Assignment 2

Tracking a Panda in a Bamboo Grove: Exponential Smoothing vs. Running Mean

Performance – Wednesday, October 1, 2025

Due to submit a performance report – Tuesday, October 7, 2025, 23:59 p.m.

The Story

You are wildlife researchers preparing to release a panda family into a bamboo grove. To make sure the grove provides enough space and resources, you must first model how a panda moves inside it. The panda's true path is invisible to the naked eye - you only get noisy drone images, blurred by leaves, wind, and camera instability.

To succeed, you'll build and test mathematical models of the panda's walk. First, you observe during a calm walk with clear drone footage to learn the panda's step rhythm and noise levels. Then, you test your methods under harsh conditions of shaky cameras and foggy air to see which tracking technique survives the noise.

Random walk model

A panda's walk is not straight; it is stepwise and meandering. Sometimes it pauses to eat bamboo, sometimes it shifts sideways, sometimes it continues forward. This wandering style is well captured by the **random walk model**, where each new step depends on the last one plus a random variation. Thus, we describe the panda's movement as a random walk and use it to model its life in the bamboo grove.

Still, the panda's true trajectory remains hidden, however for scenario modeling and learning we need a **ground truth**. If we want to compare two tracking methods, we must know the correct answer in advance. By simulating a true path, we can test safely, just as ecologists would model different panda behaviors before introducing a real family into the grove.

Part I. Calm Walk: Learning the Panda's Rhythm

Here the panda moves calmly, and the drone camera is relatively stable. This stage is about learning the panda's step rhythm and tuning your "tracking lens". This lets you train the estimation methods:

1. Generate a true trajectory of a walking panda, X_i, in meter, using the random walk model

$$X_i = X_{i-1} + w_i \tag{1}$$

w_i is normally distributed random noise with a mean of zero and a variance of σ_w^2.

Group 1: $\sigma_w^2 = 8$;	Group 11: $\sigma_w^2 = 18$;
Group 2: $\sigma_w^2 = 9$;	Group 12: $\sigma_w^2 = 19$;
Group 3: $\sigma_w^2 = 10$;	Group 13: $\sigma_w^2 = 20$;
Group 4: $\sigma_w^2 = 11$;	Group 14: $\sigma_w^2 = 8$;
Group 5: $\sigma_w^2 = 12$;	Group 15: $\sigma_w^2 = 9$;

Group 6: $\sigma_w^2 = 13;$	Group 16: $\sigma_w^2 = 10;$
Group 7: $\sigma_w^2 = 14;$	Group 17: $\sigma_w^2 = 11;$
Group 8: $\sigma_w^2 = 15;$	Group 18: $\sigma_w^2 = 12;$
Group 9: $\sigma_w^2 = 16;$	Group 19: $\sigma_w^2 = 13;$
Group 10: $\sigma_w^2 = 17;$	Group 20: $\sigma_w^2 = 14;$

The trajectory size is defined as follows (consider two cases):

1. 3000 points.
2. 300 points.

To generate the true trajectory of the walking panda, use the initial condition $X_1 = 10\ m$.

2. Generate measurements z_i of the panda's trajectory X_i, that represent a panda walking in a bamboo grove.
In practice, we work with measurements obtained from various sensors and use models to analyze these measurements. However, for learning purposes, we will generate measurements based on a true trajectory, incorporating noise to simulate real-world conditions

$$z_i = X_i + \eta_i \tag{2}$$

η_i is normally distributed random noise with a mean of zero and a variance of σ_η^2.

Group 1: $\sigma_\eta^2 = 16;$	Group 11: $\sigma_\eta^2 = 12;$
	Group 12: $\sigma_\eta^2 = 10;$
Group 3: $\sigma_\eta^2 = 13;$	Group 13: $\sigma_\eta^2 = 15;$
Group 4: $\sigma_\eta^2 = 10;$	Group 14: $\sigma_\eta^2 = 16;$
Group 5: $\sigma_\eta^2 = 9;$	Group 15: $\sigma_\eta^2 = 12;$
Group 6: $\sigma_\eta^2 = 8;$	Group 16: $\sigma_\eta^2 = 13;$
Group 7: $\sigma_\eta^2 = 10;$	Group 17: $\sigma_\eta^2 = 10;$
Group 8: $\sigma_\eta^2 = 9;$	Group 18: $\sigma_\eta^2 = 9;$
Group 9: $\sigma_\eta^2 = 11;$	Group 19: $\sigma_\eta^2 = 8;$
Group 10: $\sigma_\eta^2 = 10;$	Group 20: $\sigma_\eta^2 = 10;$

3. Identify σ_w^2 and σ_η^2 using the identification method presented on slide 55 of *Topic_2_Quasi-optimal approximation under uncertainty.pdf*. Perform the identification for different trajectory sizes (3000 and 300 points). Compare the estimation results with the true values of σ_w^2 and σ_η^2, and evaluate the accuracy of the estimations.

4. Determine the optimal smoothing coefficient for exponential smoothing.

$$\alpha = \frac{-\chi + \sqrt{\chi^2 + 4\chi}}{2} \tag{3}$$

$$\chi = \frac{\sigma_w^2}{\sigma_\eta^2}$$

5. Perform exponential smoothing using the determined smoothing coefficient. Plot the results, including the measurements, true values of the panda's walk, and the exponentially smoothed data for comparison.

Part II – Shaky Cameras: Testing Under Harsh Conditions

Now the panda ventures deeper into the bamboo grove, where conditions worsen: the drone camera shakes in the wind, branches obscure the view, and images become extremely noisy. In harsh conditions, you see how your tracking methods hold up when reality gets messy

1. Generate a new true trajectory, X_i, of a walking panda, using the random walk model (1) with much higher noise. The trajectory size is 300 points, with an initial condition of $X_1 = 10\ \mathrm{m}$. The variance of the noise is w_i is $\sigma_w^2 = 28^2$.

2. Generate measurements z_i of the panda's trajectory X_i, that represents a panda walking in a bamboo grove, using Equation (2). The variance of noise measurement noise η_i, $\sigma_\eta^2 = 97^2$.

3. Determine the optimal smoothing coefficient α using Equation (3). There is no need to identify it again; simply use the equation for α from Part I with the given values of σ_w^2 and σ_η^2.

4. Find the matching running mean window size M so that measurement-error contributions are comparable.

   The component of the full error related to measurement errors is determined as (refer to slide 37 of *Topic_2_Quasi-optimal approximation under uncertainty.pdf*)

   Running mean (RM):

$$\sigma_{RM}^2 = \frac{\sigma_\eta^2}{M} \tag{4}$$

   Exponential smoothing (ES):

$$\sigma_{ES}^2 = \sigma_\eta^2 \frac{\alpha}{2-\alpha} \tag{5}$$

Determine the window size M (use rounded values) that ensures the equality of σ^2_{RM} and σ^2_{ES} using the determined smoothing constant α

$$\sigma_{ES}^2 = \sigma_{RM}^2$$

5. Apply the running mean using the determined window size M and the exponential mean using the determined smoothing constant α on the measurements z_i (refer to page 30 of *Topic_2_Quasi-optimal approximation under uncertainty.pdf*). Plot the panda's true trajectory X_i, measurements z_i, and both the running and exponential means.

6. Make a visual comparison of the results. Draw conclusions about which methods yield greater methodical error under conditions of equal errors caused by measurement errors for this particular generated trajectory.

7. **Make conclusions about the Assignment**
   Conclusions should be presented in the form of a learning log. Learning log should capture the key takeaways from the assignment. The learning log is not simply a diary of tasks completed but a reflective journal showcasing your personal learning, experiments, and critical insights gained throughout the process.

8. **Prepare a performance report and submit to Canvas:**
   Each assignment will be graded on a 0-100% scale and should be submitted as a PDF report. Any accompanying code in Python or Matlab must also be converted to PDF. Your report should include: title and team info, commented code, plots with labeled axes, titles, and legends. Clear conclusions about method performance.

   The breakdown is as follows:

   - ✓ 50%: Successful completion of all steps, thoroughly documented in the report.
   - ✓ 20%: Clear presentation—figures must include captions, labeled axes, legends for curves, and concise conclusions or discussions.
   - ✓ 30%: Learning log.
   - ✓ **Quizzes.** Regular written quizzes will be held throughout the course, covering key concepts and specific assignments discussed in class. Laptops and phones will not be allowed during quizzes, so please review the relevant materials and complete assigned tasks in advance. Your performance in these quizzes will contribute to your assignment grade.