Семинар 10

D - # доков
T - # тем
W - словарь

$\theta \in \mathbb{R}^{D \times T}$ - распределение тем над каждым доком

Z - тема каждого слова

$\varphi \in \mathbb{R}^{T \times W}$ - распред. темы над словами

$$p(W, Z, \theta \mid \varphi, \alpha) = \prod_{d=1}^{D} p(\theta_d \mid \alpha) \prod_{n=1}^{N_d} p(w_{dn} \mid \varphi, z_{dn})\, p(z_{dn} \mid \theta_d)$$

$t \in \mathbb{R}^{D}$ - тема каждого дока (у него будет всего 1 тема)

$p(t \mid \pi) = Cat(\pi)$

$$p(W, t \mid \varphi, \pi) = \prod_{d=1}^{D} p(t_d \mid \pi) \prod_{n=1}^{N_d} p(w_{dn} \mid \varphi, t_d) =$$

$$= \prod_{d} \prod_{k=1}^{T} \left[\pi_k \prod_{n=1}^{N_d} \varphi_{k, w_{dn}}\right]^{[k = t_d]}$$

$Count^{D \times W}$

D 0 1 2

1) E-step

$$q(t) = \prod_{d=1}^{D} q(t_d)$$

$$q(t_d = k) = p(t_d = k \mid W_d) = \frac{p(k = t_d, W_d)}{\sum_{k=1}^{T} p(t = k, W_d)} = \frac{\pi_k \prod_{n=1}^{N_d} \varphi_{k, w_{dn}}}{\sum_{i=1}^{T} \pi_i \prod_{n=1}^{N_d} \varphi_{i, w_{dn}}} = \mu_{dk}$$

$$q(t_d) = Cat(\mu_{d1}, \dots \mu_{dT})$$

2) M-step

$$\mathbb{E}_{q(t)} \log p(W, t \mid \varphi, \pi) \to \max_{\varphi, \pi}$$

$$= \mathbb{E}_{q(t)} \sum_{d=1}^{D} \sum_{k=1}^{T} [k = t_d] \left(\log \pi_k + \sum_{n=1}^{N_d} \sum_{w=1}^{W} \mathbb{1}[w_{dn} = w] \log \varphi_{kw} \right)$$

$$= \sum_d \sum_k \mu_{dk} \left(\log \pi_k + \sum_n \sum_w \mathbb{1}[w_{dn} = w] \log \varphi_{kw} \right)$$

1) $\sum_d \sum_k \mu_{dk} \log \pi_k \to \max_\pi \qquad \sum_k \pi_k = 1$

$$L = \sum_d \sum_k \mu_{dk} \log \pi_k - \lambda \left(\sum_{k=1}^{T} \pi_k - 1 \right)$$

$$\frac{\partial L}{\partial \pi_k} = \sum_{d=1} \frac{\mu_{dk}}{\pi_k} - \lambda = 0 \qquad \pi_k = \frac{\sum_d \mu_{dk}}{\lambda}$$

$$1 = \sum_k \pi_k = \frac{\sum_d \sum_k \mu_{dk}}{\lambda} \Rightarrow \lambda = D \qquad \Rightarrow \pi_k = \frac{1}{D} \sum_d \mu_{dk}$$

2) $\sum_{d=1}^{D} \sum_{k=1}^{T} \sum_{n=1}^{N_d} \sum_{w=1}^{W} \mu_{dk} [w_{dn} = w] \log \varphi_{kw} \to \max_\varphi$

$\sum_{w=1}^{W} \varphi_{kw} = 1.$

(для 1 стр. записали)

$$L = \sum_{d=1}^{D} \sum_{n=1}^{N_d} \sum_{w=1}^{W} \mu_{dk} [w_{dn} = w] \log \varphi_{kw} - \lambda \left(\sum_{w=1}^{W} \varphi_{kw} - 1 \right)$$

$$\frac{\partial L}{\partial \varphi_{kw}} = \sum_d \sum_n \mu_{dk} [w_{dn} = w] \frac{1}{\varphi_{kw}} - \lambda = 0.$$

$$\varphi_{kw} = \frac{\sum_d \sum_n \mu_{dk} [w_{dn} = w]}{\lambda}$$

$$1 = \sum_w \varphi_{kw} = \ldots \qquad \Rightarrow \varphi_{kw} = \frac{\sum_d \sum_n \mu_{dk} [w_{dn} = w]}{\sum_d \sum_n \mu_{dk} \sum_w [w_{dn} = w]}$$

$$= \frac{\sum_d \sum_n \mu_{dk} [w_{dn} = w]}{\sum_d N_d \mu_{dk}}$$

Добавлены априорные распред.

1 тема, $t \in \mathbb{R}^{D}$ + прайер на φ

$$p(W, t, \varphi \mid \pi, \beta) = \prod_{k=1}^{T} Dir(\varphi_k \mid \beta) \prod_{d=1}^{D} \prod_{i=1}^{T} \left[\pi_i \prod_{n=1}^{N_d} \varphi_{i, w_{dn}}\right]^{[t_d = i]}$$

1) $q(t, \varphi) \approx q(t)\, q(\varphi)$

$$\log q(t) \propto \mathbb{E}_{q(\varphi)} \log p(W, t, \varphi) = \sum_{d=1}^{D} \sum_{i=1}^{T} [t_d = i] \Big(\log \pi_i +$$

$$+ \sum_{n=1}^{N_d} \mathbb{E}_{q(\varphi)} \log \varphi_{i, w_{dn}} \Big)$$

$$q(t) = \prod q(t_d)$$

$$q(t_d = k) = \frac{\pi_k \prod_{n=1}^{N_d} \exp(\mathbb{E}_{\varphi} \log \varphi_{k, w_{nd}})}{\sum_{i=1}^{T} \pi_i \prod_{n=1}^{N_d} \exp(\mathbb{E}_{\varphi} \log \varphi_{i, w_{nd}})} = \mu_{dk}$$

2) $$\log q(\varphi) = \mathbb{E}_{q(t)} \log p(W, t, \varphi) = \mathbb{E}_{q(t)} \sum_{k=1}^{T} \log Dir(\varphi_k \mid \beta) +$$

$$+ \mathbb{E}_{q(t)} \sum_{d=1}^{D} \sum_{k=1}^{T} [\log \pi_k + \sum_{n=1}^{N_d} \sum_{W=1}^{W} [W = w_{dn}] \log \varphi_{kW} \; [t_d = k] + Const$$

$$= \cancel{\mathbb{E}} \sum_{k=1}^{T} (\beta - 1) \sum_{W=1}^{W} \log \varphi_{kW} + \sum_{d=1}^{D} \sum_{k=1}^{T} \mu_{dk} \sum_{k=1}^{N_d} \sum_{W=1}^{W} [W = w_{dk}] \log \varphi_{kW}$$

$$= \sum_{k=1}^{T} \sum_{W}^{W} \log \varphi_{kW} \Big[\beta - 1 + \sum_{d=1}^{D} \sum_{n=1}^{N_d} \mu_{dk} [W = w_{dn}]\Big]$$

$$q(\varphi_k) = Dir(\varphi_k \mid [\beta'_{k1}, \dots, \beta'_{kW}]) \qquad \beta'_{ki} = \beta + \sum_{d=1}^{D} \sum_{n=1}^{N_d} \mu_{dk} [W = w_{dn}]$$

ELBO:

$$\mathbb{E}_{q(\cdot)} \log p - \mathbb{E}_q \log q$$

$$= \sum_d \sum_k \mu_{dk} \left(\log \pi_k + \sum_n \sum_w \mathbb{1}[w_{dn} = w] \log \varphi_{kw} \right) -$$

$$- \sum_d \sum_k \mu_{dk} \log \mu_{dk} .$$