2.03 Continuous Distributions

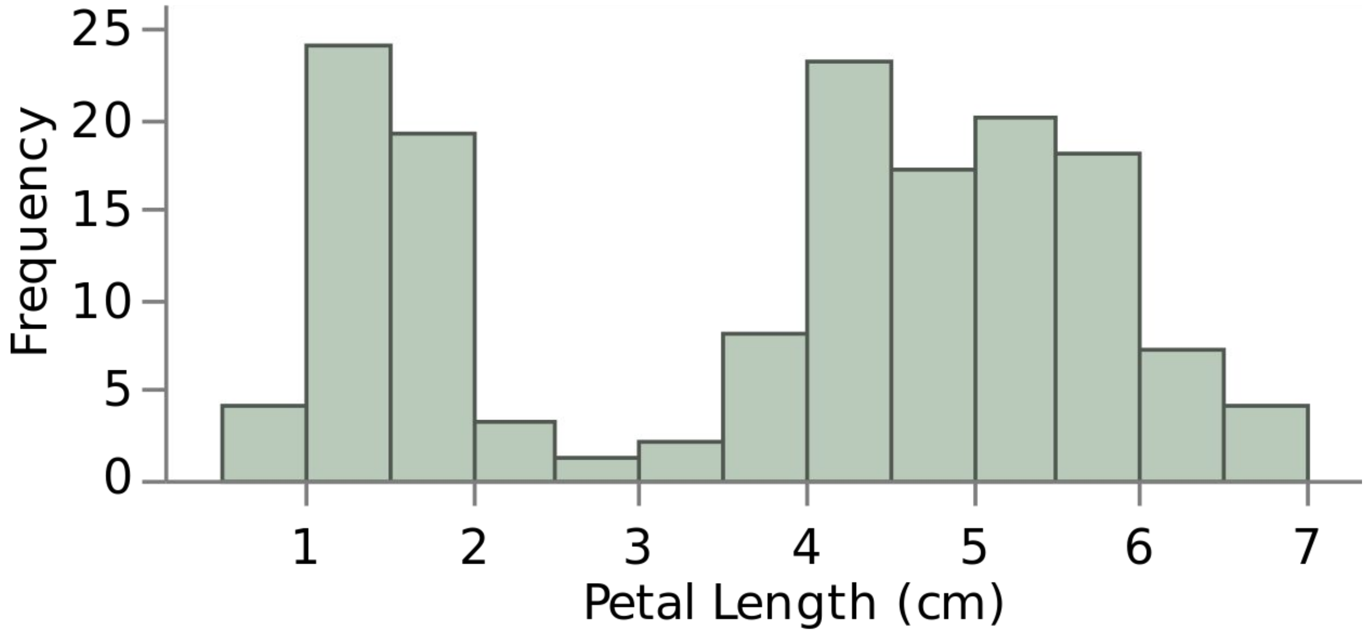
Recap on Discrete Distributions

Back to Distributions...

A probability distribution describes the probability of a random variable taking on certain values.

First, what is a Distribution?

A **distribution** is the set of all values of a variable and how frequently we observe each value.



What is a Random Variable?

A **random variable** is just a variable whose value is a numerical outcome from some random event.

Examples:

- Flip a coin 10 times and let X represent the number of times you flipped tails. X is a random variable.
- You post a picture on Instagram and are keeping track of your engagement. Let random variable X represent the number of likes you get on your post.
- You randomly select a person from a crowd. This person’s height can be represented by a random variable X.

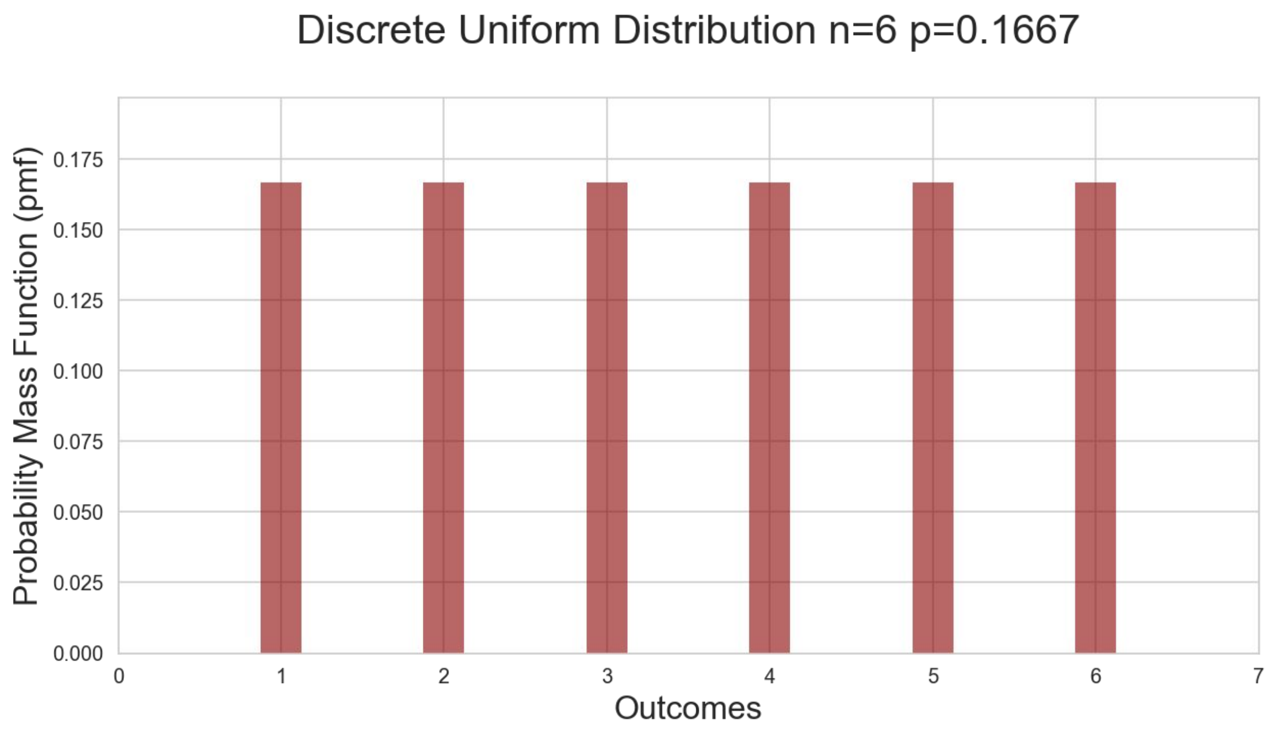
What is the difference between a continuous random variable and discrete random variable?

A **continuous random variable** takes on an uncountably infinite number of values.

A **discrete random variable** takes on a countable number of values.

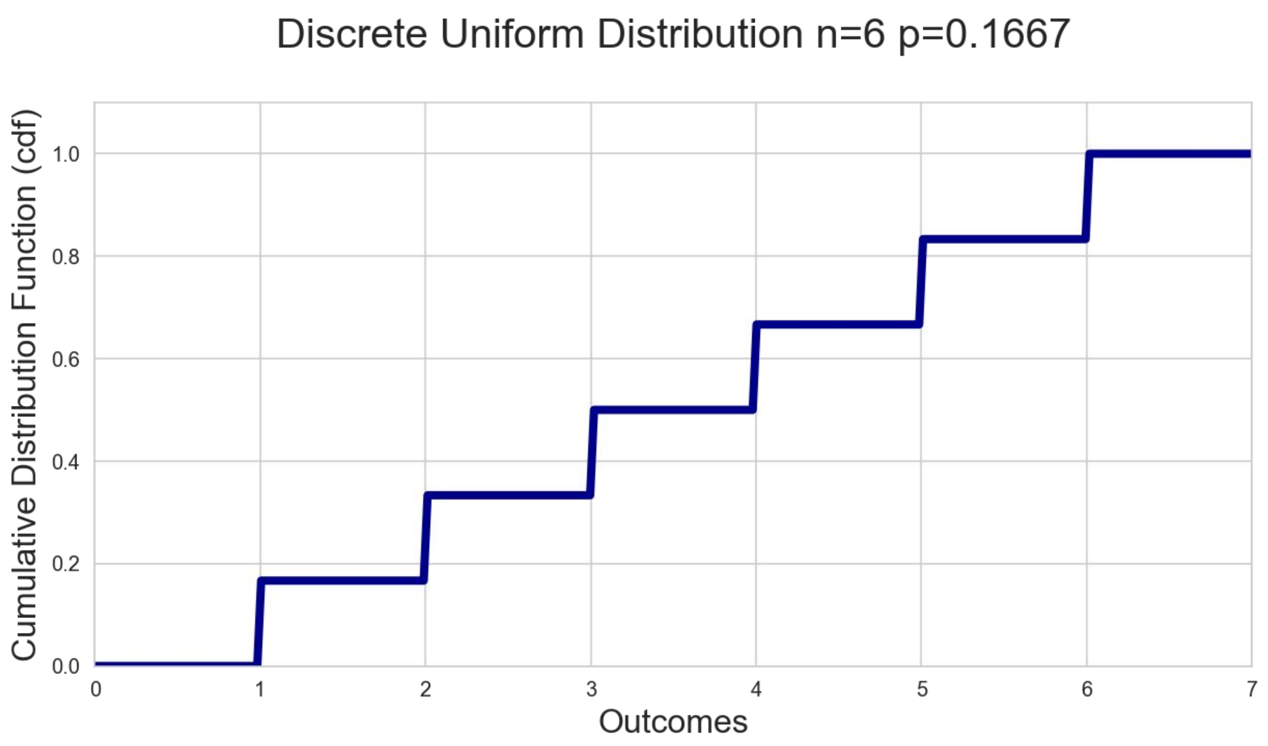
Probability Mass Function (PMF)

A function that tells us the probability that a discrete random variable is exactly equal to some value.



Cumulative Distribution Function (CDF)

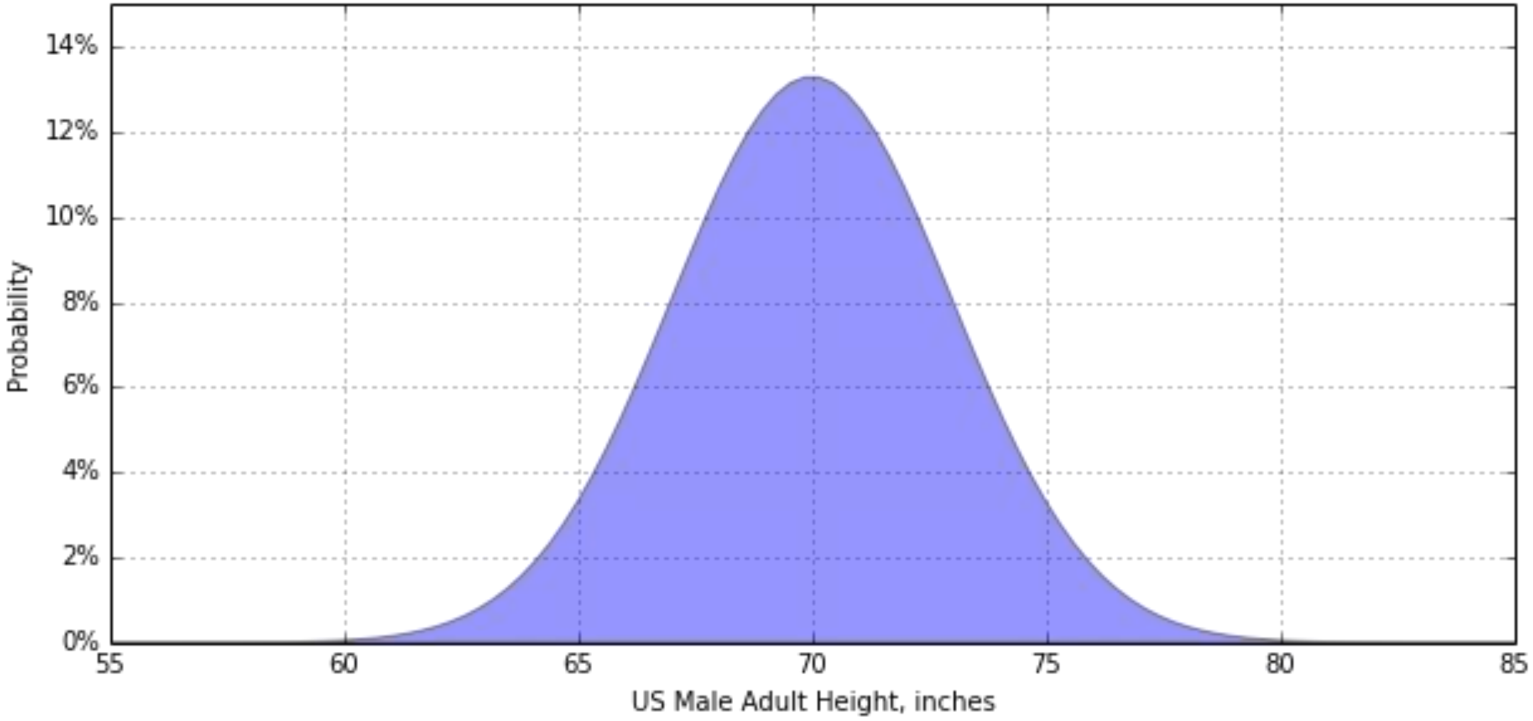
A function that describes the probability that a random variable is less than or equal to some value.



Continuous Distributions

~~Probability Mass Function (PMF)~~ Probability Density Function (PDF)

A function that tells us the ~~probability~~ relative likelihood that a ~~discrete~~ continuous random variable ~~is exactly~~ would be equal to some value.



Continuous Uniform Distribution

All values have the same probability density.

Parameters:

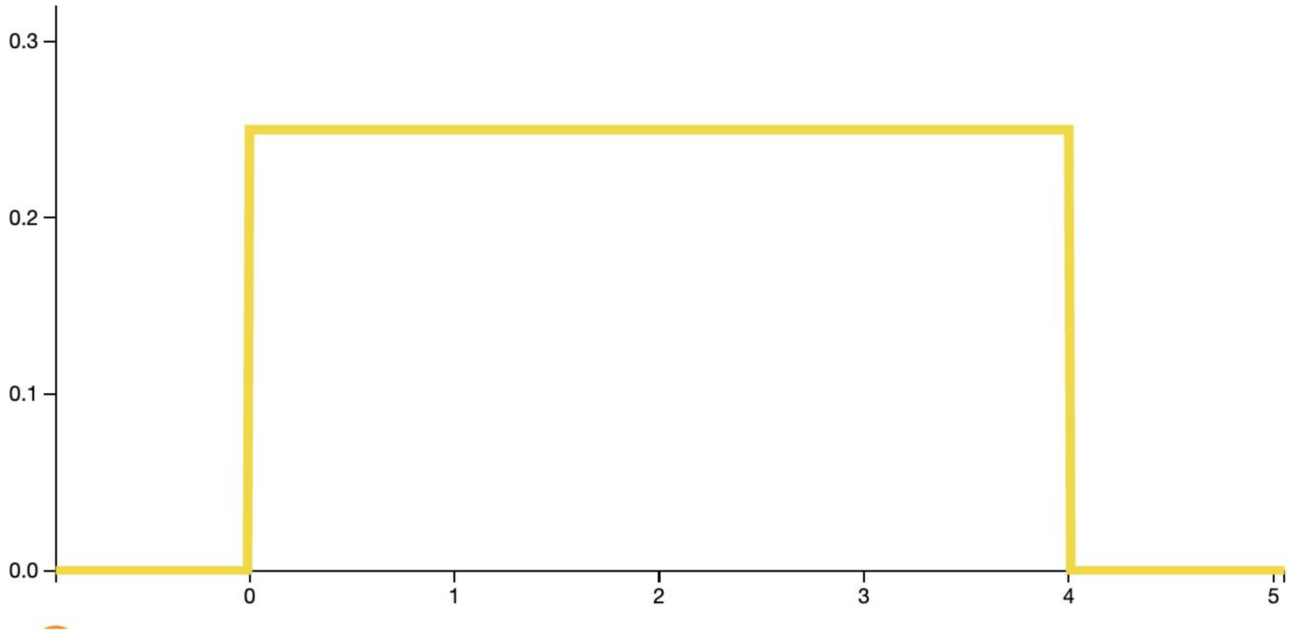
a, the minimum value of the distribution

b, the maximum value of the distribution

Continuous Uniform Distribution

Examples:

- I am thinking of a number between 1 and 10.
- My food will arrive between 25 - 35 minutes.



Exponential Distribution

We commonly use the Exponential distribution when we are interested in modeling the amount of time until an event.

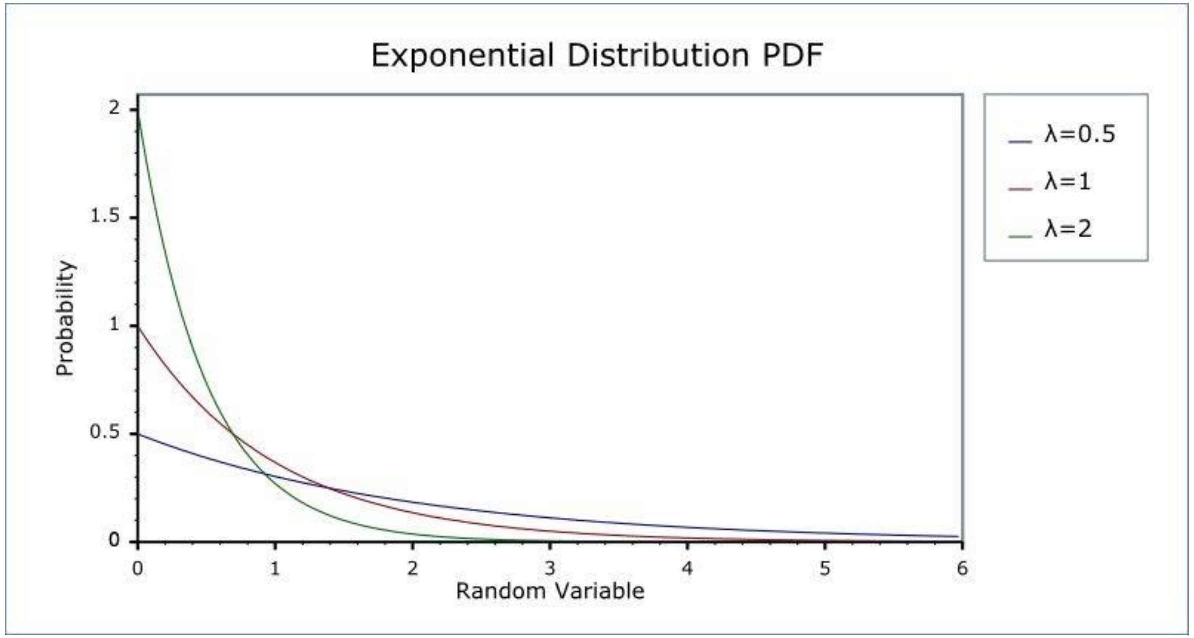
Parameters:

β, the average time to an event

Exponential Distribution

Examples:

- The amount of time an employee will spend with a customer.
- The amount of time until a new person walks into a museum.



Exponential Distribution

Another example:

Based on historical data, we see an average of 10 buses per hour. From this, how long do you think it will take on average for a new bus to arrive?

6 minutes

10 buses per hour = 1 bus every **6 minutes**

Gamma Distribution

The exponential distribution is actually a special case of the Gamma distribution. That is, if you have α exponential distributions with the same β their sum is $Gamma(\alpha, \beta)$.

Parameters:

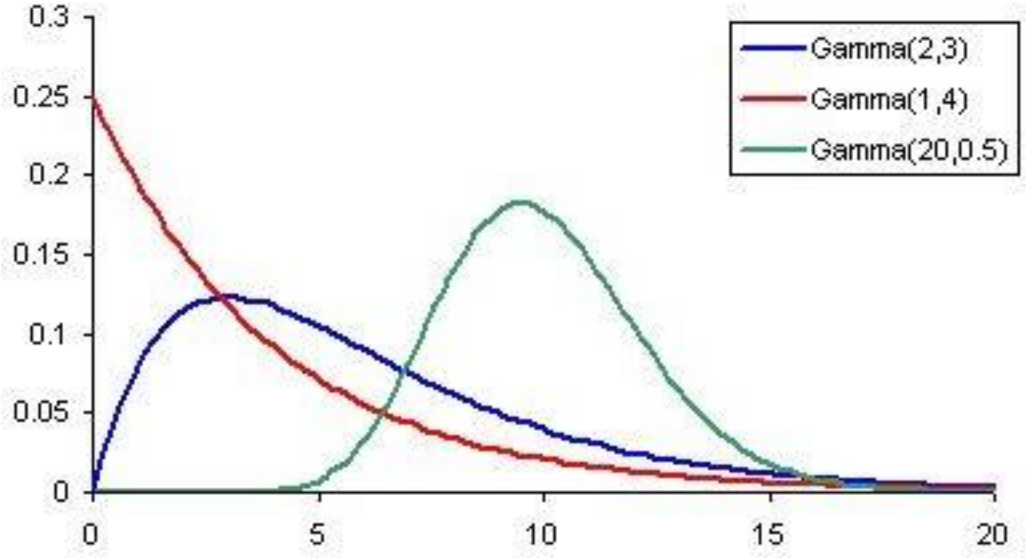
α , shape

β , the average time to an event

Gamma Distribution

Example:

- Suppose a light bulb lasts on average 12 months. Once it dies, you replace it with a light bulb of the same brand. How long will it take to go through 5 light bulbs? You might model this with Gamma(5,12).



Normal Distribution

The Normal distribution is the most well known and most important distribution. Many real-world processes can be modeled using a Normal distribution.

Parameters:

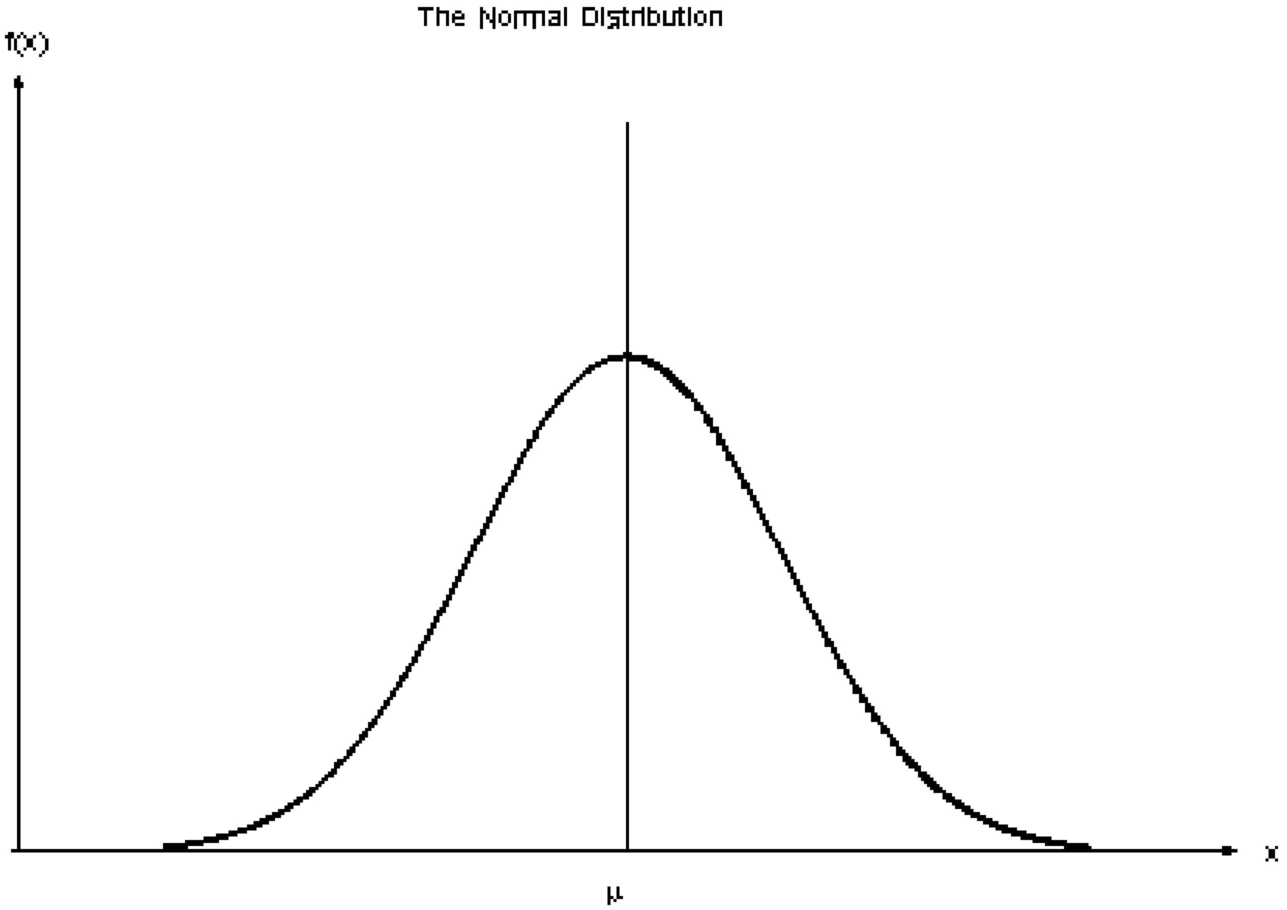
μ, the mean

σ, the standard deviation

Normal Distribution

Examples:

- Height of a population
- Test scores
- Lengths of carrots



Beta Distribution

The Beta distribution can only take on values between 0 and 1. This makes it especially useful for modeling probabilities.

Parameters:

α, β , shape parameters

Beta Distribution

Example:

- Probabilities