$$C_3 = \frac{-1}{3!\,0!}\int_0^1 (u+0)(u+1)(u+2)\,du = \frac{-1}{6}\int_0^1 (u^3+3u^2+2u)\,du = \frac{1}{6}\left[\frac{1}{4}+\frac{3}{3}+\frac{2}{2}\right] = \frac{9}{24}$$

$$\Rightarrow y_{n+4} = y_{n+3} + \int_{t_{n+3}}^{t_{n+4}} \sum_{j=0}^{3} \frac{(-1)^{3-j-1} f(t_{n+j})}{j!\,(4-j-1)!\,h^3} \prod_{\substack{i=0\\ j\neq i}}^{3} (t-t_{n+i})$$

$$= y_{n+3} + h\left(\frac{55}{24} f(t_{n+3}) - \frac{59}{24} f(t_{n+2}) + \frac{37}{24} f(t_{n+1}) - \frac{9}{24} f(t_n)\right)$$

b) Un método de Adams-Bashforth puede alcanzar el orden $s+1$
los coeficientes se hallan:

$$C_{s-j} = \frac{(-1)^j}{j!\,(s-j)!}\int_0^1 \prod_{\substack{i=0\\ i\neq j}}^{s} (u+i-1)\,du, \quad \text{para } j = 0, \dots, s$$

Para orden 3

$$C_0 = \frac{(-1)^0}{0!\,(2)!}\int_0^1 u(u+1) = \frac{1}{2}\int_0^1 [u^2+u]\,du = \frac{1}{2}\left[\frac{1}{3}+\frac{1}{2}\right] = \frac{5}{12}$$

$$C_1 = \frac{-1}{1!\,1!}\int_0^1 (u-1)(u+1) = -1\int_0^1 (u^2-1)\,du = -1\left[\frac{1}{3}-1\right] = \frac{2}{3} = \frac{8}{12}$$

$$C_2 = \frac{(-1)^2}{2!\,0!}\int_0^1 (u-1)\,u = \frac{1}{2}\int_0^1 (u^2-u) = \frac{1}{2}\left(\frac{1}{3}-\frac{1}{2}\right) = \frac{-1}{12}$$

$$y_{n+2} = y_{n+1} + \int_{t_{n+1}}^{t_{n+2}} \sum_{j=0}^{2} \frac{(-1)^{3-j} f(t_{n+j})}{j!\,(2-j)!\,h^2} \prod_{\substack{i=0\\ i\neq j}}^{2} (t_n - t_{n+i})$$

$$y_{n+2} = y_{n+1} + h\left(\frac{5}{12} f(t_{n+2}) + \frac{8}{12} f(t_{n+1}) - \frac{1}{12} f(t_n)\right)$$

Para orden 4

$$C_0 = \frac{(-1)^0}{0!\,3!}\int_0^1 u(u+1)(u+2) = \frac{1}{6}\int (u^3+3u^2+2u)\,du = \frac{1}{6}\left[\frac{1}{4}+\frac{3}{3}+\frac{2}{2}\right] = \frac{9}{24}$$

$$C_1 = \frac{(-1)}{1!\,2!}\int_0^1 (u-1)(u+1)(u+2) = \frac{-1}{2}\int_0^1 (u^3+2u^2-u-2)\,du = \frac{1}{2}\left[\frac{1}{4}+\frac{2}{3}-\frac{1}{2}-2\right] = \frac{23}{24}$$

$$C_2 = \frac{1}{2!\,1!}\int_0^1 (u-1)\,u(u+2) = \frac{1}{2}\int_0^1 u^3+u^2-2u = \frac{1}{2}\left[\frac{1}{4}+\frac{1}{3}-\frac{2}{2}\right] = -\frac{5}{24}$$

$$C_3 = \frac{1}{3!\,0!}\int_0^1 (u-1)\,u(u+1) = \frac{1}{6}\int_0^1 (u^3-u)\,du = \frac{1}{6}\left[\frac{1}{4}-\frac{1}{2}\right] = \frac{-1}{24}$$

$$\Rightarrow y_{n+3} = y_{n+2} + \int_{t_{n+1}}^{t_{n+3}} \sum_{j=0}^{3} \frac{(-1)^{4-1} f(t_{n+j})}{j!\,(2-j)!\,h^3} \prod_{\substack{i=0\\ i\neq j}}^{3} (t_n - t_{n+i})$$

$$y_{n+3} = y_{n+2} + h\left[\frac{9}{24} f(t_{n+3}) + \frac{23}{24} f(t_{n+2}) - \frac{5}{24} f(t_{n+1}) - \frac{1}{24} f(t_n)\right]$$

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2.

a) $P(t) = \sum_{j=0}^{s-1} \frac{(-1)^{s-j-1} f(t_{n+j})}{j!(s-j-1)h^{s-1}} \prod_{\substack{i=0\\ i\neq j}}^{s-1} (t - t_{n+i})$ $\qquad y' = f(t,y) = p(t)$

$\Rightarrow y_{n+s} = y_{n+s-1} + \int_{t_{n+s-1}}^{t_{n+s}} p(t)\,dt$ $\Rightarrow$ Así los coeficientes C_j se hallan a través de:

$$C_{s-j-1} = \frac{(-1)^j}{j!(s-j-1)!}\int_0^1 \prod_{\substack{i=0\\ i\neq j}}^{s-1}(u+i)\,du \quad \text{para } j=0,\dots,s-1$$

$\Rightarrow$ Para orden 3

$$C_0 = \frac{(-1)^0}{0!(3-0-1)!}\int_0^1 \prod_{\substack{i=0\\ i\neq 0}}^{3-1}(u+i)\,du = \frac{1}{(2)!}\int_0^1 (u+1)(u+2)\,du$$

$$= \frac12\int_0^1(u^2+2u+u+2)\,du = \frac12\left[\frac{u^3}{3}+\frac{3u^2}{2}+2u\right]_0^1$$

$$= \frac12\left[\frac{23}{6}\right] = \frac{23}{12}$$

$$C_1 = \frac{-1}{1!(1)!}\int_0^1(u+0)(u+2)\,du = -1\int_0^1\left[u^2+2u\right]du = -\left[\frac13+\frac22\right] = \frac43 = \frac{16}{12}$$

$$C_2 = \frac{1}{2!0!}\int_0^1(u+0)(u+1)\,du = \frac12\int_0^1\left[u^2+u\right]du = \frac12\left[\frac13+\frac12\right] = \frac12\left[\frac56\right] = \frac{5}{12}$$

$$\Rightarrow y_{n+3} = y_{n+2} + \int_{t_{n+2}}^{t_{n+3}} \sum_{j=0}^{2} \frac{(-1)^{3-j-1} f(t_{n+j})}{j!(s-j-1)h^2}\prod_{i=0}^{2}(t-t_{n+i})$$

$$y_{n+3} = y_{n+2} + h\left(\frac{23}{12}f(t_{n+2}) - \frac{16}{12}f(t_{n+1}) + \frac{5}{12}f(t_n)\right)$$

b) Para orden 4

$$C_0 = \frac{1}{0!(4-0-1)!}\int_0^1\prod_{\substack{i=0\\ i\neq 0}}^{4-1}(u+i)\,du = \frac{1}{3!}\int_0^1(u+1)(u+2)(u+3)\,du = \frac16\int_0^1(u^3+6u^2+11u+6)\,du$$

$$= \frac16\left[\frac{u^4}{4}+\frac{6u^3}{3}+\frac{11u^2}{2}+6u\right]_0^1 = \frac16\left[\frac14+\frac63+\frac{11}{2}+6\right] = \frac16\cdot\frac{55}{4} = \frac{55}{24}$$

$$C_1 = \frac{-1}{1!2!}\int_0^1(u+0)(u+2)(u+3)\,du = -\frac12\int_0^1(u^3+5u^2+6u)\,du = -\frac12\left[\frac14+\frac53+\frac62\right] = \frac{-59}{24}$$

$$C_2 = \frac{1}{2!1!}\int_0^1(u+0)(u+1)(u+3)\,du = \frac12\int_0^1\left[u^3+4u^2+3u\right]du = \frac12\left[\frac14+\frac43+\frac32\right] = \frac{37}{24}$$