# Математический анализ

## ДЗ 20

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**Группа БПМИ248**

## Задание 17.6

$$\int x\sin(x^2)dx = \frac{1}{2}\int \sin(x^2)dx^2 = -\frac{1}{2}\cos(x^2) + C$$

$$\int \frac{e^x + e^{2x}}{1-e^x}dx = \int \frac{e^x(1+e^x)}{1-e^x}dx = |u = 1-e^x, du = -e^x dx| = \int \frac{u-2}{u}du = \int 1 - \frac{2}{u}du = u - 2\ln|u| + C =$$

$$= 1 - e^x - 2\ln|1-e^x| + C$$

$$\int \frac{dx}{x(\ln x + 5)} = \int \frac{1}{\ln x + 5}d\ln x = \ln|\ln x + 5| + C$$

$$\int \frac{dx}{\cos x} = \int \frac{\cos x}{1-\sin^2 x}dx = |u = \sin x, du = \cos x dx| = \int \frac{1}{1-u^2}du = \frac{1}{2}\ln\left|\frac{1+u}{1-u}\right| + C = \frac{1}{2}\ln\left|\frac{1+\sin x}{1-\sin x}\right| + C$$

$$\int \frac{\sin 2x}{\sqrt{1-4\sin^2 x}}dx = -\int \frac{1}{\sqrt{1-(2\sin x)^2}}d\cos 2x = -\int \frac{1}{\sqrt{2\cos 2x - 1}}d\cos 2x = -\frac{1}{2}\sqrt{2\cos 2x - 1} + C$$

$$\int \sin^7 x dx = \int \left(1-\cos^2 x\right)^3 \sin x = -\int \left(1-t^2\right)^3 dt = \int t^6 - 3t^4 + 3t^2 - 1dt = \frac{1}{7}t^7 - \frac{3}{5}t^5 + t^3 - t + C =$$

$$= \frac{1}{7}\cos^7 x - \frac{3}{5}\cos^5 x + \cos^3 x - \cos x + C$$

$$\int \frac{dx}{\cos x + \sin x} = \int \frac{\cos x - \sin x}{\cos^2 x - \sin^2 x}dx = \int \frac{\cos x}{\cos 2x}dx - \int \frac{\sin x}{\cos 2x}dx = \frac{1}{\sqrt{2}}\int \frac{1}{1-2\sin^2 x}d\sqrt{2}\sin x$$

$$+\frac{1}{\sqrt{2}}\int \frac{1}{2\cos^2 x - 1}d\sqrt{2}\cos x = \frac{1}{2\sqrt{2}}\ln\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| + \frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{2}\cos x - 1}{\sqrt{2}\cos x + 1}\right| + C$$

$$\int x^2\sqrt{1-x^2}dx = |x = \sin t, dx = \cos t dt| = \int \sin^2 t\cos^2 t dt = \frac{1}{4}\int \sin^2 2t dt = \frac{1}{8}\int (1-\cos 4t)dt =$$

$$= \frac{1}{8}\left(t - \frac{\sin 4t}{4}\right) + C = \frac{1}{8}t - \frac{\sin 4t}{32} + C = \frac{1}{8}\arcsin x - \frac{\sin(4\arcsin x)}{32} + C$$

$$\int \frac{dx}{(a^2-x^2)^{\frac{3}{2}}} = |x = a\sin t, dx = a\cos t dt| = \int \frac{a\cos t dt}{(a^2\cos^2 t)^{\frac{3}{2}}} = \int \frac{a\cos t dt}{a^3\cos^3 t} = \frac{1}{a^2}\int \frac{1}{\cos^2 t}dt =$$

$$= \frac{1}{a^2}\operatorname{tg} t + C = \frac{\operatorname{tg}\left(\arcsin\frac{x}{a}\right)}{a^2} + C$$

## Задание 17.7

$$\int \arctan x dx = x \arctan x - \int x d \arctan x = x \arctan x - \int \frac{x}{1+x^2} dx = x \arctan x - \frac{1}{2} \ln\left|1+x^2\right| + C$$

$$\int \ln^2 x dx = x \ln^2 x - 2 \int \ln x dx = x \ln^2 x - 2(x \ln x - x) + C$$

$$\int x^2 \ln(1+x) dx = |u = \ln(1+x), dv = x^2 dx| = \ln(1+x) \frac{x^3}{3} - \frac{1}{3} \int \frac{x^3}{1+x} dx =$$

$$\left( \int \frac{x^3}{1+x} dx = |t = 1+x| = \int \frac{(t-1)^3}{t} dt = \int \frac{t^3 - 3t^2 + 3t - 1}{t} dt = \int t^2 - 3t + 3 - \frac{1}{t} dt = \right.$$

$$\left. = \frac{t^3}{3} - \frac{3}{2} t^2 + 3t - \ln|t| + C = \frac{(1+x)^3}{3} - \frac{3}{2}(1+x)^2 + 3(1+x) - \ln|1+x| + C \right) =$$

$$= \ln(1+x) \frac{x^3}{3} - \frac{1}{3} \left( \frac{(1+x)^3}{3} - \frac{3}{2}(1+x)^2 + 3(1+x) - \ln|1+x| \right) + C$$

$$\int \sin(\ln x) dx = x \sin(\ln x) - \int \cos(\ln x) dx = x \sin(\ln x) - \left( x \cos(\ln x) + \int \sin(\ln x) \right)$$

$$2 \int \sin(\ln x) dx = x \sin(\ln x) - x \cos(\ln x) + C$$

$$\int \sin(\ln x) dx = \frac{x \sin(\ln x) - x \cos(\ln x)}{2} + C$$

$$\int \sqrt{a^2 + x^2} dx = x\sqrt{a^2 + x^2} - \int \frac{x^2}{\sqrt{a^2 + x^2}} dx = x\sqrt{a^2 + x^2} - \int \sqrt{a^2 + x^2} - \frac{a^2}{\sqrt{a^2 + x^2}} dx =$$

$$= x\sqrt{a^2 + x^2} - \left( \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2} \ln\left|x + \sqrt{a^2 + x^2}\right| \right) + a^2 \ln\left|x + \sqrt{a^2 + x^2}\right| + C$$

$$\int e^{ax} \cos(bx) dx = |u = e^{ax}, dv = \cos(bx) dx| = e^{ax} \frac{\sin(bx)}{b} - a \int \frac{\sin(bx)}{b} e^{ax} dx$$

## Задание 18.5

$$\int \frac{dx}{x(x+1)(x+2)}dx = \int \frac{\frac{1}{2}}{x} - \frac{1}{x+1} + \frac{\frac{1}{2}}{x+2}dx = \frac{1}{2}\ln|x| - \ln|x+1| + \frac{1}{2}\ln|x+2| + C$$

$$\int \frac{x^2+5x+4}{x^4+5x^2+4}dx$$

$$\frac{x^2+5x+4}{x^4+5x^2+4} = \frac{ax+b}{x^2+1} + \frac{cx+d}{x^2+4}$$

$$x^2+5x+4 = (a+c)x^3 + (b+d)x^2 + (4a+c)x + (4b+d)$$

$$\begin{cases} a+c=0 \\ b+d=1 \\ 4a+c=5 \\ 4b+d=4 \end{cases} \iff (a,b,c,d) = \left(\frac{5}{3}, 1, -\frac{5}{3}, 0\right)$$

$$\Rightarrow \frac{x^2+5x+4}{x^4+5x^2+4} = \frac{\frac{5}{3}x+1}{x^2+1} + \frac{-\frac{5}{3}x}{x^2+4}$$

$$\int \frac{x^2+5x+4}{x^4+5x^2+4}dx = \int \frac{\frac{5}{3}x+1}{x^2+1} + \frac{-\frac{5}{3}x}{x^2+4}dx = \frac{5}{3}\int \frac{x}{x^2+1}dx + \int \frac{1}{x^2+1}dx - \frac{5}{3}\int \frac{x}{x^2+4}dx =$$

$$= \frac{5}{6}\ln\left|x^2+1\right| + \arctan x - \frac{5}{6}\ln\left|x^2+4\right| + C$$

$$\int \frac{x^2+3x-2}{(x-1)(x^2+x+1)}dx$$

$$\frac{x^2+3x-2}{(x-1)(x^2+x+1)} = \frac{a}{x-1} + \frac{bx+c}{x^2+x+1}$$

$$x^2+3x-2 = (a+b)x^2 + (a-b+c)x + (a-c)$$

$$\begin{cases} a+b=1 \\ a-b+c=3 \\ a-c=-2 \end{cases} \iff (a,b,c) = \left(\frac{2}{3}, \frac{1}{3}, \frac{8}{3}\right)$$

$$\int \frac{x^2+3x-2}{(x-1)(x^2+x+1)}dx = \frac{2}{3}\int \frac{1}{x-1}dx + \frac{1}{3}\int \frac{x+8}{x^2+x+1}dx = \frac{2}{3}\ln|x-1| + \frac{1}{3}\left(\int \frac{x+\frac{1}{2}}{x^2+x+1}dx + \right.$$

$$\left. + \frac{15}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}dx\right) = \frac{2}{3}\ln|x-1| + \frac{1}{3}\left(\frac{1}{2}\ln\left|x^2+x+1\right| + \frac{2}{\sqrt{3}}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)\right) + C$$

## Задание 18.6

$$\int \frac{dx}{3+\sin x}dx = |t = \operatorname{tg}\frac{x}{2}, \sin x = \frac{2t}{1+t^2}, dx = \frac{2dt}{1+t^2}| = \int \frac{2dt}{3t^2+2t+3} =$$

$$= \frac{2}{3}\int \frac{dt}{\left(t+\frac{1}{3}\right)^2+\frac{8}{9}} = \frac{1}{\sqrt{2}}\arctan\left(\frac{3t+1}{2\sqrt{2}}\right)+C = \frac{1}{\sqrt{2}}\arctan\left(\frac{3\operatorname{tg}\frac{x}{2}+1}{2\sqrt{2}}\right)+C$$

$$\int \frac{dx}{2\sin x+3\cos x+5} = |t = \operatorname{tg}\frac{x}{2}, \sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}, dx = \frac{2dt}{1+t^2}| =$$

$$= \int \frac{dt}{t^2+2t+4} = \int \frac{dt}{(t+1)^2+3} = \frac{1}{\sqrt{3}}\arctan\left(\frac{t+1}{\sqrt{3}}\right)+C = \frac{1}{\sqrt{3}}\arctan\left(\frac{\operatorname{tg}\frac{x}{2}+1}{\sqrt{3}}\right)+C$$

$$\int \frac{dx}{2\sin^2 x+3\cos^2 x} = \int \frac{\frac{1}{\cos^2 x}dx}{2\operatorname{tg}^2 x+3} = |t = \operatorname{tg} x, dt = \frac{1}{\cos^2 x}dx| = \frac{1}{2}\int \frac{dt}{t^2+\frac{3}{2}} =$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{\frac{3}{2}}}\arctan\left(\frac{t}{\sqrt{\frac{3}{2}}}\right)+C = \frac{1}{\sqrt{6}}\arctan\left(\frac{\sqrt{2}\operatorname{tg} x}{\sqrt{3}}\right)+C$$

## Задание 18.7

$$I_n = \int \frac{dx}{\sin^n x} = \int \frac{1}{\sin^{n-2} x} \cdot \frac{1}{\sin^2 x} dx = |u = \frac{1}{\sin^{n-2} x}, dv = \frac{1}{\sin^2 x} dx| = \left(\frac{1}{\sin^{n-2} x}(-\operatorname{ctg} x)\right) -$$

$$- \int (-\operatorname{ctg} x) \frac{-(n-2)\cos x}{\sin^{n-1} x} dx = -\frac{\operatorname{ctg} x}{\sin^{n-2} x} - (n-2) \int \frac{\cos^2 x}{\sin^n x} =$$

$$= -\frac{\operatorname{ctg} x}{\sin^{n-2} x} - (n-2) \int \frac{1 - \sin^2 x}{\sin^n x} = -\frac{\operatorname{ctg} x}{\sin^{n-2} x} - (n-2) \left( \int \frac{1}{\sin^n x} dx - \int \frac{1}{\sin^{n-2} x} dx \right) =$$

$$-\frac{\operatorname{ctg} x}{\sin^{n-2} x} - (n-2)(I_n - I_{n-2}) \Rightarrow I_n = \frac{1}{n-1} \left( -\frac{\operatorname{ctg} x}{\sin^{n-2} x} + (n-2) I_{n-2} \right)$$