EXERCISES ABOUT CONTEXT-FREE LANGUAGES (CFLS)

1 Draw a deterministic push-down automaton (DPDA) for the language:

$\{0^n 1^m 0^n \mid n \geq 0 \text{ and } m \geq 0\}$.

2 Test the determinism in the automata of previous exercises. Verify if they are deterministic automata and if they are not, identify transitions which make them non-deterministic.

3 "Given a non-deterministic PDA, there exists always an equivalent deterministic PDA, i.e., which recognizes exactly the same language." Is this sentence true? Justify.

4 Consider the CFG below. Convert it to the Chomsky Normal Form (CNF), showing the intermediate simplification steps.

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S → A1B
A → 0A | ε
B → 0B | 1B | ε
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5 Convert the following CFG into the Chomsky Normal Form, showing intermediate steps.

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S → 0S00 | 0B0 | B
B → 11B22 | 12 | C
C → 0 | ε
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6 Show, using the pumping lemma for context-free languages, that the language of the strings $a^n b^n c^i$, in which $n \leq i \leq 2n$, is not a context-free language.

7 Show, using the pumping lemma for context-free languages, that the language of the strings 0^p, with p a prime number, is not a context-free language. [Note: we have already proven that this language does not satisfy the pumping lemma for regular languages; let us use a similar strategy to prove that this language does not satisfy the pumping lemma for context-free languages.]

8 Show, using the pumping lemma for context-free languages, that the language of the strings $0^i 1^j$, in which $j=i^2$, is not a context-free language.

9 When we try to apply the pumping lemma for context-free languages to a language of this family, trying to show that it does not belong to the context-free languages, it happens that the "adversary wins" always and we cannot complete the prove. Illustrate this process with the languages L1= {00, 11} and L2= $\{0^n 1^n, n \geq 1\}$.

10 Given two string w and x, let us call *inter*(w,x) to the set of strings obtained interchanging symbols of w and x by the order they occur in w and in x. We can extend the operation to two languages L1 and L2, naming *inter*(L1, L2) to the union, for all the pairs of strings w from L1 and x from L2, of *inter*(w,x).

a) Determine the value of *inter*(00,111).

b) Determine the value of *inter*(L1, L2) with L1= L(0*) and L2= $\{0^n 1^n, n \geq 0\}$.

c) Show that if L1 and L2 are both regular languages then *inter*(L1, L2) is a regular language, as well. [Suggestion: consider the DFAs of L1 and L2.]

d) Show that if L is a context-free language and R a regular language then *inter*(L, R) is a context-free language. [Suggestion: consider a PDA for L and a DFA for R.]

11 Show that in a grammar in the Chomsky Normal Form (CNF), all the analysis trees of length n have 2n-1 inner nodes.