DD MM AA

a) $\vec{v}_{n+1} = \vec{v}_n + \frac{h}{2}\left(\vec{a}(\vec{r}_n) + \vec{a}(r_{n+1})\right).$

$a_{n+1} = a_n + h\frac{\partial a}{\partial x} + \frac{h^2}{2}\frac{\partial^2 a}{\partial^2 x^2} + O(h^3)$

$x_{n+1} = 2x_n - x_{n-1} + h^2 a_n \qquad a_n = \frac{\partial^2 x}{\partial t^2}$

Error en el paso de tiempo n+1 = sln exacta - sln aprox, para n+1.

Expresamos x_{n+1} y x_{n-1} alrededor de la sln exacta

$\epsilon_n = x_n - x_{exacta,n}$

$x_{n+1} = x_{exacta,n} + \epsilon_n + hv_{exacta,n} + \frac{h^2}{2}a_{exacta,n} + O(h^3)$

$x_{n-1} = x_{exacta,n} - \epsilon_{n-1} + hv_{exacta,n} + \frac{h^2}{2}a_{exacta,n} + O(h^3)$

$x_{n-1} = x_{exacta,n} - \epsilon_{n-1} - hv_{exacta,n} + \frac{h^2}{2}a_{exacta,n} + O(h^3)$

Sustituyendo en la definición de valor

$x_{exacta,n} + \epsilon_{n+1} = 2(x_{exacta,n} + \epsilon_n) - (x_{exacta,n} - \epsilon_{n-1}) + h^2 a_n$

$x_{exacta,n} - 2x_{exacta,n} + x_{exacta,n} = -\epsilon_{n+1} + 2\epsilon_n + \epsilon_{n-1} + h^2 a_n = 0$

$\epsilon_{n+1} = 2\epsilon_n - \epsilon_{n-1} + h^2 a_n$ Dividiendo entre h^2: $\frac{\epsilon_{n+1}}{h^2} - \frac{2\epsilon_n}{h^2} + \frac{\epsilon_{n-1}}{h^2} = a_n$

$a'_n = \frac{\partial a}{\partial x}, \Rightarrow \left(\frac{\epsilon_{n+1}}{h^2} - 2\frac{\epsilon_n}{h^2} + \frac{\epsilon_{n-1}}{h^2} = a'_n\epsilon_n\right.$ multiplicando x h^2

$\epsilon_{n+1} - (2 + h^2 a'_n)\epsilon_n + \epsilon_{n-1} = 0$

b) Para el oscilador armónico clásico: $a = -w^2 x$, $R = \frac{h^2 w^2}{2}$

$a' = \frac{\partial a}{\partial x} = -w^2$

$\epsilon_{n+1} - (2 + h^2(-w^2))\epsilon_n + \epsilon_{n-1} = 0$

$\Rightarrow \epsilon_{n+1} - 2\left(1 - \frac{h^2 w^2}{2}\right)\epsilon_n + \epsilon_{n-1} = 0 \Rightarrow \epsilon_{n+1} - 2(1-R)\epsilon_n + \epsilon_{n-1} = 0$

c) $\epsilon_n = \epsilon_0\lambda^n \Rightarrow (\epsilon_0\lambda^{n+1} - 2(1-R)\epsilon_0\lambda^n + \epsilon_0\lambda^{n-1} = 0) \div \epsilon_0\lambda^{n-1}$

$\lambda^2 - (2-2R)\lambda + 1 = 0$

$\lambda = \frac{-(-(2-2R)) \pm \sqrt{(2-2R)^2 - 4(1)(1)}}{2(1)} = \frac{2(1-R) \pm \sqrt{4 - 4R + 4R^2 - 4}}{2}$

$\lambda = (1-R) \pm \sqrt{1 - R + R^2 - 1} = (1-R) \pm \sqrt{(R^2 - R)}$