1 Objectives

2 Work

3 Derivations

3.1 Multivariate Normal Distribution Representation

This is the part that is confusing. With the change of variables, $p(\mathbf{z}|\mu)$ goes from a normal distribution in two-dimensional vector variable $\mathbf{z}$ to a normal distribution in one-dimensional scalar variable μ with two-dimensional mean and covariance matrix.

$$p(\mathbf{z}|\mu) = \mathcal{N}_{\mathbf{z}}(\boldsymbol{\mu}_\nu, \boldsymbol{\Sigma}_\nu) = \mathcal{N}_\mu\left(\boldsymbol{\mu}_{\nu,2}, \boldsymbol{\Sigma}_{\nu,2}\right) \tag{1}$$

3.2 Complex Normal Distribution Representation

$$\mathbf{Z} = \mathbf{X} + \mathrm{i}\mathbf{Y} \qquad\qquad \mathbf{X}, \mathbf{Y} \in \mathbb{R}^k \tag{2}$$

Bar represents complex conjugate.

$$\boldsymbol{\mu} = \mathrm{E}[\mathbf{Z}] \qquad\qquad \boldsymbol{\Gamma} = \mathrm{E}\left[(\mathbf{Z}-\boldsymbol{\mu})\left(\bar{\mathbf{Z}}-\bar{\boldsymbol{\mu}}\right)^T\right] \qquad\qquad \mathbf{C} = \mathrm{E}\left[(\mathbf{Z}-\boldsymbol{\mu})(\mathbf{Z}-\boldsymbol{\mu})\right] \tag{3}$$

$$\frac{1}{\pi^k\sqrt{\det(\boldsymbol{\Gamma})\det\left(\bar{\boldsymbol{\Gamma}}-\bar{\mathbf{C}}^T\boldsymbol{\Gamma}^{-1}\mathbf{C}\right)}}\exp\left\{-\frac{1}{2}\begin{pmatrix}\left(\bar{\mathbf{Z}}-\bar{\boldsymbol{\mu}}\right)^T & (\mathbf{Z}-\boldsymbol{\mu})^T\end{pmatrix}\begin{pmatrix}\boldsymbol{\Gamma} & \mathbf{C}\\ \bar{\mathbf{C}}^T & \bar{\boldsymbol{\Gamma}}\end{pmatrix}\begin{pmatrix}\mathbf{Z}-\boldsymbol{\mu}\\ \bar{\mathbf{Z}}-\bar{\boldsymbol{\mu}}\end{pmatrix}\right\} \tag{4}$$

$$\boldsymbol{\Gamma} = \mathbf{V_{xx}} + \mathbf{V_{yy}} + \mathrm{i}\left(\mathbf{V_{yx}} - \mathbf{V_{xy}}\right) \tag{5}$$

$$\boldsymbol{\Gamma} = \mathbf{V_{xx}} - \mathbf{V_{yy}} + \mathrm{i}\left(\mathbf{V_{yx}} + \mathbf{V_{xy}}\right) \tag{6}$$

$$\mathbf{R} = \bar{\mathbf{C}}^T\boldsymbol{\Gamma}^{-1} \tag{7}$$

$$\mathbf{P} = \bar{\boldsymbol{\Gamma}} - \mathbf{R}\mathbf{C} \tag{8}$$

$$\mathbf{z}|\mu \sim \mathcal{N}\left(\mathcal{G}(\mu,\mathbf{k}), \boldsymbol{\Sigma}_\nu\right) \qquad \boldsymbol{\mu}_\nu = \begin{bmatrix}\mathrm{Re}\left(\mathcal{G}(\mu,\mathbf{k})\right)\\ \mathrm{Im}\left(\mathcal{G}(\mu,\mathbf{k})\right)\end{bmatrix} = \begin{bmatrix}\boldsymbol{\mu}_{\nu,r}\\ \boldsymbol{\mu}_{\nu,i}\end{bmatrix} \qquad \boldsymbol{\Sigma}_\nu = \begin{bmatrix}\sigma^2_{\nu,r} & 0\\ 0 & \sigma^2_{\nu,r}\end{bmatrix} = \begin{bmatrix}\sigma^2_\nu & 0\\ 0 & \sigma^2_\nu\end{bmatrix} \tag{9}$$

$$\boldsymbol{\mu} \sim \mathcal{N}\left(\boldsymbol{\mu}_\mu, \boldsymbol{\Sigma}_\mu\right) \qquad\qquad \boldsymbol{\mu}_\mu = \begin{bmatrix}\mu_\mu\\ 0\end{bmatrix} \qquad\qquad \boldsymbol{\Sigma}_\mu = \begin{bmatrix}\sigma^2_\mu & 0\\ 0 & 0\end{bmatrix} \tag{10}$$

$$\begin{gathered}\mathbf{z}|\mu \sim \mathcal{CN}\left(\mu_\nu, \Gamma_\nu, C_\nu\right)\\ \mu_\nu = \mu_{\nu,r} + \mathrm{i}\mu_{\nu,i}\\ \Gamma_\nu = \sigma^2_\nu + \mathrm{i}\sigma^2_\nu\\ C_\nu = 0\\ R_\nu = 0\\ P_\nu = \sigma_n u^2 - \mathrm{i}\sigma^2_\nu\end{gathered} \tag{11}$$

$$\begin{gathered}\boldsymbol{\mu} \sim \mathcal{CN}\left(\mu_\mu, \Gamma_\mu, C_\mu\right)\\ \mu_\mu = \mu_\mu\\ \Gamma_\mu = \sigma^2_\mu\\ C_\mu = \sigma^2_\mu\\ R_\mu = 1\\ P_\mu = 0\end{gathered} \tag{12}$$

4 Directions