UNIMORE
UNIVERSITÀ DEGLI STUDI DI
MODENA E REGGIO EMILIA

Basic principles of data analytics and optimization for logistics and operations

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Inventory System
Problem statement

- Objective: compare alternative ordering policies
- A company sells a single product
- How many items it should have in inventory for each of the next n months?

Inventory System
Problem statement

- Demand interrarival times are IID exponential random variables with mean 0.1 month
- Demand size (indipendent of demad time realization)
- $$D = \begin{cases} 1 & w.p.\ 1/6 \\ 2 & w.p\ 1/3 \\ 3 & w.p.\ 1/3 \\ 4 & w.p.\ 1/6 \end{cases}$$

Inventory System
Problem statement

- At beginning of each month, company reviews the inventory level and decides how many items to order from its supplier.
- If company orders Z items, it incurs costs of
  - $C = K + i \cdot Z$
  - K is the *setup* cost
  - i is the *incremental* cost
- Lead time (delivery lag) uniformly distributed

Inventory System
Problem statement

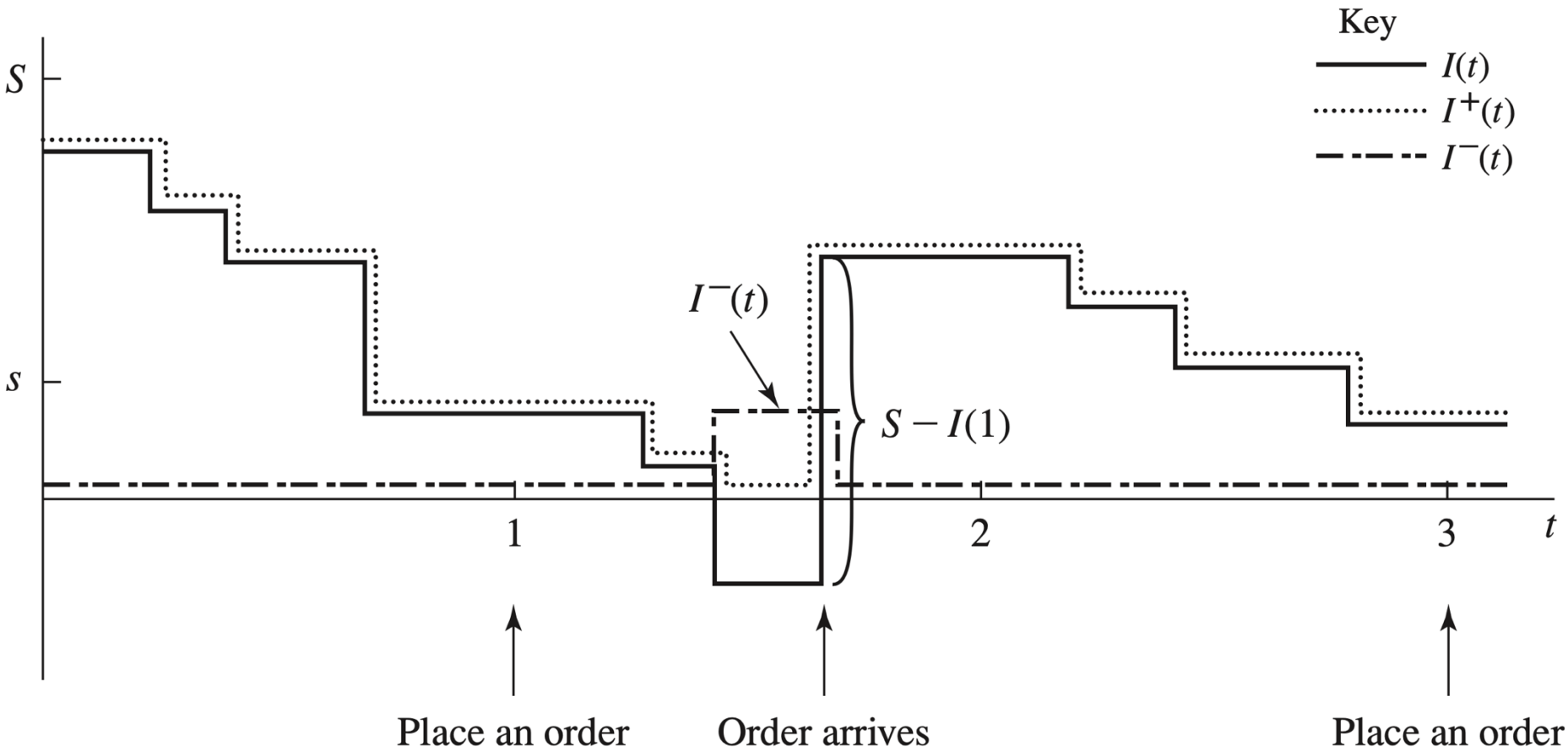
$$Z = \begin{cases} S - I & if\ I < s \\ 0 & if\ I \geq s \end{cases}$$

- **Stationary (s, S) Replenishment Policy**
- Z := how much to order from supplier
- s, S := parameters of the policy
- Note: I is the inventory level at the beginning of the month

Inventory System
Problem statement

- **When demand occurs:**
  - Satisfied immediately if inventory level at least as large as demand: $I(t) \geq D_t$
  - If demand exceeds inventory level, the excess is backlogged and satisfied by future deliveries: $I(t) < D_t$
- **When delivery occurs**:
  - At first used to satisfy backlogged demand (if any)
  - Remainder of the delivery (if any) is added to the inventory
- $I(t)$ **can be positive or negative**

Inventory System
Problem statement



Inventory System
Costs

- **Ordering** costs: $C = K + i \cdot Z$
- **Holding** costs: $\mathrm{H} = h \cdot I^{+}$
- **Shortage** costs: $\mathrm{S} = \pi \cdot I^{-}$
- where:
  - $I^{+}(t) = \max\{I(t), 0\}$
  - $I^{-}(t) = \max\{-I(t), 0\}$
  - $I^{+} = \frac{\int_0^n I^{+}(t)}{n}$
  - $I^{-} = \frac{\int_0^n I^{-}(t)}{n}$

Warehouse rental, insurance, taxes, maintenance, opportunity costs

Record keeping, loss of customers' goodwill

Time-averages

Inventory System
Parameters

- Demand interrarival times: exponential random variable, $\mu = 0.1$
- $K = 32; i = 3$
- Leadtime $U(0.5; 1)$
- $h = 1; \pi = 5$
- $I(0) = 60$
- $n = 120\, months$

Demand size: $D = \begin{cases} 1 & w.p.\ 1/6 \\ 2 & w.p\ 1/3 \\ 3 & w.p.\ 1/3 \\ 4 & w.p.\ 1/6 \end{cases}$

Inventory System
Objective

- Objective: compare nine different inventory policies

s	20	20	20	20	40	40	40	60	60
S	40	60	80	100	60	80	100	80	100

- Performance measure: average total cost per month

$$C_{Tot} = C + H + S$$