Introduction to Linear Algebra, DGD 10

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Topics for today:

- determinants and their transformations
- finding characteristic polynomial
- finding eigenspaces
- matrix diagonalization

Problem 1: If

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$$

find

$$\begin{vmatrix} b+4c & e+4f & h+4i \\ -3c & -3f & -3i \\ 5a & 5d & 5g \end{vmatrix}$$

Problem 1: If

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Put -3 and 5 out of the determinant $\rightarrow$ *get rid of the linear combination of two rows* $\rightarrow$ *swap two rows* $\rightarrow$ *swap two rows* $\rightarrow$ *transpose* $\rightarrow$ $-15 \cdot 2 = -30$

Problem 2:
If

$$\begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix} = 2$$

Find

$$\begin{vmatrix} a & e+2a & m & 3i-m \\ b & f+2b & n & 3j-n \\ c & g+2c & o & 3k-o \\ d & h+2d & p & 3l-p \end{vmatrix}$$

Problem 2:

If

$$\begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix} = 2$$

Find

$$\begin{vmatrix} a & e+2a & m & 3i-m \\ b & f+2b & n & 3j-n \\ c & g+2c & o & 3k-o \\ d & h+2d & p & 3l-p \end{vmatrix} = -6$$

Problem 3.1:

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 1 & 3 & 1 \\ 1 & 0 & 4 \end{bmatrix}$$

Compute the characteristic polynomial

Problem 3.2:

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 1 & 3 & 1 \\ 1 & 0 & 4 \end{bmatrix}$$

$$|A - \lambda I| = (3 - \lambda)(\lambda - 3)(\lambda - 2)$$

Find a bases of $E_2 = \{v \in \mathbb{R}^3 \mid Av = 2v\}$ and $E_3 = \{v \in \mathbb{R}^3 \mid Av = 3v\}$

Problem 3.2:

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 1 & 3 & 1 \\ 1 & 0 & 4 \end{bmatrix}$$

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Find a bases of $E_2 = \{v \in \mathbb{R}^3 \mid Av = 2v\}$ and $E_3 = \{v \in \mathbb{R}^3 \mid Av = 3v\}$

$$E_2 = \ker(A - 2I) \quad E_3 = \ker(A - 3I)$$

Problem 3.2:

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 1 & 3 & 1 \\ 1 & 0 & 4 \end{bmatrix}$$

$$|A - \lambda I| = (3 - \lambda)(\lambda - 3)(\lambda - 2)$$

Find a bases of $E_2 = \{v \in \mathbb{R}^3 \mid Av = 2v\}$ and $E_3 = \{v \in \mathbb{R}^3 \mid Av = 3v\}$

$$E_2 = \text{span}\{(-2, 1, 1)\} \quad E_3 = \text{span}\{(-1, 0, 1), (0, 1, 0)\}$$

Problem 3.3:

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 1 & 3 & 1 \\ 1 & 0 & 4 \end{bmatrix}$$

$$|A - \lambda I| = (3 - \lambda)(\lambda - 3)(\lambda - 2)$$

Find a bases of $E_2 = \{v \in \mathbb{R}^3 \mid Av = 2v\}$ and $E_3 = \{v \in \mathbb{R}^3 \mid Av = 3v\}$

$$E_2 = \text{span}\{(-2, 1, 1)\} \;\; E_3 = \text{span}\{(-1, 0, 1), (0, 1, 0)\}$$

Now find a matrices P and D such, that $P^{-1}AP = D$ and D is diagonal

Problem 4:

$$B = \begin{bmatrix} 1 & 0 & -2 \\ 1 & 3 & 1 \\ 1 & 0 & 4 \end{bmatrix}$$

Same for this matrix:

- Find characteristic polynomial
- Find bases of eigenspaces
- If possible, find P and D, such that $P^{-1}AP = D$ and D is diagonal

Problem 4:

$$B = \begin{bmatrix} 2 & 0 & -3 \\ 0 & -1 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

Same for this matrix:

- Find characteristic polynomial
- Find bases of eigenspaces
- If possible, find P and D, such that $P^{-1}AP = D$ and D is diagonal

Problem 5:

$$B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Same for this matrix:

- Find characteristic polynomial
- Find bases of eigenspaces
- If possible, find P and D, such that $P^{-1}AP = D$ and D is diagonal
- If possible, find another pair of P and D

Thank you and have a nice evening!