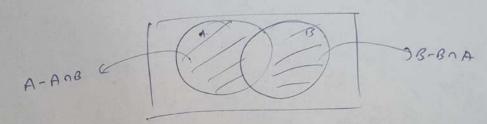HARSHIT SHAKYA
(210427)

① - Given $P(A) = 0.3$, $P(B) = 0.4$, $P(A \cap B) = 0.2$

(a) P(exactly one of A or B) = $P(A \cap B^c) + P(A^c \cap B)$

   $= P(A) - P(A \cap B) + P(B) - P(A \cap B)$

   $= 0.3 + 0.4 - 2(0.2)$

   $= \underline{0.3}$

$A - A \cap B$ ← (Venn diagram with A and B circles) → $B - B \cap A$

(b) P(at least one of A or B) = $P(A \cup B)$

   $= P(A) + P(B) - P(A \cap B)$

   $= 0.3 + 0.4 - 0.2$

   $= \underline{0.5}$

(c) P(none of A or B) = $1 - P(A \cup B)$

   $= 1 - 0.5 = \underline{0.5}$

③ Let, A → 3 drawn balls are red

before drawing Let, B_1 → All ball in the bag were red

B_2 → 5 balls in the bag were red and one were not red

B_3 → 4 balls in the bag were red and 2 were not red

B_4 → 3 balls in the bag were red and 3 were not red

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Now, B_1, B_2, B_3, B_4 are equally likely

⟹ $P(B_1) = P(B_2) = P(B_3) = P(B_4) = \dfrac{1}{7} = P(B_5)$

$= P(B_6) = P(B_7)$

$P(B_1 | A) = \dfrac{P(B_1)\, P(A|B_1)}{P(B_1)\,P(A|B_1) + P(B_2)\,P(A|B_2) + P(B_3)\,P(A|B_3) + P(B_4)\,P(A|B_4)}$

$= \dfrac{\frac{1}{7} \times 1}{\frac{1}{7} \times 1 + \frac{1}{7} \times \frac{1}{2} + \frac{1}{7} \times \frac{1}{5} + \frac{1}{7} \times \frac{1}{20}}$

$\left(\begin{array}{l} P(A|B_5) = 0 \\ P(A|B_6) = 0 \\ P(A|B_7) = 0 \end{array} \right)$

$= \dfrac{1}{1 + \frac{1}{2} + \frac{1}{5} + \frac{1}{20}}$

$= \dfrac{20}{35} = \boxed{\dfrac{4}{7}}$

④

$$P_x(x) = \begin{cases} 0.1 & x = 0.2 \\ 0.2 & x = 0.4 \\ 0.2 & x = 0.5 \\ 0.3 & x = 0.8 \\ 0.2 & x = 1 \\ 0 & o/w \end{cases}$$

(i) $P(X < 0.5) = \sum_{x < 0.5} P(X = x)$

$= P(X = 0.2) + p(0.4)$

$= 0.1 + 0.2 = \boxed{0.3}$

(ii) $P(0.25 < X < 0.75)$

$= P(X = 0.4) + P(X = 0.5)$

$= 0.2 + 0.2 = \boxed{0.4}$

(iii) $P(X = 0.2 \mid X < 0.6) = \dfrac{P(X = 0.2 \cap X < 0.6)}{P(X < 0.6)}$

$= \dfrac{P(X = 0.2)}{P(X = 0.2) + P(X = 0.4) + P(X = 0.5)}$

$= \dfrac{0.1}{0.1 + 0.2 + 0.2} = \dfrac{0.1}{0.5} = \boxed{1/5}$

⑤ $F(x) = \begin{cases} 0 & x < 0 \\ 2/3 & 0 \le x < 1 \\ \dfrac{7 - 6c}{6} & 1 \le x < 2 \\ \dfrac{4c^2 - 9c + 6}{4} & 2 \le x < 3 \\ 1 & x > 3 \end{cases}$

(i) since $F(\cdot)$ is right continuous and increasing

$F(3) = F(3^+)$

$$\frac{4c^2 - 9c + 6}{4} = 1$$

$4c^2 - 9c + 6 = 4$

$4c^2 - 9c + 2 = 0$

$4c^2 - 8c - c + 2 = 0$

$4c(c-2) - 1(c-2) = 0$

$(4c-1)(c-2) = 0$

$c = 1/4$ or $c = 2$

$\frac{7 - 6c}{6} = \frac{7 - 6(2)}{6} = -5/6 \not< 2/3$

$\therefore$ $\boxed{c = 1/4}$

(ii) ~~$P(1 < x < 2)$~~ = $\therefore$ $F(x) = \begin{cases} 0 & x < 0 \\ 2/3 & 0 \le x < 1 \\ 11/12 & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$

$P(1 \le x < 2) = F(2^-) - F(1^-)$

$\qquad = 11/12 - 11/12 = \boxed{0}$

$P(2 \le x < 3) = F(3^-) - F(2^-)$

$\qquad = 1 - 11/12 = \boxed{1/12}$

$P(0 < x \le 1) = F(1) - F(0^+)$

$\qquad = 11/12 - 2/3 = \boxed{1/4}$

$P(1 \le x \le 2) = F(2) - F(1^-)$

$\qquad = 1 - 2/3 = \boxed{1/3}$

$$P(X \geq 3) = F(3) - F(3^-) = \boxed{0}$$

(6) $X \sim U(0,1)$

$$\therefore \quad f_X(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & o/w \end{cases}$$

(iv) $E[X^2 + Y^2] = 1$ & $Var(Y) = 5/9$

(i) $E(X) = \int_0^1 x\, dx = \frac{x^2}{2}\Big|_0^1 = \boxed{1/2}$

(ii) $Var(X) = E(X^2) - (E(X))^2$

$$E(X^2) = \int_0^1 x^2\, dx = \frac{x^3}{3}\Big|_0^1 = 1/3$$

$$Var(X) = 1/3 - 1/4 = \boxed{1/12}$$

(iii) $E(X^2 + Y^2) = 1$

$$E(Y^2) = 1 - E(X^2) = 1 - 1/3 = 2/3$$

$$Var(Y) = 5/9$$

$$E(Y^2) - (E(Y))^2 = 5/9$$

$$(E(Y))^2 = -5/9 + 2/3 = 1/9$$

$$E(Y) = \boxed{1/3}$$

(iv) $E(X+Y) = E(X) + E(Y)$

$$= 1/2 + 1/3 = \boxed{5/6}$$

Summary :-

① (a) 0.3 (b) 0.5 (c) 0.5

② was not able to solve

③ $4/7$

④ (i) $P(X < 0.5) = 0.3$

(ii) $P(0.25 < X < 0.75) = 0.4$

(iii) $P(X = 0.2 \mid X < 0.6) = 1/5$

⑤ (i) $c = 1/4$

(ii) (a) 0 (b) $7/12$ (c) $1/4$ (d) $1/3$ (e) 0

⑥ (i) $E(X) = 1/2$

(ii) $Var(X) = 1/12$

(iii) $E(Y) = 1/3$

(iv) $E(X+Y) = 5/6$