1 Goals

2 Work

$$I(\mu; z) = H(z) - H(z|\mu) = \frac{1}{2}\ln\left((2\pi e)^2 \cdot |\Sigma_z|\right) - \frac{1}{2}\ln\left((2\pi e)^2 \cdot |\Sigma_\nu|\right) \tag{1}$$

$$\Sigma_\nu = \begin{bmatrix} \sigma_{\nu,r}^2 & 0 \\ 0 & \sigma_{\nu,i}^2 \end{bmatrix} \tag{2}$$

The quantities $\sigma_{\nu,r}^2$ and $\sigma_{\nu,i}^2$ are measured.

$$|\Sigma_\nu| = \sigma_{\nu,r}^2 \sigma_{\nu,i}^2 \tag{3}$$

$$\Sigma_z = \mathrm{E}\left[(\vec{z} - \mathrm{E}[\vec{z}])(\vec{z} - \mathrm{E}[\vec{z}])^T\right] = \mathrm{E}\left[\begin{pmatrix} z_r - \bar{z}_r \\ z_i - \bar{z}_i \end{pmatrix}\begin{pmatrix} z_r - \bar{z}_r & z_i - \bar{z}_i \end{pmatrix}\right] \tag{4}$$

$$\Sigma_z = \begin{bmatrix} \mathrm{E}\left[(z_r - \bar{z}_r)^2\right] & \mathrm{E}\left[(z_r - \bar{z}_r)(z_i - \bar{z}_i)\right] \\ \mathrm{E}\left[(z_i - \bar{z}_i)(z_r - \bar{z}_r)\right] & \mathrm{E}\left[(z_r - \bar{z}_r)^2\right] \end{bmatrix} \tag{5}$$

$$|\Sigma_z| = \mathrm{E}\left[(z_r - \bar{z}_r)^2\right]\mathrm{E}\left[(z_i - \bar{z}_i)^2\right] - \left(\mathrm{E}\left[(z_r - \bar{z}_r)(z_i - \bar{z}_i)\right]\right)^2 \tag{6}$$

$$p(z) = \int p(z|\mu)\, p(\mu)\, d\mu = \int S\frac{1}{\sqrt{2\pi\sigma_{tot}^2}}\exp\left(-\frac{(\mu - \mu_{tot})^2}{2\sigma_{tot}^2}\right) d\mu \tag{7}$$

$$x = \frac{\mu - \mu_{tot}}{\sqrt{2}\sigma_{tot}} \tag{8}$$

$$\mu = \sqrt{2}\sigma_{tot}x + \mu_{tot} \tag{9}$$

$$d\mu = \sqrt{2}\sigma_{tot}dx \tag{10}$$

$$\sigma_{tot}^2 = \left(\frac{a_r^2}{\sigma_{\nu,r}^2} + \frac{a_i^2}{\sigma_{\nu,i}^2} + \frac{1}{\sigma_\mu^2}\right)^{-1} \tag{11}$$

$$\mu_{tot} = \left(\frac{\mu_\mu}{\sigma_\mu^2} + \frac{(z_r - b_r)/a_r}{\sigma_{\nu,r}^2/a_r} + \frac{(z_i - b_i)/a_i}{\sigma_{\nu,i}^2/a_i}\right)\sigma_{tot} \tag{12}$$

$$S = \left[2\pi\sqrt{\frac{(\sigma_{\nu,r}/a_r)^2(\sigma_{\nu,i}/a_i)^2\sigma_\mu^2}{\sigma_{tot}^2}}\right]^{-1}\exp\left[-\frac{1}{2}\left(\frac{(z_r - b_r)^2}{\sigma_{\nu,r}^2} + \frac{(z_i - b_i)^2}{\sigma_{\nu,i}^2} + \frac{\mu_\mu^2}{\sigma_\mu^2} - \frac{\mu_{tot}^2}{\sigma_{tot}^2}\right)\right] \tag{13}$$

$$p(z) = S\frac{\sqrt{2}\sigma_{tot}}{\sqrt{2\pi\sigma_{tot}^2}}\int e^{-x^2}dx = \frac{S}{\sqrt{\pi}}\left(\frac{1}{2}\sqrt{\pi}\mathrm{erf}(x) + C\right) \tag{14}$$

3 Directions

The fact that $p(z)$ exists for values of $z > 0$ causes problems. This results from $p(\mu) \sim \mathcal{N}$. Gauss-Hermite quadrature might not be an option for finite integration bounds. May need to redo the calculation using a uniform distribution for $p(\mu)$.