1 Objectives

The objective is to solve the mutual information calculation for the complex signal model and two tissue types. This will be extended to N tissue types in the next document.

2 Work

2.1 Probability Distribution Definitions

Calculate mutual information using numerical integration for a complex, linearized signal model $\mathbf{z}(\mu, \mathbf{k})$.

$$\mathbf{z}(\mu, \mathbf{k}) = \int_{\Omega} M(\mathbf{x}) e^{-s(\mu,\mathbf{x})} e^{-2\pi i \mathbf{k}\cdot\mathbf{x}} d\mathbf{x} + \nu = \mathcal{G}(\mu, \mathbf{k}) + \nu \qquad \nu \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_\nu) \qquad \mathbf{\Sigma}_\nu = \begin{bmatrix} \sigma_{\nu,r}^2 & 0 \\ 0 & \sigma_{\nu,i}^2 \end{bmatrix} \tag{1}$$

$$s(\mu, \mathbf{x}) = \frac{T_E}{T_2^*(\mathbf{x})} + i\left[2\pi\gamma\alpha B_0 T_E \Delta u(\mu, \mathbf{x}) + T_E \Delta\omega_0(\mathbf{x})\right]$$

Therefore, the probability distribution for $p(\mathbf{z}|\mu)$ is

$$p(\mathbf{z}|\mu) = \mathcal{N}(\mathcal{G}(\mu), \mathbf{\Sigma}_\nu) \tag{2}$$

Assume normal distribution for the model parameter, optical attenuation coefficient μ.

$$\mu \sim \mathcal{N}(\bar{\mu}, \mathbf{\Sigma}_\mu) \tag{3}$$

Also of note, because the real and imaginary components of z are assumed independent, the covariance matrix $\mathbf{\Sigma}_z$ is diagonal, and the following simplification results.

$$p(\mathbf{z}|\mu) = \mathcal{N}_{\mathbf{z}}(\mathcal{G}(\mu), \mathbf{\Sigma}_\nu) = \mathcal{N}_{z_r}(\mathcal{G}_r(\mu), \mathbf{\Sigma}_\nu)\, \mathcal{N}_{z_i}(\mathcal{G}_i(\mu), \mathbf{\Sigma}_\nu) = p(z_r|\mu)\, p(z_i|\mu) \tag{4}$$

The tissue properties can described by the following piecewise functions.

$$\mu(\mathbf{x}) = \sum_{n=1}^{N} \mu_n U(\mathbf{x} - \Omega_n) \tag{5}$$

$$\bigcup_{n=1}^{N} \Omega_n = \Omega \qquad \Omega_n \cap \Omega_m = \varnothing \tag{6}$$

$$U(\mathbf{x} - \Omega_n) = \begin{cases} 1, & x \in \Omega_n \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

2.2 Problem Statement

The most approachable way to solve mutual information is to begin with the difference of entropies definition.

$$I(\boldsymbol{\mu}; \mathbf{z}) = H(\mathbf{z}) - H(\mathbf{z}|\boldsymbol{\mu}) = \frac{1}{2}\ln\left((2\pi e)^2 \cdot |\mathbf{\Sigma}_z|\right) - \frac{1}{2}\ln\left((2\pi e)^2 \cdot |\mathbf{\Sigma}_\nu|\right) \tag{8}$$

Assuming $\sigma_{\nu,r}^2 = \sigma_{\nu,i}^2$, then

$$|\mathbf{\Sigma}_\nu| = \sigma_\nu^2 \sigma_\nu^2 - 0 = \sigma_\nu^4 \tag{9}$$

Calculation of $\mathbf{\Sigma}_z$ is less straightforward.

$$\mathbf{\Sigma}_z = \begin{bmatrix} \mathrm{E}\left[(z_r - \mathrm{E}[z_r])^2\right] & \mathrm{E}\left[(z_r - \mathrm{E}[z_r])(z_i - \mathrm{E}[z_i])\right] \\ \mathrm{E}\left[(z_i - \mathrm{E}[z_i])(z_r - \mathrm{E}[z_r])\right] & \mathrm{E}\left[(z_i - \mathrm{E}[z_i])^2\right] \end{bmatrix} \tag{10}$$

$\mathrm{E}[z_r]$, $\mathrm{E}[z_i]$, $\mathrm{E}\left[(z_r - \mathrm{E}[z_r])^2\right]$, $\mathrm{E}\left[(z_i - \mathrm{E}[z_i])^2\right]$, and $\mathrm{E}[(z_r - \mathrm{E}[z_r])(z_i - \mathrm{E}[z_i])]$ all need to be calculated numerically.

Alternatively, $\int p(z|\mu)p(\mu)$ can probably be calculated analytically using [Bromiley 2003]. The product of three normal distributions is a scaled normal distribution, so after converting N_{z_i} and N_{z_r} to distributions in terms of μ for the integration, the normal distribution integrates to 1, and the scaling factor comes out as a function of z_r and z_i, $S_{fgh}(z_r, z_i)$, for the remaining two integrals. This leaves potentially very difficult numerical integrations at this step.

2.3 Gauss-Hermite Quadrature

Gaussian quadrature:

$$\int_{-\infty}^{\infty} \exp^{-x^2} f(x)\, dx \approx \sum_{i=1}^{N} \omega_i f(x_i) \tag{11}$$

$$\omega_i = \frac{2^{n-1} n! \sqrt{\pi}}{n^2 \left[H_{n-1}(x_i)\right]^2} \tag{12}$$

where n is the number of sample points used, $H_n(x)$ is the physicists' Hermite polynomial, x_i are the roots of the Hermite polynomial, and ω_i are the associated Gauss-Hermite weights.

Substitution for normal distributions using Gauss-Hermite quadrature:

$$\mathrm{E}[h(y)] = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right) h(y)\, dy \tag{13}$$

h is some function of y, and random variable Y is normally distributed.

$$x = \frac{y-\mu}{\sqrt{2}\sigma} \Leftrightarrow y = \sqrt{2}\sigma x + \mu \tag{14}$$

$$\mathrm{E}[h(y)] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} \exp\left(-x^2\right) h\left(\sqrt{2}\sigma x + \mu\right) dx \tag{15}$$

$$\mathrm{E}[h(y)] \approx \frac{1}{\sqrt{\pi}} \sum_{i=1}^{N} \omega_i h\left(\sqrt{2}\sigma x_i + \mu\right) \tag{16}$$

2.4 Mutual Information Calculation

$$I(\boldsymbol{\mu}; \mathbf{z}) = H(\mathbf{z}) - H(\mathbf{z}|\boldsymbol{\mu}) = \frac{1}{2}\ln\left((2\pi e)^2 \cdot |\boldsymbol{\Sigma}_z|\right) - \frac{1}{2}\ln\left((2\pi e)^2 \cdot |\boldsymbol{\Sigma}_\nu|\right) \tag{17}$$

$$|\boldsymbol{\Sigma}_\nu| = \sigma_\nu^2 \sigma_\nu^2 - 0 = \sigma_\nu^4 \tag{18}$$

$$\boldsymbol{\Sigma}_z = \begin{bmatrix} \mathrm{E}\left[(z_r - \mathrm{E}[z_r])^2\right] & \mathrm{E}[(z_r - \mathrm{E}[z_r])(z_i - \mathrm{E}[z_i])] \\ \mathrm{E}[(z_i - \mathrm{E}[z_i])(z_r - \mathrm{E}[z_r])] & \mathrm{E}\left[(z_i - \mathrm{E}[z_i])^2\right] \end{bmatrix} \tag{19}$$

$$p(\mathbf{z}) = \int p(\mathbf{z}|\boldsymbol{\mu})\, p(\boldsymbol{\mu})\, d\boldsymbol{\mu} \tag{20}$$

$$p(\mathbf{z}|\boldsymbol{\mu}) = \mathcal{N}_{\mathbf{z}}\left(\mathcal{G}(\boldsymbol{\mu}), \boldsymbol{\Sigma}_\nu\right) = \mathcal{N}_{z_r}\left(\mathcal{G}_r(\boldsymbol{\mu}), \sigma_\nu\right) \mathcal{N}_{z_i}\left(\mathcal{G}_i(\boldsymbol{\mu}), \sigma_\nu\right) = p(z_r|\boldsymbol{\mu})\, p(z_i|\boldsymbol{\mu}) \tag{21}$$

$$p(\boldsymbol{\mu}) = \mathcal{N}_{\boldsymbol{\mu}}(\mathbf{m}_{\mu}, \boldsymbol{\Sigma}_{\mu}) = \prod_{i=1}^{N} \mathcal{N}_{\mu_i}(m_{\mu_i}, \sigma_{\mu_i}) = \prod_{i=1}^{N} p(\mu_i) \tag{22}$$

The signal measurement probability distribution $p(\mathbf{z})$ can be approximated using Gauss-Hermite quadrature. The calculation is performed separately for the real and imaginary components, $p(z_r)$ and $p(z_i)$, because z_r and z_i are assumed independent.

$$p(z_r) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(z_r|\mu_1,\mu_2)\, p(\mu_1)\, p(\mu_2)\, d\mu_1 d\mu_2 \tag{23}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{\mu_1}} \exp\left(-\frac{(\mu_1 - m_{\mu_1})^2}{2\sigma_{\mu_1}^2}\right) \frac{1}{\sqrt{2\pi}\sigma_{\mu_2}} \exp\left(-\frac{(\mu_2 - m_{\mu_2})^2}{2\sigma_{\mu_2}^2}\right) p(z_r|\mu_1,\mu_2)\, d\mu_1 d\mu_2 \tag{24}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} \exp\left(-x_1^2\right) \frac{1}{\sqrt{\pi}} \exp\left(-x_2^2\right) p\left(z_r \middle| \left(\sqrt{2}\sigma_{\mu} x_1 + \bar{\mu}, \sqrt{2}\sigma_{\mu} x_2 + \bar{\mu}\right)\right) dx_1 dx_2 \tag{25}$$

$$\approx \frac{1}{\pi}\sum_{p=1}^{P}\sum_{q=1}^{Q} \omega_p \omega_q p\left(z_r \middle| \left(\sqrt{2}\sigma_{\mu_1} x_p + m_{\mu_1}, \sqrt{2}\sigma_{\mu_2} x_q + m_{\mu_2}\right)\right) = \frac{1}{\pi}\sum_{p=1}^{P}\sum_{q=1}^{Q} \omega_p \omega_q p(z_r|\mu_p,\mu_q) \tag{26}$$

$$\begin{cases} \mu_p = \sqrt{2}\sigma_{\mu_1} x_p + m_{\mu_1} \\ \mu_q = \sqrt{2}\sigma_{\mu_2} x_q + m_{\mu_2} \end{cases} \tag{27}$$

An identical calculation for the imaginary component results in

$$p(z_i) = \frac{1}{\pi}\sum_{p=1}^{P}\sum_{q=1}^{Q} \omega_p \omega_q p(z_i|\mu_p,\mu_q) \tag{28}$$

$$\begin{cases} \mu_p = \sqrt{2}\sigma_{\mu_1} x_p + m_{\mu_1} \\ \mu_q = \sqrt{2}\sigma_{\mu_2} x_q + m_{\mu_2} \end{cases} \tag{29}$$

Calculate expectation values for real and imaginary components by definition and approximating $p(\mathbf{z})$ with Gauss-Hermite quadrature as shown above.

$$\mathrm{E}[z_r] = \int z_r p(z_r)\, dz_r \approx \int z_r \frac{1}{\pi}\sum_{p=1}^{P}\sum_{q=1}^{Q} \omega_p \omega_q p(z_r|\mu_p,\mu_q)\, dz_r = \frac{1}{\pi}\sum_{p=1}^{P}\sum_{q=1}^{Q} \omega_p \omega_q \int z_r p(z_r|\mu_p,\mu_q)\, dz_r \tag{30}$$

$$= \frac{1}{\pi}\sum_{p=1}^{P}\sum_{q=1}^{Q} \omega_p \omega_q \mathrm{E}[z_r|\mu_p,\mu_q] = \frac{1}{\pi}\sum_{p=1}^{P}\sum_{q=1}^{Q} \omega_p \omega_q \mathcal{G}_r(\mu_p,\mu_q) \tag{31}$$

Similarly for the imaginary component,

$$\mathrm{E}[z_i] \approx \frac{1}{\pi}\sum_{p=1}^{P}\sum_{q=1}^{Q} \omega_p \omega_q \mathrm{E}[z_i|\mu_p,\mu_q] = \frac{1}{\pi}\sum_{p=1}^{P}\sum_{q=1}^{Q} \omega_p \omega_q \mathcal{G}_i(\mu_p,\mu_q) \tag{32}$$

The variance can be calculated as follows:

$$\mathrm{E}\left[(z_r - \mathrm{E}[z_r])^2\right] = \mathrm{E}\left[z_r^2\right] - (\mathrm{E}[z_r])^2 = \int z_r^2 p(z_r)\, dz_r - (\mathrm{E}[z_r])^2 \tag{33}$$

$$\approx \int z_r^2 \frac{1}{\pi}\sum_{p=1}^{P}\sum_{q=1}^{Q} \omega_p \omega_q p(z_r|\mu_p,\mu_q)\, dz_r - (\mathrm{E}[z_r])^2 \tag{34}$$

$$= \frac{1}{\pi}\sum_{p=1}^{P}\sum_{q=1}^{Q} \omega_p \omega_q \int z_r^2 p(z_r|\mu_p,\mu_q)\, dz_r - (\mathrm{E}[z_r])^2 \tag{35}$$

$$= \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \int z_r^2 \frac{1}{\sqrt{2\pi}\sigma_\nu} \exp\left(-\frac{\left(z_r - \mathcal{G}_r\left(\mu_p, \mu_q\right)\right)^2}{2\sigma_\nu^2} \right) dz_r - \left(\mathrm{E}\left[z_r\right]\right)^2 \tag{36}$$

$$= \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \int \left(x^2 \sigma_\nu^2 + 2x\sigma_\nu \mathcal{G}_r\left(\mu_p, \mu_q\right) + \mathcal{G}_r^2\left(\mu_p, \mu_q\right) \right) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2} \right) dx - \left(\mathrm{E}\left[z_r\right]\right)^2 \tag{37}$$

$$= \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \left(\sigma_\nu^2 + \mathcal{G}_r^2\left(\mu_p, \mu_q\right) \right) - \left(\mathrm{E}\left[z_r\right]\right)^2 \tag{38}$$

$$= \frac{\sigma_\nu^2}{\pi} + \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \mathcal{G}_r^2\left(\mu_p, \mu_q\right) - \left(\mathrm{E}\left[z_r\right]\right)^2 \tag{39}$$

$$\left\{ \begin{array}{l} \mu_p = \sqrt{2}\sigma_{\mu_1} x_p + m_{\mu_1} \\ \mu_q = \sqrt{2}\sigma_{\mu_2} x_q + m_{\mu_2} \end{array} \right. \tag{40}$$

Again, an identical calculation for the imaginary component yields

$$\mathrm{E}\left[\left(z_i - \mathrm{E}\left[z_i\right]\right)^2 \right] = \frac{\sigma_\nu^2}{\pi} + \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \mathcal{G}_i^2\left(\mu_p, \mu_q\right) - \left(\mathrm{E}\left[z_i\right]\right)^2 \tag{41}$$

$$\left\{ \begin{array}{l} \mu_p = \sqrt{2}\sigma_{\mu_1} x_p + m_{\mu_1} \\ \mu_q = \sqrt{2}\sigma_{\mu_2} x_q + m_{\mu_2} \end{array} \right. \tag{42}$$

The off-diagonal elements of Σ_z are equal.

$$\mathrm{E}\left[\left(z_r - \mathrm{E}\left[z_r\right]\right)\left(z_i - \mathrm{E}\left[z_i\right]\right) \right] = \mathrm{E}\left[\left(z_i - \mathrm{E}\left[z_i\right]\right)\left(z_r - \mathrm{E}\left[z_r\right]\right) \right] \tag{43}$$

$$\mathrm{E}\left[\left(z_r - \mathrm{E}\left[z_r\right]\right)\left(z_i - \mathrm{E}\left[z_i\right]\right) \right] = \int \left(z_r - \mathrm{E}\left[z_r\right]\right)\left(z_i - \mathrm{E}\left[z_i\right]\right) p\left(\mathbf{z}\right) d\mathbf{z} \tag{44}$$

$$= \int \left(z_r - \mathrm{E}\left[z_r\right]\right)\left(z_i - \mathrm{E}\left[z_i\right]\right) \int p\left(\mathbf{z} | \boldsymbol{\mu}\right) p\left(\boldsymbol{\mu}\right) d\mathbf{z} \tag{45}$$

$$\approx \int \left(z_r - \mathrm{E}\left[z_r\right]\right)\left(z_i - \mathrm{E}\left[z_i\right]\right) \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q p\left(\mathbf{z} | \mu_p, \mu_q\right) d\mathbf{z} \tag{46}$$

$$= \int \left(z_r - \mathrm{E}\left[z_r\right]\right)\left(z_i - \mathrm{E}\left[z_i\right]\right) \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q p\left(z_r | \mu_p, \mu_q\right) p\left(z_i | \mu_p, \mu_q\right) d\mathbf{z} \tag{47}$$

$$= \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \int \left(z_r - \mathrm{E}\left[z_r\right]\right) p\left(z_r | \mu_p, \mu_q\right) dz_r \int \left(z_i - \mathrm{E}\left[z_i\right]\right) p\left(z_i | \mu_p, \mu_q\right) dz_i \tag{48}$$

$$= \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \int \left(z_r - \mathrm{E}\left[z_r\right]\right) \frac{1}{\sqrt{2\pi}\sigma_\nu} \exp\left(-\frac{\left(z_r - \mathcal{G}_r\left(\mu_p, \mu_q\right)\right)^2}{2\sigma_\nu^2} \right) dz_r \tag{49}$$

$$\cdot \int \left(z_i - \mathrm{E}\left[z_i\right]\right) \frac{1}{\sqrt{2\pi}\sigma_\nu} \exp\left(-\frac{\left(z_i - \mathcal{G}_i\left(\mu_p, \mu_q\right)\right)^2}{2\sigma_\nu^2} \right) dz_i \tag{50}$$

$$= \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \int \left(\sigma_\nu x + \mathcal{G}_r\left(\mu_p, \mu_q\right) - \mathrm{E}\left[z_r\right] \right) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2} \right) dx \tag{51}$$

$$\cdot \int \left(\sigma_\nu y + \mathcal{G}_i\left(\mu_p, \mu_q\right) - \mathrm{E}\left[z_i\right] \right) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2} \right) dy \tag{52}$$

$$= \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \left(\left(\mathcal{G}_r\left(\mu_p, \mu_q\right) - \mathrm{E}\left[z_r\right] \right) \left(\mathcal{G}_i\left(\mu_p, \mu_q\right) - \mathrm{E}\left[z_i\right] \right) \right) \tag{53}$$

$$= \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \left(\mathcal{G}_r\left(\mu_p, \mu_q\right) \mathcal{G}_i\left(\mu_p, \mu_q\right) - \mathcal{G}_r\left(\mu_p, \mu_q\right) \mathrm{E}\left[z_i\right] - \mathcal{G}_i\left(\mu_p, \mu_q\right) \mathrm{E}\left[z_r\right] + \mathrm{E}\left[z_r\right] \mathrm{E}\left[z_i\right] \right) \tag{54}$$

$$= \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \mathcal{G}_r\left(\mu_p, \mu_q\right) \mathcal{G}_i\left(\mu_p, \mu_q\right) - \mathrm{E}\left[z_r\right] \mathrm{E}\left[z_i\right] \tag{55}$$

$$\Sigma_z = \begin{bmatrix} \frac{\sigma_\nu^2}{\pi} + \frac{1}{\pi} \sum\limits_{p=1}^{P} \sum\limits_{q=1}^{Q} \omega_p \omega_q \mathcal{G}_r^2\left(\mu_p, \mu_q\right) - \left(\mathrm{E}\left[z_r\right]\right)^2 & \frac{1}{\pi} \sum\limits_{p=1}^{P} \sum\limits_{q=1}^{Q} \omega_p \omega_q \mathcal{G}_r\left(\mu_p, \mu_q\right) \mathcal{G}_i\left(\mu_p, \mu_q\right) - \mathrm{E}\left[z_r\right] \mathrm{E}\left[z_i\right] \\ \frac{1}{\pi} \sum\limits_{p=1}^{P} \sum\limits_{q=1}^{Q} \omega_p \omega_q \mathcal{G}_r\left(\mu_p, \mu_q\right) \mathcal{G}_i\left(\mu_p, \mu_q\right) - \mathrm{E}\left[z_r\right] \mathrm{E}\left[z_i\right] & \frac{\sigma_\nu^2}{\pi} + \frac{1}{\pi} \sum\limits_{p=1}^{P} \sum\limits_{q=1}^{Q} \omega_p \omega_q \mathcal{G}_i^2\left(\mu_p, \mu_q\right) - \left(\mathrm{E}\left[z_i\right]\right)^2 \end{bmatrix} \tag{56}$$

$$\begin{aligned} \left|\Sigma_z\right| = & \left(\frac{\sigma_\nu^2}{\pi} + \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \mathcal{G}_r^2\left(\mu_p, \mu_q\right) - \left(\mathrm{E}\left[z_r\right]\right)^2 \right) \left(\frac{\sigma_\nu^2}{\pi} + \frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \mathcal{G}_i^2\left(\mu_p, \mu_q\right) - \left(\mathrm{E}\left[z_i\right]\right)^2 \right) \\ & - \left(\frac{1}{\pi} \sum_{p=1}^{P} \sum_{q=1}^{Q} \omega_p \omega_q \mathcal{G}_r\left(\mu_p, \mu_q\right) \mathcal{G}_i\left(\mu_p, \mu_q\right) - \mathrm{E}\left[z_r\right] \mathrm{E}\left[z_i\right] \right)^2 \end{aligned} \tag{57}$$

$$I\left(\mu; z\right) = H\left(z\right) - H\left(z|\mu\right) = \frac{1}{2} \ln\left(\left(2\pi e\right)^2 \cdot \left|\Sigma_z\right| \right) - \frac{1}{2} \ln\left(\left(2\pi e\right)^2 \cdot \left|\Sigma_\nu\right| \right) \tag{58}$$

3 Derivations

4 Directions