$$y = \left(\frac{L}{2}\right)\cos\theta$$

$$\frac{dy}{dt} = \frac{L}{2}\frac{d\theta}{dt}\operatorname{sen}\theta$$

centro masas

$$m\ddot{a} = N - mg$$

$$-m\frac{L}{2}\cos\theta\left(\frac{d\theta}{dt}\right)^2 - m\frac{L}{2}\operatorname{sen}\theta\frac{d^2\theta}{dt^2} = N - mg$$

$$I_c\frac{d^2\theta}{dt^2} = N\frac{L}{2}\operatorname{sen}\theta$$

$$I_c = \frac{1}{12}mL^2$$

$$\frac{1}{12}mL^2\ddot{\theta} = N\frac{L}{2}\operatorname{sen}\theta$$

$$\frac{1}{6}\frac{mL^2}{\operatorname{sen}\theta}\ddot{\theta} = N$$

$$-m\frac{L}{2}\cos\theta\left(\frac{d\theta}{dt}\right)^2 - m\frac{L}{2}\operatorname{sen}\theta\,\ddot{\theta} = \frac{1}{6}\frac{mL^2}{\operatorname{sen}\theta} - mg$$

$$\frac{d\theta}{dt} = \dot{\theta}$$

$$-m\frac{L}{2}\cos\theta\,\dot{\theta}^2 - m\frac{L}{2}\sin\theta\,\ddot{\theta} = \frac{1}{6}m\frac{L^2}{\sin\theta} - mg$$

despejando para

$$\ddot{\theta} = \frac{(2g/L - \dot{\theta}^2\cos\theta)\sin\theta}{1/3 + \sin^2\theta}$$