Data, Environment and Society: Lecture 11: Multiple Regression

Instructor: Duncan Callaway
GSI: Seigi Karasaki

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Announcements

Today

- x

Reading

- x

Mid term and final project

Midterm: What to expect.

Final project: What you'll be doing.

Review

Confidence intervals, redux

- Cover distributions – which one are we describing with the standard error?
- Derive standard errors?

What if the confidence interval contains zero?

For example, if

$$-10.3 < \beta_1 < 24.8?$$

This implies there is more than a remote chance that there is no significant relationship between the dependent and independent variables.

p-values

p-values measure the probability that the estimated coefficients arose by chance from a data generating process that actually has *no* relationship between the inputs and outputs.

p = 0.05 implies a 5% chance that the true parameter value is *zero*.

A small p-value indicates that it is unlikely to observe such a substantial association between the predictor and the response due to chance.

p-hacking?

What's wrong with these practices:

- Stop collecting data once $p < 0.05$
- Analyze many independent variables, but only report those for which $p < 0.05$
- Collect and analyze many data samples, but only report those with $p < 0.05$
- Exclude participants to get $p < 0.05$.
- Transform the data to get $p < 0.05$.

(credit to Leif Nelson, UCB Haas)

The trouble with p-hacking...

...is that by looking for the data set and the models that give low p-values, you could just be looking for those 5% “chances” where the real relationship is non-existent.

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Some estimates suggest that this practice leads to false positive rates of 61%!

Model accuracy: R^2

$$R^2 = \frac{TSS - RSS}{TSS} = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2 - \sum_{i=1}^{n} e_i^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} = 1 - \frac{\sum_{i=1}^{n} e_i^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

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R^2 measures the fraction of variation in the dependent variable that is captured by the model.

Multivariate regression

This is exactly the same process as single (independent) variable regression. Parameters solutions can be found by

- Gradient search
- Normal equations
- Setting partial derivatives of MSE to zero and solving – but now for $\beta_0, \beta_1, \beta_2, \ldots, \beta_d$ (*d* is the number of features, a.k.a. independent variables).

The mechanics of finding parameters is easy. The real challenge is: Which features to include?

Model selection

The challenge: Don’t include variables in your model that lead to over-fit.
Two basic methods:

- Computationally heavy and theoretically robust to
- Easy to implement (no need for significant computing)

The easy-to-implement method

Akaike information criterion (AIC):

1. Construct all the models you have time for using *all* the data to train the models.
2. Then, choose the model with the lowest AIC, where

$$\text{AIC} = \frac{1}{n\hat{\sigma}^2}(\text{RSS} + 2d\hat{\sigma}^2) = \frac{1}{\hat{\sigma}^2}\left(\frac{\text{RSS}}{n}\right) + \frac{2d}{n}$$

As you can see, AIC “penalizes” models with a high value of *d*.

What the heck is AIC?

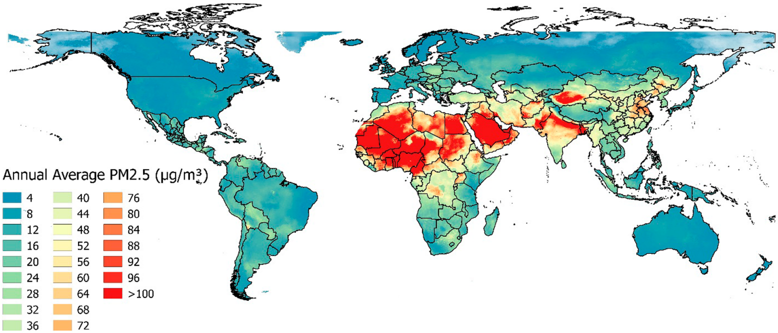
It actually has a rigorous theoretical underpinning. Understanding the derivation requires background in information theory and more time than we have here.

But:

- It gives unbiased estimate of the MSE you'd get if you *did* use a test data set (as long as the errors are Gaussian)
- It's ok to just work with the intuition that choosing models that minimize AIC is analogous to
  - choosing models that minimize MSE ...
  - plus a penalty for the number of features.

Prediction application: Land use regression

- Suppose we'd like to know pollutant concentrations at a fine spatial resolution
- We only have pollutant measurements at low resolution (coarse spatial scale)
- But we have other measurements at finer spatial resolution
- This is an ideal job for forecasting.
- But rather than forecast in *time* we will forecast in *space*.



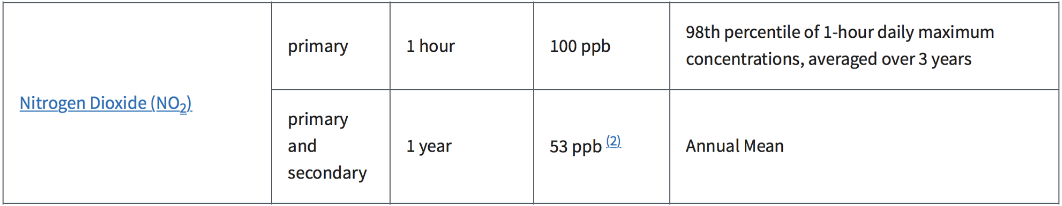
(From Shaddick *et al* ES&T 2018)

Nitrogen dioxide

NO_2:

- Direct product of fossil fuel combustion
- Used as an indicator for larger group of nitrogen oxides.
- Health impact: Contributes to development of and aggravates asthma
- Environmental impact: Haze, acid rain, nutrient pollution in coastal waters

EPA Regulates NO2:



Nitrogen Dioxide (NO_2)	primary	1 hour	100 ppb	98th percentile of 1-hour daily maximum concentrations, averaged over 3 years
	primary and secondary	1 year	53 ppb [(2)]	Annual Mean

Novotny *et al* setup

- NO_2 concentrations are known where monitors are present.
- But we don't have monitors everywhere
- Can we *predict* concentrations where monitors are absent?

"Remote sensing" data from satellites can be useful:

- Aurora satellite "Ozone Monitoring Instrument" provides tropospheric NO2 column abundance (units: ppb; Called "WRF+DOMINO" in data set we'll work with).

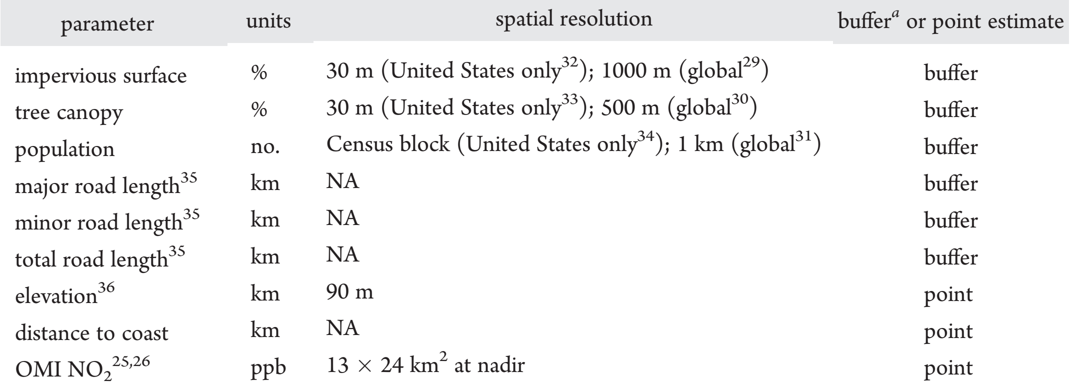
But!

- Measurements are for entire column of air above a location, not ground-level
- Spatial resolution is low

Land use regression for NO_2

Dependent variable: Hourly NO_2 concentrations from EPA sensors.

Independent variables to consider:



parameter	units	spatial resolution	buffer[a] or point estimate
impervious surface	%	30 m (United States only[32]); 1000 m (global[29])	buffer
tree canopy	%	30 m (United States only[33]); 500 m (global[30])	buffer
population	no.	Census block (United States only[34]); 1 km (global[31])	buffer
major road length[35]	km	NA	buffer
minor road length[35]	km	NA	buffer
total road length[35]	km	NA	buffer
elevation[36]	km	90 m	point
distance to coast	km	NA	point
OMI NO_2[25,26]	ppb	13×24 km^2 at nadir	point

Novotny *et al* Table 1.

Regressing against remote-sensed data only

Run this model.

Let’s include other predictors!

Supplemental

Important Questions for Multiple Linear Regression

- Is at least one of the predictors X1 , X2 , . . . , Xp useful in predicting the response?
  - cover only briefly
- Do all the predictors help to explain Y, or is only a subset of the predictors useful?
  - Review variable selection.
  - Cue attention to Marshall et al approach.
- How well does the model fit the data?
  - Return to question of Rsq
- Given a set of predictor values, what response value should we predict, and how accurate is our prediction?
  - Prediction intervals (contrast to confidence intervals)

Basic sketch of the Novotny et al paper:

“Stepwise multivariate regression”:

1. The independent variable most correlated with the dependent variable is added to the model first
2. Of the remaining variables, the one most correlated with model residuals is selected as the next independent variable
3. Step 2 repeats on each new model with the remaining variables.
    - Variables are not kept in the model if $p > 0.05$
    - Variables are not kept in the model if they are collinear with others (we’ll discuss this Tuesday).

Basic sketch for Novotny, ctd

Choosing the model:

- Model-building using a random sample of 90% of the monitoring data
- Tested the model's ability to predict the remaining 10%.
- Create 500 different random samples of training data – calculate R2, error, and bias for all 500 iterations.

Question: Why build the model with 90% of the data only?

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Question: Why build the model with 90% of the data only?

Answer: to avoid choosing a model that over-fits the data.

In HW9, we'll go through a process similar (and arguably superior) to this one.

In HW6 – next week – we'll use a different method, known as AIC. We'll go over that today.