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# Circumventing Dynamic Modeling: Evaluation of the Error-State Kalman Filter applied to Mobile Robot Localization*

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## Abstract

*The mobile robot localization problem is treated as a two-stage iterative estimation process. The attitude is estimated first and is then available for position estimation. The indirect (error state) form of the Kalman filter is developed for attitude estimation when applying gyro modeling. The main benefit of this choice is that complex dynamic modeling of the mobile robot and its interaction with the environment is avoided. The filter optimally combines the attitude rate information from the gyro and the absolute orientation measurements. The proposed implementation is independent of the structure of the vehicle or the morphology of the ground. The method can easily be transfered to another mobile platform provided it carries an equivalent set of sensors. The 2D case is studied in detail first. Results of extending the approach to the 3D case are presented. In both cases the results demonstrate the efficacy of the proposed method.*

## 1 Introduction

On July 4th 1997, the Mars Pathfinder mission deployed an autonomous micro-rover on the surface of Mars. Due to challenges such as round trip communication delay time, the robot was equipped to perform a high degree of autonomous goal seeking and hazard avoidance behavior. Future missions to Mars involve longer traverses of rovers to sites of scientific interest kilometers apart [11]. In order to autonomously navigate, such rovers need to know their exact position and orientation; i.e. to localize themselves. Localization is a key problem in autonomous mobile robotics. Different techniques have been developed to tackle this problem; these can be sorted into two main categories:

**Relative (local) localization** : consists of evaluating the position and the orientation through integration of information provided by encoders or inertial sensors with knowledge of the initial conditions.

**Absolute (global) localization** : is the technique that permits the vehicle to determine its position directly using navigation beacons, active or passive landmarks, map matching or a Global Positioning System (GPS).

Relative localization is also known as dead-reckoning and uses two sets of sensors: odometric sensors and inertial navigation systems (INS). In most mobile robots, odometry is implemented by means of optical encoders that monitor the wheel revolutions and steering angles of the wheels. Using simple geometry (a kinematic model of the vehicle), the encoder data are used to compute the position of the vehicle relative to a known starting position. INS are widely used in aviation and lately in outdoor robots [1]. They consist of gyroscopes and accelerometers that provide angular rate and velocity rate information. By integrating this information, the position and orientation of the vehicle is calculated. Dead-reckoning is widely used because of its simplicity but is unsuitable for long distances due to errors associated with noise, slip and modeling inaccuracies.

For the above reasons, there is error in the calculation of the vehicle's position and orientation which generally grows unbounded with time. Substantial improvement is provided by applying Kalman filtering

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techniques [9] which have been used successfully in position estimation problems for the last four decades.

The next step to limit the error growth relies on absolute localization techniques using global sensor measurements or maps of the environment. This can drastically increase the accuracy of the estimate and keep the associated uncertainty within certain bounds.

The **key contributions** of this paper are:

- We list the drawbacks related to applying dynamic modeling for attitude estimation in the case of outdoor mobile robots. The cumbersome modeling of the specific vehicle and its interaction with a dynamic environment is avoided by selecting gyro modeling instead. The gyro signal appears in the system (instead of the measurement) equations and thus the formulation of the problem requires an Indirect (error-state) Kalman filter approach.

- The equations for the linear estimator for the 2D case are derived in detail and the resulting Indirect Kalman filter is studied as a signal processing unit that suppresses the effect of bias and optimally combines the gyro signal with the signal from the absolute orientation sensor[1]. The results for the 3D estimator are also shown.

In the next section we discuss related previous work; in section 3 we discuss the rationale behind dynamic model replacement; in 4 we discuss various forms of the Kalman filter concluding in favor of the Indirect form; in section 5 we discuss the case of planar motion with a single bias compensated gyro as well as extensions to the 3D case. We conclude the paper with a summary and a discussion of future work.

## 2 Previous Work

In order to deal with systematic errors in indoor applications, a calibration technique called the UMBmark test is given in [3]. In [4] the same authors discuss a technique they call gyrodometry, which uses odometry data most of the time, while substituting gyro data only during brief instances (e.g. when the vehicle goes over a bump) during which gyro and odometry data differ drastically. This way the system is kept largely free of the drift associated with the gyroscope.

A complementary Kalman filter [5] is used in [8] to estimate the robot's attitude from the accelerometer signal during low frequency motion and the gyro signal during high frequency motion. The attitude information is then used to calculate a position increment.

In [1] the authors use a low cost INS system (3 gyroscopes, and a triaxial accelerometer) and 2 tilt sensors. Their approach is to incorporate in the system *a priori* information about the error characteristics of the inertial sensors and to use this directly in an Extended Kalman Filter (EKF) to estimate position.

Examples of absolute localization include [14] in which the basic localization algorithm is formalized as a vehicle-tracking problem, employing an EKF to match beacon observations to a map in order to maintain an estimate of the position of the mobile robot; [2] in which the authors use an EKF to fuse odometry and angular measurements of known landmarks and [25] in which a Bayesian approach is used to learn useful landmarks for localization.

Most of the above approaches limit themselves to the case of planar motion. In addition, their accuracy depends heavily on the presence of some form of an absolute positioning system. We propose an estimation algorithm that is capable of incorporating absolute position measurements but is also able to provide reliable estimates in the absence of externally provided positioning information. The focus of our method is on attitude estimation [2] using an Indirect Kalman filter that operates on the error state.

## 3 Dynamic Model Replacement

In our approach we avoid dynamic modeling. The reasons are as follows:

- The most elementary reason is that every time a modification is made to the robot (i.e. a mass changes, or a part is relocated, or dimensions are altered) the dynamic modeling has to be redone. The produced estimator is tailored for a specific structure. A slightly different vehicle would require a new estimator.

- A more practical reason is that dynamic modeling would require a very large number of states (consider for example Rocky 7, a Mars rover prototype, which has 6 wheels, 4 steering joints, 3 bogey joints). An estimator has to be practical as far as its computational needs are concerned. The size of the estimated state can have large computational demands with very little gain in precision.

- Dynamic modeling and the added complexity does not always produce the expected results. One ex-

[1] This measurement can be from a magnetic compass or a sun sensor in this two dimensional example. From now on we will assume that this sensor measures the absolute orientation of the vehicle.

[2] Updating the position based on the updated attitude is straightforward and will not be considered further.

ample is an attempt by Lefferts and Markley [12] to model the attitude dynamics of the NIMBUS-6 spacecraft, which indicated that dynamic modeling with elaborate torque models could still not give acceptable attitude determination accuracy. For this reason most attitude estimation applications in the aerospace domain use gyros in a dynamic model replacement mode [27].

- Modeling a mobile robot moving on rough terrain is more complicated than modeling a spacecraft essentially because of the interaction between the vehicle and the ground. The external forces on a spacecraft traveling on a ballistic trajectory in space, are precisely described by the law of gravitational forces. The interaction of a rover with the ground depends on many parameters and requires assumptions to be made (the point contact assumption is frequently used). The modeling of the vehicle-terrain dynamic effects as the wheel impact, wheel slippage, wheel sinkage, requires prior knowledge of the ground parameters (i.e. friction coefficients as a function of wheel slippage ratio, soil shear strength, elastic modulus etc). In addition, the lateral slippage is not observable and can not be accounted for in the dynamic model. Precise modeling of the motors is also required to obtain the real values of the torques acting on each of the wheels.

- Looking at the problem from another perspective, most vehicles have a suspension system whose purpose is to decouple the vehicle's motion from the terrain morphology. The "ideal" suspension system would support a motion of the vehicle that would imitate to some extent, the motion of a hovercraft. The robot then could move in a very smooth fashion. A motion like this is prone to be effectively estimated following methods that rely on the use of inertial navigation systems as a dynamic model replacement, and are independent of the particular terrain.

## 4 Forms of the Kalman Filter

As mentioned before, Kalman filtering has been widely used for localization. The kinds that usually appear in mobile robot applications are the linear Kalman filter and the Extended Kalman filter (EKF) forms of the full state Kalman filter. In this work we choose to use the error-state form of both the linear Kalman filter and EKF. In section 5 we derive the equations needed for such a formulation. In this section we examine the reasons to select the Indirect-feedback form over others commonly found in the robotics literature.

### 4.1 Indirect vs. Direct Kalman Filter

A very important aspect of the implementation of a Kalman filter in conjunction with inertial navigation systems (INS) is the use of the **indirect** instead of the direct form, also referred to as the **error state** and the total state formulation respectively [15]. As the name indicates, in the total state (direct) formulation, total states such as orientation are among the variables in the filter, and the measurements are INS outputs, such as from a gyro, and external source signals. In the error state (indirect) formulation, the *errors* in orientation are among the estimated variables, and each measurement presented to the filter is the difference between the INS and the external source data.

There are some serious drawbacks to the direct filter implementation. Being in the INS loop and using the total state representation, the filter would have to maintain explicit, accurate awareness of the vehicle's angular motion i.e. incorporate a dynamic model, as well as attempt to suppress noisy and erroneous data at a relatively high frequency.

In addition, the dynamics involved in the total state description of the filter include a high frequency component and are well described only by a non-linear model. The development of a Kalman filter is predicated upon an adequate linear system model, and such a total state model does not exist.

Another disadvantage of the direct filter design is that if the filter fails (as by a temporary computer failure) the entire navigation algorithm will fail. The INS is useless without the filter. From the reliability point of view it would be desirable to provide an emergency degraded performance mode in such a case of failure. The Indirect filter in case of a failure can continue to provide estimates by acting as an integrator on the INS data. The direct filter would be ideal for fault detection and identification purposes [22, 23].

The Indirect (error-state) Kalman filter estimates the errors in the navigation and attitude information using the difference between the INS and external sources of data. The INS itself is able to follow the high frequency motions of the vehicle very accurately, and there is no need to model these dynamics explicitly in the filter. Instead, the dynamics upon which the Indirect filter is based are a set of inertial system error propagation equations which are low frequency and very adequately represented as linear. Because the filter is out of the INS loop and is based on low frequency dynamics, its sampling rate can be much lower than that of the direct filter. In fact, an effective Indirect filter can be developed with a sample period (of the external source) of the order of minutes [15]. For these reasons, the error state formulation is used in essentially all terrestrial aided inertial navigation systems [24].

### 4.2 Feedforward vs. Feedback Indirect Kalman Filter

The basic difference between the feedforward and feedback Indirect Kalman filters is mainly in the way they handle the updated error estimate. In the first case the updated error estimate is fed forward to correct the current orientation estimate without updating the INS. In the feedback formulation the correction is actually fed back to the INS to correct its "new" starting point, i.e. the state that the integration for the new time step will start from. In a sense the difference between the feedforward and feedback forms is equivalent to the difference between the Linearized Kalman filter and the Extended Kalman filter [16]. In the second one the state propagation starts from the corrected (updated) state right after a measurement while in the Linearized filter the propagation continues at the state that the propagation has reached when the measurement appeared, thus ignoring the correction just computed. The Linearized Kalman filter and the feedforward Indirect Kalman filter are thus free to drift with unbounded errors.

## 5 Planar Motion with a Single Bias Compensated Gyro

The greatest difficulty in all attitude estimation approaches that use gyros, is the low frequency noise component, also referred to as bias or drift that violates the white noise assumption required for standard Kalman filtering. This problem has attracted the interest of many researchers since the early days of the space program [17, 10, 6]. Inclusion of the gyro noise model in a Kalman filter by suitably augmenting the state vector has the potential to provide estimates of the sensor bias when the observability requirement is satisfied. Early implementations of gyro noise models in Kalman filters can be found in [19, 18, 26].

As we have already mentioned, an estimate of $\theta$ (the vehicle yaw in the planar case) would imply the derivation of an equation of the form:

$$\ddot{\theta} = f(\tau, \dot{\theta}, \theta, v, \alpha, \vec{p}_{vehicle}, \vec{p}_{ground}) \qquad (1)$$

where $\tau$ is an actuator torque, $\dot{\theta}$ and $v$ are the angular and translational velocities respectively, and $\alpha$ is the linear acceleration. $\vec{p}_{vehicle}$ is the vehicle parameter vector and $\vec{p}_{ground}$ parameterizes the ground morphology, soil friction etc. In order to avoid developing such an equation for all the reasons mentioned in section 3 we develop a replacement for Equation 1. The replacement relates the gyro output signal to the bias and the angular velocity of the vehicle.

In our approach we use the simple and realistic model due to [7]. In this model the angular velocity $\omega = \dot{\theta}$ is related to the gyro output $\omega_m$ according to the equation:

$$\dot{\theta} = \omega_m + b + n_r \qquad (2)$$

where $b$ is the drift-rate bias and $n_r$ is the drift-rate noise. $n_r$ is assumed to be a Gaussian white-noise process with covariance $N_r$.

The drift-rate bias $b$ is not a static quantity but is driven by a second Gaussian white-noise process, the gyro drift-rate ramp noise $n_w$. Thus $\dot{b} = n_w$ with covariance $N_w$. The two noise processes are assumed to be uncorrelated.

As previously mentioned, we study the simple case of attitude estimation when the vehicle moves in a plane. It is assumed that only one gyro is used and that the absolute orientation information (the yaw) is provided directly by another sensor at frequent intervals.

### 5.1 The Feedback Indirect Kalman Filter Formulation

From Equation (2) the true angular velocity is:

$$\dot{\theta}_{true} = \omega_m + b_{true} + n_r \qquad (3)$$

where $\dot{b}_{true} = n_w$.

Rearranged in matrix form we have:

$$\frac{d}{dt}\begin{bmatrix} \theta_{true} \\ b_{true} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \theta_{true} \\ b_{true} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\omega_m + \begin{bmatrix} n_r \\ n_w \end{bmatrix} \qquad (4)$$

The absolute orientation measurement is:

$$z = \theta_m = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \theta_{true} \\ b_{true} \end{bmatrix} + n_\theta \qquad (5)$$

where $n_\theta$ is the noise for the sensor measuring the absolute orientation. The assumption is that $n_\theta$ is a Gaussian white-noise process with covariance $N_\theta$.

The term $\omega_m$ is like a driving (control) input to the system shown in Equation 4 and needs to be eliminated. This can be done in two ways. The first way would be to add it to the state and estimate it. This would imply developing an expression for $\dot{\omega}$ via Equation 1. The second option (chosen here) is to formulate the estimation algorithm as an Indirect Kalman filter. The orientation estimate obtained by integrating the gyro signal (assuming constant bias) is given by:

$$\dot{\theta}_i = \omega_m + b_i \tag{6}$$

where the rate of the estimated bias is $\dot{b}_i = 0$.

Thus, the state equations for the integrator are:

$$\frac{d}{dt}\begin{bmatrix} \theta_i \\ b_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \theta_i \\ b_i \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\omega_m \tag{7}$$

Subtracting equations (3) and (6) the error in orientation can be written as:

$$\Delta\dot{\theta} = \Delta b + n_r \tag{8}$$

and $\omega_m$ is now omitted. $\Delta\theta$ is the error in orientation and $\Delta b$ is the bias error. Subtracting the equations for $\dot{b}_{true}$ and $\dot{b}_i$ the bias error can be written as $\Delta\dot{b} = n_w$. These error propagation equations for the Indirect (error state) Kalman filter can be rearranged as:

$$\frac{d}{dt}\begin{bmatrix} \Delta\theta \\ \Delta b \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \Delta\theta \\ \Delta b \end{bmatrix} + \begin{bmatrix} n_r \\ n_w \end{bmatrix} \tag{9}$$

or in a more compact form as:

$$\frac{d}{dt}\Delta x = F\Delta x + n \tag{10}$$

We assume that the measurement provided to the Indirect Kalman filter is:

$$\Delta z = \theta_m - \theta_i = \theta_{true} + n_\theta - \theta_i = \Delta\theta + n_\theta \tag{11}$$

where $\theta_i$ is available through the gyro signal integration and $\theta_m$ is the absolute orientation measurement. This equation in matrix form becomes:

$$\Delta z = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \Delta\theta \\ \Delta b \end{bmatrix} + n_\theta \tag{12}$$

or in a more compact form:

$$\Delta z = H\Delta x + n_\theta \tag{13}$$

The continuous Kalman filter equation for the covariance is:

$$\dot{P} = FP + PF^T + Q - PH^T R^{-1} HP \tag{14}$$

where $P$ is the covariance matrix, $F$ is the system matrix, $H$ is the measurement matrix, $Q$ is the system noise covariance matrix and $R$ is the measurement noise covariance matrix. Consider this equation in the steady state case where $lim_{t\to\infty}(\dot{P}) = 0$:

$$\mathbf{0} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} + \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} N_r & 0 \\ 0 & N_w \end{bmatrix} - \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} N_\theta \end{bmatrix}^{-1}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \tag{15}$$

Solving for the elements of matrix $P$ we find:

$$p_{11} = \sqrt{N_\theta}\sqrt{N_r + 2\sqrt{N_w N_\theta}}\ , \quad p_{12} = \sqrt{N_w N_\theta}$$
$$p_{22} = \sqrt{N_w}\sqrt{N_r + 2\sqrt{N_w N_\theta}} \tag{16}$$

The steady state Kalman gain is:

$$K = PH^T R^{-1} = \begin{bmatrix} \sqrt{\frac{N_r + 2\sqrt{N_w N_\theta}}{N_\theta}} \\ \sqrt{\frac{N_w}{N_\theta}} \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \tag{17}$$

The Indirect Kalman filter estimates the error states. The estimate propagation equation with the added correction is:

$$\frac{d}{dt}\begin{bmatrix} \widehat{\Delta\theta} \\ \widehat{\Delta b} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \widehat{\Delta\theta} \\ \widehat{\Delta b} \end{bmatrix} + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}\left( \Delta z - \widehat{\Delta\theta} \right) \tag{18}$$

Substituting the error state estimates

$$\widehat{\Delta\theta} = \hat{\theta} - \theta_i \ , \quad \widehat{\Delta b} = \hat{b} - b_i \tag{19}$$

in the estimate propagation equation (18) we have

$$\frac{d}{dt}\begin{bmatrix} \hat{\theta} - \theta_i \\ \hat{b} - b_i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \hat{\theta} - \theta_i \\ \hat{b} - b_i \end{bmatrix} + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}\left( \Delta z - \widehat{\Delta\theta} \right) \tag{20}$$

Separating the estimated and integrated quantities in order to get the feedback formulation we have:

$$\frac{d}{dt}\begin{bmatrix} \hat{\theta} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \hat{\theta} \\ \hat{b} \end{bmatrix} + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}\left( \Delta z - \widehat{\Delta\theta} \right) + \left( \frac{d}{dt}\begin{bmatrix} \theta_i \\ b_i \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \theta_i \\ b_i \end{bmatrix} \right) \tag{21}$$

Notice from equations (11) and (19):

$$\Delta z - \widehat{\Delta\theta} = (\theta_m - \theta_i) - (\hat{\theta} - \theta_i) = \theta_m - \hat{\theta} \tag{22}$$

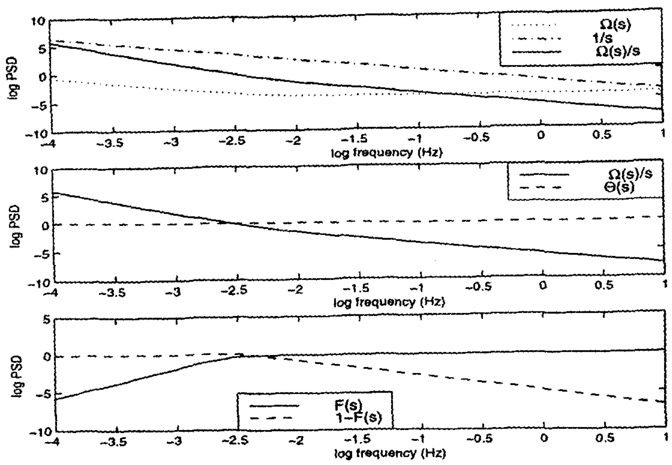

Figure 1: In the first plot the power spectral density of the gyro noise is displayed before and after integration. In the second plot the power spectral densities of the integrated gyro noise and of the absolute orientation sensor noise are shown. The third plot displays the power spectral densities of $F(s)$ which filters the integrated gyro noise, and $1 - F(s)$ which filters the absolute orientation sensor noise.

Now substituting in (21) from (7) and (22) we have

$$\frac{d}{dt}\begin{bmatrix}\hat{\theta}\\ \hat{b}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\hat{\theta}\\ \hat{b}\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}\omega_m + \begin{bmatrix}k_1\\ k_2\end{bmatrix}(\theta_m - \hat{\theta}) \tag{23}$$

Taking the Laplace transform and solving for $\hat{\Theta}(s)$:

$$\hat{\Theta}(s) = \frac{s^2}{s^2 + k_1 s + k_2}\frac{\Omega_m(s)}{s} + \frac{k_1 s + k_2}{s^2 + k_1 s + k_2}\Theta_m(s) \tag{24}$$

which can be rewritten as

$$\hat{\Theta}(s) = F(s)\frac{\Omega_m(s)}{s} + (1 - F(s))\Theta_m(s) \tag{25}$$

This reveals that the Indirect Kalman filter weighs the two different sources of information (i.e. the integrated gyro signal and the absolute orientation measurement) in a complementary fashion according to their noise characteristics. To acquire more intuition on how the Indirect Kalman filter deals with noise, we examine the case where the sensor inputs are only noise, (i.e. the sensor signals do not contain any useful information at any frequency).

In Figure 1, on the first plot we see the effect of the integration process $1/s$ on the gyro noise. The already strong noise components in the lower frequencies are amplified because of the integration process and thus any useful low frequency information will be contaminated by this noise. The obvious conclusion is that we cannot rely on the gyroscope integration to estimate low frequency motion. At higher frequencies the effect of the integrator is to suppress the gyro noise and therefore the gyro is reliable for high frequency motion. The ideal situation would be to fuse the integrated gyro information with an absolute orientation sensor (such as a sun sensor or a compass) that has a complementary noise profile (i.e. the noise is small at low frequencies and large at high frequencies). Failing that, even a constant noise profile (the same for all frequencies) will improve the estimate. The second case has been assumed and the noise profiles of both the integrated gyro and the absolute orientation sensor are depicted in the second plot of Figure 1.

At this point we should clarify how the Indirect Kalman filter optimally combines the information from the two different sources by examining it as a signal processing entity. The explanation comes from the third plot of Figure 1. The function $F(s)$ which filters the integrated gyro signal works as a high pass filter. It suppresses the low frequency noise content and allows the higher frequencies to pass undistorted. To the contrary, the function $1 - F(s)$ which filters the absolute orientation sensor acts like a low pass filter. It allows the critical low frequency information from the absolute orientation sensor to pass undistorted while reducing the higher frequency content. This is de

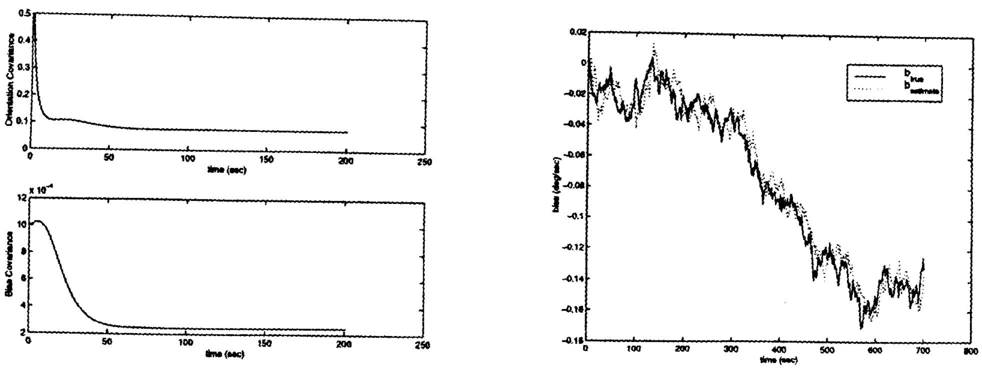

Figure 2: a. In the first plot the orientation covariance is displayed. In the second plot the bias covariance is shown. Both of these covariances reach a low steady state. This is expected since the system is observable, b. The solid line represents the true value of the gyro bias. The dotted line is the estimate of the bias from the Indirect Kalman filter. Though the estimate follows the true value it is obvious that there is a lag. This is because the absolute orientation sensor does not measure the bias directly. It only measures its effect on the orientation error. In all the results shown here the accuracy of the absolute orientation sensor was 3°.

able since the estimator performs better if it relies on the gyro in the high frequency range.

## 5.2 The 3D Case

The error state equation in the 3D case is given by:

$$\frac{d}{dt}\begin{bmatrix}\delta\vec{q}\\ \Delta\vec{b}\end{bmatrix}=\begin{bmatrix}[[\ \vec{\omega}_m\ ]] & -(1/2)I_{3\times3}\\ 0_{3\times3} & 0_{3\times3}\end{bmatrix}\begin{bmatrix}\delta\vec{q}\\ \Delta\vec{b}\end{bmatrix}+\begin{bmatrix}-(1/2)I_{3\times3} & 0_{3\times3}\\ 0_{3\times3} & I_{3\times3}\end{bmatrix}\begin{bmatrix}\vec{n}_r\\ \vec{n}_w\end{bmatrix} \quad (26)$$

The quaternion notation is following [13]. The quaternion error between the true attitude $q_{true}$ and the estimated (integrated) attitude $q_i$ is:

$$\delta q \simeq [\delta\vec{q}\ \ 1]^T \quad (27)$$

This error is given by the quaternion composition (product) $q_{true} = \delta q \otimes q_i$. As in the 2D case, the bias error is defined as the difference between the true and estimated bias $\Delta\vec{b} = \vec{b}_{true} - \vec{b}_i$. Following steps similar to the 2D case, an indirect Kalman filter for the 3D case has been developed. Due to space constraints, we do not derive the 3D formulation here - the reader is referred to [20, 21] for that. The extension to 3D motion shows the same benefits as the 2D case and a tracking example of one of the quaternion components is shown in Figure 3.

## 6 Conclusions

In this paper we decompose the localization problem into attitude estimation and, subsequently, position estimation. We focus on obtaining a good attitude estimate without building a model of the vehicle dynamics. The dynamic model was replaced by gyro modeling. An Indirect (error state) Kalman filter that optimally incorporates inertial navigation and absolute measurements was developed for this purpose. The linear form of the system and measurement equations for the planar case derived here allowed us to examine the role of the Kalman filter as a signal processing unit. The extension of this formulation to the 3D case shows the same benefits. A tracking example in the 3D case was also shown in this paper.

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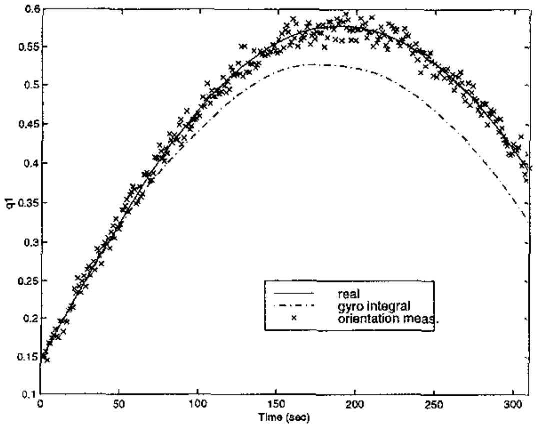

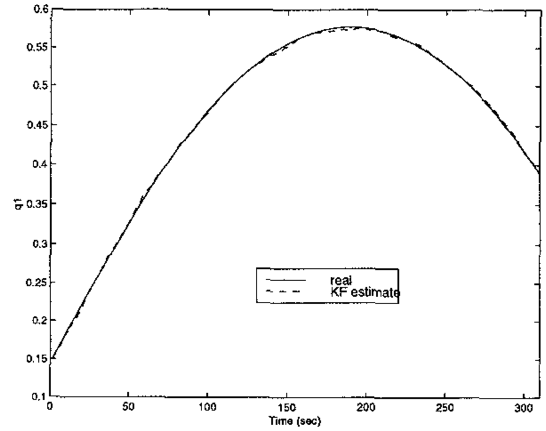


Figure 3: In both plots only the first element of the quaternion vector is shown. The first plot presents the real value of $q_1$, its estimate derived from the integration of the gyro signal, and its measured value using an absolute orientation sensor such as a compass. The second plot presents the real value of $q_1$ again for comparison purposes and its estimate as calculated by the Indirect Kalman filter.


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