

$$X_{n+1} = X_n + V_n \Delta t + \frac{1}{2} a_n \Delta t^2$$

$$V_{n+1} = V_n + \frac{1}{2} (a_n + a_{n+1}) \Delta t$$

$$\frac{\partial X_{n+1}}{\partial X_n} = 1$$

$$\frac{\partial X_{n+1}}{\partial V_n} = \Delta t$$

$$\frac{\partial V_{n+1}}{\partial X_n} = 0$$

$$\frac{\partial V_{n+1}}{\partial V_n} = 1 + \frac{\Delta t}{2} \frac{\partial a_{n+1}}{\partial V_n}$$

$$\frac{\partial a_{n+1}}{\partial V_n} = - \frac{\partial^2 V}{\partial X_{n+1} \partial V_{n+1}}$$

$$J = \frac{\partial X_{n+1}}{\partial X_n} \frac{\partial V_{n+1}}{\partial V_n} - \frac{\partial X_{n+1}}{\partial V_n} \frac{\partial V_{n+1}}{\partial X_n}$$

$$= 1 \left(1 + \frac{\Delta t}{2} \frac{\partial a_{n+1}}{\partial V_n} \right) - \Delta t \cdot 0$$

$$= 1 - \frac{\Delta t}{2} \frac{\partial^2 V}{\partial X_{n+1} \partial V_{n+1}} = 1$$