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To Bayes or Bootstrap – that is the question

BAYESDAYS LIVERPOOL 2019

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Bayesian inference is both coherent and flexible, and not much more difficult that other things we do

Bootstrap

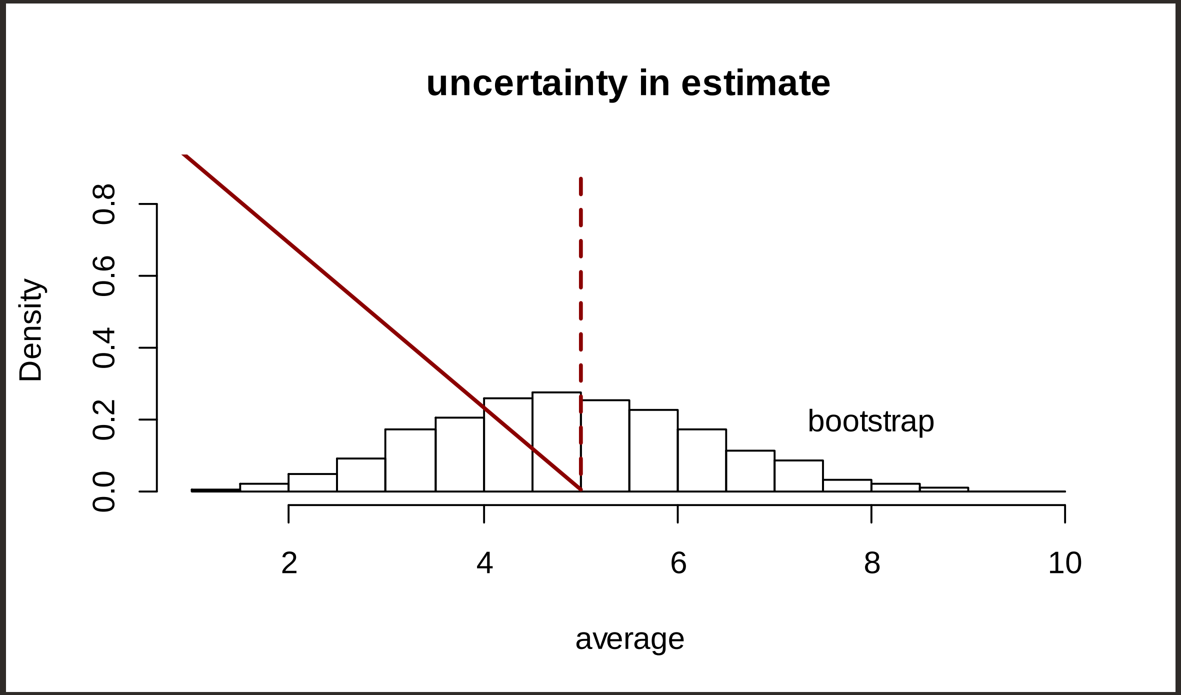
$\hat{\mu} = 5 \qquad V(\hat{\mu}) = 1.94$

2 5 1 9 10 3	average	5

Samples with replacement

10 5 10 1 10 2	average	6.33
5 3 5 5 3 10	average	5.17
1 3 3 10 3 3	average	3.83

…



Bayes

2 5 1 9 10 3 average 5

$$\hat{\mu} = E(\mu|data)$$

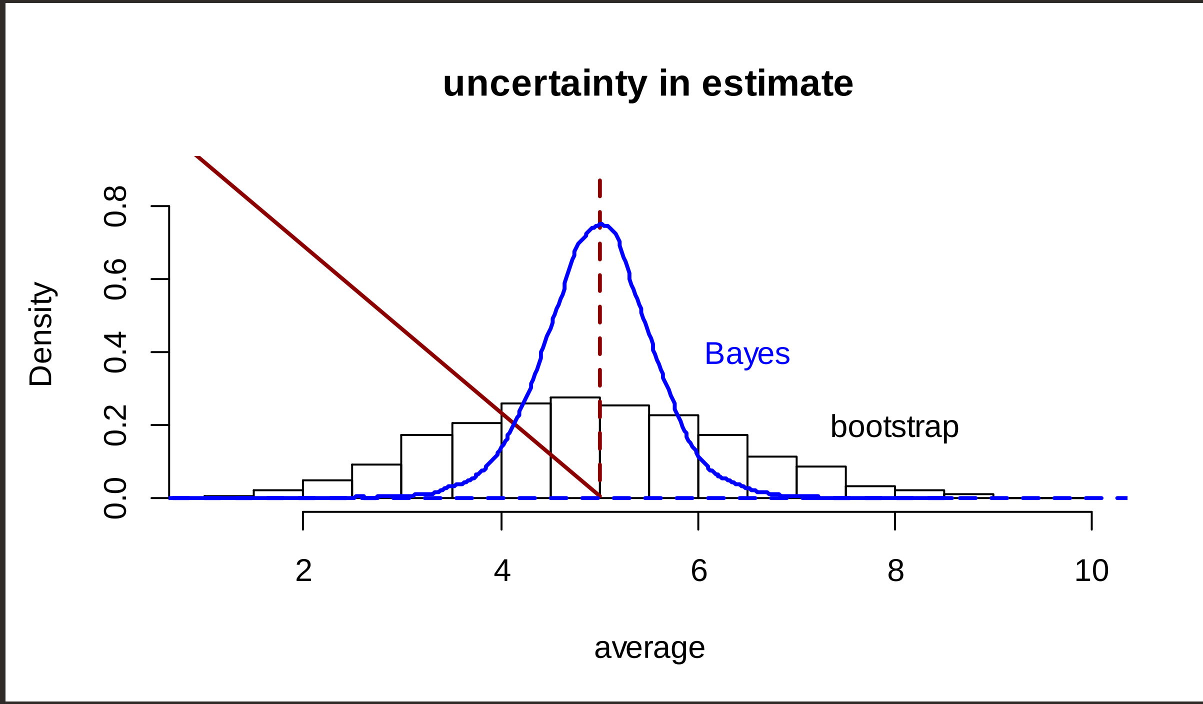
$$\mathrm{V}(\hat{\mu}) = V(\mu|data)$$

likelihood $data \sim N(\mu, \sigma)$

prior $\mu|\sigma \sim N(\mu_0, \frac{\sigma}{\sqrt{n}})$

$\sigma \sim InvGa(\alpha, \beta)$

posterior $\mu|data \sim .$



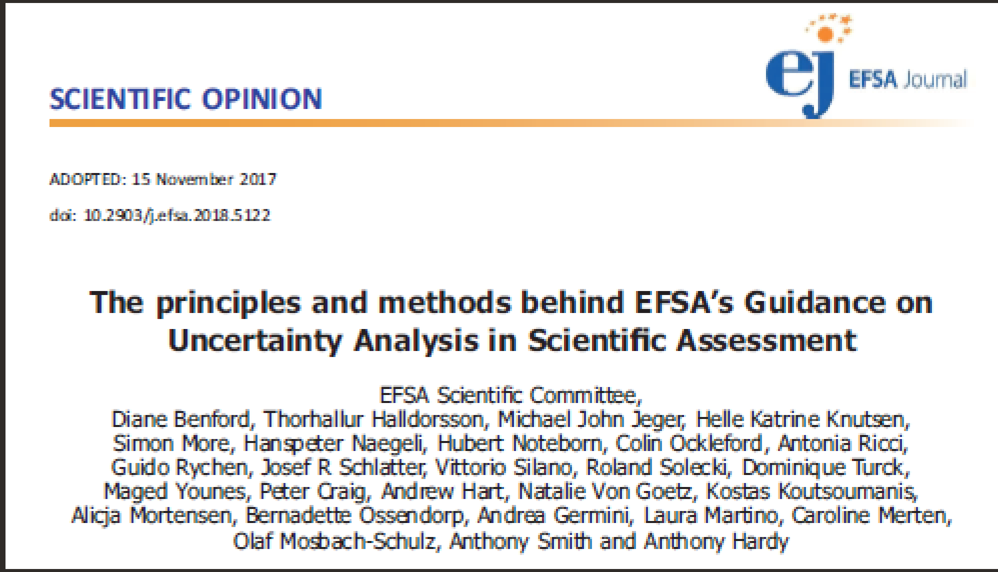
Bayes vs Bootstrap

- What are the differences between these two?
- When do they give similar results?
- Which one should I use?
- How to argue for one over another?

European Food Safety Authority

The quantitative methods reviewed includes

- The bootstrap quantifying uncertainty about parameters in a statistical model on the basis of data.
- Bayesian inference quantifying uncertainty about parameters in a statistical model on the bases of data and expert judgement about the values of the parameters.


SCIENTIFIC OPINION

EFSA Journal

ADOPTED: 15 November 2017

doi: 10.2903/j.efsa.2018.5122

The principles and methods behind EFSA's Guidance on Uncertainty Analysis in Scientific Assessment

EFSA Scientific Committee,
Diane Benford, Thorhallur Halldorsson, Michael John Jeger, Helle Katrine Knutsen, Simon More, Hanspeter Naegeli, Hubert Noteborn, Colin Ockleford, Antonia Ricci, Guido Rychen, Josef R Schlatter, Vittorio Silano, Roland Solecki, Dominique Turck, Maged Younes, Peter Craig, Andrew Hart, Natalie Von Goetz, Kostas Koutsoumanis, Alicja Mortensen, Bernadette Ossendorp, Andrea Germini, Laura Martino, Caroline Merten, Olaf Mosbach-Schulz, Anthony Smith and Anthony Hardy

The basic output of the bootstrap is a sample of possible values for the estimator(s) obtained by applying the estimator(s) to hypothetical data sets, of the same size as the original data set, obtained by resampling the original data with replacement. This provides a measure of the sensitivity of the estimator(s) to the sampled data. It also provides a measure of uncertainty for estimators for which standard confidence interval procedures are unavailable without requiring advanced mathematics. The bootstrap is often easily implemented using Monte Carlo.

Various methods can be applied to the basic output to obtain a confidence interval for the 'true' value of an estimator: the value which would be obtained by applying the estimator to the whole distribution of the variable. Each of the methods is approximate and makes some assumptions which apply well in some situations and less well in others. As for all confidence intervals, they have the weakness that the confidence interval probability needs reinterpretation before being used as a subjective probability (see Section 11.2.1).

Although the basic output from the bootstrap is a sample from a probability distribution for the estimator, that distribution does not directly represent uncertainty about the true value of the estimator using subjective probability and is subject to a number of biases which depend on the model, data and estimator used. However, in many cases, it may be reasonable for assessors to make the judgement that the distribution does approximately represent uncertainty. In doing so, assessors would be adopting the distribution as their own expression of uncertainty. In such situations, the bootstrap output might be used as an input to subsequent calculations to combine uncertainties, for example, using either probability bounds analysis or Monte Carlo (see Section 11.4).

Potential role in main elements of uncertainty analysis: can be used to obtain limited probabilistic information, and in some cases full probability distributions relevant to uncertainty, about general summaries of variability.

Form of uncertainty expression: range with approximate confidence level or distribution (represented by a sample) which does not directly represent uncertainty.

Principal strengths: can be used to evaluate uncertainty for non-standard estimators, even in non-parametric models, and provides a probability distribution which assessors may judge to be an adequate representation of uncertainty for an estimator.

Principal weaknesses: the distribution from which the output is sampled does not directly represent uncertainty and expertise is required to decide whether or not it does adequately represent uncertainty.

11.2.3. Bayesian inference (Annex B.12)

Bayesian inference is a method for quantifying uncertainty about parameters in a statistical model on the basis of data and expert judgements about the values of the parameters. The ingredients are a statistical model for some form of variability, a prior distribution for the parameters of the model, and data which may be considered to have arisen from the model. The prior distribution represents uncertainty about the values of the parameters in the model prior to observing the data. The prior distribution should preferably be obtained by expert knowledge elicitation (see Section 11.3). For some models, there exist standard choices of prior distribution which are intended to represent lack of knowledge. If such a prior is used, it should be verified that the probability statements it makes are acceptable to relevant experts for the parameter in question. The result of a Bayesian inference is a (joint) probability distribution for the parameters of the statistical model. That distribution combines the information provided by the prior distribution and the data and is called the posterior distribution. It represents uncertainty about the values of the parameters and incorporates both the information provided by the data and the prior knowledge of the experts expressed in the prior distribution. It is a good idea in general to assess the sensitivity of the posterior distribution to the choice of prior distribution. This is particularly important if a standard prior distribution was used, rather than a prior elicited from experts.

The posterior distribution from a Bayesian inference is suitable for combination with subjective probability distributions representing other uncertainties (see Section 11.4).

Potential role in main elements of uncertainty analysis: provides a quantitative assessment of uncertainty, in the form of a probability distribution, about parameters in a statistical model.

Form of uncertainty expression: distribution (for a quantity of interest) or probability (for a question of interest), often represented in practice by a large sample.

Principal strengths: output is a subjective probability distribution representing uncertainty and which may incorporate information from both data and expert judgement.

Principal weakness: limited familiarity with Bayesian inference amongst EFSA assessors – likely to need specialist support.

Example –Farmland birds

- Farmland Bird Index
  - Monitoring conservation efforts in Europe
  - Evaluate effects of interventions e.g. change of subsidy to farmers
- Abundance of farmland birds depends on amount of agricultural land and how resources are distributed in the landscape

Stjernman, M., Sahlin, U., Olsson, O., and Smith, H. G.. 2019. Estimating effects of arable land use intensity on farmland birds using joint species modeling. *Ecological Applications* 29(4):e01875. 10.1002/eap.1875

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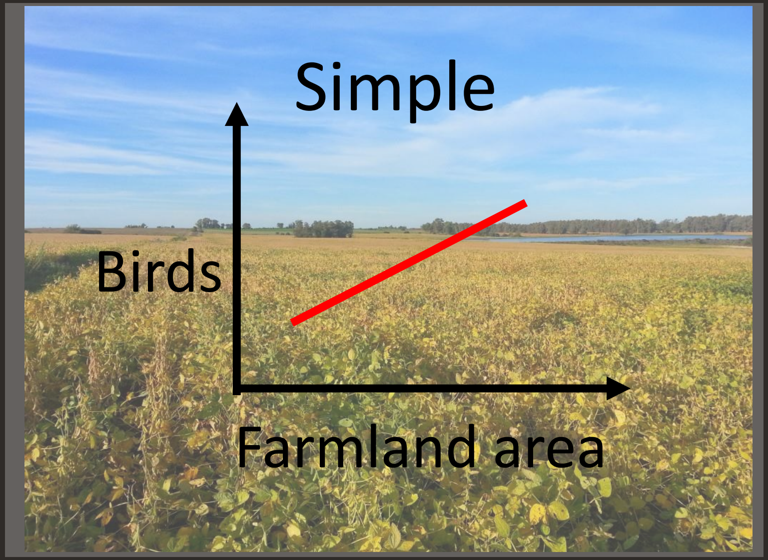

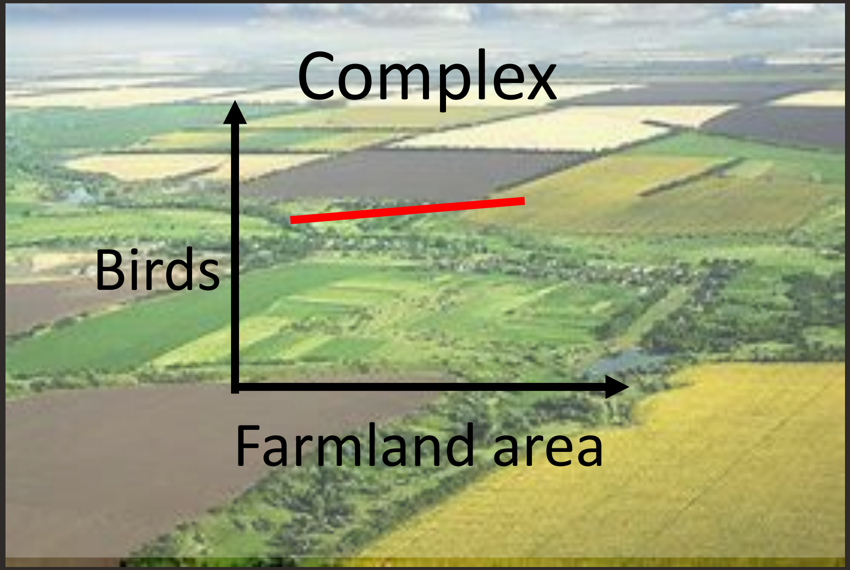

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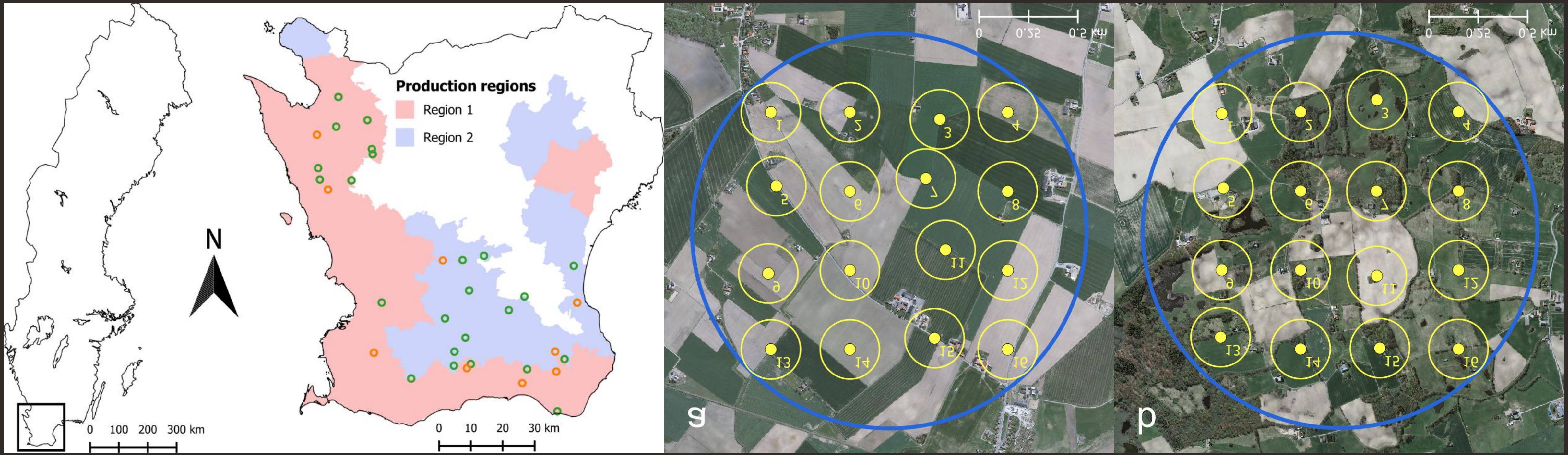
Example –Farmland birds

- Derive a predictive statistical model for FBI
- Data collected in a designed field study 2011
- Landscape complexity measured by a Landscape Heterogeneity Index (LHI)



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The statistical model – a GLMM

- Generalized Linear Mixed Model
- Count data

Response

$$y_i \sim Po(\lambda_i) \qquad \log(\lambda_i) \sim N(\cdot, \sigma)$$

In the dot:

Intercept

$$\beta_0$$

+

Fixed effects

$$\beta_1 Farml.area + \beta_2 LHI + \beta_{12} Farml.area * LHI$$

+

Random effects

site variability

$$\gamma_{s(i)} \qquad \gamma_s \sim N(0, \sigma_\gamma)$$

observer variability

$$\delta_{o(i)} \qquad \delta_o \sim N(0, \sigma_\delta)$$

Bayes vs Bootstrap to quantify uncertainty

- Likelihood given by the GLMM
- Bayes
  - Add prior distributions to parameters
  - Bayesian updating using MCMC sampling
  - Summarize
- Bootstrap
  - Estimate parameters using e.g. Maximum Likelihood estimation
  - Sample the estimator of interest using the bootstrap method
  - Summarize
  - Assign an interpretation

ML + Bootstrap

lme4-package {lme4} R Documentation

Linear, generalized linear, and nonlinear mixed models

Description

`lme4` provides functions for fitting and analyzing mixed models: linear (`lmer`), generalized linear (`glmer`) and nonlinear (`nlmer`.)

Differences between nlme and lme4

lme4 covers approximately the same ground as the earlier **nlme** package. The most important differences are:

- **lme4** uses modern, efficient linear algebra methods as implemented in the `Eigen` package, and uses reference classes to avoid undue copying of large objects; it is therefore likely to be faster and more memory-efficient than **nlme**.
- **lme4** includes generalized linear mixed model (GLMM) capabilities, via the `glmer` function.

bootMer {lme4} R Documentation

Model-based (Semi-)Parametric Bootstrap for Mixed Models

Description

Perform model-based (Semi-)parametric bootstrap for mixed models.

Usage

```
bootMer(x, FUN, nsim = 1, seed = NULL, use.u = FALSE, re.form=NA,
        type = c("parametric", "semiparametric"),
        verbose = FALSE, .progress = "none", PBargs = list(),
        parallel = c("no", "multicore", "snow"),
        ncpus = getOption("boot.ncpus", 1L), cl = NULL)
```

Arguments

`x`	a fitted `merMod` object: see `lmer`, `glmer`, etc.
`FUN`	a function taking a fitted `merMod` object as input and returning the *statistic* of interest, which must be a (possibly named) numeric vector.
`nsim`	number of simulations, positive integer; the bootstrap *B* (or *R*).

Bayesian inference

brms-package {brms} R Documentation

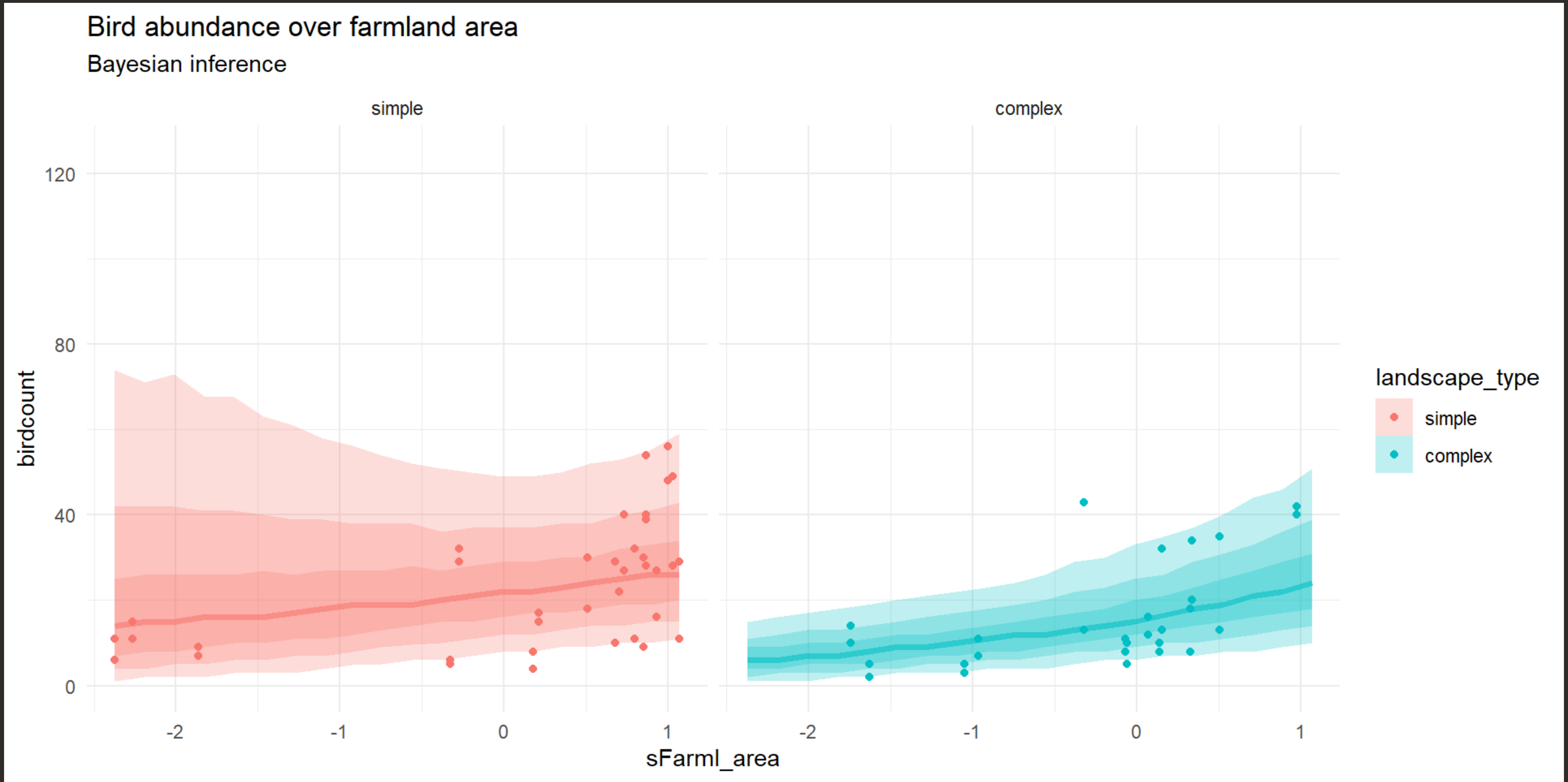
Bayesian Regression Models using 'Stan'

Description

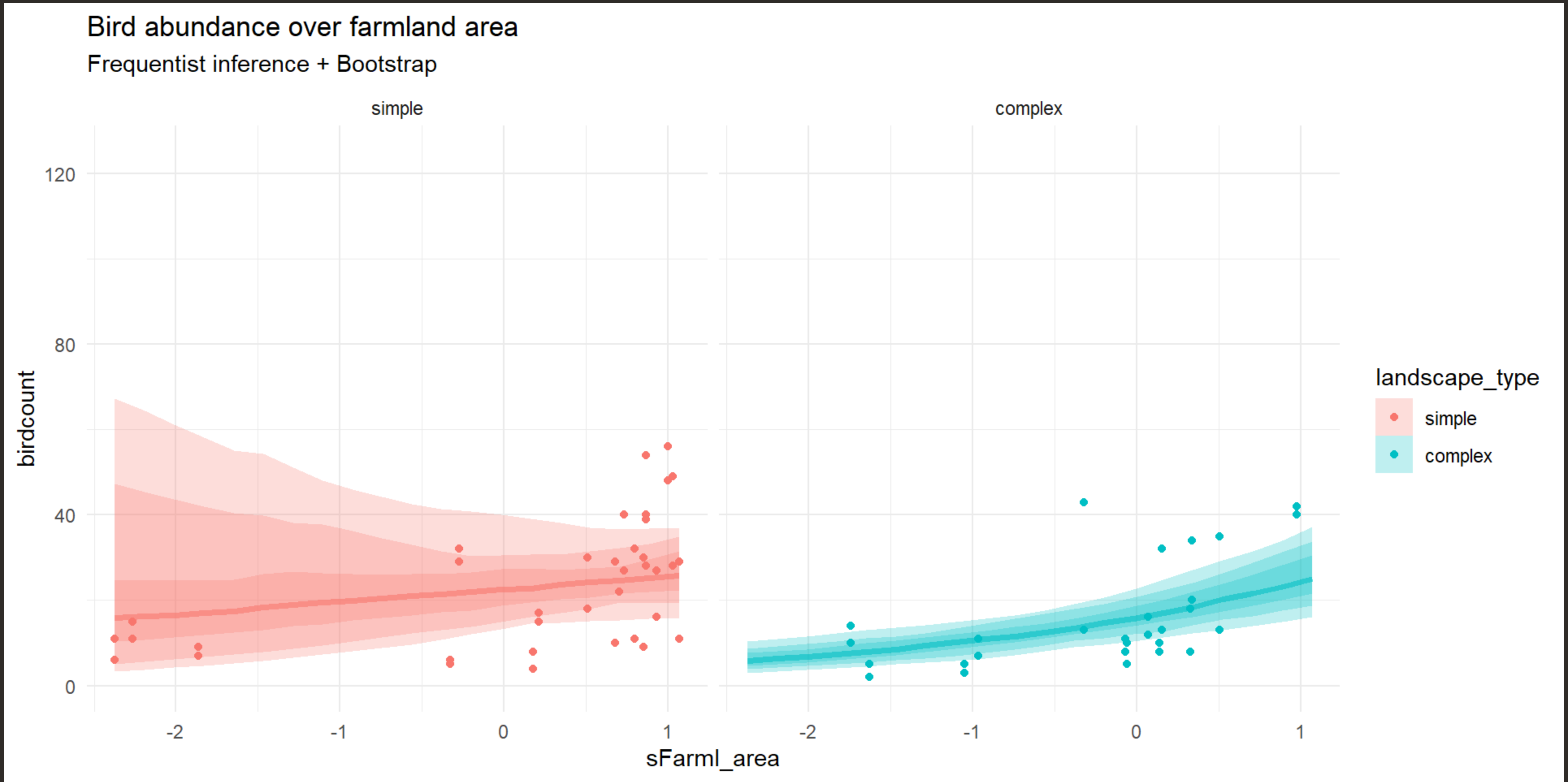
Stan Development Team

The **brms** package provides an interface to fit Bayesian generalized multivariate (non-)linear multilevel models using **Stan**, which is a C++ package for obtaining full Bayesian inference (see http://mc-stan.org/). The formula syntax is an extended version of the syntax applied in the **lme4** package to provide a familiar and simple interface for performing regression analyses.

Walk through the R-code



Bird abundance over farmland area
Bayesian inference
simple
complex
120
80
40
0
birdcount
-2
-1
0
1
-2
-1
0
1
sFarml_area
landscape_type
simple
complex


Bird abundance over farmland area
Frequentist inference + Bootstrap
simple
complex
birdcount
120
80
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0
-2
-1
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sFarml_area
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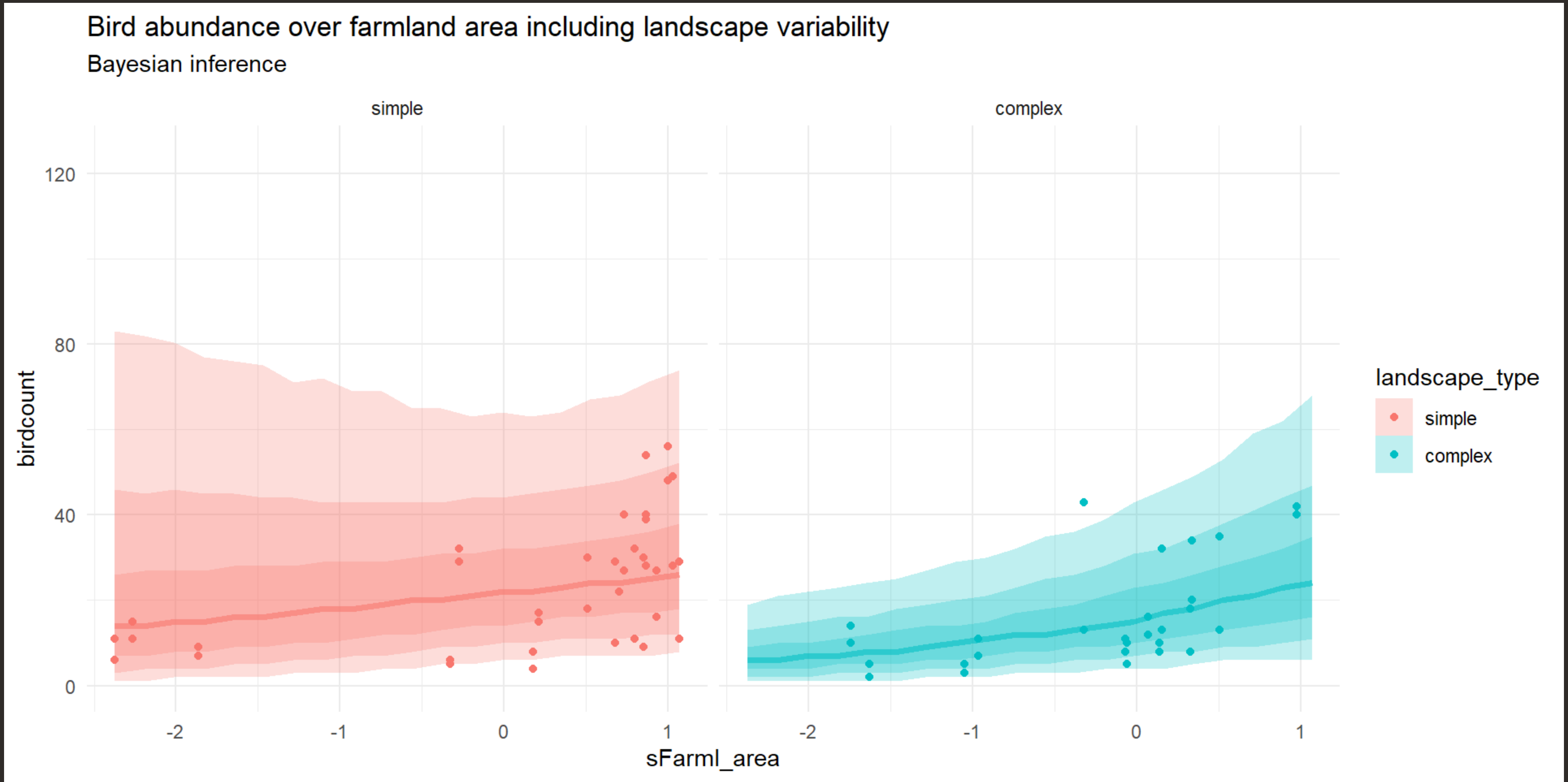

Bird abundance over farmland area including landscape variability
Bayesian inference
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0
-2
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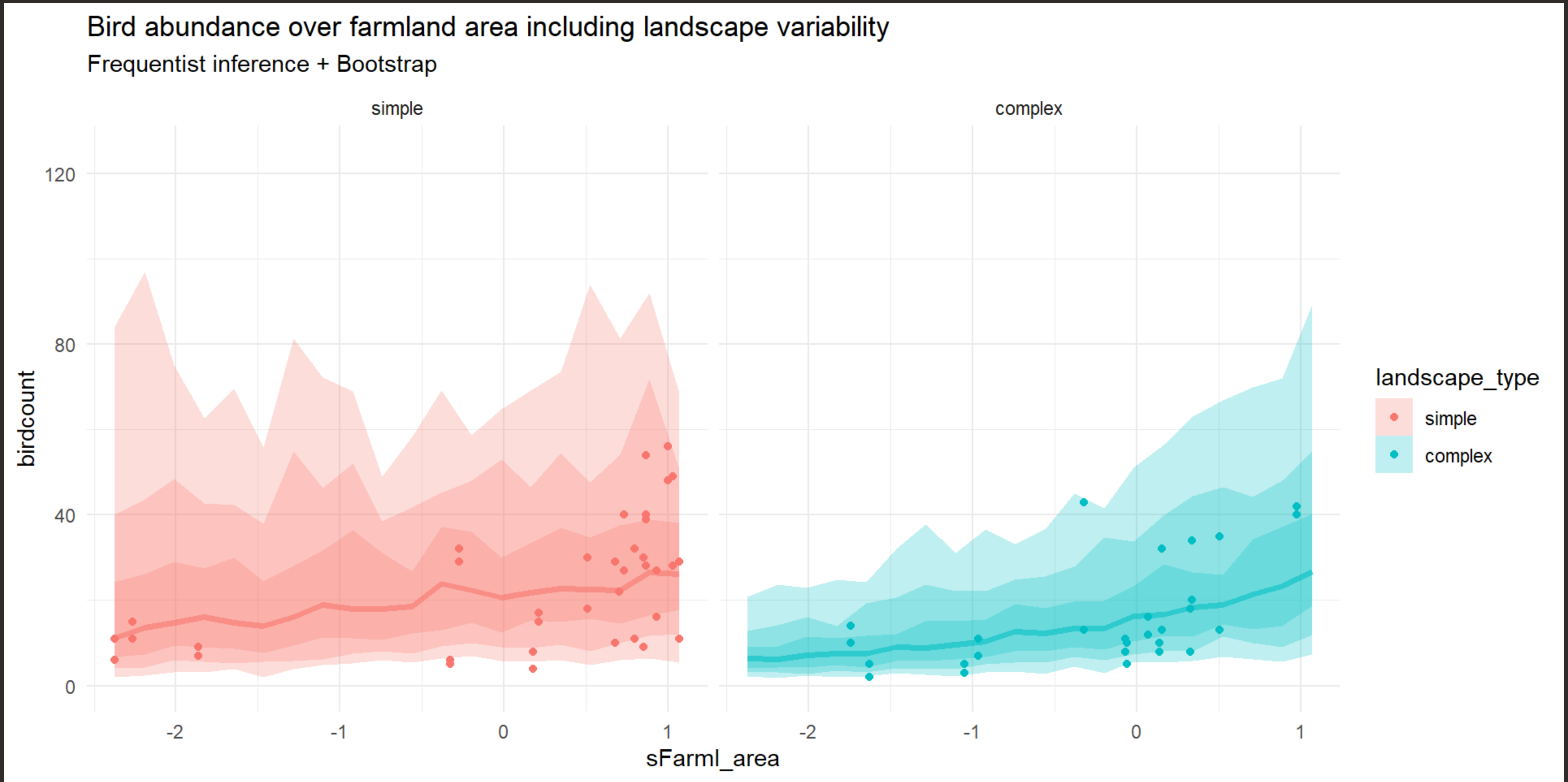
0
1
sFarml_area
landscape_type
simple
complex



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landscape_type
simple
complex

Bayes vs Bootstrap

- Conceptually simple
- Flexible
- Widely applicable
- Gives you a distribution

- Bootstrap
  - Assumes data is all there is
  - Mixture of principles
  - No clear interpretation of the distribution
- Bayesian
  - Open up to use expert judgement
  - Coherent principle
  - Subjective probability

Rasmus Bååth lecture on Bayesian bootstrapping
https://youtu.be/VceFc5hsMw8

Bayesian inference is both coherent and flexible, and not much more difficult that other things we do

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https://github.com/dmi3kno/BayesBootstrap

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