

$$3) \quad x^3 y' = x^4 y^2 - 2x^2 y - 1$$

$$\text{Substituiere } y_1 = x^{-2}$$

$$y_1' = -2x^{-3} = -2x^{-3}$$

$$x^3 (-2x^{-3}) = x^4 (x^{-2})^2 - 2x^2 x^{-2} - 1$$

$$-2 = 1 - 2 - 1$$

$$-2 = -2$$

$$P_0(x) \frac{dy}{dx} + P_1(x) y = Q(x) y^2 + R(x)$$

Change of variable: $y = y_1 + u^{-1}$

$$y = y_1 + u^{-1}$$

$$y = x^{-2} + u^{-1}$$

$$y' = -2x^{-3} - u^{-2} u'$$

$$x^3 (-2x^{-3} - u^{-2} u') = x^4 (x^{-2} + u^{-1})^2 - 2x^2 (x^{-2} + u^{-1}) - 1$$

$$-2 - x^3 u^{-2} u' = x^4 [x^{-4} + 2x^{-2} u^{-1} + u^{-2}] - 2x^0 - 2x^2 u^{-1} - 1$$

$$-2 - x^3 u^{-2} u' = \underbrace{x^0}_{=1} + 2x^2 u^{-1} + x^4 u^{-2} - 2 - 2x^2 u^{-1} - 1$$

$$\therefore -x^3 u^{-2} u' = x^4 u^{-2}$$

$$u' = -x$$

$$\Rightarrow \frac{du}{dx} = -x$$

$$u = \int -x dx = -\frac{x^2}{2} + C$$

$$y = x^{-2} + \left(-\frac{x^2}{2} + C\right)^{-1}$$