Assigning Dielectric Properties to MRI-based Models

This is a brief guide explaining how users can assign dielectric properties to the MRI-based axillary region models of this repository. We first detail how a Debye model is defined, and then, we present two alternatives to assign dielectric properties to the MRI-segmented tissues using Debye models.

A Debye model can be used to simulate the frequency-dependent dielectric properties of biological tissues and can be written as:

$$\varepsilon^*(\omega) = \varepsilon_\infty + \frac{\sigma_s}{j\omega\varepsilon_0} + \frac{\Delta\varepsilon}{1 + j\omega\tau} \tag{1}$$

where ω is the angular frequency, $\Delta\varepsilon = \varepsilon_s - \varepsilon_\infty$, ε_∞ is the permittivity at $\omega = \infty$, ε_s and σ_s are the static permittivity and conductivity at $\omega = 0$, and τ is the relaxation time constant.

The $\varepsilon^*(\omega)$ is a complex number which can be written as $\varepsilon'(\omega) - j\varepsilon''(\omega)$. The real part $\varepsilon'(\omega)$ corresponds to the relative permittivity and the conductivity can be calculated from the imaginary part as $\sigma = \omega\varepsilon_0\varepsilon''(\omega)$.

1. Recommended Debye Models of Dielectric Properties

The dielectric properties of the tissues included in the axillary region models can be inserted in the electromagnetic simulation software inputting the Debye models parameters, as shown in Table I.

Table I. Parameters of Debye models [1, 2, 3] (valid from 1 to 20 GHz).

Tissues	ε_∞	σ_s (S/m)	$\Delta\varepsilon$	τ (ps)
Skin	15.93	0.83	23.83	13.00
Adipose	3.12	0.05	1.59	13.00
Lung	7.62	0.81	14.03	13.00
Muscle	21.66	0.89	33.24	13.00
Healthy Lymph Nodes	11.05	0.49	24.74	13.00
Metastasised Lymph Nodes	14.40	0.74	36.63	13.00

2. Creation of Dielectric Property Maps

Some software packages for electromagnetic simulations may allow the input of dielectric property maps for each frequency. In order to apply this method, each model is represented by two files which can be used to create these maps: the *label_map* and *weight_map* files.

The *label_map* file has a matrix where each voxel is represented by one of 7 labels (background and 6 groups of tissues). Each label corresponds to a tissue type or group of tissues, and defines which dielectric properties curves should be used to interpolate the voxel intensities into the corresponding dielectric properties, as shown in Table II.

The *weight_map* file includes the weight assigned to each voxel which was calculated from the linear interpolation using the voxel intensities:

$$w = \frac{v - v_{min}}{v_{max} - v_{min}} \tag{2}$$

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where v is the voxel intensity, and v_{min} and v_{max} are the minimum and maximum voxel intensity of the corresponding cluster, respectively. w is a value between 0 and 1.

The corresponding dielectric property value for each voxel can be calculated using the following equation:

$$dp(f) = w * c_{upper}(f) + (1 - w) * c_{lower}(f) \tag{3}$$

where c_{upper} and c_{lower} are the dielectric property values at the upper and lower curves considered for interpolation, respectively, at a given frequency *f*.

Table II. Correspondence between labels and Debye models for interpolation [2, 3] (valid from 1 to 20 GHz).

Label	Tissue	ε_∞	σ_s (S/m)	$\Delta\varepsilon$	τ (ps)
-2	Skin	15.93	0.83	23.83	13.00
-1	Lung	7.62	0.81	14.03	13.00
0	Background	-	-	-	-
1	Adipose Min	2.31	0.005	0.09	13.00
	Adipose Q1	2.85	0.05	1.10	13.00
2	Adipose Q1	2.85	0.05	1.10	13.00
	Adipose Q3	3.99	3.55	0.08	13.00
3	Fibroglandular Q1	12.99	0.40	24.40	13.00
	Fibroglandular Q3	14.20	0.82	40.49	13.00
4	Fibroglandular Q3	14.20	0.82	40.49	13.00
	Fibroglandular Max	23.20	1.31	46.05	13.00

3. Example of a Python script for permittivity map calculation

```
import numpy as np

def calculate_properties_from_debye(debye_parameters, curve, frequency):
    """
    Function to calculate dielectric properties curve based on parameters
of Debye model.
    Inputs:
        - "debye_parameters": parameters of Debye model for each
        curve
        - "curve": string of the curve name
        - "frequency": frequency value(s), in Hz
    Outputs:
        - "e_r": relative permittivity values
        - "sigma_eff": effective conductivity values
    """
    e0 = 8.85e-12
    w = 2*np.pi*frequency
    e = debye_parameters[curve]['e_inf'] + \
            debye_parameters[curve]['delta_e']/ \
            (1+(1j*w*debye_parameters[curve]['tau'])**(1-
debye_parameters[curve]['alpha'])) + \
            debye_parameters[curve]['sigma']/(1j*w*e0)

    e_r = np.real(e)
    sigma_eff = w*e0*np.absolute(np.imag(e))
    return e_r, sigma_eff
```


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```python
# Load the files

# model = LOAD MAT OR RAW label_map FILE
# w = LOAT MAT OR RAW weight_map FILE


# Define parameters of Debye models
debye_parameters = {}
debye_parameters['Skin'] = {'e_inf':15.93, 'delta_e':23.83, 'tau':13e-12,
'alpha':0, 'sigma':0.831}
# [...] apply same rationale for remaining curves



# Calculation of relative permittivity and conductivity of Debye model
curve for the specified frequency
# e_r is the relative permittivity
# sigma_eff is the effective conductivity
# f is the frequency of interest in Hz

f = 5
e_r = {}
sigma_eff = {}
for tissue_name in debye_parameters.keys():
    e_r[tissue_name], sigma_eff[tissue_name] =
calculate_properties_from_debye(debye_parameters, tissue_name, f)


# Create matrix of zeros with the same dimensions of the model
permittivity_map = np.zeros(model.shape)

# For loop scanning each voxel
for x in range(model.shape[0]):
    for y in range(model.shape[1]):
        for z in range(model.shape[2]):
            if model[x,y,z] == 0:
                c_upper = 0
                c_lower = 0
            elif model[x,y,z] == -2:
                c_upper = e_r['Skin']
                c_lower = e_r['Skin']
            elif model[x,y,z] == -1:
                c_upper = e_r['Lung']
                c_lower = e_r['Lung']
            elif model[x,y,z] == 1:
                c_upper = e_r['Adipose Min']
                c_lower = e_r['Adipose Q1']
            elif model[x,y,z] == 2:
                c_upper = e_r['Adipose Q1']
                c_lower = e_r['Adipose Q3']
            elif model[x,y,z] == 3:
                c_upper = e_r['Fibroglandular Q1']
                c_lower = e_r['Fibroglandular Q3']
            elif model[x,y,z] == 4:
                c_upper = e_r['Fibroglandular Q3']
                c_lower = e_r['Fibroglandular Max']

            permittivity_map[x,y,z] = w[x,y,z]*c_upper + (1-
w[x,y,z])*c_lower
```

4. References

[1] D. M. Godinho et al., "Development of MRI-based axillary numerical models and estimation of axillary lymph node dielectric properties for microwave imaging", Med. Phys., vol. 48, no. 10, pp. 5974–5990, 2021.

[2] S. Gabriel, R. W. Lau, and C. Gabriel, "The dielectric properties of biological tissues: III Parametric models for the dielectric spectrum of tissues", Phys. Med. Biol., vol. 41, no. 11, pp. 2271–2293, 1996.

[3] M. Lazebnik et al., "A large-scale study of the ultrawideband microwave dielectric properties of normal breast tissue obtained from reduction surgeries", Phys. Med. Biol., vol. 52, no. 10, pp. 2637–3656, 2007.