Methods of classification and dimensionality reduction - Report 1

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SVD

We get a real $n \times d$ dimensional matrix Z. (We may assume that $n \geq d$.) The aim is to approximate Z by a matrix of smaller rank. Precisely, we want to find matrix $\tilde{Z}_r$ of rank r and $r << rank(Z)$, so that $\|Z - \tilde{Z}_r\|$ is small.

From the theorem on the lecture we can present Z as follows

$$Z = U\Lambda^{\frac{1}{2}}V^T,$$

where

- U is $n \times d$ matrix such that $UU^T = I$,
- V is a $d \times d$ matrix such that $VV^T = I$,
- Λ is a $d \times d$ diagonal matrix with values $\lambda_1, \ldots, \lambda_d$ on diagonal such that $\lambda_1 \geq \cdots \geq \lambda_d \geq 0$.

We construct $\tilde{Z}_r$ as

$$\tilde{Z}_r = U_r\Lambda_r^{\frac{1}{2}}V_r^T,$$

where

- U_r is $n \times r$ matrix such that it is a truncation of matrix U to its first r columns,
- V_r is $r \times d$ matrix such that it is truncations of matrix V to its first r columns,
- Λ_r is $r \times r$ diagonal matrix with $\lambda_1, \ldots, \lambda_r$ on diagonal.