proc reg data=cent train;
701
                    model y=income white veterans firms inthouseholdeducation intwhiteeducation
702
                             intincomefirms intfirmsveterans;
703
                   run:
704
          quit;
705
706
          proc reg data=cent_train;
                    model y=income white veterans Firms inthouseholdeducation intwhiteeducation
707
                            intincomefirms intincomeeducation;
708
                    run;
709
710
          proc reg data=cent_train;
711
                    model y=income white veterans firms inthouseholdeducation intwhiteeducation
712
                            intincomefirms intincomeeducation;
713
                    run:
714
                    /* we choose intwhitehousehold as a candidate interaction varible, now check for it */
715
                    /*r:0.5412 vif<10*/
716
          proc reg data=cent_train;
717
                    model y=income white veterans firms inthouseholdeducation intwhiteeducation
718
                             intincomefirms intwhitehousehold/ vif;
719
                    run:
720
721
                    /* 3.10 add intwhitehousehold, determine the fifth interaction variable*/
722
          proc reg data=cent_train;
723
                    model y=income white veterans firms inthouseholdeducation intwhiteeducation
                            intincomefirms intwhitehousehold intwhiteincome;
724
                    run;
725
          quit;
726
727
          proc reg data=cent train;
728
                    model y=income white veterans firms inthouseholdeducation intwhiteeducation
                             intincomefirms intwhitehousehold intwhitegender;
729
```

```
730
         run;
731
    quit;
732
    proc reg data=cent train;
733
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
734
             intincomefirms intwhitehousehold intwhitefirms;
735
         run:
736
    quit;
737
738
    proc reg data=cent_train;
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
739
             intincomefirms intwhitehousehold intwhiteveterans;
740
         run:
741
    quit;
742
743
    proc reg data=cent_train;
744
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
745
             intincomefirms intwhitehousehold intincomehousehold;
746
    quit;
747
748
    proc reg data=cent_train;
749
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
750
             intincomefirms intwhitehousehold intincomegender;
751
         run:
752
    quit;
753
754
    proc reg data=cent_train;
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
755
             intincomefirms intwhitehousehold intincomeeducation;
756
         run:
757
    quit;
758
759
    proc reg data=cent_train;
760
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
761
             intincomefirms intwhitehousehold intincomeveterans;
762
         run;
763
    quit;
764
    proc reg data=cent_train;
765
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
766
            intincomefirms intwhitehousehold inthouseholdgender;
767
         run:
768
    quit;
769
770
    proc reg data=cent_train;
771
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
            intincomefirms intwhitehousehold inthouseholdfirms;
772
         run:
773
    quit;
774
775
    proc reg data=cent_train;
776
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
777
            intincomefirms intwhitehousehold inthouseholdveterans;
778
779
    quit;
780
    proc reg data=cent train;
781
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
782
            intincomefirms intwhitehousehold intgendereducation;
783
         run:
784
    quit;
785
786
    proc reg data=cent_train;
787
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
             intincomefirms intwhitehousehold intgenderfirms;
788
         run;
789
    quit;
790
791
    proc reg data=cent_train;
792
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
793
             intincomefirms intwhitehousehold intgenderveterans;
794
        run;
795
    quit:
796
    proc reg data=cent_train;
797
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
798
             intincomefirms intwhitehousehold inteducationfirms;
799
         run;
800
    quit;
801
    proc reg data=cent train;
802
```

```
803
        model y=income white veterans firms inthouseholdeducation intwhiteeducation
804
            intincomefirms intwhitehousehold inteducationveterans;
805
    quit:
806
807
    proc reg data=cent_train;
808
        model y=income white veterans firms inthouseholdeducation intwhiteeducation
809
            intincomefirms intwhitehousehold intfirmsveterans;
810
811
    quit;
812
    proc reg data=cent train;
813
        model y=white Income veterans Firms inthouseholdeducation intwhiteeducation
814
            intincomefirms intwhitehousehold intincomeeducation;
815
         run:
816
817
         /* we choose intincomehousehold as a candidate interaction varible, now check for it */
818
    proc reg data=cent_train;
819
        model y=white income veterans firms inthouseholdeducation intwhiteeducation
            intincomefirms intwhitehousehold intincomehousehold/ vif;
820
821
822
         /* 3.11 add intincomehousehold, determine the sisth interaction variable*/
823
    proc reg data=cent_train;
824
        model y=income white veterans firms inthouseholdeducation intwhiteeducation
825
            intincomefirms intwhitehousehold intincomehousehold intwhiteincome;
826
        run;
827
    quit;
828
    proc reg data=cent_train;
829
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
830
            intincomefirms intwhitehousehold intincomehousehold intwhitegender;
831
        run:
832
    quit;
833
834
    proc reg data=cent_train;
835
        model y=income white veterans firms inthouseholdeducation intwhiteeducation
836
            intincomefirms intwhitehousehold intincomehousehold intwhitefirms;
837
    quit;
838
839
    proc reg data=cent train;
840
        model y=income white veterans firms inthouseholdeducation intwhiteeducation
841
            intincomefirms intwhitehousehold intincomehousehold intwhiteveterans;
842
        run:
843
    quit;
844
    proc reg data=cent_train;
845
        model y=income white veterans firms inthouseholdeducation intwhiteeducation
846
            intincomefirms intwhitehousehold intincomehousehold intincomegender;
847
        run;
848
    quit;
849
850
    proc reg data=cent_train;
851
        model y=income white veterans firms inthouseholdeducation intwhiteeducation
852
            intincomefirms intwhitehousehold intincomehousehold intincomeeducation;
        run;
853
    quit:
854
855
    proc reg data=cent_train;
856
        model y=income white veterans firms inthouseholdeducation intwhiteeducation
857
            intincomefirms intwhitehousehold intincomehousehold intincomeveterans;
858
        run:
859
    quit;
860
    proc reg data=cent_train;
861
        model y=income white veterans firms inthouseholdeducation intwhiteeducation
862
            intincomefirms intwhitehousehold intincomehousehold inthouseholdgender;
863
        run:
864
    quit;
865
866
    proc reg data=cent_train;
867
        model y=income white veterans firms inthouseholdeducation intwhiteeducation
868
            intincomefirms intwhitehousehold intincomehousehold inthouseholdfirms;
869
    quit;
870
871
    proc reg data=cent_train;
872
        model y=income white veterans firms inthouseholdeducation intwhiteeducation
873
            intincomefirms intwhitehousehold intincomehousehold inthouseholdveterans;
874
    quit;
875
```

```
876
877
          proc reg data=cent_train;
                  model y=income white veterans firms inthouseholdeducation intwhiteeducation
878
                           intincomefirms intwhitehousehold intincomehousehold intgendereducation;
879
                  run;
880
          quit;
881
882
          proc reg data=cent_train;
883
                  model y=income white veterans firms inthouseholdeducation intwhiteeducation
224
                           intincomefirms intwhitehousehold intincomehousehold intgenderfirms;
885
          quit;
886
887
          proc reg data=cent_train;
888
                  model y=income white veterans firms inthouseholdeducation intwhiteeducation
889
                           intincomefirms intwhitehousehold intincomehousehold intgenderveterans;
890
                  run:
891
          quit;
892
          proc reg data=cent_train;
893
                  model y=income white veterans firms inthouseholdeducation intwhiteeducation
894
                           intincomefirms intwhitehousehold intincomehousehold inteducationfirms;
895
                  run:
896
          quit;
897
898
          proc reg data=cent_train;
899
                  model y=income white veterans firms inthouseholdeducation intwhiteeducation
900
                           intincomefirms intwhitehousehold intincomehousehold inteducationveterans;
901
          quit;
902
903
          proc reg data=cent_train;
904
                  \begin{tabular}{lll} \textbf{model} & \textbf{y=} \textbf{income} & \textbf{white} & \textbf{veterans} & \textbf{firms} & \textbf{inthouseholdeducation} & \textbf{intwhitee} \\ \textbf{ducation} & \textbf{one} & \textbf{one} \\ \textbf{ducation} & \textbf{one} \\ \textbf{du
905
                           intincomefirms intwhitehousehold intincomehousehold intfirmsveterans;
906
                  run:
907
          quit;
908
          /st we choose intincomegender as a candidate interaction varible, now check for it st/
909
          /*r:0.5531 vif<10*/
910
          proc reg data=cent_train;
911
                  model y=white income veterans firms inthouseholdeducation intwhiteeducation
912
                           intincomefirms intwhitehousehold intincomehousehold intincomegender / vif;
913
914
915
                   /* 3.12 add intincomegender, determine the seventh interaction variable*/
916
          proc reg data=cent_train;
917
                  model y=income white veterans firms inthouseholdeducation intwhiteeducation
                           intincomefirms intwhitehousehold intincomehousehold intincomegender
918
                           intwhiteincome;
919
                  run;
920
          quit;
921
922
          proc reg data=cent_train;
923
                  model y=income white veterans firms inthouseholdeducation intwhiteeducation
924
                           intincomefirms intwhitehousehold intincomehousehold intincomegender
925
                           intwhitegender;
                  run;
926
          quit;
927
928
          proc reg data=cent_train;
929
                  model y=income white veterans firms inthouseholdeducation intwhiteeducation
930
                           intincomefirms intwhitehousehold intincomehousehold intincomegender
931
                           intwhitefirms;
932
                  run:
933
          quit;
934
          proc reg data=cent_train;
935
                  model y=income white veterans firms inthouseholdeducation intwhiteeducation
936
                           intincomefirms intwhitehousehold intincomehousehold intincomegender
937
                           intwhiteveterans:
938
                  run:
939
          quit;
940
941
          proc reg data=cent_train;
                  model y=income white veterans firms inthouseholdeducation intwhiteeducation
942
                           intincomefirms intwhitehousehold intincomehousehold intincomegender
943
                           intincomeeducation;
944
                  run;
945
          quit;
946
947
          proc reg data=cent train;
                  model y=income white veterans firms inthouseholdeducation intwhiteeducation
948
```

```
949
                              intincomefirms intwhitehousehold intincomehousehold intincomegender
  950
                              intincomeveterans;
                     run:
  951
            quit;
  952
  953
            proc reg data=cent_train;
  954
                     model y=income white veterans firms inthouseholdeducation intwhiteeducation
  955
                              intincomefirms intwhitehousehold intincomehousehold intincomegender
  956
                              inthouseholdgender:
  957
                     run:
            quit;
  958
  959
            proc reg data=cent_train;
  960
                     model y=income white veterans firms inthouseholdeducation intwhiteeducation
  961
                              intincomefirms intwhitehousehold intincomehousehold intincomegender
  962
                              inthouseholdfirms;
  963
                     run:
  964
            quit;
  965
            proc reg data=cent_train;
  966
                     model y=income white veterans firms inthouseholdeducation intwhiteeducation
  967
                              intincomefirms intwhitehousehold intincomehousehold intincomegender
  968
                              inthouseholdveterans;
  969
                     run;
  970
            auit:
  971
  972
            proc reg data=cent_train;
  973
                     model y=income white veterans firms inthouseholdeducation intwhiteeducation
                              intincomefirms intwhitehousehold intincomehousehold intincomegender
  974
                              intgendereducation;
  975
                     run;
  976
            quit;
  977
  978
            proc reg data=cent_train;
  979
                     model v=income white veterans firms inthouseholdeducation intwhiteeducation
  980
                              intincomefirms intwhitehousehold intincomehousehold intincomegender
  981
                              intgenderfirms;
  982
                     run:
            quit;
  983
  984
            proc reg data=cent_train;
  985
                     model y=income white veterans firms inthouseholdeducation intwhiteeducation
  986
                              intincomefirms intwhitehousehold intincomehousehold intincomegender
  987
                              intgenderveterans;
  988
                     run:
  989
            quit;
  990
            proc reg data=cent_train;
  991
                     model y=income white veterans firms inthouseholdeducation intwhiteeducation
  992
                              intincomefirms intwhitehousehold intincomehousehold intincomegender
  993
                              inteducationfirms;
  994
                     run;
  995
            auit:
  996
  997
            proc reg data=cent_train;
                     \begin{tabular}{lll} \textbf{model} & \textbf{y=} \textbf{income} & \textbf{white} & \textbf{veterans} & \textbf{firms} & \textbf{inthouseholdeducation} & \textbf{intwhitee} \\ \textbf{ducation} & \textbf{one} & \textbf{one} \\ \textbf{ducation} & \textbf{one} \\ \textbf{du
  998
                              intincomefirms intwhitehousehold intincomehousehold intincomegender
  999
                              inteducationveterans;
1000
                     run;
1001
            quit;
1002
1003
             proc reg data=cent_train;
1004
                     model y=income white veterans firms inthouseholdeducation intwhiteeducation
1005
                              intincomefirms intwhitehousehold intincomehousehold intincomegender
1006
                              intfirmsveterans;
                     run:
1007
            quit;
1008
1009
             /* we choose inteducationveterans as a candidate interaction varible, now check for it */
1010
             /*r_square:0.5555 vif<10 */
1011
            proc reg data=cent_train;
1012
                     model y=white income veterans firms inthouseholdeducation intwhiteeducation
1013
                              intincomefirms intwhitehousehold intincomehousehold intincomegender
1014
                              inteducationveterans/ vif;
1015
1016
                      /* 3.13 add inteducationveterans, determine the eighth interaction variable*/
1017
             proc reg data=cent_train;
1018
                     model y=income white veterans firms inthouseholdeducation intwhiteeducation
1019
                              intincomefirms intwhitehousehold intincomehousehold intincomegender
1020
                              inteducationveterans intwhiteincome;
                     run:
1021
```

```
1022
     quit;
1023
     proc reg data=cent train;
1024
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1025
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1026
              inteducationveterans intwhitegender;
1027
          run:
1028
     quit;
1029
1030
     proc reg data=cent_train;
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1031
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1032
              inteducationveterans intwhitefirms;
1033
          run;
1034
      quit;
1035
1036
      proc reg data=cent_train;
1037
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1038
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1039
              inteducationveterans intwhiteveterans;
          run;
1040
     quit;
1041
1042
     proc reg data=cent_train;
1043
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1044
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1045
              inteducationveterans intincomeeducation;
1046
          run:
1047
     quit;
1048
      proc reg data=cent_train;
1049
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1050
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1051
              inteducationveterans intincomeveterans;
1052
          run:
1053
     quit;
1054
1055
      proc reg data=cent_train;
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1056
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1057
              inteducationveterans inthouseholdgender;
1058
1059
     quit;
1060
1061
     proc reg data=cent_train;
1062
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1063
              intincomefirms intwhitehousehold intincomehousehold intincomegender
              inteducationveterans inthouseholdfirms;
1064
          run:
1065
     quit;
1066
1067
      proc reg data=cent_train;
1068
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1069
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1070
              inteducationveterans inthouseholdveterans;
1071
          run:
     quit;
1072
1073
      proc reg data=cent_train;
1074
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1075
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1076
              inteducationveterans intgendereducation;
1077
          run;
1078
     quit;
1079
     proc reg data=cent_train;
1080
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1081
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1082
              inteducationveterans intgenderfirms;
1083
          run:
1084
     quit;
1085
1086
     proc reg data=cent train;
1087
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1088
              intincomefirms intwhitehousehold intincomehousehold intincomegender
              inteducationveterans intgenderveterans;
1089
          run;
1090
     quit;
1091
1092
      proc reg data=cent_train;
1093
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1094
```

```
1095
              inteducationveterans inteducationfirms;
1096
          run:
     quit;
1097
1098
     proc reg data=cent_train;
1099
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1100
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1101
              inteducationveterans intfirmsveterans;
1102
          run:
1103
     auit:
1104
      /* we choose intwhitefirms as a candidate interaction varible, now check for it */
1105
      /*r_square:0.5577 vif<10*/
     proc reg data=cent_train;
1107
          model y=white income veterans firms inthouseholdeducation intwhiteeducation
1108
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1109
              inteducation veterans intwhitefirms / vif:
1110
1111
          /* 3.14 add intwhitefirms, determine the ninth interaction variable*/
1112
     proc reg data=cent_train;
1113
         model y=income white veterans firms inthouseholdeducation intwhiteeducation
1114
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1115
              inteducationveterans intwhitefirms intwhiteincome;
1116
          run:
1117
      quit;
1118
1119
     proc reg data=cent_train;
1120
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1121
              inteducationveterans intwhitefirms intwhitegender;
1122
          run:
1123
      quit;
1124
1125
     proc reg data=cent_train;
1126
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1127
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1128
              inteducation veterans intwhitefirms intwhiteveterans;
1129
          run;
     quit;
1130
1131
      proc reg data=cent train;
1132
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1133
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1134
              inteducationveterans intwhitefirms intincomeeducation;
1135
          run;
1136
     quit;
1137
      proc reg data=cent train;
1138
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1139
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1140
              inteducationveterans intwhitefirms intincomeveterans;
1141
         run:
1142
     quit;
1143
1144
     proc reg data=cent_train;
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1145
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1146
              inteducationveterans intwhitefirms inthouseholdgender;
1147
          run:
1148
      quit;
1149
1150
      proc reg data=cent train;
1151
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1152
              inteducationveterans intwhitefirms inthouseholdfirms;
1153
          run;
1154
     quit;
1155
1156
     proc reg data=cent_train;
1157
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1158
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1159
              inteducationveterans intwhitefirms inthouseholdveterans;
1160
          run;
     quit;
1161
1162
      proc reg data=cent_train;
1163
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1164
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1165
              inteducationveterans intwhitefirms intgendereducation;
1166
          run;
     quit;
1167
```

```
1168
1169
     proc reg data=cent train;
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1170
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1171
              inteducationveterans intwhitefirms intgenderfirms;
1172
1173
     quit;
1174
1175
     proc reg data=cent train;
1176
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1177
              inteducationveterans intwhitefirms intgenderveterans;
1178
          run:
1179
     quit;
1180
1181
      proc reg data=cent_train;
1182
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1183
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1184
              inteducationveterans intwhitefirms inteducationfirms;
          run;
1185
     quit;
1186
1187
      proc reg data=cent_train;
1188
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1189
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1190
              inteducation veterans intwhitefirms intfirms veterans;
1191
          run;
1192
     quit;
1193
      /* we choose intfirmsveterans as a candidate interaction varible, now check for it */
1194
      /*r_square:0.5589 vif<10 */
1195
     proc reg data=cent train;
1196
          model y=white income veterans firms inthouseholdeducation intwhiteeducation
1197
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1198
              inteducation veterans intwhitefirms intfirms veterans / vif:
1199
          run;
1200
1201
          /* 3.15 add intfirmsveterans, determine the tenth interaction variable*/
      proc reg data=cent_train;
1202
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1203
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1204
              inteducationveterans intwhitefirms intfirmsveterans intwhiteincome;
1205
          run:
1206
     quit;
1207
1208
      proc reg data=cent_train;
1209
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1210
              inteducationveterans intwhitefirms intfirmsveterans intwhitegender;
1211
          run:
1212
     quit;
1213
1214
     proc reg data=cent train;
1215
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1216
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1217
              inteducationveterans intwhitefirms intfirmsveterans intwhiteveterans;
          run;
1218
     quit:
1219
1220
     proc reg data=cent_train;
1221
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1222
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1223
              inteducationveterans intwhitefirms intfirmsveterans intincomeeducation;
1224
          run:
1225
     quit;
1226
     proc reg data=cent_train;
1227
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1228
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1229
              inteducationveterans intwhitefirms intfirmsveterans intincomeveterans;
1230
          run:
1231
     quit;
1232
1233
     proc reg data=cent_train;
1234
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1235
              inteducationveterans intwhitefirms intfirmsveterans inthouseholdgender;
1236
          run;
1237
      quit;
1238
1239
      proc reg data=cent train;
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1240
```

```
1241
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1242
              inteducationveterans intwhitefirms intfirmsveterans inthouseholdfirms;
          run:
1243
     quit:
1244
1245
      proc reg data=cent_train;
1246
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1247
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1248
              inteducationveterans intwhitefirms intfirmsveterans inthouseholdveterans;
1249
          run:
     quit;
1250
1251
      proc reg data=cent_train;
1252
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1253
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1254
              inteducationveterans intwhitefirms intfirmsveterans intgendereducation;
1255
         run:
1256
     quit;
1257
     proc reg data=cent_train;
1258
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1259
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1260
              inteducationveterans intwhitefirms intfirmsveterans intgenderfirms;
1261
          run;
1262
     auit:
1263
1264
     proc reg data=cent train;
          model y=income white veterans firms inthouseholdeducation intwhiteeducation
1265
1266
              intincomefirms intwhitehousehold intincomehousehold intincomegender
              inteducationveterans intwhitefirms intfirmsveterans intgenderveterans;
1267
          run:
1268
     quit;
1269
1270
      proc reg data=cent_train;
1271
          model v=income white veterans firms inthouseholdeducation intwhiteeducation
1272
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1273
              inteducationveterans intwhitefirms intfirmsveterans inteducationfirms;
1274
          run:
      quit;
1275
1276
      /* all the p value > 0.05 and r_square is 0.5589, so the final model is \ st/
1277
     proc reg data=cent_train;
1278
          model y=white income veterans firms inthouseholdeducation intwhiteeducation
1279
              intincomefirms intwhitehousehold intincomehousehold intincomegender
1280
              inteducationveterans intwhitefirms intfirmsveterans/ vif;
1281
          run;
1282
          *#################;
          *### STEP 4: Final Model verify
1283
          *############;
1284
          %let reg x= white income veterans firms inthouseholdeducation intwhiteeducation intincomefirms intwhitehouseho
1285
1286
          /*1. Check multicollinearity*/
1287
     proc reg data=cent_train;
1288
         model y=&reg_x / vif;
1289
         run;
1290
     quit;
1291
      /*2. Check the assumptions */
1292
      /* Normality: verify qqplot of (studentised) residuals */
1293
     proc reg data=cent_train;
1294
          model y=&reg_x;
1295
          output out=resid r=rman p=pman student=student;
1296
         run:
1297
     quit;
1298
     /* Linearity: verify plot of (studentised) residuals vs predicted values*/
1299
      /* Homoscedasticity: verify (squared) residuals vs predicted values */
1300
     data resid2;
1301
          set resid;
1302
          rman2=rman**2;
1303
     run:
1304
1305
     proc sgplot data=resid2;
1306
          scatter x=pman y=rman2;
1307
          refline 0 / axis=y lineattrs=(color=red);
1308
1309
      /*3. Check for outliers */
1310
     proc reg data=cent_train noprint;
1311
          model y=&reg_x / r;
1312
          output out=cookdis cookd=cdist;
          run;
1313
```

```
1314
     quit;
1315
     data cookdis2;
1316
         set cookdis;
1317
         n=_n_;
1318
1319
1320
     title "Cook's distance threshold 4/n=0.00255";
1321
1322
     proc sgplot data=cookdis2;
         scatter x=n y=cdist;
1323
         refline 0.0025 / axis=y lineattr=(color=red);
1324
     run;
1325
1326
     title;
1327
1328
      /*Remove outlier according to Cook's distance threshold 4/n=0.00255;*/
1329
     data cent_training_outlier_removed;
1330
         set cookdis2;
         where cdist<0.00255;
1331
1332
         /* 93 outlier removed*/
1333
     run;
1334
1335
     /*Build model after removing outlier*/
1336
      /*r_square:0.5714,intwhitehousehold(p=0.4253) and intfirmsveterans(p=0.5833)*/
1337
     proc reg data=cent_training_outlier_removed;
1338
         model y=&reg_x / vif;
1339
         run;
     quit;
1340
1341
      /*Remove intwhitehousehold and intfirmsveterans and check outlier again*/
1342
     /*Only 2 outliers and they are very close to the transhold, r_square:0.5714*/
1343
1344
     %let reg x= white income veterans firms inthouseholdeducation intwhiteeducation intincomefirms intincomehousehold
1345
1346
     proc reg data=cent_training_outlier_removed noprint;
1347
         model y=&reg_x / r;
         output out=cookdis cookd=cdist;
1348
         run:
1349
     quit;
1350
1351
     data cookdis2;
1352
         set cookdis;
1353
         n=_n_;
1354
     run;
1355
     title "Cook's distance threshold 4/n=0.00255";
1356
1357
     proc sgplot data=cookdis2;
1358
         scatter x=n y=cdist;
1359
         refline 0.0025 / axis=y lineattr=(color=red);
1360
     run;
1361
1362
     title;
1363
      /*Export the dataset for Logistic Regression*/
1364
     proc export data=cent training outlier removed (keep=y county &reg x)
1365
             outfile="/folders/myfolders/StatmodProject2020/cent_train.csv" dbms=csv replace;
1366
     run;
1367
1368
     proc export data=cent_test (keep=y county &reg_x)
1369
             outfile="/folders/myfolders/StatmodProject2020/cent test.csv" dbms=csv replace;
1370
     run;
     *###########;
1372
      *### STEP 5: Testing Final Model
1373
     *############;
1374
1375
     proc reg data=cent_training_outlier_removed outest=train_estimate noprint;
1376
         model y=&reg_x;
1377
         run:
1378
1379
     proc score data=cent_test score=train_estimate out=test_result type=parms
1380
             predict;
         var y &reg_x;
1381
     run;
1382
1383
     ods listing gpath=&dir;
1384
     ods graphics on;
1385
     ods select FitPlot;
1386
```

```
1387
      proc reg data=test_result;
1388
          model y=model1;
          plot model1*y / pred conf;
1389
          run;
1390
          ods graphics off;
1391
          ods listing;
1392
1393
1394
1395
1396
1397
1398
1399
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US election final

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#import data#
mypath = '.'
setwd(mypath)
cent_train<-read.csv('cent_train.csv')</pre>
cent_test<-read.csv('cent_test.csv')</pre>
whole_dataset<- rbind(cent_train,cent_test)</pre>
str(cent_train)
str(cent_test)
plot(cent_train$y,xlab = 'series',ylab = 'vote_percent')
#6.categorize y#
cent_train$cat_y <-ifelse(cent_train$y>0.5,1,0)
cent_test$cat_y <-ifelse(cent_test$y>0.5,1,0)
#fit with categorized y# state what we have found when compare two models # #
fit.raw.train <-lm(y ~ white+income+veterans+firms+inthouseholdeducation+
                      intwhiteeducation+intincomefirms+intincomehousehold+
                      intincomegender+inteducationveterans+intwhitefirms,
                    data=cent_train)
fit.cat.train <- glm(cat_y ~ white+income+veterans+firms+inthouseholdeducation+
                        \verb|intwhitee ducation+intincome firms+intincome household+|\\
                        intincomegender+inteducationveterans+intwhitefirms,
                      family = binomial,data=cent_train)
fit.raw.train.summary<-summary(fit.raw.train)</pre>
fit.cat.train.summary<-summary(fit.cat.train)</pre>
beta1 <- fit.cat.train$coefficients[2]</pre>
con.odds.white <- exp(beta1)</pre>
plot(effects::effect('white', fit.cat.train))
glm.pred.raw <- predict(fit.raw.train, newdata = cent_test, type = "response")</pre>
glm.pred.cat <- ifelse(glm.pred.raw >0.5,1,0)
glm.pred.cat<-as.numeric(glm.pred.cat)</pre>
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#test result#
p_vs_a<-as.matrix(table(actual=cent_test$cat_y,predict=glm.pred.cat))</pre>
n = sum(p_vs_a) # number of instances
nc = nrow(p_vs_a) # number of classes
diag = diag(p_vs_a) # number of correctly classified instances per class
rowsums = apply(p_vs_a, 1, sum) # number of instances per class
colsums = apply(p_vs_a, 2, sum) # number of predictions per class
p = rowsums / n # distribution of instances over the actual classes
q = colsums / n # distribution of instances over the predicted classes
accuracy = sum(diag) / n
precision = diag / colsums
recall = diag / rowsums
overlapEst(cent_test$cat_y, glm.pred.cat)
#difference------
t.test(glm.pred.cat,cent_test$cat_y,paired = T)
var.test(glm.pred.cat, cent_test$cat_y, alternative = "two.sided")
fit.raw.whole <-lm(y ~ white+income+veterans+firms+inthouseholdeducation+
                    intwhiteeducation+intincomefirms+intincomehousehold+
                    intincomegender+inteducationveterans+intwhitefirms,
                  data=whole_dataset)
glm.pred.whole <- predict(fit.raw.whole, newdata = whole_dataset, type = "response")</pre>
glm.pred.whole<-as.numeric(glm.pred.whole)</pre>
glm.pred.whole.cat <- ifelse(glm.pred.whole >0.52, 'win',
                           ifelse(glm.pred.whole<0.48, 'lose', 'undecided'))</pre>
whole_dataset$result<-glm.pred.whole.cat
undecided <- whole_dataset%>%
 subset(result == 'undecided')
factor(whole_dataset$county)
factor(undecided$county)
table(undecided$county,undecided$result)
```